Wilson to Lindblad

Late-time Obstacles to Reliable EFT Calculations with Horizons

Enervac19
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CPB, Kaplanek, Rummel
1806.11415 & 1907.xxx

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Contents

- Motivation

- Secular terms in de Sitter space

  - Secular effects near dS space

- Qubits in dS

  - late-time resummation evaluated explicitly; critical slowing down; non-Markovian evolution

- Towards black holes:

  - Secular thermal effects in flat space; accelerated qubits in flat space

The Punchline

- **Gravity as an EFT**

  low-energy quantum effects in gravity can be understood as a Wilsonian EFT

The Punchline

e.g. Hawking radiation calculation
relies on Q<<M$_p$ with Q = 1/r$_s$

$$\mathcal{L} \sim \sqrt{-g}\left[M_p^2 R + c_1 R^2 + \frac{c_2}{m^2}R^3 + \cdots\right]$$

$$\mathcal{A}_E(Q) \sim \left(\frac{Q^2}{M_p^{E-2}}\right)\left(\frac{Q}{4\pi M_p}\right)^{2L}\prod_{n,d>2}\left[\frac{Q^2}{M_p^2}\left(\frac{Q}{m}\right)^{d-4}\right]^{V_{nd}}$$

Leading order: tree-level GR

Next order: 1-loop GR

tree GR with one R² insertion

see e.g. gr-qc/0311082

The Punchline

- BUT: there are hints EFT methods fail at late times even at small curvatures

  - secular growth and IR sensitivity in cosmological perturbations

 - information loss at late times for BHs

Can domain of validity of EFT sometimes require more than small curvatures?

The Punchline

- Will argue yes: EFT with gravity can differ from ordinary Wilsonian EFT

  - similar to effective description of particles in a medium

  - can involve issues of open systems (particularly in presence of horizons)

  - generic problems with late-time perturbative predictions (tho resummation methods exist)

The Punchline

- Late-time breakdown of perturbation theory illustrated by geometrical optics regime
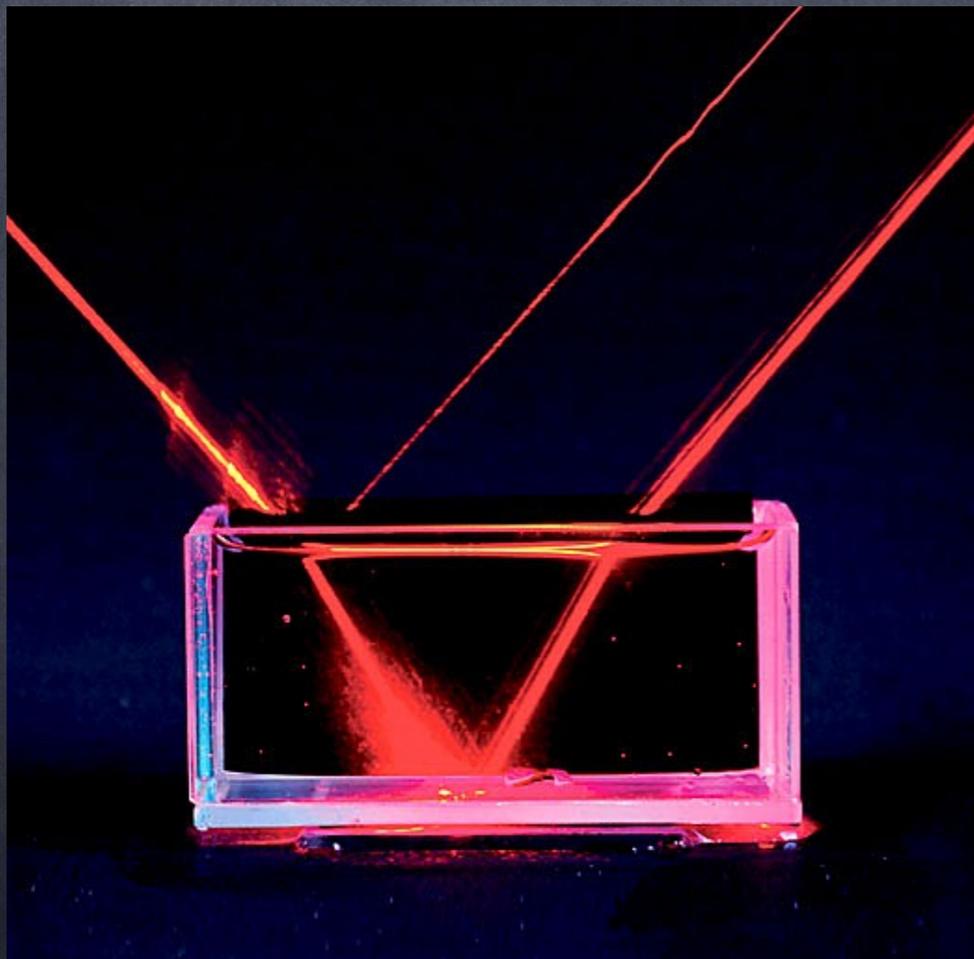
Naive perturbation theory fails at late times

