

Non Linear Conductivity in Coulomb Glasses

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KITP Program: The Electron Glass

- Introduction. Experiments on nonlinear conductivity
- Model and numerical techniques
- Effective temperature
- Non linear conductivity. Hot electron model
- Dissipated power in hopping systems
- Switching transition
- Open questions

Mainly Monte Carlo simulations of 2D Coulomb glasses

Non linear effects in hopping

Field effect models

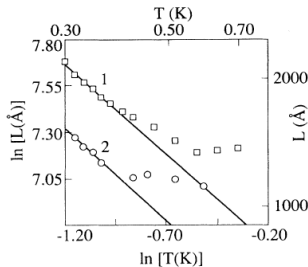
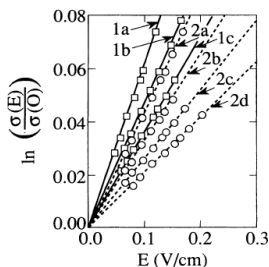
Field effects models are predictions for noninteracting systems (trivially extended to interacting systems)

$$\sigma(T, E) = \sigma(T, 0) \exp \{ eEl_h/kT \}$$

where l_h is a characteristic hopping length.

- Hill and Pollak and Riess $l_h \propto T^{-1/2}$
- Shklovskii $l_h \propto T^{-1}$

(Grannan et al, 92)



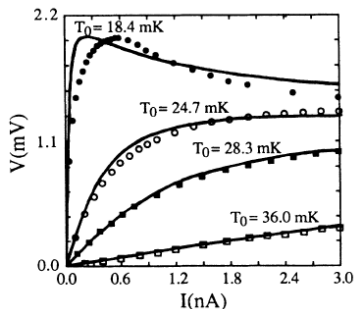
Non linear effects in hopping

Hot electron model

Wang et al (90) interpreted results on NTD Ge in terms of a hot electron model:

$$\sigma(T, E) = \sigma(T_{\text{eff}}, 0) \propto \exp\{- (T_0/T_{\text{eff}})^{1/2}\}$$

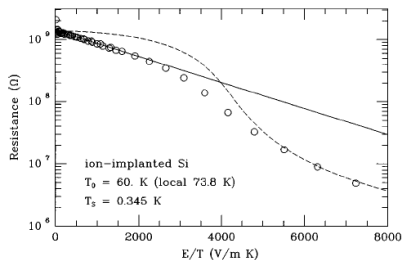
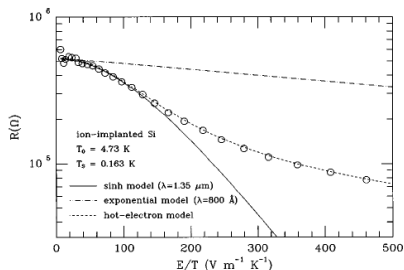
$$P = a(T_{\text{eff}}^\beta - T^\beta)$$



Non linear effects in hopping

Zhang et al (98) studied systematically non linear effects in doped Si and Ge and concluded that

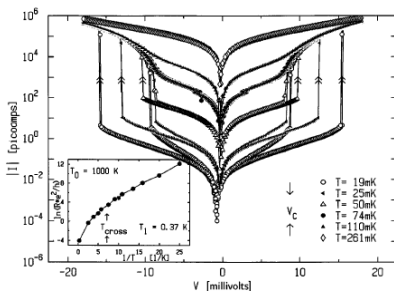
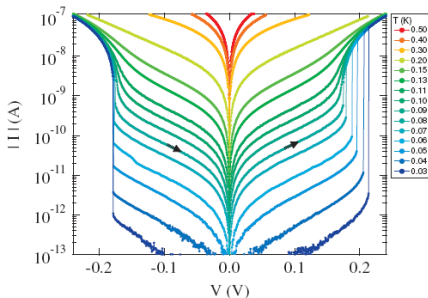
- If $T_0/T < 135$ hot-electron models provide a better fit
- If $T_0/T > 135$ field effect models are better



Switching transition

Hot electron model

Altshuler et al (09) interpreted the switching transition observed in InO by Ovadia et al (09) as a bistability of the effective temperature. Ladieu et al (96) had observed a similar effect in a-YSi.



