ELECTRON GLASS: Experiments, Models and Questions

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KITP, Electroglass10

A common type of experiment in glasses:
Excite the system and measure the relaxation by measuring the time dependence of some suitable response function.

In electron glasses
Excitation done by change of gate voltage, by application of high electric field, by electromagnetic radiation (infra red).

The response function used is conductance – it responds to excitat the conductance is higher in an excited state.

Results (groups of Ovadyahu, Grenet, Frydman and others):
The relaxation from an excited state is extremely slow, logarithmic in time, typically with a decay of several percent per decade of time. (A good measure of decay rate is τ – time of decay to half amplitude). The log t dependence is observed from ~ 1sec for durations of a day or so. THE GLASS IS NON-ERGODIC.

The ergodic condition

$$< D >_{\text{time}} = (1/\tau)\int_0^{\tau} D(t)\mathrm{d}t =$$

$$= < D >_{\text{ensemble}} = \int_{\Omega} D(\xi)\, p(\xi) d\xi =$$

D – Any dynamical variable (or operator)

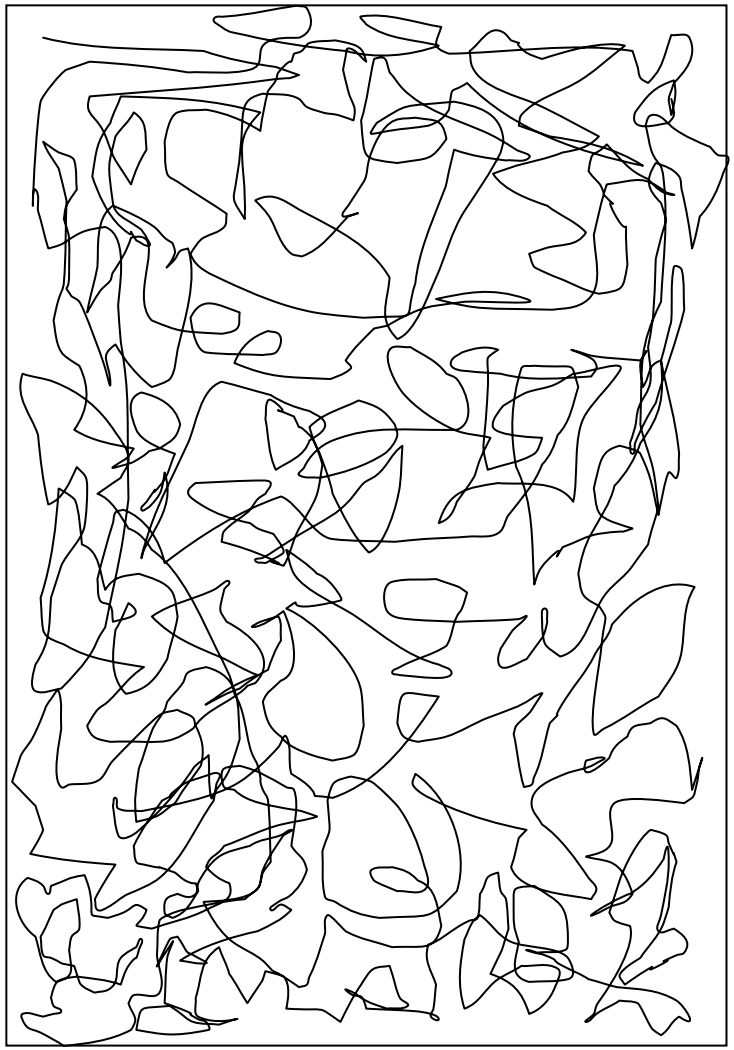
ξ - microscopic state (point in phase space or state in Hilbert space).

