destruction of superconductivity by disorder

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examples of disordered ($T \ll T_c^o$) superconductor-metal transitions:

S-wave superconductor-normal metal transition in the presence of magnetic field

superconducting droplets embedded into Normal metal with electron repulsion

D- and P- wave superconductors in the presence of disorder

$$\Delta(\vec{r},\vec{r}\,') = \int \Delta(\vec{k}) e^{i\vec{k}(\vec{r}-\vec{r}\,')} d\vec{k}$$

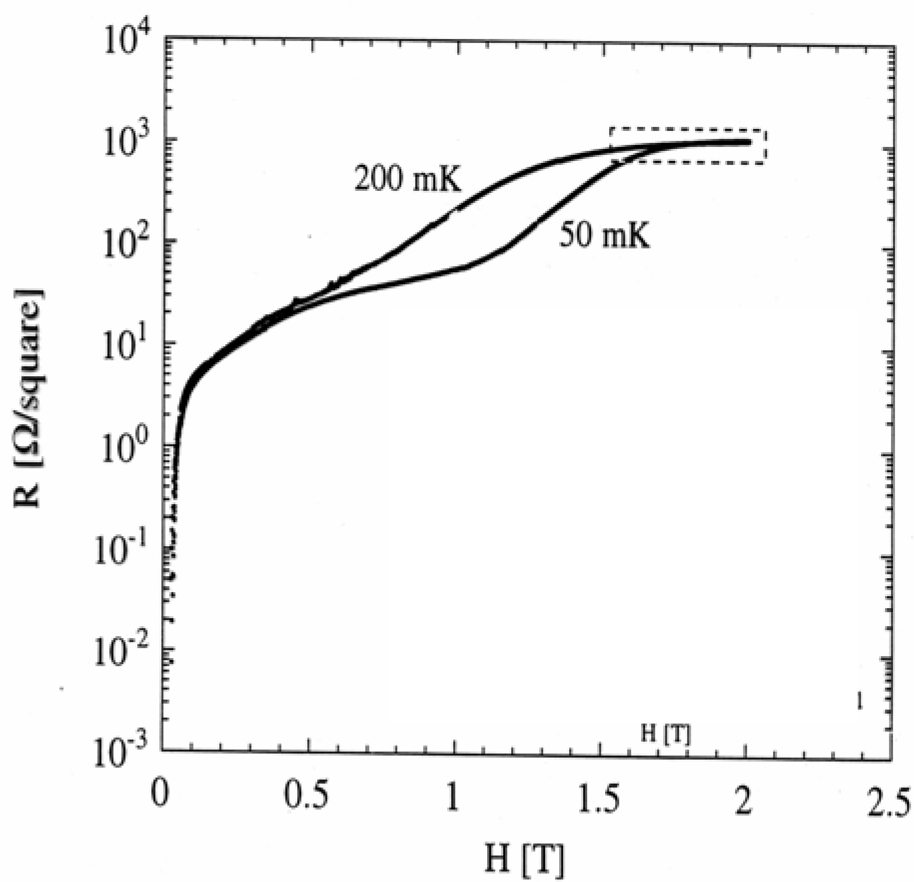
in S-wave superconductors $\Delta(\vec{r}=\vec{r}\,') = \Delta(\vec{r})$

in D- and P-wave superconductors $\Delta(\vec{r}=\vec{r}\,') = 0$

elastic electron scattering destroys D- and P-wave superconductivity when the electron mean free path l $\sim \xi$ becomes of order of the superconducting coherence length.

an assumption: e-e interaction in D (or P)-channel is attractive, while in S-channel it is repulsive

T=0 superconductor-metal transition in a perpendicular magnetic field



N. Masson,
A. Kapitulnik

There are conductors whose T=0 conductance is four order of magnitude larger than the Drude value.

superconductor – glass transition in a magnetic field parallel to the film

W. Wu, P. Adams,

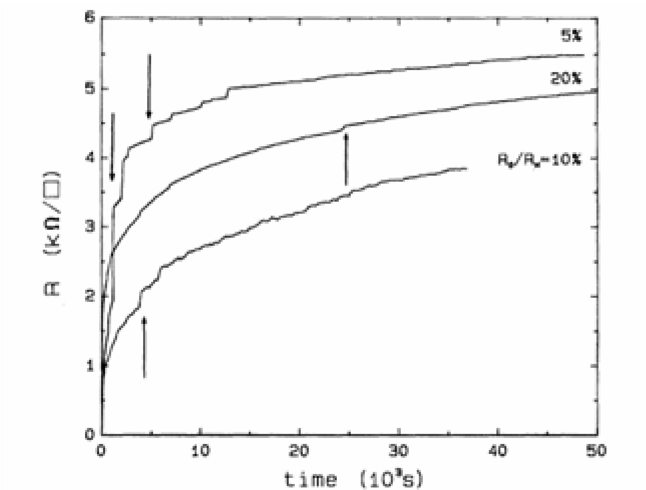
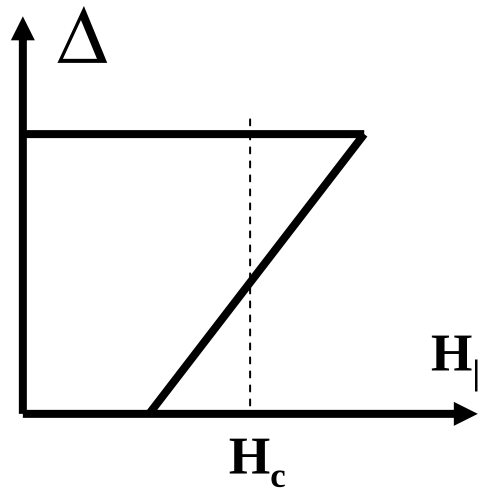


