Avalanche-Size Distributions: beyond toy models and mean field

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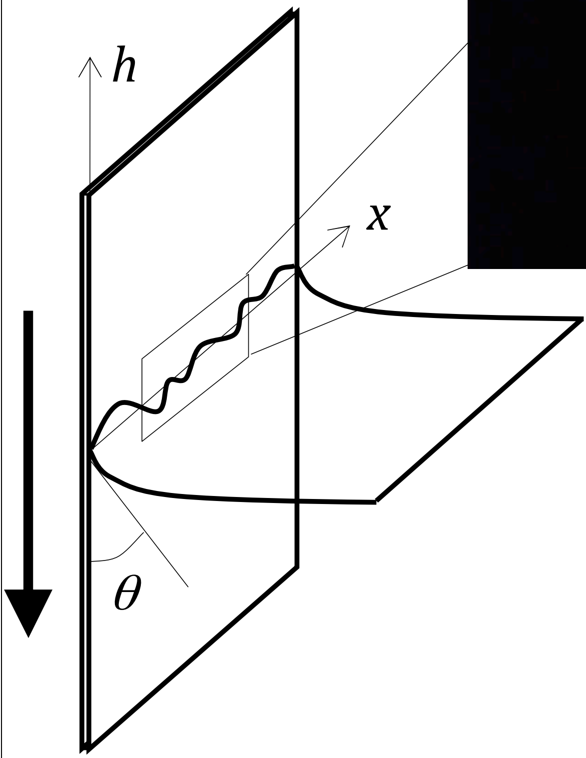
KITP, 2.9.2010

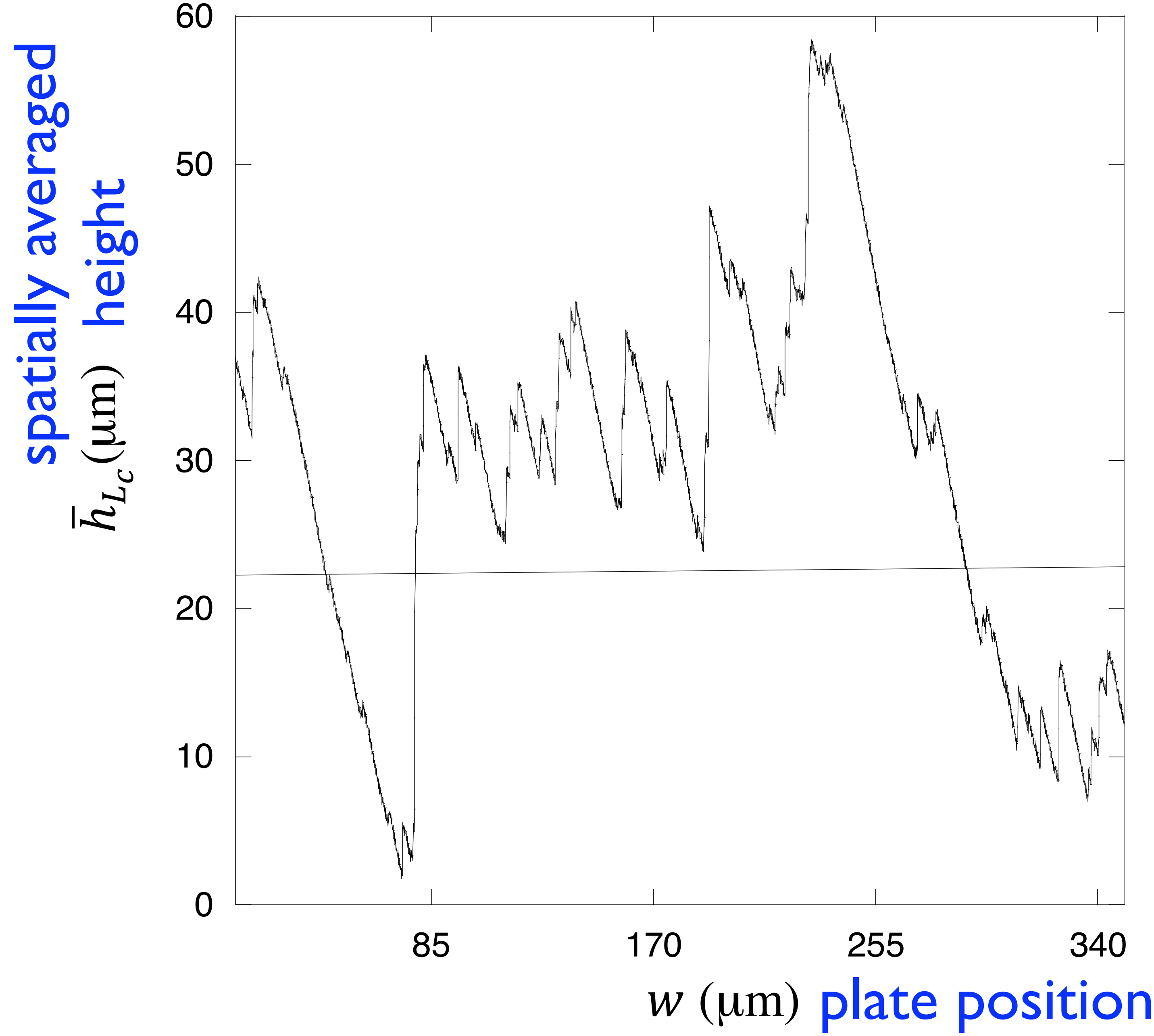
EPL, 87 (2009) 56001, Phys. Rev. E 79, 051106 (2009)

Contact line wetting



- isobutanol on a randomly silanized silicon wafer
- hydrogen on disordered Cesium substrate

The height as a function of the plate position



What can we learn out of this?

The model

Displacement field $\qquad x \in \mathbb{R} \longrightarrow u(x) \in \mathbb{R}$

Elastic energy: $\qquad \mathcal{H}_{\mathrm{el}} = \frac{1}{2} \int \frac{\mathrm{d}^d k}{2\pi} \left|\tilde{u}_k\right|^2 \varepsilon_k + \int_x \frac{m^2}{2} [u(x) - w]^2$

for contact angle $\theta = 90^\circ$: $\qquad \varepsilon_k \approx \sqrt{k^2 + \kappa^2} - \kappa$

$\kappa^{-1} = m^{-2}$ kapillary length $\qquad$ (instead of $\varepsilon_k = k^2$)

Disorder energy $\qquad \mathcal{H}_{\mathrm{DO}} = \int \mathrm{d}^d x V(x, u(x))$

with correlations $\qquad \overline{V(x,u)V(x',u')} = \delta^d(x - x')R(u - u')$

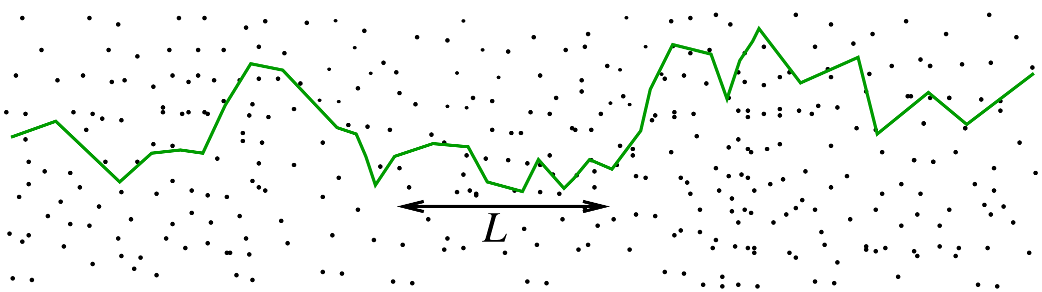
Simple theory for zero temperature $T = 0$

Suppose $R(u)$ is analytic. Then to all orders in perturbation theory:

$$\left\langle [u(x) - u(0)]^2 \right\rangle \sim -R''(0) x^{4-d} + O(T)$$

shift in dimension by two from thermal 2-point function
$\left\langle [u(x) - u(0)]^2 \right\rangle = T x^{2-d}$: dimensional reduction.

Experimentally wrong beyond Larkin length:



elastic energy $\mathscr{E}_{\rm el} = c\, L^{d-2}$

disorder $\mathscr{E}_{\rm DO} = \bar{f} \left(\frac{L}{r}\right)^{d/2}$

$$\mathscr{E}_{\rm el} = \mathscr{E}_{\rm DO} \quad \Rightarrow \quad L_c = \left(\frac{c^2}{\bar{f}^2} r^d\right)^{\frac{1}{4-d}}$$

critical dimension is $d_c = 4$

u dimensionless in $d_c = 4$ $\Rightarrow$ all powers of u relevant!

Need functional RG!

