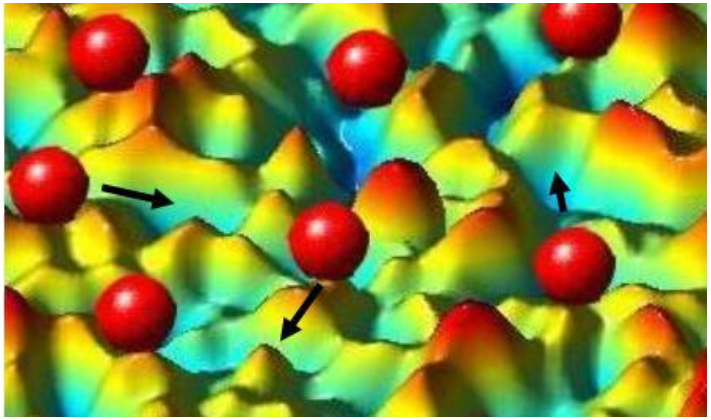
Slow relaxations and aging in electron glasses

Yuval Oreg



מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

Together with

Ariel Amir and Joe Imry

Outline

- Electron glass model
  - slow relaxations and aging
  - statics (Coulomb gap) & steady-states (Variable Range Hopping)

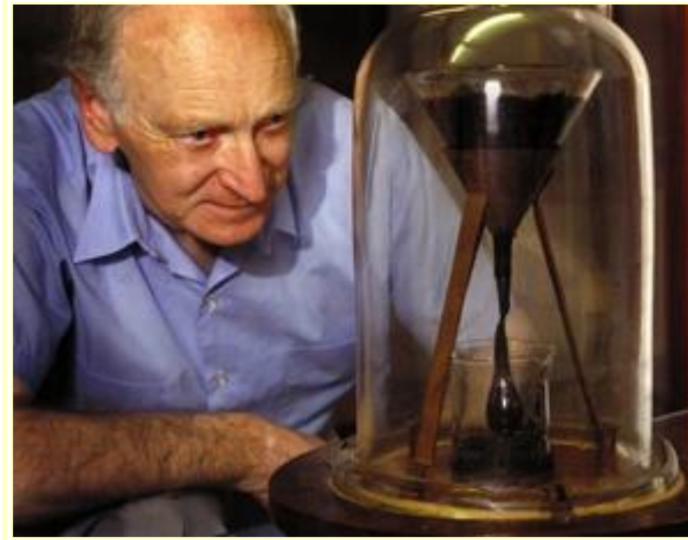
  Dynamics:
  - mapping to a new class of Random Matrix Theory (RMT)

- Definition of Random Distance Matrices
- Solution of model: *eigenvalue distribution* through moment calculation
- Solution of model : RG approach → *localization properties*
- Implications: slow relaxations, localized phonons
- Conclusions

Huge viscosity of glasses

- Disordered system – a snapshot looks identical to a liquid.
- It flows extremely slowly (huge viscosity).
- Similar to ‘real’ glass (S_iO_2) / pitch:

Year	Event
1930	The stem was cut
1938(Dec)	1st drop fell
1947(Feb)	2nd drop fell
1954(Apr)	3rd drop fell
1962(May)	4th drop fell
1970(Aug)	5th drop fell
1979(Apr)	6th drop fell
1988(Jul)	7th drop fell
2000(Nov)	8th drop fell



R. Edgeworth, B. J. Dalton and T. Parnell,

Eur. J. Phys (1984) : visocity = 10^{11} *water viscosity

http://www.smp.uq.edu.au/pitch/

Common features of glassy models

• Quenched disorder.

• Rugged energy landscape: Many states close to ground state.

• Aging and memory effects: relaxation slower if perturbation lasts longer.

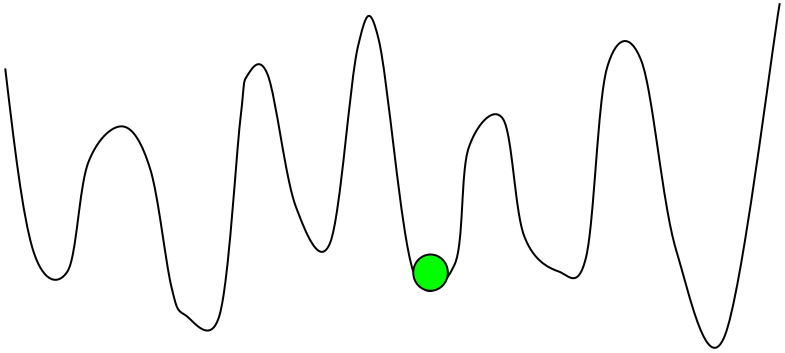
Physical examples

• Structural glass (Window glass)

• Various magnetic materials.

• Packing of hard spheres.

• Electron glass!   (long-ranged interactions, but not infinite!)



General questions: structure of states, out-of-equilibrium dynamics?

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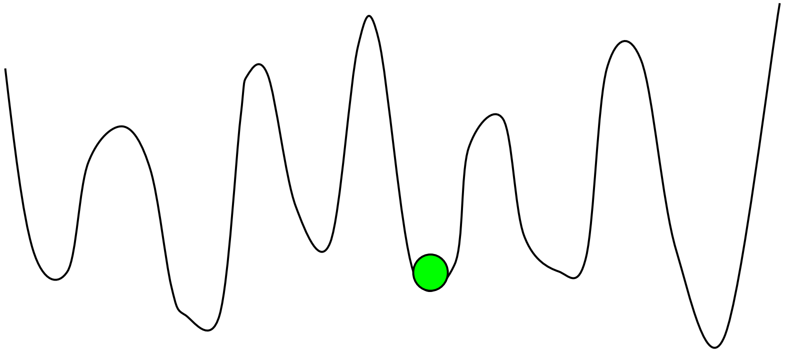
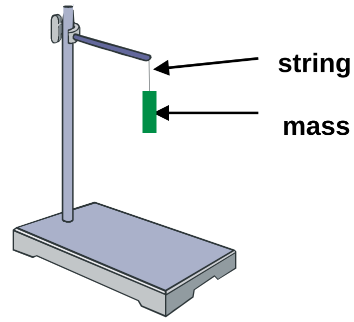


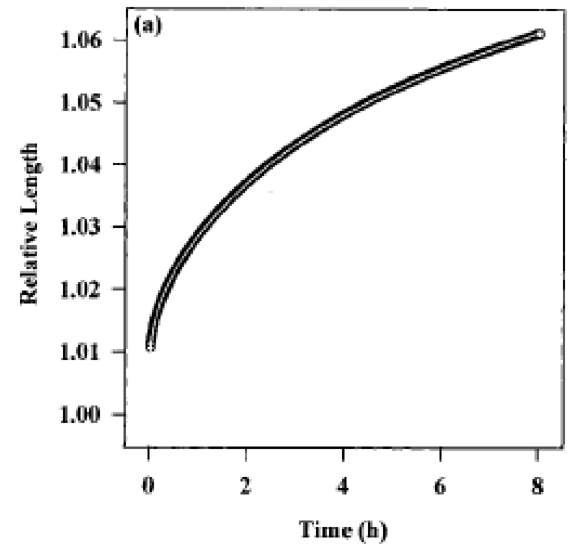
General questions: structure of states, out-of-equilibrium dynamics?

Slow relaxations in nature

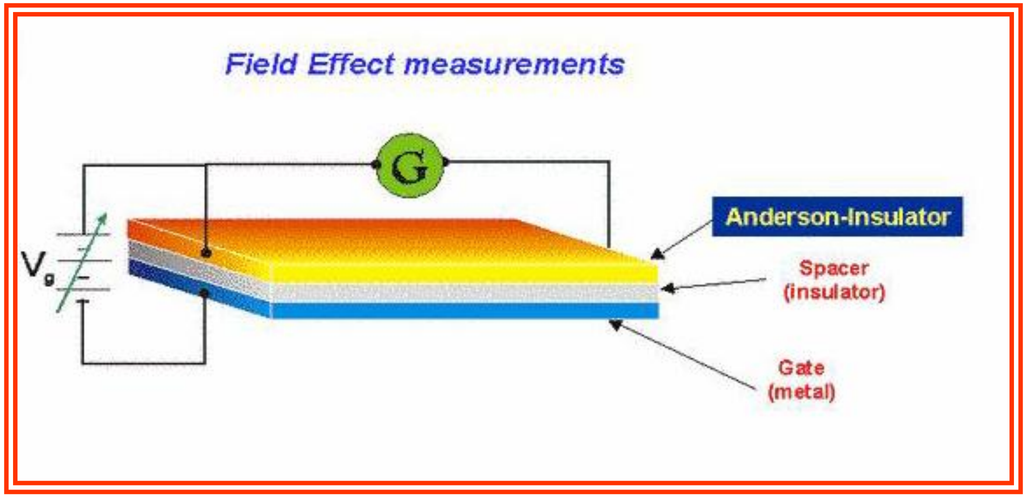
W. Weber, *Ann. Phys.* (1835)

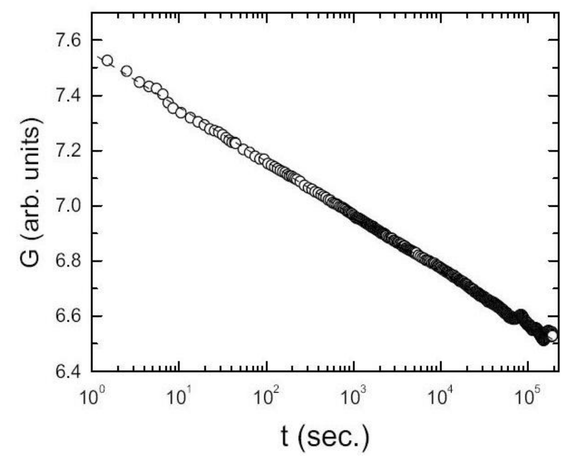
D. S. Thompson, *J. Exp. Bot.* (2001)





Electron glass- Experimental system





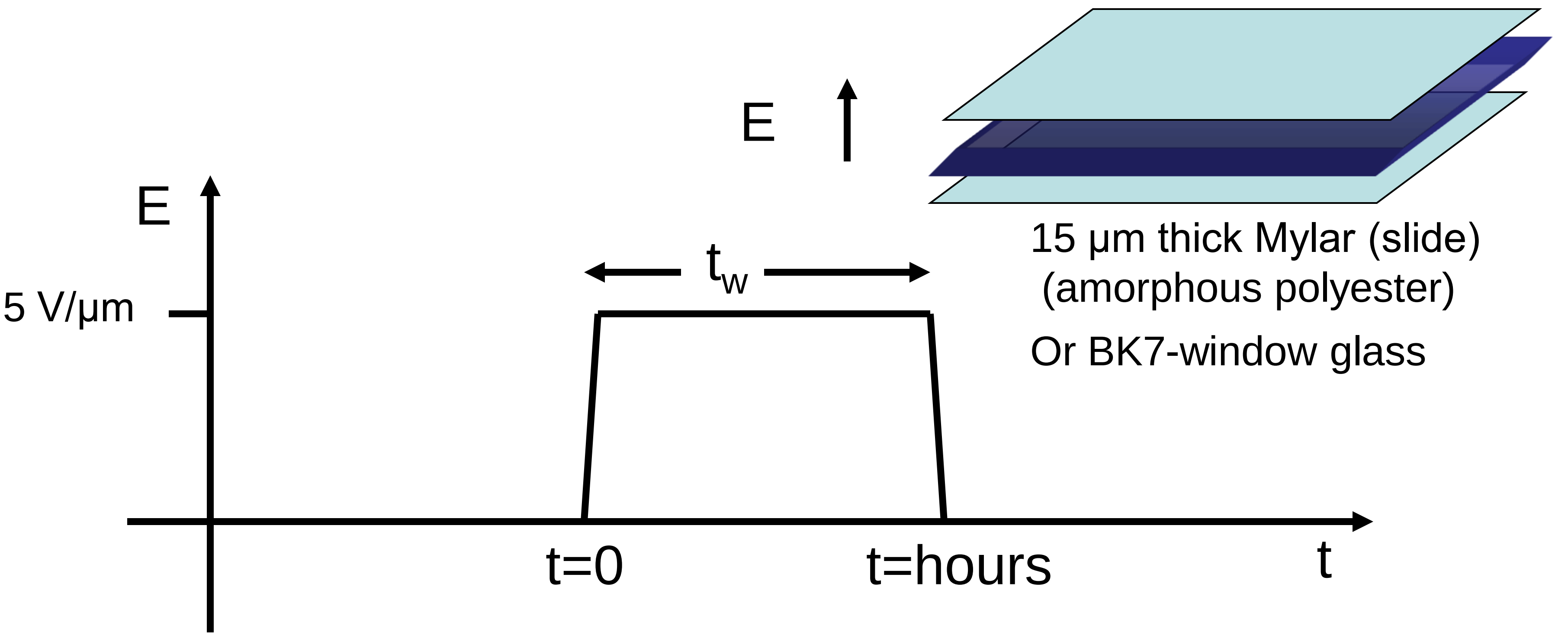
What are the ingredients leading to slow relaxations?

Ovadyahu et al.

Logarithmic relaxations for 5 days!

An Aging Protocol

- Step I: Cool the system and let it relax for a long time (days)
- Step II: Switch on "the perturbation"
- Step III: After aging time t_w – Switch the perturbation off
- Throughout the experiment a physical property (dielectric constant) is measured as a function of time



Ludwig, Nalbach, Rosenberg and Osheroff, PRL (2003)

Electron glass aging– experimental protocol

A. Vaknin and Z. Ovadyahu and M. Pollak, PRL 2000

Step I

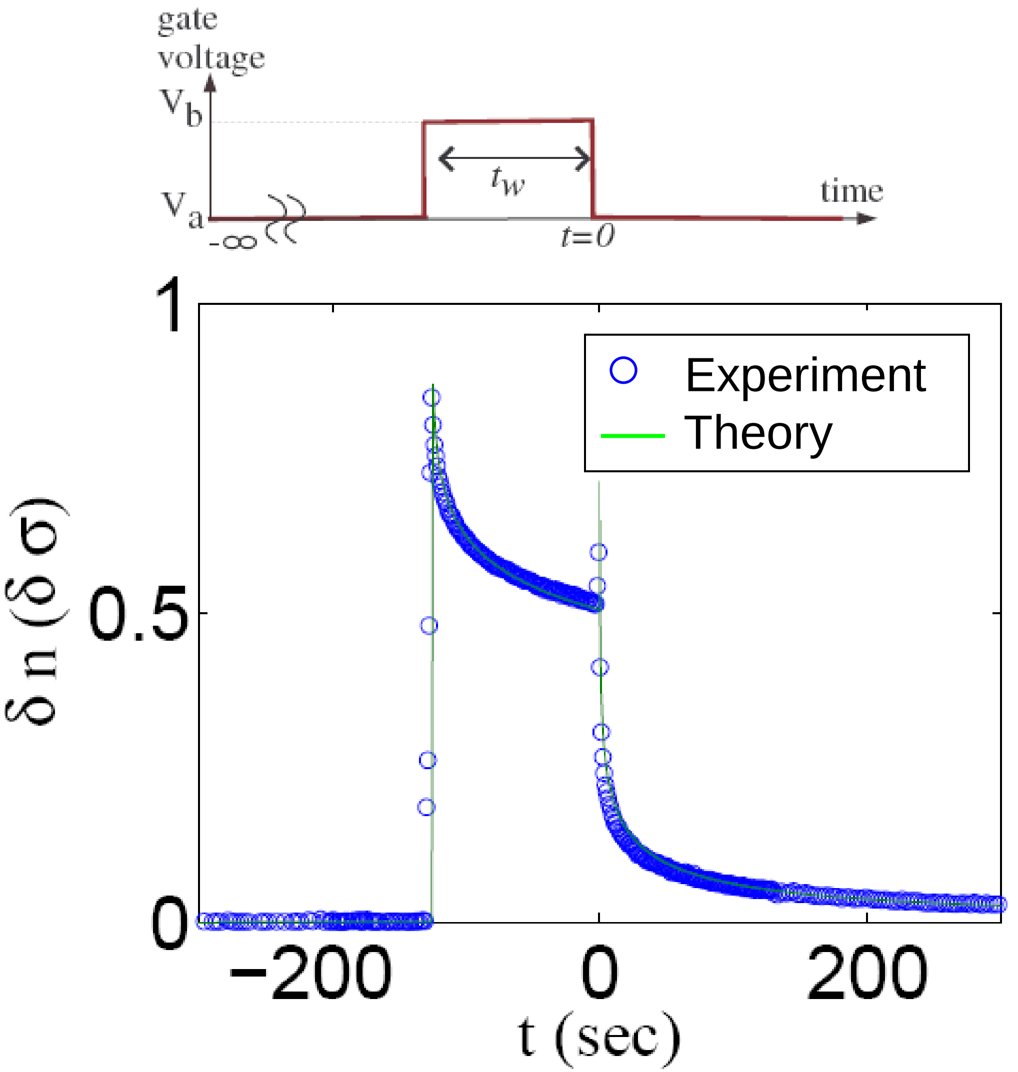
System equilibrates for long time

Step II

V_g is changed, for a time of t_w.

