Orbital orders glasses

Zohar Nussinov

Batista, LANL arXiv:cond-mat/04

Biskup, L. Chayes, UCLA; J. van den Br

Xiv:cond-mat/0309691(Comm M

0309692 (EPL)

Fradkin, UIUC arXiv:cond-mat/041(

tiz, E. Cobanera, Indiana arXiv:cond-r

Conclusions (new resu

Orbital systems can order by **thermal** \
disorder" **fluctuations** even in their clas
(no (1/S) zero point quantum fluctuatior
necessary).

Similar to charge and spin driven quant
behavior, it is theoretically possible to h
order driven quantum critical behavi
(Prediction.)

**Orbital glasses appear in exactly sol
models**

Orbital systems can exhibit topological
dimensional reductions due to their u

at are orbital orders?

dels for orbital order

"Order by disorder" in orbital systems

ital order driven quantum criticality ar

xact solutions as a theoretical proof of co

metries and topological order

imensional gauge like symmetries and d

tions: experimentally testable selection r

Vhat are orbital orders? (old)

lodels for orbital order (old)

"Order by disorder" in orbital system

ctuations) (new)

Orbital order driven quantum criticality

Exact solutions as a theoretical proof o

ymmetries and topological order

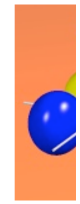
dimensional gauge like symmetries an

els in $3d$ shell split by crystal field.

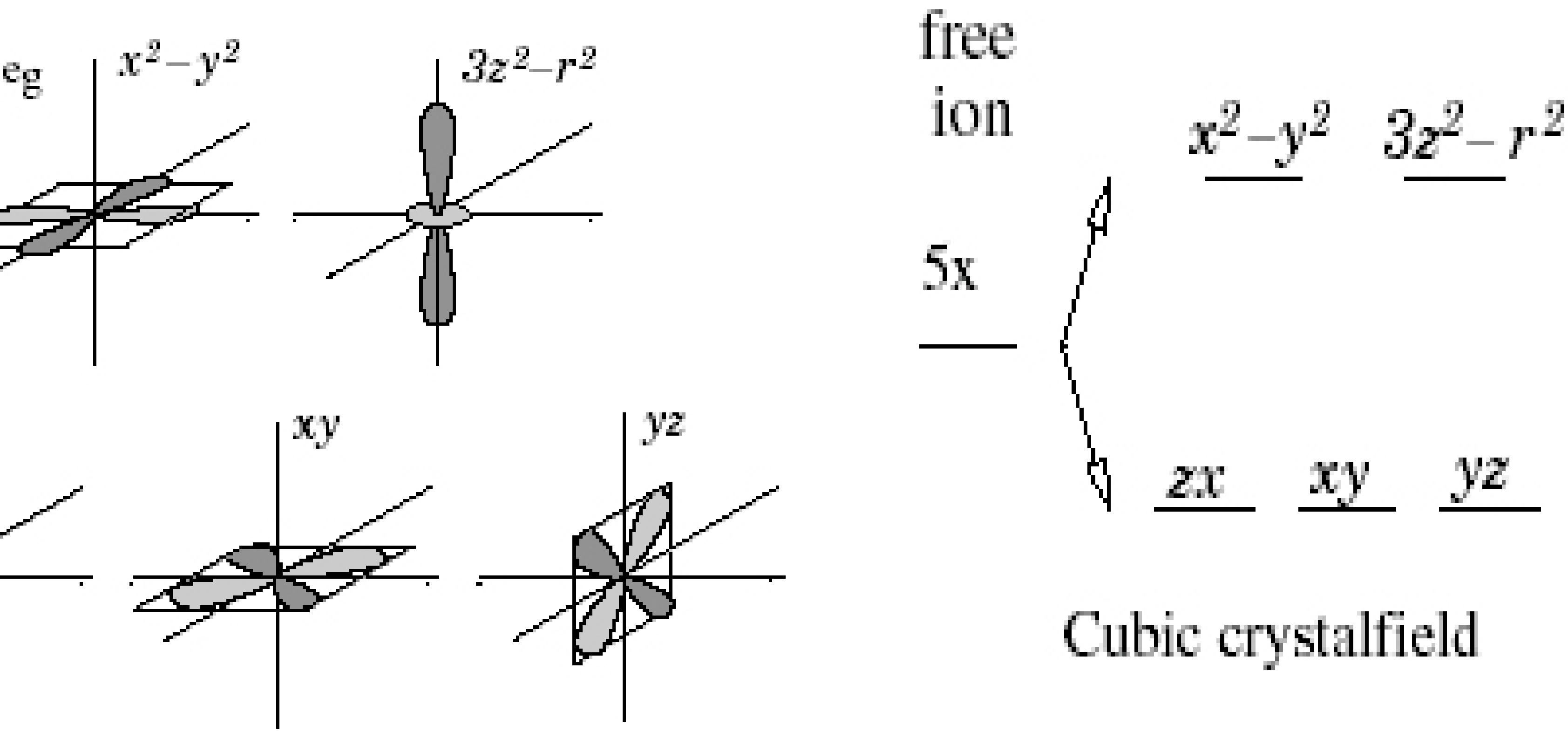
e_g

gle itinerant electron @ each site
multiple *orbital* degrees of freedom.

