

# A $\nu$ Solution to the Strong CP Problem

based on work performed in collaboration with  
M. Carena, D. Liu, J. Liu, N.R. Shah and X. Wang, arXiv:1904.05860

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Kavli Institute for Theoretical Physics

**Origin of the vacuum energy and electroweak scales**


# Standard Model

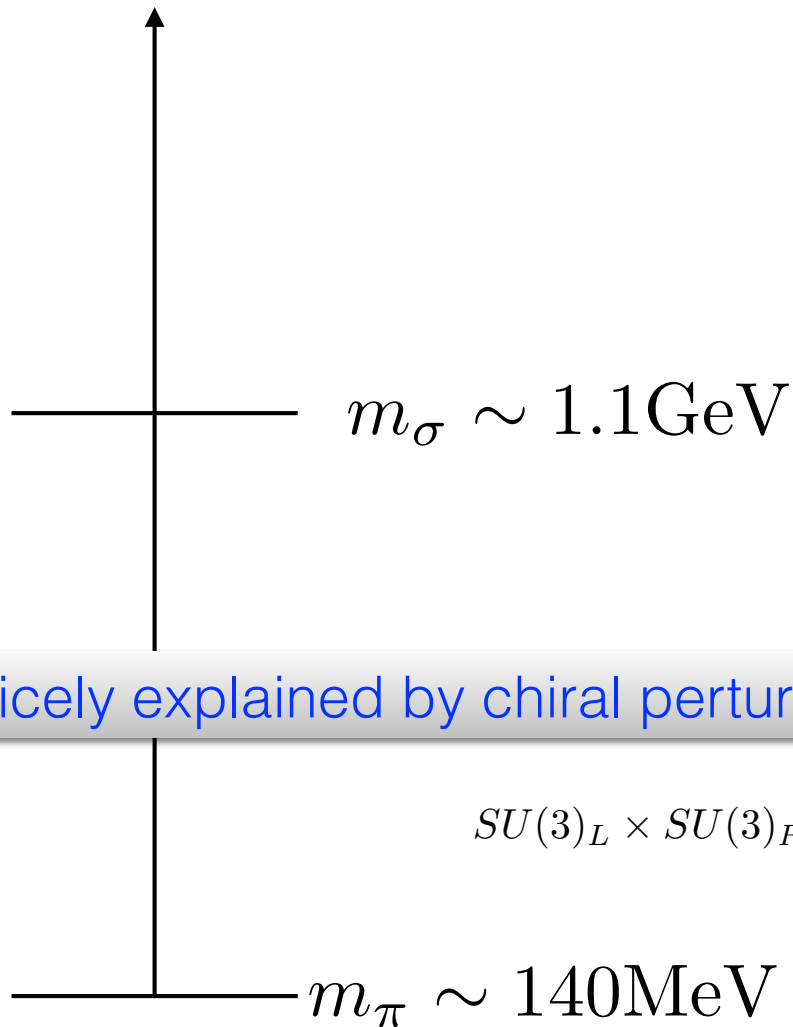
- Presents a consistent description of particle interactions based on
  - Lorentz Invariance
  - Gauge Invariance
  - Renormalizability
- Gauge theory is chiral. Masses are obtained via the introduction of the Higgs.
- CP symmetry is broken and the effect may be understood as proceeding from arbitrary complex Yukawa coupling that lead, after diagonalization, to CP violating phases in weak interactions.
- Electromagnetic and strong interactions are CP-invariant at tree-level. No CP-violating effects mediated by these interactions have been observed.

# Other Properties of the SM

- No tree-level Flavor Changing Neutral Currents
- GIM Suppression at the loop level
- Since the right-handed quarks do not feel the charged weak interactions, after diagonalization all the phases in the diagonal mass terms may be eliminated by redefinition of the right-handed quark fields, with no tree-level consequences.
- Problem at tree-level : Large hierarchy of fermion masses. In particular, neutrino masses
- Problem at the quantum-level :
  - CP-violation in strong interaction is induced : Strong CP Problem
  - Higgs mass parameter is ultraviolet sensitive : Hierarchy Problem
  - Hypercharge interactions are not asymptotically free : Energy of the associated Landau Pole too high for the SM to be valid as an effective theory at those scales.

# Other Properties of the SM

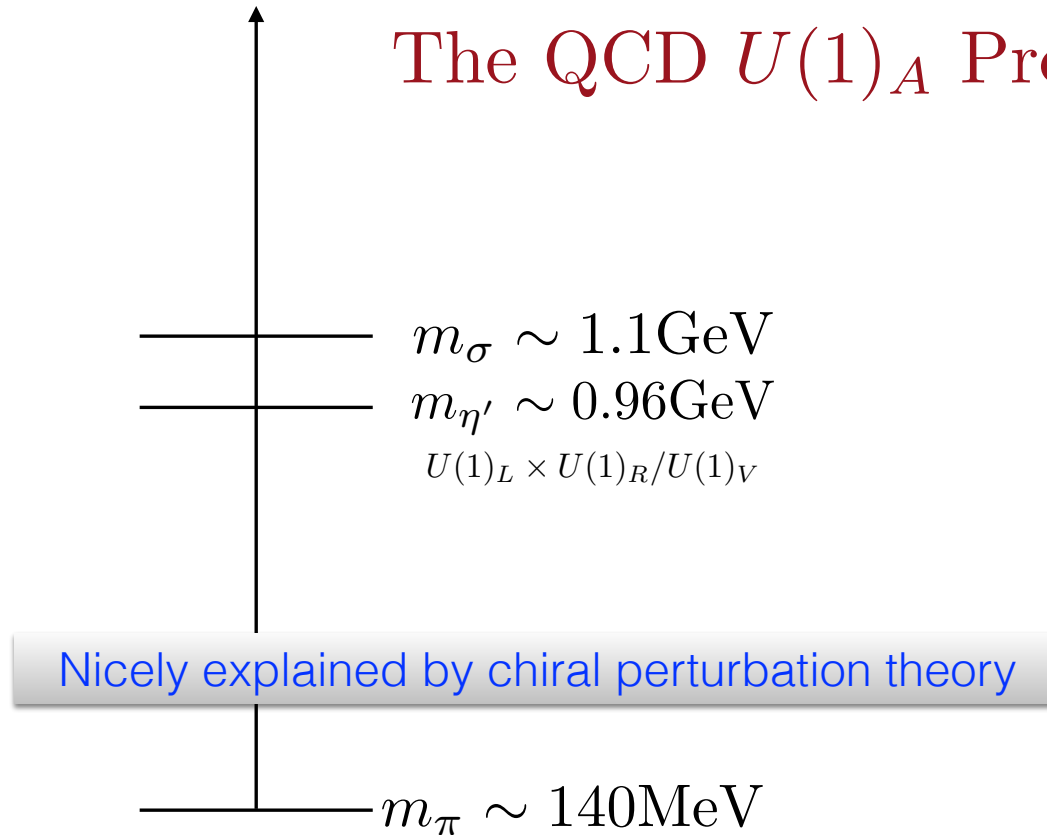
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- 