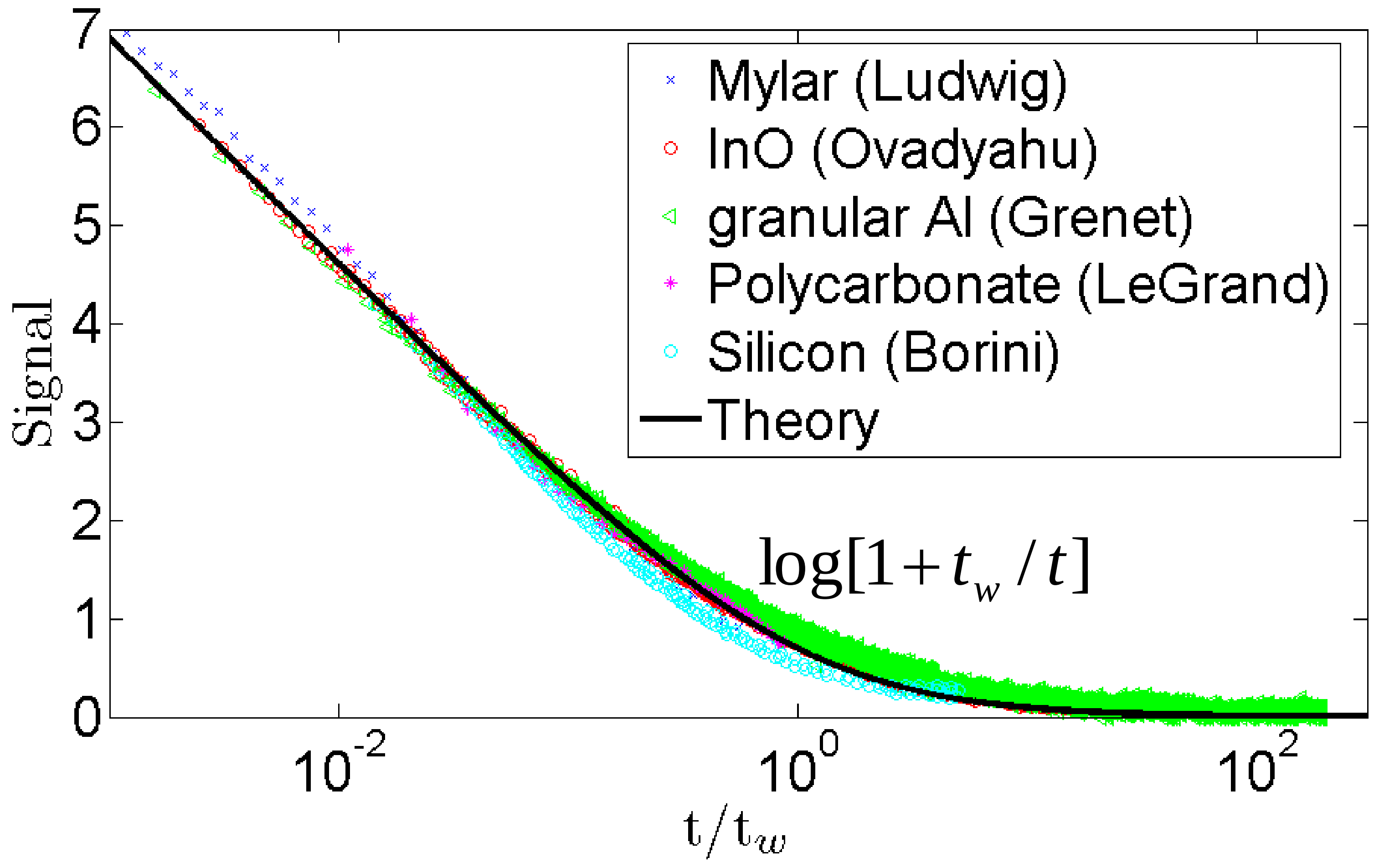
Throughout the experiment

Conductance is measured as a function of time.



Data: *Ovadyahu et al.*

Aging and universality

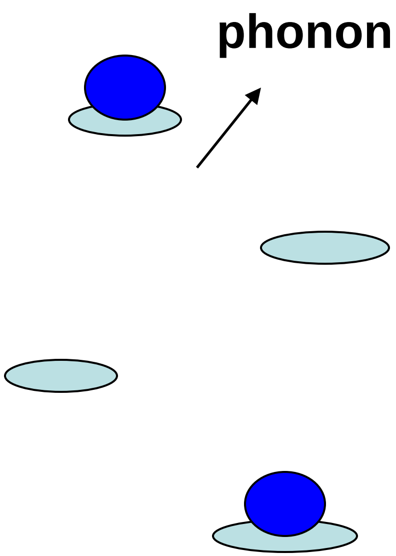


Amir, Oreg and Imry, to be published

The model

- Strong localization due to disorder

  → randomly positioned sites, on-site disorder.
- Coulomb interactions are included
- “Phonons” induce transitions between configurations.
- Interference (quantum) effects neglected.



e.g:

Pollak (1970)

Shklovskii and Efros (1975)

Ovadyahu and Pollak (2003)

Muller and Ioffe (2004)

“Local mean-field” approximation - Dynamics

AA, Oreg and Imry, PRB (2008)

$$n_i \to \langle n_i \rangle, \qquad \frac{dn_i}{dt} = \sum_j -\gamma_{i,j} + \gamma_{j,i}$$

$$\gamma_{i,j} = \exp(-2r_{ij}/\xi) n_i (1-n_j)[N(|\Delta E|) + \theta(\Delta E)]$$

- ΔE includes the interactions
- N is the Bose-Einstein distribution
- ξ - the localization length

$$\left(E_i = \varepsilon_i + \sum_j \frac{n_j}{r_{ij}} \right)$$

At long times (Statics):

- The system reaches a locally stable point (metastable state).
- Many metastable states, each manifesting a Coulomb gap

(“Pseudo-ground-states”, *Baranovski et al., J. Phys. C, 1979*)

"Local mean-field" approximation - Equilibrium

- Detailed balance leads to Fermi-Dirac statistics ($n_j \rightarrow f_j$).
- Self-consistent set of equations for the energies:

$$E_i = \varepsilon_i - \sum_j \frac{1}{2}\frac{e^2}{r_{ij}} \tanh\left(\frac{E_j}{2T}\right)$$

(assuming half filling).

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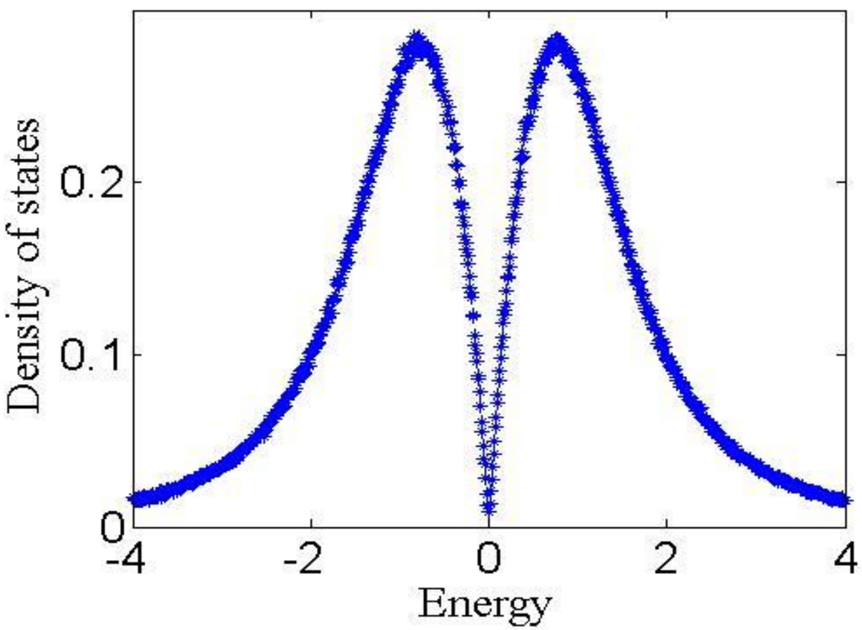
Yields linear **Coulomb gap**

(+ temperature dependence)



M. Pollak, Discuss. Faraday Soc (1970)

A.L. Efros and B.I. Shklovskii,

J. Phys. C (1975)

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M. Grunewald et al., J. Phys. C. (1982)

A. A. Mogilyanskii and M.E. Raikh (1989)

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AA et al., PRB (2008)

Surer et al., PRL (2009)

Goethe et al., PRL (2009)

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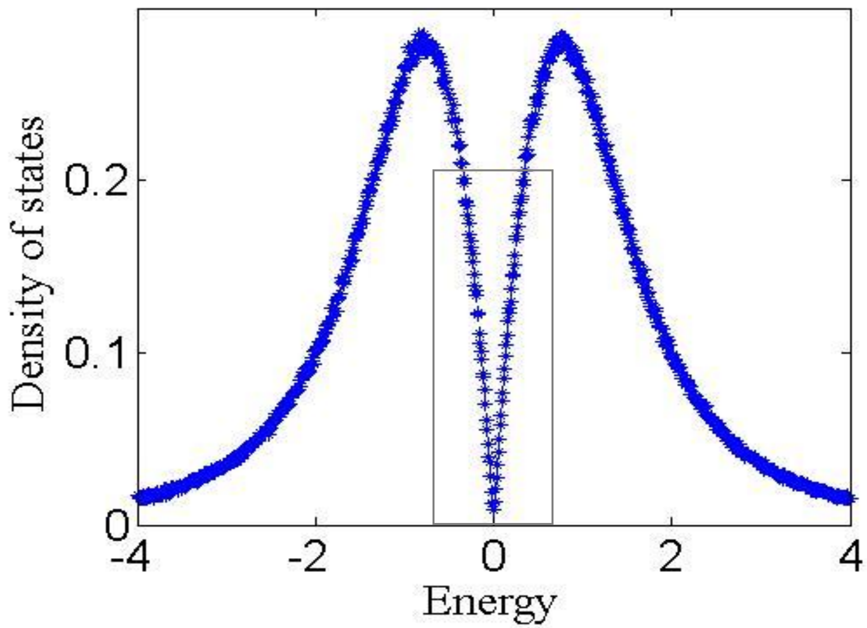

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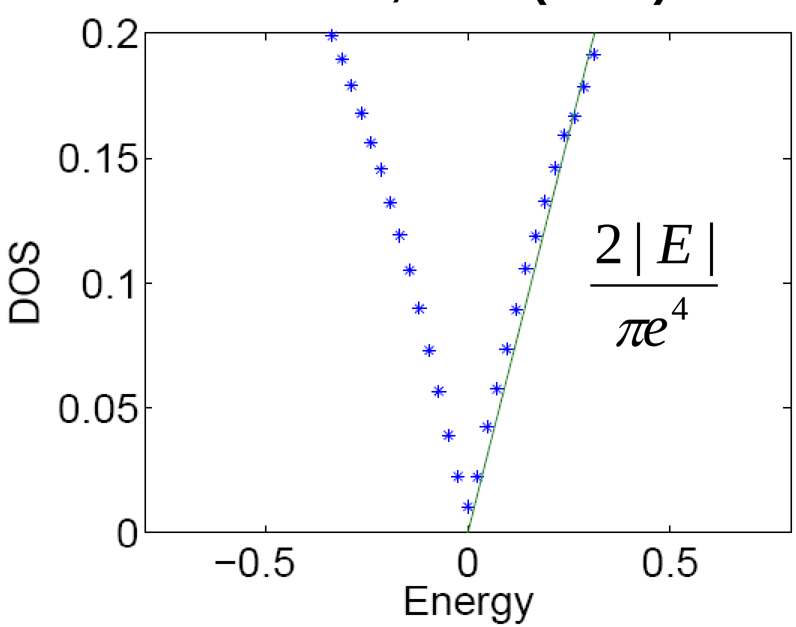

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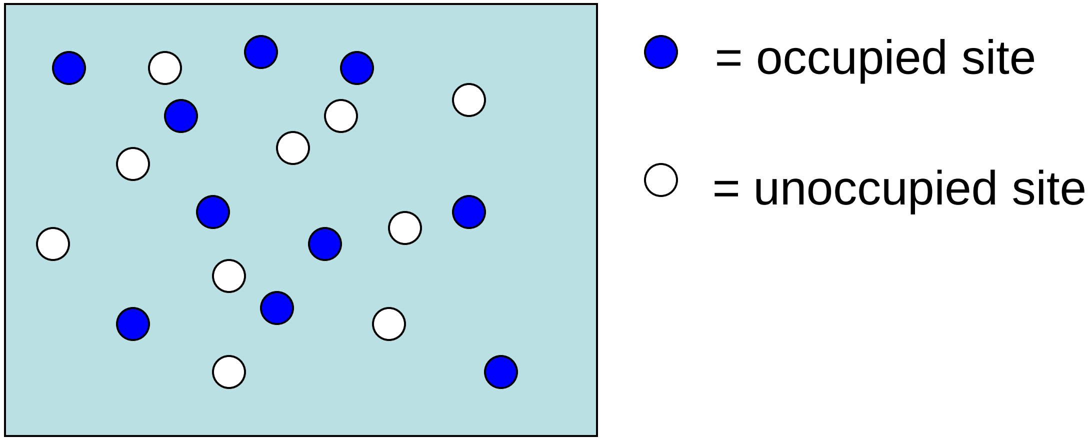
Efros-Shklovskii argument , T=0



Cost of moving an electron: $\Delta E = E_i - E_j - \dfrac{e^2}{r_{ij}}$

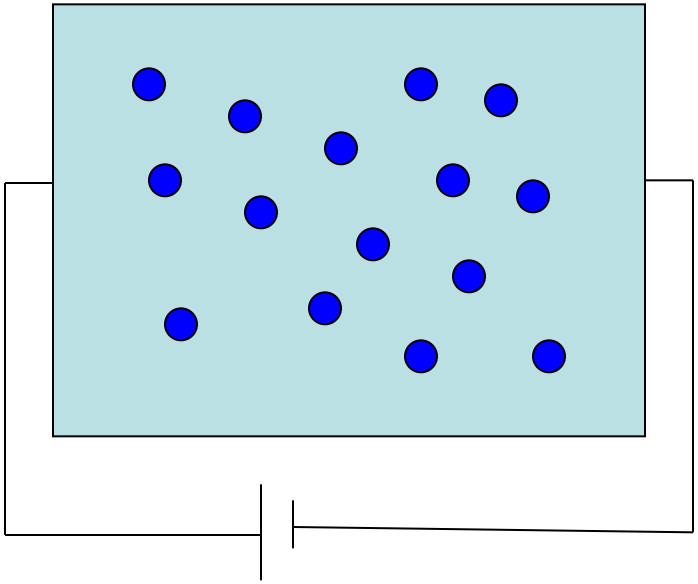
For ground state: $\Delta E \geq 0$

Assume finite density of states at E_f $\longrightarrow$ Contradiction.