t_{2g}

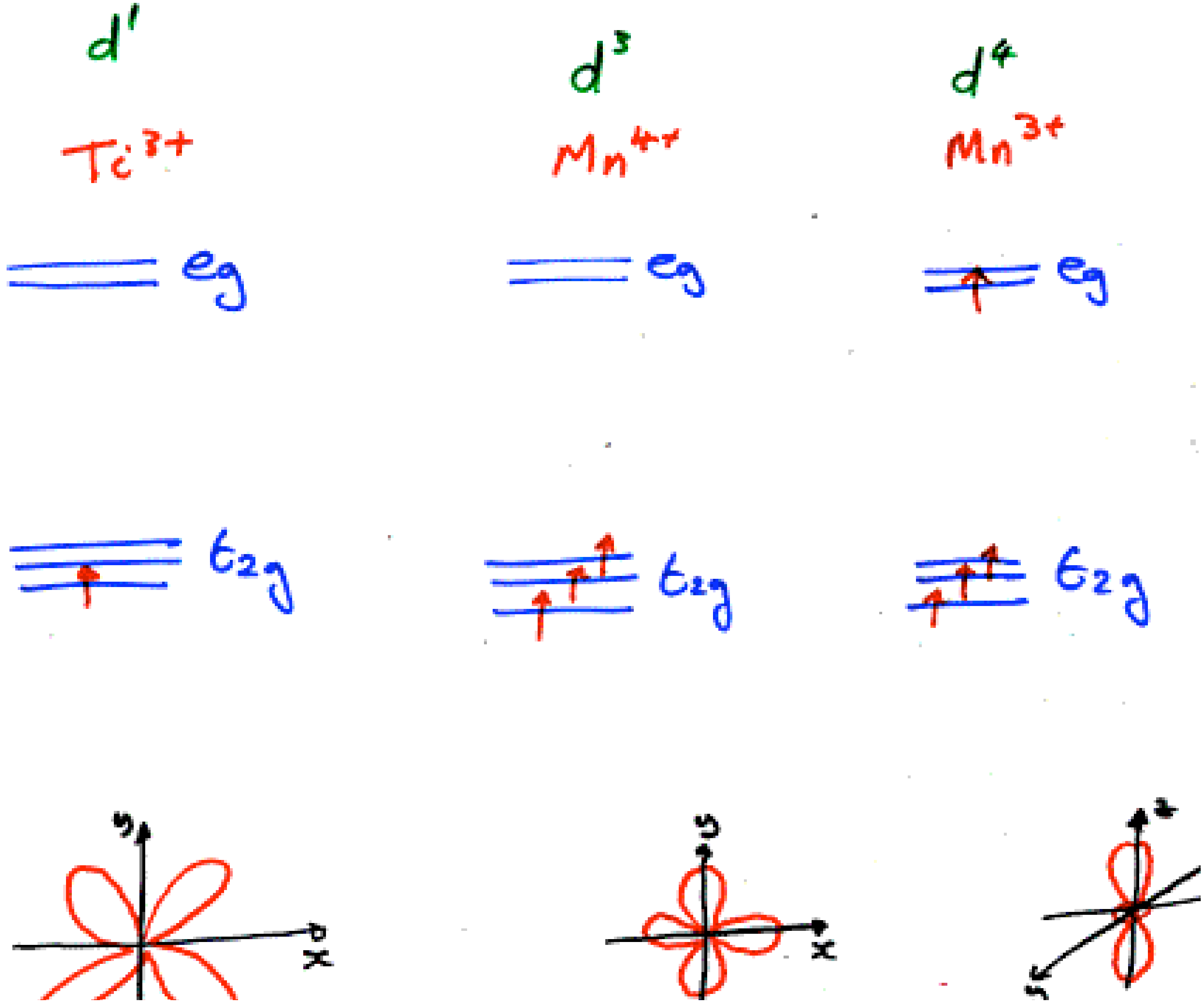


The 3d orbitals

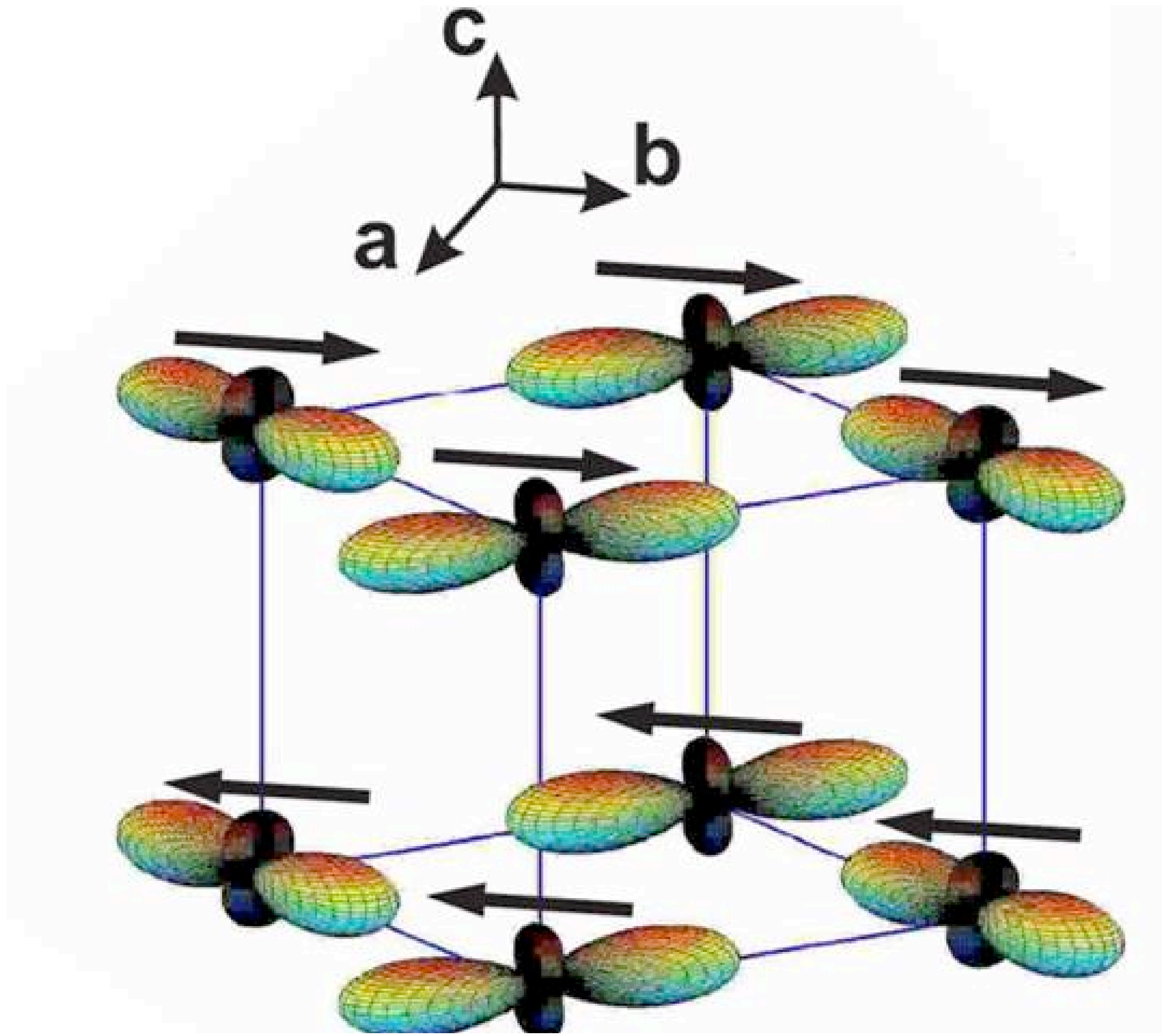


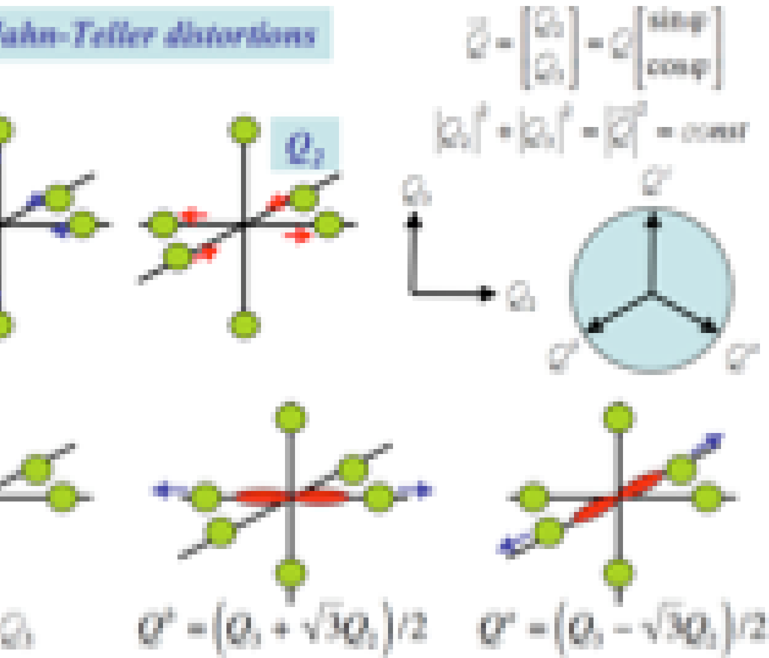
The five 3d orbital states share the

Function. Their angular depen



d1
Ti3+
eg
t2g
d3
Mn4+
eg
t2g
d4
Mn3+
eg
t2g
y
x
y
x
z
y


c
b
a



$$\left|x^2 - y^2\right\rangle = \left|S = -m = 1/2\right\rangle \equiv$$

$$\left|xy\right\rangle = \left|S = 1, m = 0\right\rangle; \left|yz\right\rangle = 2$$
$$\left|zx\right\rangle = -i2^{-1/2}\left(\left|1\,1\right\rangle - \left|1\,-1\right\rangle\right)$$

The Hilbert space of the e_g orbitals is spanned by two
es. The associated Jahn-Teller distortions can be expres
rs on a two dimensional unit disk (linear combinations o
endent distortions $Q_{2,3}$). An effective pseudo-spin S=1/
esentation. There is an angle of *120 degrees* between th

Vhat are orbital orders? (old)

lodels for orbital order (old)

"Order by disorder" in orbital system

ctuations) (new)

rbital order driven quantum criticality

Exact solutions as a theoretical proof o

ymmetries and topological order

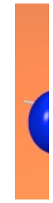
dimensional gauge like symmetries an

Unlike spins, orbitals live in real sp
The orbital interactions
are <u>not</u> isotropic. Reduced symme
