Ultraslow dynamics in disordered superconducting nanowires

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in collaboration with

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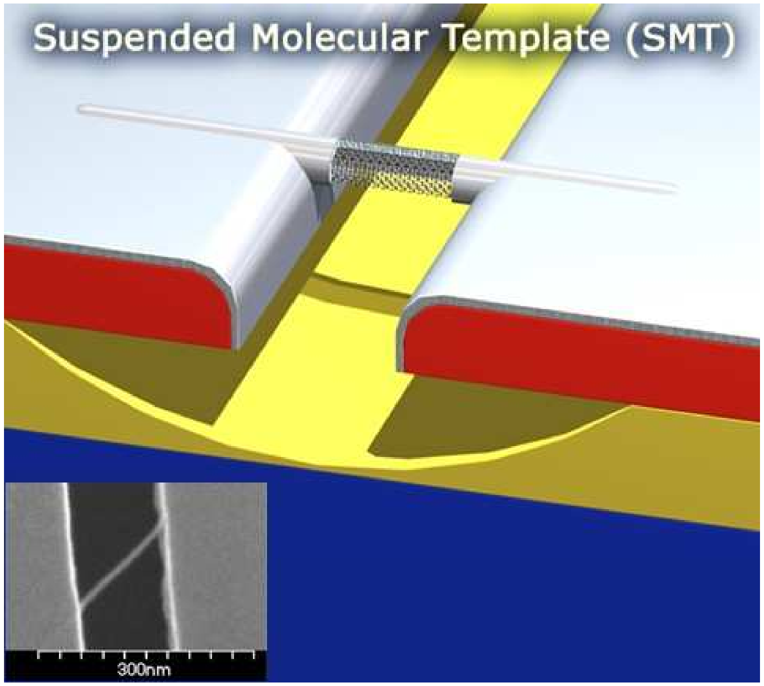
Outline

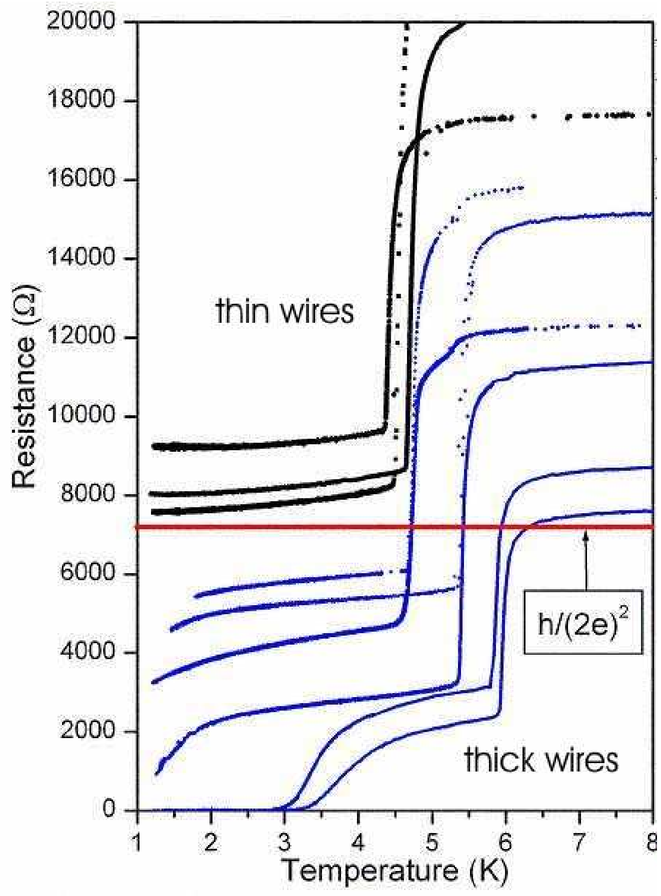
- Experiment: superconducting nanowires
- Order parameter field theory of the superconductor-metal QPT
- Strong-disorder renormalization group
- Quantum criticality in the presence of disorder and dissipation
- Back to nanowires: dynamical conductivity

Phys. Rev. Lett. **99**, 230601 (2007), Phys. Rev. B **79**, 024401 (2009), arXiv:1006.3793

Superconductivity in ultrathin nanowires

- ultrathin MoGe wires (width $\sim$ 10 nm)
- produced by molecular templating using a single carbon nanotube
  [A. Bezryadin et al., Nature 404, 971 (2000)]



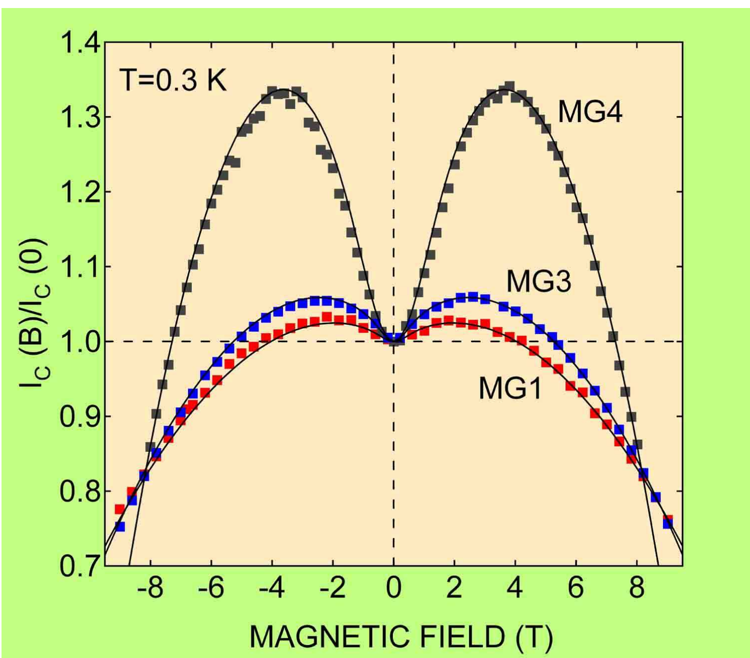


- thicker wires are superconducting at low temperatures
- thinner wires remain metallic