Pions are pseudo-Goldstone bosons. They would be massless if quarks were massless

# The relevance of $\theta$ in QCD

## The QCD $U(1)_A$ Problem



- The solution to this problem is associated with the anomalous nature of the axial symmetry  $U(1)_A$ , which is therefore not a good symmetry at the quantum level.
- t'Hooft solution to this problem relied also on the complexity of the QCD vacuum, which is associated with a new, dimensionless parameter  $\theta_{\text{QCD}}$

# The $\theta$ vacuum

- Vacua in QCD associated with pure gauge fields belonging to different homotopy classes. There are field configurations mediating transitions between these vacua

$$n = n_+ - n_- = \frac{g_s^2}{32\pi^2} \int d^4x G_{\mu\nu,a} G^{\mu\nu,a}$$

- The true, gauge invariant and physical vacuum of QCD is none of those vacua, but a particular combination

$$|\theta\rangle = \sum_n \exp(in\theta) |n\rangle$$

- And actually, given two different values of this new parameter and can simply show that the matrix element of gauge invariant operators

$$\langle \theta' | T(O_1 O_2 \dots O_n) | \theta \rangle \propto \delta(\theta - \theta')$$

- This means that these vacua are stable under gauge invariant perturbations. It also means that while quantizing the QCD theory one should add an additional term

$$S_{\text{QCD}} \supset -\theta \frac{g_s^2}{32\pi^2} \int d^4x G_{\mu\nu,a} G^{\mu\nu,a}$$

# Chiral Anomaly

- In the massless limit, the anomaly is connected with the divergence of the axial current

$$J_5^\mu = \sum_{i=1}^{N_f} \bar{q}_i \gamma^\mu \gamma_5 q_i,$$

$$\partial_\mu J_5^\mu = 2N_f \left( \frac{g^2}{32\pi^2} G^{\mu\nu,a} \tilde{G}_{\mu\nu,a} \right) \longrightarrow \Delta Q_5 = 2N_f \int d^4x \left( \frac{g^2}{32\pi^2} G^{\mu\nu,a} \tilde{G}_{\mu\nu,a} \right)$$

- As is well known, the right is a total derivative, and one can define a new, conserved current ( $\partial_\mu \tilde{J}_5^\mu = 0$ ), which is however not gauge invariant

$$\tilde{J}_5^\mu = J_5^\mu - 2N_f \left( \frac{g^2}{32\pi^2} K^\mu \right)$$

$$K^\mu = \epsilon_{\mu\alpha\beta\gamma} A_a^\alpha \left[ F_a^{\beta\gamma} - \frac{g}{3} \epsilon_{abc} A_b^\beta A_c^\gamma \right]$$

- Indeed, considering

$$\Omega_1 |n\rangle = |n+1\rangle, \text{ then } \Omega_1 \tilde{Q}_5 \Omega_1^\dagger = \tilde{Q}_5 + 2N_f$$

- Moreover,  $\exp(i\alpha \tilde{Q}_5) |\theta\rangle = |\theta + 2N_f \alpha\rangle$ , consistent with the fact that

$$\exp(i\alpha) q_{R,L} \text{ implies } \theta \rightarrow (\theta \pm \alpha)$$



# CP-Violation in the Strong Sector

- The new term ,  $-\theta \int d^4x \left( \frac{g^2}{32\pi^2} G^{\mu\nu,a} \tilde{G}_{\mu\nu,a} \right)$  , explicitly violates CP.
- Actually, as we shown before any chiral rotation of the quark fields would lead to a redefinition of the new parameter  $\theta$ , implying that the only, physical, parametrization invariant quantity is given by

$$\theta_{\text{QCD}} = \theta + \arg[\det[M_q]]$$

- Here, the mass terms have been defined as

$$\mathcal{L} \supset - \sum_q m_q (\bar{q}_L q_R + h.c.)$$

- Therefore, the physical parameter can be identified with the  $\theta$  term in the Lagrangian when all mass parameters are real.
- One can, by proper chiral transformations, redefine  $\theta$  away, while shifting it to the [argument of one of the quark masses, for instance, the up quark mass.](#)

# CP-Violation and Chiral Perturbation Theory

- The study of the consequences of CP-violation in chiral perturbation theory may be better performed by removing the  $\theta$  term and transferring it to the imaginary component of the mass terms.
- One should do it carefully, so that the new CP-violating mass operators do not mix with the Goldstone bosons. The result is straightforward, resulting in the three flavor case, in a CP-violating Lagrangian,

$$\mathcal{L}_{\text{CP}} = - \frac{\text{Im}[m_u m_d m_s \exp(i\theta)]}{|m_u m_d| + |m_u m_s| + |m_d m_s|} (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s)$$