Old idea: Wegner, Houghton (1973)

for disordered systems: D.S. Fisher (1985)

Functional renormalization group (FRG)

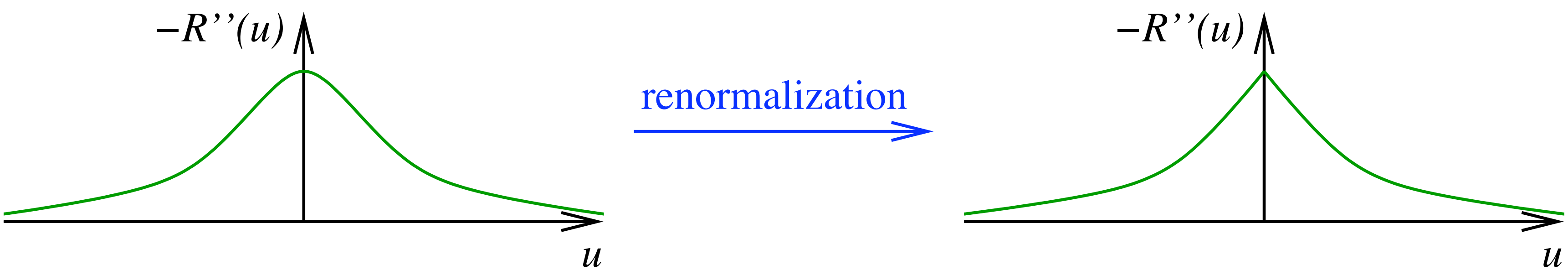
(D. Fisher 1986)

$$\frac{\mathscr{H}[u]}{T} = \frac{1}{2T}\sum_{\alpha=1}^{n}\left[\int_k \varepsilon_k |\tilde{u}_k^\alpha|^2 + \int_x m^2(u^\alpha(x)-w)^2\right]$$
$$-\frac{1}{2T^2}\int_x \sum_{\alpha,\beta=1}^{n} R\big(u^\alpha(x)-u^\beta(x)\big)$$

Functional renormalization group equation (FRG) for the disorder correlator $R(u)$ at 1-loop order:

$$-\frac{m\mathrm{d}}{\mathrm{d}m}R(u) = (\varepsilon-4\zeta)R(u) + \zeta u R'(u) + \frac{1}{2}R''(u)^2 - R''(u)R''(0)$$

Solution for force-force correlator $-R''(u)$:



Cusp: $R''''(0) = \infty$ appears after finite RG-time (at Larkin-length)

FRG at 2-loop order

$$\partial_\ell R(u) = (\varepsilon - 4\zeta)R(u) + \zeta u R'(u) + \frac{1}{2}R''(u)^2 - R''(u)R''(0)$$
$$+\frac{1}{2}\left[R''(u) - R''(0)\right]R'''(u)^2 + \lambda\frac{1}{2}R'''(0^+)^2 R''(u)$$

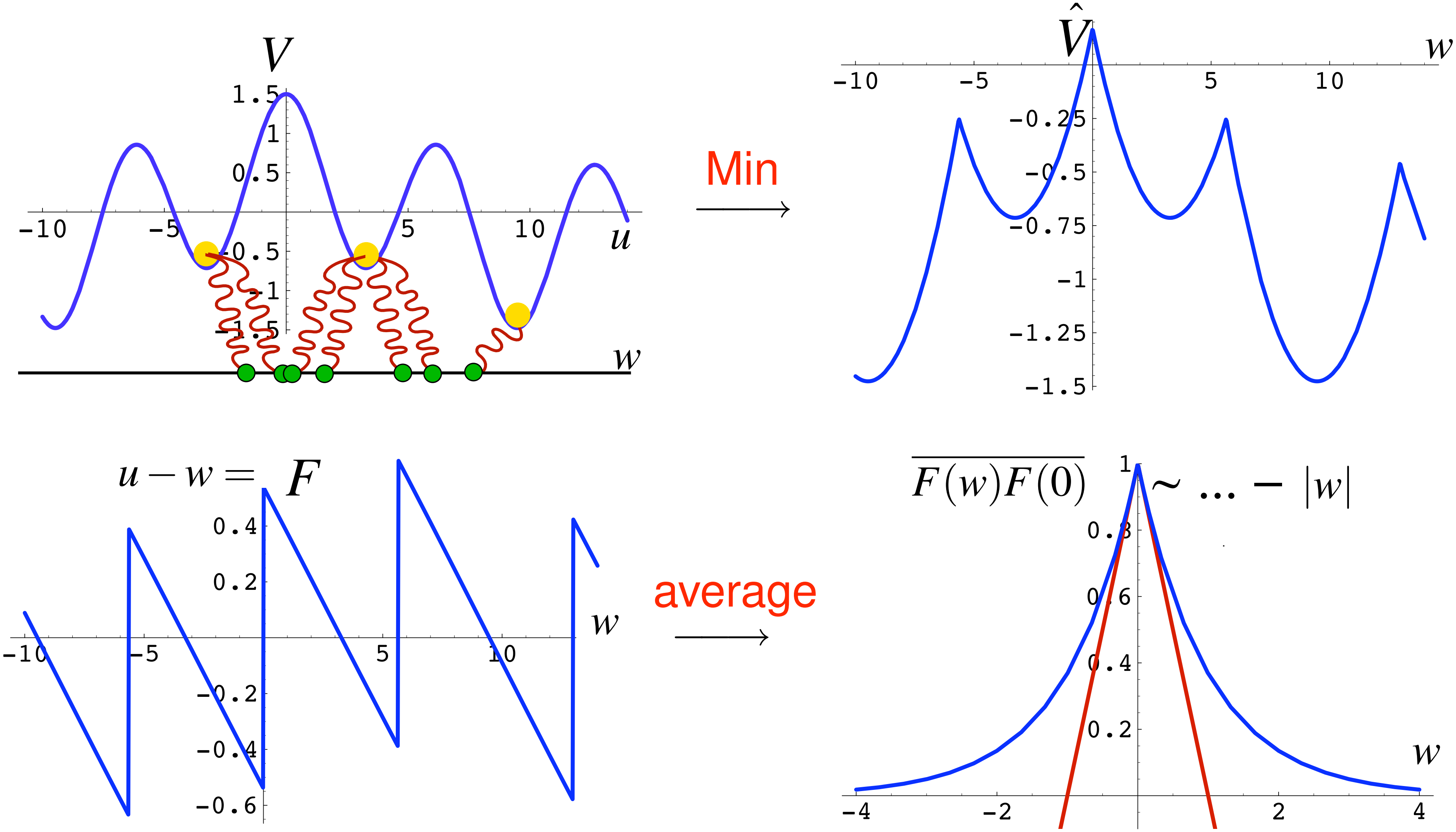
$\lambda = -1$ statics, $\lambda = 1$ (depinning)

Universality classes

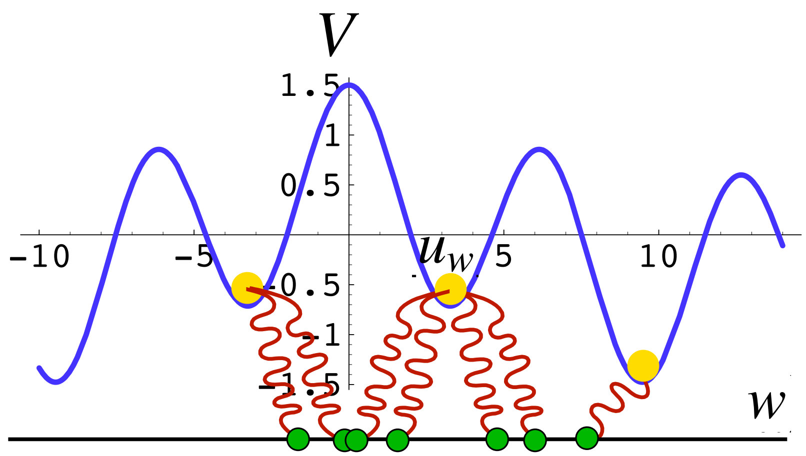
- periodic disorder
- random field disorder: $\Delta(u) = -R''(u)$ short-ranged
  statics: $\zeta = \frac{\varepsilon}{3}$ (exact), depinning $\zeta = \frac{\varepsilon}{3}(1 + 0.14331\varepsilon + \ldots)$
- random bond: $R(u)$ short-ranged
  statics: $\zeta = 0.20829804\varepsilon + 0.006858\varepsilon^2$, dynamics $\to$ RF

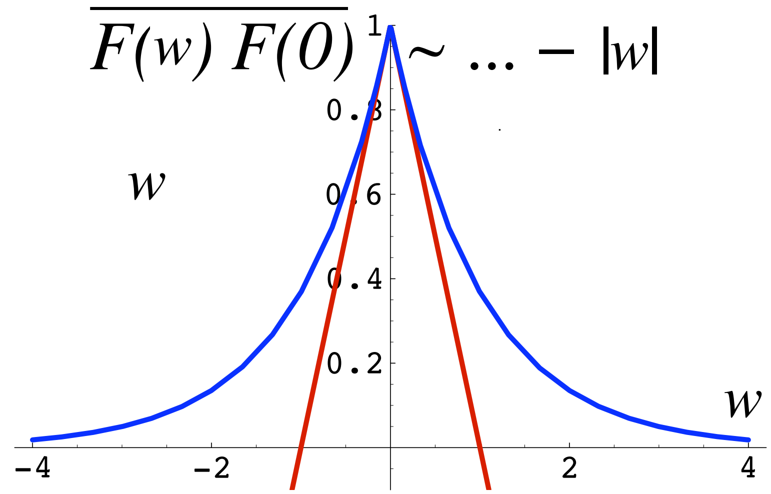
Why is a cusp necessary?