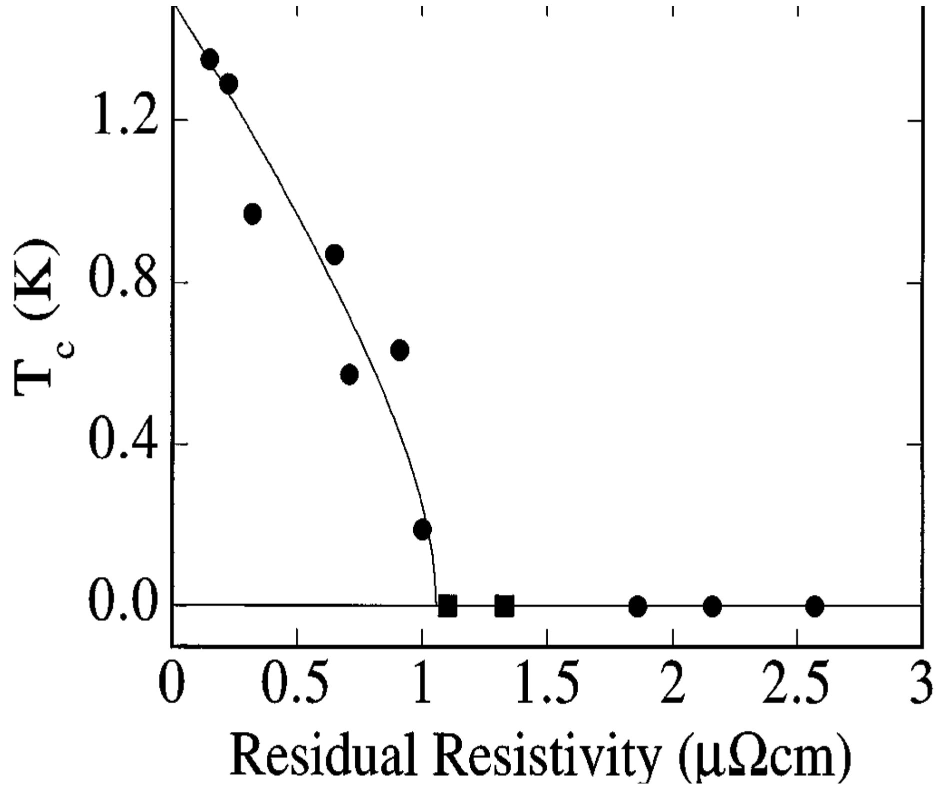
FIG. 2. R versus time after $H_\parallel$ was held constant when R_0/R_N reached desired values during field-up sweeps. Arrows indicate some of the avalanches. Note that the $R_0/R_N = 5\%$ curve actually jumped above the $R_0/R_N = 20\%$ curve.



The mean field theory:
The phase transition is of first order.

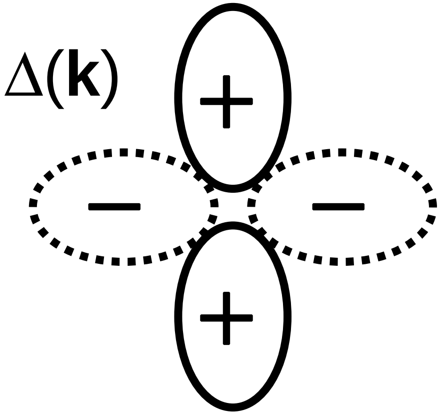
There are long time (hours) relaxation processes reflected in the time dependence of the resistance.

superconducting Sr_2RuO_4 is a strong suspects for P-wave superconductivity



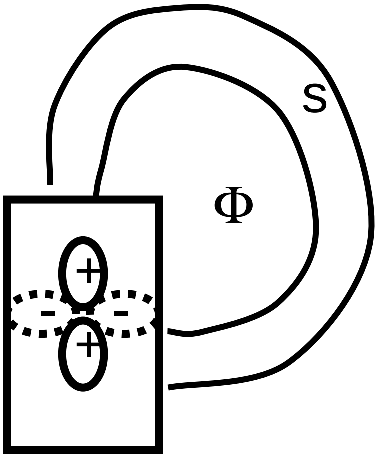
A. P. Mackenzie at all

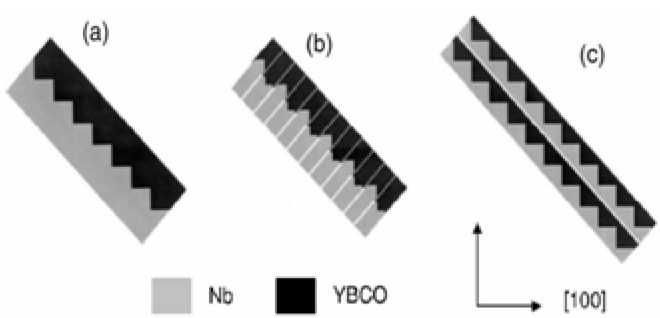
D-wave order parameter in pure superconductors