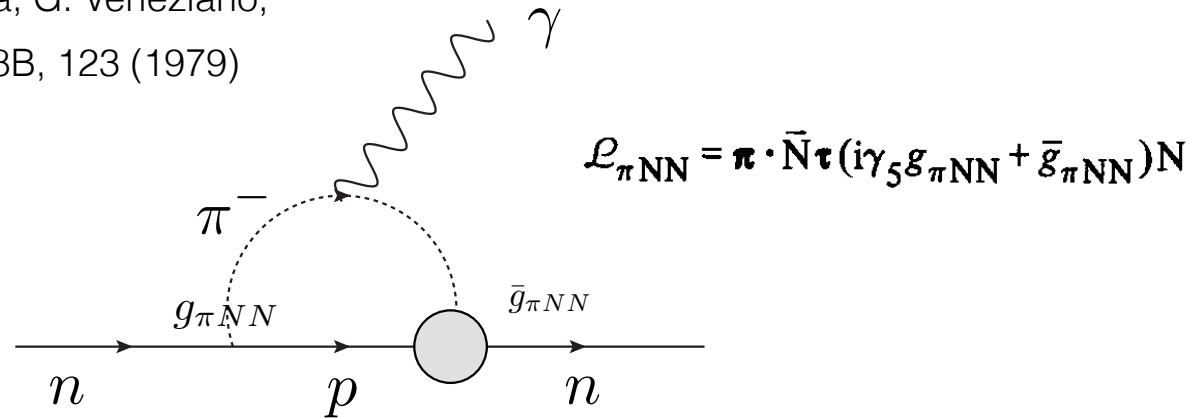
- This expression contains all the right properties, showing that for vanishing masses the CP-violation is not present.
- The presence of this CP-violating term induces a calculable neutron electric dipole moment in chiral perturbation theory. Observe that the mass factor is just

$$m_{\text{eff}} \sin(\theta_{\text{QCD}}) \quad \text{with} \quad m_{\text{eff}} = \frac{|m_u m_d m_s|}{|m_u m_d| + |m_u m_s| + |m_s m_d|}$$

# The neutron electric dipole moment

- In chiral perturbation theory, taking the strange mass to be much larger than the up and down masses,

R. J. Crewther, P. Di Vecchia, G. Veneziano,  
and E. Witten, Phys. Lett. 88B, 123 (1979)



$$\frac{d_n}{e} \sim \frac{g_{\pi NN} \bar{g}_{\pi NN}}{4\pi^2 M_N} \ln \frac{M_N}{m_\pi}, \quad \bar{g}_{\pi NN} \sim \theta_{QCD} \frac{m_{eff}}{F_\pi}, \quad m_{eff} \sim \frac{m_u m_d}{m_u + m_d}$$

- Similar results may be obtained with QCD sum rules,

$$d_n \simeq \theta_{QCD} \times (2.4 \pm 0.7) \times 10^{-16} e \text{ cm}$$

M. Pospelov and A. Ritz, arXiv:9908508

# The Strong CP Problem

- Current bounds on the neutron electric dipole moment show that the physical parameter  $\theta_{\text{QCD}}$  should be very small, namely

$$d_n < 3 \times 10^{-26} \text{e cm}$$

- This implies that

$$\theta_{\text{QCD}} < 1.3 \times 10^{-10}$$

- But from our discussion above, there is no real reason why this parameter should be small. The mass parameters, after all, can in principle carry arbitrary phases, and one would expect in general that

$$\theta_{\text{QCD}} \sim \mathcal{O}(1)$$

- This constitutes the so-called Strong CP-problem.

# Old solutions to the strong CP Problem

- Make  $\theta$  a Dynamical field : The axion solution. Axion effective potential is such that the vacuum solution is associated with an effective  $\theta_{\text{QCD}} = 0$

**R.D. Peccei and H.R. Quinn'77**  
**F. Wilczek'78, S. Weinberg'78**

- Make CP an exact symmetry broken spontaneously in such a way that the determinant of the quark matrix remains real.

**A. Nelson'84 and S.M. Barr'84**

- Up quark is massless

**H. Georgi and I. Mc Arthur'81**  
**K. Choi, C.W. Kim and W.K. Sze'88**  
**T. Banks, Y. Nir and N. Seiberg'94**  
**W. A. Bardeen'19**

# Rephrasing the Problem

- When discussing the neutron electric dipole moment we discovered that it depended on the combination

$$[m_u m_d m_s \exp(i\theta)] = |m_u m_d m_s| \exp(i\theta_{\text{QCD}})$$

- This combination is physical and invariant under field redefinitions. One can make appropriate field redefinitions to eliminate  $\theta$  and make the down and strange quark masses real.
- In such a basis, only the up quark mass is complex and the bound on the electric dipole moment becomes a bound on the imaginary component of the up quark mass. **In this basis**, that we will just use as a guidance,

$$\theta_{\text{QCD}} = \arg[m_u]$$

- And the bound translates to a bound on the imaginary component of  $m_u$

$$\text{Im}[m_u(1\text{GeV})] < 10^{-3} \text{eV}$$

# General Basis

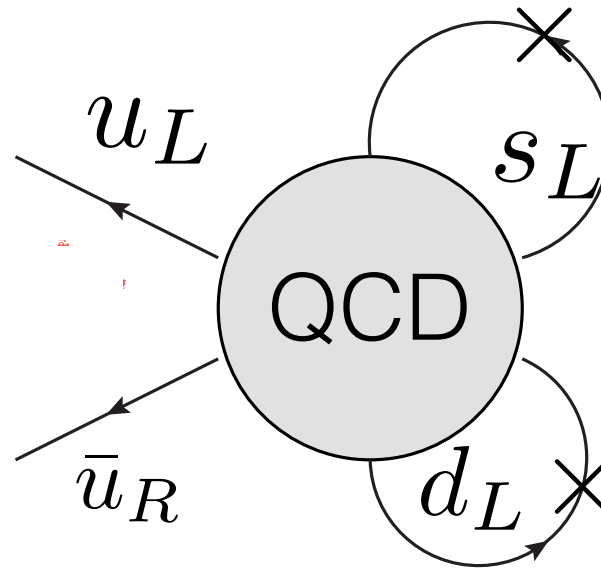
- The bound on the imaginary part of the up quark mass in that particular basis (let's call it canonical basis) can be rephrased in a more general basis :
- What we need is the bulk of the contribution to the up quark mass to be real in this basis, or in a general basis, not to contribute to  $\theta_{\text{QCD}}$
- On the other hand, there could be a smaller, arbitrary complex contribution in the canonical basis, provided it remains small, satisfying the above bound.
- These constraints are fulfilled if, for instance, the bulk of the up quark mass comes from instanton contributions. Indeed,

