Universal Dynamics and Entanglement near the Many Body Localization Transition

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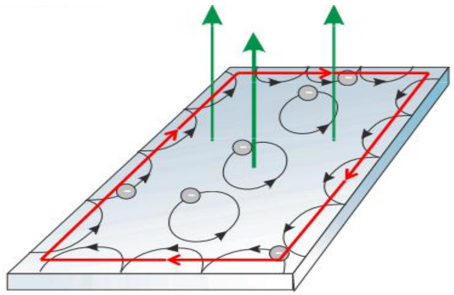
erc UQUAM ultracold quantum matter

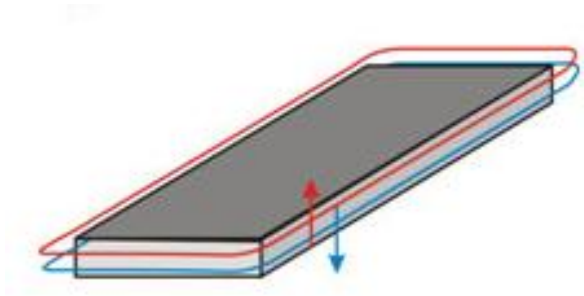
BSF United States – Israel Binational Science Foundation

Conventional wisdom:
Quantum physics is manifest only in/near ground states

Quantum Hall effect:

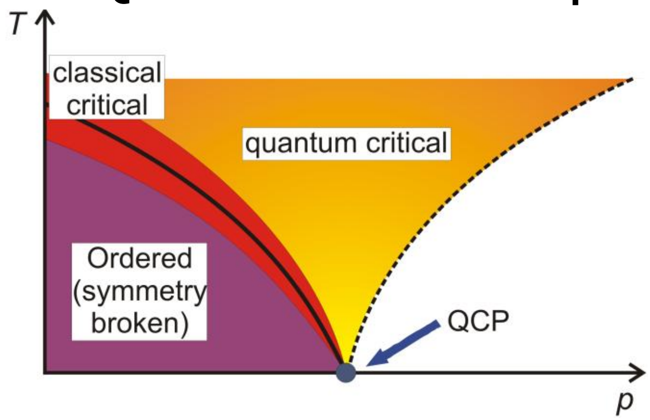


Topological insulators:



Fermi liquid:

Quantum critical points



This talk: quantum behavior and entanglement at high energy

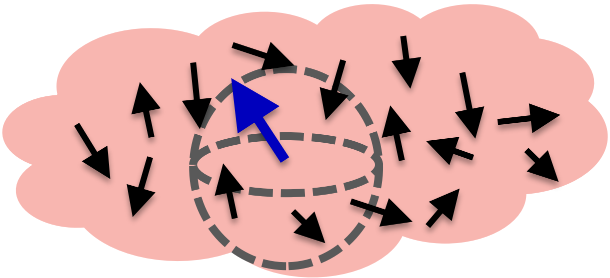
Quantum dynamics at high energies: two generic paradigms

Thermalization

Quantum correlations in local d.o.f are rapidly lost as these get entangled with the rest of the system.

Classical hydro description of remaining slow modes (conserved quantities, and order parameters).

Many-body localization



Local quantum information persists indefinitely.

Need a fully **quantum** description of the long time dynamics!

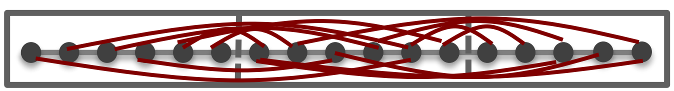
?

The many-body localization transition = Sharp interface between quantum and classical

Alternative perspective: structure of eigenstates at high energies

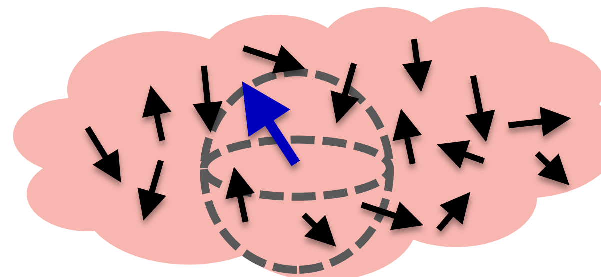
Thermalizing

Eigenstate thermalization: large entanglement

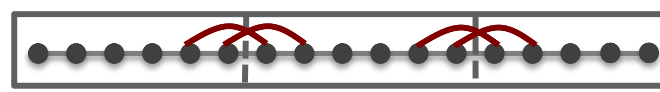


$S_A \sim L^d$ (volume law)

Many-body localized

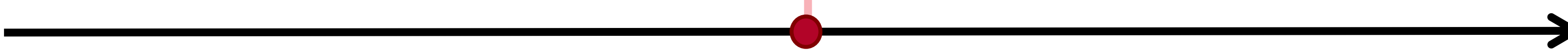


Eigenstates have low entanglement



$S_A \sim L^{d-1}$ (area law)

?



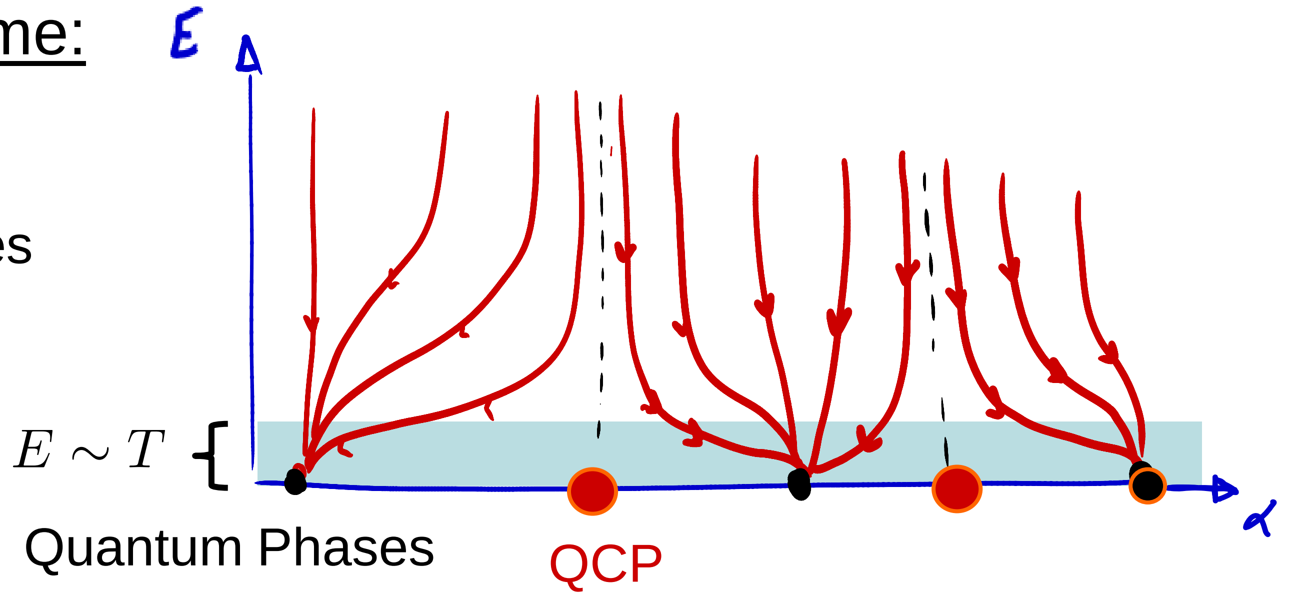
<u>Localization transition:</u> fundamental change in entanglement pattern. More radical than in any other phase transition we know !

Can we use a RG framework to understand dynamics and entanglement at high energy?

<u>Conventional scheme:</u>

Low temperature = sampling low energies

$$P(E) \sim e^{-E/T}$$



<u>Universality out of equilibrium?</u>

- All energies play a role. Cannot focus on low energies!
- Need a different RG framework.

Outline

- Effective description of the many-body localized phase
  RG, Quasi-local integrals of motion

- The many-body localization phase transition
  RG approach: transport, entanglement scaling and a surprise!