$\longrightarrow$ Upper bound is $g(E) \leq \alpha \, |E|^{d-1}$

“Local mean-field” approximation – Steady State

Miller-Abrahams resistance network (no interactions)



$$R_{ij} = \frac{T}{e^2 \gamma^0_{ij}}$$

Equilibrium rates,

obeying detailed balance

A. Miller and E. Abrahams, (Phys. Rev. 1960)

Generalization

1) Find n_i and E_i such that the systems is in steady state.

2) Construct resistance network.

"Local mean-field" approximation – Steady State

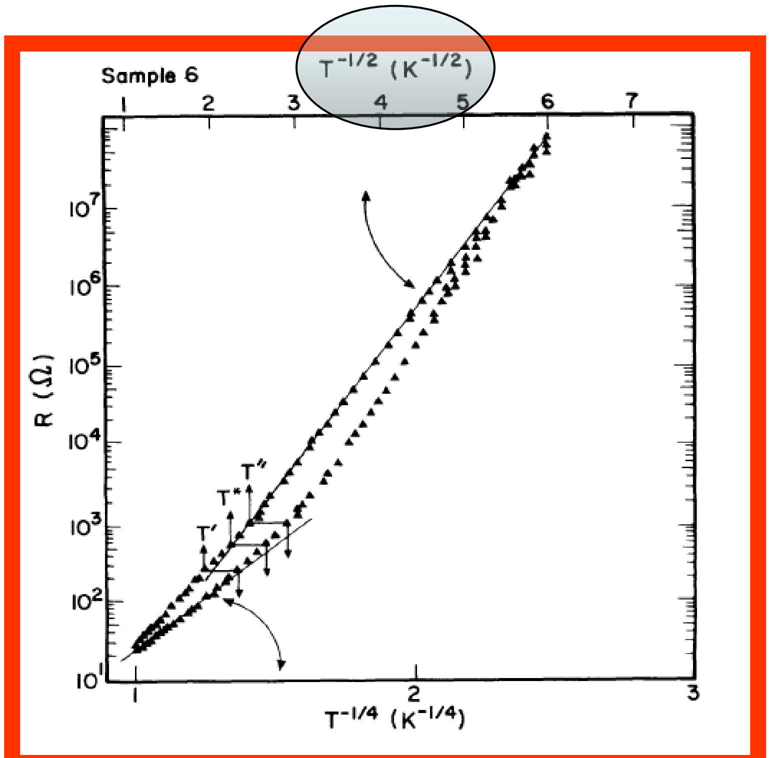
With interactions (ES) | Without interactions (Mott, VRH)

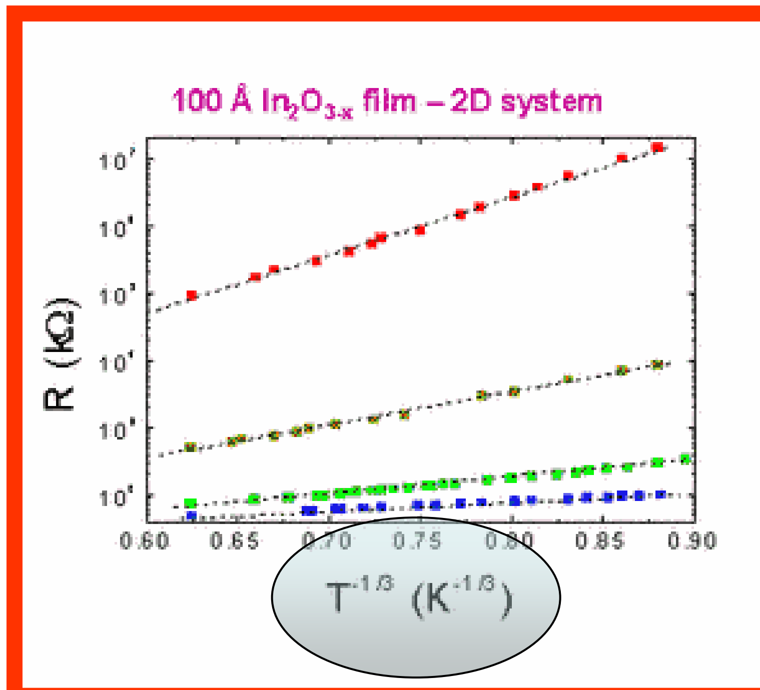
$$\sigma \sim e^{-\left(\frac{T_{ES}}{T}\right)^{1/2}} , T_{ES} \sim e^2/\xi$$

$$\sigma \sim e^{-\left(\frac{T_M}{T}\right)^{1/(d+1)}} , T_M \sim e^2/[\upsilon \xi^d]$$

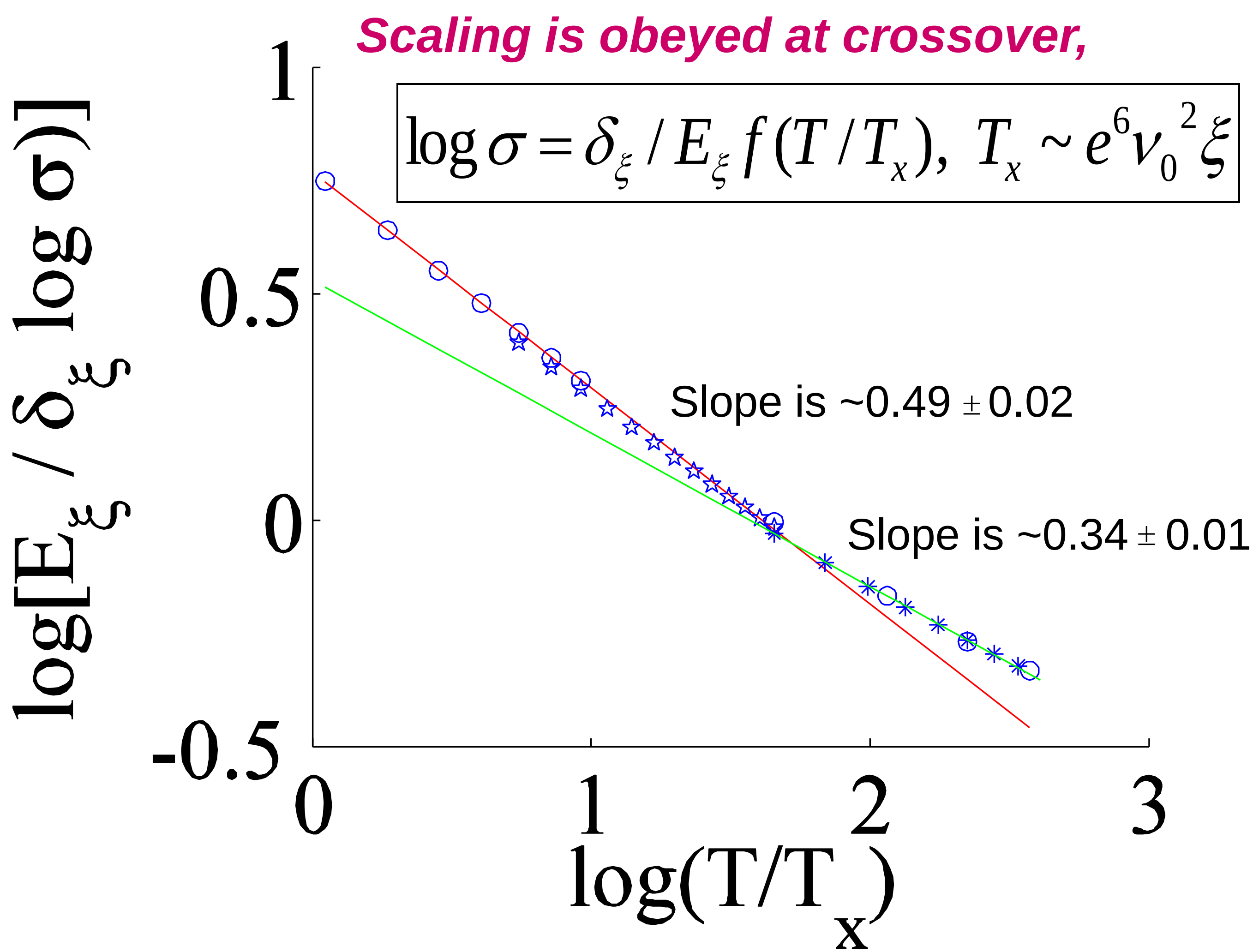
GeAs samples Shlimak, Kaveh, Yosefin, Lea and Fozooni (1992)

Ovadyahu (2003)





VRH (Mott) to E-S Crossover



Amir, Oreg and Imry , PRB (2009)

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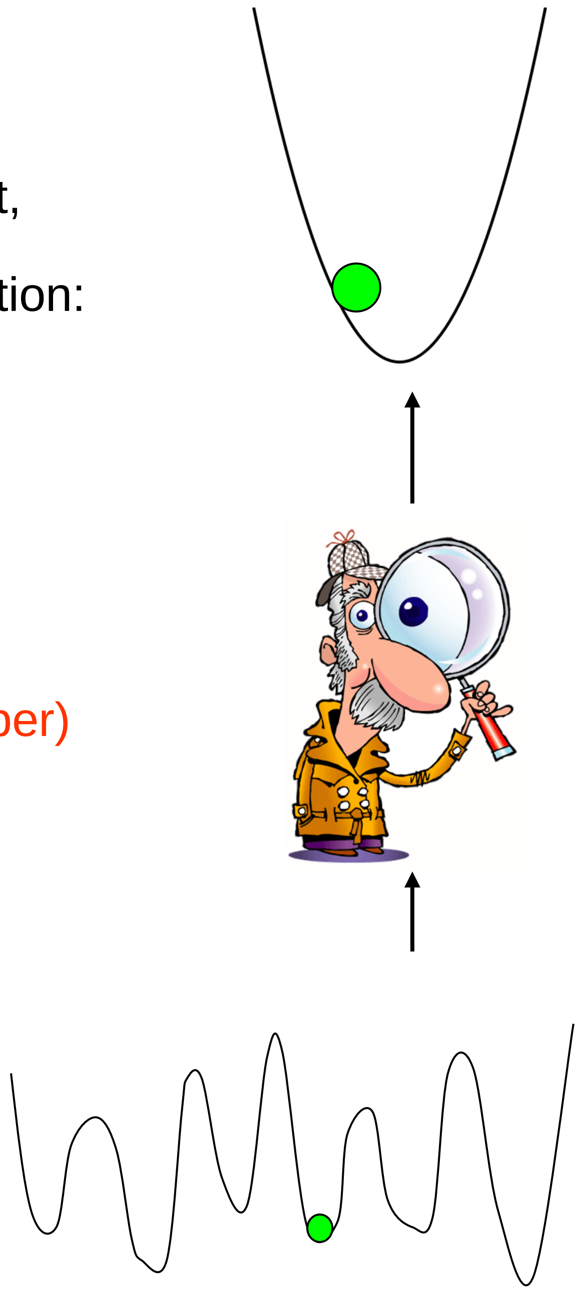
$$\gamma_{i,j} = \exp(-2r_{ij}/\xi) n_i (1-n_j)[N(|\Delta E|) + \theta(\Delta E)]$$

We saw: approach works well for statics & steady-states

Moving on to dynamics…

Solution near locally stable point



Close enough to the equilibrium (locally) stable point,

one can linearize the equations, leading to the equation:

$$\frac{d\delta\vec{n}}{dt} = A \cdot \delta\vec{n}$$

$$A_{i,j} = \frac{\gamma^0_{i,j}}{n^0_j(1-n^0_j)} - \frac{e^2}{T}\sum_{l\neq i,j}\gamma^0_{i,k}\left(\frac{1}{r_{i,j}} - \frac{1}{r_{i,k}}\right), \qquad (i \neq j)$$

Sum of columns vanishes (particle conservation number)

$A = \gamma \cdot \beta$, β^{-1} is equal-time correlation matrix

$\gamma^0_{i,j} \sim e^{-\frac{2r_{ij}}{\xi}}$ (Anderson Localization)

For low temperatures, near a local minimum,

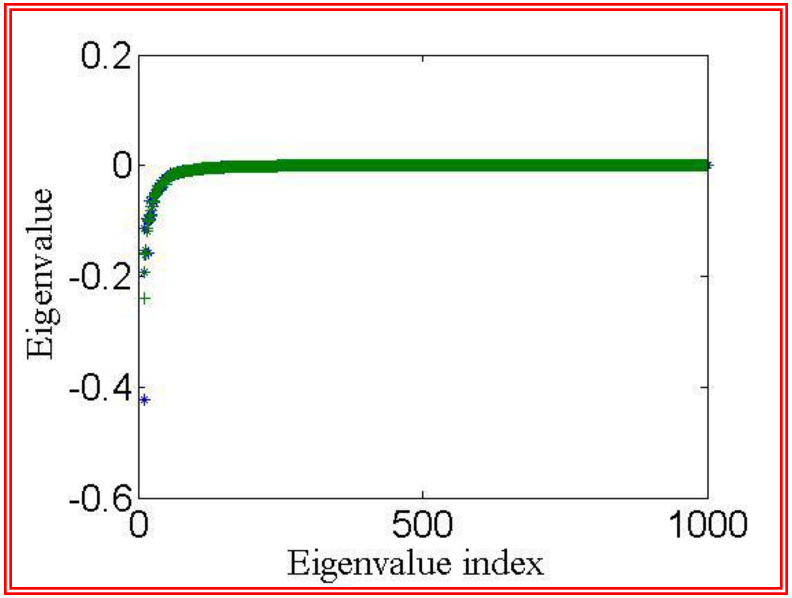
second term is negligible →

All eigenvalues are real and negative

Eigenvalue Distribution

Solving numerically shows a distribution proportional to $1/\lambda$:

Eigenvalues



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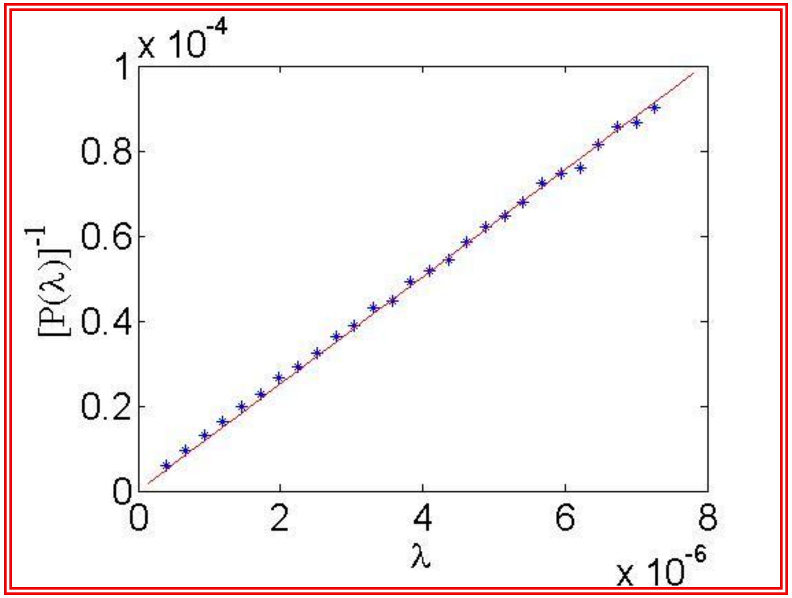
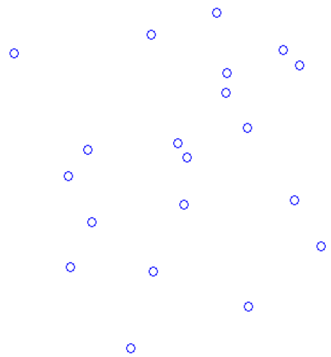


$$\sum_{\lambda} e^{-\lambda t} \longrightarrow \int P(\lambda) e^{-\lambda t} d\lambda \sim -\gamma_E - \log(\lambda_{\min} t)$$

Digression: What are Random Distance Matrices?

1) Choose N points randomly and uniformly in a d-dimensional cube.



I. M. Lifshitz, *Adv. Phys* (1964).

Mezard, Parisi and Zee, *Nucl. Phys.* (1999)

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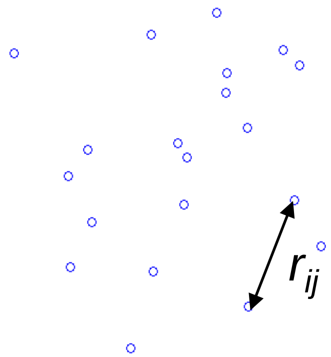
2) Define the off-diagonals of our matrix as:

$$A_{i,j} = f(r_{ij}) \ , \quad f(r) = e^{-r/\xi}$$

(Euclidean distance)

$$\varepsilon = \xi / \langle r \rangle$$



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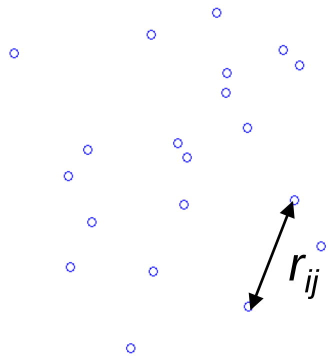

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$$A_{i,i} = -\sum_{j \neq i} A_{i,j}$$

sum of every column vanishes

(will come from a conservation law)

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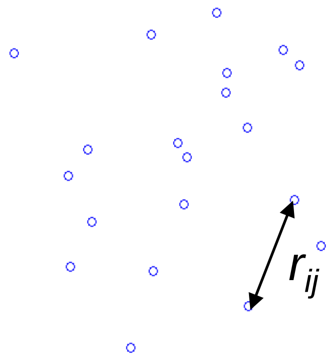

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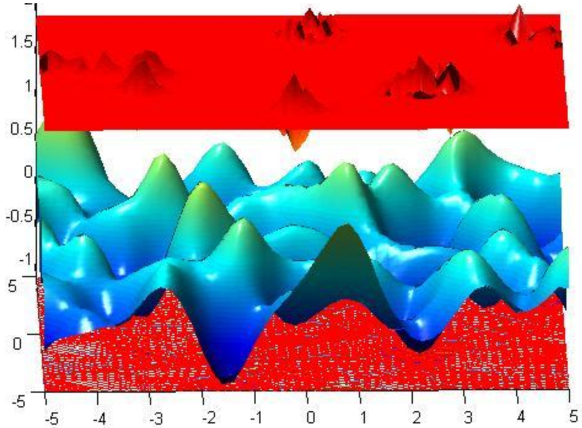
Q: What is the eigenvalue distribution?

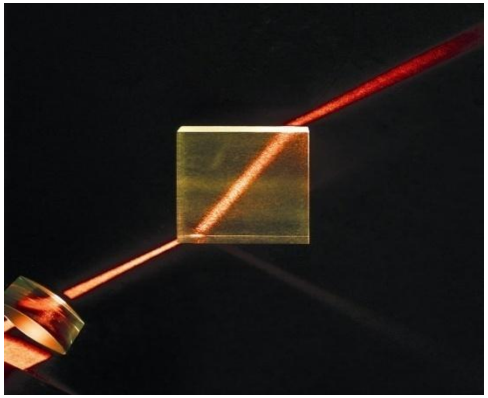
What are the eigenmodes?

Distance matrices – Motivation

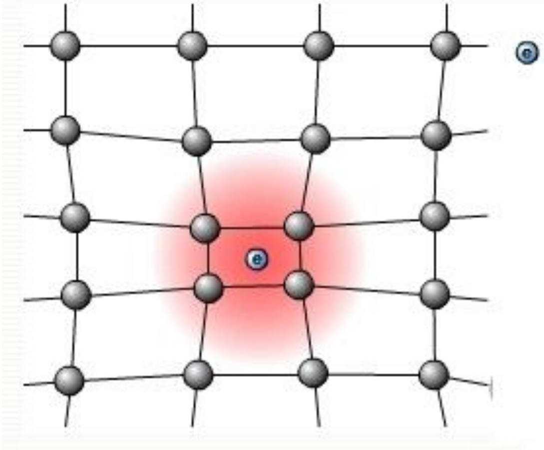
Relaxation in electron glasses
Amir, Oreg and Imry, PRB 2008



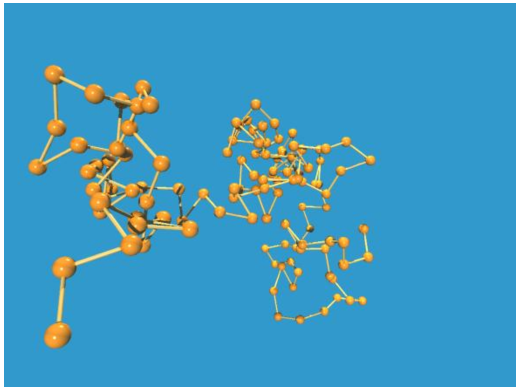
Photon propagation in a gas
Akkermans, Gero and Kaiser, PRL 2008



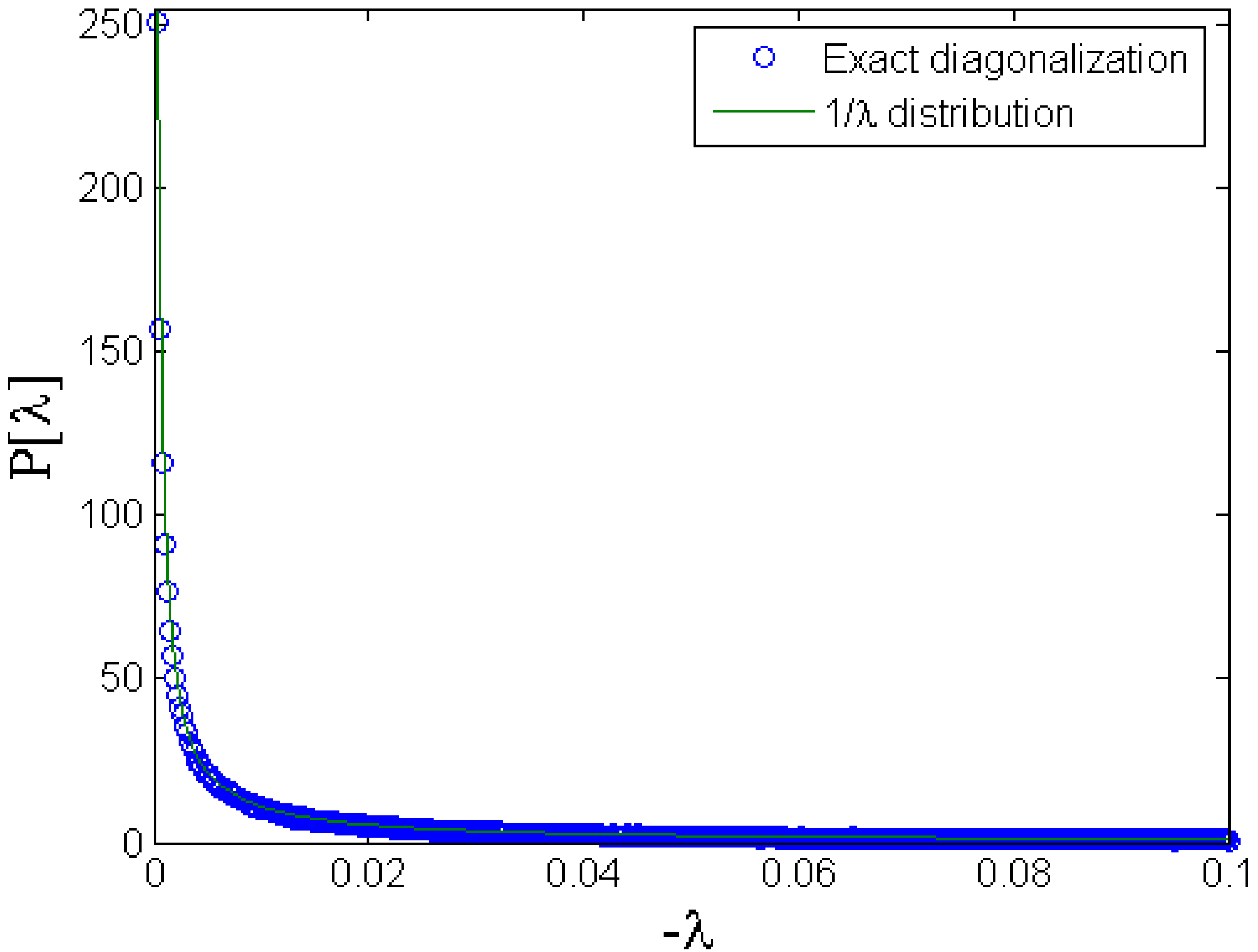
Localization of phonons
Ziman, PRL 1982
Vitelli et al., PRE 2010



Anomalous diffusion
Scher and Montroll, PRB 1975
Metzler, Barkai and Klafter, PRL 1999

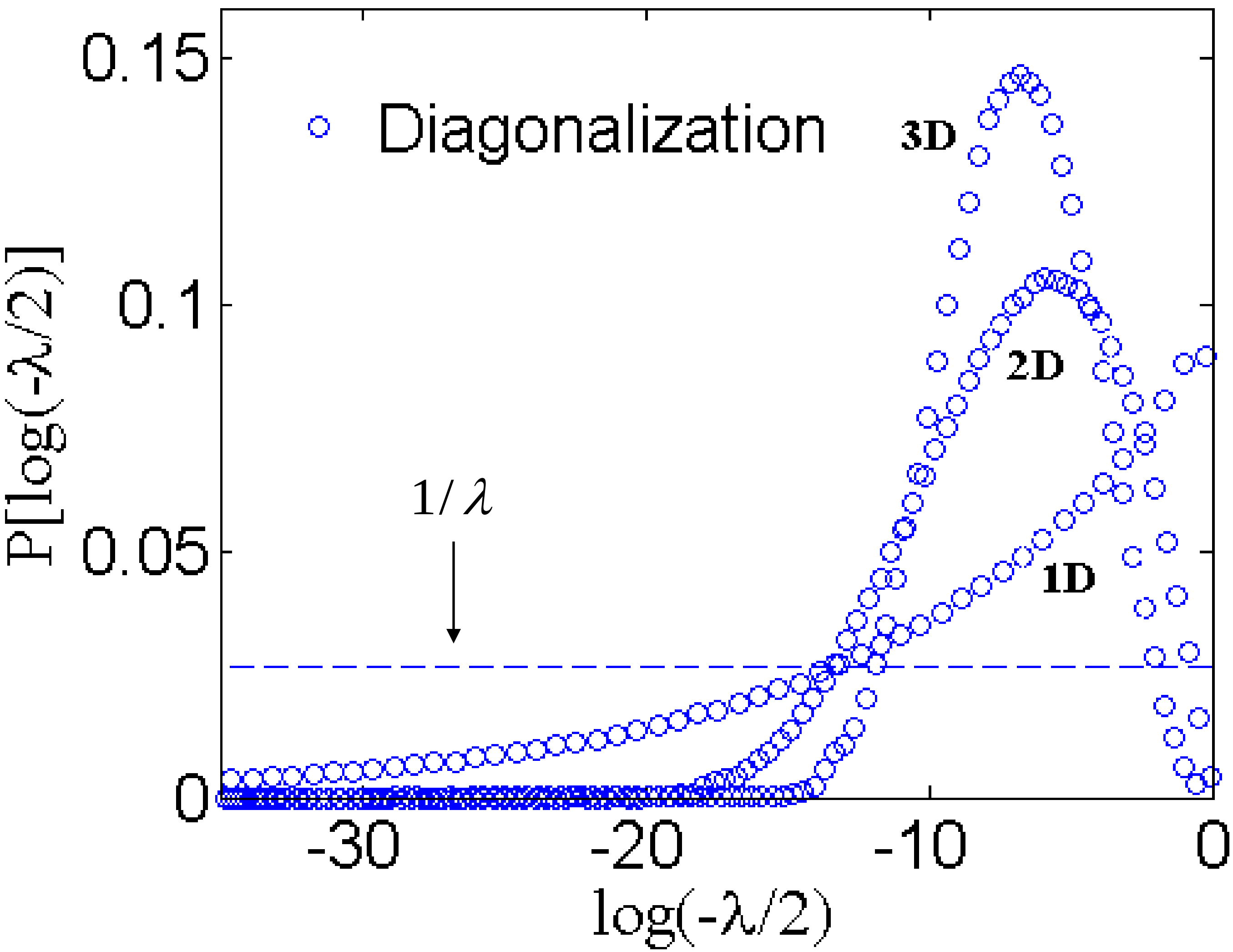


Results – 2D



Exact diagonalization
1/λ distribution
P[λ]
-λ
250
200
150
100
50
0
0
0.02
0.04
0.06
0.08
0.1