and frustration.

vels in $3d$ shell split by crystal field.

e_g

t_{2g}



gle itinerant electron @ each site
multiple *orbital* degrees of freedom.

exchange approximation (and neglect of strain–field induced int

$$\sum_{<r,r'>} H^{r,r'}_{\mathrm{orb}}(\mathbf{s}_r \cdot \mathbf{s}_{r'} + \tfrac{1}{4})$$

$$H^{r,r'}_{\mathrm{orb}} = J[4\hat{\pi}^{\alpha}_{r}\hat{\pi}^{\alpha}_{r'} - 2\hat{\pi}^{\alpha}_{r}$$

gel–Khomskii Hamiltonian]

$$\hat{\pi}^{x}_{r} = \tfrac{1}{4}(-\sigma^{z}_{r} + \sqrt{3}\sigma^{x}_{r})$$

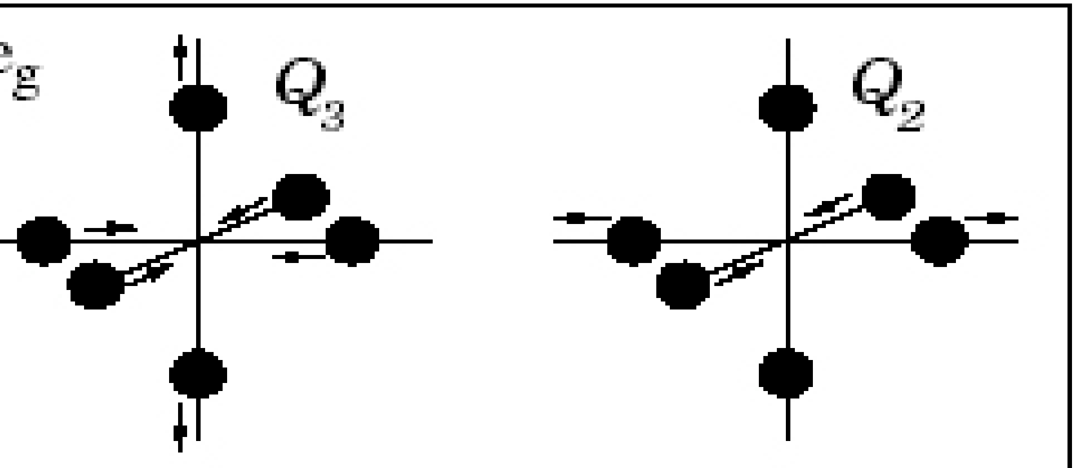
$$\hat{\pi}^{z}_{r} =$$

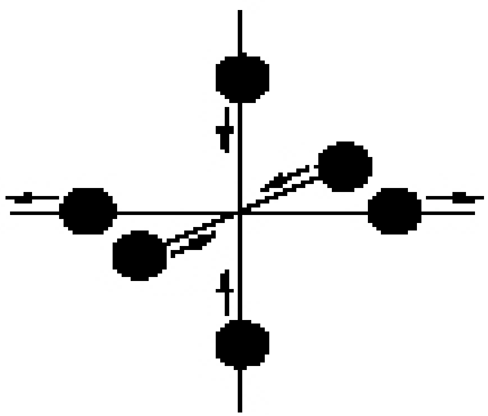
120°–model (e_g–compounds)

V_2O_3, $LiVO_2$, $LaVO_3$,$LaMnO_3$, …

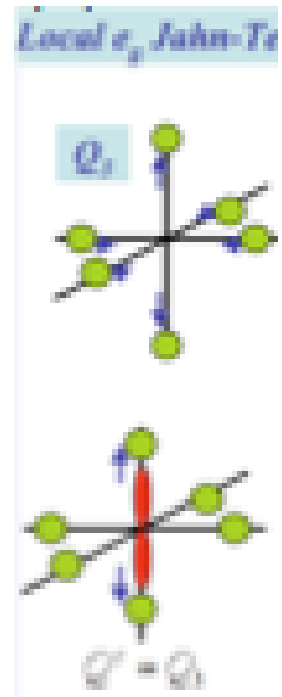
Jahn-Teller distortior

listortions preferred by different c





$Q_3\ cos\phi + Q_2\ sin\phi$



The orbital only interact

he orbital component of the orbital depend
exchange as well as the direct Jahn-Teller o
interactions have a similar form:

$$H_{orb} = J \sum_{\alpha} \sum_{r} \pi_r^{\alpha} \pi_{r+e_{\alpha}}^{\alpha}$$

al only approximation: Neglect spin degre

120° Hamiltonian:

$$= J\sum_r (S_r^{[a]}S_{r+e_x}^{[a]} + S_r^{[b]}S_{r+e_y}^{[b]} + S_r^{[c]}S_{r+e_z}^{[c]})$$