$$m_u^{\text{inst}} = \exp(-i\theta) \frac{m_d^* m_s^*}{\Lambda_{\text{inst}}}$$

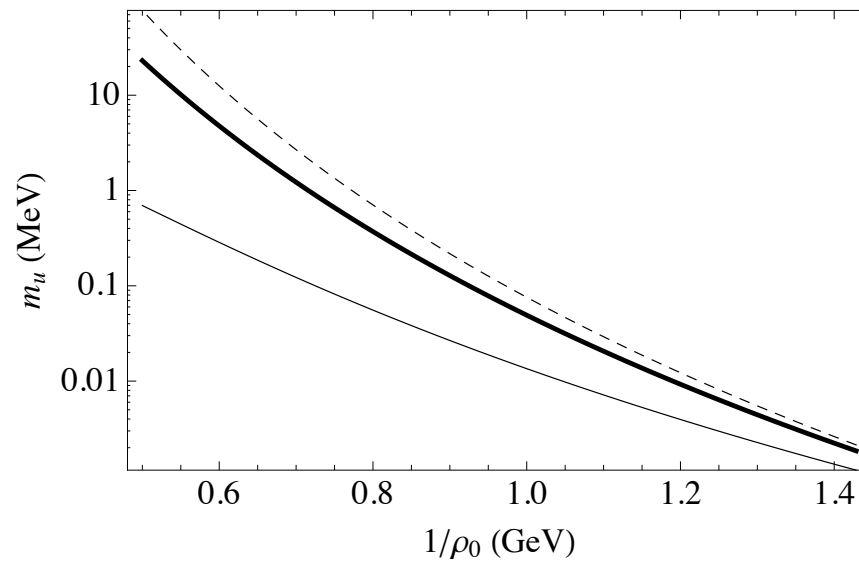
- One can easily see that  $m_u^{\text{inst}}$  does not contribute to  $\theta_{\text{QCD}}$ . It is real in the canonical basis. It could potentially solve the strong CP problem for a massless up-quark.

**K. Choi, C.W. Kim and W.K. Sze'88**

# Diagrammatic representation of the Instanton contributions



$$m_u^{\text{inst}} = \exp(-i\theta) \frac{m_d^* m_s^*}{\Lambda_{\text{inst}}}$$



**Draper and Dine'14**

These contributions  
can potentially be large  
enough



$$\theta_{\text{QCD}}$$

In general, one would expect a non-vanishing tree-level contribution

$$m_u = m_u^H + m_u^{\text{inst}}$$

In the canonical basis, assuming instanton contribution dominant

$$|m_u^H| \ll |m_u| \simeq |m_u^{\text{inst}}|$$

$$\theta_{\text{QCD}} \simeq \sin \theta^H \frac{|m_u^H|}{|m_u|} (1 \text{ GeV})$$

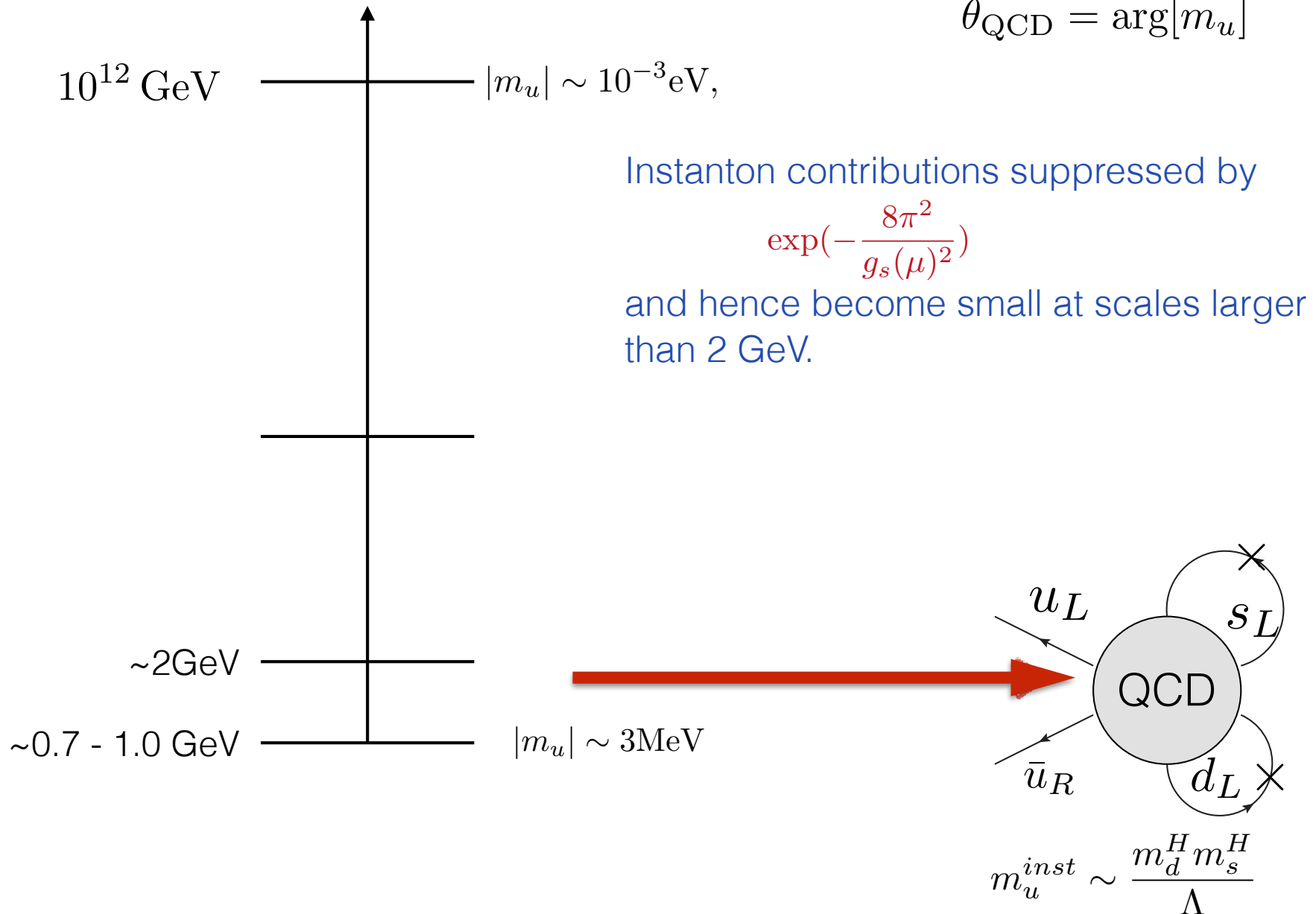
Bound on  $\theta_{\text{QCD}}$  leads to a bound on tree-level up quark mass