…calculate effective action for single degree of freedom…



Generalization for a manifold



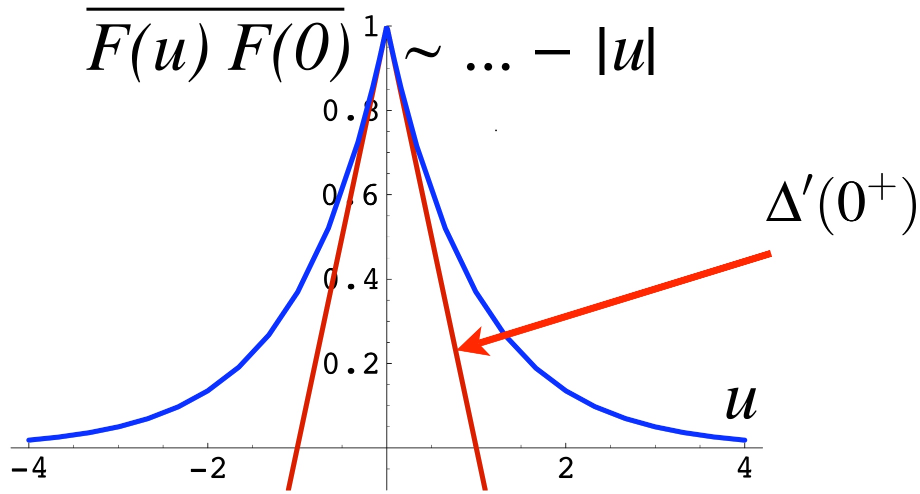


center-of-mass position of the interface: $u_w := \frac{1}{L^d}\int_x u_w(x)$. Then

$$\overline{h_w h_{w'}}^c = \overline{[u_w - w][u_{w'} - w']}^c = m^{-4}L^{-d}\Delta_m(w-w') \equiv -m^{-4}L^{-d}R''_m(w-w')$$

(Le Doussal 2006)

Disorder Force-Force Correlator versus Avalanche Moments

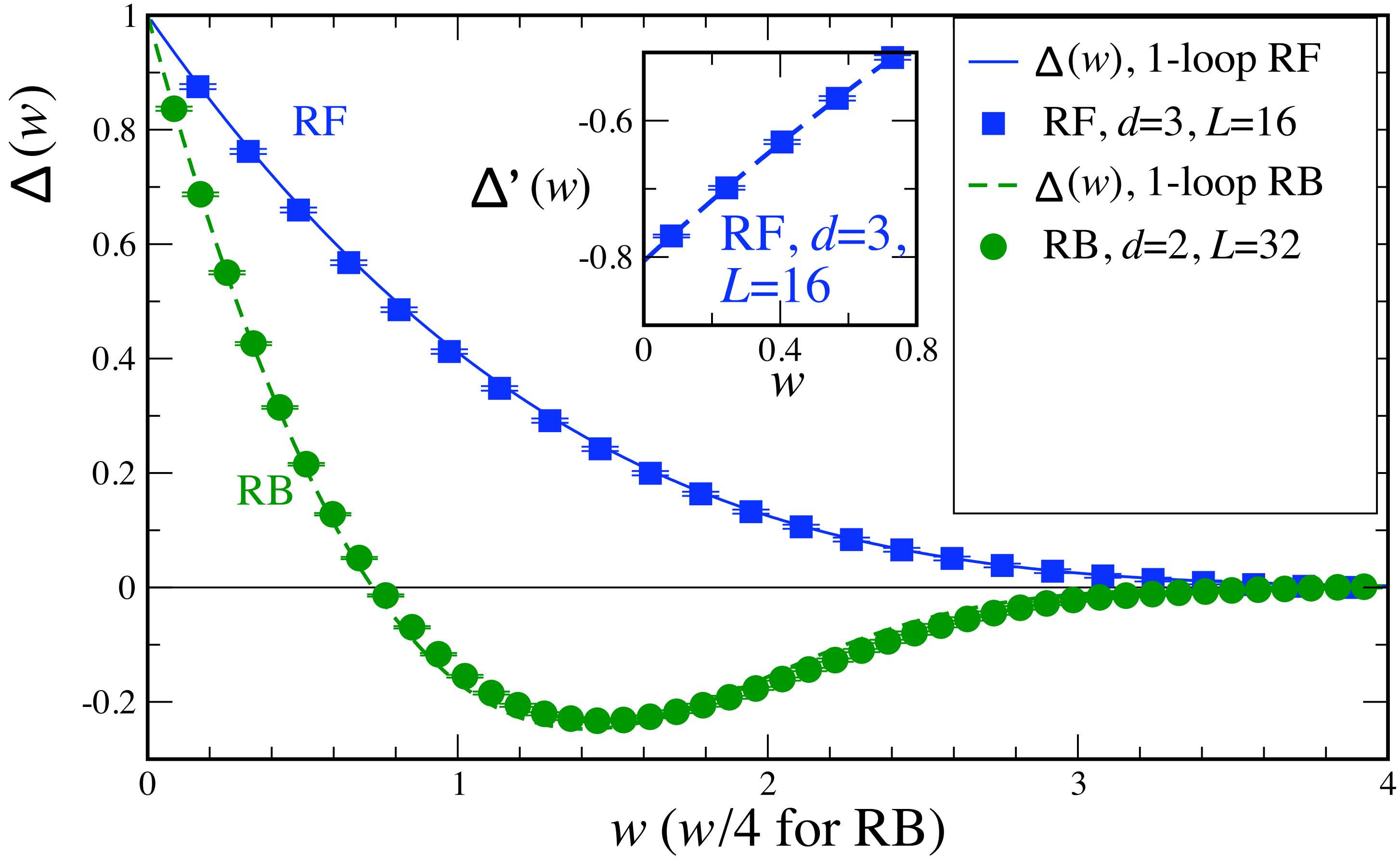


$$\frac{|\Delta'(0^+)|}{m^4} = \frac{\langle S^2 \rangle}{2\,\langle S \rangle}$$