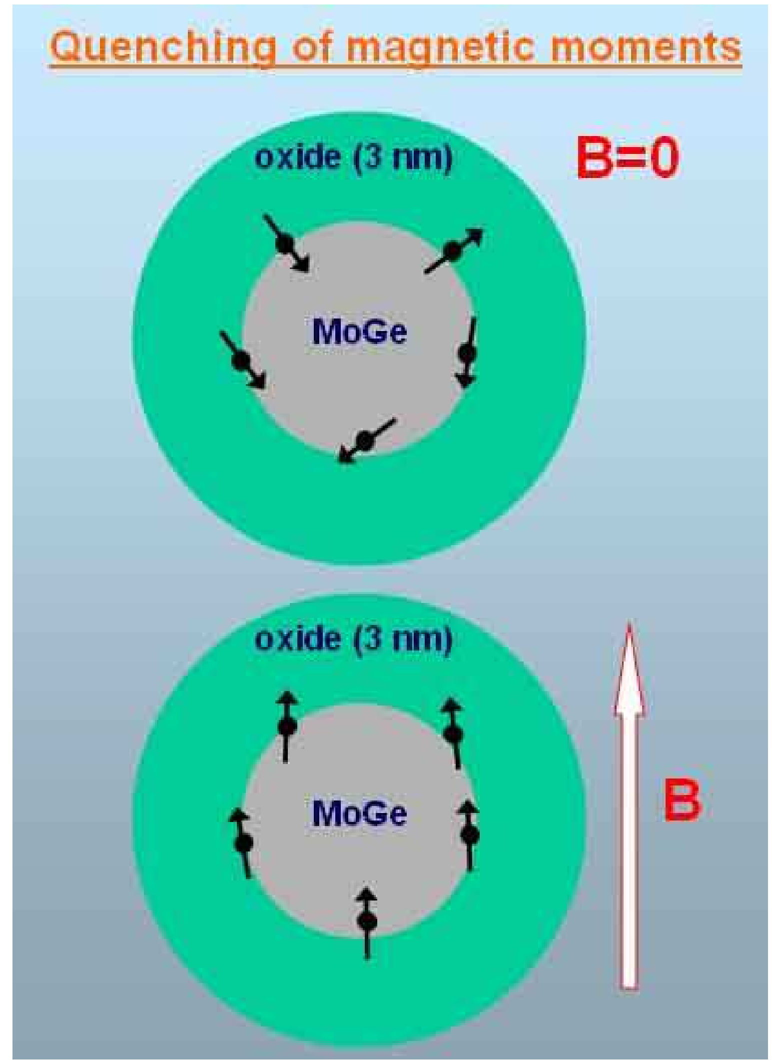
superconductor-metal QPT as function of wire thickness

Pairbreaking mechanism

- pair breaking by surface magnetic impurities
- random impurity positions
  $\Rightarrow$ quenched **disorder**
- gapless excitations in metal phase
  $\Rightarrow$ Ohmic **dissipation**



weak field enhances superconductivity



magnetic field aligns the impurities and reduces magnetic scattering

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Quasi-onedimensional behavior

For wires of width of about 10nm:

- number of transport channels (states transverse to wire) is large, $N_\perp \gg 1$
  $\Rightarrow$ motion of (unpaired) electrons is **three-dimensional**
- width is less than superconducting coherence length
  $\Rightarrow$ superconducting fluctuations are **one-dimensional**

Cooper pair propagator:

(Lopatin, Shah, Vinokur 2005)

$$C(q,\omega_n) = \frac{1}{r + \xi_0^2 q^2 + \gamma|\omega_n|}$$

ξ_0 = bare correlation (coherence) length
r = distance from criticality, related to strength of pair breaking
γ = Ohmic dissipation due to coupling to normal unpaired electrons of metal

Dissipative $O(N)$ order parameter field theory

N-component $(N > 1)$ order parameter field $\varphi(\mathbf{x}, \tau)$ in d dimensions
derived by standard methods (Hubbard-Stratonovich transformation etc.)

$$S = T \sum_{\mathbf{q},\omega_n} \left(r + \xi_0^2 \mathbf{q}^2 + \gamma \, |\omega_n| \right) |\varphi(\mathbf{q}, \omega_n)|^2 + \frac{u}{2N} \int \mathrm{d}^d x \mathrm{d}\tau \; \varphi^4(\mathbf{x}, \tau)$$

Disorder: { distance r from criticality; bare correlation length ξ_0; Ohmic dissipation constant γ } random functions of position

- Superconductor-metal quantum phase transition in nanowires $(d = 1, N = 2)$
  $\varphi(\mathbf{x}, \tau)$ represents local Cooper pair operator (Sachdev, Werner, Troyer 2004)
- Hertz' theory of itinerant quantum Heisenberg antiferromagnets $(d = 3, N = 3)$
  $\varphi(\mathbf{x}, \tau)$ represents staggered magnetization (Hertz 1976)

What is the fate of a quantum phase transition under the combined influence of disorder and dissipation?