$$|m_u^H| (1 \text{ GeV}) \sin \theta^H < 6.5 \times 10^{-4} \text{ eV}$$

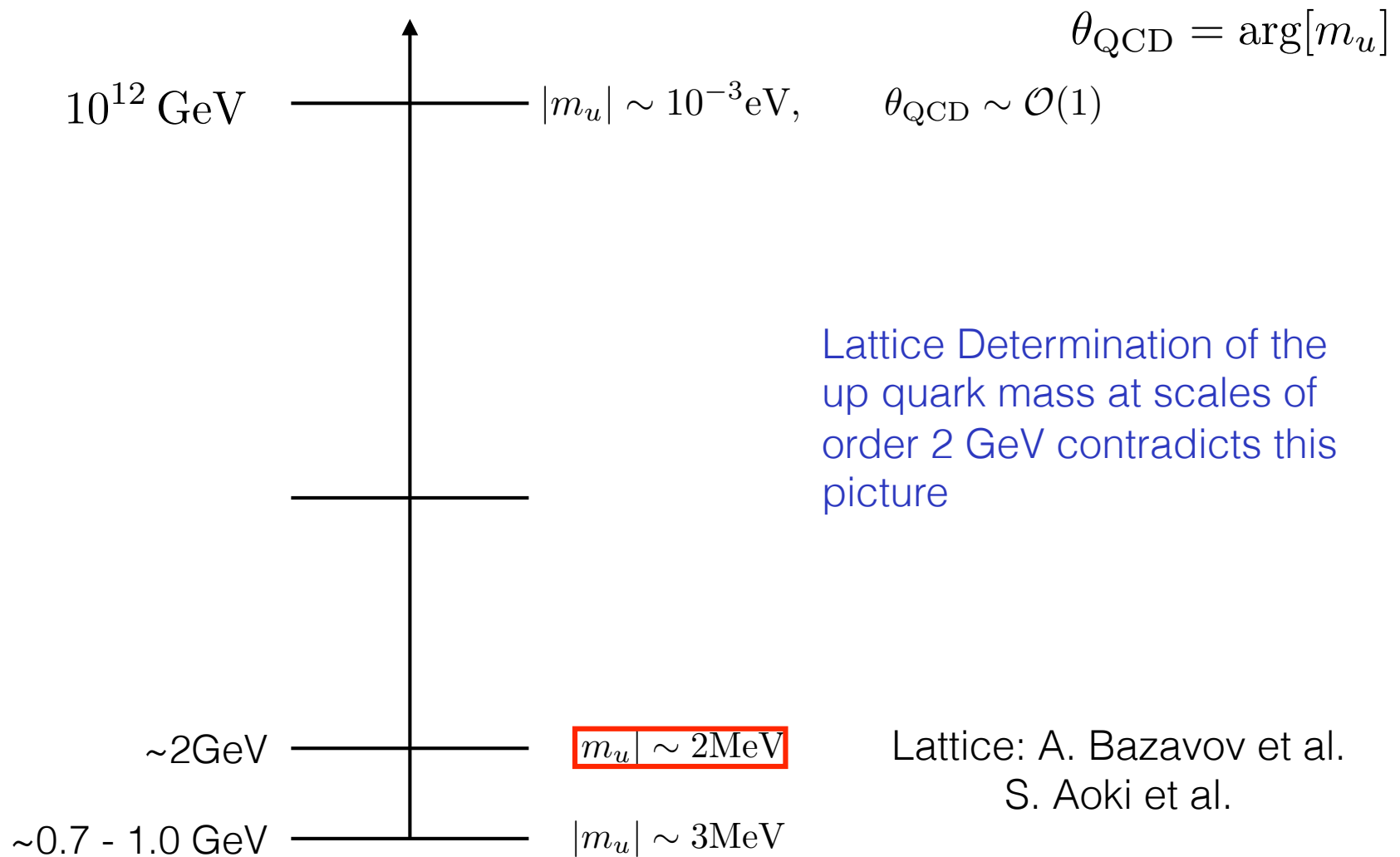
If dynamical, this would be a solution to the strong CP problem

# Ideal Situation in the canonical basis

$$\theta_{\text{QCD}} = \arg[m_u]$$



# Problems with the QCD Instanton Picture

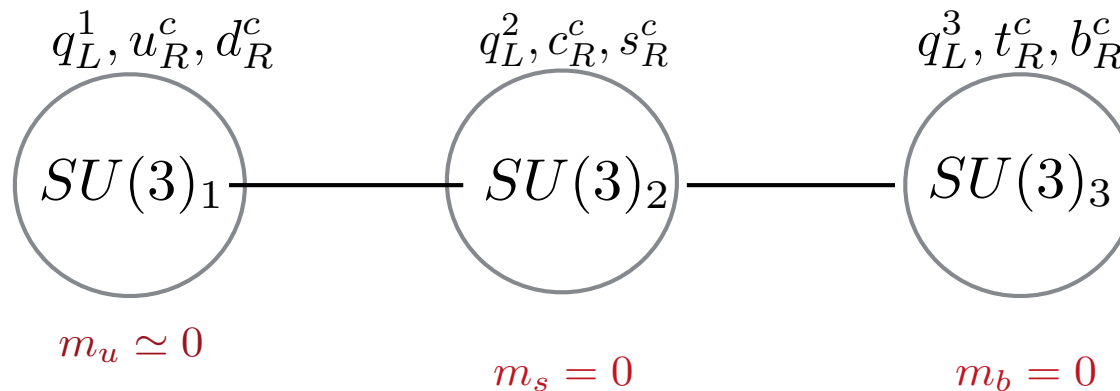


Not everybody seems to be persuaded, see Bardeen, arXiv:1812.06041

# Alternative Instanton Contributions ?

- Is there a possibility of having UV instanton contributions, different from the regular, low energy QCD ones ?

