How to Reduce the Dissipation in Glasses

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How to Reduce the Dissipation in Glasses

* Introduction to glasses at low temperatures (specific heat, thermal conductivity, two-level-systems model)
* Introduction to dissipation in glasses (how to measure dissipation and theory)
* How to reduce acoustic dissipation in glasses (high stress increases barrier heights of defects)
* Conclusion

Introduction to glasses

Structural glasses

Solids but no long range order

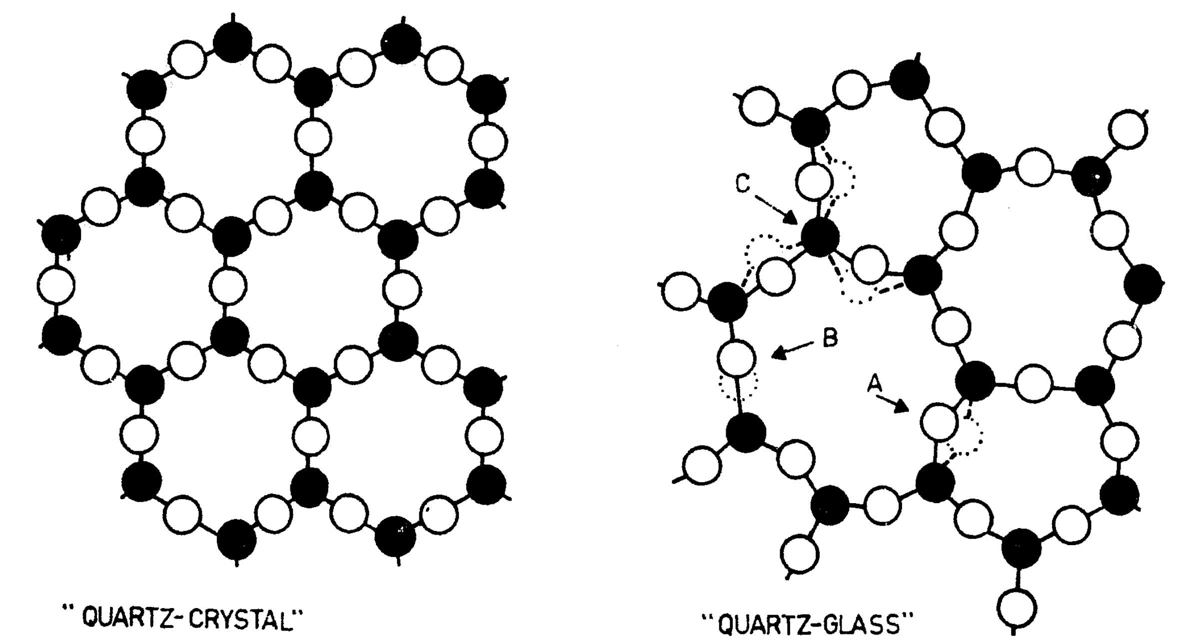


Fig. 1. Schematic representation of crystalline and glassy quartz structure with three possible types of two-state defects in the glass (A, B and C) [19].

from Jackle,et.al., J.Of.Non-cry. Solid. (1976)

Specific heat

CT^{-3}

Glass

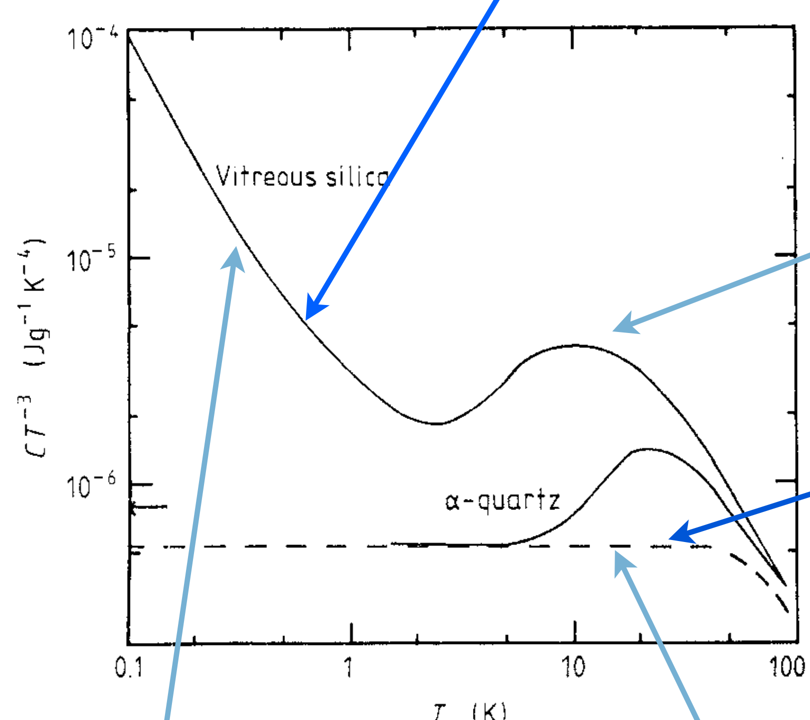


Figure 1. The heat capacity $C(T)$ of vitreous silica and crystalline quartz as a function of temperature T (Jones 1982, after Zeller and Pohl 1971), plotted as C/T^3 against T.

Boson peak

Crystal

T

linear on T

Debye theory

Crystals

Debye theory gives

$$C(T) \propto T^3$$

Glasses

Specific heat

1) is linear in T at low temperature

2) has peak at 10K

from W.A.Phillips,Rep. Prog.Phys. (1987)

Linear T term in specific heat

$$C(T) \sim T$$

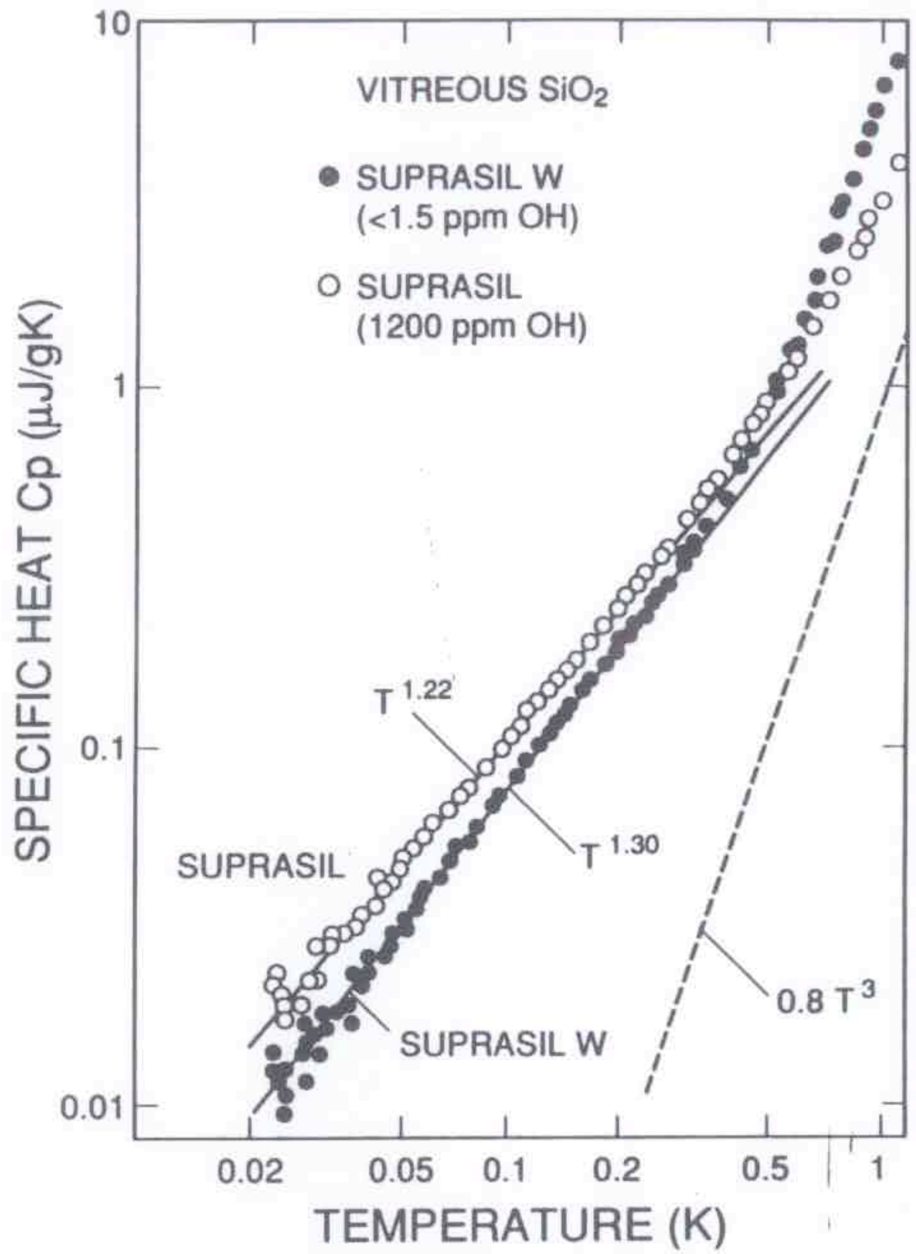


FIGURE 1 Low temperature specific heat of vitreous SiO_2 from Ref. 27. Notice that the specific heat is slightly superlinear.

Zeller and Pohl, (1972)

Seen in a wide variety of glasses

from Yu and Leggett Comments Cond. Mat. Phys. (1988)

Boson peak

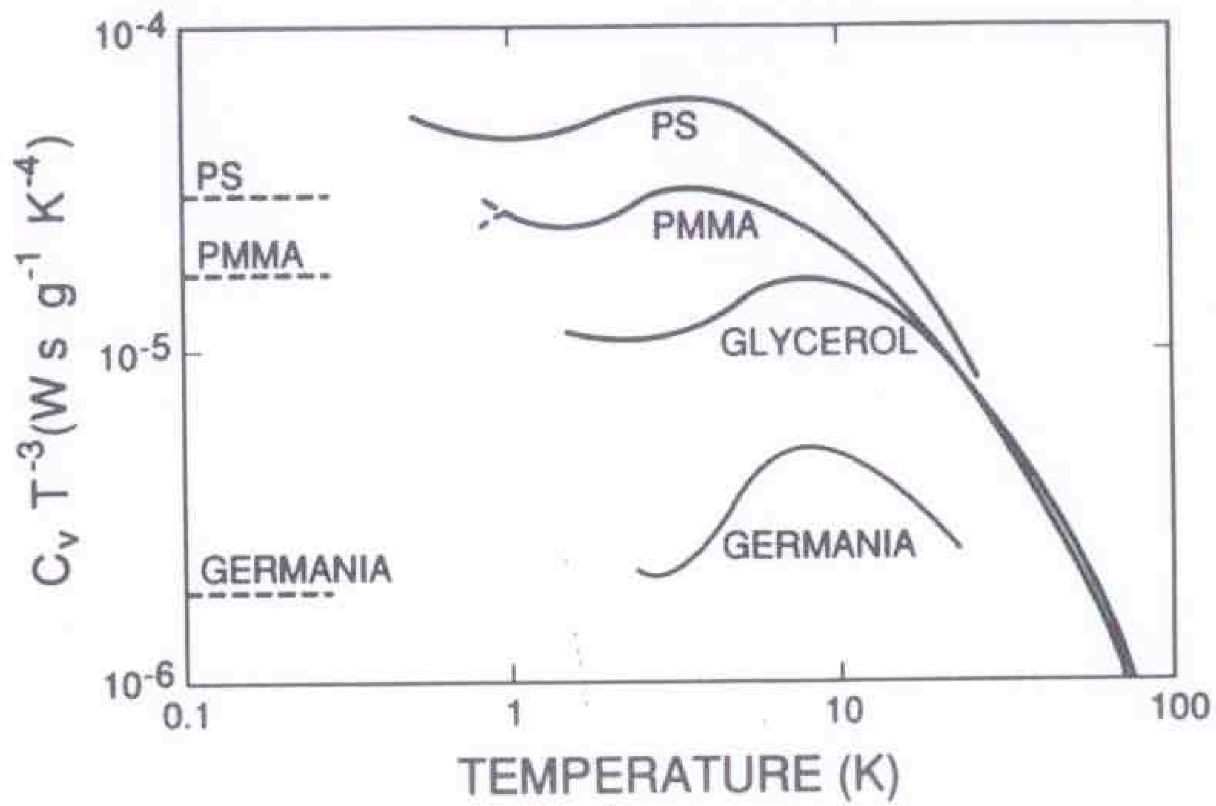


FIGURE 3 C/T^3 versus T from Ref. 5. Notice the bump is between 3 and 10 K.