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Space discretization and large-N limit

To apply real-space based strong-disorder renormalization group:

- discretize space by introducing "rotor" variables $\phi_j(\tau)$
- large-N limit of an infinite number of order parameter components

Resulting action:

$$S = T \sum_{i,\omega_n} (r_i + \lambda_i + \gamma_i |\omega_n|) \, |\phi_i(\omega_n)|^2 - T \sum_{i,\omega_n} J_i \, \phi_i(-\omega_n) \, \phi_{i+1}(\omega_n)$$

r_i, $\gamma_i > 0$, $J_i > 0$: random functions of lattice site i

λ_i: Lagrange multiplier enforcing large-N constraint $\langle \varphi_i^2(\tau) \rangle = 1$

$\epsilon_i = r_i + \lambda_i$: renormalized (local) distance from criticality

Strong-disorder renormalization group

- introduced by Ma, Dasgupta, Hu (1979), further developed by Fisher (1992, 1995)
- asymptotically exact if disorder distribution becomes broad under RG

Basic idea: Successively integrate out the local high-energy modes and renormalize the remaining degrees of freedom.

in our system

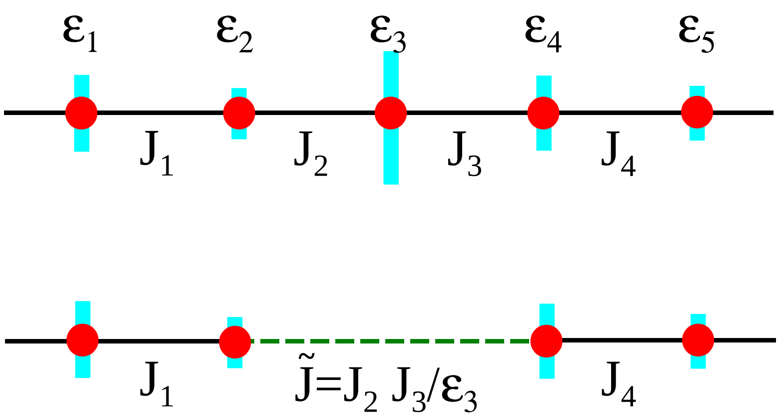
$$S = T \sum_{i,\omega_n} (\epsilon_i + \gamma_i |\omega_n|) \left|\phi_i(\omega_n)\right|^2 - T \sum_{i,\omega_n} J_i \, \phi_i(-\omega_n) \, \phi_{i+1}(\omega_n)$$

the competing local energies are:

- interactions (bonds) J_i favoring the ordered phase
- local "gaps" ϵ_i favoring the disordered phase

$\Rightarrow$ in each RG step, integrate out largest among all J_i and ϵ_i

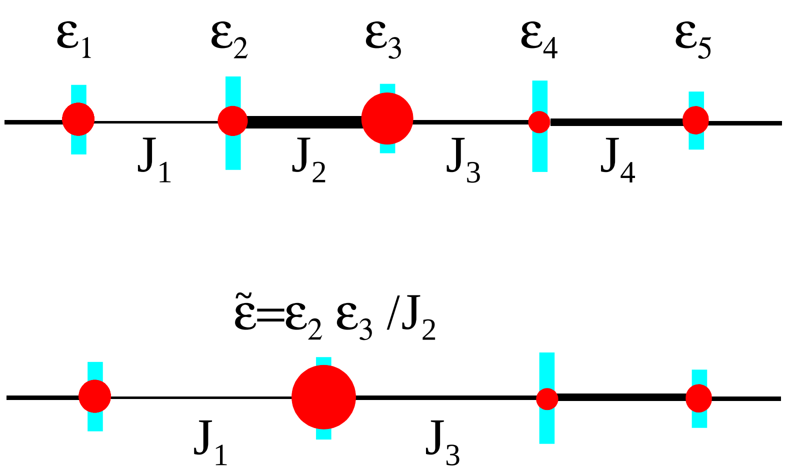
Recursion relations in one dimension



if largest energy is a gap, e.g., $\epsilon_3 \gg J_2, J_3$:

- site 3 is removed from the system
- coupling to neighbors is treated in 2nd order perturbation theory

new renormalized bond $\tilde{J} = J_2 J_3/\epsilon_3$



if largest energy is a bond, e.g., $J_2 \gg \epsilon_2, \epsilon_3$:

- rotors of sites 2 and 3 are parallel
- can be replaced by single rotor with moment $\tilde{\mu} = \mu_2 + \mu_3$

renormalized gap $\tilde{\epsilon} = \epsilon_2\epsilon_3/J_2$

Renormalization-group flow equations

- strong disorder RG step is iterated, gradually reducing maximum energy Ω
- competition between cluster aggregation and decimation
- leads to larger and larger clusters connected by weaker and weaker bonds