$$U(t) = e^{-i(H_0 + V)t}$$

Physics is not powerless in geometrical optics regime: tools exist for understanding late time evolution

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Secular Terms in dS

- Extra-Hubble modes are key to success of inflationary predictions

  - What is their EFT?

- What quantifies theoretical error?

Hubble length

wavelength
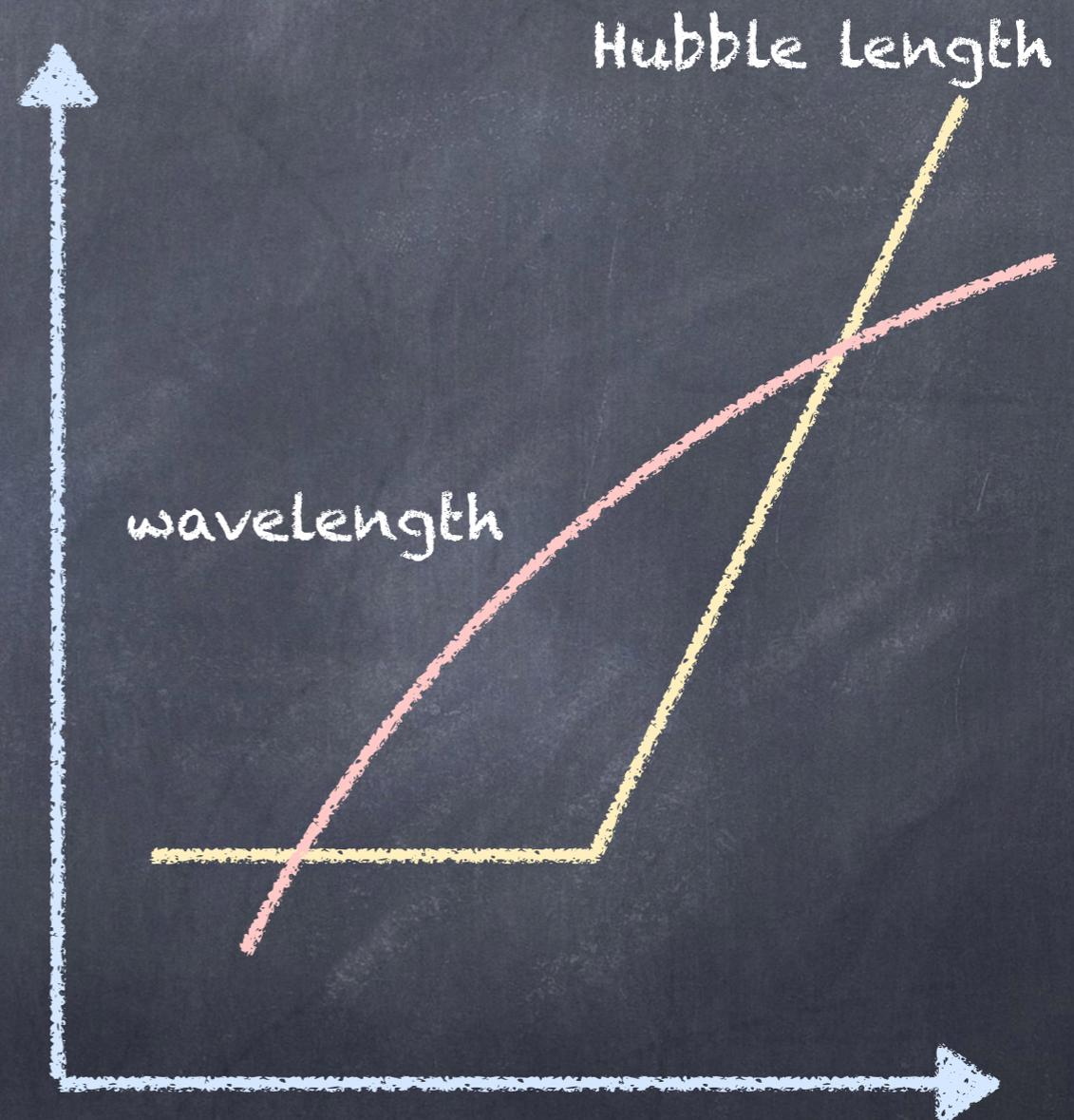
Secular Terms in dS

- Predictions: free fields on curved background

$$\mathcal{L} = -\sqrt{-g}\,\frac{1}{2}(\partial\phi)^2$$

$$\langle\phi^{2n}\rangle = (2n-1)!!\left(\frac{H^2}{4\pi^2}\,\ln a\right)^n$$

$$e.g. \quad \langle\phi^2\rangle = \frac{H^3 t}{4\pi^2}$$

Secular Terms in dS

- Predictions: free fields on curved background

- Corrections can be both IR divergent and break down at late times

$$\mathcal{L} = -\sqrt{-g}\left[\frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4!}\,\phi^4\right]$$

$$\langle\phi^{2n}\rangle = (2n-1)!!\left(\frac{H^2}{4\pi^2}\,\ln a\right)^n\left[1 - \frac{n(n+1)}{2}\left(\frac{\lambda}{36\pi^2}\right)\ln^2 a + \cdots\right]$$

Tsamis & Woodard

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Aside