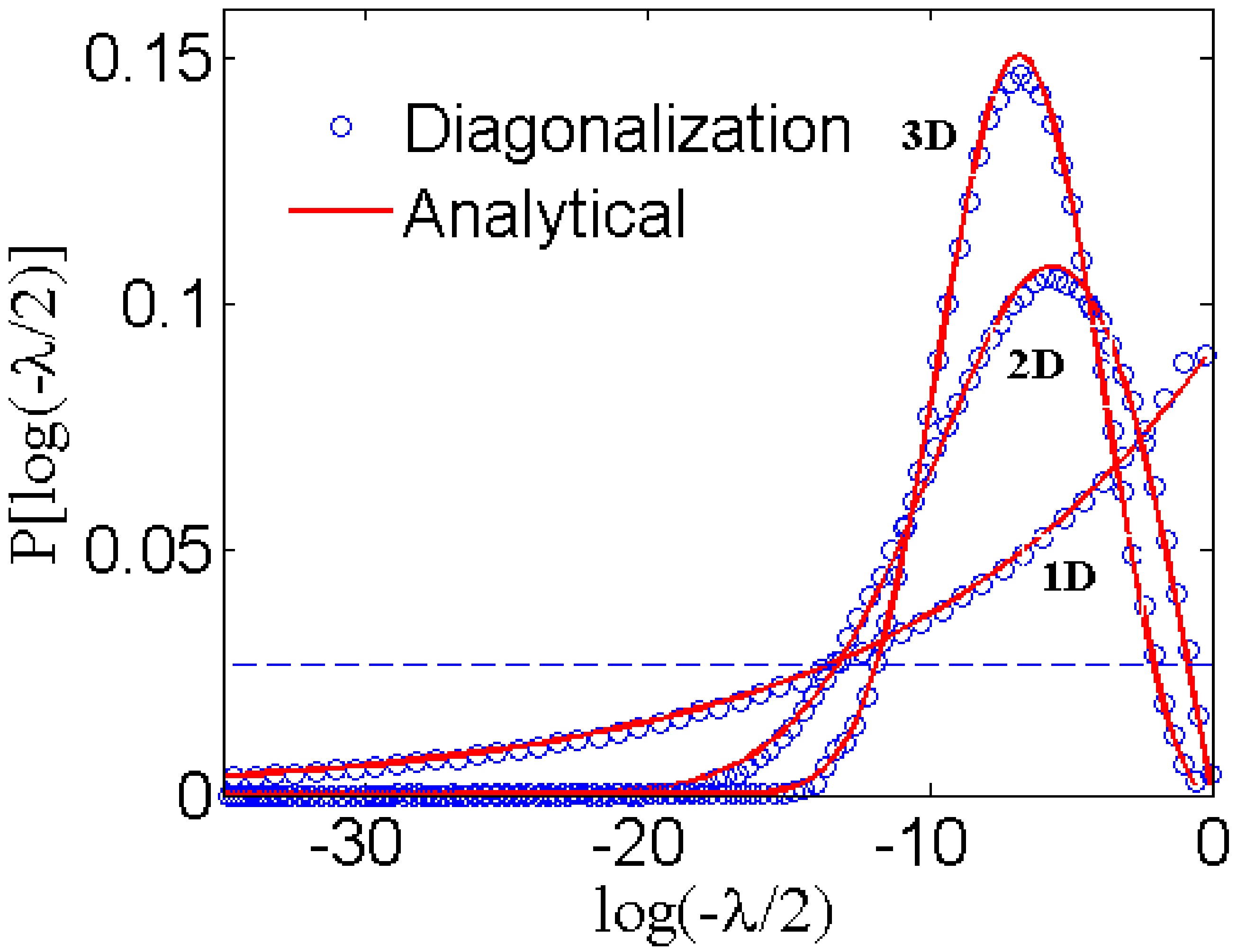
Results



Diagonalization
3D
2D
1D
1/ λ
0.15
0.1
0.05
0
-30
-20
-10
0
log(-λ/2)
P[log(-λ/2)]

Results

(no fitting parameters)



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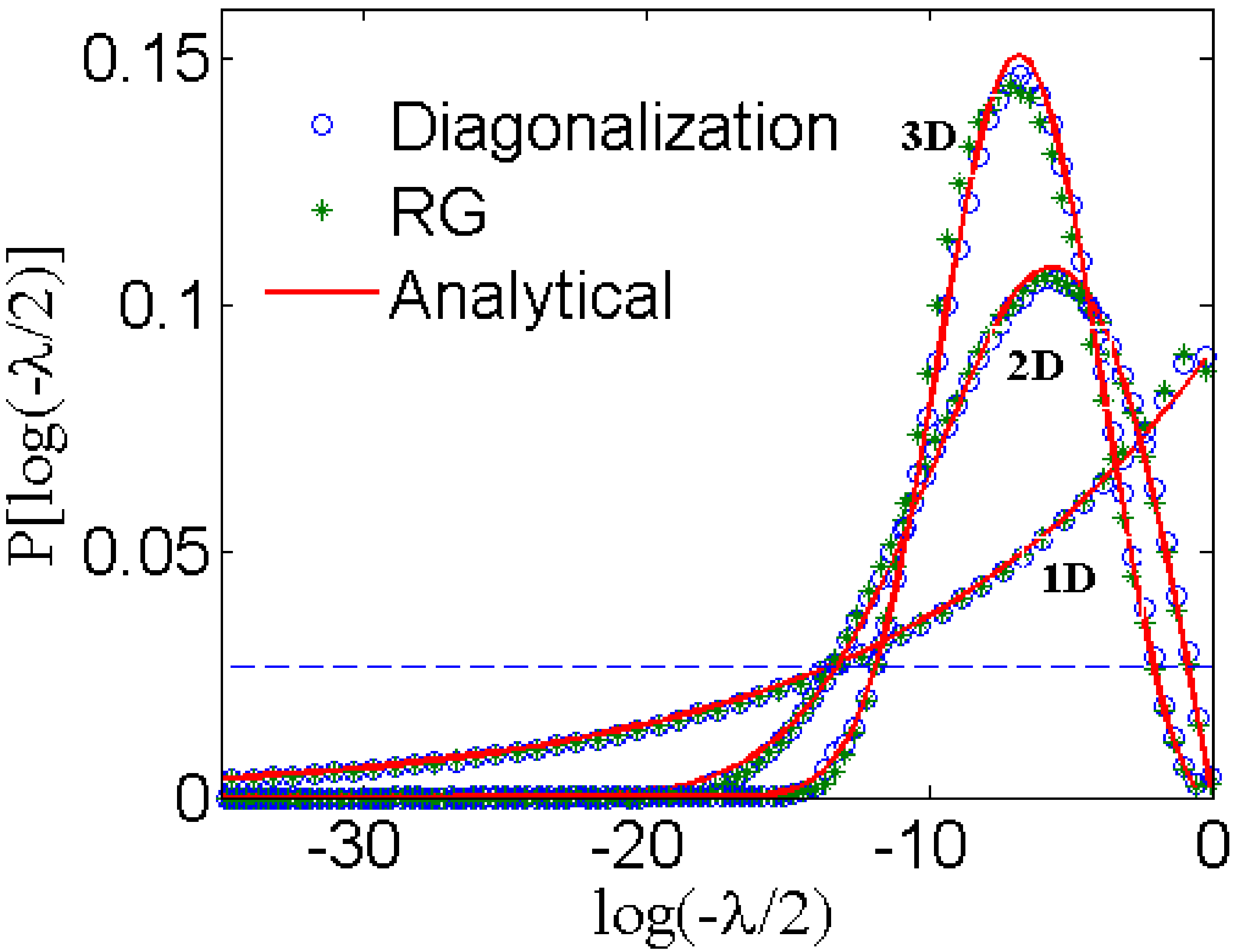


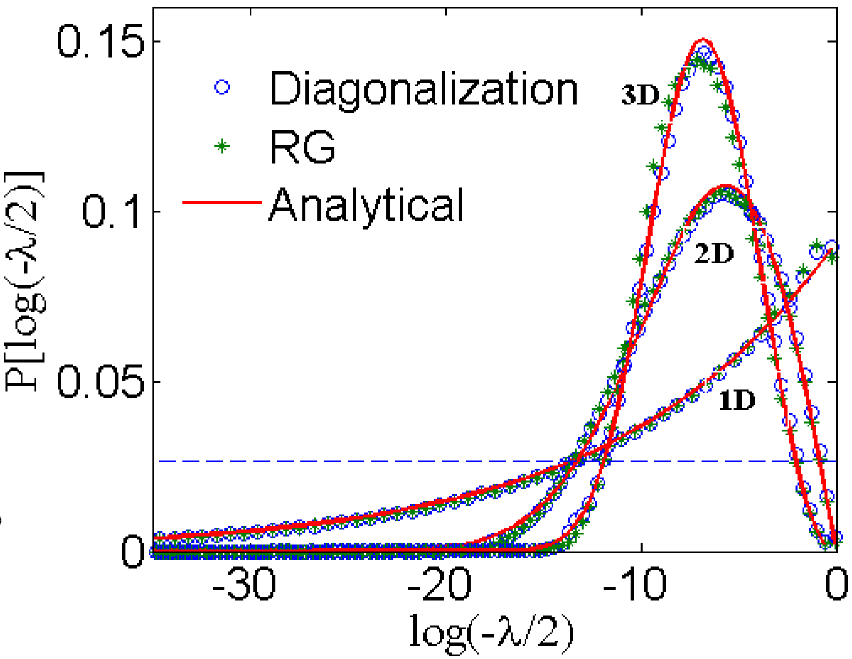
Exponential Distance Matrices- results

$$P(\lambda)=\frac{dC_d\varepsilon^d\log^{d-1}(\lambda/2)e^{-\frac{C_d}{2}\varepsilon^d\log^d(\lambda/2)}}{2\lambda}$$

(arbitrary dimension *d*)

$\varepsilon=\xi/\langle r\rangle,$ C_d*=volume of a d-dimensional sphere*

- Logarithmic corrections to $1/\lambda$
- In dimensions > 1: cutoff at $e^{-C/\varepsilon^{d/(d-1)}}$



Calculation of moments:

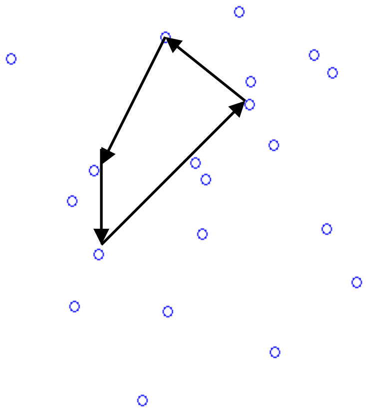
$$I_k=\int\lambda^kP(\lambda)d\lambda=\frac{1}{N}\left\langle A_{i_1,i_2}A_{i_2,i_3}\dots A_{i_k,i_1}\right\rangle$$

Amir, Oreg and Imry, **PRL (2010)**

Analytical approach in a nutshell

Part I – Moment calculation

$$I_k = \int \lambda^k P(\lambda) d\lambda = \frac{1}{N} \left\langle A_{i_1,i_2} A_{i_2,i_3} \dots A_{i_k,i_1} \right\rangle$$



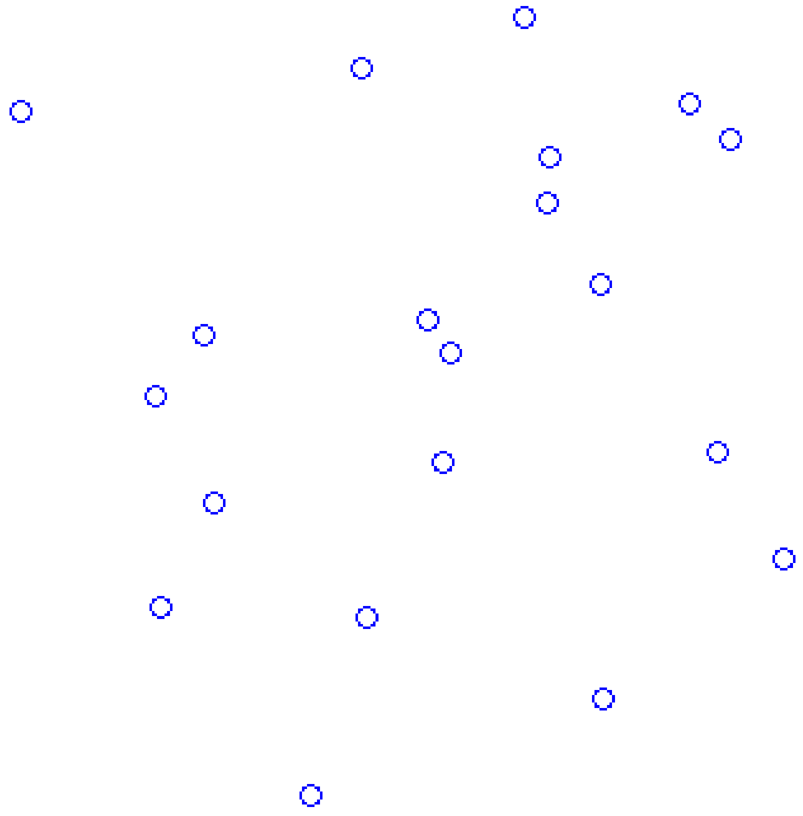
The *k*'th moment:

$$\mathrm{I_k} = 2^{\mathrm{k-1}} \mathrm{d!C_d} (\varepsilon/\mathrm{k})^{\mathrm{d}}$$

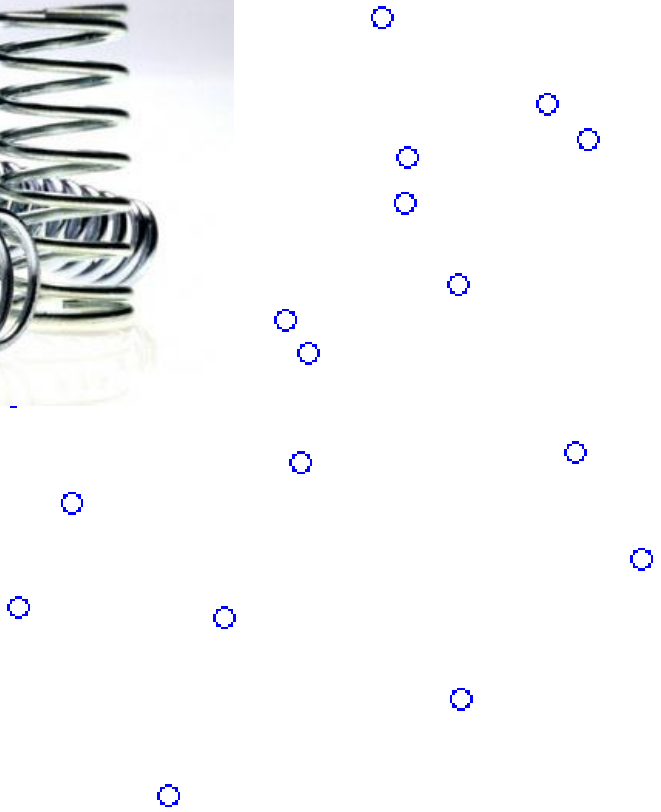
leads to the distribution function.