In both experiments the conduction is activated.

Coulomb gap

Hamiltonian

- Disorder and long range interactions

$$H = \sum_i \phi_i n_i + \sum_{i>j} \sum_j \frac{(n_i - K)(n_j - K)}{r_{ij}}$$

- $n_i = \{0, 1\}$ occupation number (very large U).
- $\phi_i \in [-W/2, W/2]$ random site energy.
- K compensation.
- Transfer energies considered in lowest possible order.
 - 0 for energies, density of states, etc.
 - 1 for conduction, relaxation, etc.
 - No exchange terms. Degenerate with respect to spin.



Coulomb gap

Density of states

- Site energies are defined as

$$\epsilon_i = \phi_i + \sum_{j \neq i} \frac{(n_j - K)}{r_{i,j}}$$

- The excitation energies are

$$\Delta_{i,j} = \epsilon_j - \epsilon_i - \frac{1}{r_{i,j}}$$

- The stability of the ground state requires

$$\Delta_{i,j} \leq 0 \quad \implies \quad r_{\min} \propto |E|^{-1}$$

Density of states $N(E) \propto |E|^{d-1}$

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- We apply an electric field

$$H = H_{CG} - \sum_i x_i E n_i$$

- Periodic boundary conditions
 - Interaction: minimum image distance.
 - Field included in the transition energies. Stationary state.
 - Field not included in the energy.
- One-electron transition rates

$$\Gamma_{i,j} = \tau_0^{-1} e^{-2r_{i,j}/\xi} \min\{1, e^{-\Delta E_{i,j}/kT}\}$$

- Many-electron transition rates

$$\Gamma_{\alpha,\beta} = \tau_0^{-1} \Xi_{\alpha,\beta} e^{-2\sum_{i,j} r_{i,j}/\xi} \min\{1, e^{-\Delta E_{\alpha,\beta}/kT}\}$$

- Shortest possible transition
- For two electrons $\Xi_{\alpha,\beta} = \frac{1}{r_{1,3}} + \frac{1}{r_{2,4}} - \frac{1}{r_{1,4}} - \frac{1}{r_{2,3}}$

Monte Carlo simulation

- We choose a pair i, j with probability $e^{-2r_{i,j}/\xi} / \sum_{i,j} e^{-2r_{i,j}/\xi}$ and then accept or reject the transition according to its energy.
- One-electron jumps.
- High temperature regime.
- Joakim is implementing relevant two-electron transitions in a 2D lattice.

Master equation

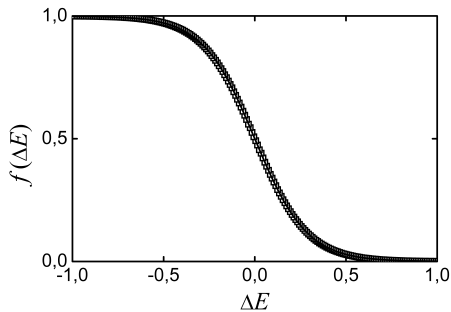
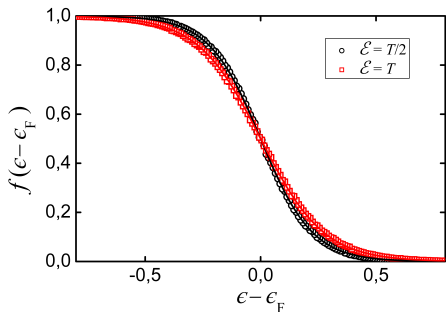
- We first obtain low-energy many-particle configurations and then calculate transition rates between them.
- Many-particle transitions are easily included.
- Very low values of E, T .