$p(\xi)$ – Ensemble probability that system is in state ξ

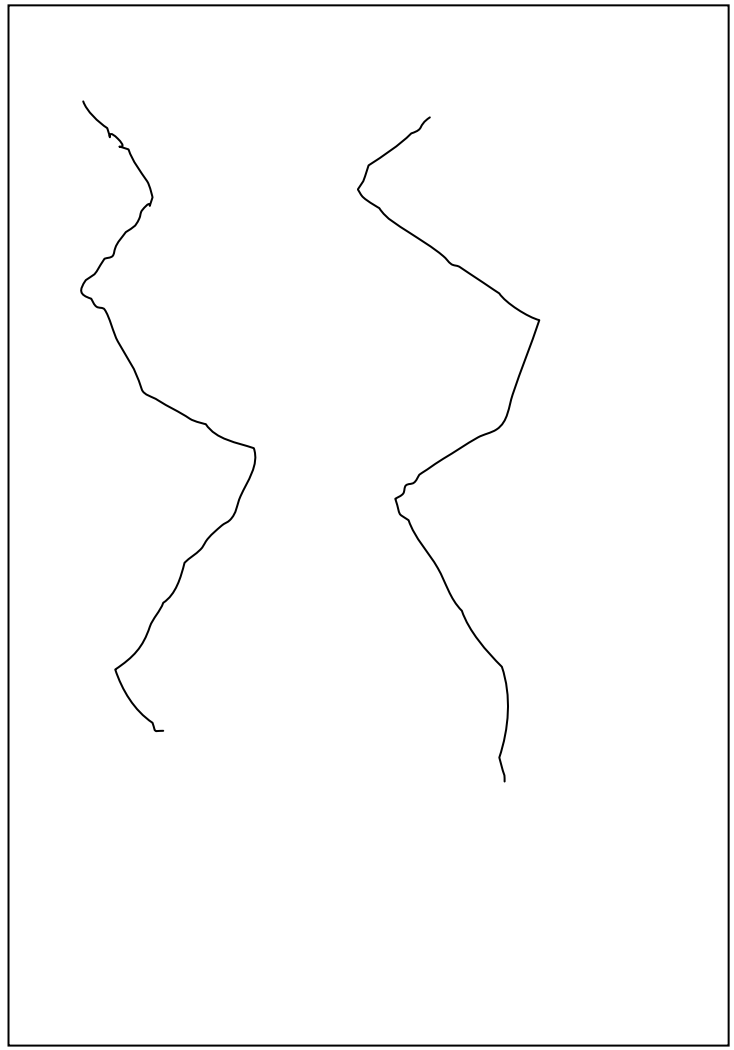
In theory $\tau \rightarrow \infty$,

More practical is to make τ > a very long experimental time

A necessary condition for ergodicity: rapid transitions between states, so the system can visit all relevant states during experimental time (e.g. all states at a fixed energy for an isolated system).