$$\boxed{P(\lambda) = \frac{d C_d \varepsilon^d \log^{d-1}(\lambda/2) e^{-\frac{C_d}{2}\varepsilon^d \log^d(\lambda/2)}}{2\lambda}}$$

Renormalization group approach

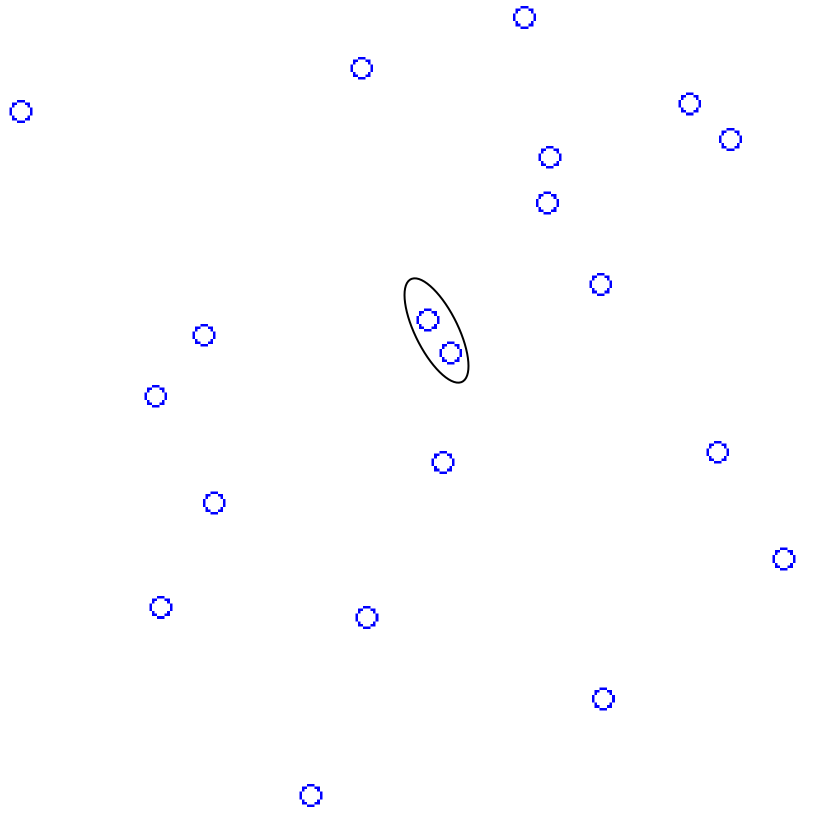


Renormalization group approach



Mechanical intuition: *network of masses and springs*

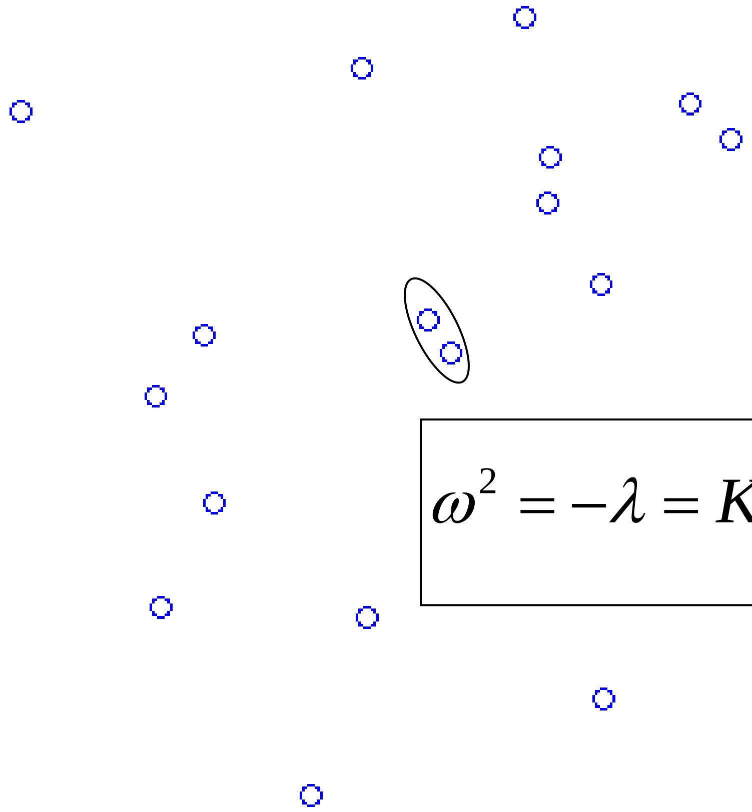
Renormalization group approach



Dasgupta and Ma, PRB 1980

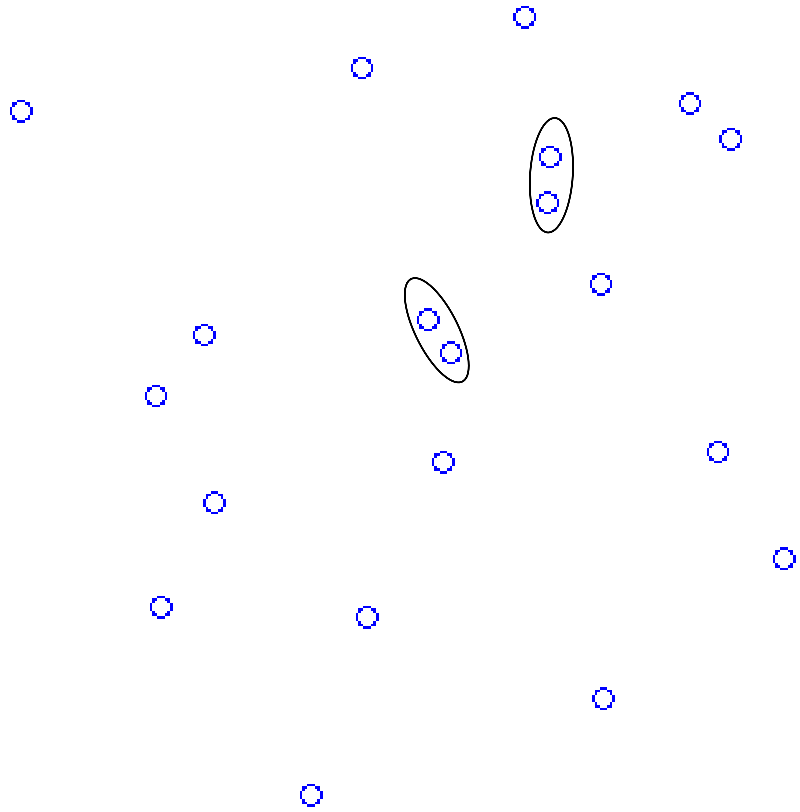
Fisher , PRL (1992) etc.

Renormalization group approach

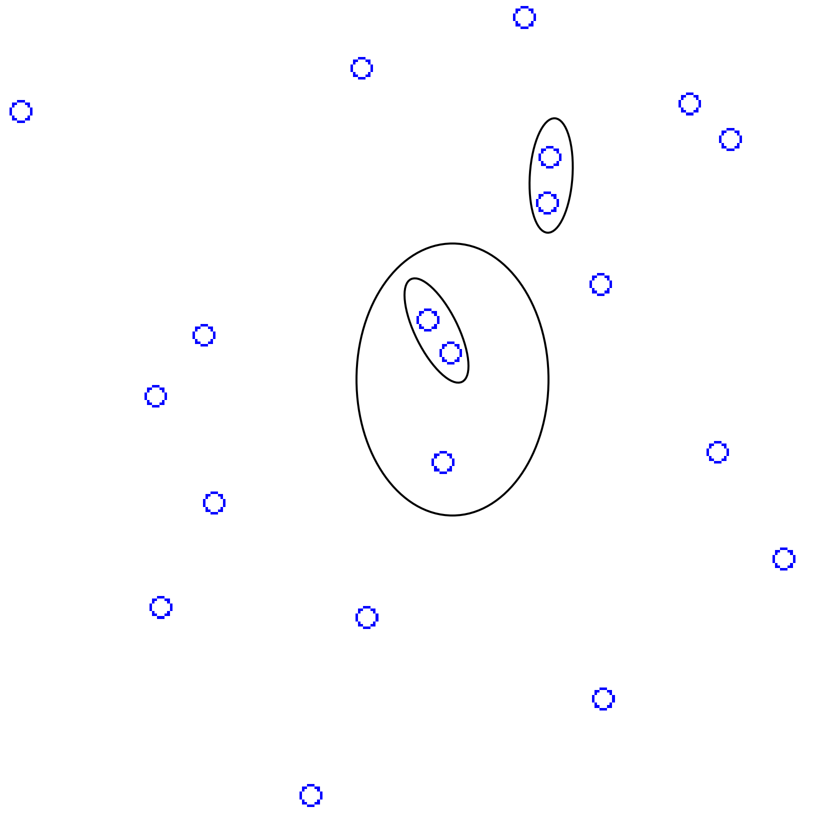


$$\boxed{\omega^2 = -\lambda = K/\mu \ , \ \frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}}$$

Renormalization group approach

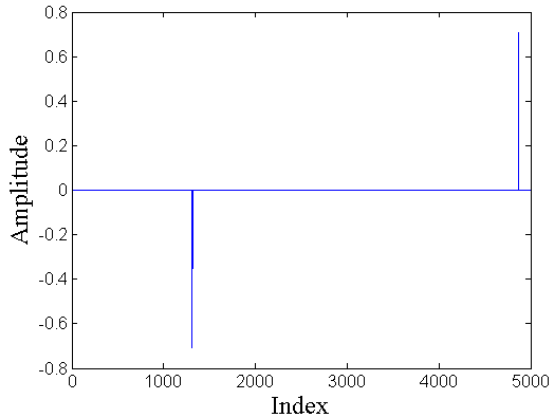


Renormalization group approach

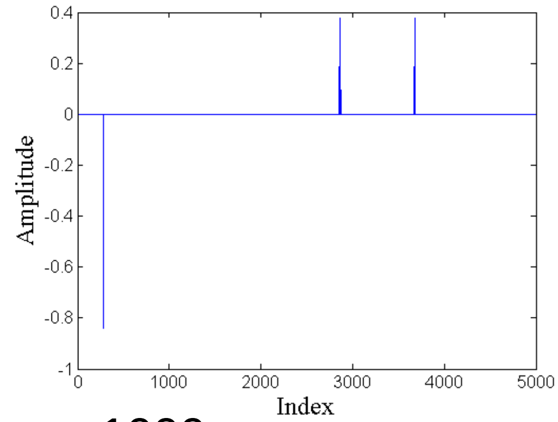


RG procedure yields growing clusters

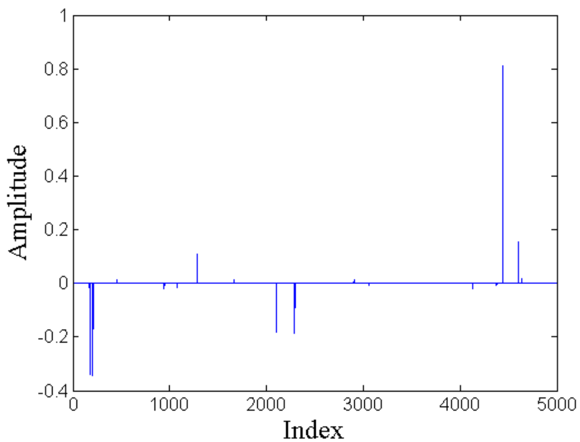
Structure of eigenmodes



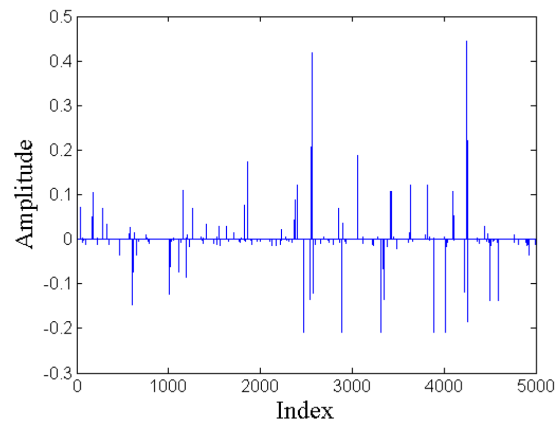
3'rd $\lambda \sim -1.86$



1000 $\lambda \sim -0.05$



3000 $\lambda \sim -9.6 \cdot 10^{-4}$



4000 $\lambda \sim -8.5 \cdot 10^{-5}$

Examples of eigenmodes of a 5000X5000 matrix

Renormalization group approach

Eigenvalue distribution + RG approach

$$\longrightarrow \quad n_c \sim e^{\frac{C_d}{2}\varepsilon^d |\log^d(-\lambda/2)|}$$

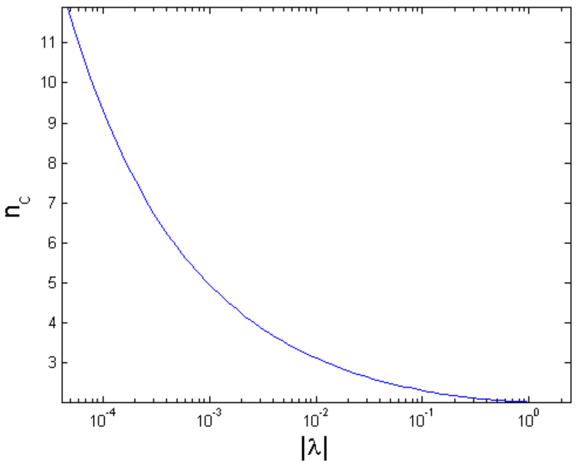
Number of points in a cluster of a given eigenvalue

Renormalization group approach

Eigenvalue distribution + RG approach

$$\longrightarrow \quad n_c \sim e^{\frac{C_d}{2}\varepsilon^d |\log^d(-\lambda/2)|}$$

Number of points in a cluster of a given eigenvalue



- Eigenmodes are localized clusters ("phonon localization")
- Size of clusters diverges at low frequencies

Amir, Oreg and Imry**,** *Localization, anomalous diffusion and slow relaxations: a random distance matrix approach***, PRL (2010)**