P. Agrawal and K. Howe, 1712.05803



- In this model,  $SU(3)_1 \times SU(3)_2 \times SU(3)_3 \rightarrow SU(3)_c$  through the vev's of bifundamental fields.
- Masses of up, strange and bottom quark obtained through instanton effects.
- CKM matrix may be properly obtained. However, flavor violating effects are present and push the scale of symmetry breaking to values of the order of 100 TeV or larger.

# Phenomenological Problems ?

● To contribute to the up, strange and bottom masses through instanton effects, the gauge coupling strength should be sizable.

● However, at the symmetry breaking scale, one must have

$$\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} = \frac{1}{\alpha_s}$$

● There is therefore a tension between the mass generation requirement and the fact that

$$\alpha_s(100 \text{ TeV}) \simeq 0.05$$

● Agrawal and Howe assumed the addition of one or more additional SU(3) factors, containing a massless up quark, which gets mass through instanton effects. These additional quarks get masses of the order of the TeV scale or larger for larger number of SU(3) factors.

## $m_u^H$ and the neutrino masses

- The smallness of the tree-level value of the up-quark mass in this framework is reminiscent of the problem of the smallness of neutrino masses.

- In the normal hierarchy model, it is known that

$$m_{\nu,1} \simeq 5 \times 10^{-2} \text{eV}, \quad m_{\nu,2} \simeq 8 \times 10^{-3} \text{eV}, \quad m_{\nu,3} \leq \text{few} 10^{-3} \text{eV}$$

- So, the constraints on the tree-level up-quark mass in this scenario are similar to the ones on the lightest neutrino masses ( $|m_u^H| \sin \theta^H < 10^{-3} \text{ eV}$ )

- If the bulk of the up-quark mass comes from instanton-like contributions, could there be a relation between the origins of  $m_u^H$  and  $m_\nu$  ?

- Such a relation could occur within the context of the Dirac See-saw mechanism, which provides an explanation for the smallness of the neutrino Yukawa couplings in the Dirac case.

# Dirac See-Saw Mechanism

- The standard, see-saw mechanism leading to the Weinberg operator generating **Majorana neutrino masses**, for arbitrary Yukawas

$$\frac{(yLH)(yLH)}{M_R} \rightarrow m_\nu = \frac{y^2 v^2}{M_R}$$

- In the Dirac see-saw mechanism, instead, one tries to suppress the Yukawa coupling. This can be generated assuming that neutrinos couple to a heavy Higgs, which acquires a vev via a small mixing with the standard one

$$\mathcal{L} = Y_\nu \bar{L}_L \bar{\Phi} \nu_R + Y_u \bar{Q}_L \bar{\Phi} u_R + h.c., \quad \bar{\Phi} = i\sigma_2 \phi^*$$

$$V = m_\Phi^2 \Phi^\dagger \Phi + (\rho S H^\dagger \Phi + h.c.) + \dots$$

- To ensure that neutrinos and the up-quark couple to the heavy Higgs, we assumed the presence of a Z4 symmetry, with all SM particles being neutral under it, apart from the following fields

$$Q[u_R] = Q[\nu_R] = Q[b_R] = Q[s_R] = 1, \quad Q[\Phi] = 1, \quad Q[S] = 3$$

- The charges of the right-handed bottom and strange quarks ensure that their tree-level contribution is zero. These charges would be zero in the QCD instanton scenario.

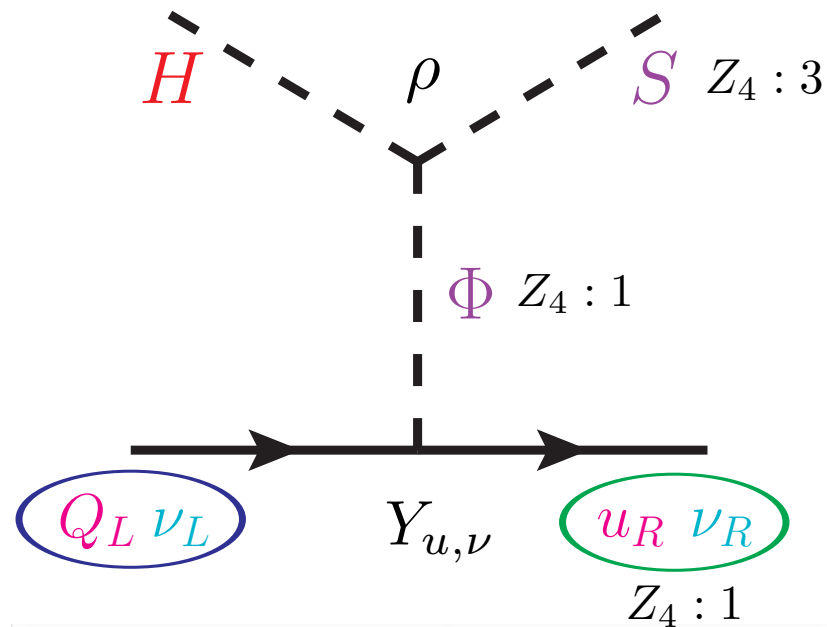
# Up-quark and Neutrino Dirac See-Saw

M. Carena, D. Liu, J. Liu, N.R. Shah, X. Wang, C.W.'19

P.-H. Gu and H.-J. He, hep-ph/061027

C. Bonilla, J. M. Lamprea, E. Peinado

and J. W. F. Valle, 1710.06498



$$-Y_\nu \frac{\rho}{m_\Phi^2} S \bar{\ell}_L \tilde{H} \nu_R$$

$$-Y_u \frac{\rho}{m_\Phi^2} S \bar{q}_L \tilde{H} u_R$$

$$m_\nu \sim Y_\nu \frac{\rho v S v}{2m_\Phi^2}, \quad m_u^H \sim Y_u \frac{\rho v S v}{2m_\Phi^2}.$$

$$\lambda_S S^4$$

No axion-like Goldstone!