- IR divergences/secular growth are gauge artefacts for single-field (single-clock) models

<div align="right">Senatore, Zaldarriagga
Giddings, Sloth</div>

- General statement (multiple scalars, other massless fields, tensor modes, etc) not known, but IR divergences should cancel from physical observables

Aside

- Normally IR divergences cancel for physical quantities (a la Bloch-Nordsieck)

- Often large logarithms survive IR cancellation, with IR scale that is system-dependent (not universal)

$$A \ln \left(\frac{\mu}{\omega} \right) + A \ln \left(\frac{m}{\mu} \right) = A \ln \left(\frac{m}{\omega} \right)$$

Secular Terms in dS

- Party-line understanding: Since $\langle \phi^2 \rangle \propto t$ resembles random walk, suggests setting up Fokker-Planck equation to capture the evolution of probability density $P[\phi, t]$

$$\frac{\partial P}{\partial t} = \frac{H^3}{8\pi^2} \frac{\partial^2 P}{\partial \phi^2} + \frac{\lambda}{18H} \frac{\partial}{\partial \phi} \left(\phi^3 P \right)$$

Starobinsky

First term (noise) and second term (drift) designed to ensure the correct evolution of the variance $\partial_t \langle \phi^2 \rangle$ and the mean $\partial_t \langle \phi \rangle$