- Experimental observation (with I. Bloch’s group)

Real space RG for dynamics and high energy states

R. Vosk and EA PRL (2013), PRL (2014)

Pekker, Refael, E.A., Demler, Oganesyan PRX 2014 – RG for spectrum

$$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right] \qquad e^{-iHt} | \Psi_0 \rangle$$

$|\Psi_0\rangle =$ ↓↑↓↓↓↓↑↓↓↓↑↓↑↑↓↑↓↑↑↑↓↑↑↓↓↑↑↓↓↑↓↓↑↓↑↓↓↑↓↑

Pick out largest couplings $\Omega = \max\left(J_i^z, h_i\right)$ H_{fast}

Short times ($t \approx 1/\Omega$): System evolves according to H_{fast}

Other spins essentially frozen on this timescale.

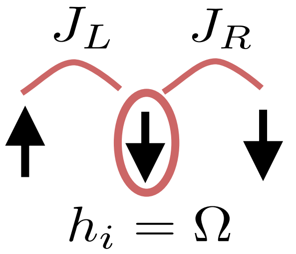
Longer times ($t >> 1/\Omega$): Eliminate fast modes (order Ω) perturbatively to obtain effective evolution for longer timescales.

Similar to philosophy of SDRG for the ground state (Dasgupta-Ma 78).

But here we focus on small frequencies (E diff.) rather than absolute E.

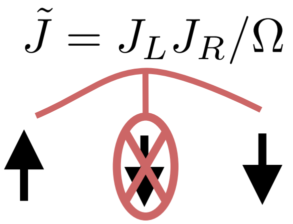
Outcome of RG: integrals of motion = (frozen spins)

Example: strong transverse field



$$H = \sum_i \left[J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + V_i \sigma_i^x \sigma_{i+1}^x \right]$$

$$H_{\text{eff}} = e^{-iS} H e^{iS}$$



$$H_{\text{eff}} = \Omega \tilde{\sigma}_i^x + V_L \tilde{\sigma}_i^x \sigma_L^x + V_R \tilde{\sigma}_i^x \sigma_R^x + \frac{J_L J_R}{\Omega} \sigma_L^z \tilde{\sigma}_i^x \sigma_R^z$$

In this RG scheme degrees of freedom are not eliminated but rather frozen into *quasi-local integrals of motion*:

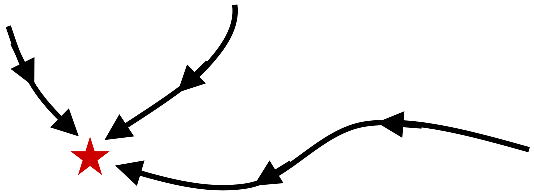
$$\tilde{\sigma}_i^x = Z \sigma_i^x + \text{exponential tail}$$

The MBL phase is a stable RG fixed point

R. Vosk & EA, PRL (2013); PRL (2014); EA & Vosk ARCMP (2015)

$$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right]$$

Fixed point characterized by infinite local integrals of motion:



$$H_{FP} = \sum_i \tilde{h}_i \tilde{\sigma}_i^x + \sum_{ij} V_{ij} \tilde{\sigma}_i^x \tilde{\sigma}_j^x + \sum_{ijk} V_{ijk} \tilde{\sigma}_i^x \tilde{\sigma}_j^x \tilde{\sigma}_k^x + \ldots$$

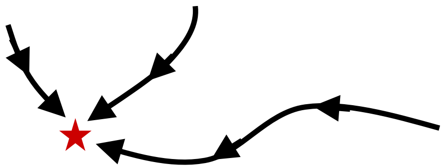
This was independently postulated as a phenomenological description of MBL
Huse, Nandkishore, Oganesyan (2013,2014), Serbyn, Papic and Abanin (2013)

Note the analogy with Fermi-liquid theory!

$$H_{FL} = \sum_{k \sim k_F} \epsilon_k \hat{n}_k + \sum_{k,k'} f_{kk'} \hat{n}_k \hat{n}_{k'} \qquad [H_{FL}, \hat{n}_k] = 0$$

The MBL phase is a stable RG fixed point

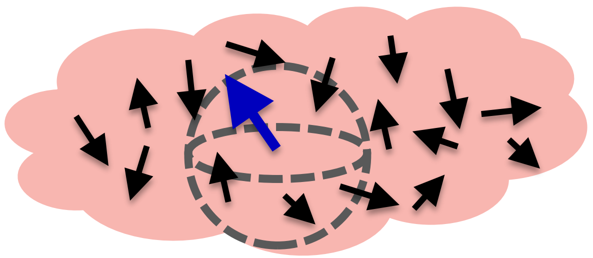
$$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right]$$



$$H_{FP} = \sum_i \tilde{h}_i \tilde{\sigma}_i^x + \sum_{ij} V_{ij} \tilde{\sigma}_i^x \tilde{\sigma}_j^x + \sum_{ijk} V_{ijk} \tilde{\sigma}_i^x \tilde{\sigma}_j^x \tilde{\sigma}_k^x + \ldots$$

Reveals surprisingly rich dynamics in MBL phase:

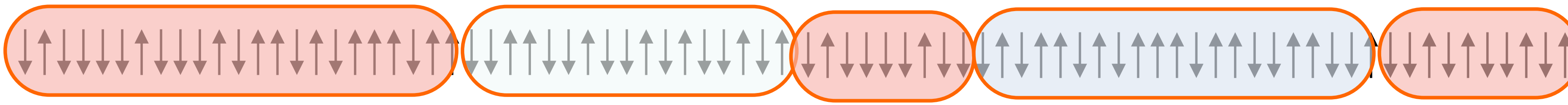
- Slow log(t) growth of the entanglement entropy.
- Anomalous relaxation of observables.
  Vasseur et. al. (2014), Serbyn et. al. (2014)
- Distinct localized phases
  (glass, paramagnetic, topological …)
  Huse et. al. 2013, Vosk and EA 2013, Pekker et. al. 2013
- Persistent quantum coherence, spin echos
  Bahri etal 2013, Serbyn etal 2013



But cannot address the MBL transition using this approach!

Theory of the many-body localization transition

Vosk, Huse and E.A. arXiv:1412.3117



Spin chain fragmented into puddles of different types: incipient insulators and incipient metals.
Modeled as coupled random matrices:

$\Delta_i\,, \Gamma_i \;\Rightarrow\; g_i = \Gamma_i/\Delta_i$

Δ_i Mean level spacing in the block

$\Gamma_i^{-1} = \tau_i$ Time for entanglement to spread across the block

$g_i \ll 1$ “insulating block”

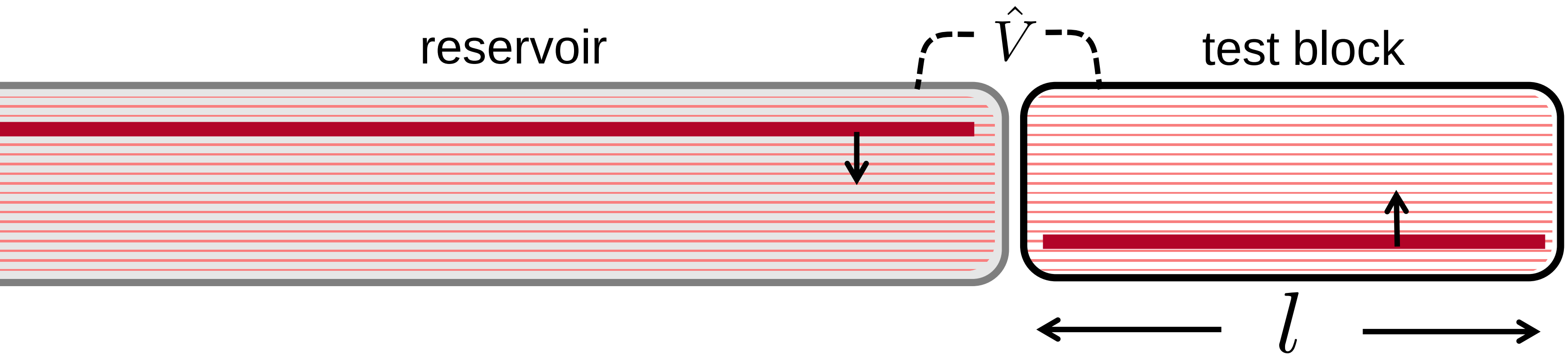
(Poisson level statistics)

$g_i \gg 1$ “thermalizing block”

(Wigner-Dyson statistics)

RG flow: itteratively join matrices that entangle with each other at running cutoff scale. At the end of the flow we are left with one big block that is either insulating or thermalizing