P. Le Doussal + K.J. Wiese, PRE 79 (2009) 050101

Measuring the cusp = effective action

PLD+KW+A. Middleton

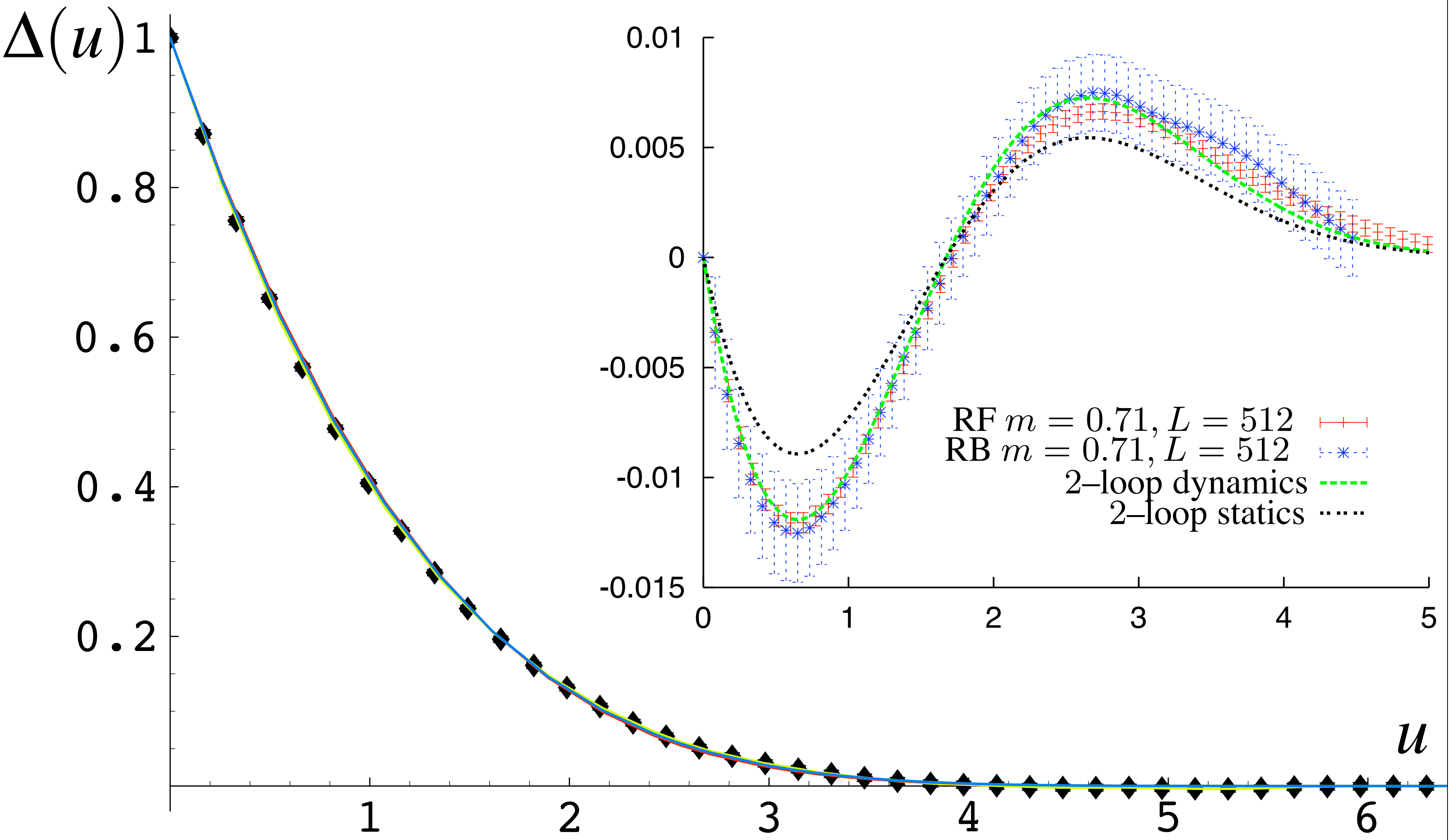


$$\Delta(w - w') = m^4 L^d \overline{[u_w - w] [u_{w'} - w']}$$

Δ = renormalized disorder correlator

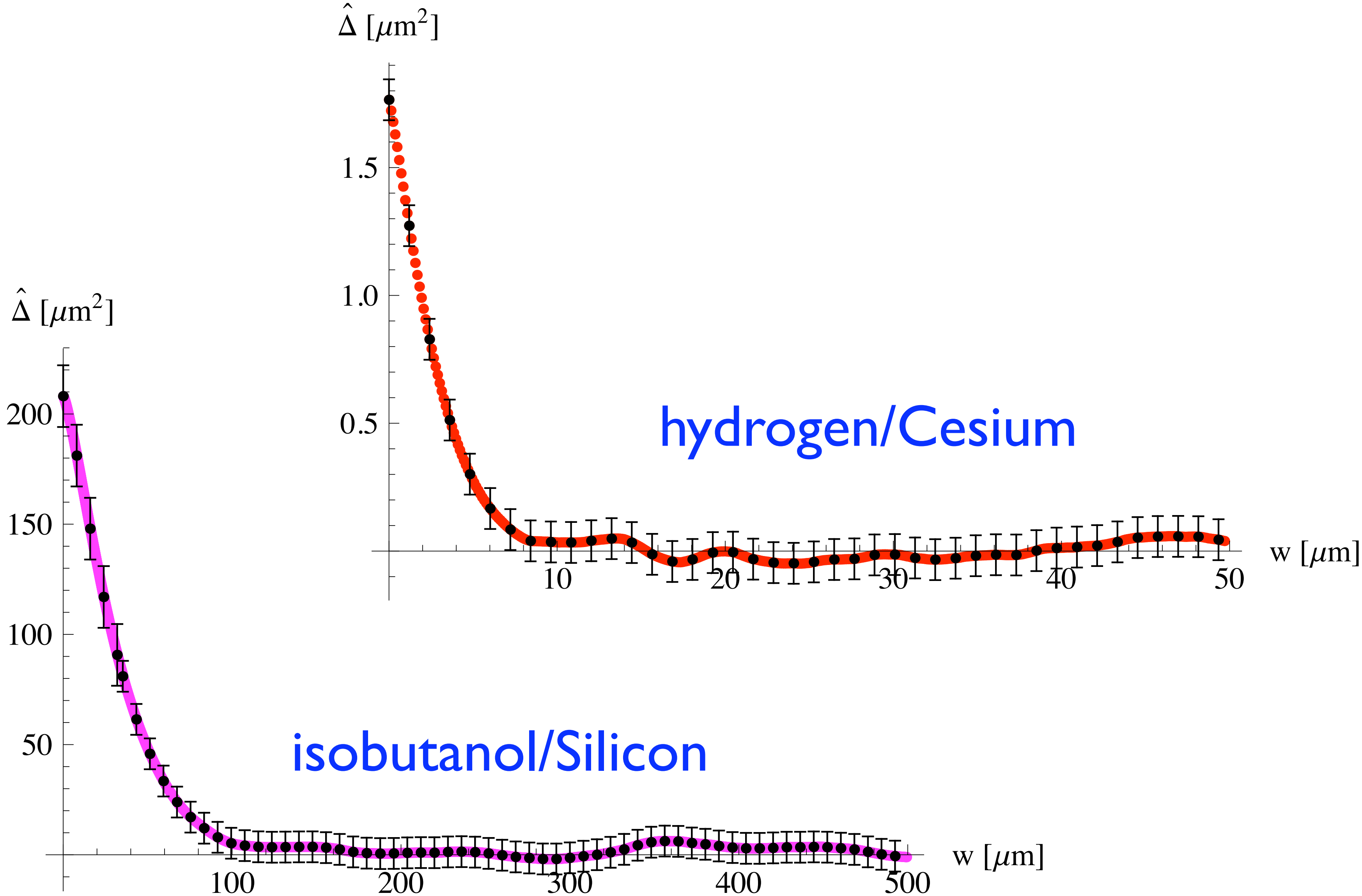
Depinning in 1+1 dimensions

$\zeta = \frac{\varepsilon}{3} + 0.04777\varepsilon^2$: 1.0 (1 loop), 1.2 ± 0.2 (2 loop), 1.25 (numerics).