$\Rightarrow$ **flow equations** for the full probability distributions $P(J)$ and $R(\epsilon)$

$$
\begin{aligned}
-\frac{\partial P}{\partial \Omega} &= [P(\Omega) - R(\Omega)]\, P + R(\Omega) \int dJ_1 dJ_2\, P(J_1) P(J_2)\, \delta\left(J - \frac{J_1 J_2}{\Omega}\right) \\
-\frac{\partial R}{\partial \Omega} &= [R(\Omega) - P(\Omega)]\, R + P(\Omega) \int d\epsilon_1 d\epsilon_2\, R(\epsilon_1) R(\epsilon_2)\, \delta\left(\epsilon - \frac{\epsilon_1 \epsilon_2}{\Omega}\right)
\end{aligned}
$$

Flow equations are identical to those of the **random transverse-field Ising chain**

Note symmetry between J and ϵ!

Fixed points

If bare distributions do not overlap:

$\langle \ln \epsilon \rangle > \langle \ln J \rangle$: no clusters formed – disordered phase

$\langle \ln \epsilon \rangle < \langle \ln J \rangle$: all sites connected – ordered phase

If bare distributions do overlap:

$\langle \ln \epsilon \rangle > \langle \ln J \rangle$: rare clusters – disordered Griffiths phase

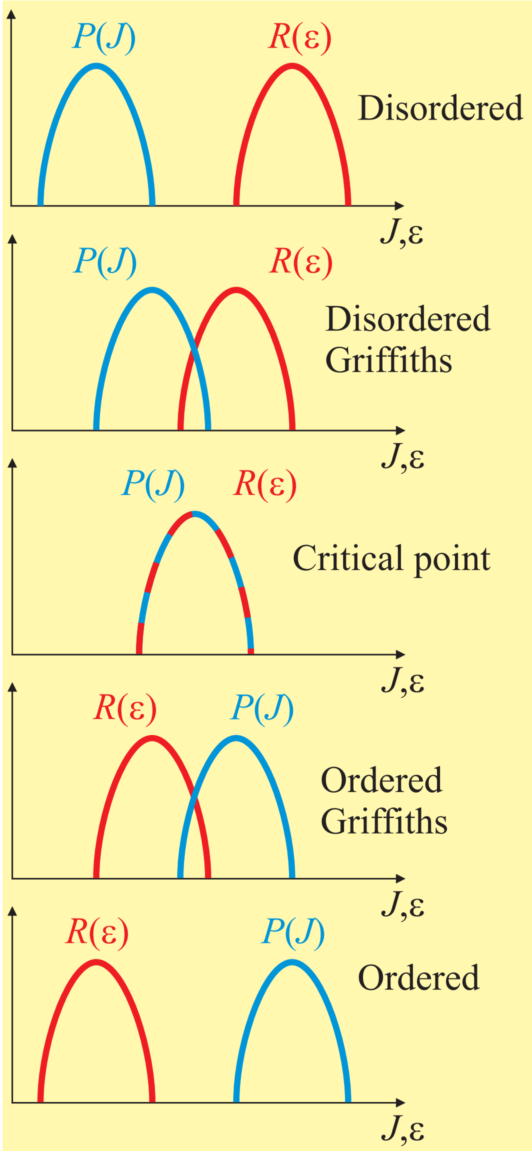
$\langle \ln \epsilon \rangle < \langle \ln J \rangle$: rare "holes" – ordered Griffiths phase

$\langle \ln \epsilon \rangle = \langle \ln J \rangle$: cluster aggregation and decimation balance at all energies – **critical point**

$$\mathcal{P}(\zeta) = \frac{1}{\Gamma} e^{-\zeta/\Gamma}, \quad \mathcal{R}(\beta) = \frac{1}{\Gamma} e^{-\beta/\Gamma}$$

log. variables $\zeta = \ln(\Omega/J)$, $\beta = \ln(\Omega/\epsilon)$, $\Gamma = \ln(\Omega_0/\Omega)$

Distributions become infinitely broad at critical point



initial (bare) distributions

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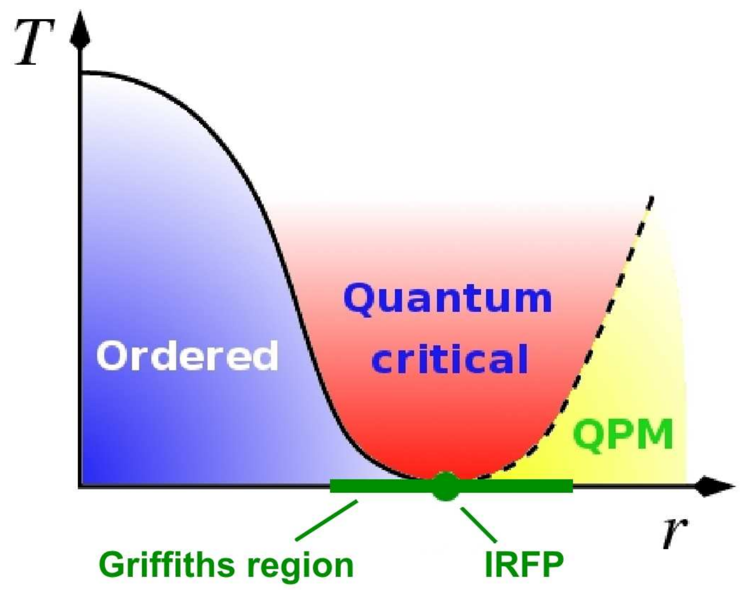
Quantum critical point