Digression: entanglement vs. transport time



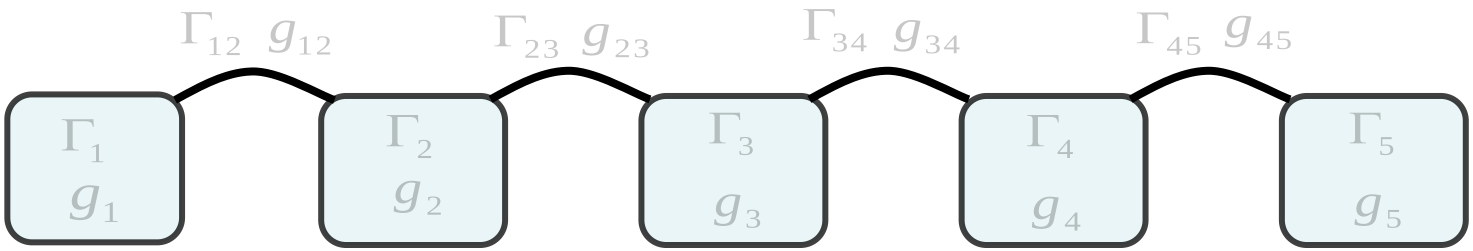
Transition due to V which changes the energy by O(1) is sufficient to "thermalize" block in the entanglement sense because there are $\sim 2^l$ possible final states within such an energy window.

$$\tau^{-1} = \Gamma \sim \sum_{r,b} |\langle r, b|V|in\rangle|^2 \delta(E_r + E_b - E_{in})$$

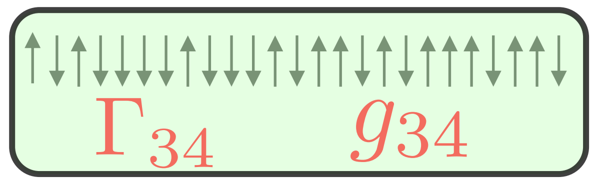
However l transitions are needed to equilibrate the energy density in the reservoir and test block!

$$\tau_{tr} = l\tau \quad \Rightarrow \quad z = z_{ent} + 1$$

Starting point for RG: chain of coupled blocks



Meaning of
the link variables:



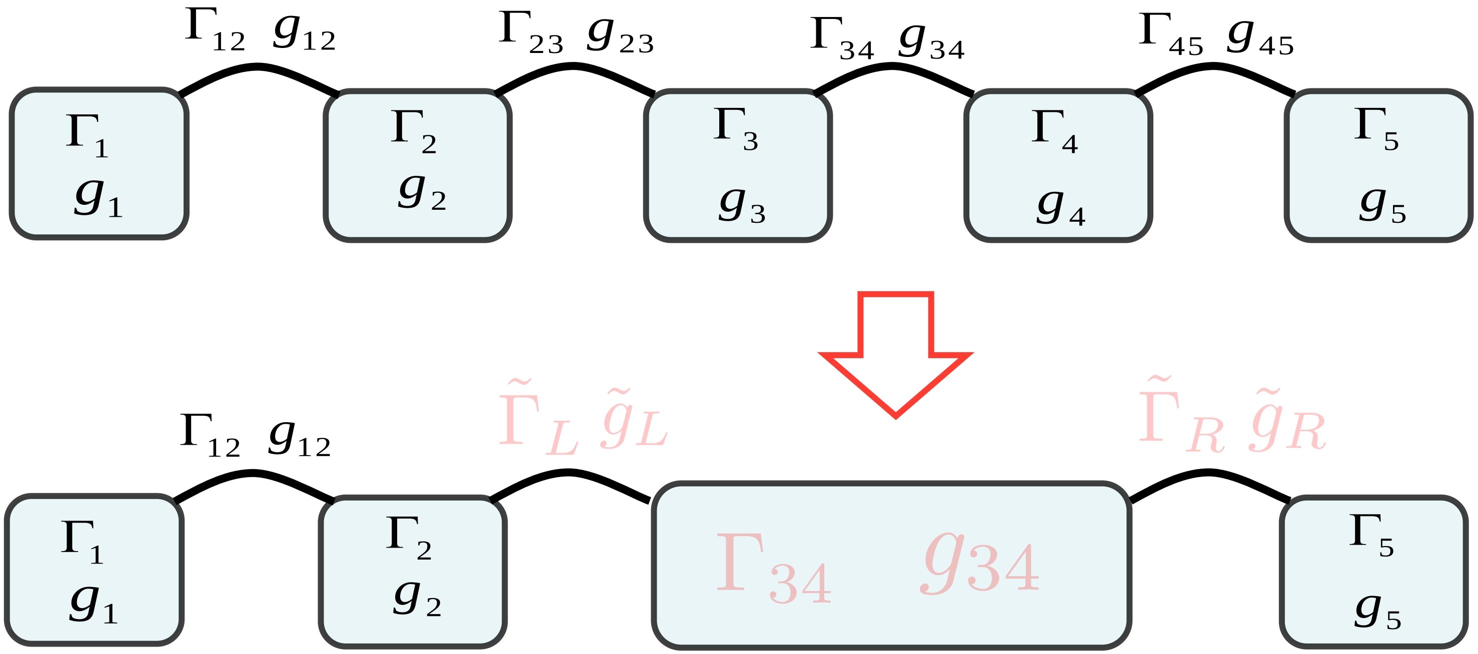
Γ_{ij} = far end-to-end entanglement rate of adjacent blocks
(Γ of the two blocks if they were considered as a single block)

Δ_{ij} = Mean level spacing of the two block system $\Delta_{ij} \sim 2^{-l_{ij}}$

$g_{ij} >> 1$ ➡ ‘effective’ link (‘thermalizing’)

$g_{ij} << 1$ ➡ ‘ineffective’ link (‘insulating’)

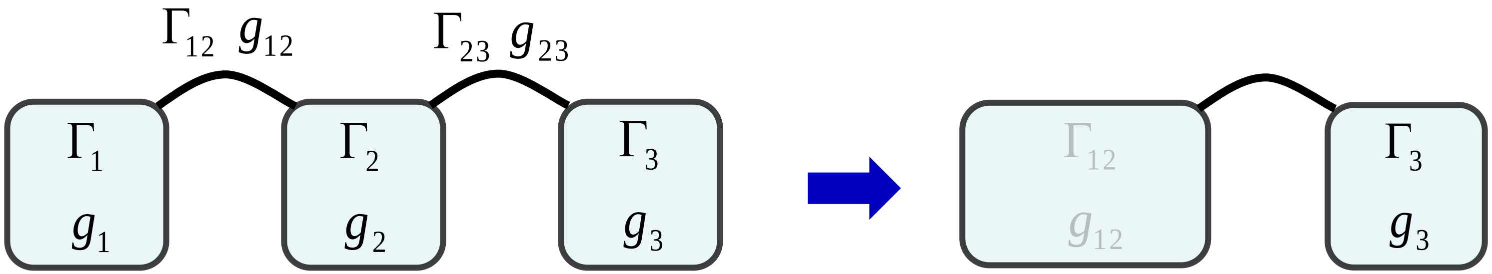
Schematics of the RG



Join blocks which entangle with each other on the fastest scale. Then compute renormalized couplings to the left and right.

Computing the flow will tell us whether we end up with one big thermalizing matrix (g>>1) or a big insulator (g<<1) at large scales

RG scheme



The simplest limits:

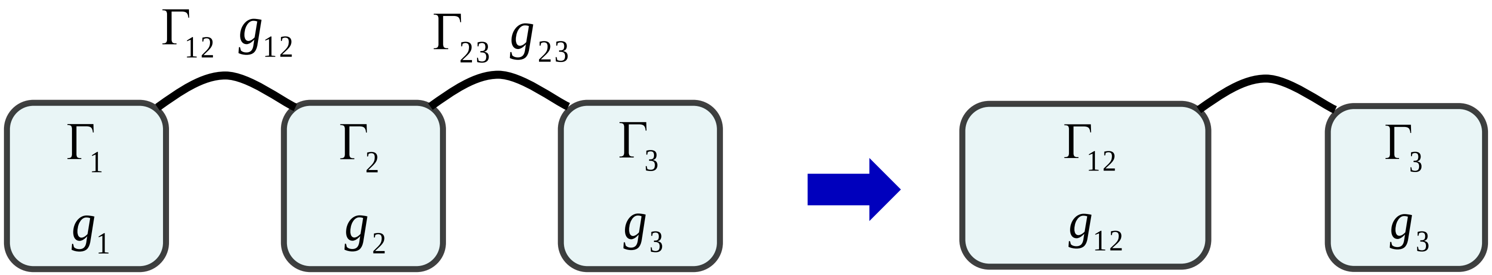
(i) Two ‘insulating’ links, i.e. g_{12}<<1 and g_{23}<<1 $\Rightarrow$ $\Gamma_R = \frac{\Gamma_{12}\Gamma_{23}}{\Gamma_2}$