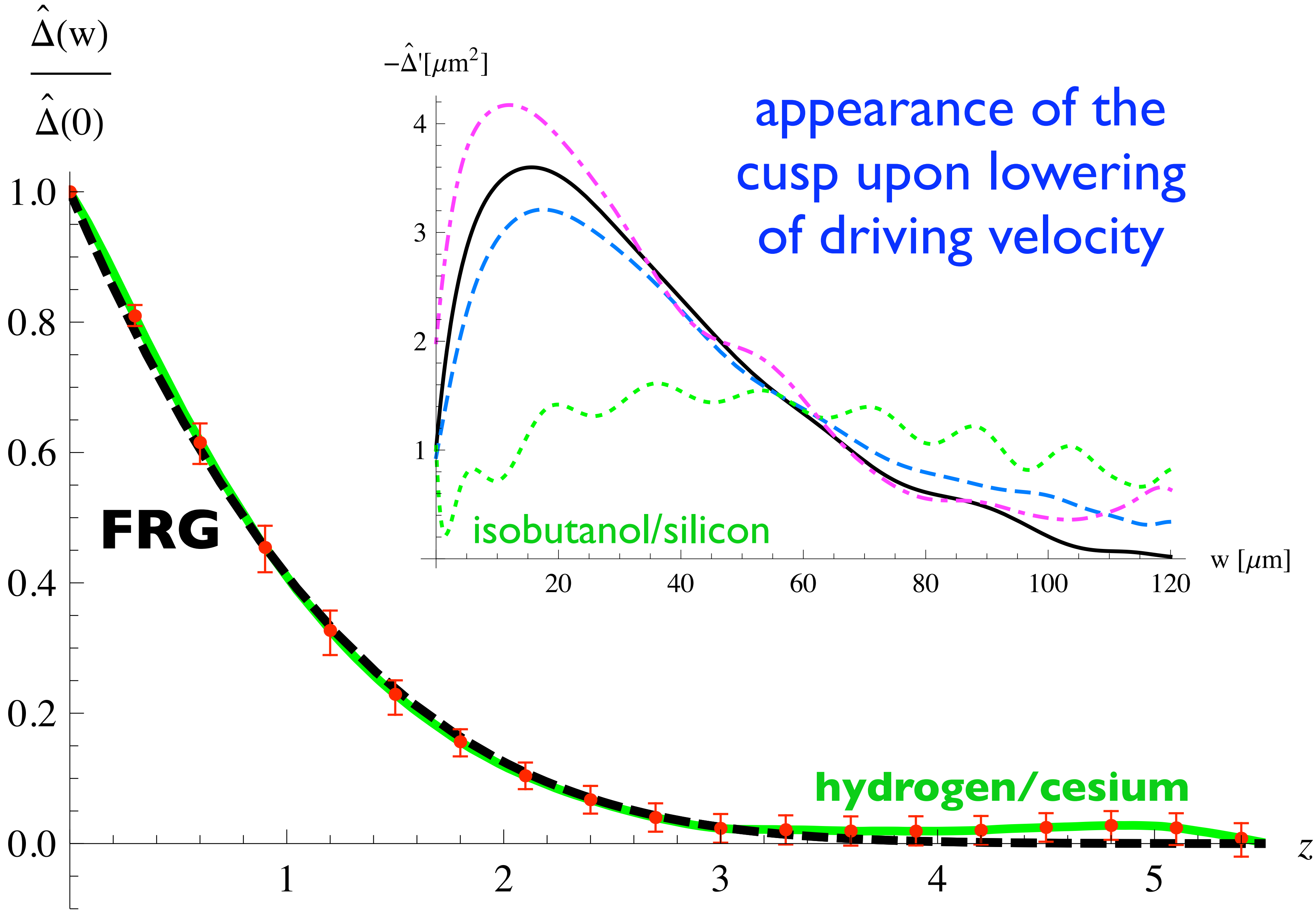


A. Rosso, P. Le Doussal, KW, PRB 75 (2007) 220201

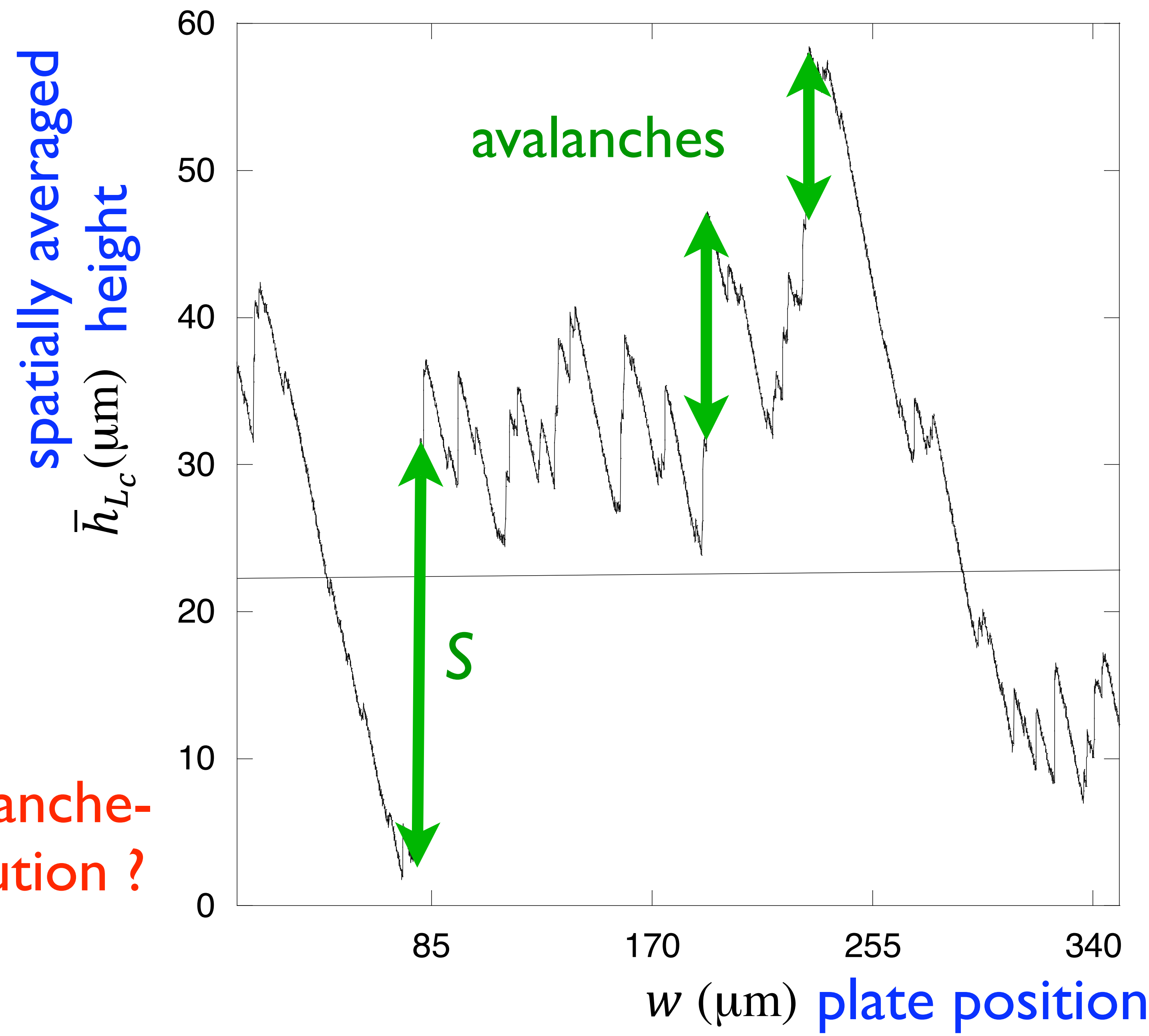
Experiments on contact line



The renormalized force-force correlator

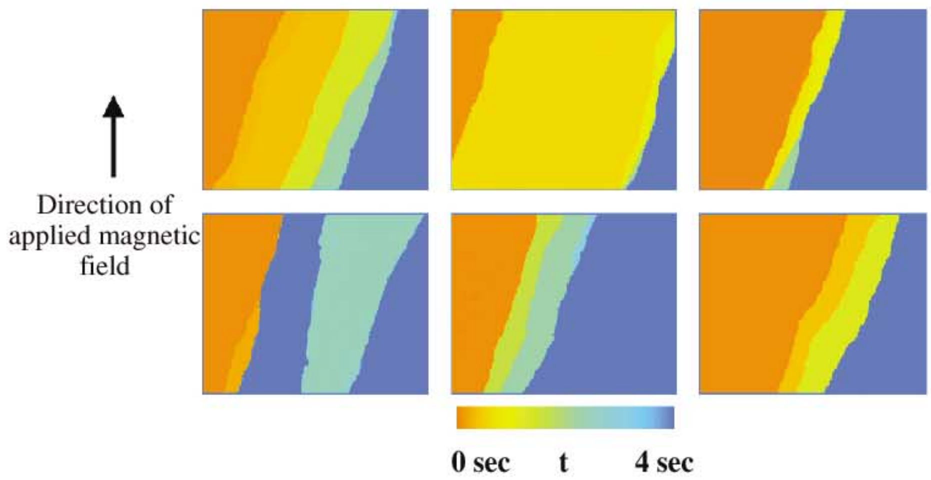


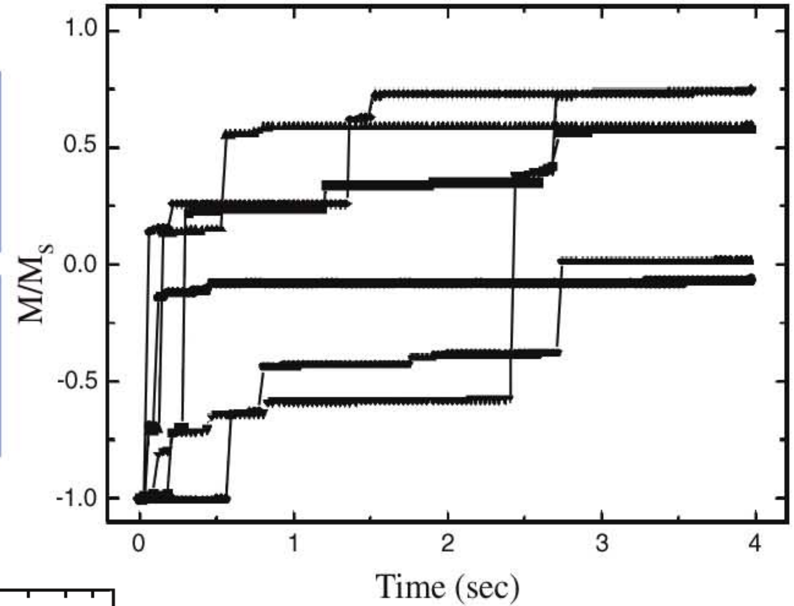
height jumps = avalanches

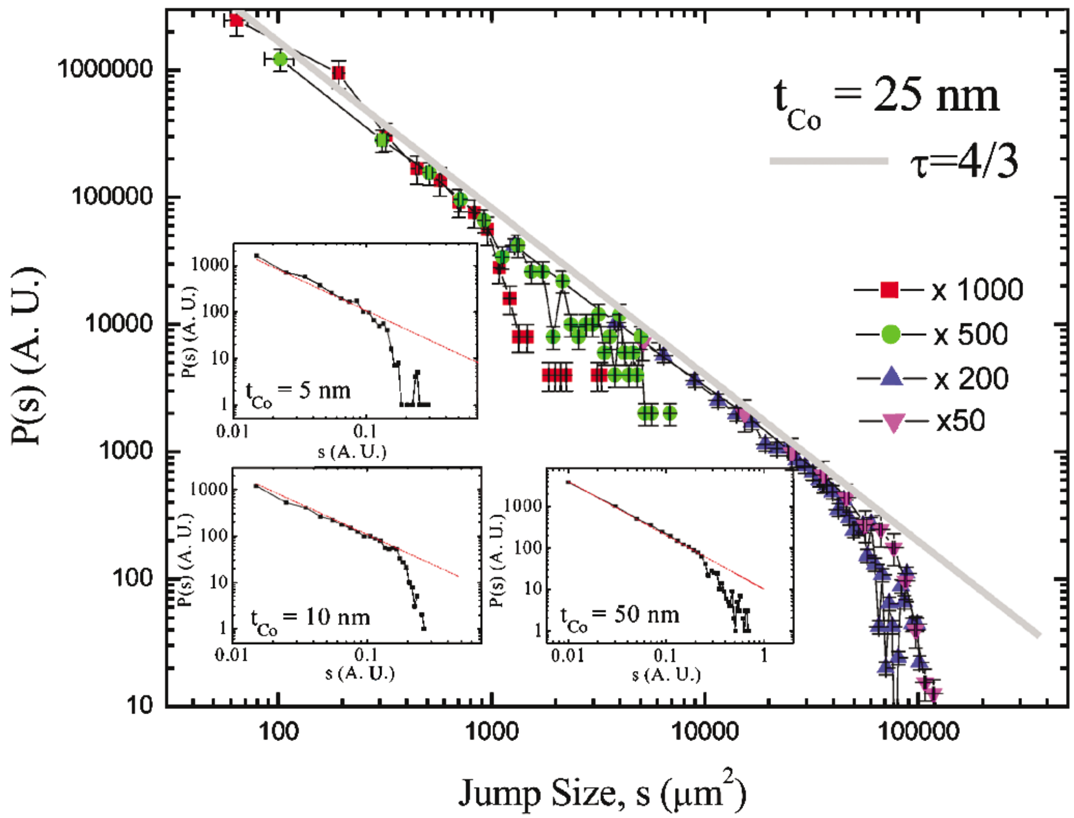


what is avalanche-size distribution ?

$$P(S)$$





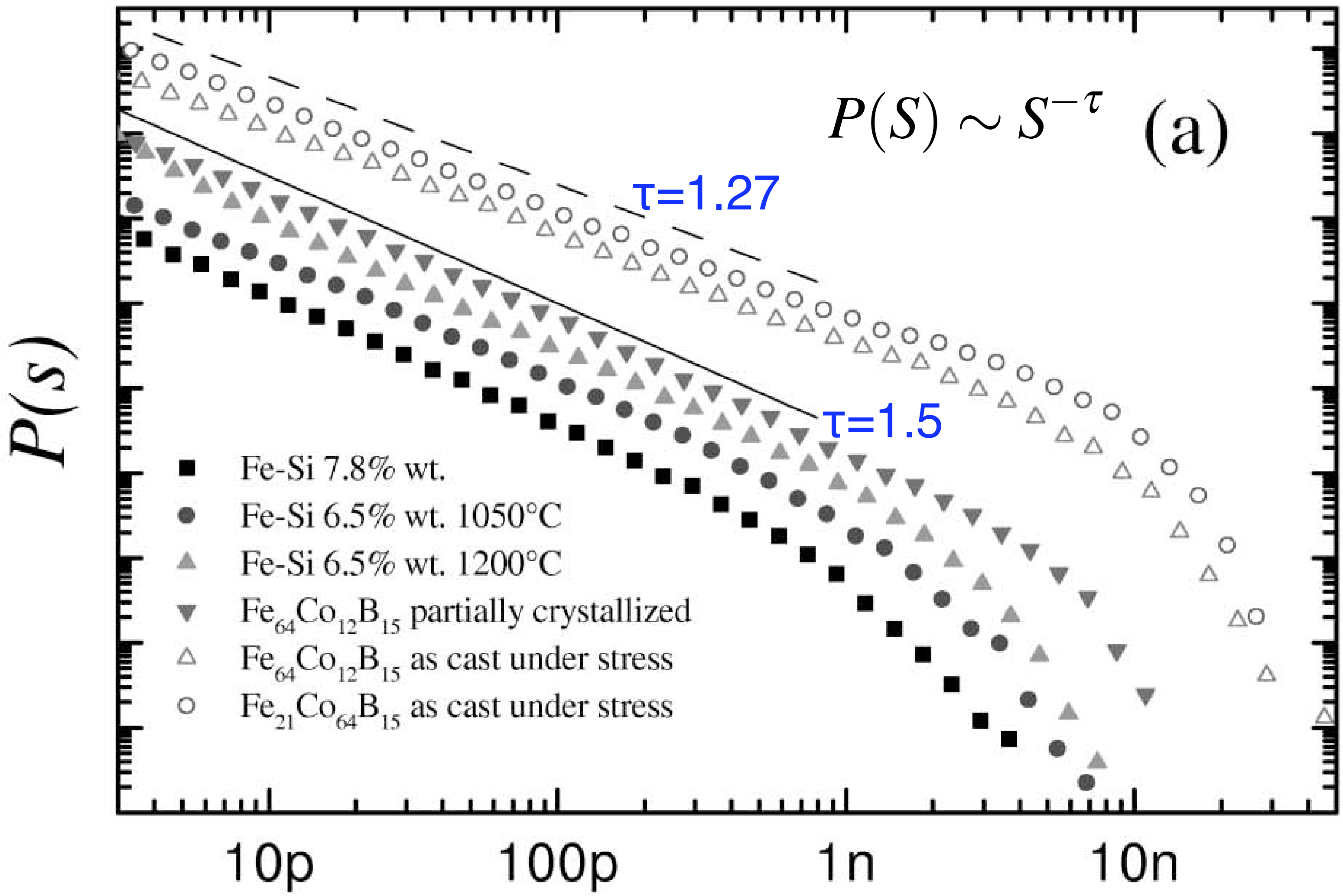


Kim, Choe, Shin, PRL 90 (2003) 87203

$$P(S) \sim S^{-\tau}$$

for small avalanches

Barkhausen Noise Experiments



Durin, Zapperi, PRL 84 (2000) 4705

Spin Glasses (SK model)