Electron glass aging– experimental protocol

Step I

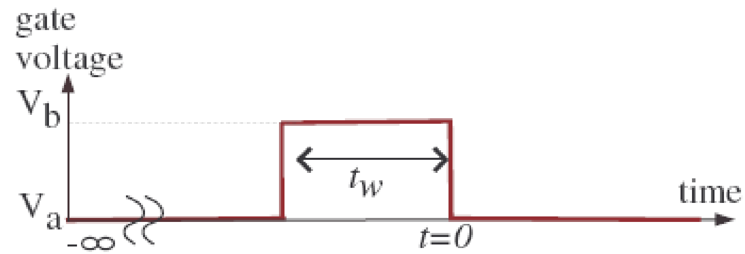
System equilibrates for long time

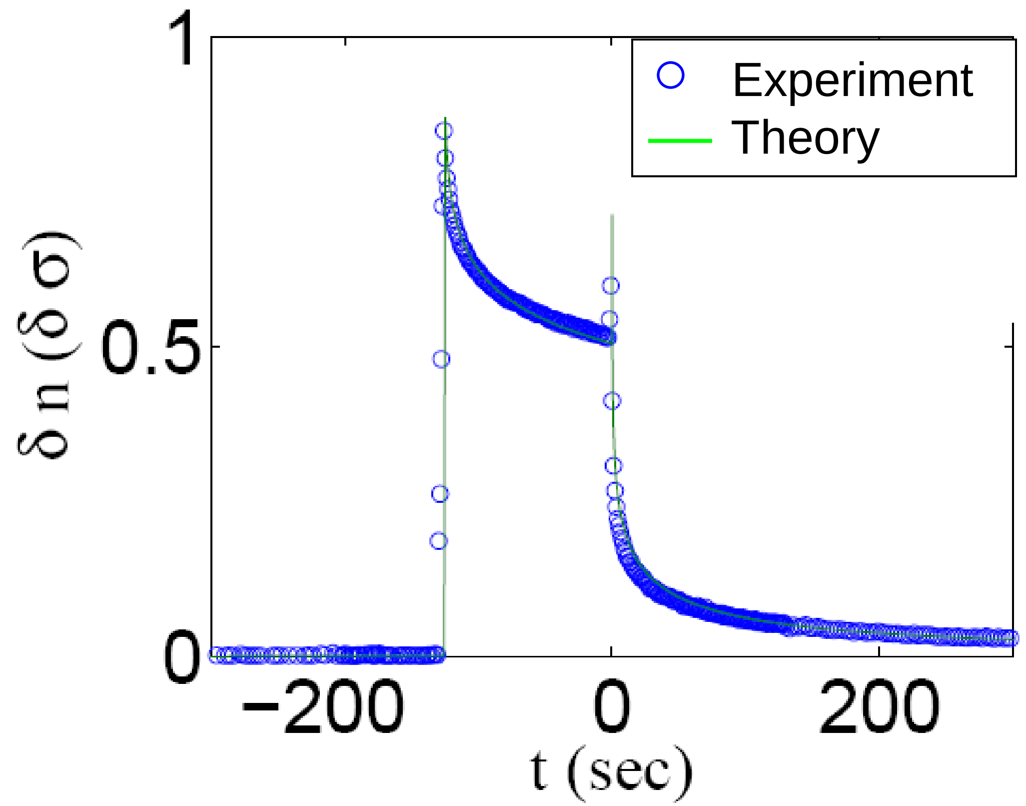
Step II

V_g is changed, for a time of t_w.

Throughout the experiment

Conductance is measured as a function of time.



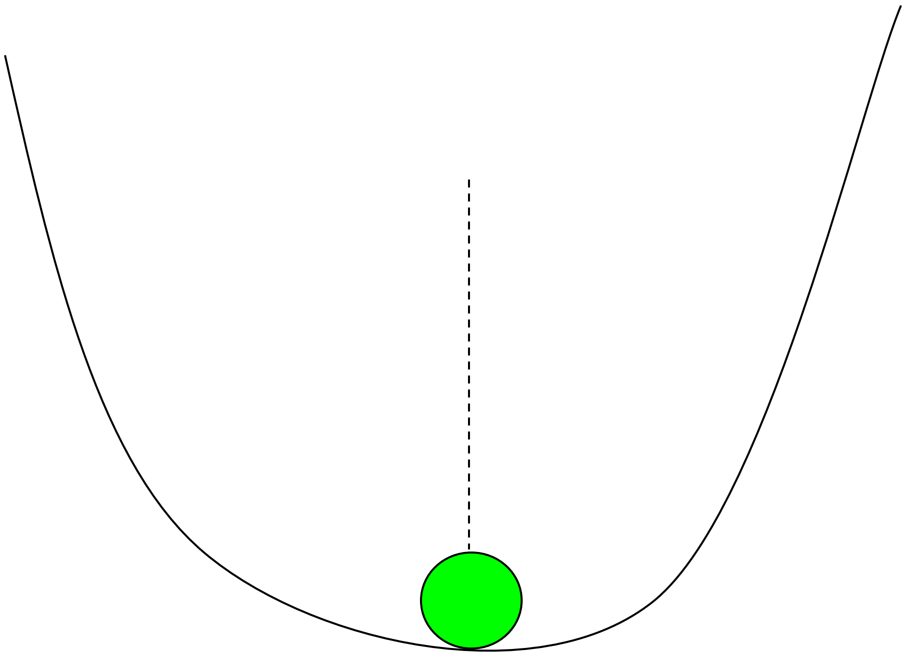


Data: *Ovadyahu et al.*

Aging – physical picture

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the repsonse?

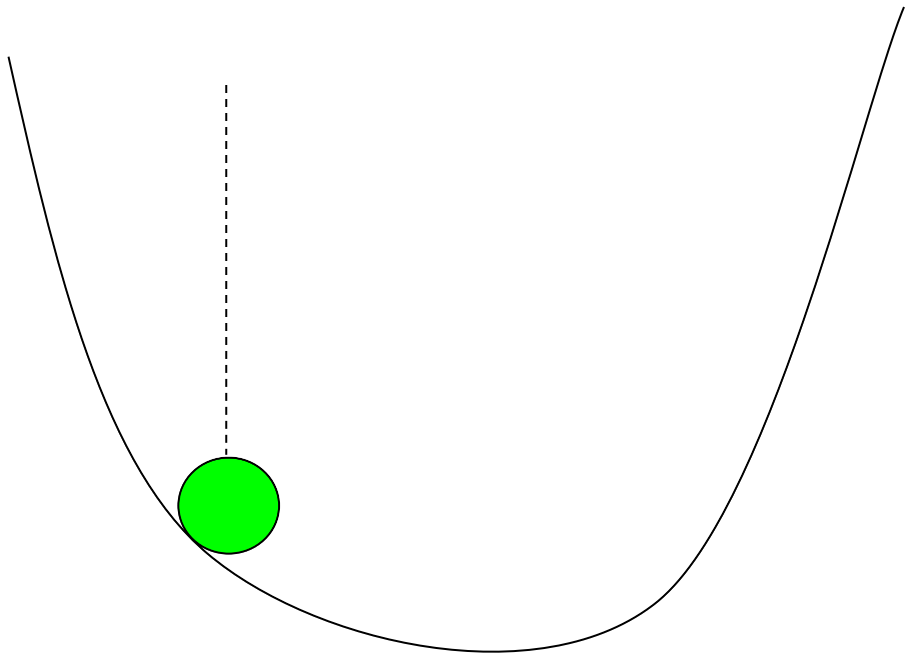


Initially, system is at some local minimum

Aging – physical picture

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the repsonse?



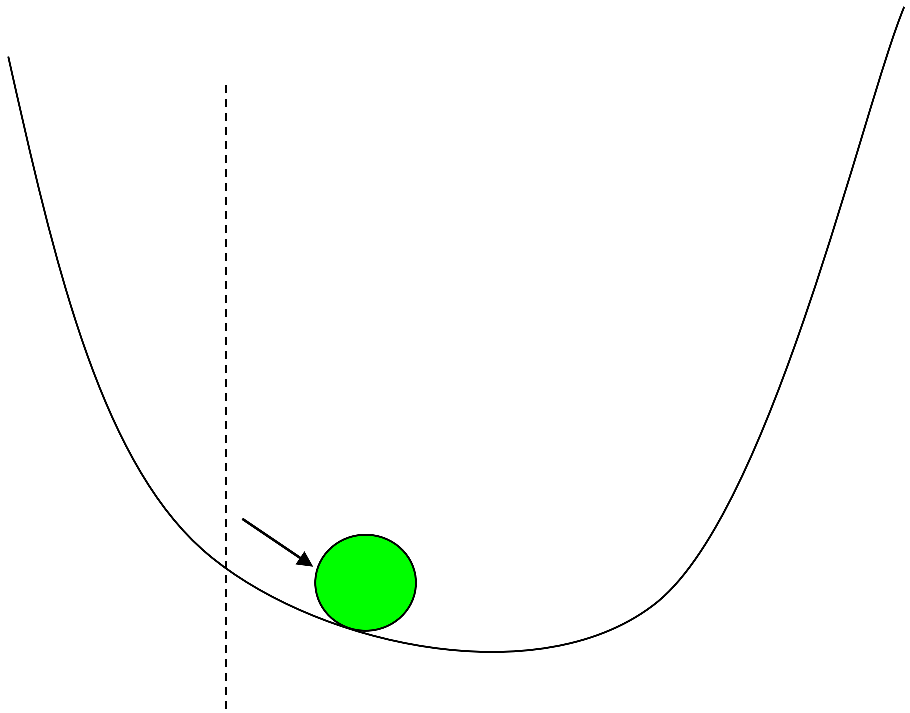
At time t=0 the potential changes,

and the system begins to roll towards the new minimum

Aging – physical picture

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the repsonse?

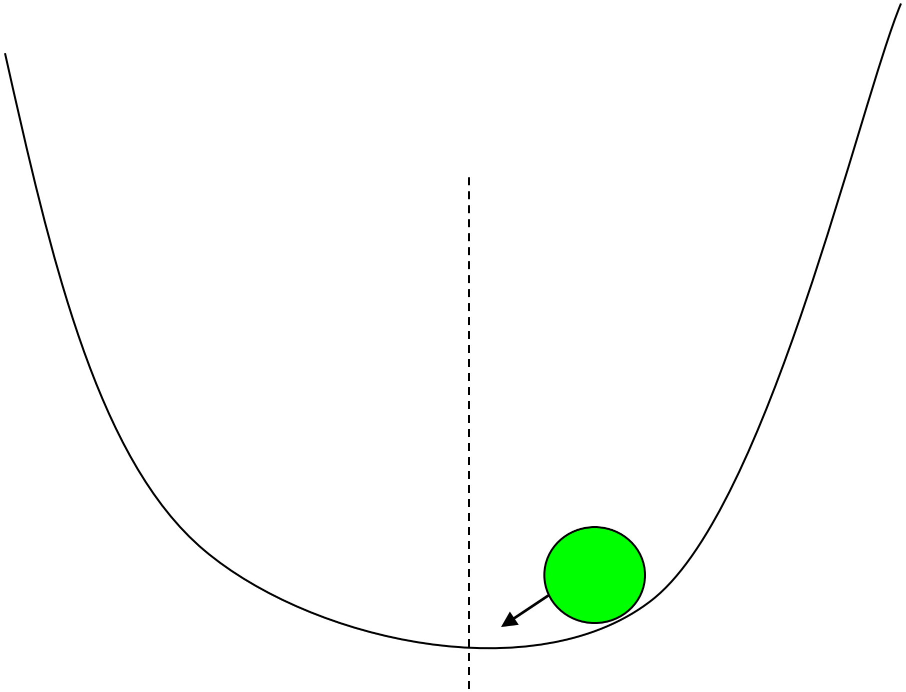


At time t_w the system reached some new configuration

Aging – physical picture

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the repsonse?



Now the potential is changed back to the initial form-

the particle is not in the minimum!

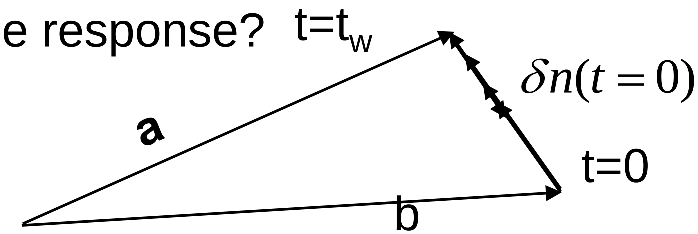
The longer t_w, the further it got away from it.

It will begin to roll down the hill.

Aging – Analysis

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the response?



Sketch of calculation

If a and b configurations are close enough in phase space:

$$\delta n(t=t_w) \sim \sum_{\text{eigenmodes}_\alpha} \chi_\alpha e^{-\lambda_\alpha t_w} \left|V_\alpha\right\rangle \Rightarrow \sum_{\text{eigenmodes}_\alpha} e^{-\lambda_\alpha t_w} =$$

modes are independent and contribute uniformly

$\boxed{\text{Logarithmic relaxation during step II}}$

Time t after the perturbation is switched off:

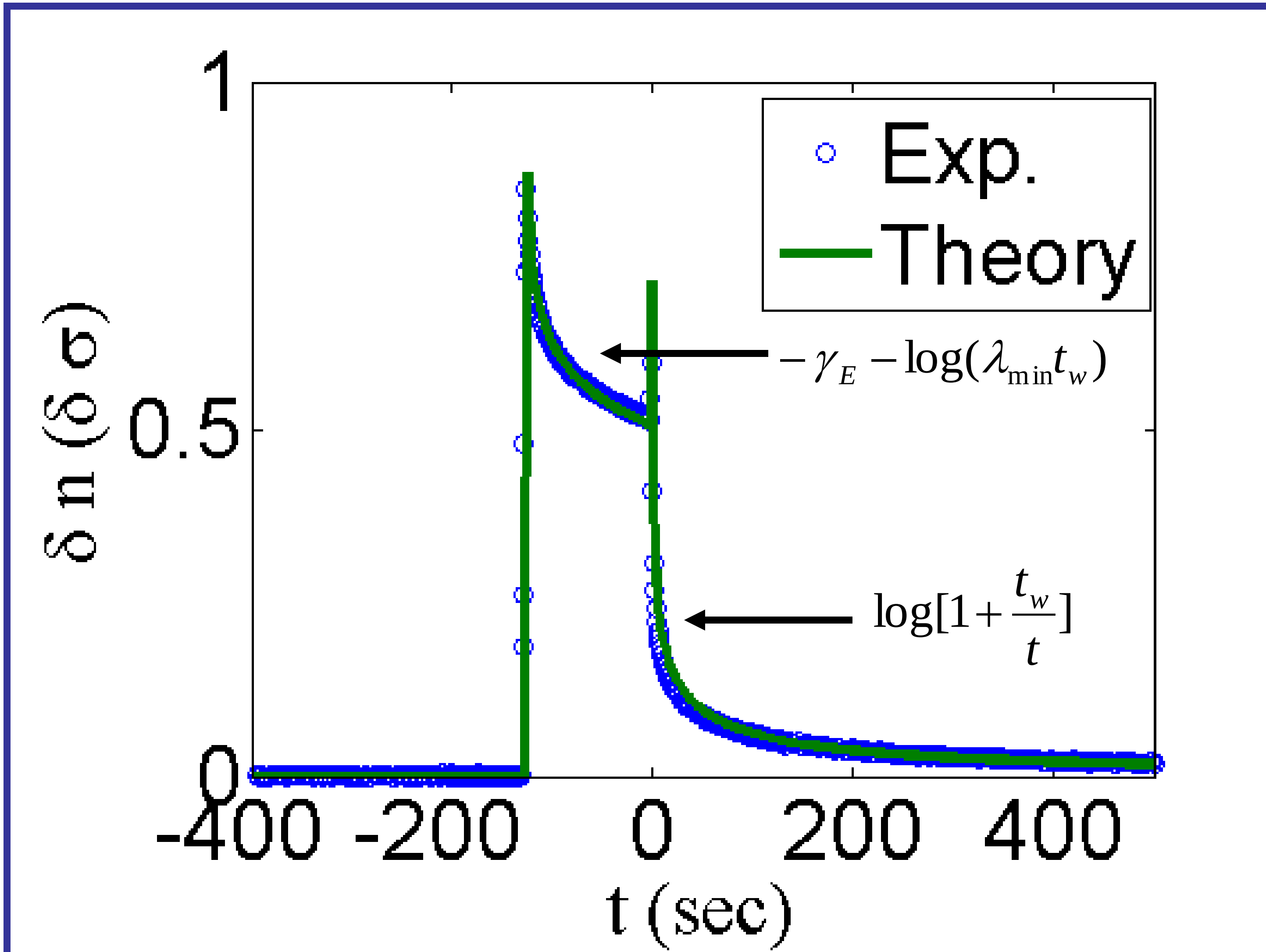
$$\delta n(t) \sim \sum_{eigenmodes\ \alpha} \chi_\alpha (1-e^{-\lambda_\alpha t_w}) e^{-\lambda_\alpha t} \, |V_\alpha> = f(t+t_w) - f(t)$$