ergodic
equilibrium



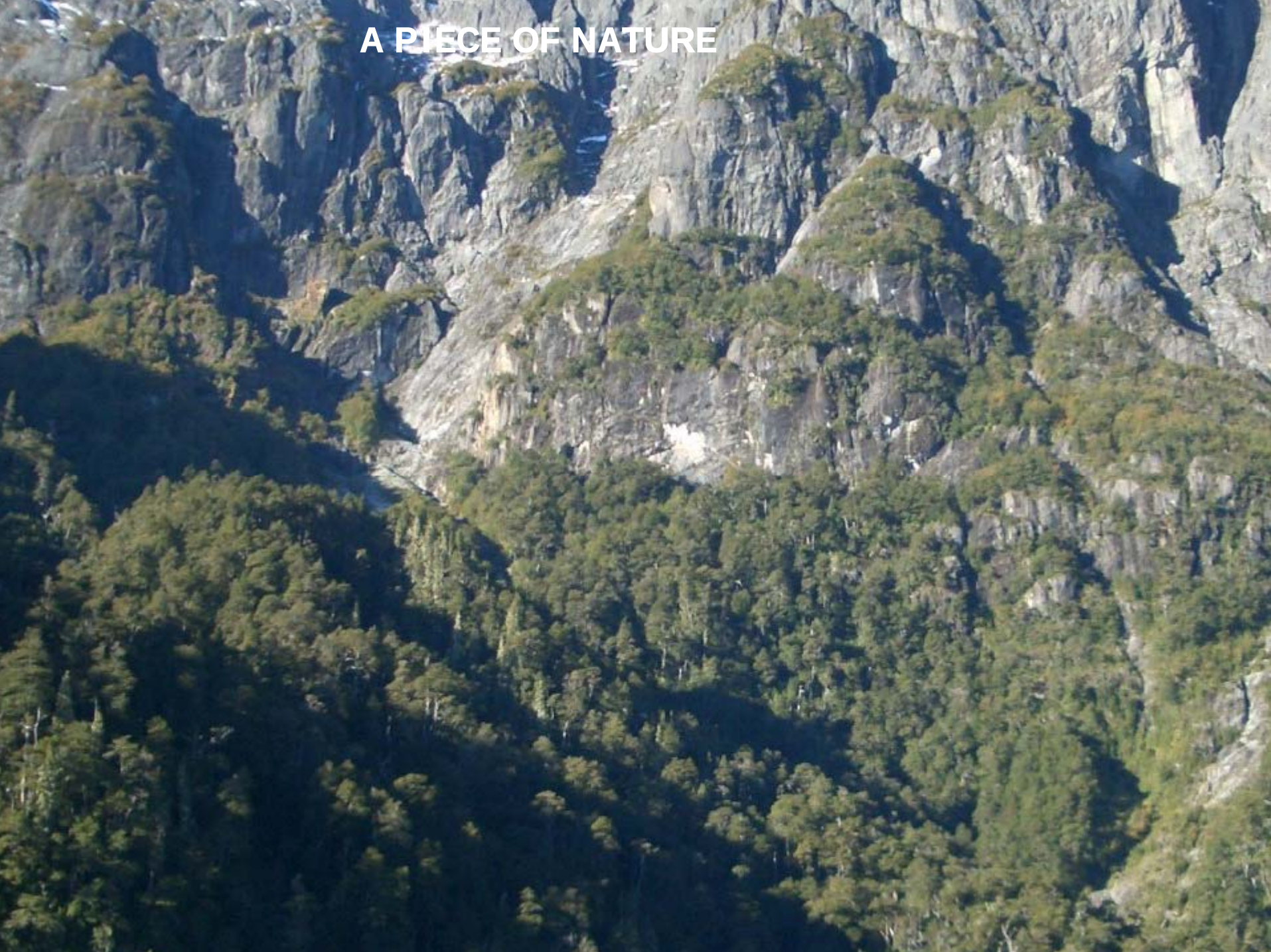
non-ergodic – no decay to

Violation of time homogeneity.

Results of a measurement depend only on $(t-t_0)$ where t_0 is the time the experiment started.

The response of the system depends on its internal state which in the glass changes extremely slowly. So measurement at t depends also on the state of the system at time t_o.

Most solids in nature are non-ergodic.


A PIECE OF NATURE

How can the very light electrons form a non-ergodic glass?

1. **Disorder - Anderson localization.**

   the motion of the electrons is reduced to hopping.

2. **Further slowing of motion - Coulomb interactions.**

The DOS near the Fermi level is dramatically reduced (Coulomb gap).

Single particle transport suppressed.

Many-particle transitions not inhibited by Coulomb gap but much slowed down by collective motion.

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Quantum effects ought to be important (in distinction to many other glasses)

Nevertheless, there exist infidels who still don't believe in the existence of intrinsic electron glasses but think of extrinsic effects to account for the glassy behavior, i.e that the observed glassy effects are due to some other entity (e.g. ions) that couple to the electrons.

There is good evidence that at least a number of electron glasses are intrinsic – certain glassy properties depend entirely on the electron concentration n (e.g, the relaxation rate).

THE "STANDARD" MODEL HAMILTONIAN

$$H = \sum_i \varepsilon_i n_i + \sum_{i,j} (e^2 / \kappa r_{ij})(n_i - K)(n_j - K)$$

energy of an electron on site *i*:

$$E_i = \varepsilon_i + \Sigma_j (e^2/\kappa r_{ij})(n_j - K)$$

K is often taken to be ½.

THE MAXIMUM OCCUPATION OF A SITE IS ONE ELECTRON
(due to large inter-site Coulomb interaction)

ALL LOCALIZED STATES ARE IDENTICAL.

NOTICE THAT THE MODEL HAMILTONIAN IS CLASSICAL

TRANSITION RATES BETWEEN SITES

$$w_{ij} = w_0 \exp(-E_{ij} / kT) \exp(-2r_{ij} / \xi)$$

r_{ij} – distance between sites i, j ; $E_{ij} = 0$ for downward hop; >0 for upward hop

Quantum mechanics enters in the lowest order – the golden rule

TRANSITION RATES BETWEEN CONFIGURATIONS

$$w_{IJ} = w_0 g^{n-1} \exp(-E_m / kT) \exp(-\Sigma_{shortest} 2r / \xi)$$

g – a factor that depends on the strength of interaction
n – the number of electrons changing position
E_m – $\max(E_i, E_j)$

M. Pollak and Z. Ovadyahu, Phys. Status Solidi C 2003

A QUASI-PARTICLE MODEL

Two crucial ingredients: Interaction and disorder

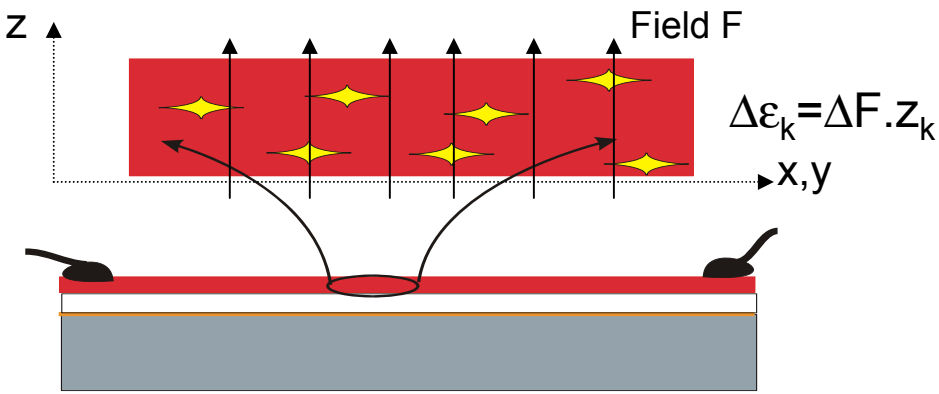
Interaction causes formation of quasi-particles that reduce the energy of the system.
Disorder causes their formation to be slow – formation is by (possibly collective) hopping