$\bar{S}$

simil

unit ve

Orbital compass Hamiltonian:

$$= J\sum (S_r^{[x]}S_{r+e_x}^{[x]} + S_r^{[y]}S_{r+e_y}^{[y]} + S_r^{[y]}S_{r+e_z}^{[z]})$$

$\vec{S}_r =$

– us

$\vec{S}_r \in S_1$, write $\vec{S}_r = \left(S_r^{[x]}, S_r^{[y]}\right)$. $S_r^{[a]} = \vec{S}_r \cdot$

$$H = J \sum_{r \in \Lambda_L} (S_r^{[a]} S_{r+e_x}^{[a]} + S_r^{[b]} S_{r+e_y}^{[b]} + S_r^{[c]} S_{r+e_z}^{[c]})$$

$$\frac{J}{2} \sum_{r \in \Lambda_L} \Big((S_r^{[a]} - S_{r+e_x}^{[a]})^2 + (S_r^{[b]} - S_{r+e_y}^{[b]})^2 + (S_r^{[c]} - S_{r+}^{[c]}$$

active couplings (ferromagnetic).

ples in x–direction with projection along a–component.
ples in y–direction with b–component.
ples in z- direction with c-component

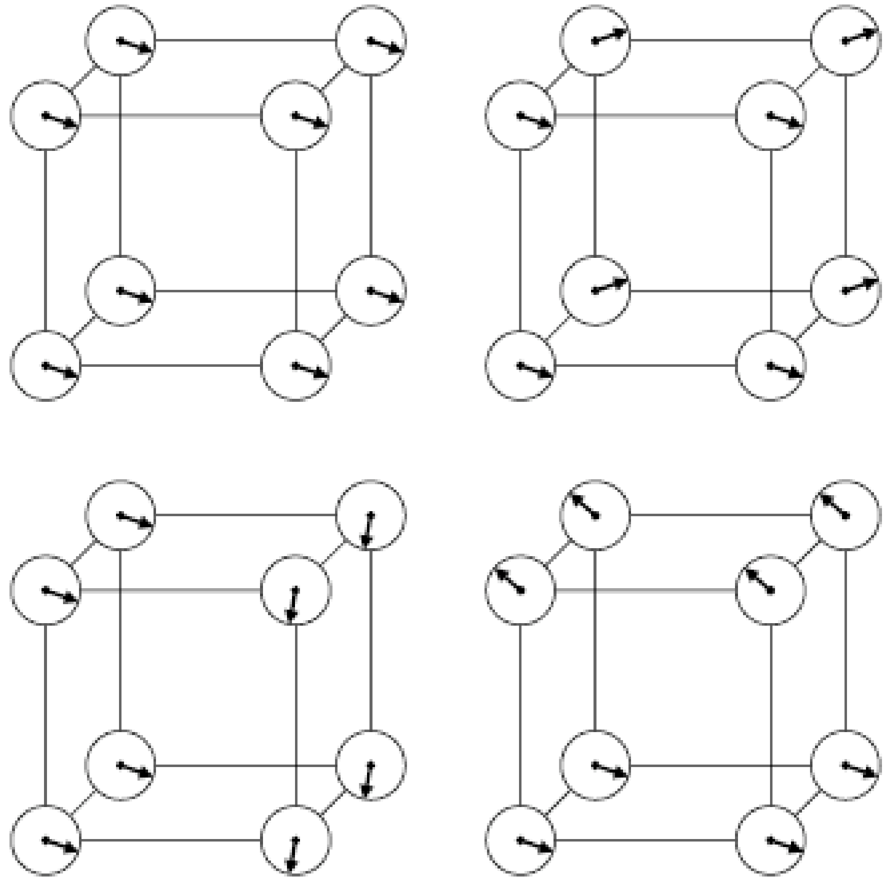
ear: Any constant spin–field is a classical grou

$$G(k,\omega=0) \quad \alpha \quad \frac{\Delta_a+\Delta_b+\Delta_c}{\Delta_a\Delta_b+\Delta_a\Delta_c+\Delta_b\Delta_c}$$

Fix k_z $$G(k,\omega=0) \quad \alpha \quad \frac{1}{\Delta_a+\Delta_b}$$

Very IR divergent.

ov, M. Biskup,
and J. v. d. Brink
EPL)



Ising-type discrete emerg

L x L
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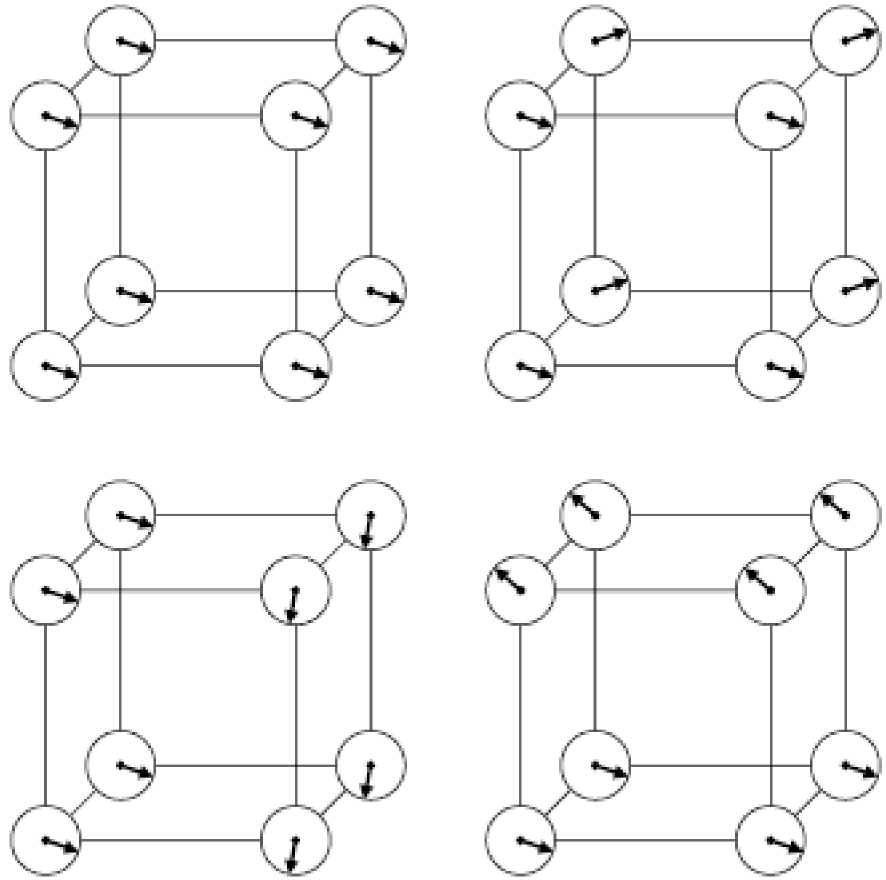

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litional discrete degeneracy fa

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y, *Ordering Due to Disorder in a Frustrated Vector*
agnet, Phys. Rev. Lett. **62** (1989) 2056–2059.