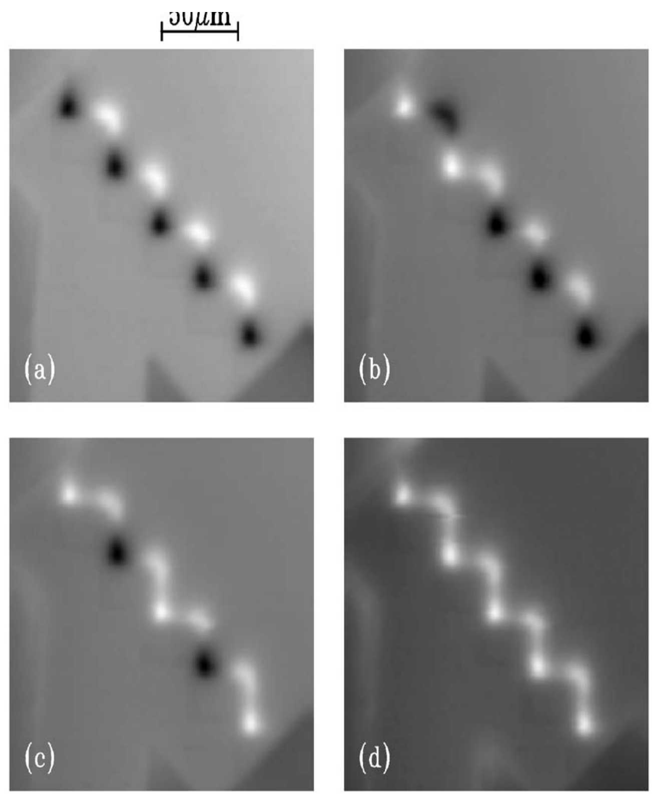


schematic picture of the order parameter in D-wave superconductors “corner SQUID” experiment which demonstrate d-wave symmetry of the order parameter in HTC

D. Geshkenbein, A.I. Larkin;
J.R. Kirtley at all



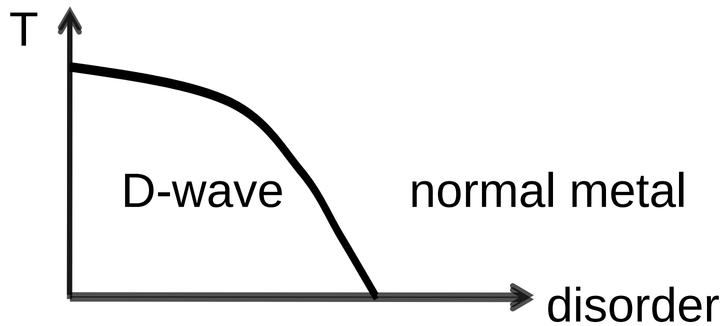




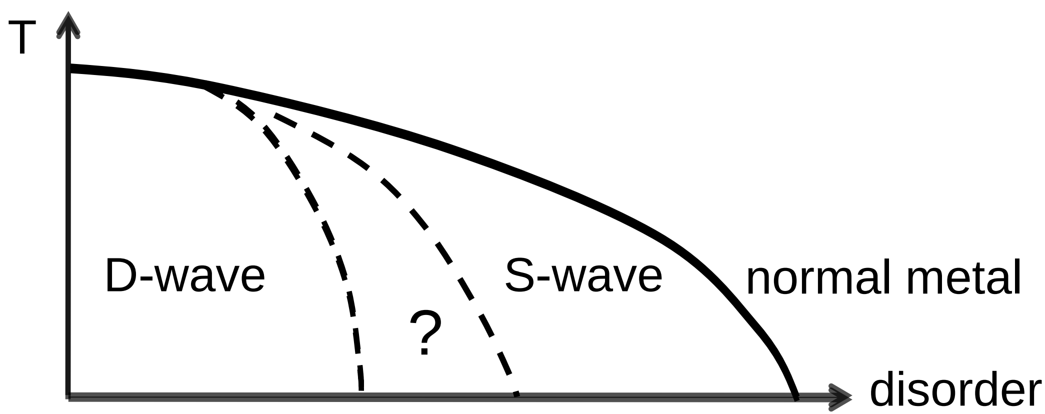
at sufficiently strong disorder all “unconventional” superconductors with even in **k** order parameter (for example D-wave) have S-wave symmetry

at sufficiently strong disorder all “unconventional” superconductors with odd in **k** order parameter (for example P-wave) either have the same Symmetry as in pure case, or they are glasses