Decay of conduction is due to increase in correlations as quasiparticles form.

Let's think for example of excitation by gate voltage

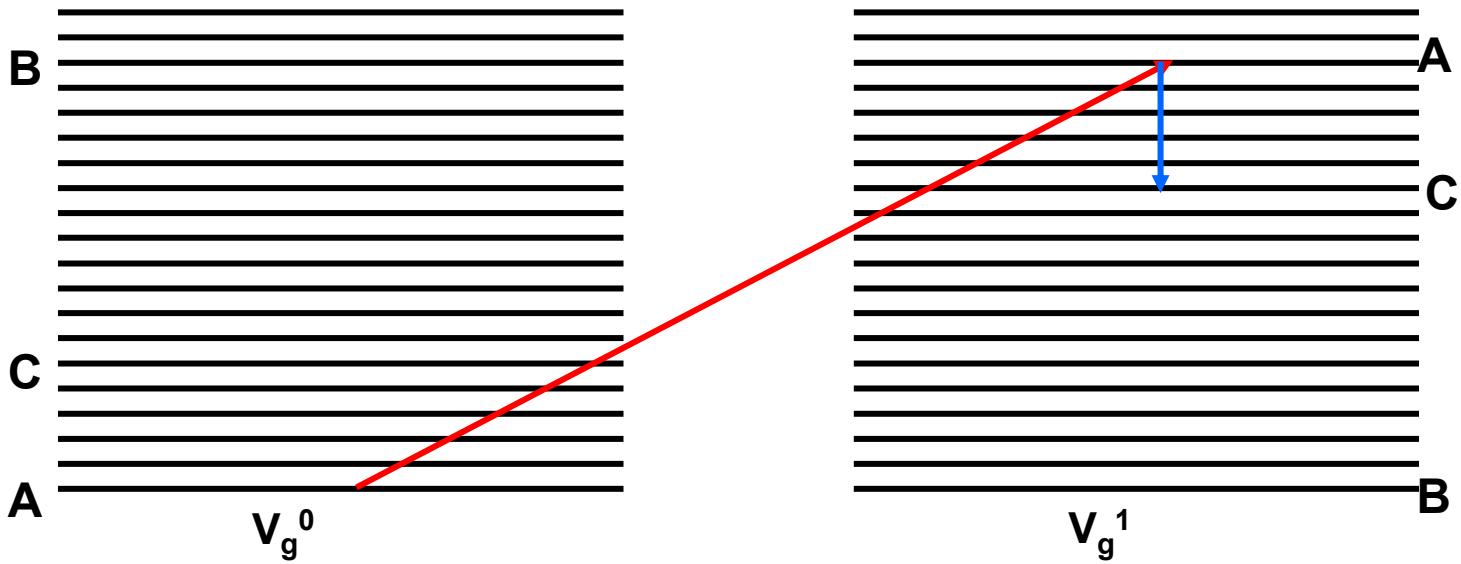
Changing gate voltage excites the electronic system in 2 ways:

1. By introducing extra electrons (or holes) at higher than thermal energies,
2. **By changing the random site potentials of the localized states.**



A QUASI-PARTICLE MODEL

SKETCH OF RESPONSE TO GATE VOLTAGE CHANGE



From ~equilibrium at V^0, a switch from V^0 to V^1 makes state A into an excited state

change of ε_i changes the Hamiltonian and so changes state energies)

Staying some time at V^1 relaxes (by partial quasiparticle formation) A into C

A QUASI-PARTICLE MODEL

RELAXATION

The transition rates *w* are exponential functions of a random variable $x=ar+b\varepsilon$

The distribution N(*w*) of the rates *w* is related to the distribution N(*x*) by

$$w = w_0 \cdot \exp[-x], \quad x=-\ln(w/w_o)$$

$$N(w)dw=N(x)dx,$$

$$N(w) = N(x) \bullet dx(w)/dw = (w_0/w) \bullet N[-\ln(w/w_0)]$$

The argument of N changes little for a large change of *w*.