Can be derived for insulators from first principles.
But also simply understood by taking a log of the two sides:

$$\log \tau_{tot} = \log \tau_{12} + \log \tau_{23} - \log \tau_2$$

$$l_{tot} = l_{12} + l_{23} - l_2$$

RG scheme



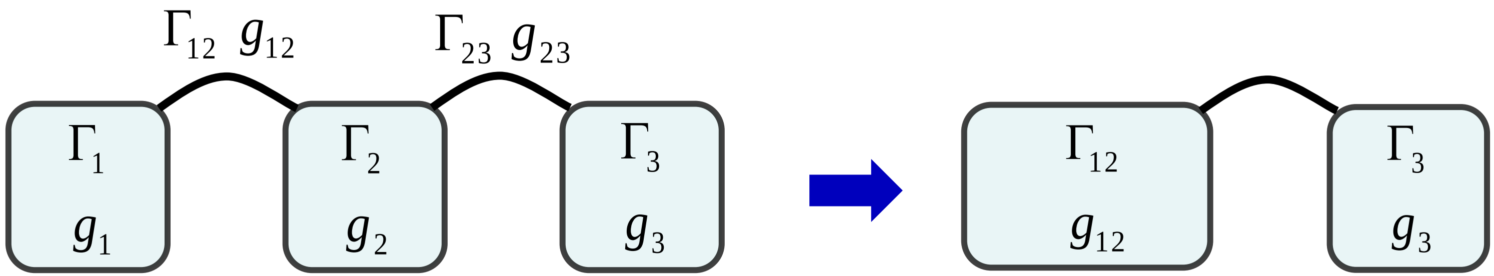
The simplest limits:

(ii) Two thermalizing links, i.e. $g_{12}, g_{23} >> 1$

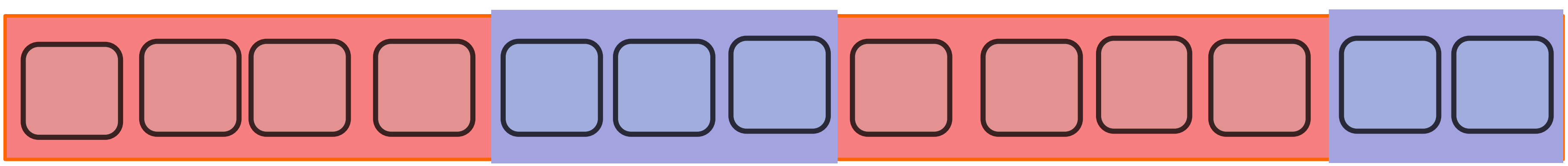
Γ_R cannot be derived perturbatively in this case.
But we know: energy transport is diffusive
and (therefore) entanglement propagates ballistically.

$$\frac{1}{\Gamma_R} = \frac{1}{\Gamma_{12}} + \frac{1}{\Gamma_{23}} - \frac{1}{\Gamma_2}$$

RG scheme

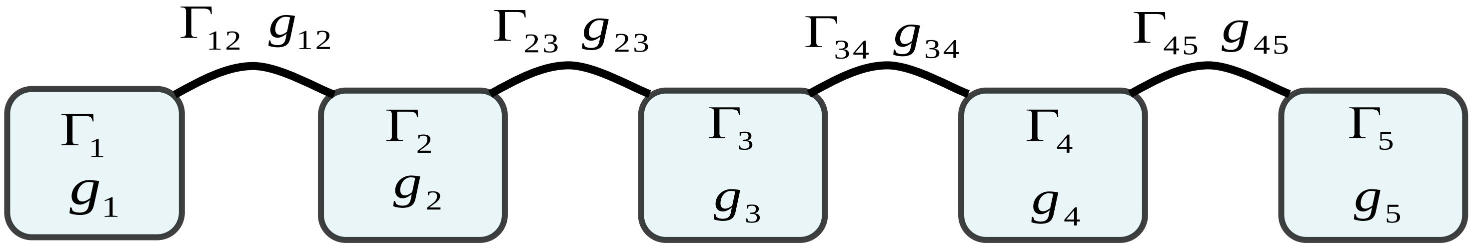


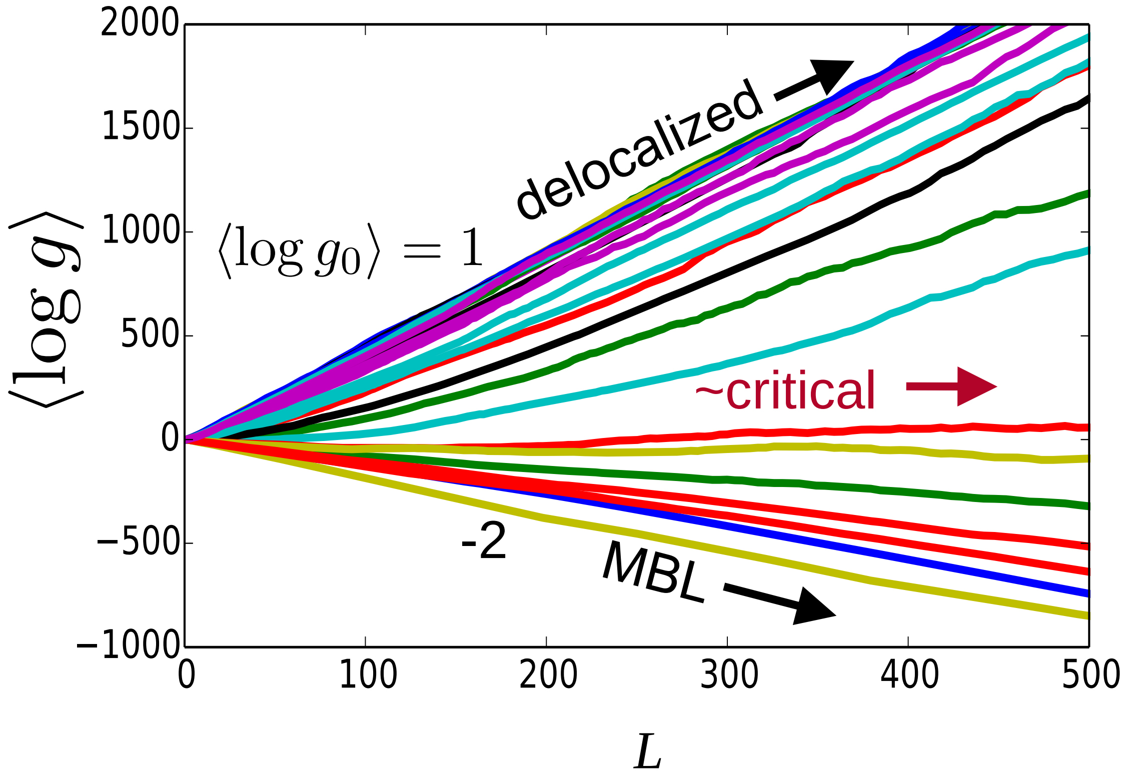
These rules capture scaling in big insulating (i) or conducting regions (ii).



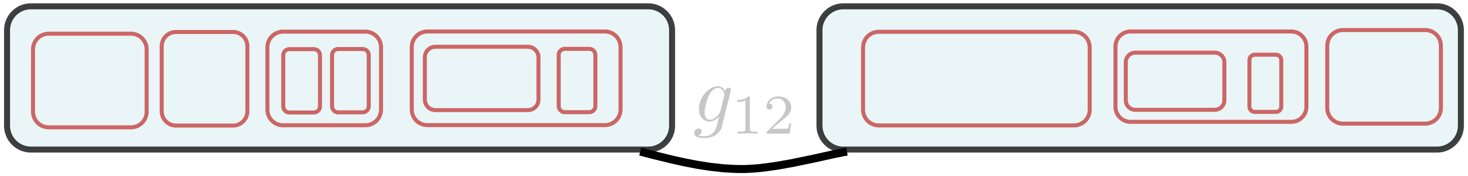
The same rules can be applied when the three sites sit at interfaces provided all individual blocks and links have extreme values of g, i.e. g>>1 or g<<1.