$$\rho_{\overline{h}}(\Delta m) = \Delta m \int_{q_m^-}^{q_c} dq\, \nu_{\overline{h}}(q) \frac{\exp[-\frac{(\Delta m)^2}{4(q_c - q)}]}{\sqrt{4\pi(q_c - q)}} \theta(\Delta m)$$

$$P(S) \sim S^{-1}$$

uses Mean Field methods with RSB

Le Doussal, Müller, Wiese, arXive: 1007.2069, EPL to appear.

Avalanches

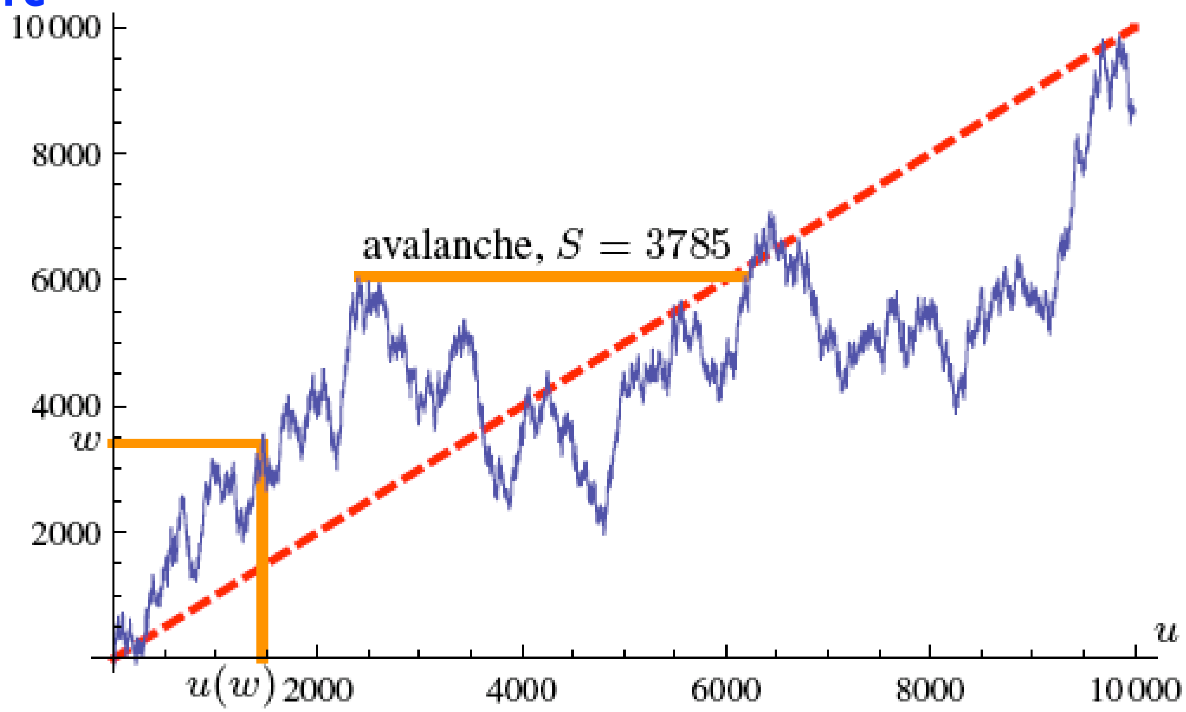
- avalanches appear in many systems: contact-lines, vortex lattices, domain walls, earthquakes, spin glaces, etc.
- Self-Organized Criticality (SOC)
- Abelian Sandpile Model (ASM) is best-known example
- conjecture by Middleton-Narayan that Charge-Density Waves (CDW) are equivalent to ASM
- Galton process = Mean-Field (MF) = ABBM
- conjecture by Narayan-Fisher:

$$P(S) \sim S^{-\tau}\,, \qquad \tau = 2 - \frac{2}{d+\zeta}$$ short-range elasticity

$$\tau = 2 - \frac{1}{d+\zeta}$$ long-range elasticity

The Galton process

- old question: survival probability of male line (Galton, Watson1873)
- equivalent: driven particle in random force landscape which itself is a Brownian = records with drift



$$P(S) \sim S^{-\frac{3}{2}}\,\mathrm{e}^{-S}$$

FRG calculation

Second Kolmogorov cumulant:

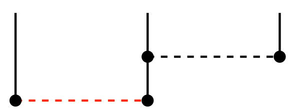
$$\hat{K}^{(2)}(w) := \overline{[u(w)-u(0)-w]^2}^{c} = \mathscr{K}$$

$$= 2\,[\Delta(0)-\Delta(w)] = -2\Delta'(0^+)|w| + \mathscr{O}(w^2)$$

$$= \frac{\langle S^2\rangle}{\langle S\rangle}|w| + \mathscr{O}(w^2)$$

Third Kolmogorov cumulant ($w>0$):

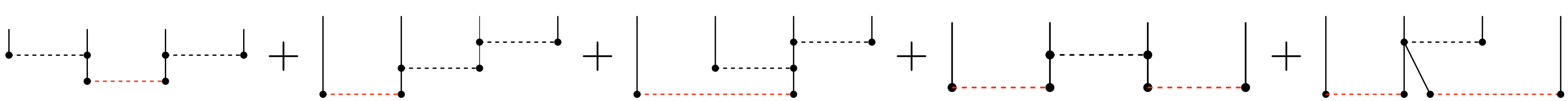
$$m^2\hat{K}^{(3)}(w) = \mathscr{K}$$



$$= -12\,[\Delta(0)-\Delta(w)]\,\Delta'(w) \approx 12\Delta'(0^+)^2 w + \mathscr{O}(w^2)$$

$$= \frac{\langle S^3\rangle}{\langle S\rangle}|w| + \mathscr{O}(w^2)$$

Fourth cumulant ($w>0$):

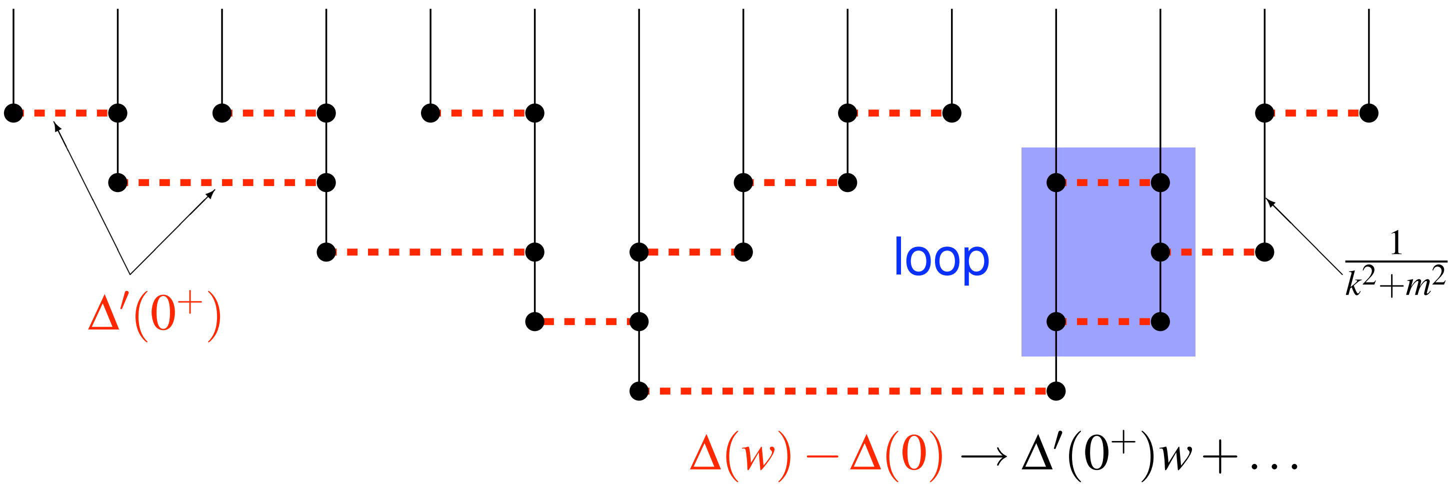


FRG-calculation

calculate the generating function $Z(\lambda)$ of avalanche-sizes S:

$$Z(\lambda) = \frac{1}{\langle S \rangle}\left(\langle e^{\lambda S}\rangle - 1 - \lambda \langle S \rangle\right)$$

$$\overline{e^{\lambda[u(w)-w-u(0)]}} - 1 = Z(\lambda)w + O(w^2) \quad \text{for} \quad w > 0\ .$$