Effective temperature

Distribution function

- Site energies near E_F follow FD statistics with a T_{eff} .
- Long excitations at T_{eff} .
- Short excitations at T .

$$\frac{P(E, r > 10)}{P(E, r > 10) + P(-E, r > 10)}$$



Effective temperature

Fluctuation dissipation theorem

Extension of the fluctuation dissipation theorem:

Spin glass (Jorge, Leticia, etc.)

External perturbation

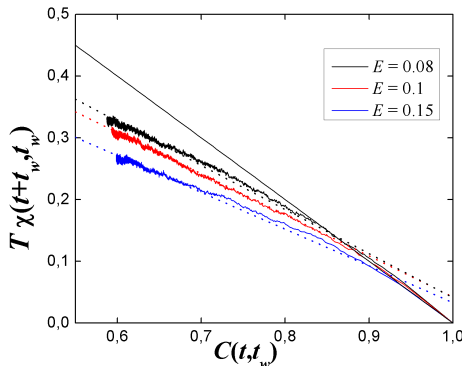
$$\delta\phi_i = \lambda(t)\varphi_i$$

Response $\delta n(t) = \lambda\chi(t, t_w)$

At equilibrium $T\chi(t) = 1 - C(t)$
where

$$C(t, t_w) = \frac{4}{N} \sum_i \langle \delta n_i(t) \delta n_i(t_w) \rangle$$

These results are compatible with our procedure.



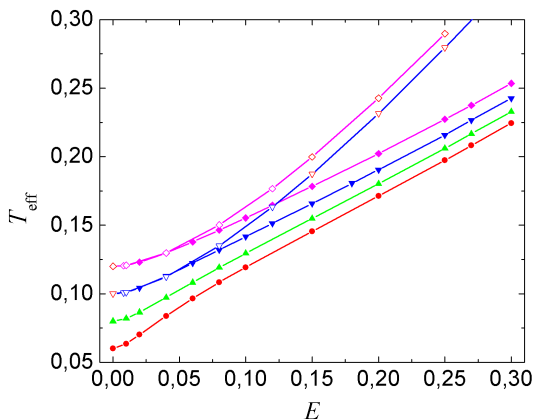
Effective temperature

E and T dependence

Effective temperature, obtained from long excitations probabilities, as a function of E and T

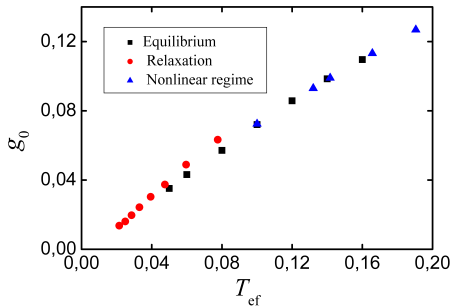
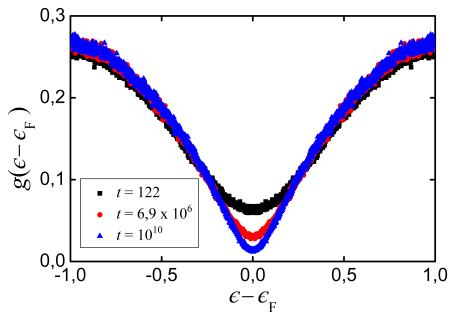
At low T , results not compatible with