Secular Terms in dS

○ Stochastic evolution correctly captures IR part of $\partial_t \langle \phi^{2n} \rangle$ & in particular the secular growth of the full field theory including subleading corrections in powers of λ

$$\frac{\partial P}{\partial t} = \frac{H^3}{8\pi^2} \frac{\partial^2 P}{\partial \phi^2} + \frac{\lambda}{18H} \frac{\partial}{\partial \phi} \left(\phi^3 P \right)$$

Tsamis & Woodard

$$\langle \phi^{2n} \rangle = (2n-1)!! \left(\frac{H^2}{4\pi^2} \ln a \right)^n \left[1 - \frac{n(n+1)}{2} \left(\frac{\lambda}{36\pi^2} \right) \ln^2 a + \cdots \right]$$
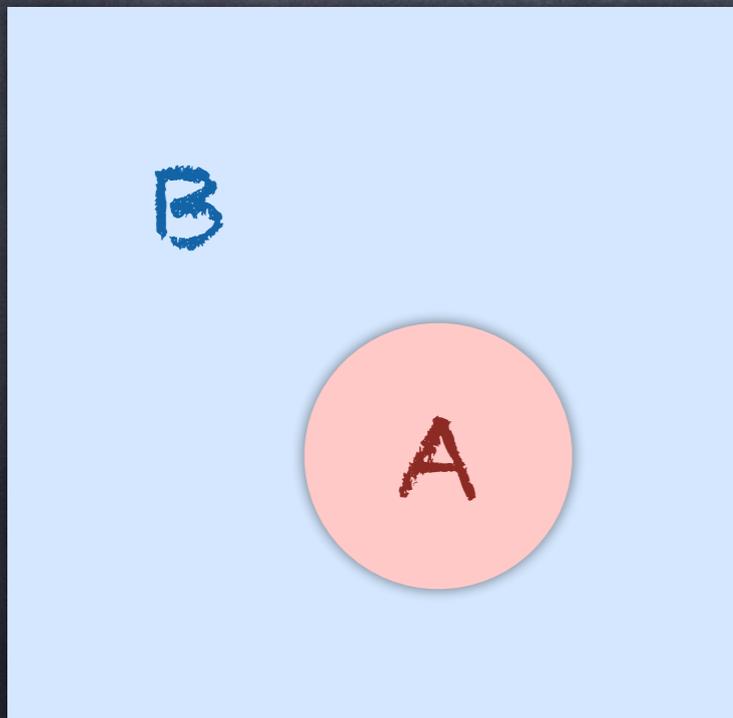
PLUS: Fokker-Planck eq predicts late-time behaviour