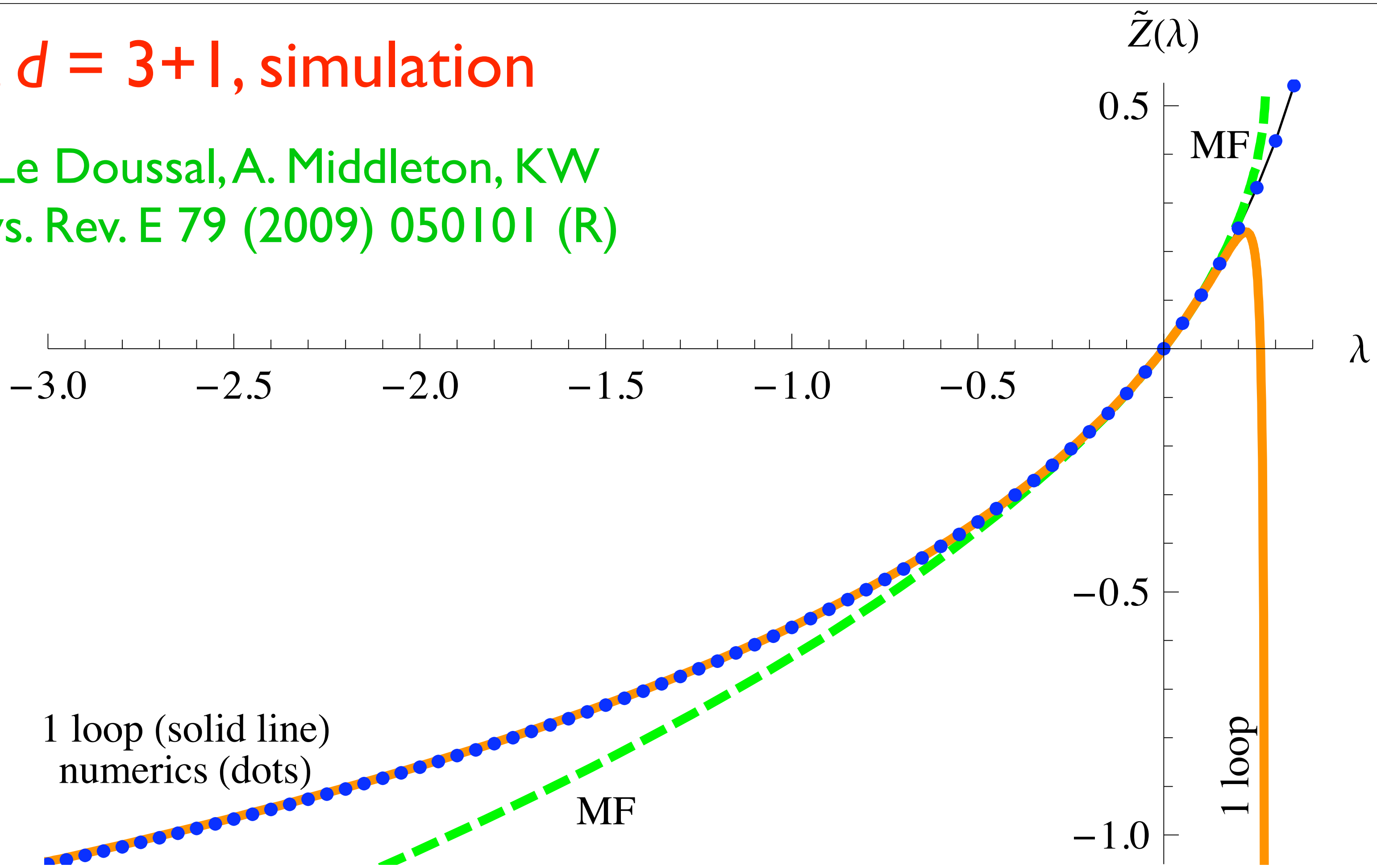
$Z(\lambda) = \sum$ 

Recursion Relation:

$$Z(\lambda) = \lambda - \underbrace{\Delta'(0^+)Z(\lambda)^2}_{\text{trees}} + \frac{\Delta''(0)}{\Delta'(0^+)}\sum_{n\geq 3}(n+1)2^{n-2}\int_k \underbrace{\frac{[-\Delta'(0^+)\,Z(\lambda)]^n}{(k^2+1)^n}}_{\text{loops with } n \text{ outgoing legs}}\ ,$$

RF, d = 3+1, simulation

P. Le Doussal, A. Middleton, KW
Phys. Rev. E 79 (2009) 050101 (R)



MF = trees

$$Z(\lambda) = \frac{1}{2}\left[1-\sqrt{1-4\lambda}\right] - \frac{\Delta''(0)}{4\sqrt{1-4\lambda}}\left[\log(1-4\lambda)(3\lambda+\sqrt{1-4\lambda}-1)-2(2\lambda+\sqrt{1-4\lambda}-1)\right]+\ldots$$

1 loop

Avalanche-size distribution and comparison to experiment without fitting

measure $P(S)$, $\langle S \rangle$ and $\langle S^2 \rangle$

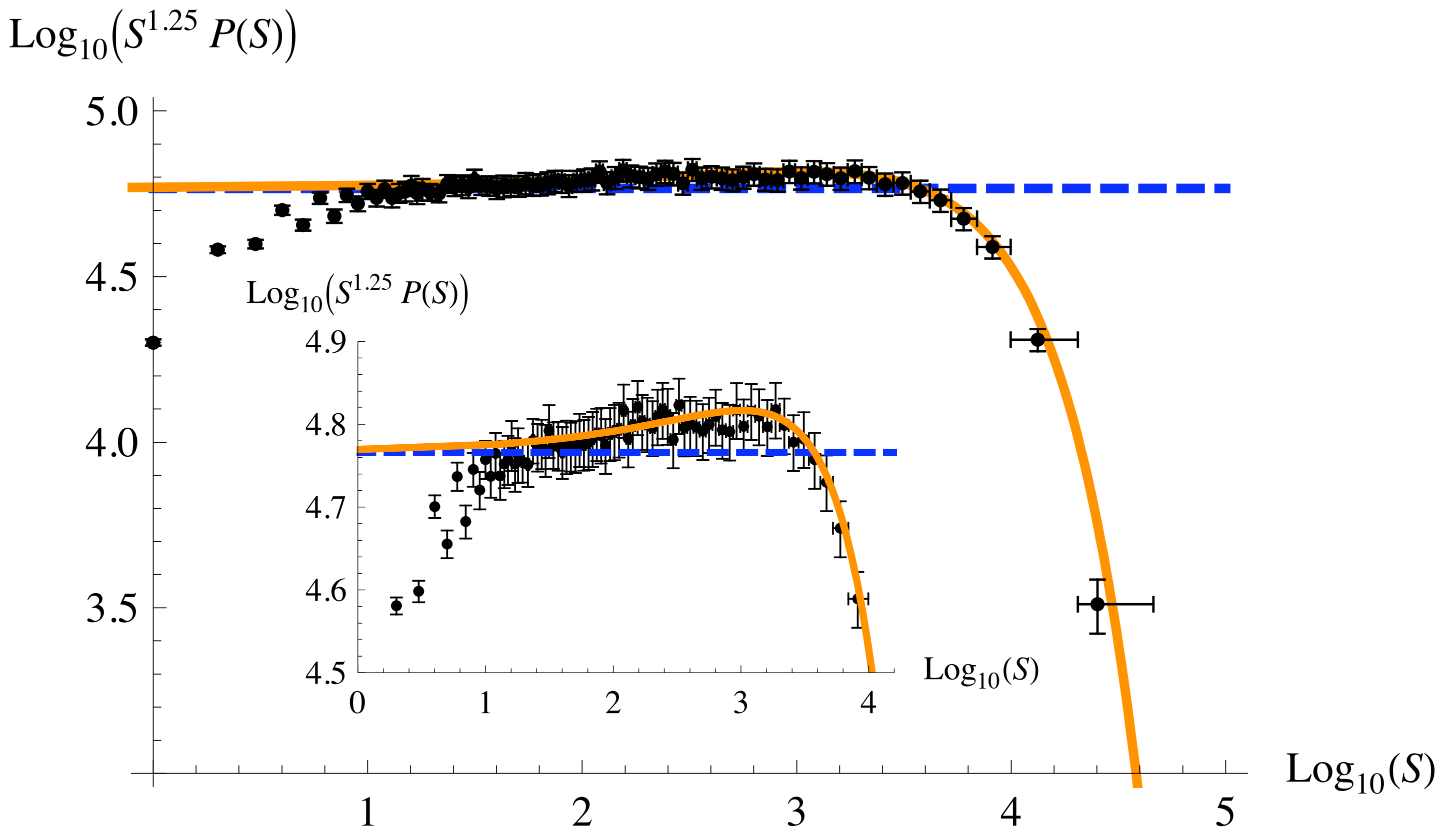
define $$S_m := \frac{\langle S^2 \rangle}{2\langle S \rangle}$$

$$p(s)\,\mathrm{d}s := \frac{S_m}{\langle S \rangle} P(sS_m)\,\mathrm{d}S$$

then $p(s)$ is universal, i.e. $\langle s \rangle_p = 1$, $\langle s^2 \rangle_p = 2$

no fitting parameter!