Outcome of the RG flow

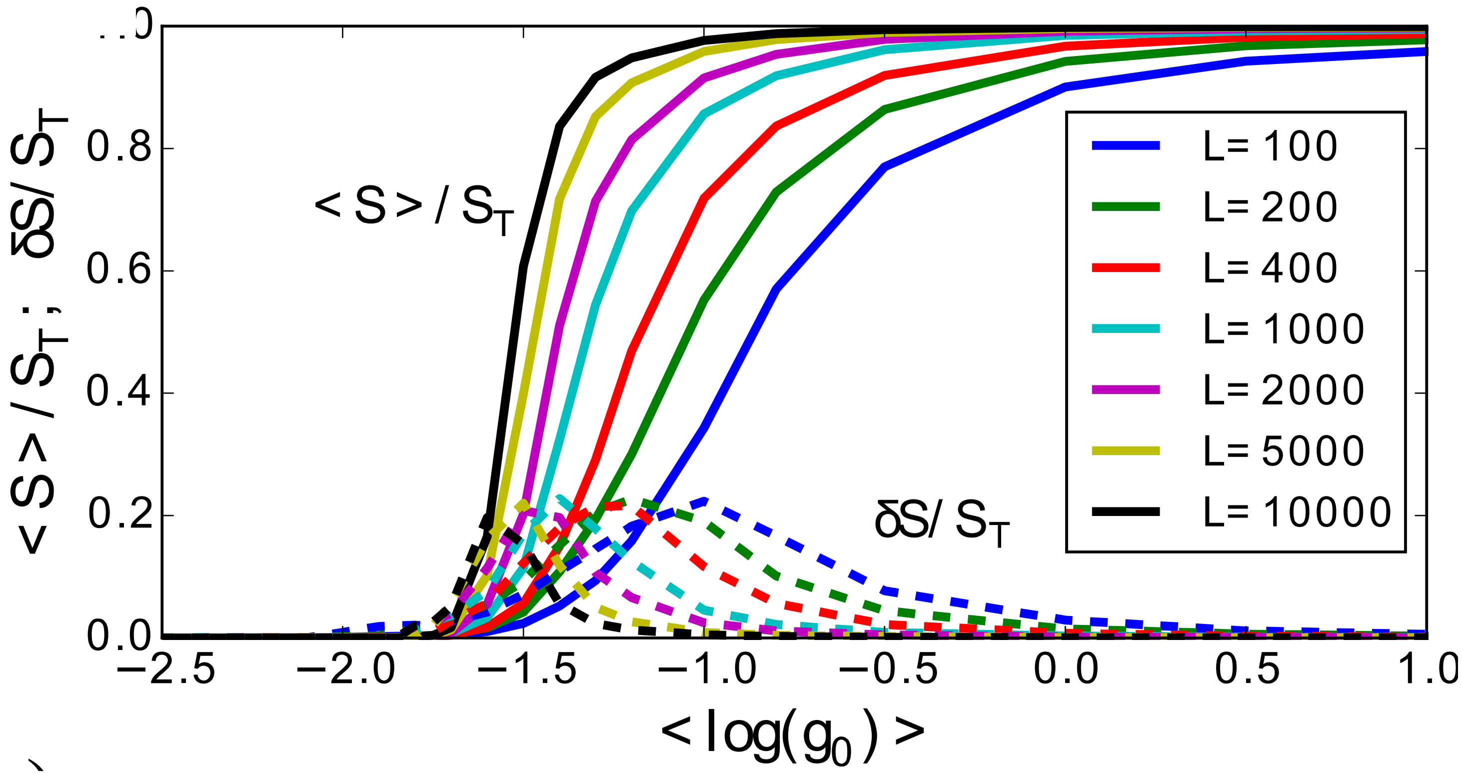




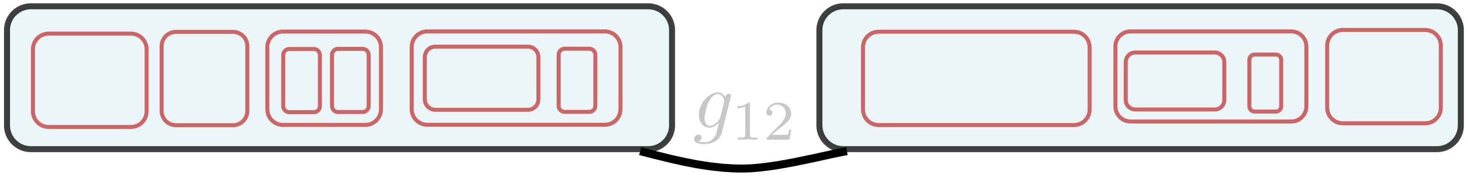
Proxy of eigenstate entanglement



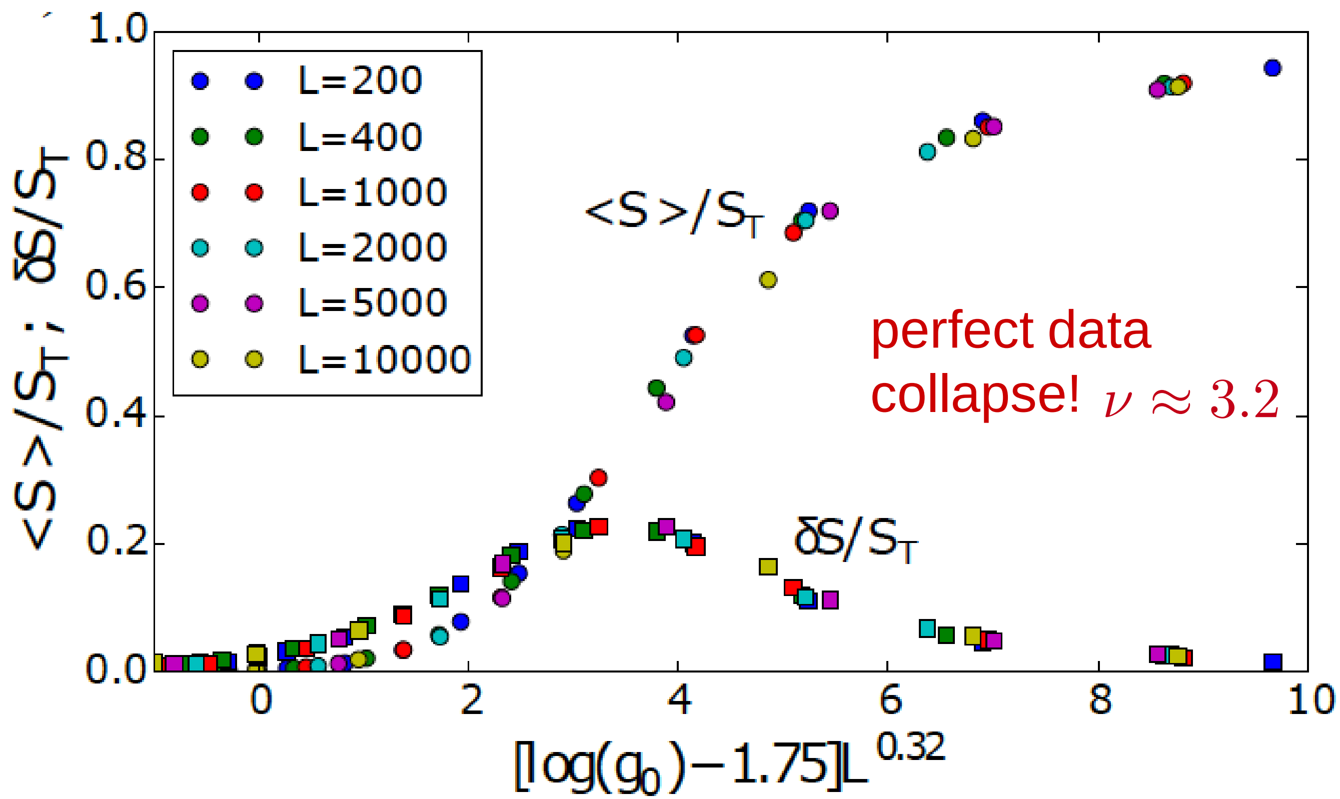
$$S_E(L/2) \sim \log_2 [g(L) + 1]$$



Proxy for eigenstate entanglement

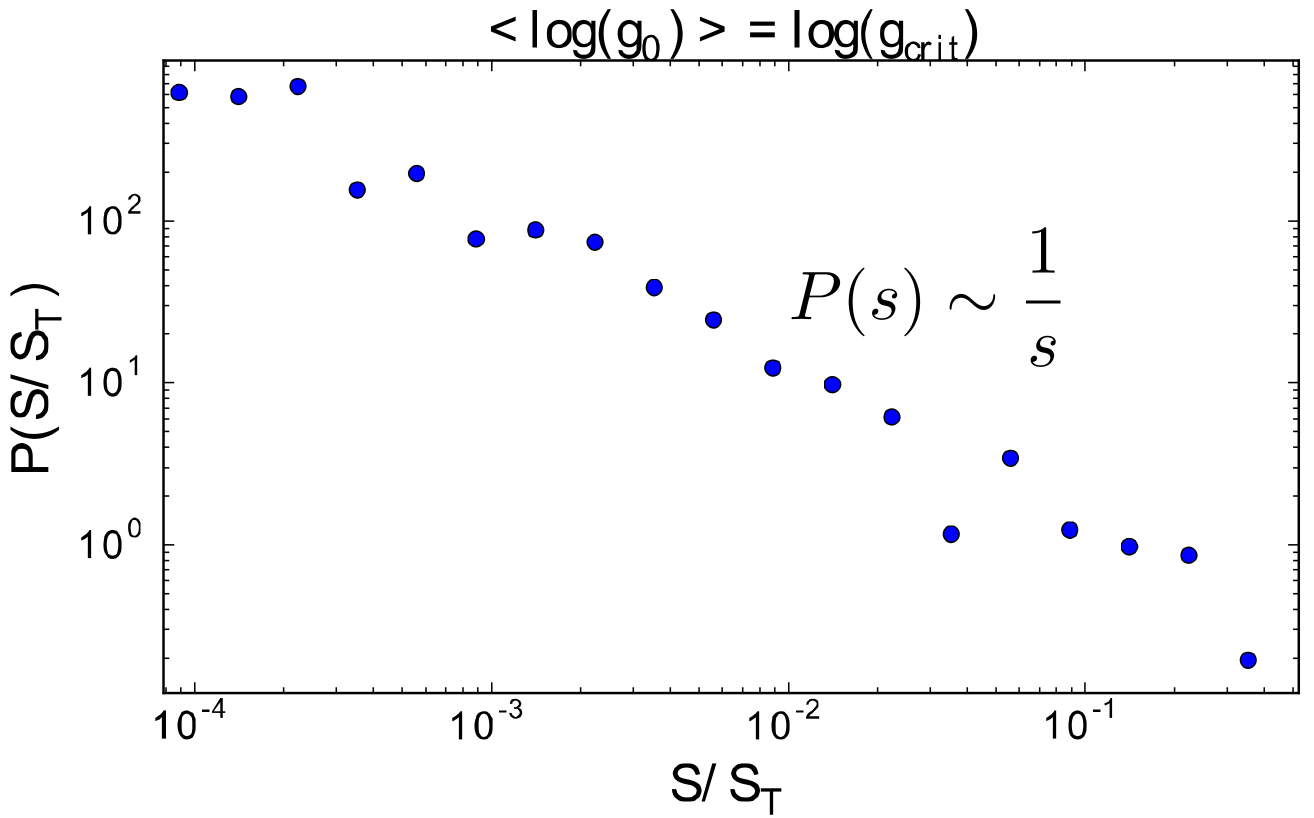


$$S_E(L/2) \sim \log_2 [g(L) + 1]$$



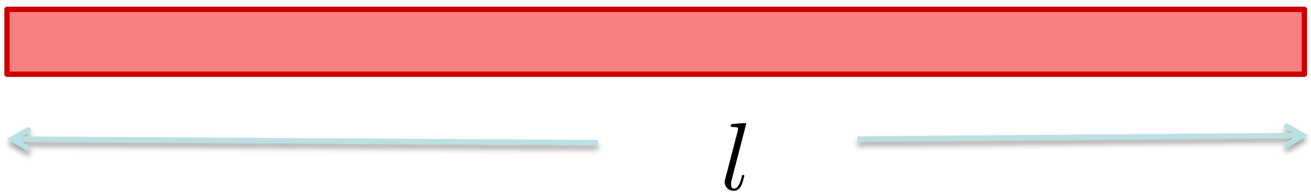
- Universal jump to full thermal entropy → Griffiths phase is thermal

Broad entropy distribution at criticality