Seen around 10 K

from Yu and Leggett
Comments Cond. Mat. Phys.
(1988)

Thermal conductivity

Crystal

T^{-1} Umklapp scattering

plateau

boundary scattering

T^3

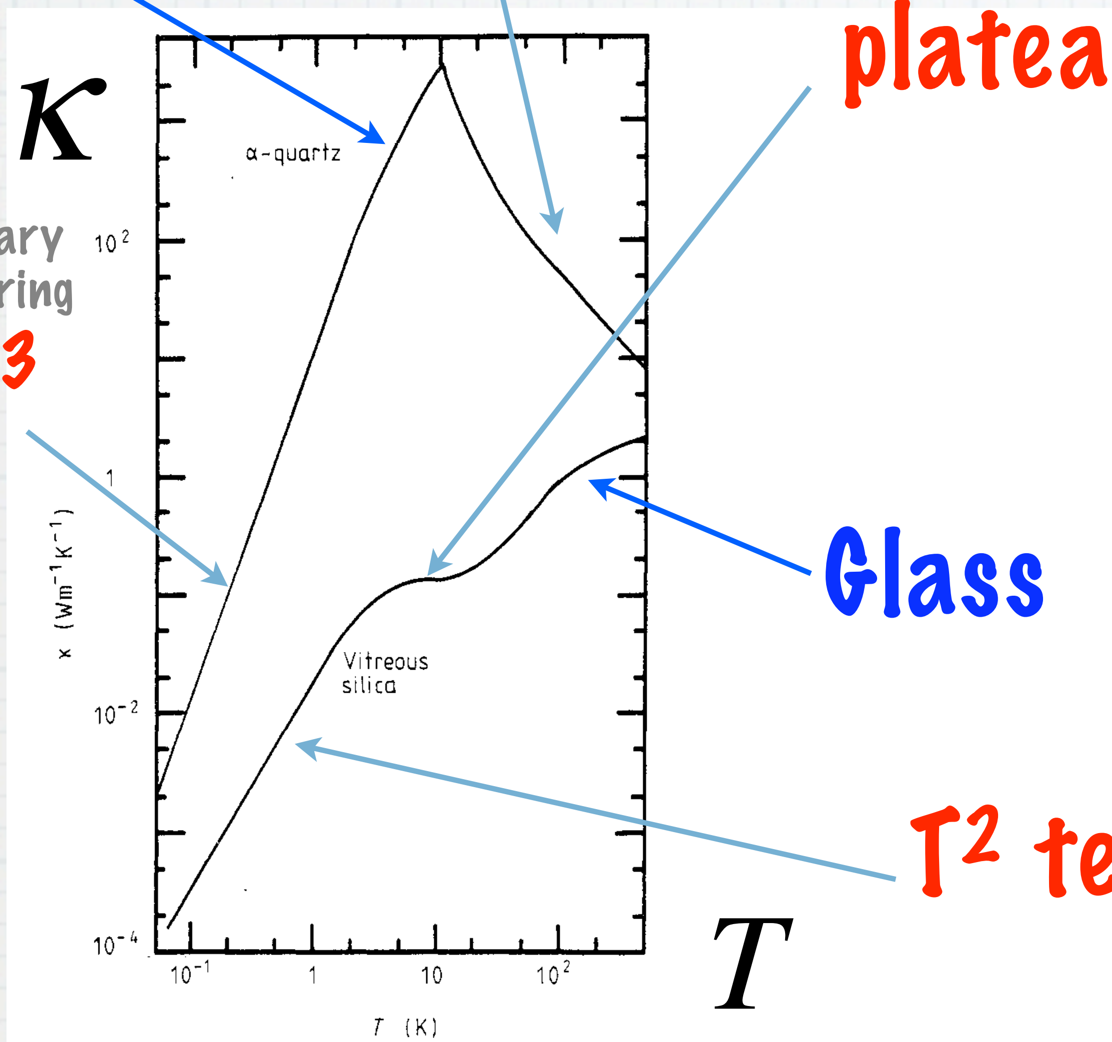
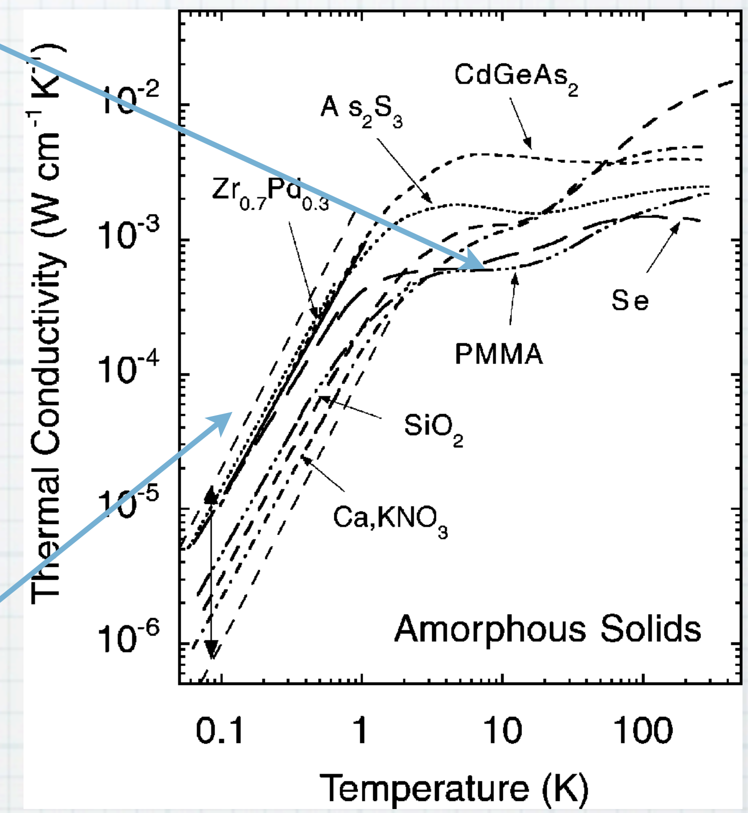


Figure 2. The thermal conductivity $\kappa(T)$ of vitreous silica and crystalline quartz (Jones 1982, after Zeller and Pohl 1971), plotted logarithmically.

Glass

T^2 term

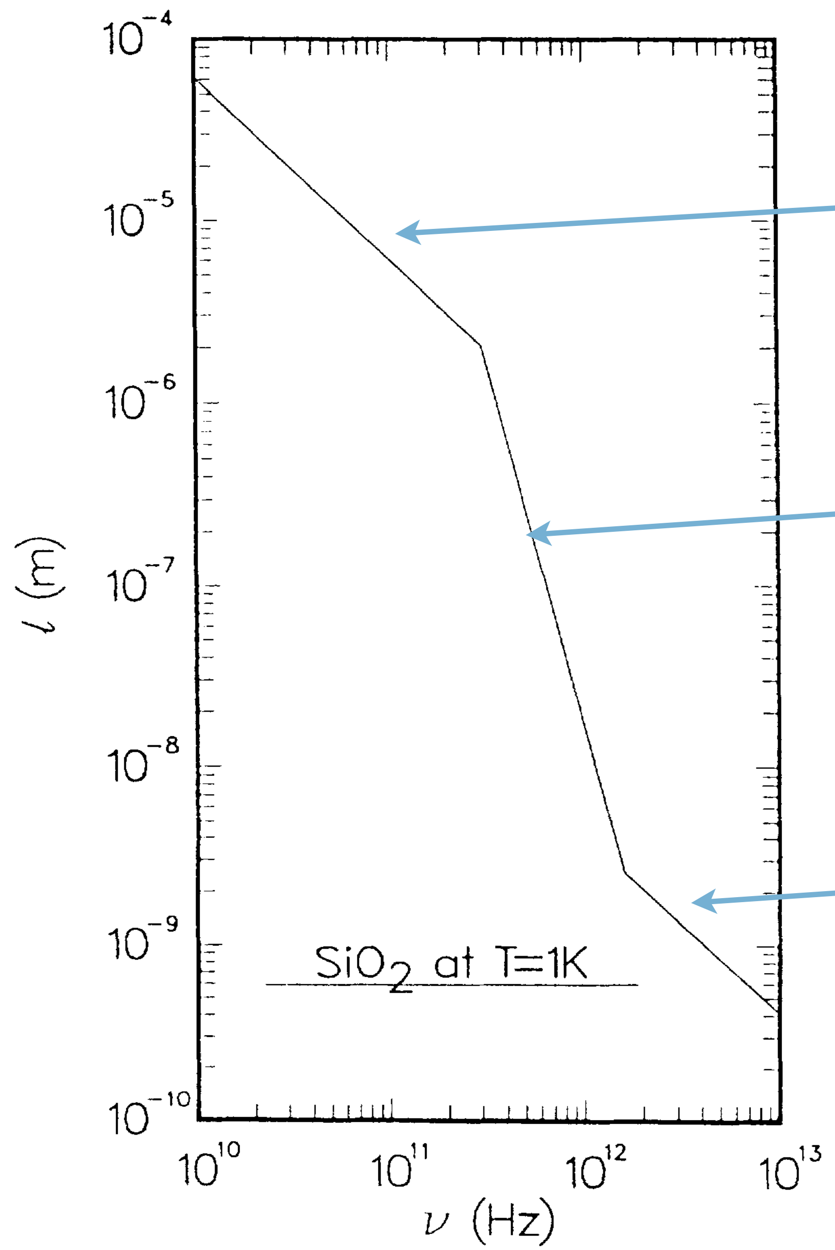


from W.A.Phillips,Rep. Prog.Phys. (1987)

from R.O.Pohl,et.al.,RMP (2002)

Mean free path of phonons deduced from thermal conductivity

$$\kappa(T) = \frac{1}{3}\int C_{ph}(\omega)\ \upsilon\ \ell_{MFP}(\omega)\ d\omega$$



$$\ell_{MFP} \approx 150\lambda$$

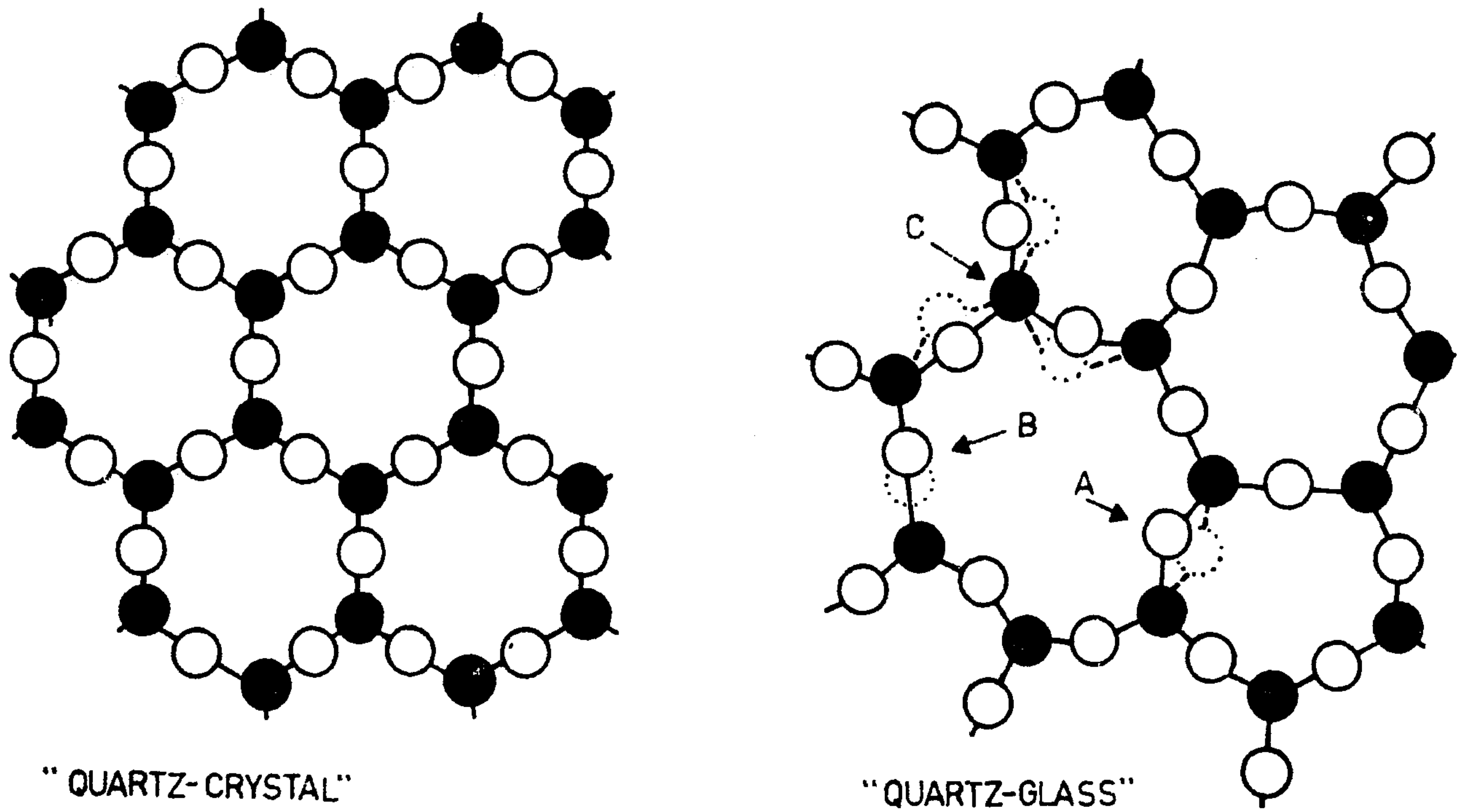
$$\ell_{MFP} \propto \lambda^4$$

$$\ell_{MFP} \propto \lambda$$

from Freeman,et.al., PRB (1986)