The important feature of N(w) is the existence of a minimal value of *w*, say w_m.

ΔE(*t*), the energy reduction due to partial quasi-particle formation at t, is then

$$\Delta E(t) = \int_{w_m}^{w_{\max}} \Delta\ N(w)\exp(-wt)dw = -\Delta w_0 \int_{w_m t}^{\infty} \frac{1}{wt}\exp(-wt)d(wt) = \Delta\ w_0[\gamma - \ln(w_m t) - \sum(-1)^n (w_m t)^n / nn!]$$

As long as $w_m t \ll 1$ the dominant t-dependent term is the logarithm. Then

$$\Delta E(t) \propto - \ln (w_m t).$$

The relaxation in energy must be related to the relaxation of the excess conductance $\Delta G(t)$ measured experimentally.

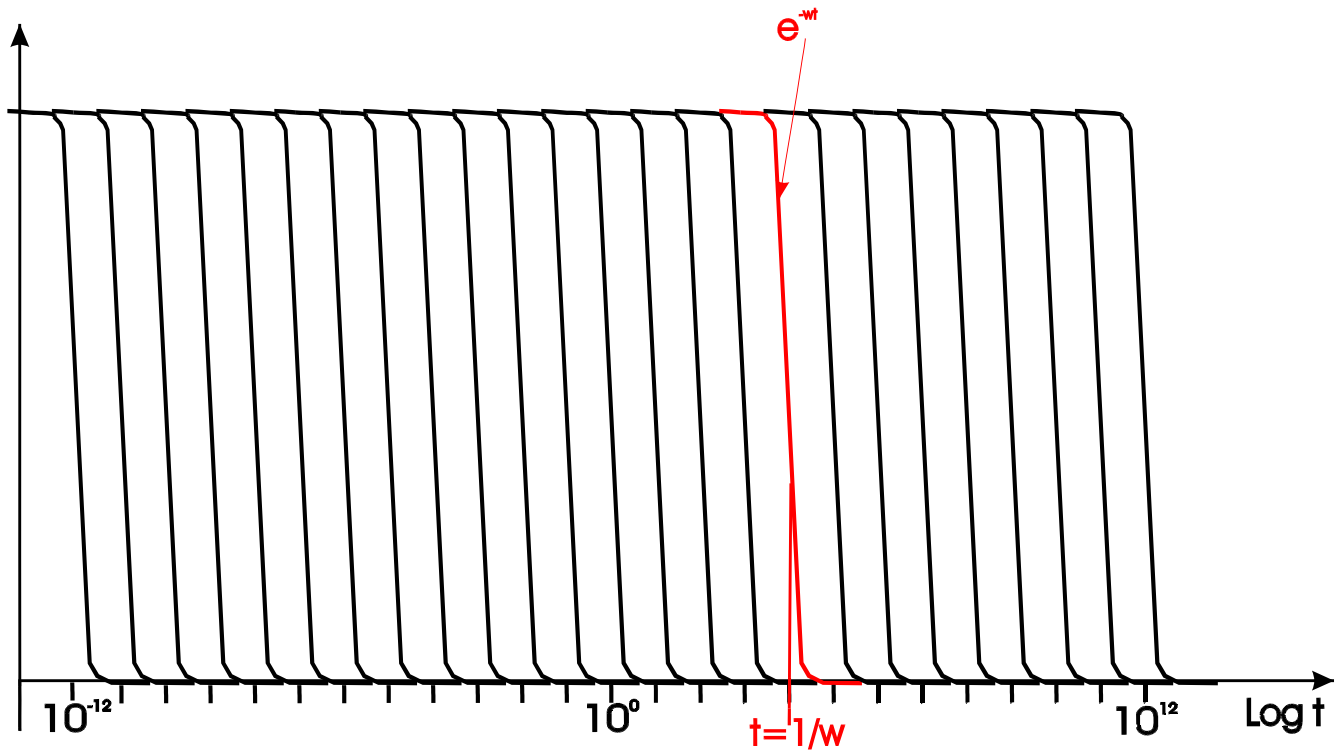
$\Delta G(t)$ decreases as the system relaxes so ΔG is a monotonic function of ΔE.