RG results I – dynamical scaling for transport

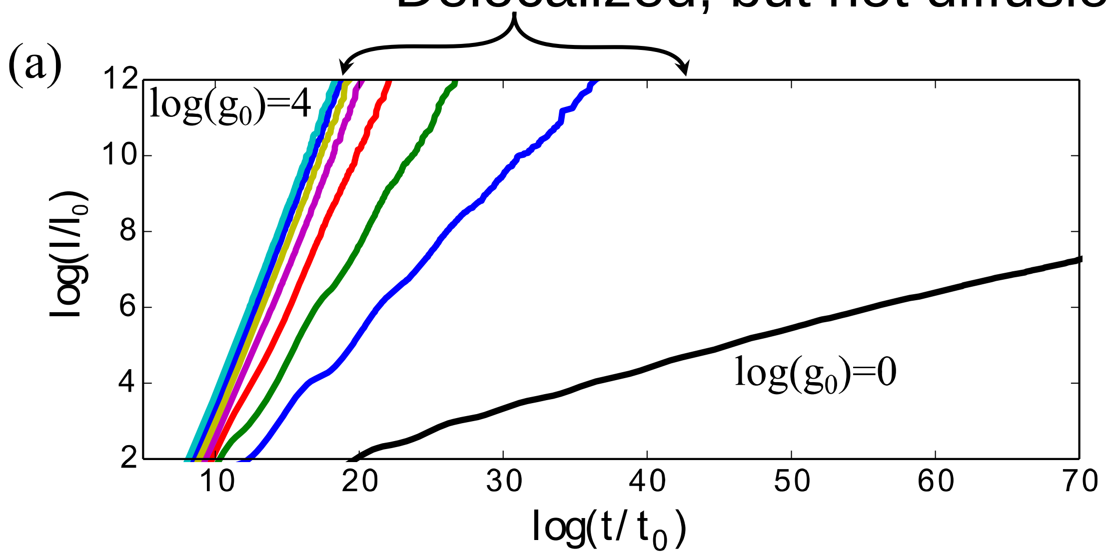
Relation between transport time τ_{tr} and length l of blocks:



Diffusion: $\tau_{tr} = l^2$ $\quad$ $l_{tr} = (D\tau)^{\cancel{1/2}}$

$\alpha < \frac{1}{2}$

Delocalized, but not diffusion

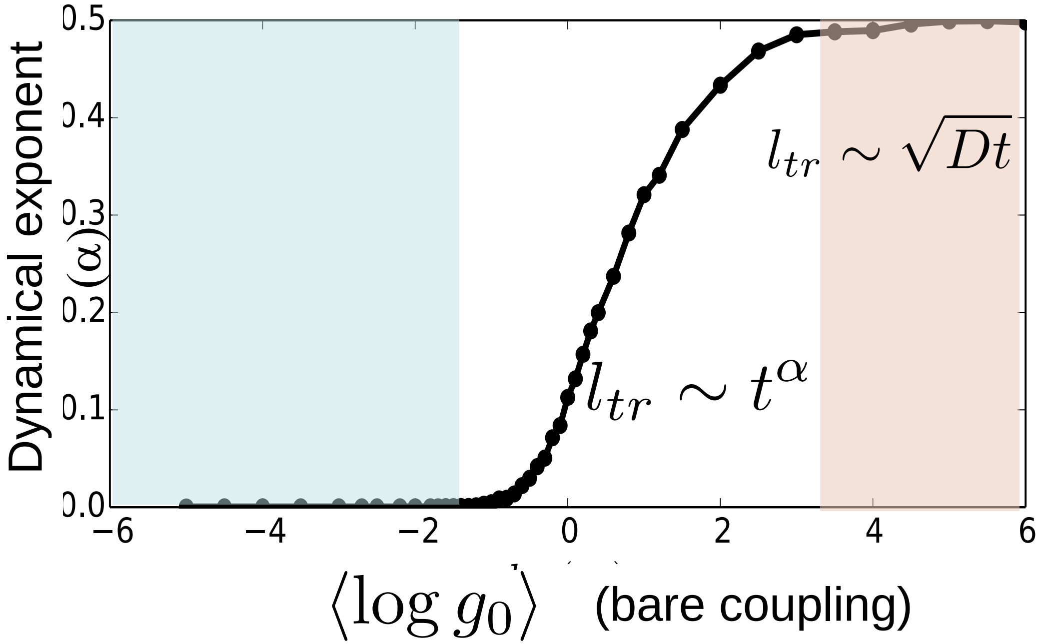


RG results I – dynamical scaling for transport

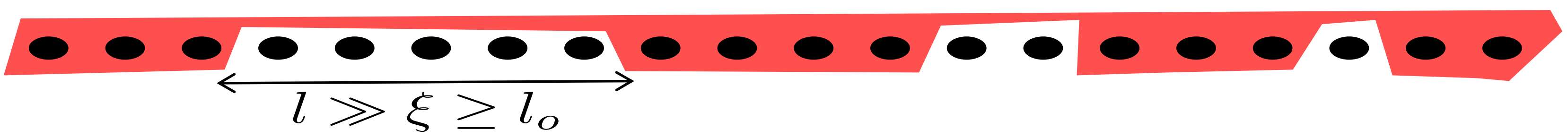
Relation between transport time τ_{tr} and length l of blocks:

Surprise! The transition is from localized to anomalous diffusion.