$$P \propto \exp \left[-\frac{8\pi^2 V}{3H^4} \right]$$

Starobinsky & Yokohama

Open EFTs

- Open EFTs: consider the evolution of a subset A of a larger system B

    eg: light in glass or neutrinos in Sun or super-Hubble modes during inflation

$$\rho_A = \mathrm{Tr}_B \, \rho$$

$$\frac{\partial \rho}{\partial t} = -i \left[\rho, H_{\mathrm{int}} \right]$$

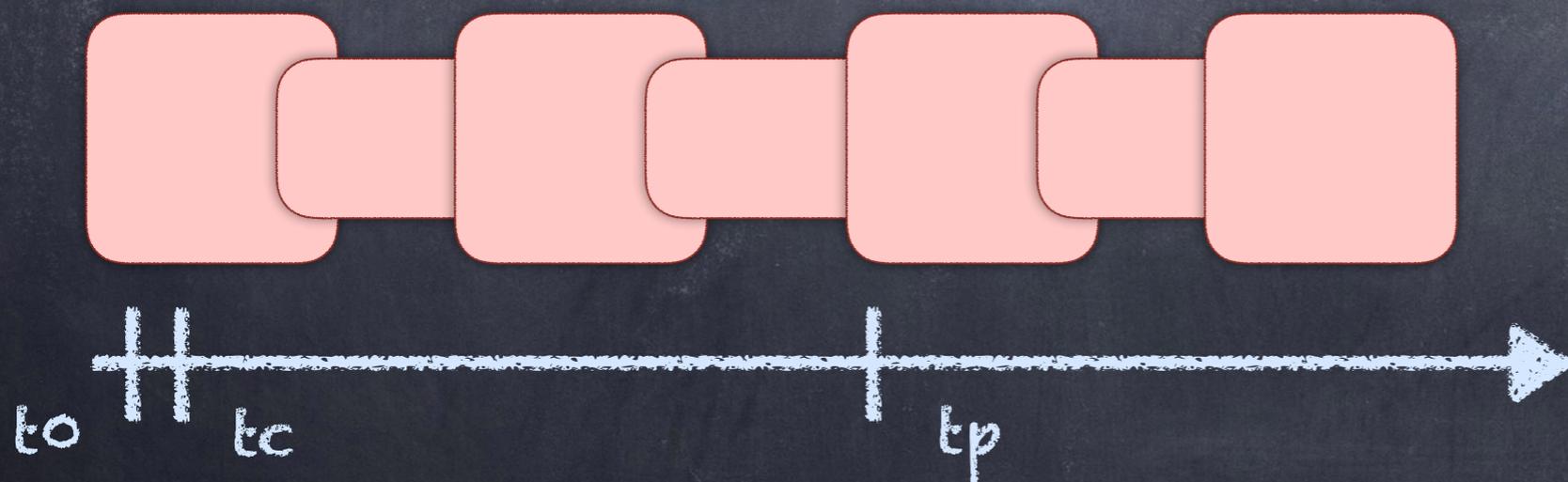$$\frac{\partial \rho_A}{\partial t} \simeq F[\rho_A, \langle H_{\mathrm{int}}(t) \rangle, \langle H_{\mathrm{int}}(t) H_{\mathrm{int}}(t') \rangle, \cdots]$$

EFT part: evolution often simplifies for t much longer than typical correlation time

B

A

Open EFTs

- Simplest example of late-time resummation: exponential decay

$$n(t) = n_0\, e^{-\Gamma t} \quad vs \quad n(t) \simeq n_0\big(1 - \Gamma t\big) \qquad \Gamma = O(g^2)$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\Gamma n$$

to   tc   tp

all orders in $g^2 t$

Open EFTs

* Starobinsky (86)
Salopek & Bond (91)
Starobinsky Yokohama (94)
Vennin, Starobinsky (15)
CPB, Holman, Tasinato (15)
Collins, Holman, Vardanyan (17)

- As applied to inflationary cosmology captures stochastic methods

System consists of super-Hubble modes; environment is Hubble and sub-Hubble modes

$$\frac{\partial \rho_A}{\partial t} \simeq i[\overline{H}_{\text{int}}, \rho_A]$$

Leading terms give stochastic inflation + corrections

$$P[\varphi] = \langle \varphi | \varrho | \varphi \rangle \qquad \frac{\partial P}{\partial t} = \frac{\partial^2}{\partial \varphi^2}(NP) + \frac{\partial}{\partial \varphi}(FP)$$

Schrodinger evolution becomes stochastic in WKB limit

see e.g. 1512.00169

Lesgourges, Polarski & Starobinsky
CB, Holman, Tasinato, Williams

Open EFTs

- As applied to inflationary cosmology also captures corrections and decoherence

System consists of super-Hubble modes; environment is Hubble and sub-Hubble modes

$$\frac{\partial \rho_A}{\partial t} \simeq i[\overline{H}_{\text{int}}, \rho_A] + \sum_{ij} c_{ij} \left[2L_i^* \rho_A L_j - L_i^* L_j \rho_A - \rho_A L_i^* L_j \right]$$