Really clarified
firm foundatic

itely many papers (mostly quantum) in which specific ca
bital order work focused on zero point 1/S fluctuations.

lt: orbital order is robust and persists for infinite S. Zero
ns are not needed to account for the observed orbital ord

Veighting of various ground states

take into account more than just energetics:

Fluctuations of spins will contribute to overall statis and about the uniform state: θ

$$\vartheta_r \equiv \theta_r - \theta^* \qquad H_{SW} = \frac{J}{2}\sum_{r,\alpha} q_\gamma(\theta^*)(\vartheta$$

$$q_c(\theta^*) = \sin^2\theta^*, \quad q_{a,b}(\theta^*) = \sin^2(\theta^* \pm \frac{2}{}$$

$$\log Z(\theta^*) = -\frac{1}{2}\sum_{k\neq 0}\log\left(\sum_\alpha \beta J q_\alpha(\theta^*)\Delta(k\right.$$

$$\Delta(k_\alpha) = 2 - 2\cos k_\alpha$$

e free energy has strict minima

$$F(\theta^*) = \int_{k\in B.Z.} \frac{d^3k}{(2\pi)^3} \log\det(\beta J \Pi_k)$$

$$\Pi_k = \begin{pmatrix} q_1\Delta_1 + q_+\Delta_+ & q_-\Delta_- \\ q_-\Delta_- & q_1\Delta_1^* + q_+\Delta_+^* \end{pmatrix}$$

$$q_\alpha \equiv q_\alpha(\theta^*) \quad \Delta_\alpha \equiv \Delta_\alpha(k)$$

$$\Delta_\alpha^* = \Delta_\alpha(k + \pi e_\alpha)$$

$$q_\pm = \frac{1}{2}(q_2 \pm q_3)$$

$$\Delta_\pm = \Delta_2 \pm \Delta_3$$

flection Positivity (chessboard estimates): $P_\beta(A)$

Using Reflection Positivity along
a Peierls argument, we readily establi
at sufficiently low temperatures,
one of the six low free energy
states is spontaneously chosen.

Interesting feature: Limiting behavior (

For the t_{2g} orbital compass type m
orm order cannot appear. By symmetry
it is established that $\langle S_r \rangle = 0$. Instea
"orbital nematic order"
(e.g., $\left\langle (S_r^x S_{r+e_x}^x - S_r^y S_{r+e_y}^y) \right\rangle \neq 0$ in the 2D orbit
can be proven to onset at suffici
low yet finite temperatures.

What are orbital orders? (old)

lodels for orbital order (old)

"Order by disorder" in orbital system

ctuations) (new)

rbital order driven quantum criticality

Exact solutions as a theoretical proof o

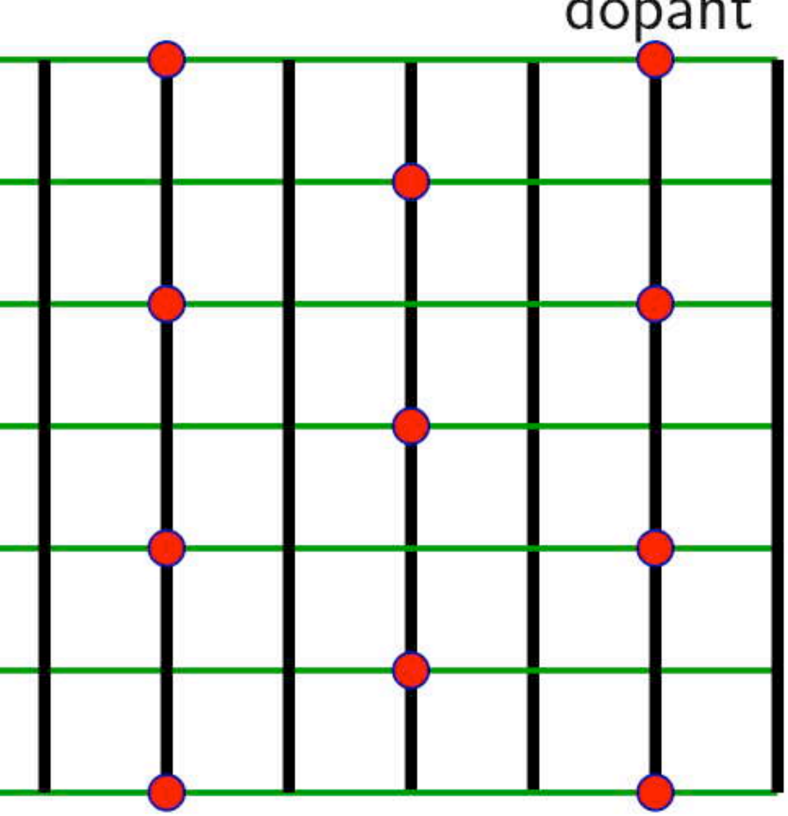
ymmetries and topological order

dimensional gauge like symmetries an

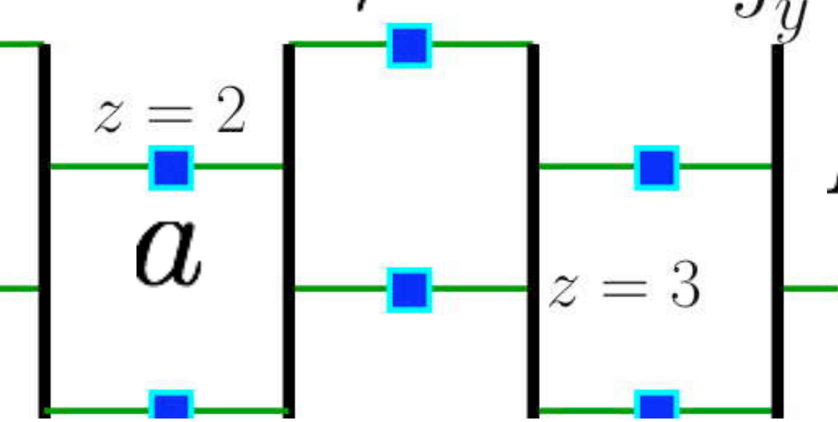
Fact: Quantum criticality can be associated wit
nd spin driven orders. The transition metal oxic
interplay of charge/superconducting, spin, anc

on: Can there be an entirely new family of "orb
quantum critical points"? Glasses?

ver: This is not forbidden and may occur theore
some simple yet exactly solvable models, the
ler driven quantum critical points (driven in the
by doping/dilution and/or uni-axial press