Explanation of these unusual properties

Low energy excitation in glasses



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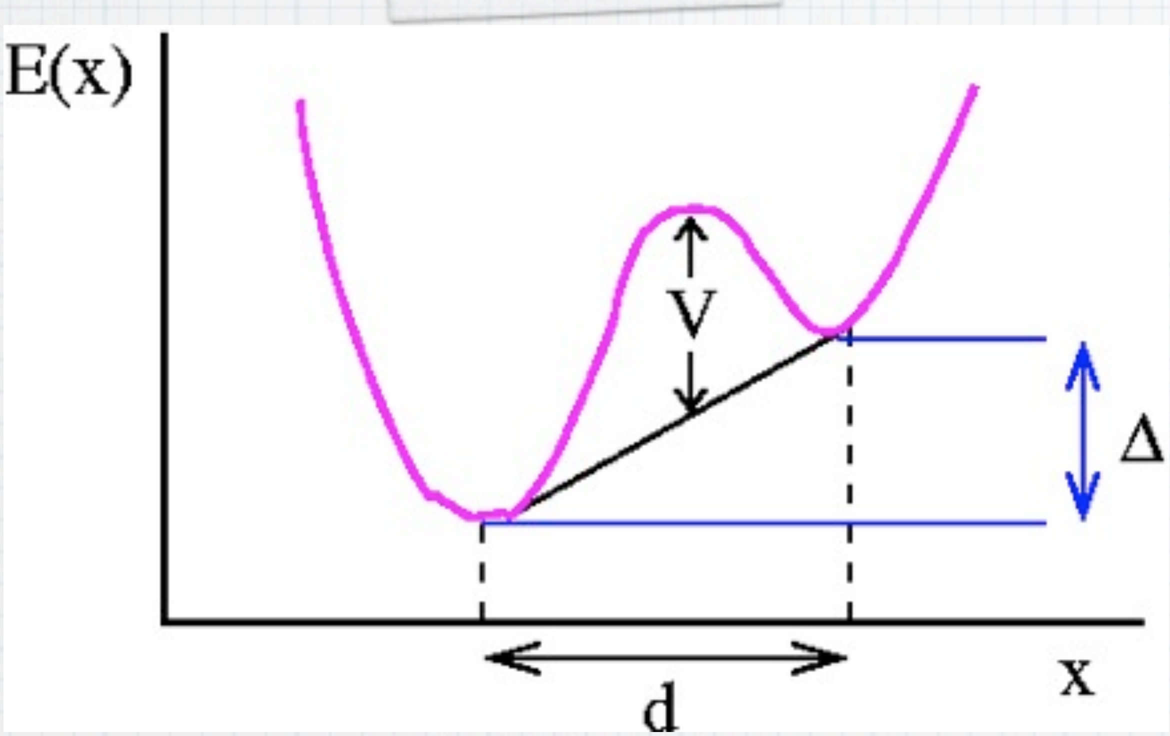
from J.Jackle,et.al., J.Of.Non-cry. Solid. (1976)

Two-level systems model

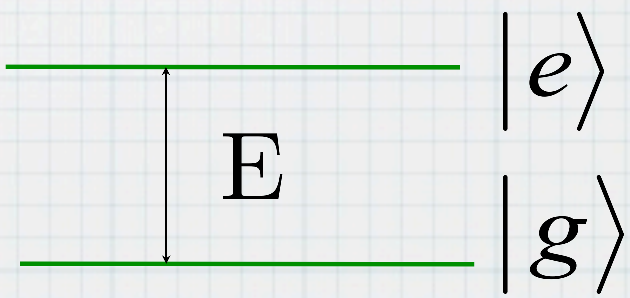


$$H = \begin{pmatrix} \Delta & \Delta_0 \\ \Delta_0 & -\Delta \end{pmatrix}$$

Asymmetry energy Δ

Tunneling energy Δ_0

$$E = \sqrt{\Delta^2 + \Delta_0^2}$$



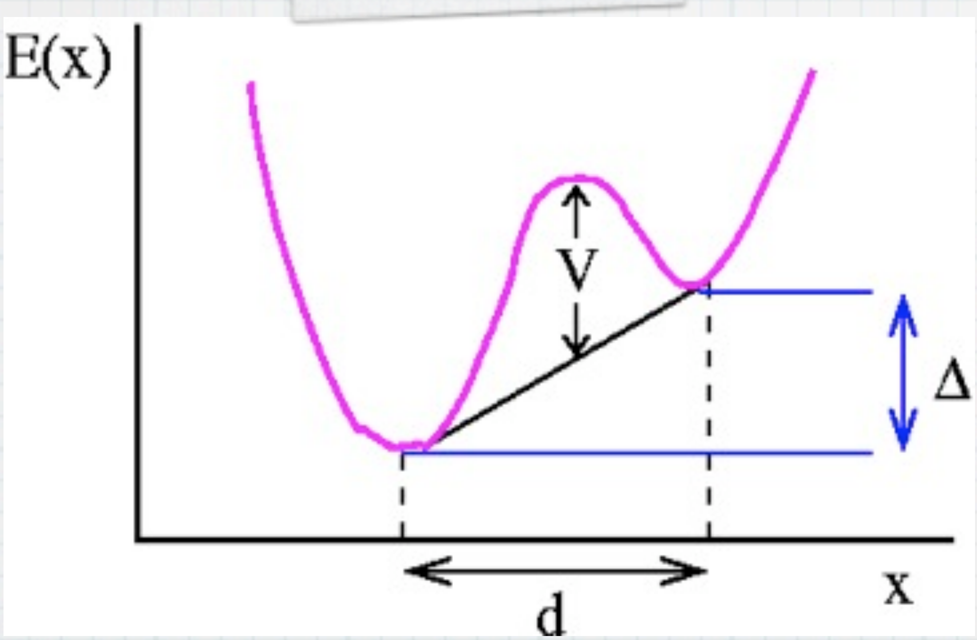
Anderson, et al., (1972), Phillips (1972)

Two-Level Systems in glasses (Open question: What is the microscopic nature of TLS ?)

Δ Asymmetry energy : uniform distribution

$$\Delta_0 = \omega_0 e^{-\frac{\sqrt{2mV}d}{\hbar}}$$

Tunneling energy: barrier heights V satisfy uniform distribution (we use Gaussian distribution instead)



ω_0 Zero point energy of single well

Anderson, et al., (1972), Phillips (1972),

Specific heat: linear T term

Simple derivation:

occupation number

energy of TLS

Average energy

$$\overline{E}(T) = \int dE\, f(E)\, E \quad n(E)$$

density of state of TLS

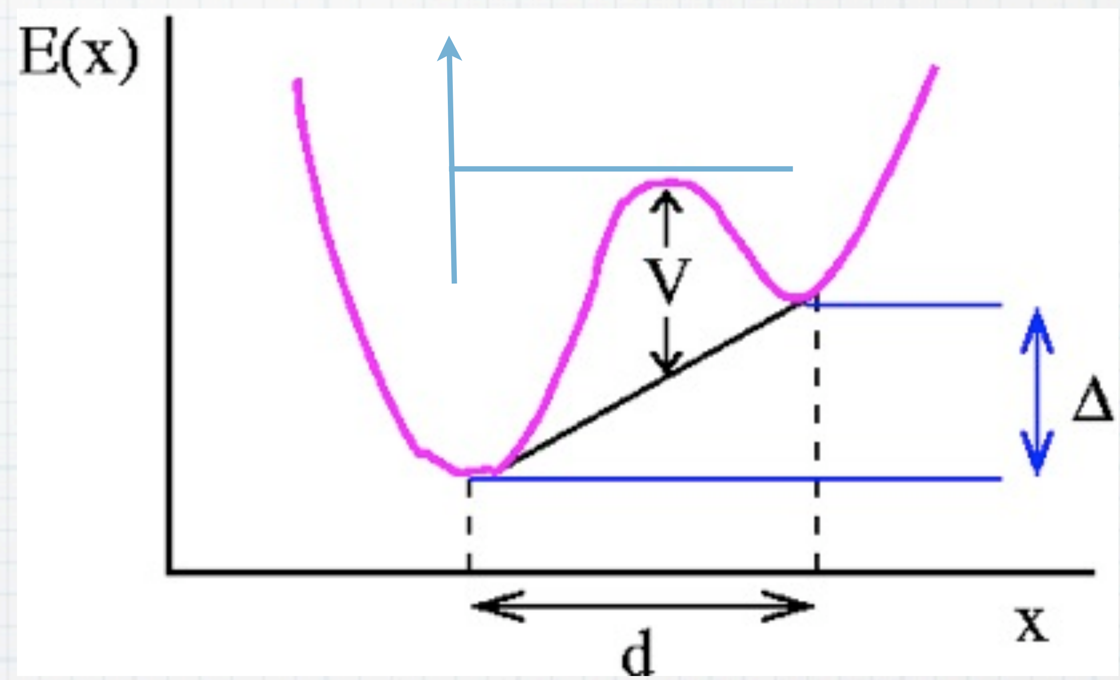
$$= n_0 \int dE \frac{1}{e^{E/k_B T} + 1} E \propto n_0 \quad T^2$$

$$C(T) = \frac{\partial \overline{E}}{\partial T} \propto n_0 T$$

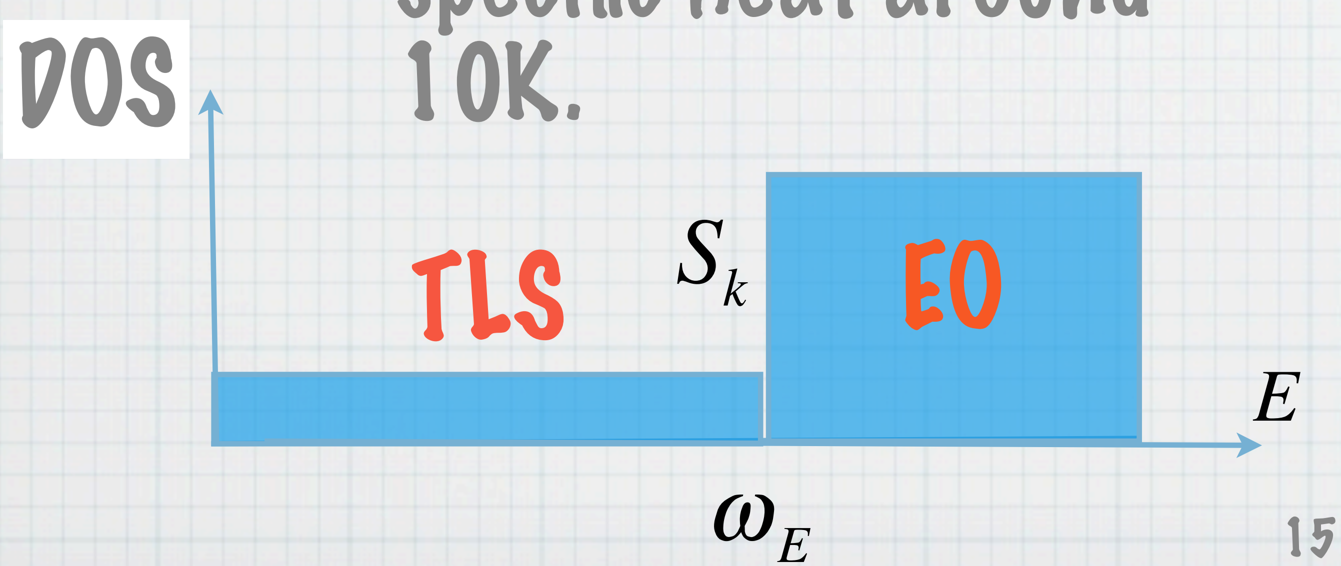
Anderson, et al., (1972),
Phillips (1972)

Extend model to higher T: Einstein oscillators

- Approximate excitations at high energy by harmonic oscillators (**Einstein oscillators = EO**).
- EO is responsible for the peak in the specific heat around 10K.