“Conventional” phase diagram of disordered D-wave superconductors

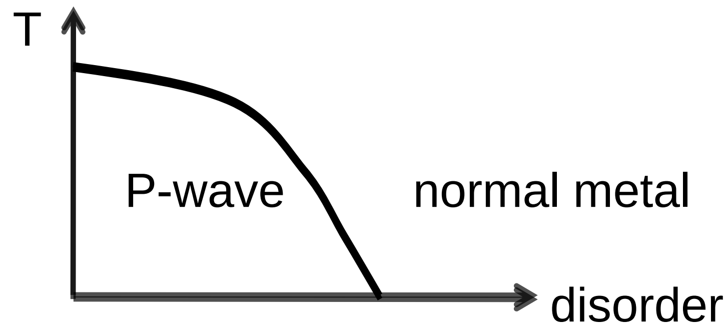


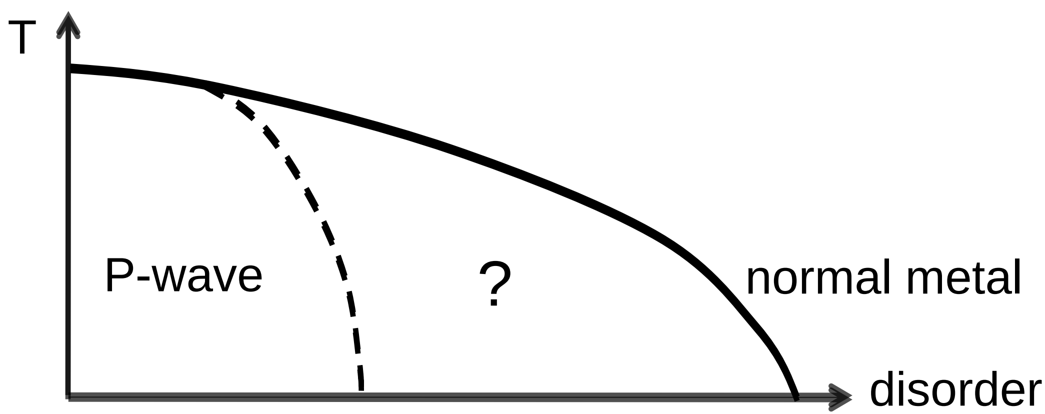
The phase diagram of disordered D-wave superconductors



near the point of the transition the order parameter has global S-wave symmetry

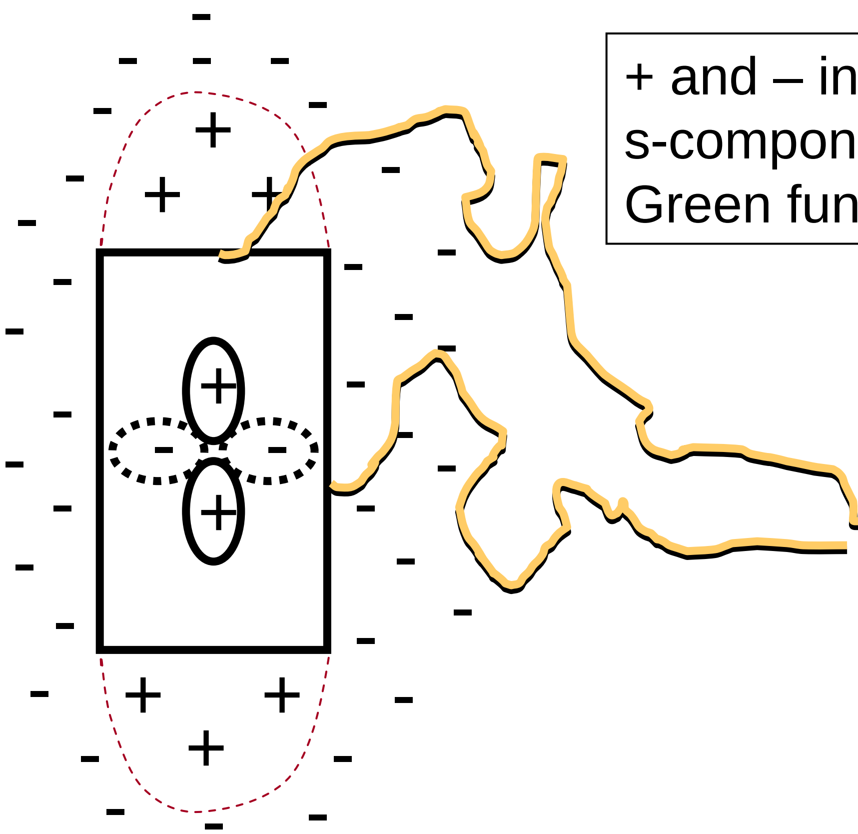
The phase diagram of disordered P-wave superconductors





near the point of the transition the order parameter has either global P-wave, or "glass symmetry"

d-wave superconducting puddle embedded into disordered normal metal.
outside the puddle s-wave component of the order parameter is generated. Only this component survives on distances larger than elastic mean free path l