$\delta = 1/4$



$$H_{\mathsf{OCM}} = -\sum_j$$

After doping: New

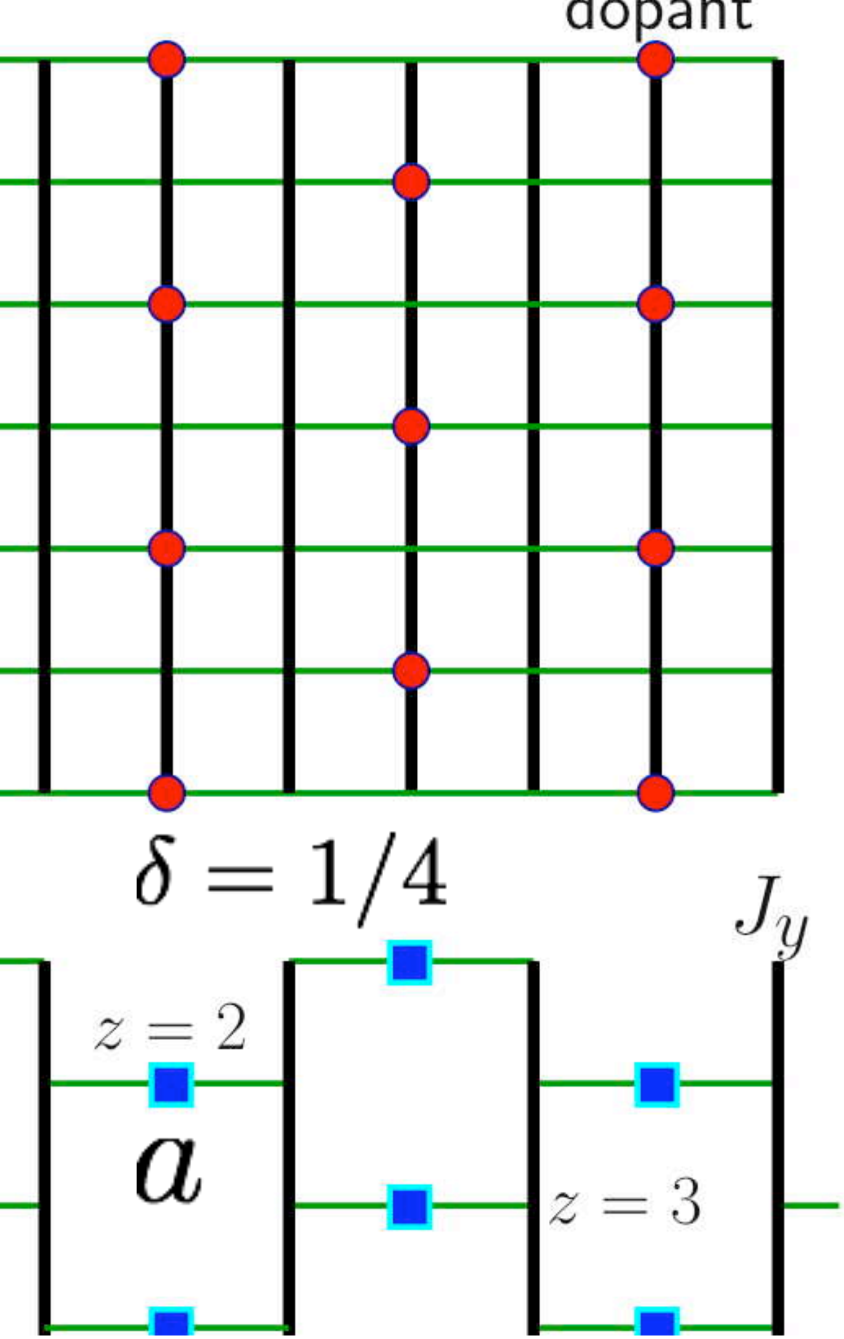
$$\hat{O}_a = \sigma_a^x, \ \ [H_{\mathsf{D}}$$

$$\bar{H}_{\mathsf{DOCM}} \equiv \hat{P}_\ell$$

$$\hat{P}_\ell = \prod_{a=1}^{N/3} \left(\frac{\mathbb{1} + \eta_a \sigma_a^x}{2} \right)$$

$$\bar{H}_{\mathsf{DOCM}} = -\sum_b \Big(J_x \eta_a$$

$$Z = \mathrm{tr} \ e^{-\beta H_{\mathsf{DOC}}}$$



$$H_{\mathrm{OCM}} = -\sum_j$$

After doping: New

For a system w
exchange cou
replicating t
steps mutatis
leads to the
Transverse F

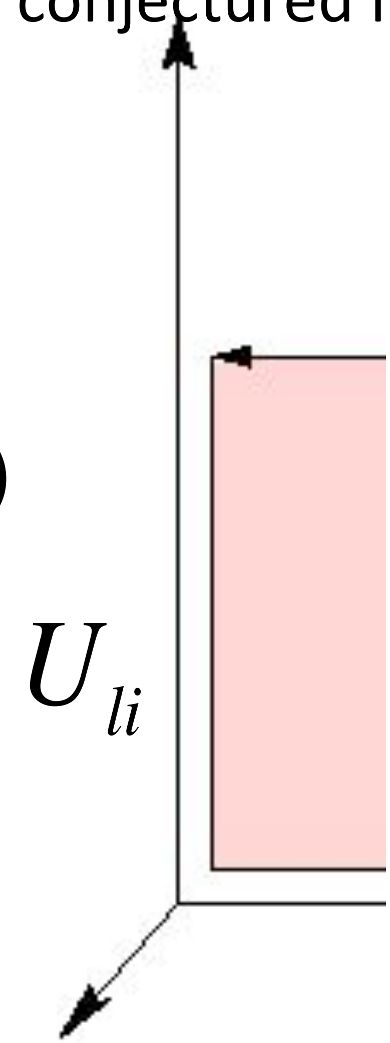
he exchange constants and magnetic
can vary with random dilution/pres

A uniaxial pressure (as we will discuss)
a magnetic field along the z direction. F
uniaxial pressures- random longitudina

By duality, the transverse field and the
exchange can be interchanged with one
a transverse field Ising chain. Random
in the vertical direction amount to rand
longitudinal exchange. Random coupli

ermezzo: using the same idea, we can solve many other mod
ebras" (Z. Nussinov and G. Ortiz, 0812.4309) and derive ma
uding a new exact self duality (E. Cobanera, G. Ortiz, and Z.
all Z_N gauge theories in 3+1 dimensions (earlier conjectured
self-dual). With 't Hooft ideas in mind,
merous authors studied of Wilson's action
Lattice Gauge Field Theories

$$\frac{1}{g^2}\left(\sum \mathrm{Re}(Tr(U_{ij}U_{jk}U_{kl}U_{li}-1))\right)$$



stricting the fields to
th roots of unity (Z_N)