- at critical FP, disorder scales to ∞
  $\Rightarrow$ **infinite-randomness critical point**
- activated dynamical scaling $\ln(1/\Omega) \sim L^{\psi}$ with tunneling exponent $\psi = 1/2$
- moments of surviving clusters grow like $\mu \sim \ln^{\phi}(1/\Omega)$ with $\phi = (1 + \sqrt{5})/2$
- average correlation length diverges as $\xi \sim |r|^{-\nu}$ with $\nu = 2$



dissipative $O(N)$ order parameter is in universality class of **dissipationless** random transverse-field Ising model.

finite-temperature phase boundary and crossover line take unusual form

$$T_c \sim \exp(-\text{const}\,|r|^{-\nu\psi})$$

Quantum Griffiths regions:

- power-law dynamical scaling with nonuniversal exponent

Quantum-critical thermodynamics

to calculate thermodynamic properties at temperature T:
run RG down to energy scale $\Omega = T$ and consider remaining clusters as free

Static order parameter susceptibility:

each surviving cluster contributes μ^2/T

$$\chi(r,T) = \frac{1}{T} n(\Omega = T) \mu^2(\Omega = T) = \frac{1}{T} \left[\ln(1/T)\right]^{2\phi - d/\psi} \Theta_\chi \left(r^{\nu\psi} \ln(1/T)\right)$$

Specific heat:

each surviving cluster contributes T to the total energy

$$C(r,T) = \frac{\partial}{\partial T} [T n(\Omega = T)] = \left[\ln(1/T)\right]^{-d/\psi} \Theta_C \left(r^{\nu\psi} \ln(1/T)\right)$$

Quantum Griffiths singularities

in disordered Griffiths phase:

thermodynamics is characterized by **nonuniversal power laws**

local OP susceptibility $\chi^{\mathrm{loc}}(r,T) \sim T^{d/z'-1}$

specific heat $C(r,T) \sim T^{d/z'-1}$

order parameter in external field $\langle \phi(r,H) \rangle \sim H^{d/z'}$

dynamical exponent

$z' \sim r^{-z\nu}$ **diverges** at infinite-randomness critical point

Dynamical susceptibility

to calculate dynamic OP susceptibilities at external frequency ω (and $T = 0$):
run RG down to energy scale $\Omega = \gamma_{\rm eff}\omega = \gamma\mu(\Omega)\omega$

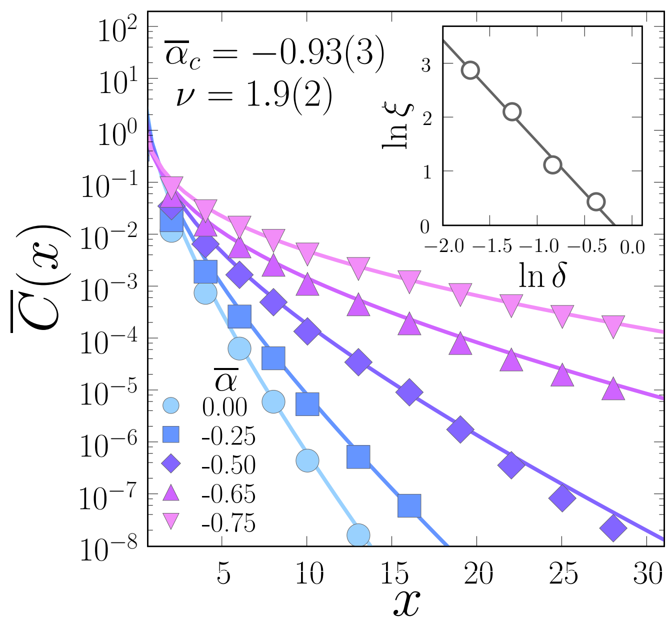
single-cluster contributions:

$$\chi_j(\omega + \mathrm{i}\delta) = \frac{\mu_j^2}{\epsilon - \mathrm{i}\mu_j\gamma\omega}, \quad \chi_j^{\rm loc}(\omega + \mathrm{i}\delta) = \frac{\mu_j}{\epsilon - \mathrm{i}\mu_j\gamma\omega}$$

Dynamic susceptibilities at $T = 0$:

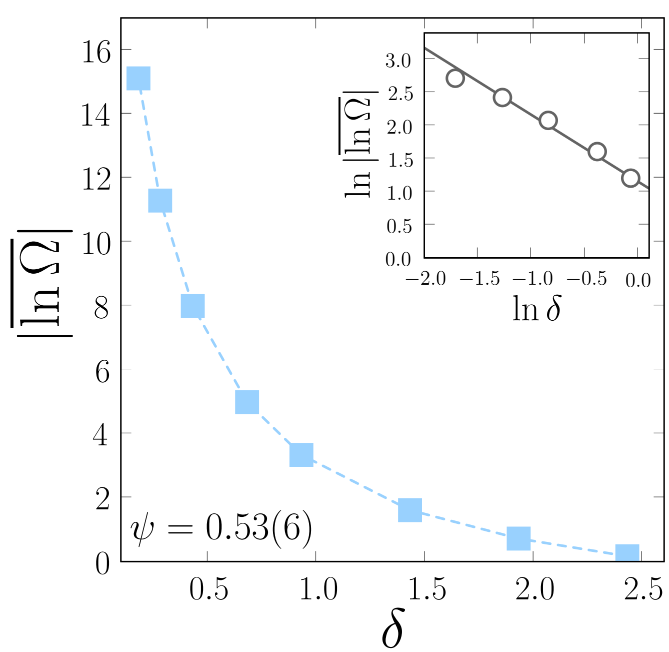
$$\begin{aligned}
\mathrm{Im}\chi(r,\omega) &\sim \frac{1}{\omega}\left[\ln(1/\omega)\right]^{\phi - d/\psi} X\left(r^{\nu\psi}\ln(1/\omega)\right) \\
\mathrm{Im}\chi^{\rm loc}(r,\omega) &\sim \frac{1}{\omega}\left[\ln(1/\omega)\right]^{-d/\psi} X^{\rm loc}\left(r^{\nu\psi}\ln(1/\omega)\right)
\end{aligned}$$

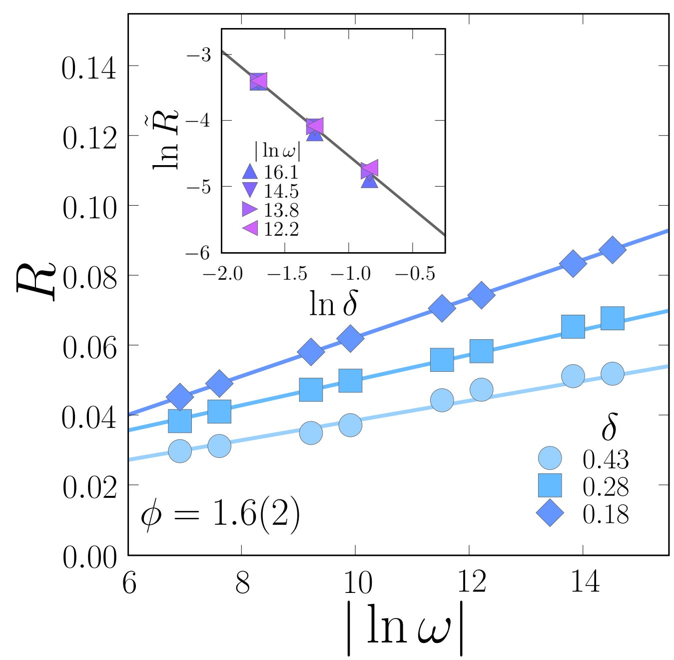
Numerical confirmation



- A. Del Maestro et al. (2008) solved disordered large-N problem numerically exactly
- calculated equal time correlation function C, energy gap Ω, and ratio R of local and order parameter dynamic susceptibilities

	ν	ψ	ϕ
SDRG	2	1/2	$(\sqrt{5}+1)/2$
Numerics	1.9(2)	0.53(6)	1.6(2)





Order parameter symmetry

- our explicit calculations are for an infinite number of OP components, $N = \infty$

Are the results valid for the physical cases $N = 2$ (superconductor-metal transition) and $N = 3$ (Hertz' antiferromagnetic transition)?

Analysis:

- infinite-randomness FP is due to **multiplicative** structure of recursion relations
- bond renormalization $\tilde{J} = J_2 J_3 / \epsilon_3$ follows from 2nd order perturbation theory, does not depend on N
- multiplicative structure of gap renormalization $\tilde{\epsilon} = \epsilon_2 \epsilon_3 / J_2$ corresponds to **exponential** dependence of the gap on the cluster size
- applies to all **continuous symmetry** cases $N > 1$ (Mermin-Wagner)
- Ising OPs are different with even stronger disorder effects

Infinite-randomness critical point for all continuous symmetry cases $N > 1$

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Kubo conductivity

dynamical (optical) conductivity ($\hbar\omega \gg k_B T$):

$$\sigma(\omega) = -\frac{i}{\hbar\omega}\left[\sum_{k,l}\int d\tau\,\langle j_k(\tau) j_l(0)\rangle\, \mathrm{e}^{i\omega\tau} - \mathcal{D}\right]_{i\omega\to\omega+i\eta}$$

- local current: $j_k(\tau) = (2ie/\gamma\hbar) J_k \left[\phi_k^*(\tau)\phi_{k+1}(\tau) - \phi_{k+1}^*(\tau)\phi_k(\tau)\right]$
- diamagnetic term $\mathcal{D} = (8e^2/\gamma\hbar)\sum_k J_k \left\langle |\phi_k(0)|^2 \right\rangle$

can be calculated within the SDRG:

- include coupling to vector potential $A_k(\tau)$ into action (source term)
- local current at each stage of the SDRG follows from $j_k(\tau) \sim -\delta S/\delta A_k(\tau)$
- evaluation of the Kubo formula within order parameter theory gives Aslamazov-Larkin fluctuation correction to conductivity