Yu and Freeman, PRB (1987)

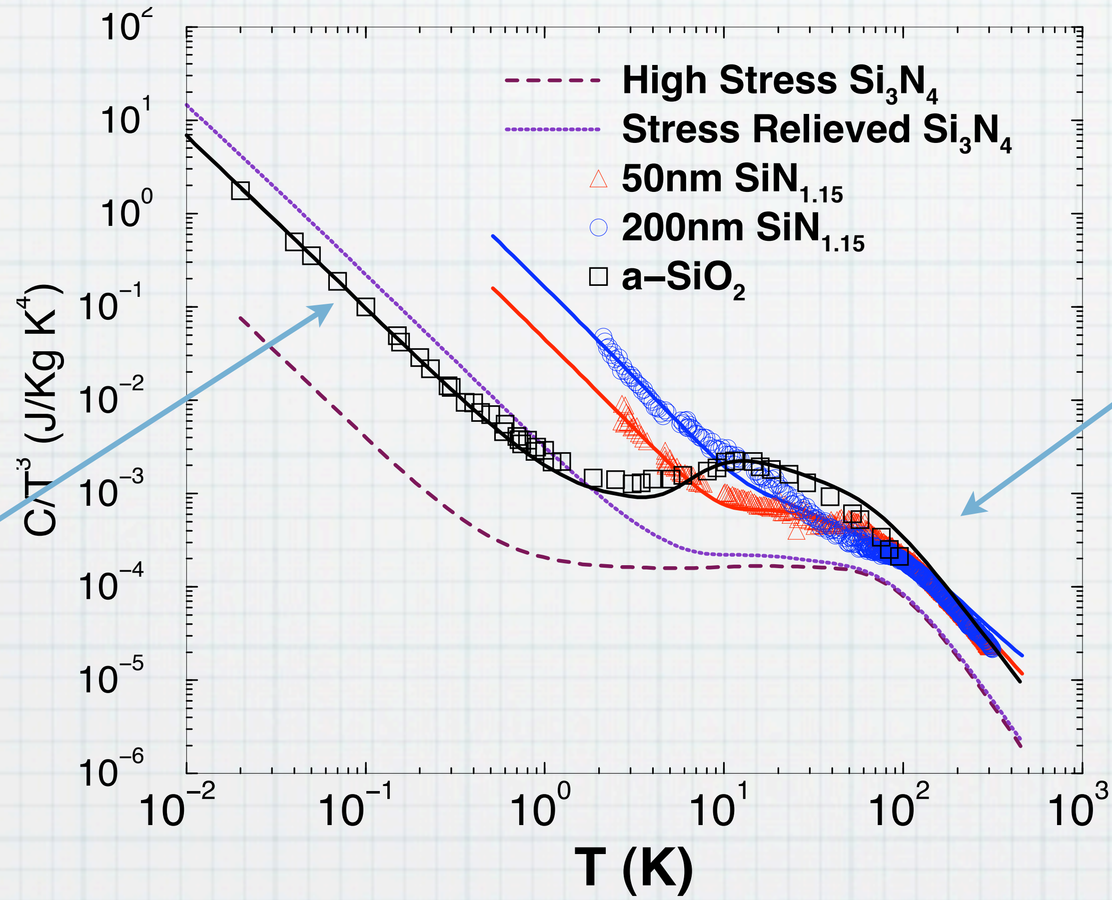


$$n(E) = n_0[1 + S_k \Theta(E - \hbar\omega_E)]$$

Specific heat over a broad temperature regime

$$C = C_{Debye} + C_{TLS} + C_{EO}$$

Einstein Oscillator (Boson peak)



Two-Level -Systems

Solid and dash lines: Predicted

$SiN_{1.15}$: from Southworth, et al., PRL (2009)

a-SiO_2: from Yu and Freeman, RPB (1987)

Thermal conductivity

Heat is transported by phonons in glasses (Zaitlin and Anderson, 1975)

$$\kappa(T) = \frac{1}{3}\int C_{ph}(\omega)\ \upsilon\ \ell_{MFP}(\omega)\ d\omega$$

How much energy a phonon can carry

How fast a phonon goes

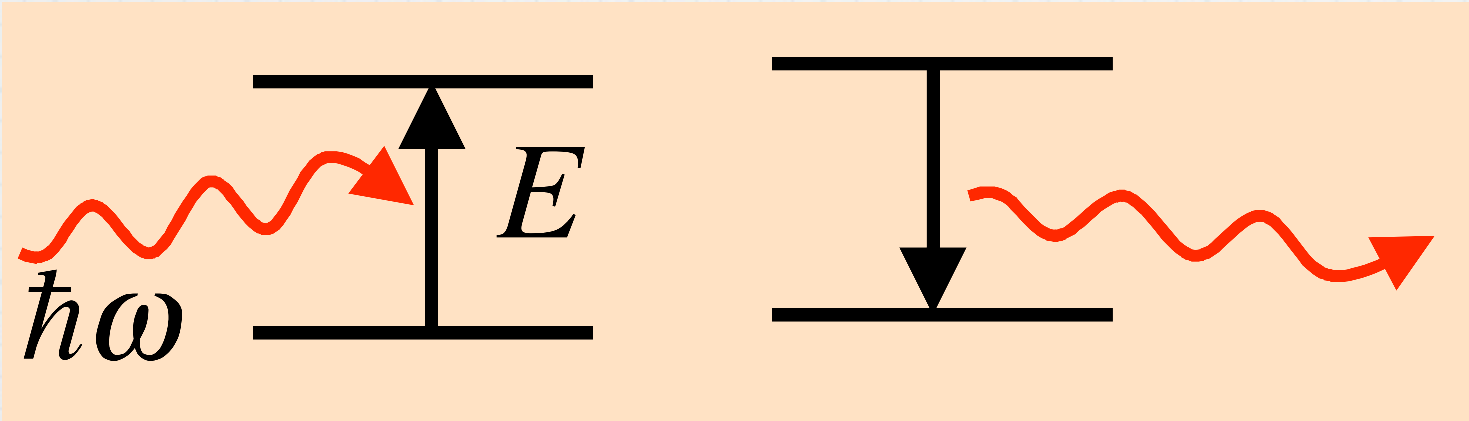
How far a phonon can go before it hits something

Resonant scattering of TLS

Phonons (photons) are absorbed and emitted when a TLS is excited and de-excited

$$\ell^{-1}_{MFP,\text{Resonance}} = \alpha\omega \tanh\left(\frac{\hbar\omega}{2k_BT}\right) \sim \omega \text{ for } \hbar\omega \gtrsim k_BT$$

$$\alpha = \frac{\pi \overline{P} \gamma^2}{\rho v^3}$$



$$\hbar\omega = E$$

$\overline{P}$ spectral density (density of TLS)

γ deformation potential (coupling of TLS and phonons)

Thermal conductivity goes as T^2 at low temperatures

$$\kappa(T) = \frac{1}{3}\int C_{ph}(\omega)\ \ \upsilon\ \ \ell_{MFP}(\omega)\ \ d\omega$$

Debye theory

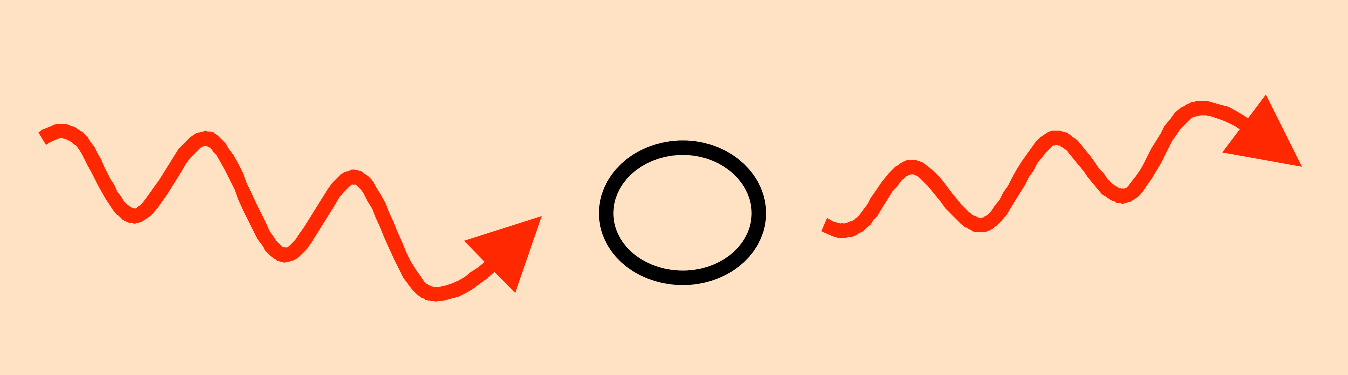
$$T^3$$

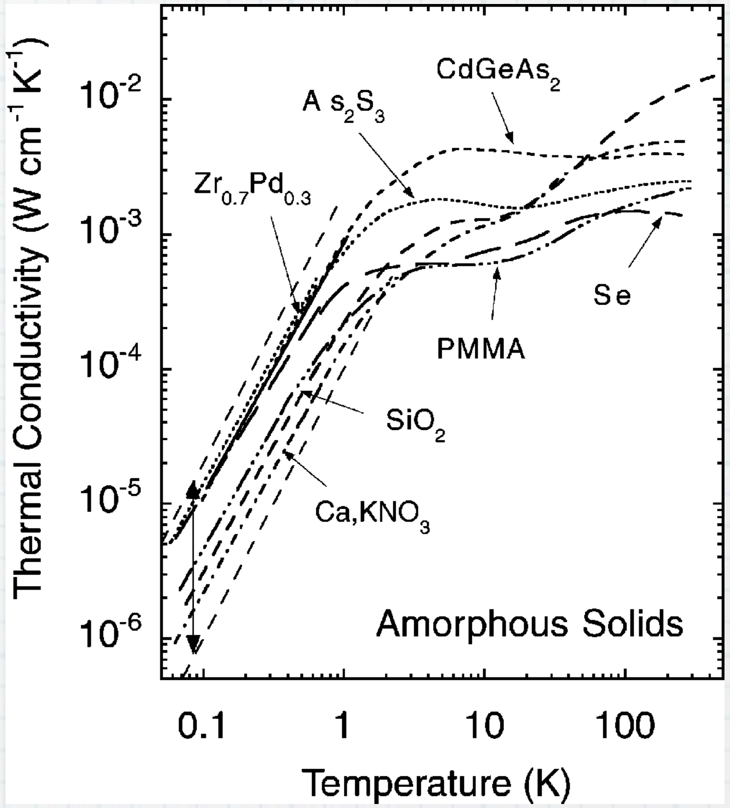
$$\ell_{MFP} \propto \frac{1}{\omega} \sim \frac{1}{T}$$

$$\kappa(T) \propto T^2$$

Plateau in thermal conductivity

Plateau dominated by Rayleigh scattering of phonons





Phonons (photons) are scattered by atoms or small size defects

$$\ell_{MFP,\mathrm{Rayleigh}}^{-1} = B\omega^4$$

Four mechanisms contribute to phonon scattering

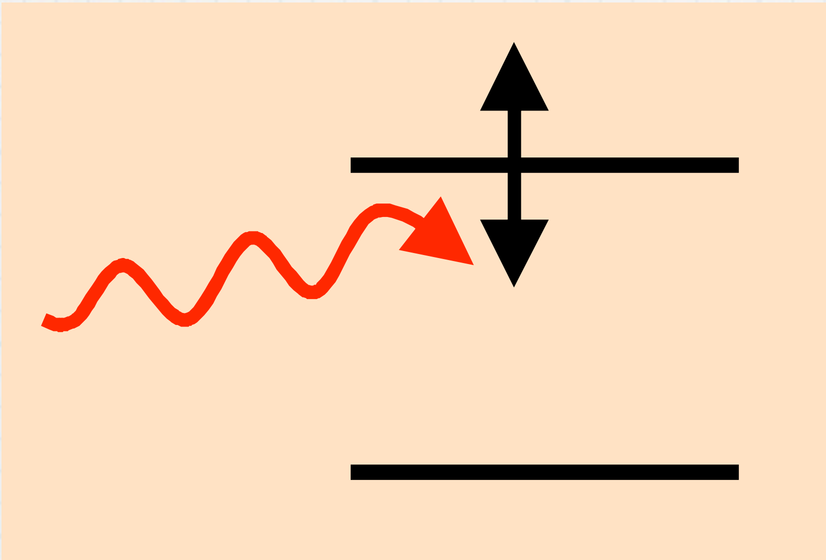
* Resonant scattering of phonons from TLS
* TLS relaxation
* Rayleigh scattering
* Scattering from Einstein oscillators

$$\ell_{MFP}^{-1} = \begin{cases} \ell_{MFP,\text{Resonant}}^{-1} + \ell_{MFP,\text{ Relaxation}}^{-1} + \ell_{MFP,\text{Rayleigh}}^{-1} & \omega < \omega_E \\ \ell_{MFP,\text{Resonant}}^{-1} + \ell_{MFP,\text{Relaxation}}^{-1} + \ell_{MFP,\text{Einstein}}^{-1} & \omega > \omega_E \end{cases}$$