$$T_{\text{eff}}^2 - T^2 = aE^2$$



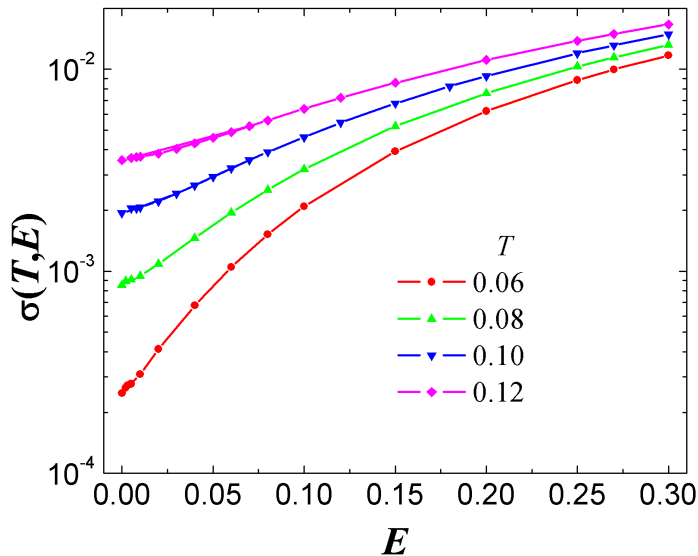
Effective temperature

Density of states



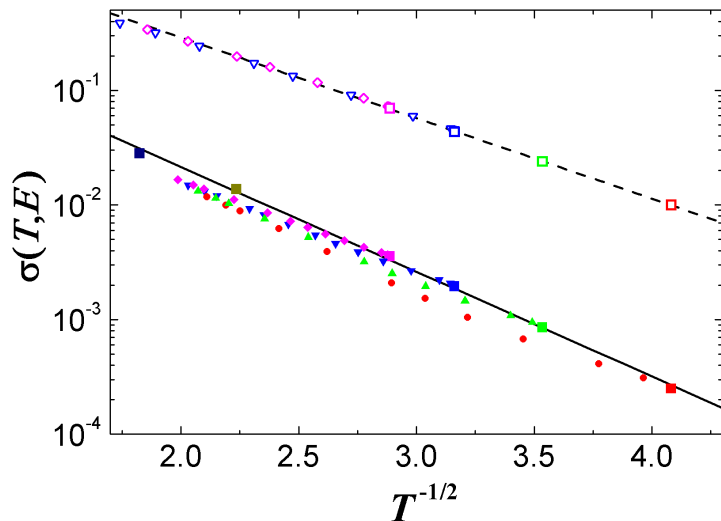
Non linear effects

Conductivity



Non linear effects

Variable range hopping



Emitted and absorbed powers

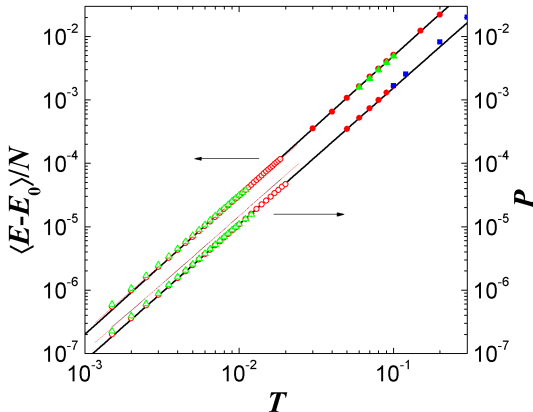
Equilibrium

At equilibrium

$$\langle E - E_0 \rangle \propto T^2$$

Emitted and absorbed powers also proportional to T^2

Contribution of short dipoles

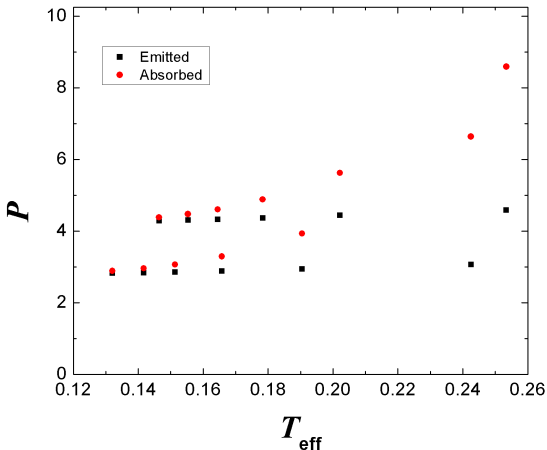


Emitted and absorbed powers

With electric field

Large common T
dependent contribution

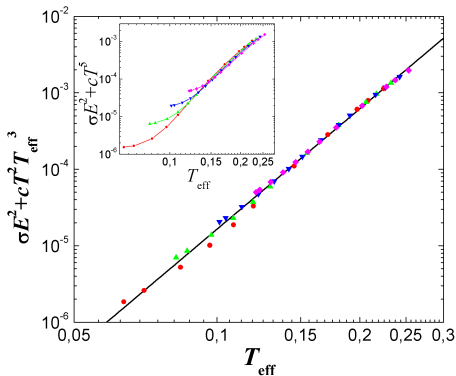
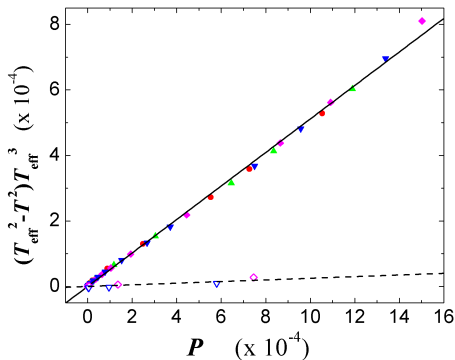
The emitted power
depends weakly on T



Matteo obtains the full range with exchange Monte Carlo (equilibrium). 

Hot electron model

Dissipated power vs T_{eff}

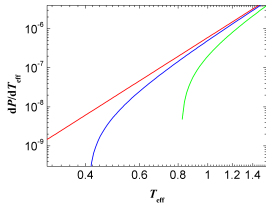
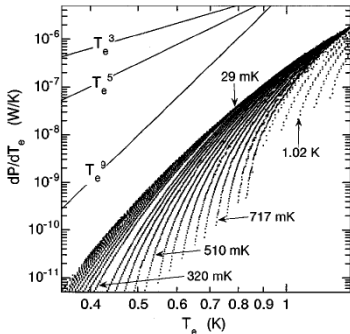