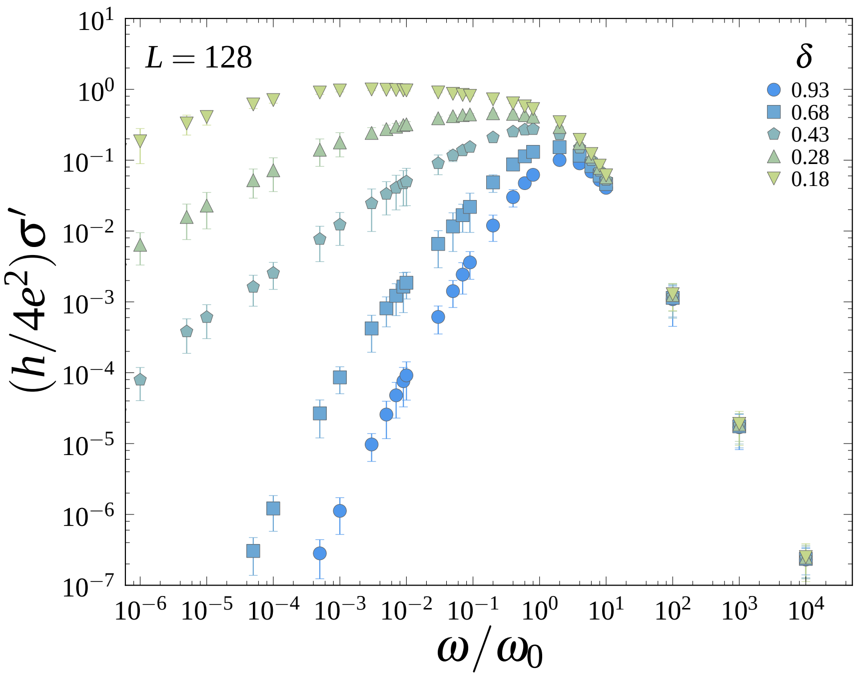
Results: fluctuation corrections to $\sigma'(\omega)$

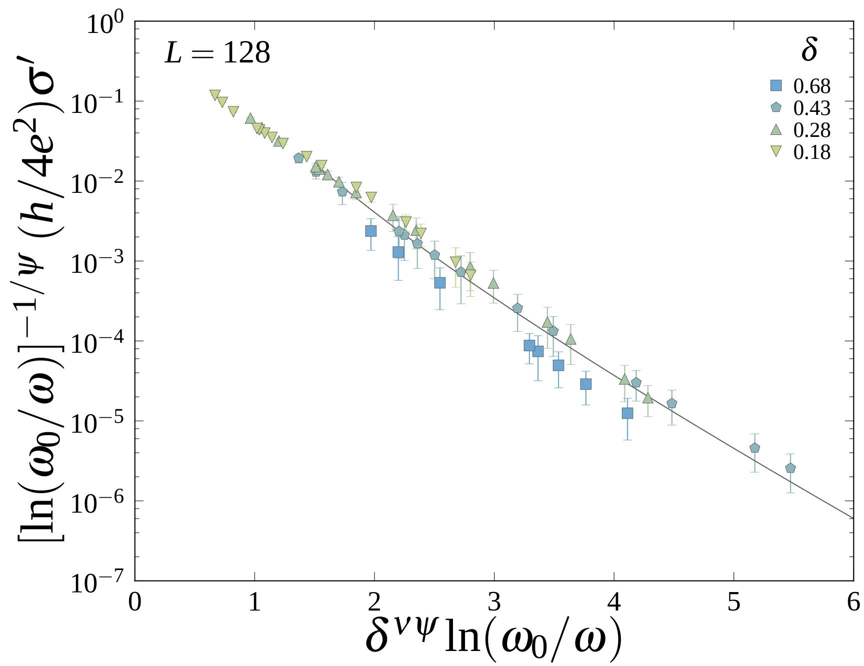
at criticality: $$\sigma'(\omega) \sim [\ln(\omega_0/\omega)]^{1/\psi} = [\ln(\omega_0/\omega)]^2$$

off criticality, in metallic Griffiths phase:

$$\sigma'(r,\omega) = \frac{4e^2}{h}\left(\ln\frac{\omega_0}{\omega}\right)^{1/\psi}\Phi_\sigma\left(r^{\nu\psi}\ln\frac{\omega_0}{\omega}\right)$$

- reflects exotic activated scaling at infinite-randomness CP





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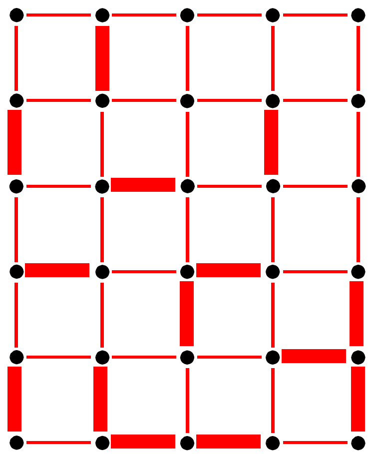
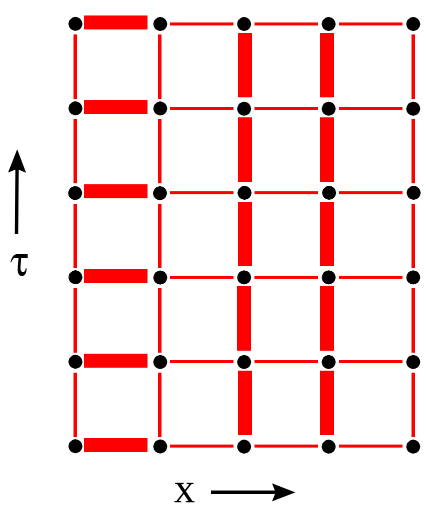
- What is the reason for the exotic infinite-randomness critical behavior?