Avalanche distribution



$$P(S) = \frac{\langle S \rangle}{2\sqrt{\pi}} S_m^{\tau-2} A S^{-\tau} \exp\left(C\sqrt{\frac{S}{S_m}} - \frac{B}{4}\left[\frac{S}{S_m}\right]^{\delta} \right)$$

$$\tau = \frac{3}{2} - \frac{1}{8}(\varepsilon - \zeta) + \ldots$$

$$\delta = 1 + \frac{1}{4}(\varepsilon - \zeta) + \ldots$$

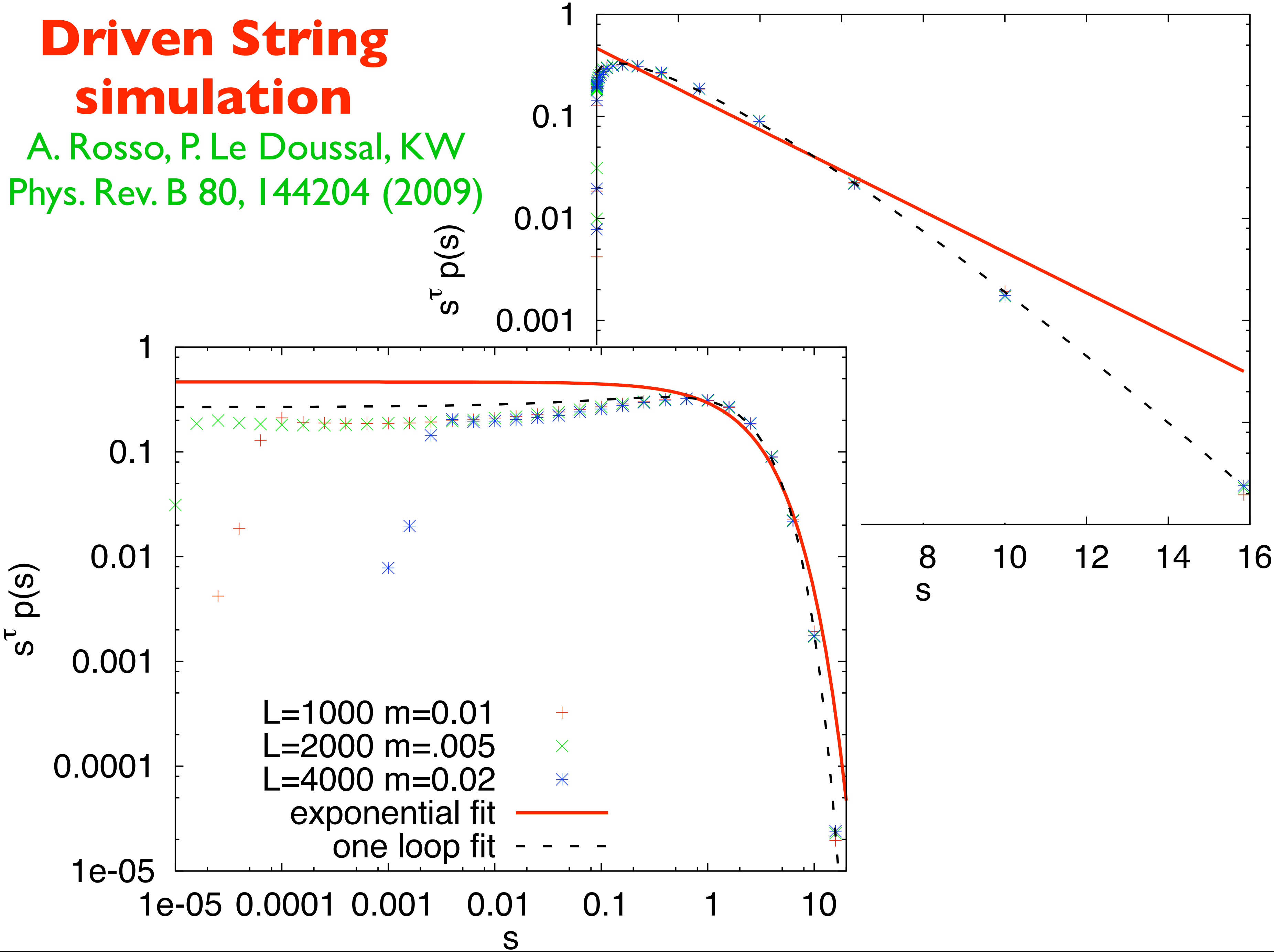
Numerical results for RF interface in $d = 3 + 1$

P. Le Doussal, A. Middleton, KW
Phys. Rev. E 79 (2009) 050101 (R)

Driven String simulation

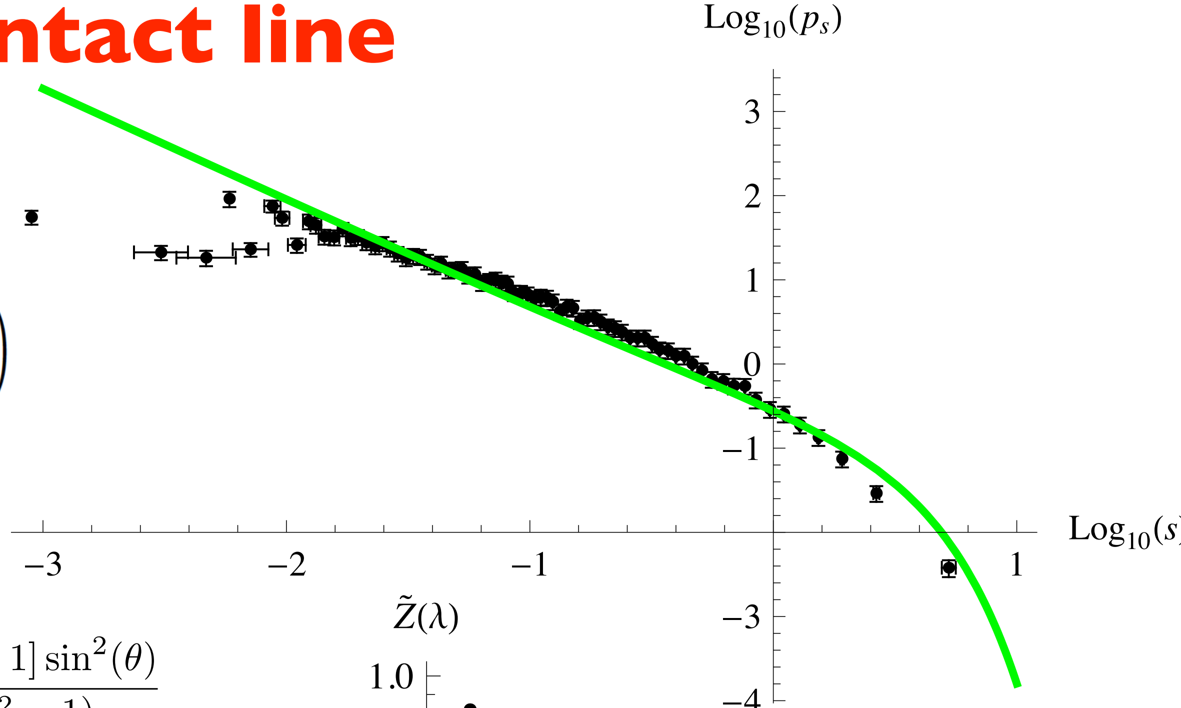
A. Rosso, P. Le Doussal, KW

Phys. Rev. B 80, 144204 (2009)



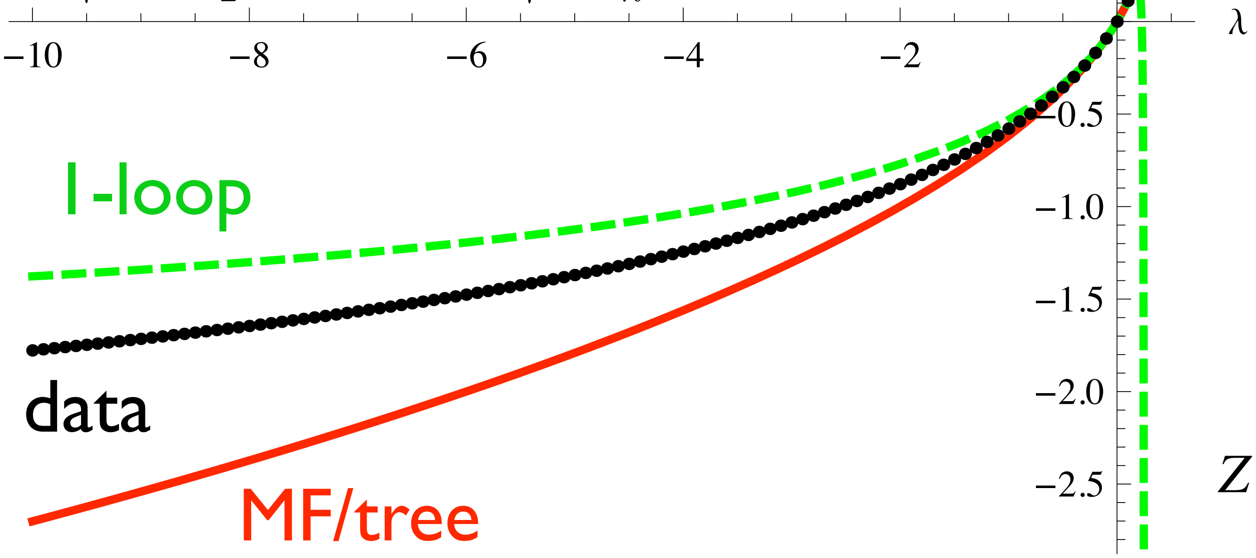
Experiments contact line

$$P(S) = A' \frac{\langle S \rangle}{2\sqrt{\pi}} S_m^{-2} \left[\left(\frac{S}{S_m} \right)^{-\tau} + D' \right] \times \exp\left(C' \sqrt{\frac{S}{S_m}} - \frac{B'}{4} \left[\frac{S}{S_m} \right]^{\delta'} \right)$$



$$\frac{\epsilon_q}{\kappa\gamma} = \frac{\sin(\theta)\cos(\varphi)}{t} + \frac{(r^2-1)\,[t(r+t)+1]\sin^2(\theta)}{t\,(r^2+3rt+3t^2-1)}$$

$$t = \sqrt{\frac{\sin(\theta+\varphi)+1}{2}} \quad , \quad r = \sqrt{1+\frac{q^2}{\kappa^2}}$$



$$Z(\lambda) := \int_0^\infty \mathrm{d}S\, p(S) \left[e^{\lambda S} - 1 \right]$$

Conclusions

- FRG provides a quantitative theory to calculate the distribution of avalanche sizes from first principles
- temporal evolution of avalanches in progress

Movie by A. Kolton