Combining two models: Yu-Freeman,PRB (1987) & Hunklinger, PRB (1992)

Phonon scattering due to TLS relaxation (dominates at low frequencies)

Phonons (photons) modulate TLS energy splitting. TLS population redistributes to achieve new equilibrium.



$$P(\Delta,V)=\frac{\overline{P}}{E_0}e^{-(V-V_0)^2/2\sigma_0^2}$$

$$\ell^{-1}_{MFP,\text{Relaxation}}=\frac{\alpha\omega}{\pi}\int dV\int_0^{2V}d\Delta P(\Delta,V)\operatorname{sech}^2\left(\frac{\hbar\omega}{2k_BT}\right)\left(\frac{\Delta}{E}\right)^2\frac{\omega\tau}{1+(\omega\tau)^2}$$

Two processes of TLS relaxation

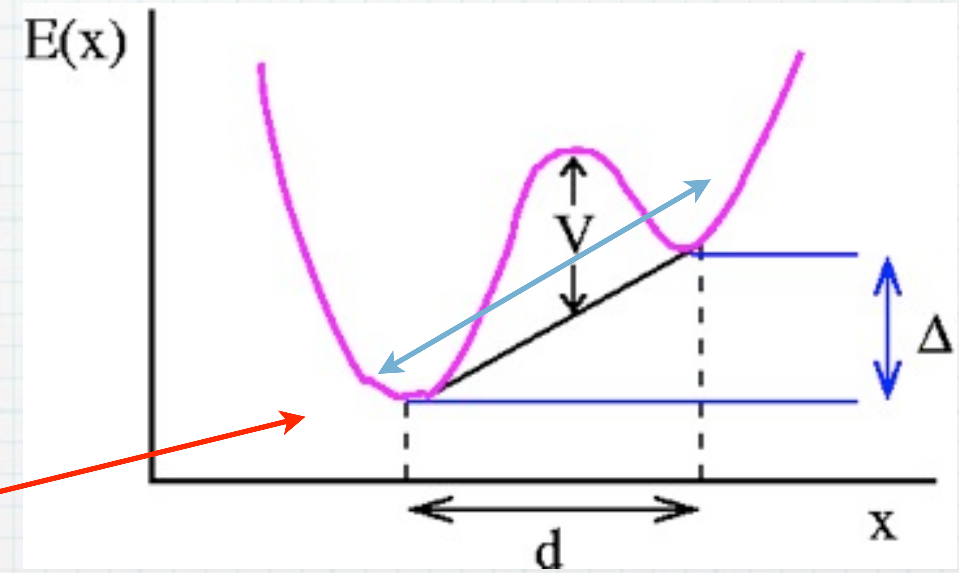
Relaxation time

$$\tau^{-1} = \tau^{-1}_{Tunneling} + \tau^{-1}_{Thermal\quad Activation}$$

Tunneling

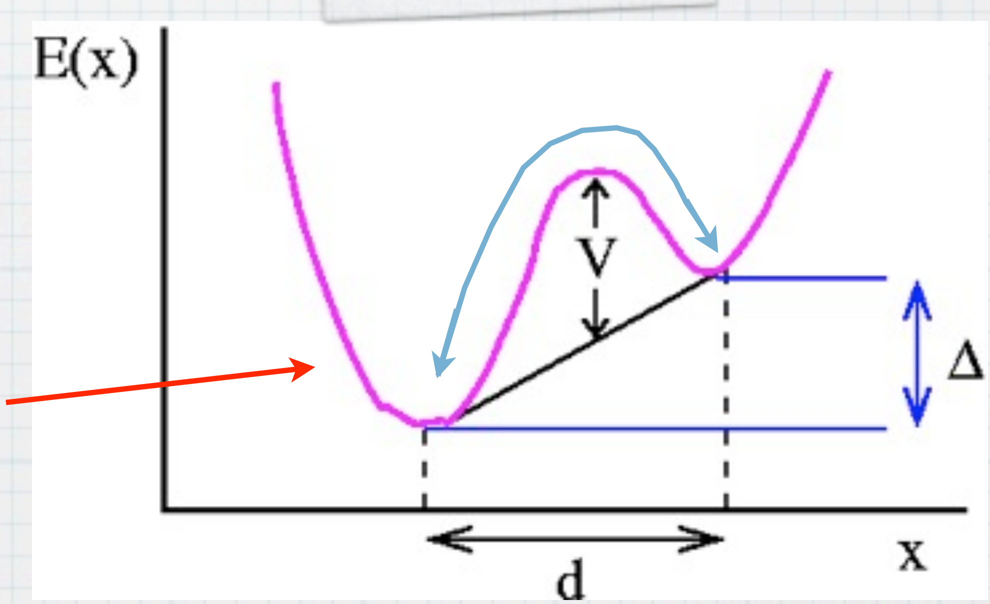
$$\Delta_0 = \omega_0 e^{-\frac{\sqrt{2mV}d}{\hbar}}$$

$$\tau^{-1}_{Tunneling} = A\Delta_0^2 E \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$



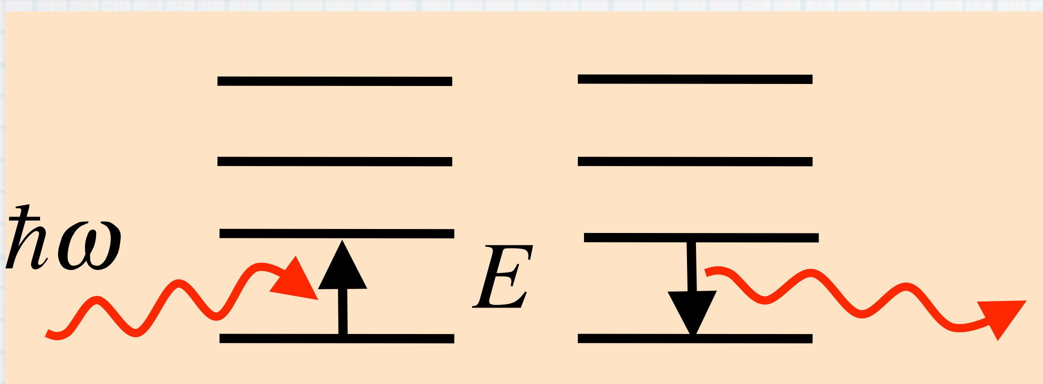
Thermal activation

$$\tau^{-1}_{Thermal\ Activation} = \tau_0^{-1} \cosh\left(\frac{\Delta}{2k_B T}\right) e^{-V/2k_B T}$$



Phonon scattering from Einstein oscillators

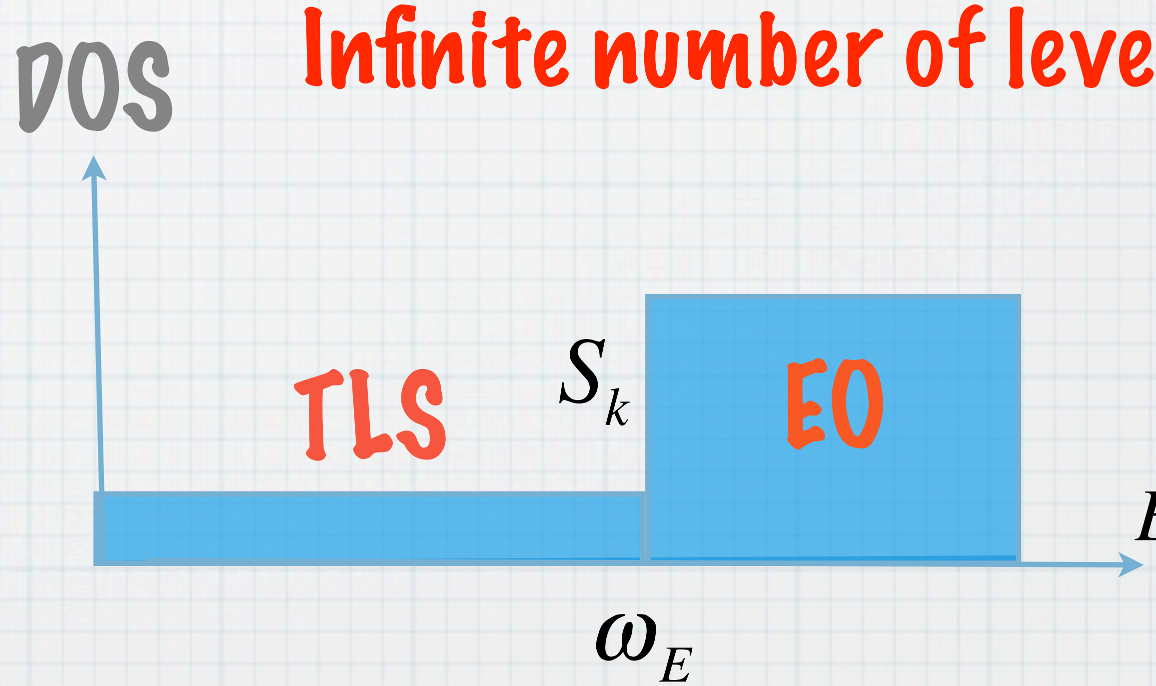
Phonons (photons) are absorbed and emitted when a harmonic oscillator is excited and de-excited