Why disorder is generically stronger at a QPT?



weak disorder at a classical PT



weak disorder at a QPT

Quenched disorder: impurities, defects, other imperfections

Weak (random-T_c) disorder:
spatial variation of coupling strength but no change in character of the ordered phase

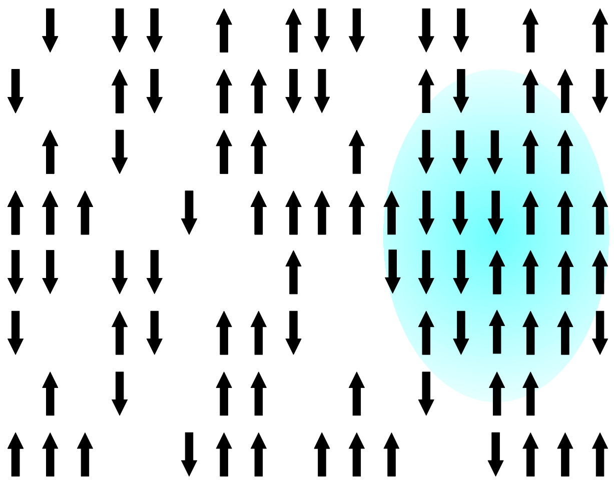
Classical phase transitions

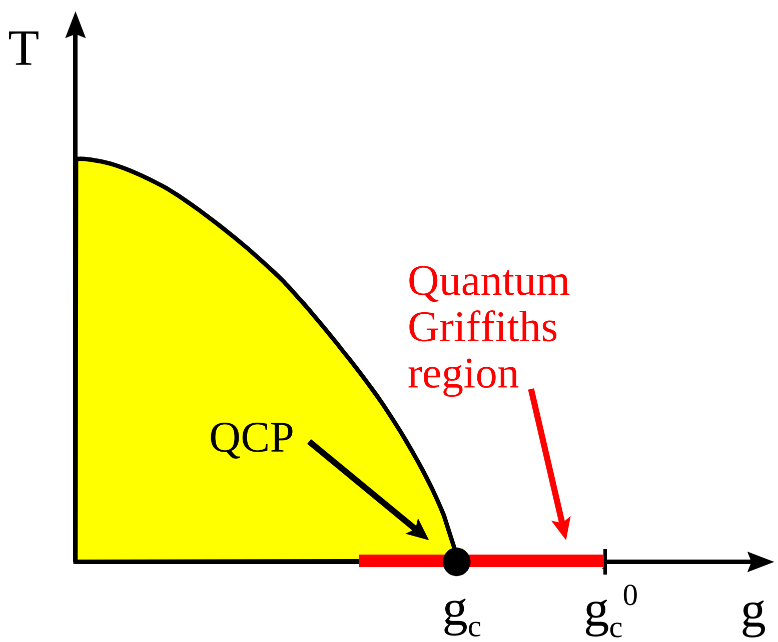
- disorder can destabilize clean critical point
- if Harris criterion $d\nu > 2$ is violated $\Rightarrow$ new critical point

Quantum phase transitions

- disorder perfectly correlated in time direction
- stronger effects at QPTs than at classical transitions
- exotic critical points with non-power-law scaling

Rare regions and Griffiths singularities





Rare regions:

- large spatial regions devoid of impurities (or more strongly coupled then the bulk)
- can be locally in the ordered phase even if bulk is disordered
- extremely slow dynamics $\Rightarrow$ large contribution to thermodynamics
- Griffiths singularities close to transition

Dissipation:

- slows down critical dynamics
- further enhances disorder effects

Classification of weakly disordered phase transitions according to importance of rare regions

T. Vojta, J. Phys. A **39**, R143–R205 (2006)

Dimensionality of rare regions	Griffiths effects	Dirty critical point	Examples (classical PT, QPT, non-eq. PT)
$d_{RR} < d_c^-$	weak exponential	conv. finite disorder	class. magnet with point defects dilute bilayer Heisenberg model
$d_{RR} = d_c^-$	strong power-law	infinite randomness	Ising model with linear defects random quantum Ising model disordered directed percolation (DP)
$d_{RR} > d_c^-$	RR become static	smeared transition	Ising model with planar defects itinerant quantum Ising magnet DP with extended defects

Conclusions

- We have performed a strong-disorder renormalization group study of the QPT in **disordered dissipative systems** with continuous symmetry order parameters
- 1D: analytical solution gives exotic **infinite-randomness** critical point in the universality class of the random transverse-field Ising model
- 2D: numerical solution displays analogous scenario, exponent values different
  3D: preliminary numerical results point in same direction

- We have applied the theory to the **superconductor-metal QPT** of nanowires
- Dynamical (optical) conductivity shows logarithmic low-frequency dependence and **activated** scaling behavior at the transition
- scaling theory generalizes to the (pair-breaking) superconductor-metal QPT in higher dimensions

For details see: Phys. Rev. Lett. **99**, 230601 (2007), Phys. Rev. B **79**, 024401 (2009), arXiv:1006.3793

Interplay between disorder and dissipation leads to exotic quantum critical behavior.