We use an expansion $\Delta G = \alpha + \beta \Delta E + \ldots$.
The constant term has no effect on the *excess* conductance.
The result of the expansion is

$$\Delta G(t) \propto - \ln (w_m t) \equiv -K \ln (w_m t)$$

A QUASI-PARTICLE MODEL

Relaxation with $N(w)$=const./w



With every decaying curve, the energy relaxes on the average by some amount δ. N(w)~1/w corresponds here to a uniform distribution. Thus **the relaxation** during an interval ΔlnT **is proportional to ΔlnT**

A.Amir, Y. Oreg and Y. Imty, PRB **77**, 165207 (2008)
PRL **103**, 126403 (2009)

MEAN FIELD MODEL

Time Dependence of Relaxation

$$f = f_{equil} + \delta f$$

probability(i occupied, j empty) transition rate i→j (if i is occupied and j empty)

$$\dot{f}_i = \dot{\delta f}_i = \sum_j [\, f_j (1 - f_i) w_{ji} - f_i (1 - f_j) w_{ij} \,] \equiv \sum_j (\gamma_{ji} - \gamma_{ij}) \quad \text{Boltzmann equation}$$

For $\delta f_k << f_{k\,equil} \equiv f_k^0$ for all k,

one can write $d\boldsymbol{\delta f}/dt = \boldsymbol{A} \cdot \boldsymbol{\delta f}$, $\quad \boldsymbol{\delta f}^{+} = [\delta f_1, \delta f_2 \, \delta f_{3\ldots} \, \delta f_N \,]$, N is the number of sites

$$A_{kk} = -\sum_{l \neq k} \gamma_{kl}^0 / f_k^0 (1 - f_k^0), \qquad A_{kl} = \gamma_{kl}^0 / f_k^0 (1 - f_k^0) - \sum_{i \neq k,l} e^2 (r_{kl}^{-1} - r_{li}^{-1}) / T$$

A bit of physics: γ_{kl}^0 is the net transition rate from k to l near equilibrium ;
$f_l^0(1-f_l^0)$ is appreciable only for l within a few kT of E_F.

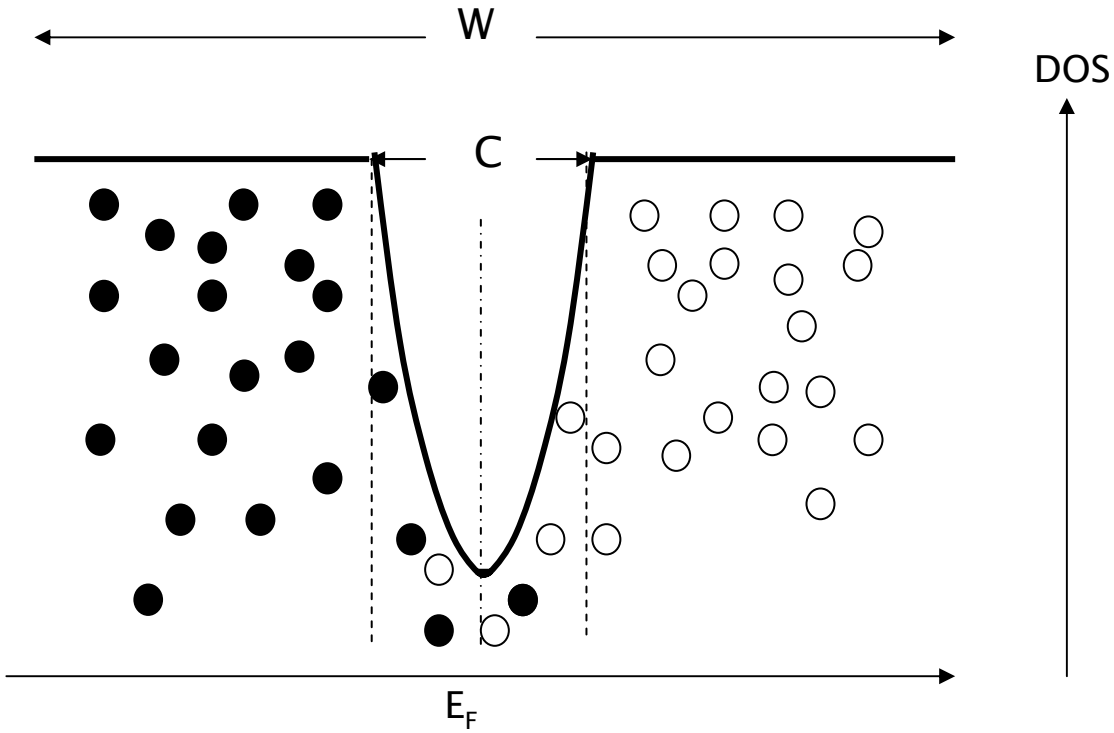
What are the eigenvalues λ of $\boldsymbol{A}$ and what is their distribution?