Non-Liouville terms give decoherence and more

$$\langle \varphi | \varrho | \varphi' \rangle$$

Quantum fluctuations rapidly decohere (in field basis) in few Hubble times

see e.g. 1408.5002

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Qubits in Space

- Evolution of two-level qubit in various spacetimes:

example of late-time resummation in qubit coupling;
shows how field interaction changes naive evolution

$$H = I \otimes H_{\text{field}} + \mathfrak{h} \otimes I + \boxed{\mathfrak{m} \otimes \int_{y(\tau)} \mathrm{d}\tau \, \phi[y(\tau)]}$$

Unruh

$$\mathfrak{h} = \frac{\omega}{2}\,\sigma_3 \qquad \mathfrak{m} = g\,\sigma_1$$

H_{int}

$$H_{\text{field}} = -\int \mathrm{d}^3 x \sqrt{-g}\,\left[\frac{1}{2}\,\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{m^2}{2}\,\phi^2 + \boxed{\frac{\lambda}{4!}\,\phi^4}\right]$$

H_λ

In dS choose qubit on co-moving trajectory

Qubits in Space

- Compute perturbative evolution in interaction picture

$$\rho_I(t) \simeq \rho(0) - i \int_0^t \mathrm{d}s_1 \left[V(s_1), \rho(0) \right] + (-i)^2 \int_0^t \mathrm{d}s_1 \int_0^{s_1} \mathrm{d}s_2 \left[V(s_2), \left[V(s_1), \rho(0) \right] \right] + \mathcal{O}(V^3)$$

$$V(s) := e^{iH_0 s} H_{\mathrm{int}} e^{-iH_0 s} \qquad\qquad \rho(0) = \left[|\Omega\rangle\langle\Omega| \right] \otimes \varrho_0$$

- Trace out field degrees of freedom to track evolution of reduced qubit density matrix

$$\varrho_I(\tau) \simeq \varrho_0 - g^2 \int_0^\tau \mathrm{d}s_1 \int_0^{s_1} \mathrm{d}s_2 \Big\{ W_\Omega(s_1, s_2) \left[\varrho_0 \, \mathfrak{m}_I(s_1), \mathfrak{m}_I(s_2) \right]$$

$$+ W_\Omega^*(s_1, s_2) \left[\varrho_0 \, \mathfrak{m}_I(s_1), \mathfrak{m}_I(s_2) \right]^\dagger \Big\} + \mathcal{O}(g^4)$$

$$\varrho(t) = \mathrm{Tr}_\phi \, \rho(t) \qquad\qquad W_\Omega(s_1, s_2) = \langle\Omega| \, \phi[y(s_1)] \, \phi[y(s_2)] \, |\Omega\rangle$$

Late Times V1.0

- Choosing qubit initially in ground state gives

$$\varrho(\tau) \simeq |\downarrow\rangle\langle\downarrow| + g^2\sigma_3 \int_0^\tau ds_1 \int_0^\tau ds_2 \; W_\Omega(s_1 - s_2)\, e^{-i\omega(s_1 - s_2)}$$

- In late-time limit ($\tau \to \infty$) the integral over s_1+s_2 is ill-defined, so compute the rate for a transition

$$\lim_{\tau\to\infty} \frac{\partial\varrho(\tau)}{\partial\tau} = g^2\sigma_3\, \mathcal{R}_\Omega(\omega)$$

Sciama & Candelas

$$\mathcal{R}_\Omega(\omega) := \int_{-\infty}^{\infty} d\tau\; W_\Omega(\tau)\, e^{-i\omega\tau}$$

Late Times V2.0

- Liouville equation and projection onto reduced density matrix are both linear processes, so can do a better job of time evolution

$$\partial_t \rho = \mathscr{L}_t(\rho) \quad \text{where} \quad \mathscr{L}_t(\rho) := -i\Big[V(t), \rho\Big]$$

$$\text{and} \quad \mathscr{P}(\mathcal{O}) := |\Omega\rangle\langle\Omega| \otimes \mathrm{Tr}_\phi(\mathcal{O}) \quad \text{so that} \quad \mathscr{P}[\rho(t)] = |\Omega\rangle\langle\Omega| \otimes \varrho(t)$$

$$\text{can check} \quad \mathscr{P}^2 = \mathscr{P} \quad \text{so that} \quad \mathcal{Q}^2 = \mathcal{Q} \quad \text{where} \quad \mathcal{Q} := 1 - \mathscr{P}$$