$$\hbar\omega = E$$

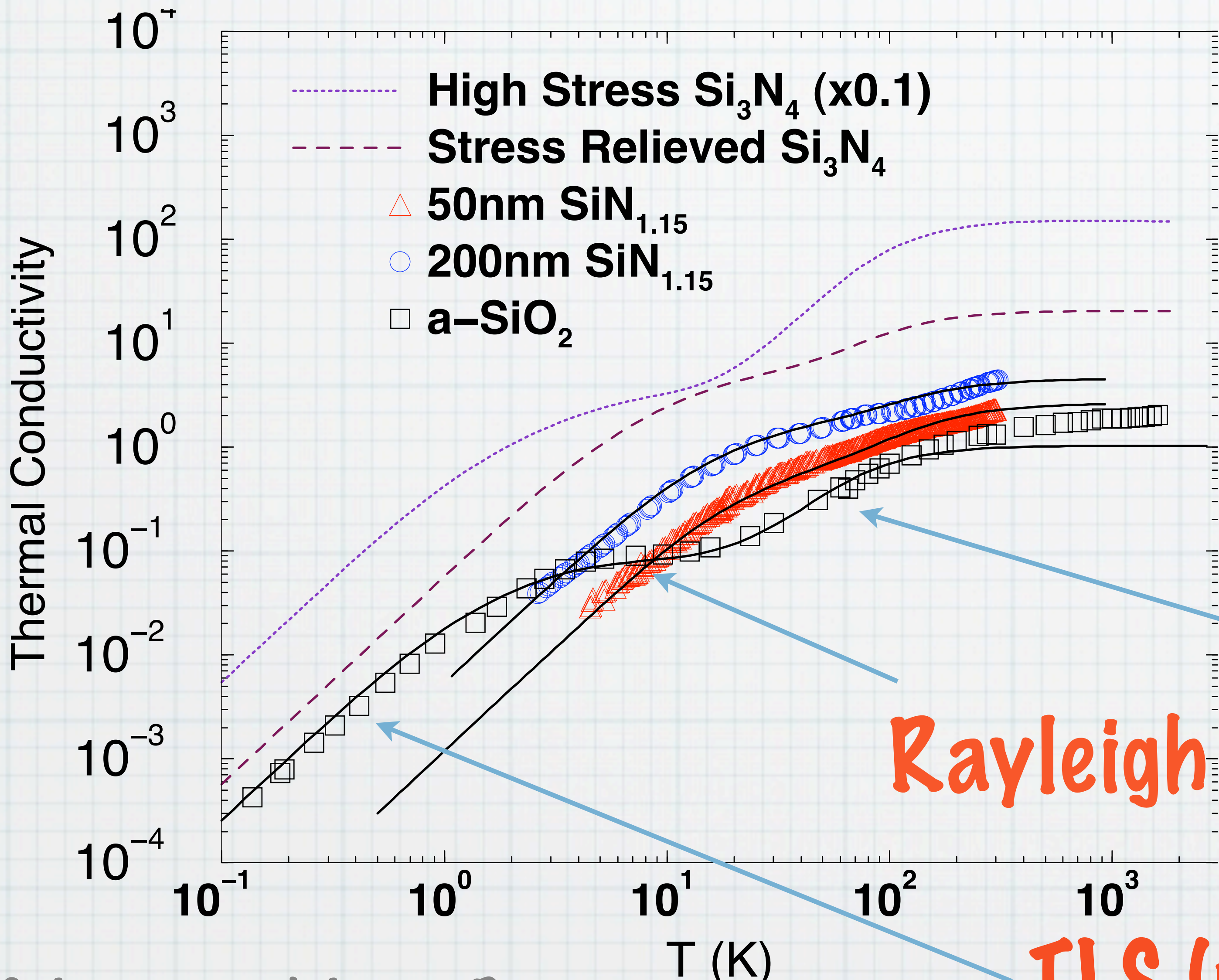
$$\ell^{-1}_{MFP,\text{Einstein}} = \frac{2\alpha S_k}{\pi}\omega$$

Infinite number of levels



$$n(E) = n_0[1 + S_k \Theta(E - \hbar\omega_E)]$$

Thermal conductivity



Rayleigh

Einstein Oscillators

TLS (resonance,relaxation)

Solid and dash lines: Predicted

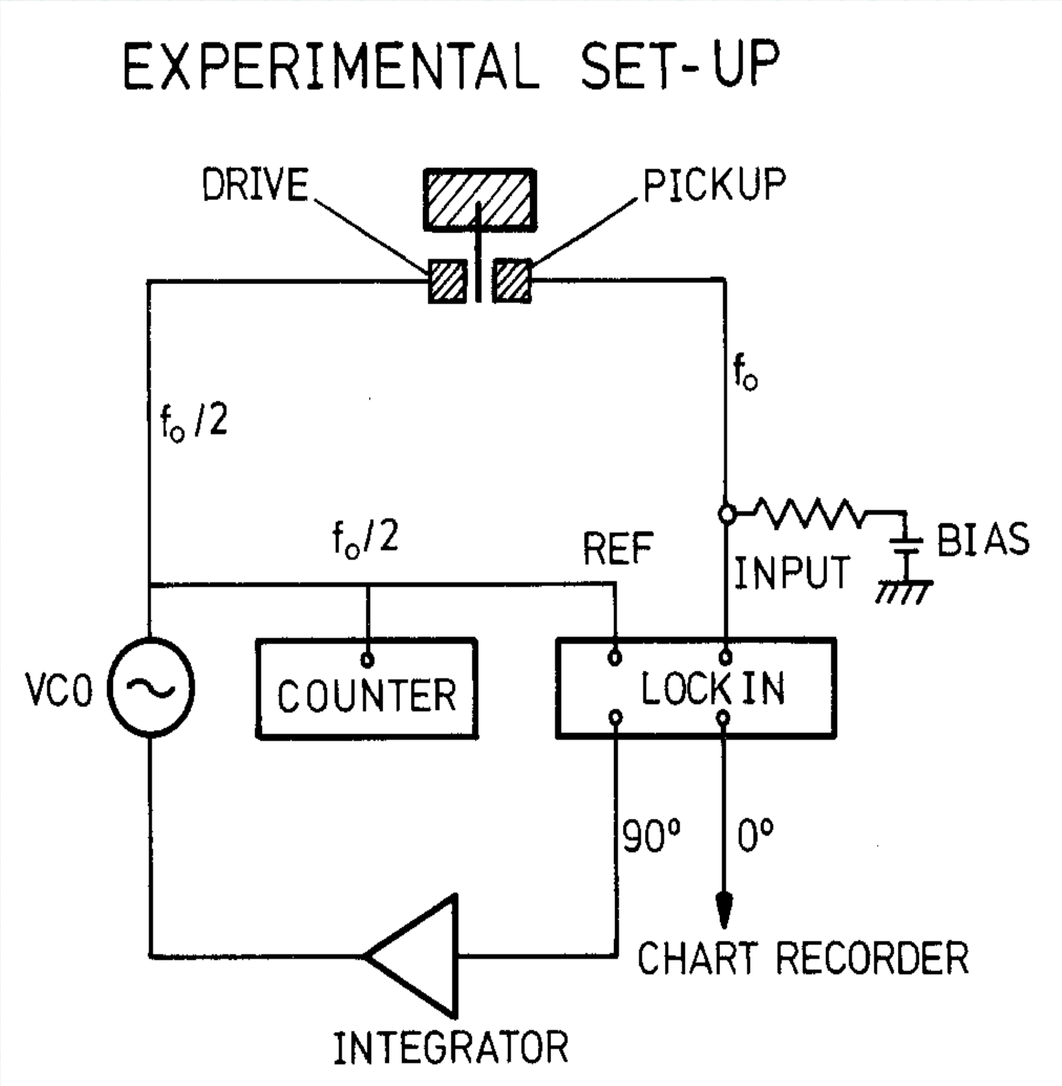
$SiN_{1.15}$: from Southworth, et al., PRL (2009)

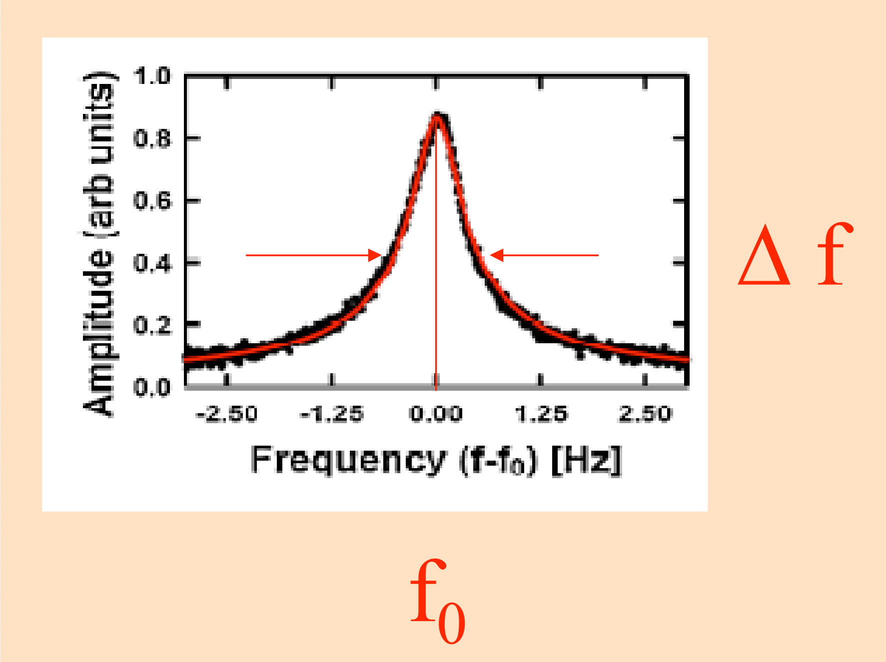
a-SiO_2: from Yu and Freeman, RPB (1987)

Introduction to acoustic dissipation in glasses

How to measure the dissipation in glasses

Measuring acoustic dissipation in glasses





Quality factor (internal friction)

$$Q^{-1} = \frac{f_0}{\Delta f}$$

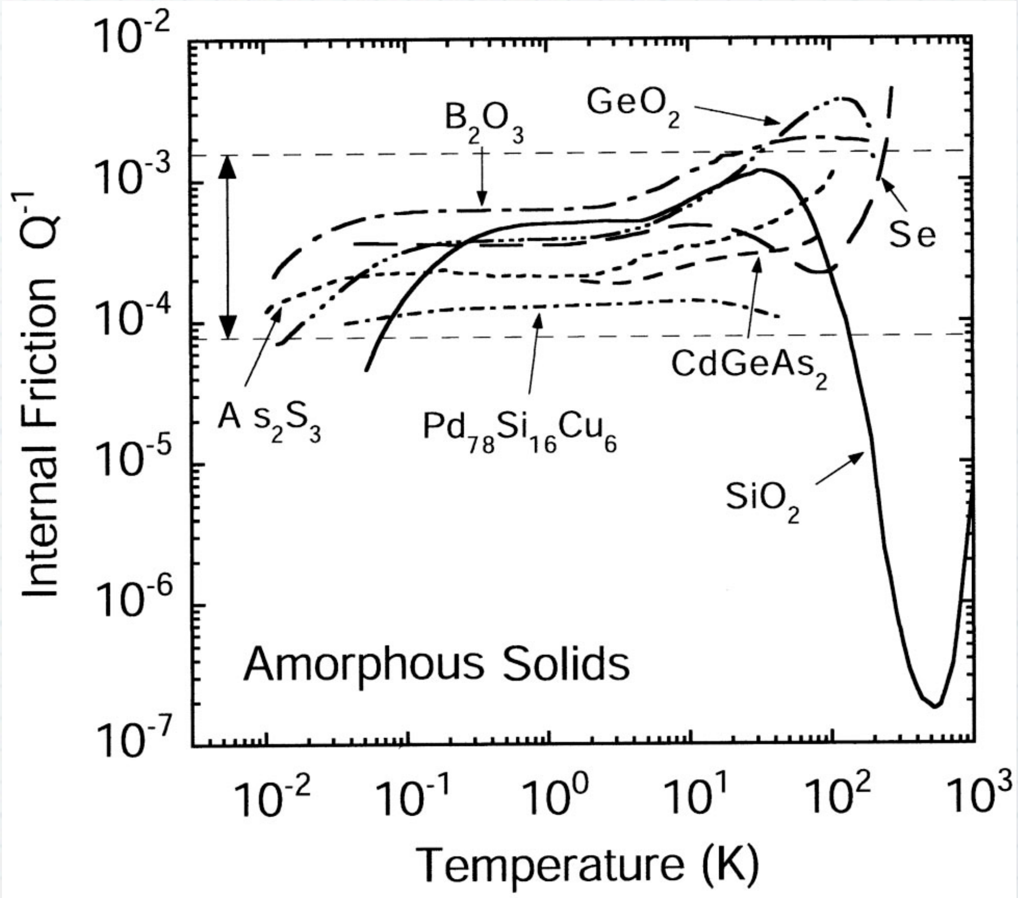
Raychaudhuri,et al. Z.Phys.B (1984)

Universal dissipation in glasses

Zeller,et al.,PRB (1971)

For various glasses such as SiO2, B2O3,...at 0.1K <T<10K

$$Q^{-1} \sim 10^{-4} - 10^{-3}$$



Due to two-level-systems (TLS) at low temperatures

Anderson, et al., (1972), Phillips (1972), Jackle (1972)

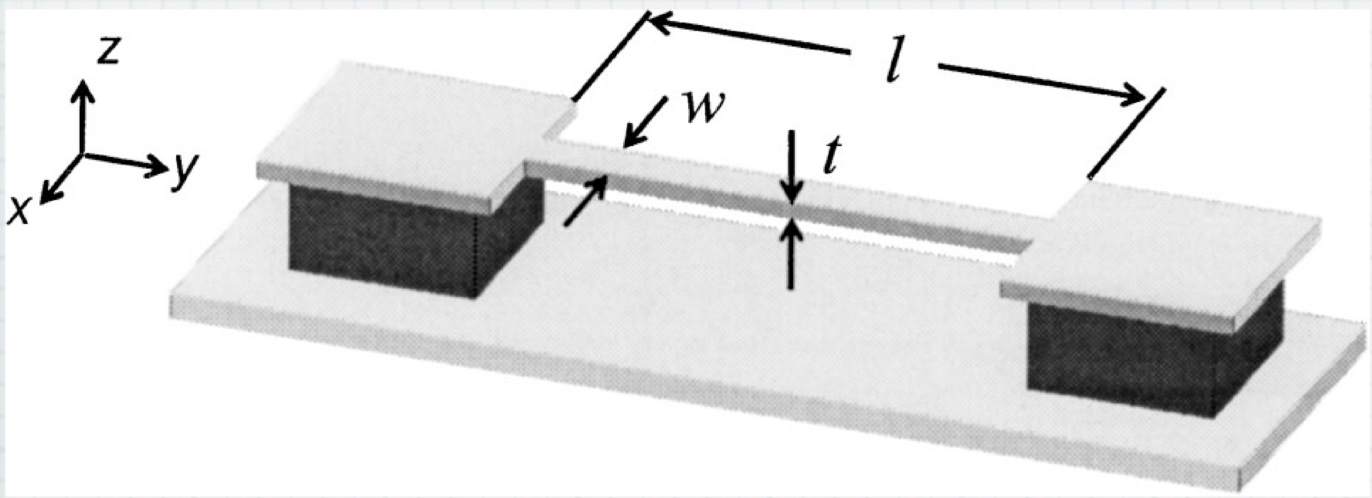
R.O.Pohl,et al.,RMP (2002)

Why do we want to reduce the dissipation in glasses?