Numerical evaluation shows a distribution $P(\lambda) \sim 1/\lambda$

The implication is a logarithmic time dependence of the relaxation

INTERACTING DIPOLE MODEL

One-Particle Density of States (DOS)



Within C: Dominant energy is interaction energy, some sites occupied, some empty

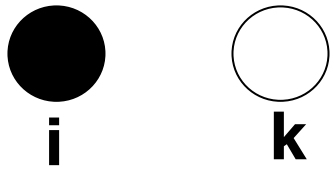
Outside C: Dominant energy is random energy,

Spatially close-by pairs are singly occupied and form dipoles.

Pairs of sites

INTERACTING DIPOLE MODEL

Dipole excitations



Two close-by sites are likely to be singly occupied because of the repulsion over close distances.

If their interaction energies with the electrons on the other sites is similar (likely if their separation is smaller than the average separation, $r_{ik} < \langle r_{nn} \rangle$) and their random energies are similar, or, more generally if

$$\sum_{j \neq l,k} n_i n_j / r_{ij} + \varepsilon_i - \sum_{j \neq l,k} n_i n_j / r_{ij} - \varepsilon_k \sim kT$$

the electron can hop easily between i and k.

INTERACTING DIPOLE MODEL

Dipole glass

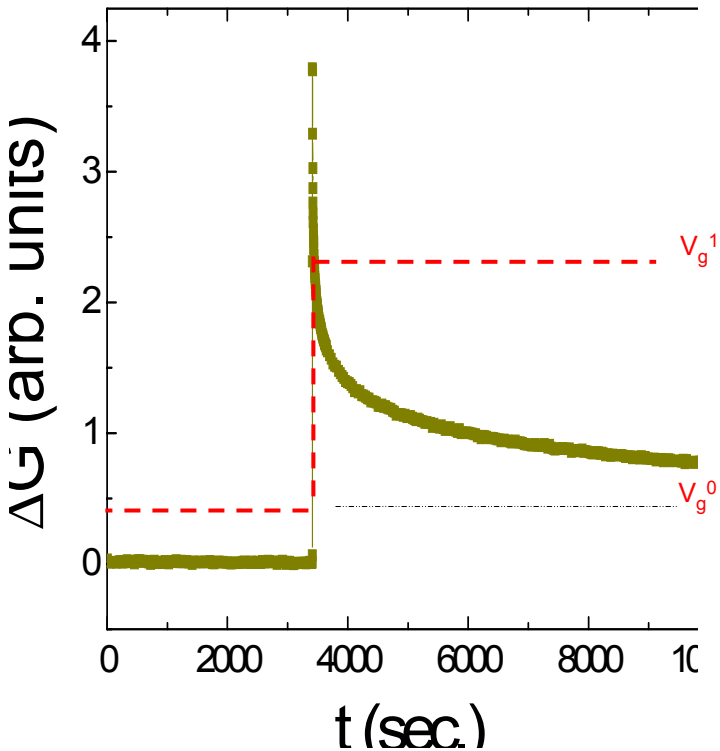
Interaction between such dipoles forms clusters which hop collectively.

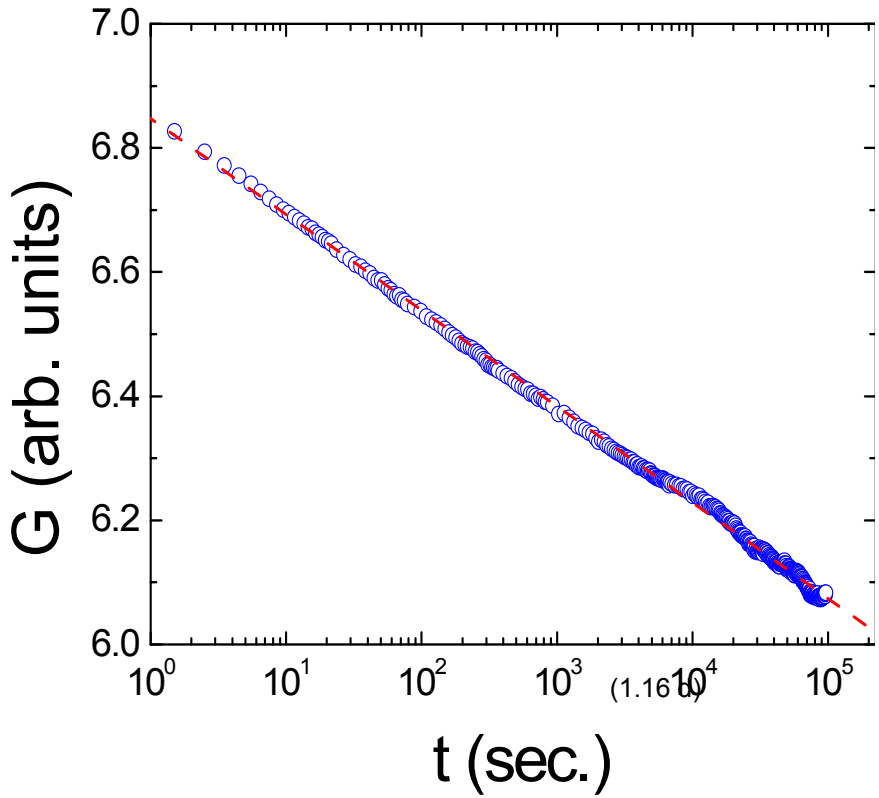
Relaxation of clusters has exponentially wide rates, so constitues a glass.