+ and – indicate signs of the s-components of the anomalous Green function $F_s(\mathbf{r},\mathbf{r})$

$$\eta = sign(F_s) = sign(\oint_S \Delta_s(\vec{r})d\vec{r}) = \pm 1$$

in diffusive metal s-component of the anomalous Green function $F_s(r)=F(r,r)$ is described by the Usadel equation

$$D_{tr}\frac{d^2\theta(\varepsilon,\vec{r})}{d^2\vec{r}}+i\varepsilon\sin\theta(\varepsilon,\vec{r})=0;\quad F_s(\vec{r},\varepsilon)=-i\sin\theta(\varepsilon,\vec{r})$$

$$F_s(\vec{r})=\int F_s(\vec{r},\varepsilon)d\,\varepsilon$$

D_{tr} *is the electron diffusion coefficient in the normal metal*

$$F_s(r)\propto\frac{1}{r^3}\qquad D=3;$$

$$F_s(r)\propto\frac{1}{r^2\ln^2 r}\qquad D=2$$

before averaging over realizations of disordered potential the order parameter $\Delta(r,r')$ (and the anomalous Green function $F(r,r')$) do not have any symmetry.

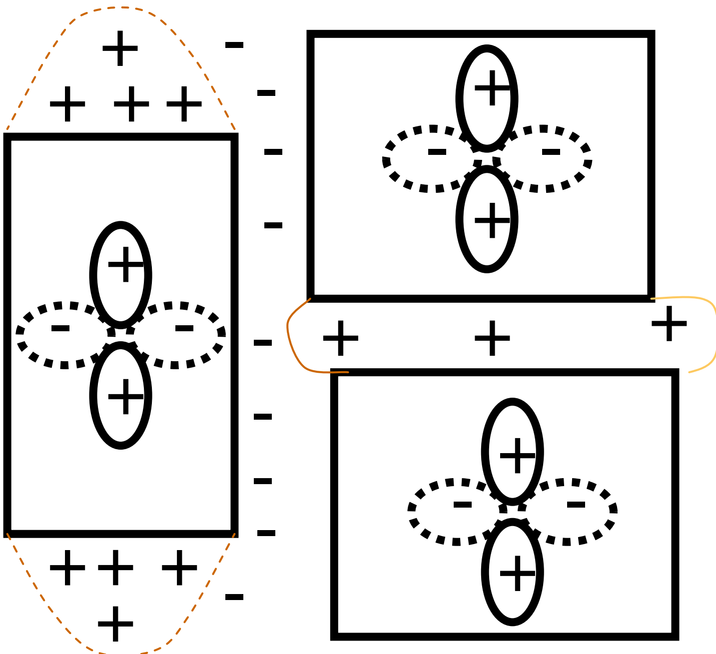
Possible definitions of the global S-wave symmetry in bulk samples :

1. corner SQUID experiment shows global s-wave symmetry of the order parameter

2. the quantity $< F_s(r)> = <F(r=r')>$ is nonzero. (The brackets stand for averaging over realizations of random potential.)

3. the system has s-wave global symmetry if $P_+ – P_- > (<)0$. P_+ and P_- are volume fractions where $F(r=r')= F_s(r)$ has positive or negative sign, respectively.

if puddle concentration is big the order parameter has global d-wave symmetry, while the s-component has random sample specific sign