Specifically, the dual coupling is given by

$$K_N\left(\frac{1}{2g^2}\right) \equiv K \qquad 4g_c^2 K_N(\cdot$$

$$\frac{1}{2}\frac{\partial F_N(K)}{\partial K} = \exp\left[-\frac{1}{2g^2}\left(1-\cos\frac{2\pi}{N}\right)\right]$$

$$F_N(K) = \sum^{N-1} \quad 2K\cos\left(\frac{2\pi n}{N}\right)$$

Pressure effects:

$$H_P = \gamma \sum_j P_\nu \sigma_j^\nu$$

$$\frac{d\vec{\sigma}_i}{dt} = \gamma \vec{\sigma}_i \times \vec{P}_i$$

$$\vec{P}_i = P_{i,\nu} e_\nu$$

rediction: In the presence of ur
ressure, the orbital state will ch

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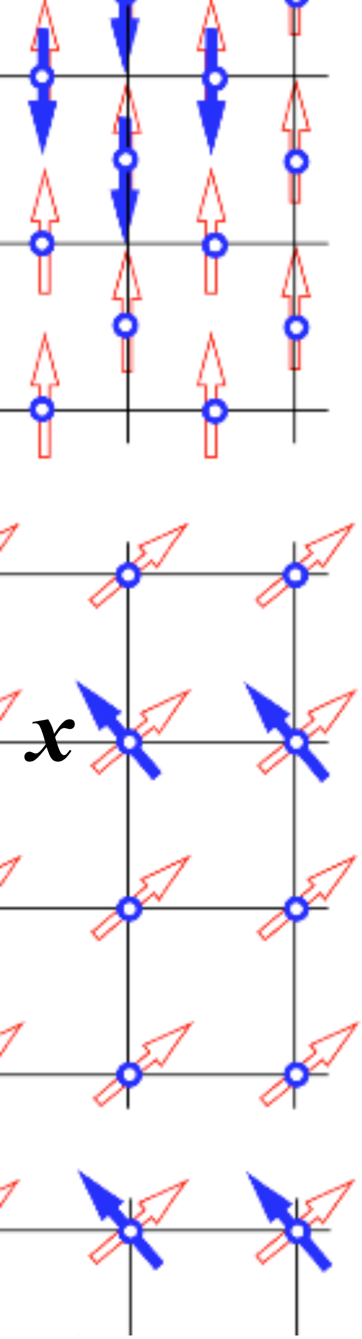


$d = 0$ (Ising Gauge Theory)

$$H = -K \sum_p \sigma^z_{ij} \sigma^z_{jk} \sigma^z_{kl} \sigma^z_{li}$$

$d = 1$ (Orbital Compass Mo

$$H = -\sum_i [J_x \sigma^x_i \sigma^x_{i+\hat{e}_x} + J_z \sigma^z_i \sigma$$

$$O^x = \prod_{j \in C_x} i\sigma^x_j$$

$d = D = 2$ (XY model)

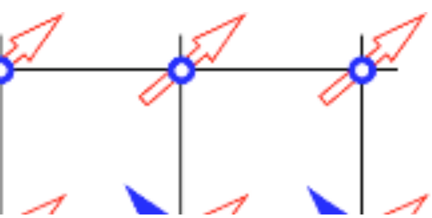
$$H = -J \sum [\sigma^x \sigma^x + \sigma^y \sigma^y]$$

re is a connection between Topological I oup generators of *d*-GLSs and its Topolo

(*D*=2 Orbital Compass Model) C_x

$$O^x = e^{i\frac{\pi}{2}\sum_{j\in C_x}\sigma_j^x} = \mathcal{P}e^{i\oint_{C_x}}$$

metries are linking operators: $O^\mu|g$



Topological defect: C_+ : op

n system with Hamiltonian H_D and *d*-G

The absolute value of the average of any qu
quantity f which is not invariant under *d*-
is bounded from above by the absolute valu
mean of the same quantity when this quasi-
quantity is computed with a *d*-dim H_d t
globally invariant under $\mathcal{G}_d$ and preserves
of the interactions in the original *D*-dim sy

c_j

n_i

$$|\langle f(\phi_i)\rangle_{H_D}| \leq |\langle f(n$$

we spontaneously break a *d*-GLS in a *D*

the Generalized Elitzur's Theorem: (fi

stren

non-$\mathcal{G}_d$-invariant quantities

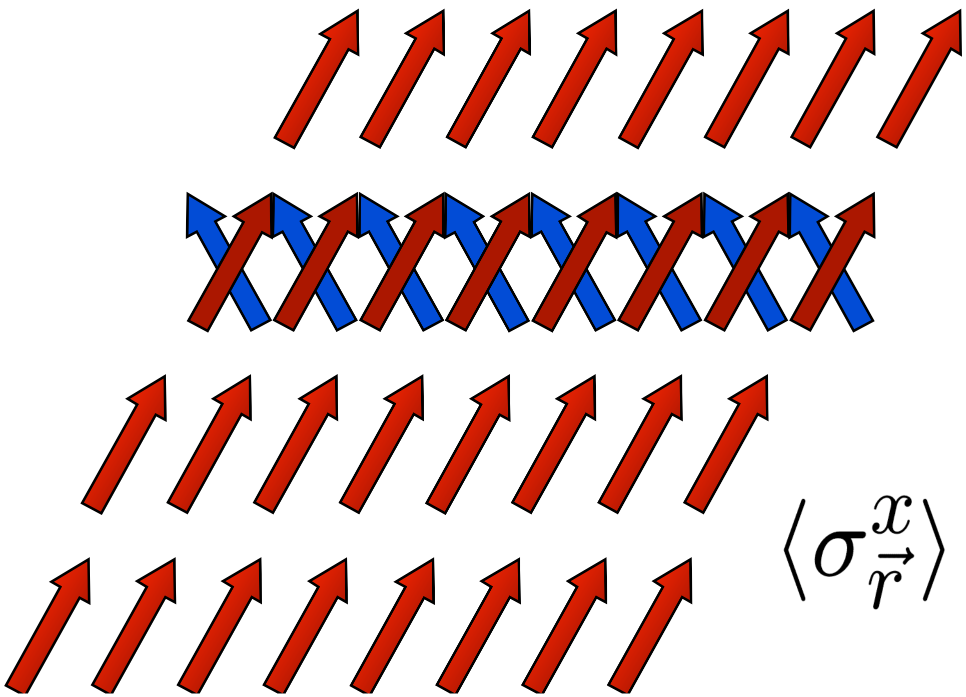
- *d=0* SSB is forbidden
- *d=1* SSB is forbidden
- *d=2* (continuous) SSB is forbidden

  d=2 (discrete) SSB may be broken

- *d=2* (continuous with a gap) SSB is f

Orbital Compass Model

$$H = J \sum_{\vec{r}} (\sigma^x_{\vec{r}} \sigma^x_{\vec{r}+\hat{e}_x} + \sigma^y_{\vec{r}} \sigma^y_{\vec{r}+\hat{e}_y}$$