- Then Liouville equation for the full density matrix can be expressed as a integro-differential equation for the reduced density matrix

$$\mathscr{P}(\partial_t \rho) = \mathscr{P}\mathscr{L}_t(\rho) = \mathscr{P}\mathscr{L}_t\mathscr{P}(\rho) + \mathscr{P}\mathscr{L}_t\mathcal{Q}(\rho)$$

$$\mathcal{Q}(\partial_t \rho) = \mathcal{Q}\mathscr{L}_t(\rho) = \mathcal{Q}\mathscr{L}_t\mathscr{P}(\rho) + \mathcal{Q}\mathscr{L}_t\mathcal{Q}(\rho)$$

Nakajima
Zwanzig

Late Times V2.0

- Weak Coupling: Evaluated to second order in perturbation theory the Nakajima-Zwanzig equation becomes

$$\partial_t \varrho(t) = -i\big[\mathfrak{m}(t), \varrho(t)\big] \langle \Omega | \mathscr{A}(t) | \Omega \rangle$$

$$+(-i)^2 \int_{t_0}^{t} ds \Big\{ \big[\mathfrak{m}(t), \mathfrak{m}(s)\varrho(s)\big] \langle \Omega | \delta\mathscr{A}(t)\delta\mathscr{A}(s) | \Omega \rangle$$

$$- \big[\mathfrak{m}(t), \varrho(s)\mathfrak{m}(s)\big] \langle \Omega | \delta\mathscr{A}(s)\delta\mathscr{A}(t) | \Omega \rangle \Big\} + \mathscr{O}(V^3)$$

where $\quad V(t) = \mathfrak{m}(t) \otimes \mathscr{A}(t)$

and $\quad \delta\mathscr{A}(t) = \mathscr{A}(t) - \langle \Omega | \mathscr{A}(t) | \Omega \rangle$

Although nonlocal in time, on both sides this refers directly only to the reduced density matrix.

Qubits in Space

- For the qubit system of interest the Nakajima-Zwanzig equation becomes (in Schrodinger picture)

$$\frac{\partial \varrho_{11}}{\partial \tau} = g^2 \int_{-\tau}^{\tau} ds\ W_\Omega(s)\, e^{-i\omega s} - 4g^2 \int_{0}^{\tau} ds\ \mathrm{Re}[W_\Omega(s)]\, \cos(\omega s)\, \varrho_{11}(\tau - s)$$

$$\frac{\partial \varrho_{12}}{\partial \tau} = -i\omega \varrho_{12}(\tau) - 4ig^2 \int_{0}^{\tau} ds\ \mathrm{Re}[W_\Omega(s)]\, \mathrm{Im}[\varrho_{12}(\tau - s)]$$

so off-diagonal and diagonal terms evolve independent of each other at this order

In general evolution is non-Markovian due to the integration over the qubit's past history ('memory effect')

Qubits in Space

- For the qubit system of interest the Nakajima-Zwanzig equation becomes (in Schrodinger picture)

$$\frac{\partial \varrho_{11}}{\partial \tau} = g^2 \int_{-\tau}^{\tau} ds \, W_\Omega(s) \, e^{-i\omega s} - 4g^2 \int_0^{\tau} ds \, \text{Re}[W_\Omega(s)] \, \cos(\omega s) \, \varrho_{11}(\tau - s)$$

$$\frac{\partial \varrho_{12}}{\partial \tau} = -i\omega \varrho_{12}(\tau) - 4ig^2 \int_0^{\tau} ds \, \text{Re}[W_\Omega(s)] \, \text{Im}[\varrho_{12}(\tau - s)]$$

so off-diagonal and diagonal terms evolve independent of each other at this order