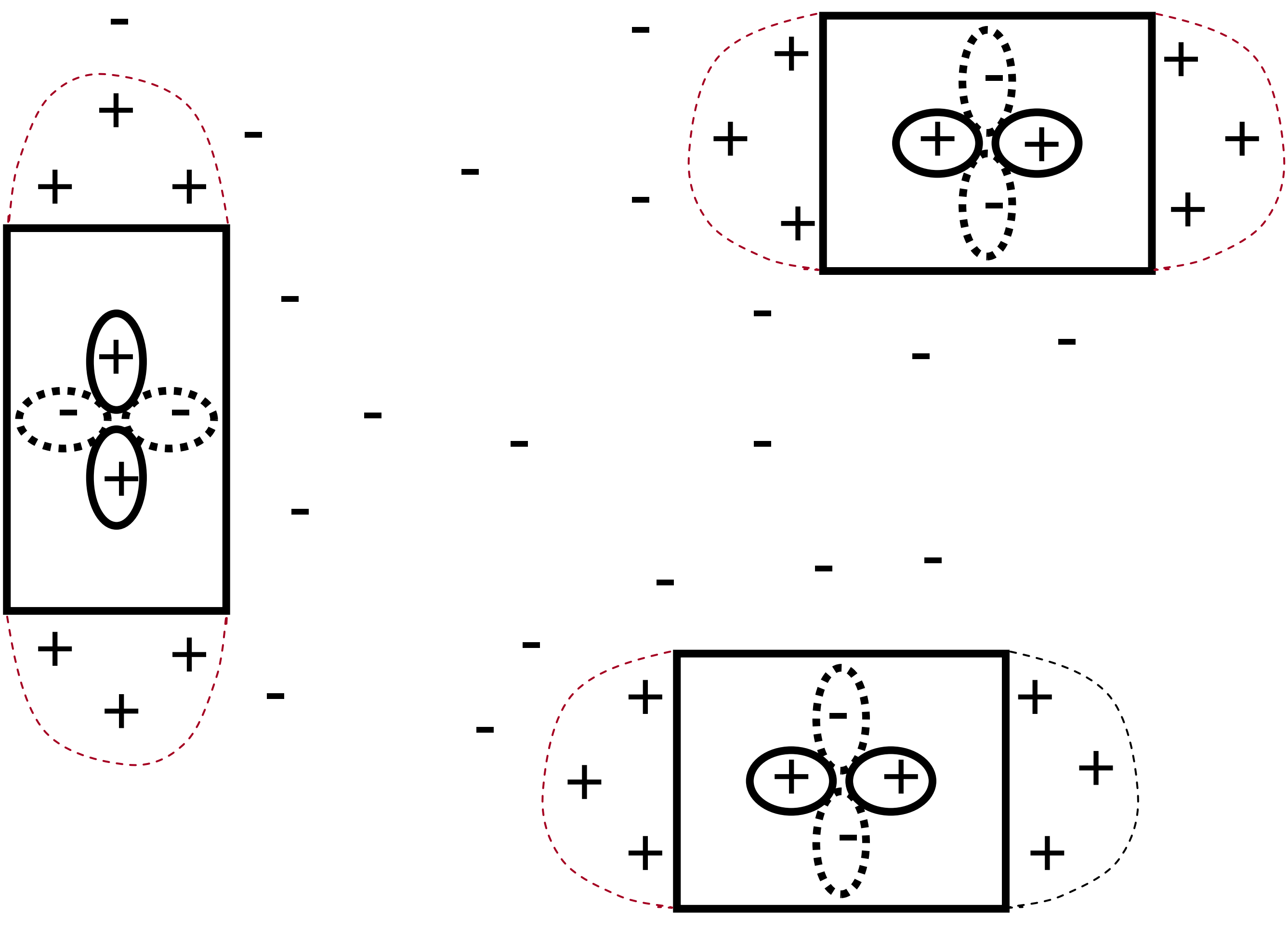


Effective mean field energy

$$E = -\sum_{ij} j_{ij}^{(d)} e^{i\left(\varphi_i - \varphi_j\right)} + c.c.$$

$J_{ij}^{(d)} > 0$ is the Joshepson coupling energy between D-wave components

if the concentration of superconducting puddles is small
the order parameter has s-wave global symmetry, while
the d-wave component has random sample specific sign



effective energy of the system is equivalent to Mattic model in the spin glass theory :

$$\eta = sign(\oint_S \Delta_s(\vec{r})d\vec{r})$$

$$E = -\sum_{ij} j_{ij}^{(s)} \eta_j \eta_i e^{i(\varphi_i - \varphi_j)} + c.c.;$$

$$\eta_i = \pm 1 \; are \; random \; in \, signs, \quad j_{ij}^{(s)} > 0$$

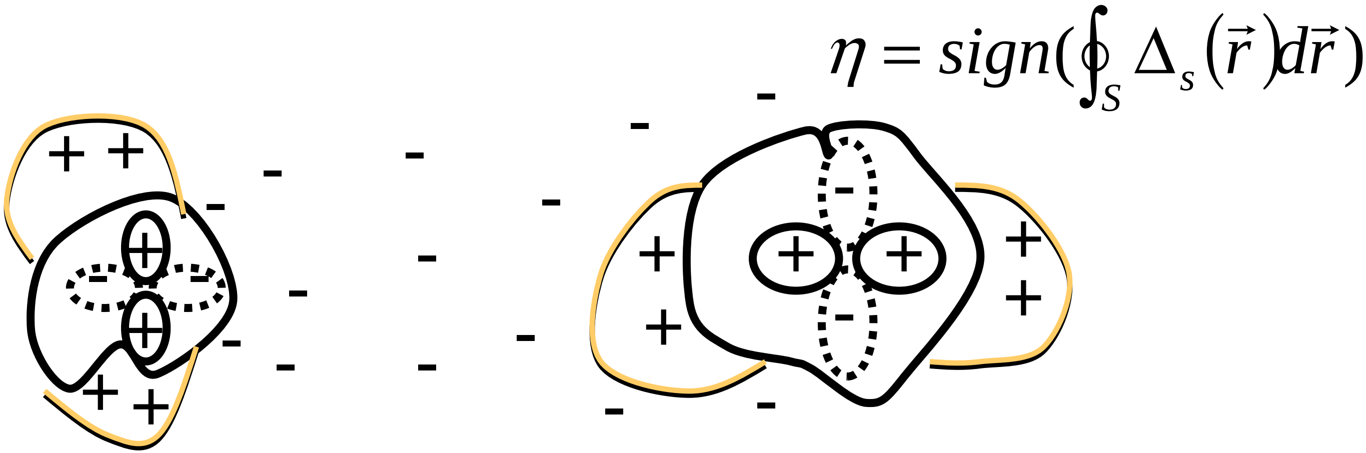
$$in \; the \; ground \; state \; e^{-i\varphi_i} = \eta_i$$

$$\Downarrow$$

$$e^{i\Phi} = \eta_i e^{i\varphi}$$

$$E = -\sum_{ij} j_{ij}^{(s)} e^{i(\Phi_i - \Phi_j)} + c.c.; \qquad j_{ij}^{(s)} > 0$$

more realistic picture superconducting puddles embedded into a metal



effective energy of the system is equivalent to Mattic model in the spin glasses theory :

$$E = -\sum_{ij} j_{ij}^{(s)} \eta_j \eta_i e^{i(\varphi_i - \varphi_j)} + c.c.; \quad \eta_i = \pm 1 ,$$