# Couplings and Scales

M. Carena, D. Liu, J. Liu, N.R. Shah, X. Wang, C.W.'19

- The up-quark and neutrino masses are therefore given by

$$m_\nu \simeq Y_\nu \frac{\rho v_s v}{2 m_\Phi^2}, \quad m_u^H \simeq Y_u \frac{\rho v_s v}{2 m_\Phi^2}$$

- Hence, to obtain the heaviest neutrino for Yukawas of order one, we need

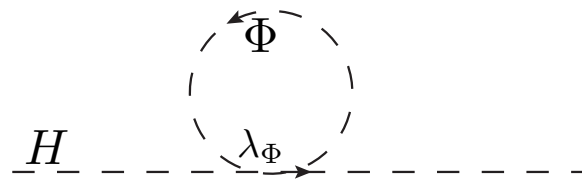
$$m_\Phi \simeq 6 \times 10^{12} \text{GeV} \left( \frac{Y_\nu}{0.1} \right) \left( \frac{\rho}{0.1 m_\Phi} \right) \left( \frac{v_s}{v} \right) \left( \frac{0.05 \text{eV}}{m_\nu} \right)$$

- On the other hand, to obtain the required small value of the tree-level up quark mass we need

$$Y_u(M_Z) < 0.1 Y_\nu \left( \frac{0.1}{\sin \theta_H} \right), \quad \text{with } \text{Im}[m_u^H] = |m_u^H| \sin \theta_H$$

# Hierarchy Problem

- Similar to the standard see-saw, the Dirac see-saw mechanism suffers from a hierarchy problem
- Physical quadratic corrections to the Higgs mass parameter, induced by the coupling  $\rho$ , for instance, would destabilize the weak scale.
- Although this is somewhat orthogonal to the CP problem, it should be eventually addressed. Also,



leads to a physical contribution to the Higgs mass

$$\delta m_H^2 \sim \frac{\lambda_\Phi}{16\pi^2} m_\Phi^2$$

Need engineering

$$\lambda_\Phi \sim 10^{-19}$$

# SUSY Extension

- One could in principle think about the supersymmetric extension
- In such a case, one could think about a Z3 symmetry.

$$W = -Y_\nu^* L \Phi_u \nu_R^c - Y_u^* Q \Phi_u u_R^c - y_e^* L H_d e_R^c - y_d^* Q H_d d_R^c \\ + \mu H_u H_d + m_\Phi \Phi_u \Phi_d + \lambda H_u \Phi_d S + \frac{\kappa}{3} S^3.$$

$$Q[u_R^c] = Q[\nu_R^c] = Q[s_R^c] = Q[b_R^c] = 1, \quad Q[\Phi_d] = 1, \quad Q[\Phi_u] = Q[S] = -1$$

- and therefore,

$$V_{\text{SUSY}} = |\mu|^2 |H_u|^2 + |\mu H_d + \lambda \Phi_d S|^2 + |m_\Phi \Phi_u + \lambda H_u S|^2 \\ + |m_\Phi|^2 |\Phi_d|^2 + |\kappa S^2 + \lambda H_u \Phi_d|^2,$$

- where we assume that  $m_\Phi \gg \mu \sim \text{TeV}$   $\rho \rightarrow \lambda m_\Phi$

$$V_{\text{soft}} = m_{\Phi_u}^2 |\Phi_u|^2 + m_{\Phi_d}^2 |\Phi_d|^2 + m_S^2 S^* S + \dots \\ + (\lambda a_\lambda H_u \Phi_d S + b_\lambda \Phi_u^\dagger H_u S + a_\kappa S^3 + \dots + h.c.),$$

# Up quark and neutrino masses and new CP violating sources

As is well known, in the absence of  $\kappa$  and  $A\kappa$  terms, there is a global PQ symmetry broken spontaneously that leads to a Goldstone boson, that would behave like an axion, something that can be avoided with these terms.

After integrating out the heavy  $\phi$  fields, one obtains

$$\mathcal{L}_{\text{eff}}^y = -Y_\nu \frac{\lambda^* S^*}{m_\Phi^*} \bar{\ell}_L \tilde{H}_u \nu_R - Y_u \frac{\lambda^* S^*}{m_\Phi^*} \bar{q}_L \tilde{H}_u u_R + \dots$$

Hence, 
$$m_\nu = \left( Y_\nu \frac{\lambda^* v_S^*}{\sqrt{2} m_\Phi^*} \right) \frac{v_u}{\sqrt{2}}, \quad m_u^H = \left( Y_u \frac{\lambda^* v_S^*}{\sqrt{2} m_\Phi^*} \right) \frac{v_u}{\sqrt{2}}$$

Estimates of the values of the necessary values of the parameters may be obtained from the non-SUSY case by changing

$$\rho \rightarrow \lambda m_\Phi \text{ and } v \rightarrow v_u$$

# Three Problems for weak scale SUSY breaking

- Now, supersymmetry breaking introduces new CP violating parameters. Assuming flavor conserving scalar mass parameters, the PQ and R symmetries of these theories imply. Physical corrections are proportional to the phases

$$\Phi_A^{if} = \arg[M_i A_f^*], \quad \Phi_B = \arg[M_g^* \mu^* (B\mu)],$$

- Weak scale SUSY breaking introduces non-decoupling corrections to the masses at the characteristic SUSY particle scale (after instanton effects have been considered)

$$\Delta_{\text{CP}} Y_{u,d} \sim \frac{1}{(4\pi)^2} \Phi_{A,B}^{if}$$

- It leads to corrections to the electric dipole moment

$$d_n^{\text{SUSY}} \sim \left( \frac{100 \text{ GeV}}{m_{\text{SUSY}}} \right)^2 \Phi_{A,B}^{if} \times 10^{-23} \text{ e cm}$$