Relaxation of the clusters deepens the Coulomb gap and thus reduces the conductance.

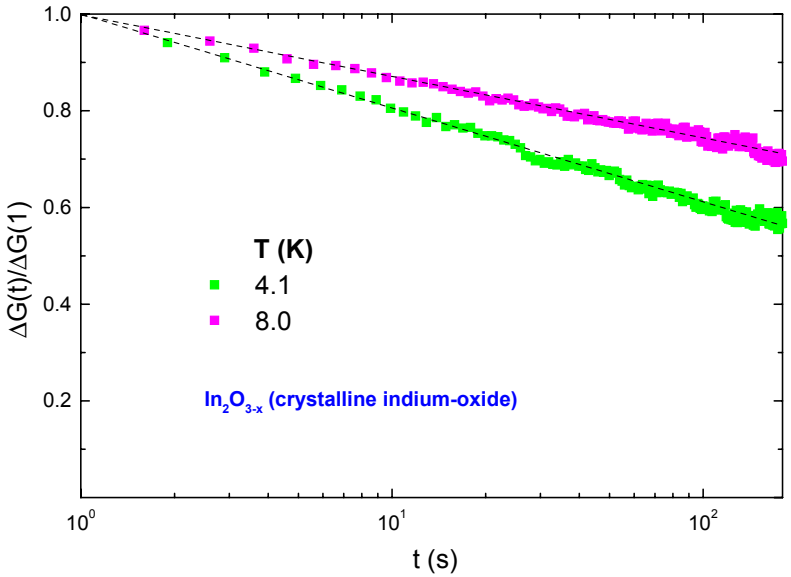
Conduction takes place near the Fermi level, i.e. by electrons in the Coulomb gap, so the “glassy sites” are separated from the sites which the current flows.

RELAXATION EXPERIMENT

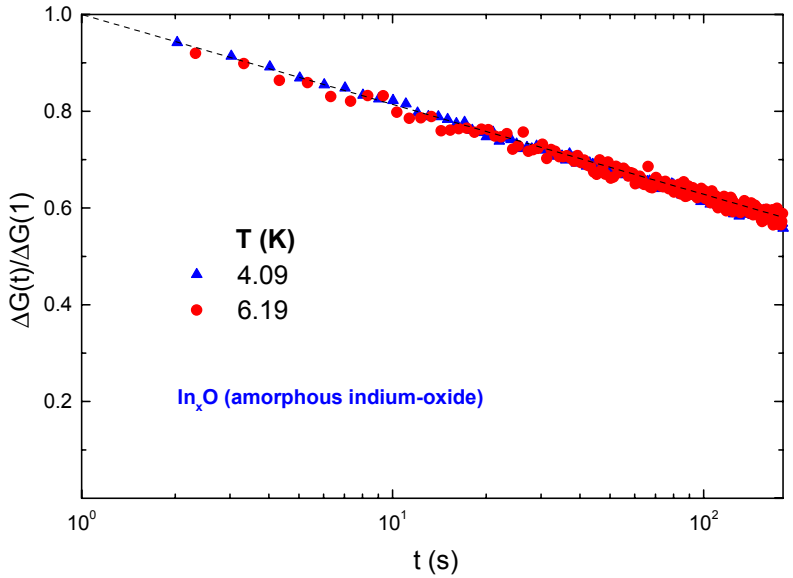




RELAXATION (T)



n=3x10^{19}



n~3x10^{19}

THIS TYPE OF T DEPENDENCE STRONGLY SUGGESTS THAT QUANTUM EFFECTS ARE IMPORTANT

OVADYAHU PROPOSED THAT QUANTUM DISSPATION IS AT WORK

Some questions:

All the materials which show non-ergodic electron glassiness have many
electrons per localized state and the localized states are not uniform.
How well can the "standard" model represent such systems?

How important in relaxation are quantum effects (e.g. quantum dissipation?)

Changes in conductance and in rate of relaxation seem to be little correlated, (e.g. change of G with change of T vs. change of τ with change of T, or
change of G with magnetic field vs. change of τ with magn. field)