Motivation: high Q (low dissipation) is important!

* Example 1: Resonant mass sensor (Q will determine the minimum mass detectable)

Si nanowire

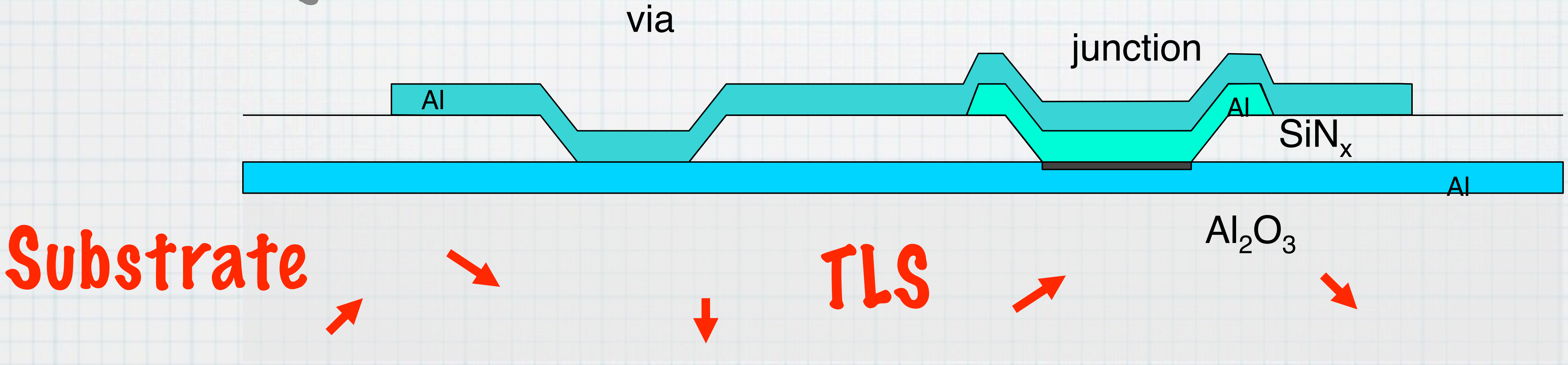


from Ekinci,et al., JAP (2004)

Motivation: high Q (low dissipation) is important!

* Example 2: SQUIDs are used as qubits; need to reduce the noise.

* Charge noise is proportional to the dielectric loss tangent of substrate.



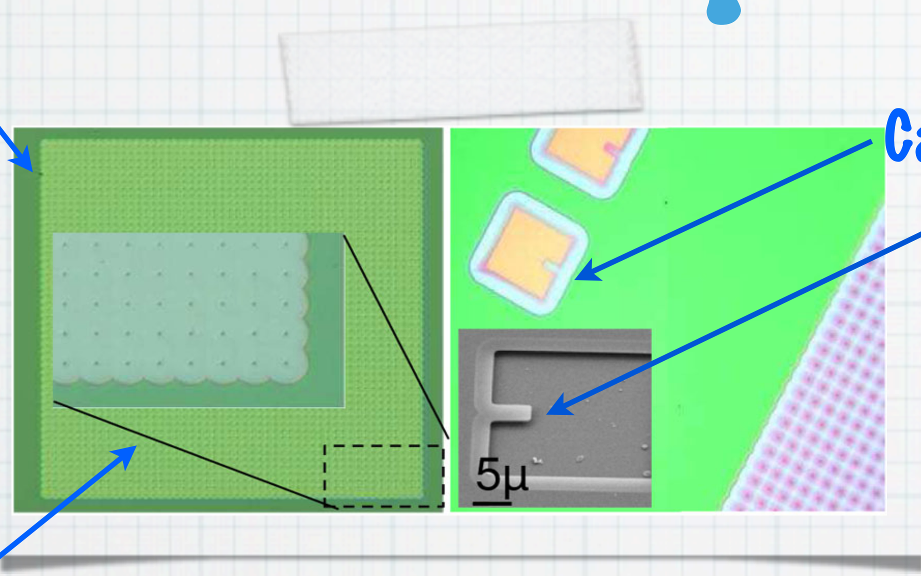
Martinis,et al., PRL (2005)

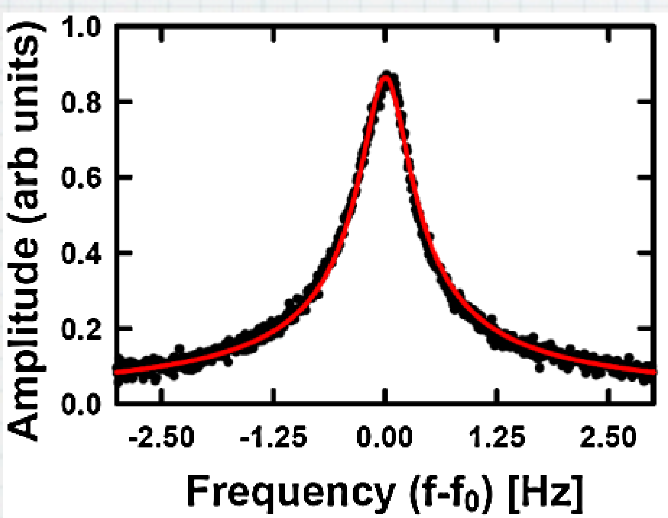
In glasses, at low temperature and low frequency, acoustic dissipation (phonons) and dielectric loss (photons) are all due to TLS.

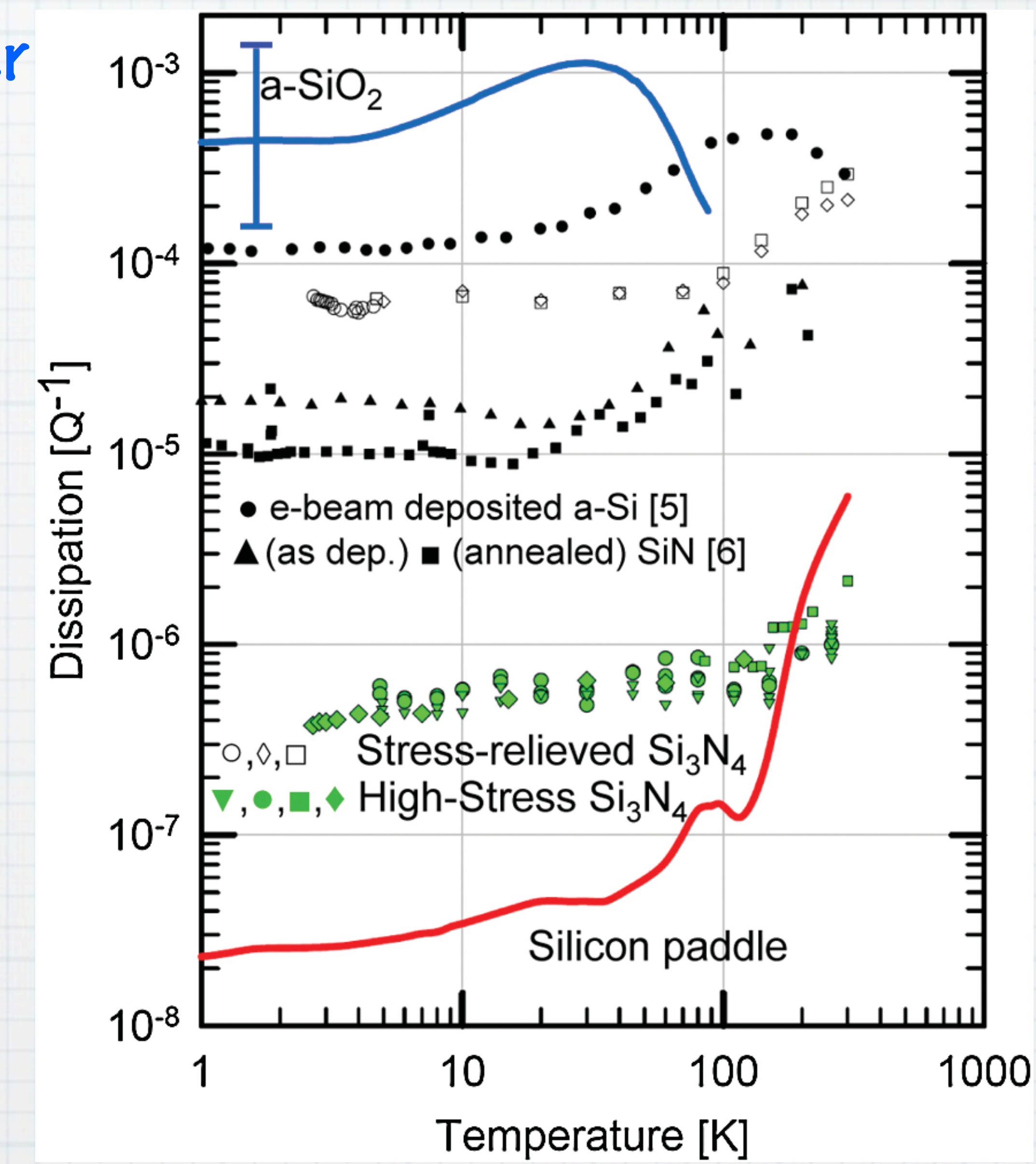
S. Hunklinger, PLTP (1984)

Since dissipation is ubiquitous in glasses, can we reduce it?

Yes! Anomalously low dissipation in Si_3N_4







D.R.Southworth, et.al., PRL (2009)

Two questions to answer

* Why does high stress reduce dissipation of Si_3N_4 so dramatically ?
* Why does stress-relieved Si_3N_4 have an order of magnitude lower in dissipation than SiO_2 ?

Why does high stress reduce the dissipation in glasses ?

Answer:

Relaxation via tunneling and thermal activation is exponentially sensitive to V.

High stress increases the barrier heights V, effectively reducing the number of defects that produce dissipation.