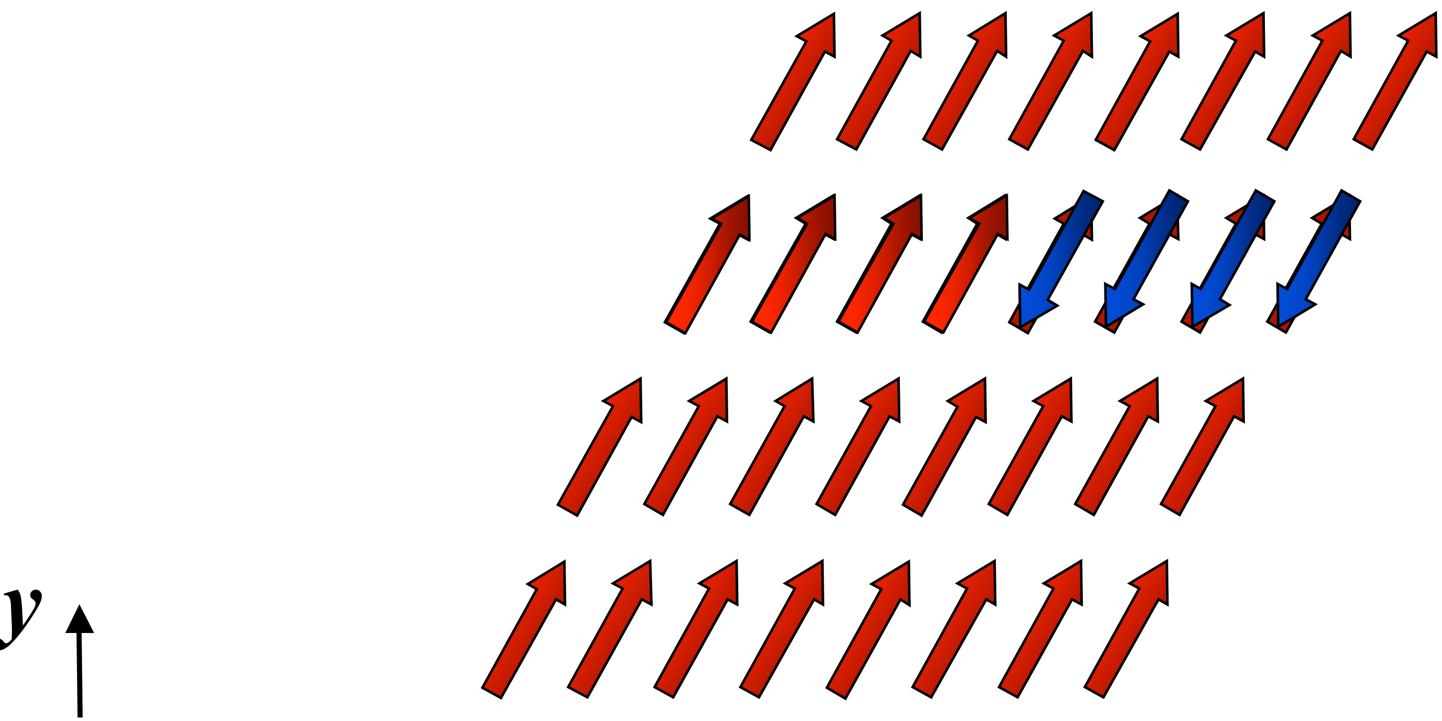


Rotation by
around the y-

$$\langle \sigma^x_{\vec{r}} \rangle = \langle \sigma^y_{\vec{r}} \rangle = \langle \sigma^z_{\vec{r}} \rangle =$$

Orbital Compass Model

$$H = J \sum_{\vec{r}} (\sigma^x_{\vec{r}} \sigma^x_{\vec{r}+\hat{e}_x} + \sigma^y_{\vec{r}} \sigma^y_{\vec{r}+\hat{e}_y}$$



What happens when the *d*-GLSs are not exact symmetries of the ful

(i.e., effect of perturbations)

Emergent Symmetries

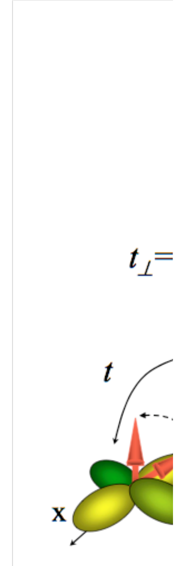
↓

I: (Exact result) Continuous $d < 2$ emerg
in a gapped system, resu

For independent d-GL
with d=1, degenerac
is exponential in the surface a
system.

ugel-Khomskii Hamiltonian H_{KK} for t_{2g} systems

A continuous symmetry
Harris et al., PRL 91, 087206 (2003))



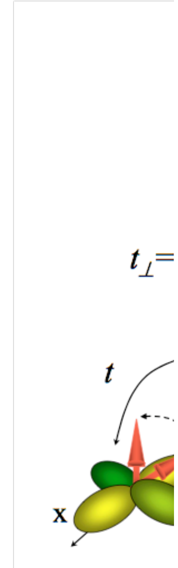
$$O_P^\gamma \equiv [\exp(i\vec{S}_P^\gamma \cdot \vec{\theta}_P^\gamma)/\hbar]$$

$$[H_{KK}, O_P^\gamma] = 0, \quad \vec{S}_P^\gamma = \sum_{r \in P} \vec{S}_r^\gamma$$

a continuous d=2 symmetry o

Kugel-Khomskii Hamiltonian

H_{KK} for t_{2g} systems.



For a system in |xy> state,

$$I(k_x, k_y, z, \omega) = \int dk_z e^{ik_z z} S(\vec{k}, \omega)$$

vanishes for non-zero z. This is so
as if two spins do not lie in the same plane (and
e a separation along the direction orthonormal to the planes
correlator is not invariant under a continuous d=2 sy
nt. must be present to account for spin order. Smilar conside
|yz> order. In general, if the KK interactions are d

$$[I(k_a, k_b, c, \omega) + I(k_b, k_c, a, \omega) + I(k_c, k_a, b, \omega)$$

Conclusions (new resu

Orbital systems can order by **thermal** "orde disorder" **fluctuations** even in their classical (1/S) zero point quantum fluctuations are ne

Similar to charge and spin driven quantum c behavior, it is theoretically possible to have **o driven quantum critical** behavior. (Predicti

Orbital systems can exhibit topological order **dimensional reductions** due to their unusu (*exact or approximate*).

A new approach to dualities.

Orbital nematic orders (from symmetry s and related selection rules

Orbital Larmor effects are predicted- peri