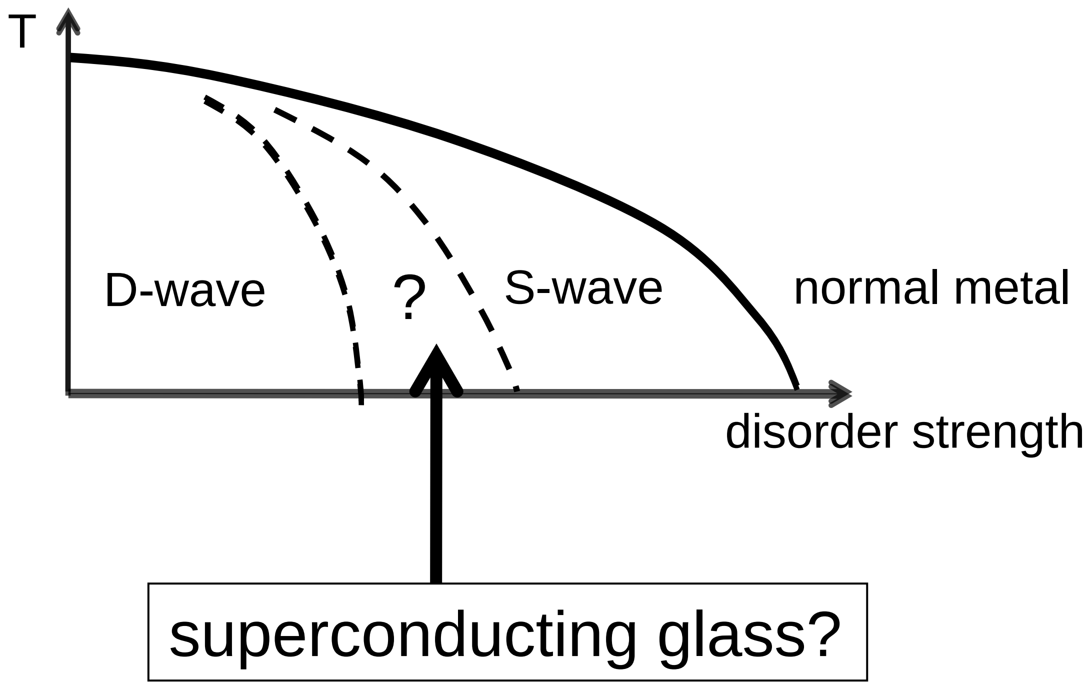
$J_{ij}^{(s)}$ is the Joshepson coupling energy between S-wave components an effective energy at intermediate concentration of superconducting puddles

$$E = -\sum_{ij}\left[j_{ij}^{(s)}\eta_j\eta_i + j_{ij}^{(d)}\right]e^{i\left(\varphi_i-\varphi_j\right)} + c.c.$$

$$\eta_i = \pm 1, \quad j_{ij}^{(s)},\ j_{ij}^{(d)} > 0$$

is there a superconducting glass phase when $J^{(s)} \sim J^{(d)}$?

The generic phase diagram of disordered D-wave superconductors



near the point of the transition to normal metal the order parameter has global S-wave symmetry

in the case of disordered p-wave superconducting puddles, spin-orbit interaction generate
the S-component of the order parameter in disordered metal. However

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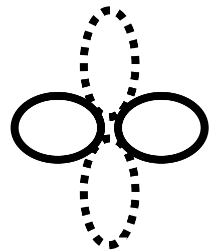
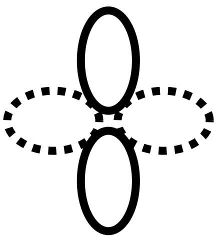
$$\oint_S \Delta_s(\vec{r})dr = 0$$

and it’s angular dependence has a dipolar character.