Theory of dissipation in glasses

$$Q^{-1} = \frac{\lambda}{\ell_{MFP}}$$

phonon wave length

Dissipation and thermal conductivity
are all related to the mean free path of phonons
(photons)

Four mechanisms contribute to the dissipation

$0.1K < T < 10K$ $\omega \sim 1$ MHz

* Resonant scattering of phonons from TLS $Q^{-1}_{\text{Resonance}} \sim 10^{-7}$
* TLS relaxation $Q^{-1}_{\text{Relaxation}} \sim 10^{-3}$ ✔
* Rayleigh scattering $Q^{-1}_{\text{Rayleigh}} \sim 10^{-18}$
* Scattering from Einstein oscillators Only involved at high T

Therefore, relaxation dominates

Dissipation due to TLS relaxation

$$Q^{-1}_{\text{Relaxation}} = \frac{2Q_0^{-1}}{\pi}\int dV \int_0^{2V} d\Delta P(\Delta,V)\,\mathrm{sec}\,h^2\left(\frac{\hbar\omega}{2k_BT}\right)\left(\frac{\Delta}{E}\right)^2 \frac{\omega\tau}{1+(\omega\tau)^2}$$

Relaxation time

$$\tau^{-1} = \tau^{-1}_{Tunneling} + \tau^{-1}_{Thermal\quad Activation}$$

Tunneling

$$\tau^{-1}_{Tunneling} = A\Delta_0^2 E \coth\left(\frac{\hbar\omega}{2k_BT}\right) \qquad \Delta_0 = \omega_0 \mathrm{e}^{-\frac{\sqrt{2\mathrm{mV}}\mathrm{d}}{\hbar}}$$

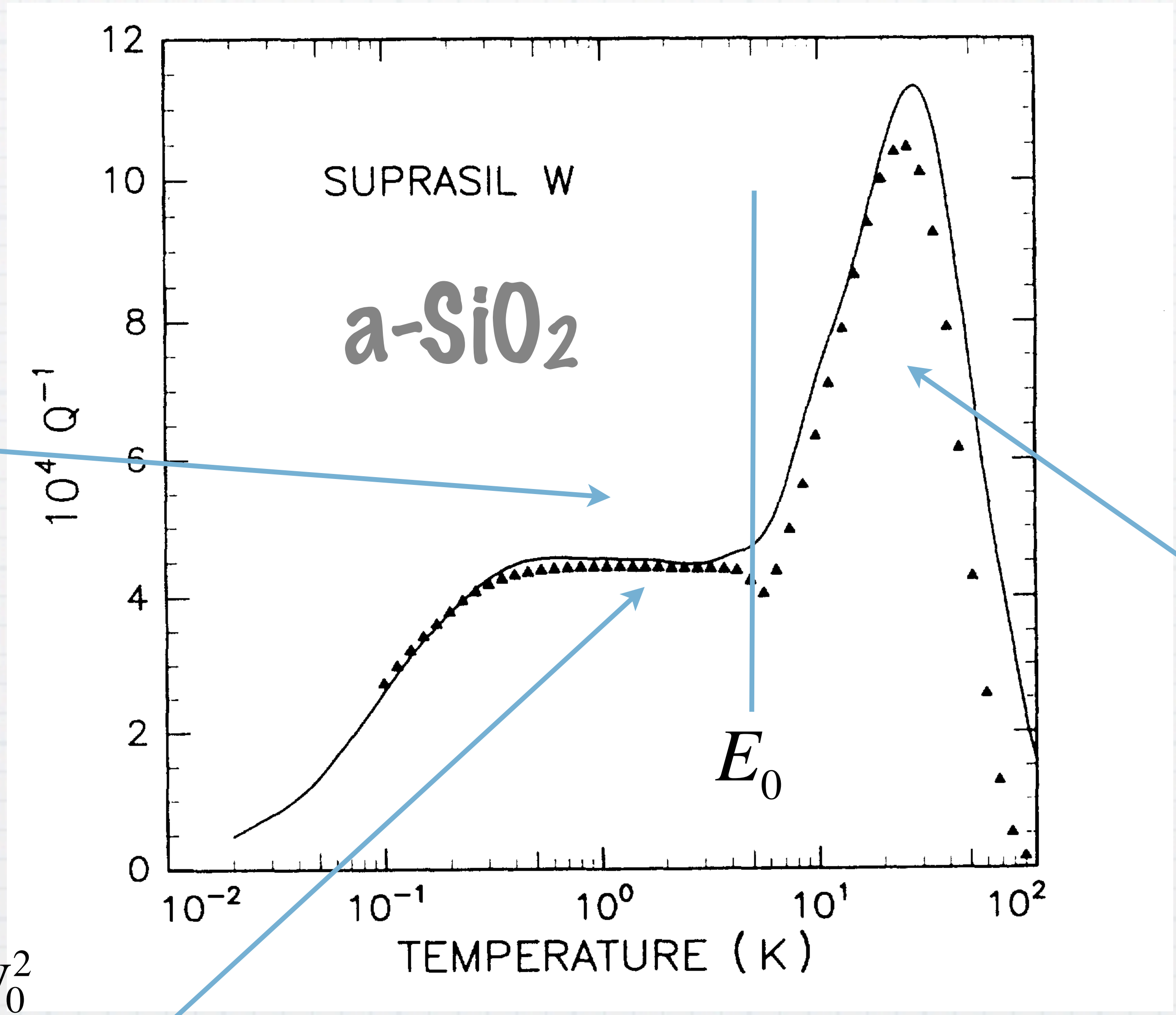
Thermal activation

$$\tau^{-1}_{Thermal\ Activation} = \tau_0^{-1}\cosh\left(\frac{\Delta}{2k_BT}\right)e^{-V/2k_BT}$$

Gaussian distribution of barrier heights V

$$P(\Delta,V) = \frac{\bar{P}}{E_0}e^{-(V-V_0)^2/2\sigma_0^2}$$

Dissipation at low temperature



Tunneling

Thermal activation

Universal

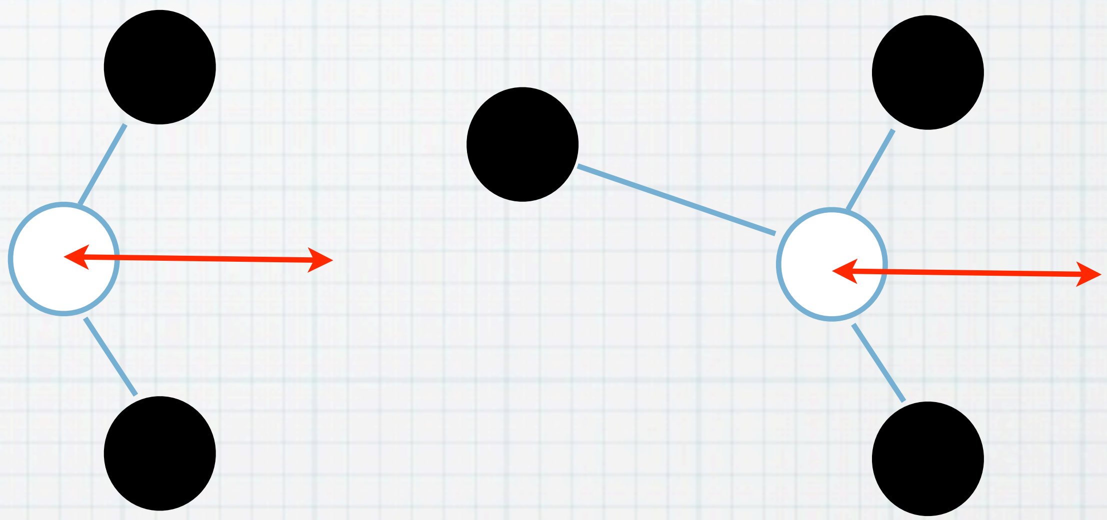
$$Q_0^{-1} \approx \frac{\pi \bar{P} \gamma^2}{2\rho v^2} e^{-\frac{V_0^2}{2\sigma_0^2}}$$

S. Hunklinger, PRB (1992)

Why low stress Si_3N_4 has low dissipation compared with SiO_2?

3- or 4-fold coordinated materials will have extra constraints, producing non-relieved strain energy, thus increasing the barrier heights. V_0 is nonzero compared to a-SiO_2

$$P(\Delta,V) = \frac{\bar{P}}{E_0} e^{-(V-V_0)^2 / 2\sigma_0^2}$$



a-SiO_2

$V_0 = 0K$

$\sigma_0 = 445K$

Si_3N_4

$V_0 = 13500K$

$\sigma_0 = 9000K$

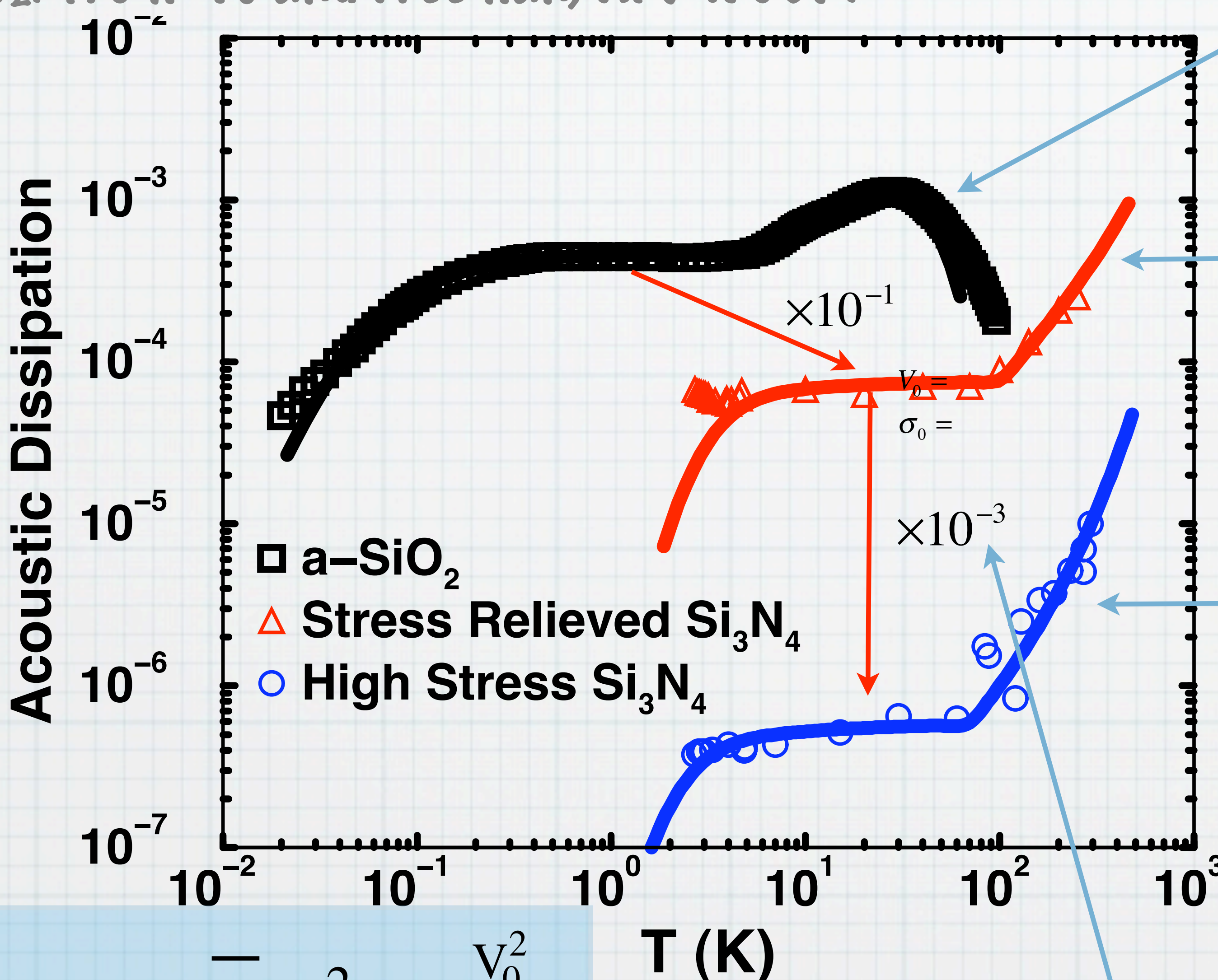
Why high stress can reduce dissipation in glasses ?

High stress increases the strain energy, thus increasing the barrier heights. V_0 is increased compared with low stress Si_3N_4

Dissipation of Si_3N_4

Si_3N_4: Queen, et al.,RSI (2009)

a-SiO_2: from Yu and Freeman, RPB (1987)



$$V_0 = 0K$$
$$\sigma_0 = 445K$$

$$V_0 = 13500K$$
$$\sigma_0 = 9000K$$

$$V_0 = 25300K$$
$$\sigma_0 = 7500K$$

$$P(\Delta,V) = \frac{\bar{P}}{E_0} e^{-(V-V_0)^2 / 2\sigma_0^2}$$

$$Q_0^{-1} \approx \frac{\pi \bar{P} \gamma^2}{2\rho v^2} e^{-\frac{V_0^2}{2\sigma_0^2}}$$

high stress

Conclusion

* Universal properties of glasses, such as dissipation, specific heat, and thermal conductivity can be well described by a **two level system and Einstein oscillator model**.
* Glasses made of **3- or 4-fold coordinated materials** and glasses in the presence of **high stress** can have very low dissipation.
* High stress increases barrier heights of defects, effectively reducing the number of defects producing dissipation.