Late-time simplicity follows if W falls off and evolution sought for times longer than falloff time

(if ω is large must also coarse grain W)

Qubits in Space

- Approximately markovian form at very late times if $W(s)$ is sufficiently sharply peaked in time since $\varrho_{ij}(\tau - s) \simeq \varrho_{ij}(\tau)$ within the integral, leading to a Lindblad equation:

Candelas-Sciama result

$$\frac{\partial \varrho_{11}}{\partial \tau} \simeq g^2 \mathcal{R}_\Omega(\omega) - 2g^2 \mathcal{C}_\Omega(\omega)\, \varrho_{11}(\tau)$$

'thermalization' time:

$$\xi_T = [2g^2 \mathcal{C}_\Omega(\omega)]^{-1}$$

frequency renormalization

$$\frac{\partial \varrho_{12}}{\partial \tau} \simeq -i\left[\omega + g^2 \Delta_\Omega\right] \varrho_{12} - g^2 \mathcal{C}_\Omega(\omega)\, \text{Im}[\varrho_{12}(\tau)]$$

'decoherence' time:

$$\xi_D = 2\xi_T$$

$$\text{with} \quad \mathcal{C}_\Omega(\omega) := 2 \int_0^\infty \mathrm{d}\tau\, \text{Re}[W_\Omega(\tau)]\, \cos(\omega\tau)$$

$$\Delta_\Omega(\omega) := 2 \int_0^\infty \mathrm{d}\tau\, \text{Re}[W_\Omega(\tau)]\, \sin(\omega\tau)$$

Describes relaxation to asymptotic static solution:

$$\text{if} \quad W_\Omega(\tau - i\beta) = W_\Omega(-\tau) \quad \text{then} \quad \varrho_{\text{static}} = \begin{bmatrix} e^{-\beta\omega} & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{e^{-\beta\omega} + 1}$$

Qubits in de Sitter

- For de Sitter space evaluate Wightman function using BD vacuum and qubit along a co-moving curve to compute relaxation rates as system approaches static solution at very late times

$$W_{\mathrm{BD}}(\tau) = \frac{H^2(\frac{1}{4} - \nu^2)}{16\pi \cos(\pi\nu)} \, {}_2F_1\left[\frac{3}{2}+\nu, \frac{3}{2}-\nu; 2; 1 + \left[\sinh\left(\frac{H\tau}{2}\right) - i\epsilon\right]^2\right]$$

$$\nu = \sqrt{\frac{9}{4} - \frac{M^2}{H^2}} = \sqrt{\frac{9}{4} - \frac{m^2}{H^2} + 12\xi}$$

conformal scalar: $\quad W_{\mathrm{BD}}(\tau) = -\frac{1}{16\pi^2} \frac{H^2}{\left[\sinh\left(H\tau/2\right) - i\epsilon\right]^2}$

- Also satisfies the thermal 'KMS' condition

$$\text{if} \quad W_\Omega(\tau - i\beta) = W_\Omega(-\tau) \quad \text{for} \quad T = \frac{1}{\beta} = \frac{H}{2\pi}$$

Qubits in de Sitter

- For de Sitter space evaluate Wightman function using BD vacuum and qubit along a co-moving curve to compute relaxation rates as system approaches static solution at very late times

e.g. for conformal scalars: $\quad \xi_D = 2\xi_T \simeq \dfrac{2\pi}{g^2\omega}\, \tanh\left(\dfrac{\pi\omega}{H}\right)$

Trust the above Markovian limit on timescales $\xi \gg H^{-1}$

For m << H the Markovian limit instead requires $\xi \gg H/m^2$
(critical slowing down)

Conclusions

- IR and secular issues likely generic for light bosons in gravitational fields, and cause perturbative failure at late times

  - Small curvatures/couplings need not be sufficient for calculation control

  - Resummation techniques available: Open EFTs

- Similar story for black hole information loss?