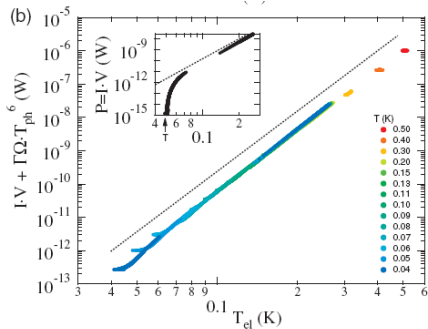


Dissipated power in the hopping regime

$$P = c (T_{\text{eff}}^\alpha - T^\alpha) T_{\text{eff}}^{\beta - \alpha}$$

Hot electron model

Low P correction



Hot electron model

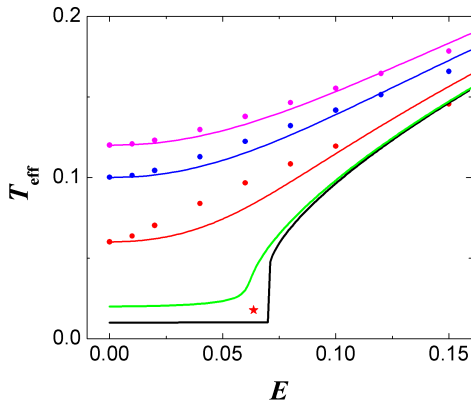
Effective temperature

$$E^2 = ce\sqrt{T_0/T_{\text{eff}}}(T_{\text{eff}}^2 - T^2)T_{\text{eff}}^3$$

$$\frac{dE^2}{dT_{\text{eff}}} = 0$$

$$\frac{d^2E^2}{dT_{\text{eff}}^2} = 0$$

$$\Rightarrow T_c = \frac{T_0}{144\sqrt{3}} \quad T_{\text{eff}} = \frac{T_0}{144}$$



Open questions

- Mechanism of thermalization of the effective temperature (relaxation)
- Explanation of the expression for the dissipated power in hopping systems
- Switching transition versus Glassy behavior.
Is activated conduction a necessary condition for the switching bistability?
- **Role of many-electron transitions in VRH (in both linear and non linear regimes)**. Many particles per localization volume.