$\boxed{\text{Full aging}}$

Only $1/\lambda$ ***distribution yields full aging!***

See also: T. Grenet et al. Eur. Phys. J B 56, 183 (2007)

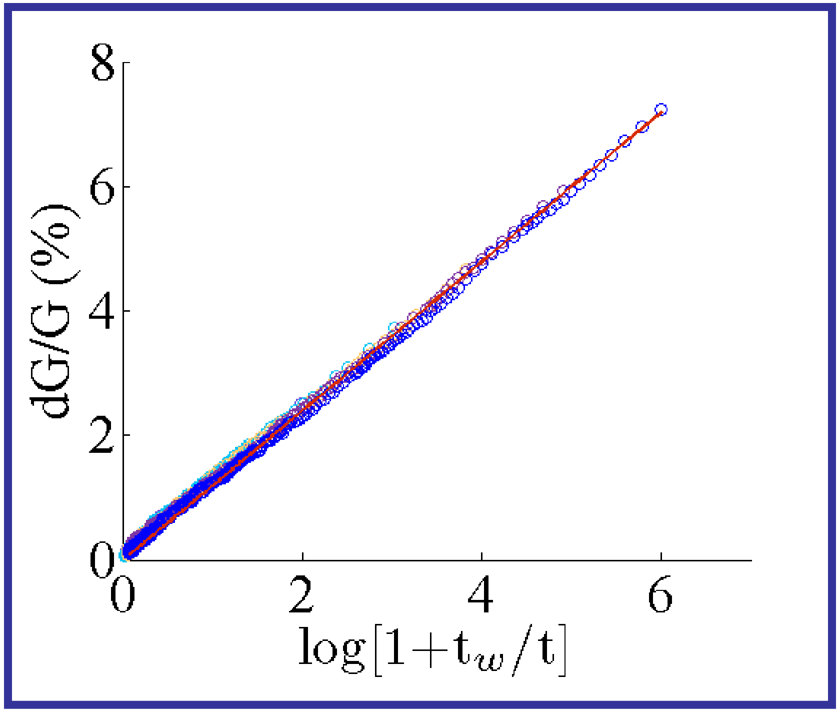
Aging Protocol - Results

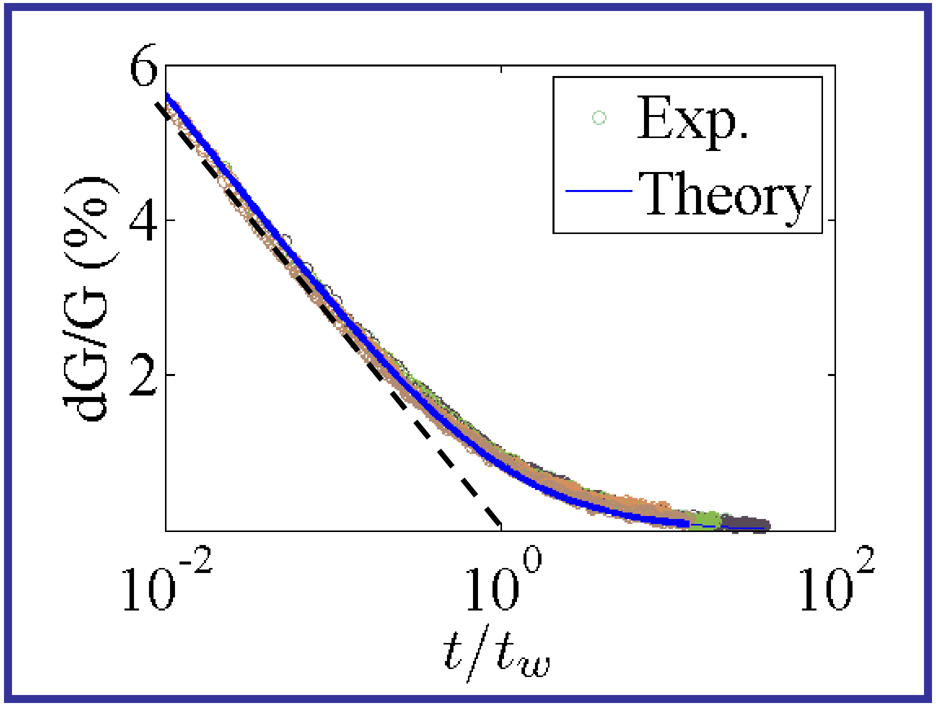


Amir, Oreg and Imry, PRL 2009

Detailed fit to experimental data

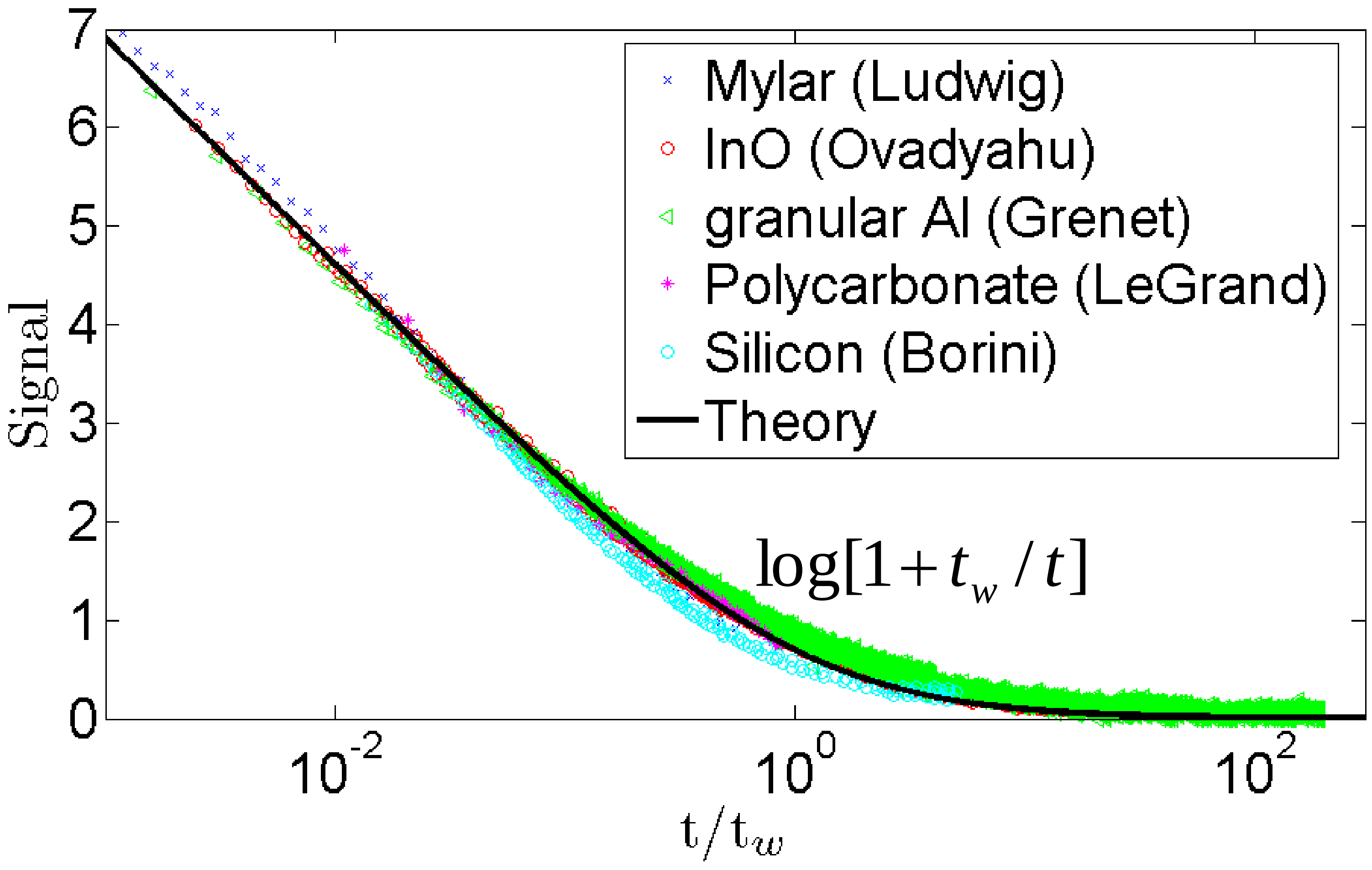
- Full aging
- Deviations from logarithm start at t / t_w





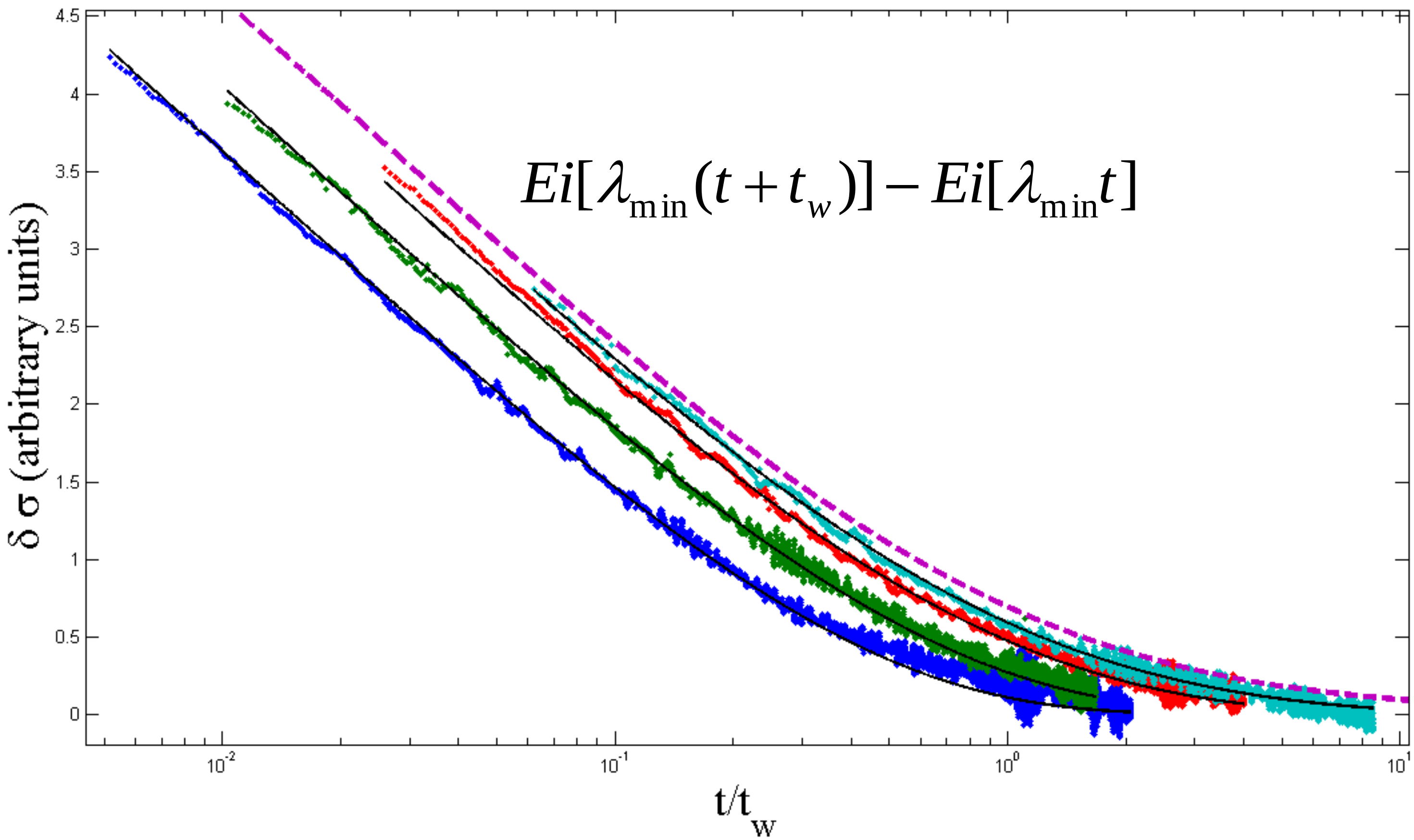
Amir, Oreg and Imry, **PRL** (2009)

Full aging and universality



Amir, Oreg and Imry, to be published

Deviations from full aging



Porous Silicon data (S. Borini)

Connection to 1/f noise?

Amir, Oreg, and Imry
arXiv:0911.5060, Ann. Phys. 2009

Langevin Noise $\longrightarrow \dfrac{d\delta\vec{n}}{dt} = A\cdot\delta\vec{n} + \vec{f}$

Equipartition theorem: each eigenmode should get $\langle E\rangle = kT/2$

The mean-field equations can be derived from a free energy:

$$F = \sum_i \varepsilon_i n_i + \sum_{i\neq j} e^2 \frac{n_i n_j}{r_{ij}} + \sum_i n_i \log n_i + (1-n_i)\log(1-n_i) + \mu N$$

From this we can find the noise correlations matrix:

$$\langle f_i f_j\rangle = -A\cdot\beta^{-1}, \beta^{-1}{}_{ij} = \delta_{ij} n_i^0 (1-n_i^0)$$

The $1/\lambda$ spectrum then leads to a 1/f noise spectrum:

$$\langle \delta n^2\rangle_f = \frac{1}{N}\sum_\lambda \frac{1/\lambda}{1+\left(\omega/\lambda\right)^2} \longrightarrow 1/f$$

B.I. Shklovskiı, Solid State Commun (1980)

K. Shtengel et al., PRB (2003)

Conclusions

• Statics: Coulomb gap, Steady-state: Variable Range Hopping

• Dynamics near locally stable point: many slow *localized* modes, $\sim 1/\lambda$ distribution.

How universal? **We believe: a very relevant RMT class.**

• One obtains full aging, with relaxation approximately of the form :

$$\delta\sigma \sim \log[1+\frac{t_w}{t}]$$

More details:

Phys. Rev. B 77, 1, 2008 (local mean-field model)

Phys. Rev. Lett. 103, 126403 (2009) (aging properties)

Phys. Rev. B 80, 245214 2009 (variable-range hopping)

Ann. Phys. 18, 12, 836 (2009) (*1/f* noise)

Phys. Rev. Lett. 105, 070601 (2010) (exponential matrices – solution)

Electron glass dynamics – Review (soon online)