Seen also in recent ED studies:
Bar-Lev et.al 2014; Agarwal et.al 2014



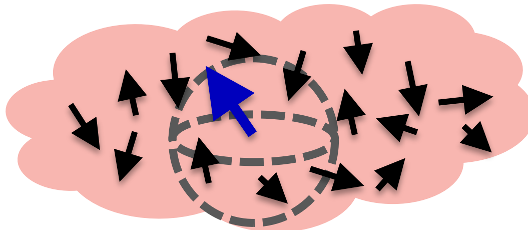
Result of Griffiths effects. long insulating inclusions inside the metal are exponentially rare but give exponentially large contribution to the transport time.



➡ Relaxation with slow power-law tails

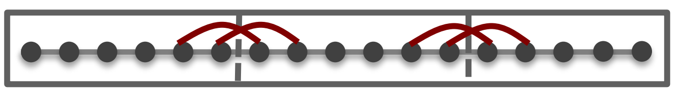
Summary

Many-body localized



Quantum coherent dynamics

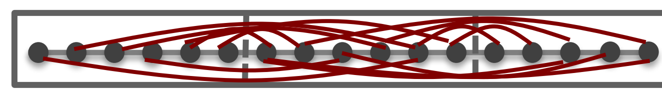
Area law entanglement

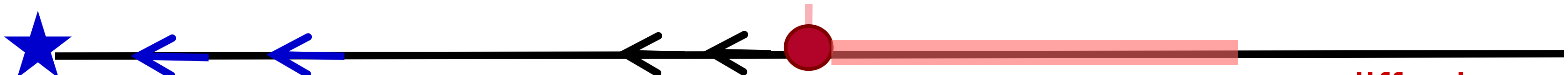


Thermalizing

"Classical" dynamics

Volume law entanglement





Dynamical RG

Random matrix RG

sub-diffusive

diffusive

Localized fixed-point

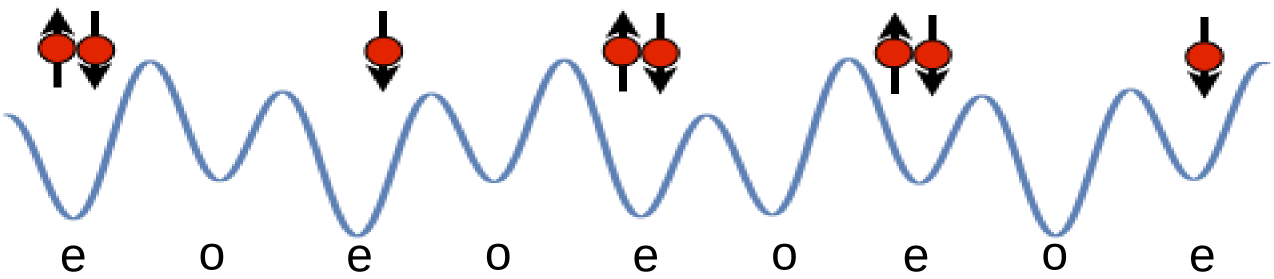
S_A broadly distributed at crit. point

Is there a gravity dual to this transition? Merging of small black holes?

Experimental observation of MBL: fermions in a quasi-random optical lattice

(Not presented in the conference talk)

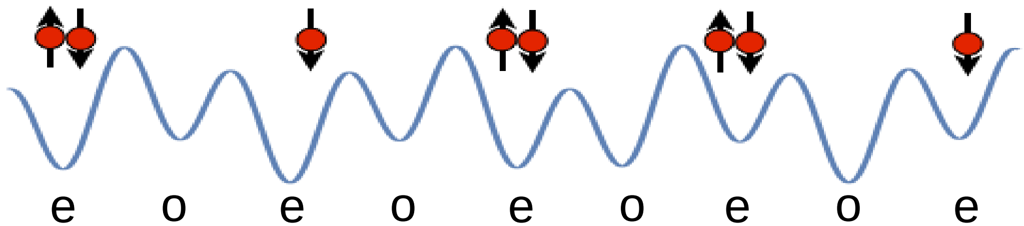
arXiv:1501.05661



With:
Immanuel Bloch’s group (Munich)
Mark Fischer and Ronen Vosk (WIS)

Quantum quench protocol

1. Fermions in optical lattice prepared in period-2 CDW

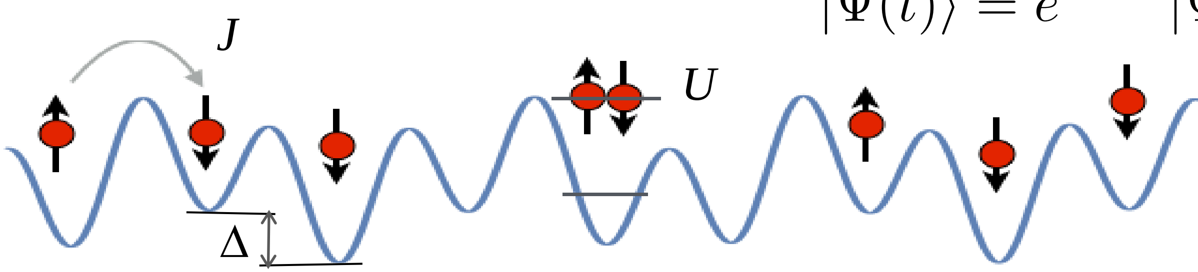


1. Evolve the state with the 1d lattice Hamiltonian:

$$\hat{H} = -J\sum_{i,\sigma}\left(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + h.c.\right) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \varphi)\hat{c}^\dagger_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}.$$

Incommensurate potential

$$|\Psi(t)\rangle = e^{-i\hat{H}t}|\Psi(0)\rangle$$



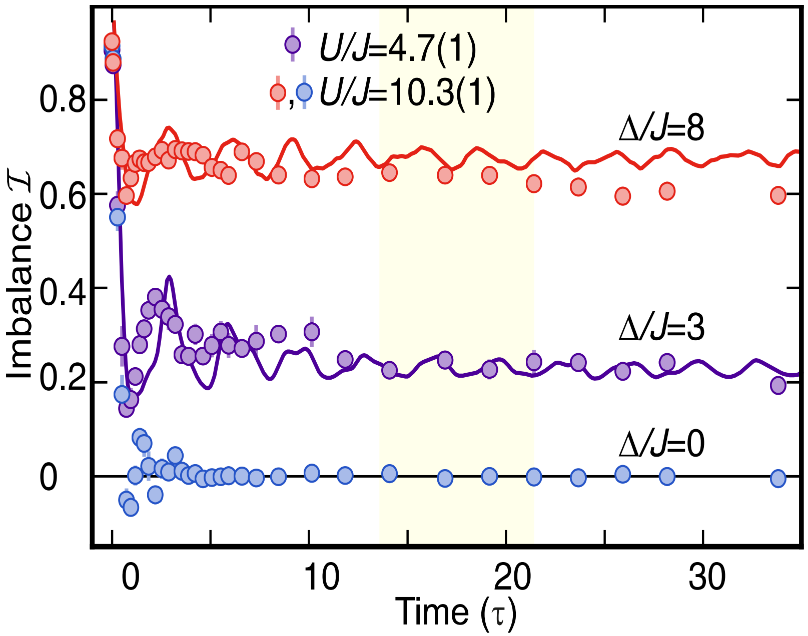
Numerics suggest that this model shows generic MBL (Iyer et. al. PRB 2013)

What to measure?