An example: p+ip symmetry of the order paramete
the case where nodes are pinned in in a particular direction

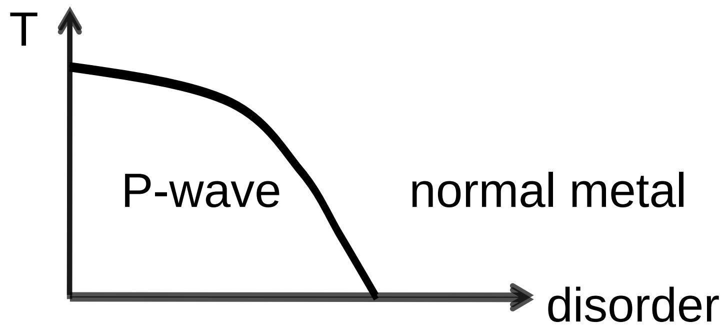
$$p_x + ip_y$$





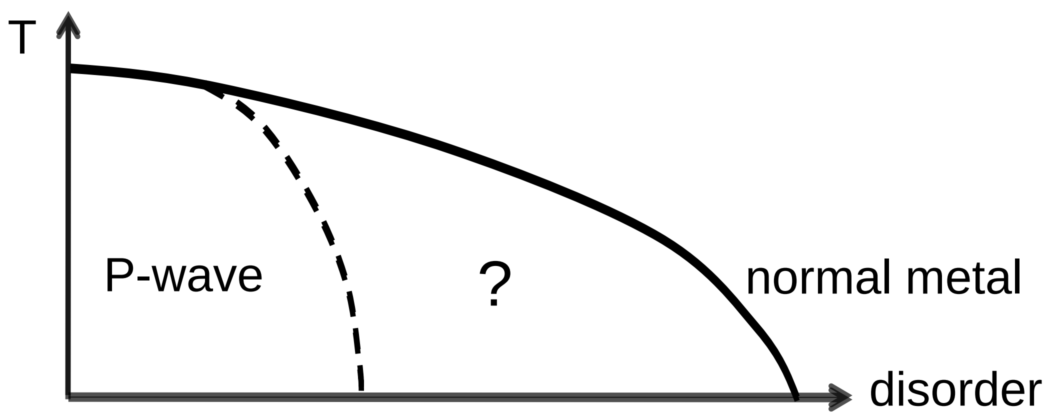
the global symmetry of the order parameter is the same as in pure case. Thus, at T=0 there is a direct quantum phase transition from P-wave superconductor to the normal metal

phase diagram of disordered P-wave superconnductor

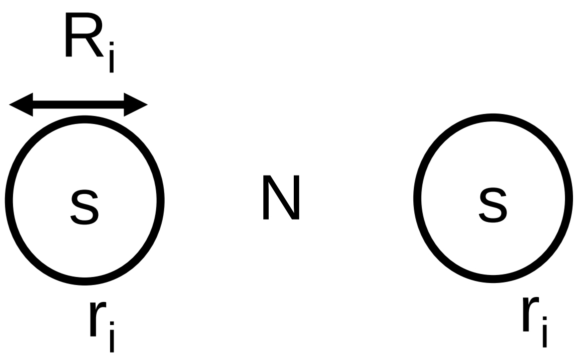


near the point of the transition the order parameter has global P-wave symmetry

In the case where anisotropy axis has random directions, near metallic phase the system is a superconducting glass



near the point of disordered quantum phase transition the system can be visualized as superconducting puddles connected by Joshepson couplings.



a generic feature of the disordered quantum transitions is that the characteristic interpuddle distance is much bigger than their characteristic size.

a criterion of a phase transition:

$$X_{ij} = \chi_i \chi_j J_{ij} J_{ji} \approx 1$$

χ_i is the susceptibility of a puddle
J_{ij} is the Joshepson coupling between puddles

χ_ι and J_{ij} are random quantities

At T=0 Joshepson coupling between the puddles decay with the inter-puddle distance slowly.

$$J_{ij} \propto \frac{1}{r_{ij}^{x}}$$

the case of disordered S-wave superconductors in the presence of magnetic field

$$\langle J \rangle \propto \exp\frac{r}{L_H}; \quad \langle J^2 \rangle \propto \frac{1}{r^4} \quad D=2$$

the case of S-wave superconducting droplets embedded in disordered metal

$$\langle J \rangle \propto \frac{1}{r^2} \quad D=2$$

the case of S-wave superconducting droplets embedded in disordered metal in the presence of parallel magnetic field

$$< J(r) > \propto \exp(-\frac{r}{L_I})\cos\frac{r}{L_I}, \qquad L_I = (D/\mu H)^{1/2}$$

Bulaevski, Buzdin

R~Rc

$$\chi = e^{G_{fff}} \qquad 3D \qquad \text{Kosterliz}$$

$$\chi = e^{\sqrt{G_{2D}}} \qquad 2D, \qquad \text{Fegelman, Larkin, Skvortsov}$$

G_{eff} and G_{2D} are conductances of a cube of normal metal of size R, and 2D normal film respectively

susceptibility is an exponential function of G >>1

Rc>>R-Rc >0

$$\chi_i \propto \Delta_0 \exp\left(\Gamma_i \frac{(R_i - R_c)}{R_c} \right); \quad \Gamma_i \approx V_i \nu \Delta_0$$

properties of the exotic metal near the quantum superconductor-metal transition:

conductivity of the "metal" is enhanced

Hall coefficient is suppressed

magnetic susceptibility is enhanced

in which sense such a metal is Fermi liquid?
For example, what is the size of quasi-particles ?
Is electron focusing at work in such metals ?

Conclusion:

Quantum disordered superconductor-normal metal transitions have unusual properties. Near the point of the transition the distance between the optimal puddles is much larger than their size