- It leads to a suppression of the mass generated by instantons

$$m_f^{\text{inst}} \propto \frac{M_{\tilde{g}}^3}{\Lambda_{\text{inst}}^3}$$

## Solution to these Problems

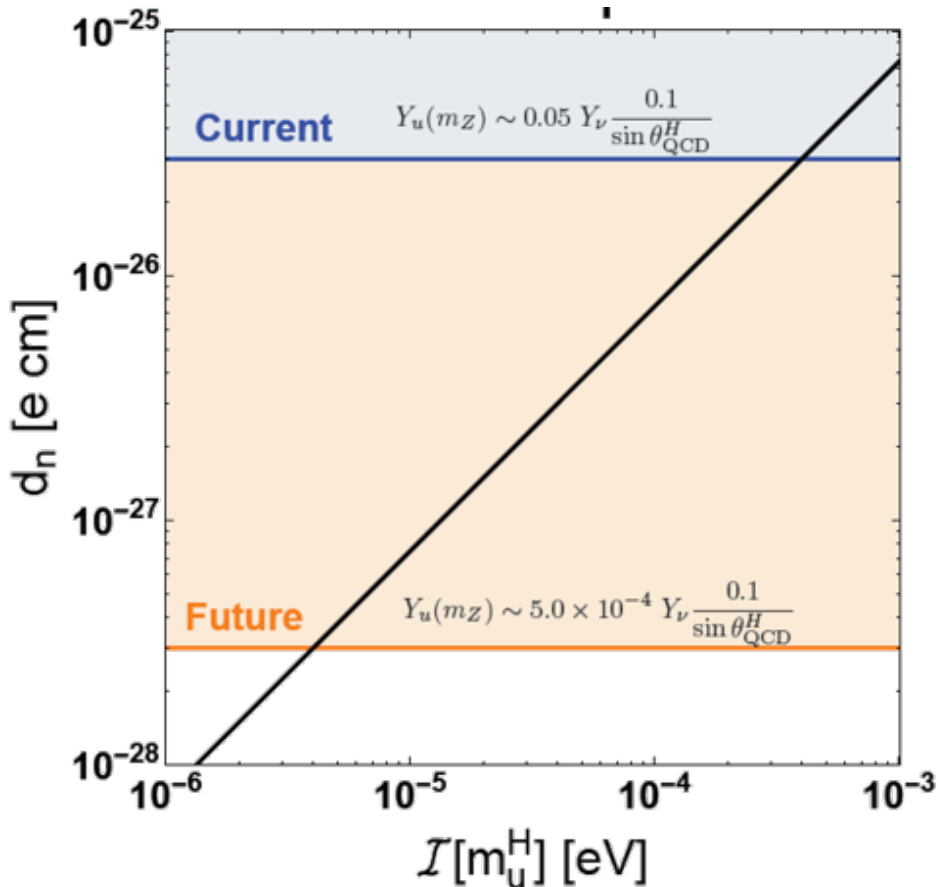
- These problems are solved if we only attempt to solve the large hierarchy problem, and not the little one.
- Assume that supersymmetry breaking occurs at scales of the order of 100 TeV, before the instanton effects take place.
- One can then integrate out the SUSY particles and provided the discrete symmetries are preserved, obtain an effective theory similar to the non-SUSY case.
- Of course, if the QCD instanton solution would be possible, only the neutron and electron electric dipole moments would be a (well known) problem.

# Consequences of this proposal

- Neutrinos would be (pseudo) Dirac. Majorana contributions, although non-zero, should be small.
- This implies no signal in near future neutrino-less beta decay experiments.
- While correlating the physical parameter  $\theta_{\text{QCD}}$  with the small tree-level up quark and neutrino masses, one obtains a contribution to the neutron electric dipole moment. In what we called the canonical basis,

$$\theta_{\text{QCD}} = \frac{\text{Im}[m_u^H]}{|m_u|}$$

# Neutron Electric Dipole Moment and the up-quark (neutrino) mass



$$d_n = \frac{\text{Im}[m_u^H]}{6.5 \times 10^{-4} \text{ eV}} 3 \times 10^{-26} \text{ e cm}$$

$$\text{Im}[m_u^H] = |m_u^H| \sin(\theta_{\text{QCD}}^H)$$

If the assumed correlation between the imaginary component of the up quark mass and neutrino masses holds, one or more of these experiments is expected to see a signal !!



# Near Future

## Neutron Electric Dipole Moment Experiments

Measurement of the electric dipole moment : Basic Idea is to measured the Larmor precession with parallel and antiparallel electric and magnetic fields

$$2\mu_n B \pm 2d_n E, \quad d_n = \frac{h\Delta\nu}{2 E}$$

Present Limit  $3 \times 10^{-26} e \text{ cm}$

Experiment	Sensitivity [e cm]
PSI	$< 1 \times 10^{-27}$
TRIUMF (TUCAN)	$1 \times 10^{-27}$
SNS	$< 3 \times 10^{-28}$
PNPI-ILL-PTI	$10^{-27} - 10^{-28}$
LANL EDM	$3 \times 10^{-27}$
Munich/ILL	$\mathcal{O}(10^{-28})$

Ultracold Neutrons  
are being used  
to eliminate systematic  
errors

# Conclusions

- We presented a possible correlation between neutrino masses and the imaginary component of the up-quark mass in a framework in which instanton like contributions form the bulk of the up-quark mass
- Dynamics : Dirac see-saw relates  $m_\nu$  and  $m_u^H$
- A non-vanishing value of the neutron electric dipole moment is predicted
- It is naturally within the reach of the next generation of experiments