CDW or imbalance between even and odd sites:

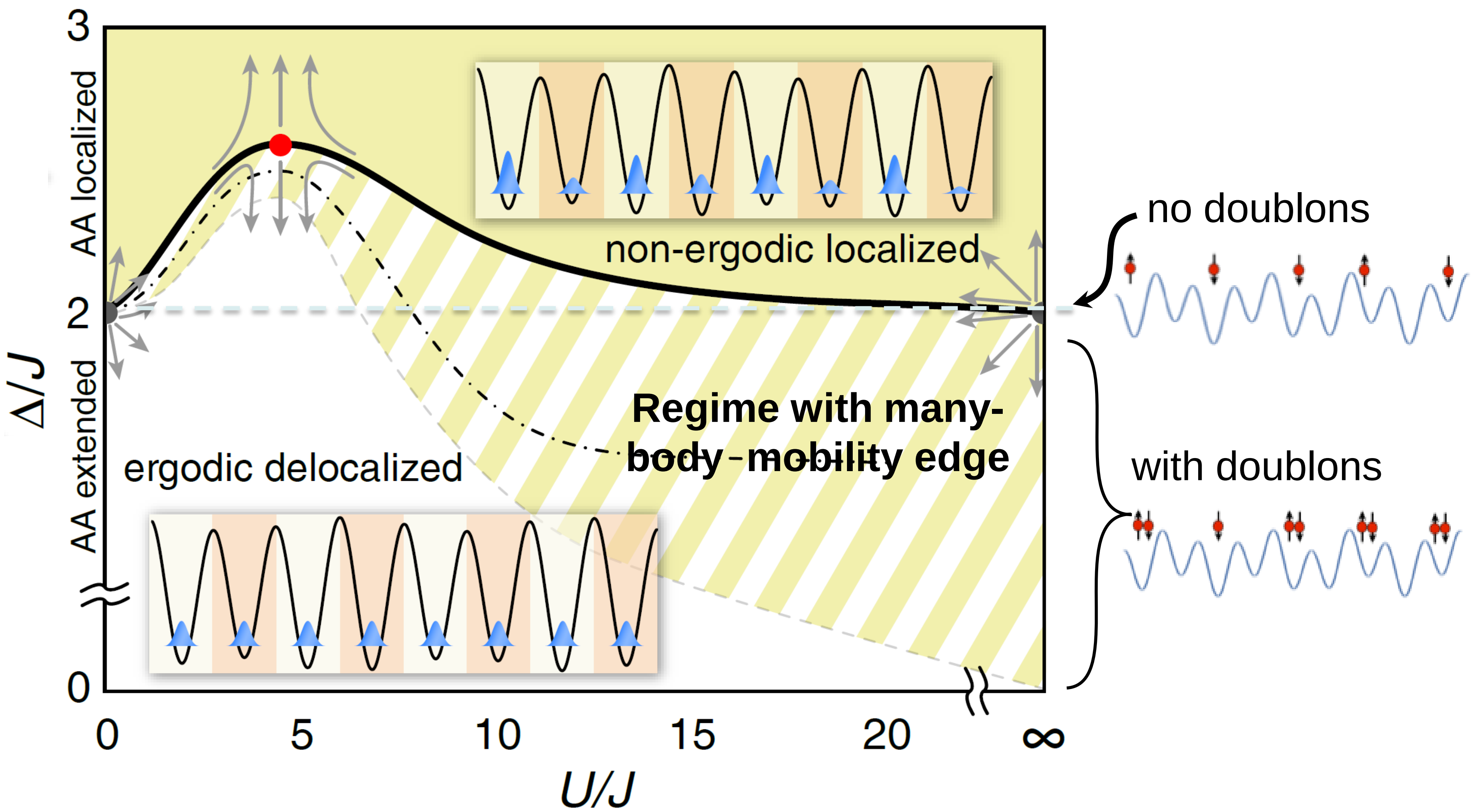
$$\mathcal{I} = \frac{1}{N}\sum_{j=1}^{L}(-1)^j\langle n_j\rangle = \frac{\langle N_e - N_o\rangle}{N_e + N_o}$$

Non complete relaxation is direct evidence for ergodicity breaking



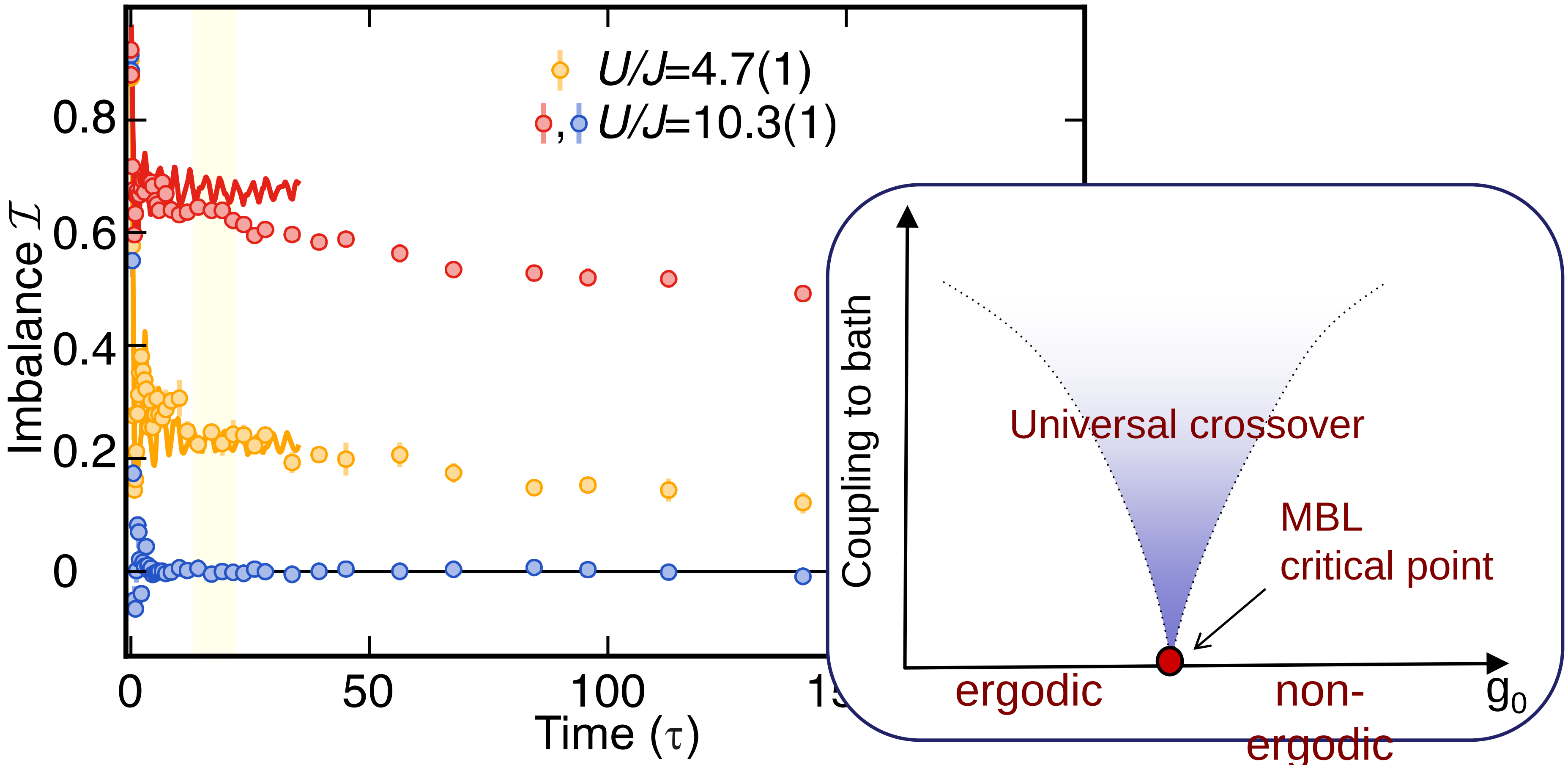
If the system is localized, the density wave operator has finite overlap with an integral of motion.

Phase diagram



Effect of inadverdant coupling to bath

Slow decay of the imbalance at long times:



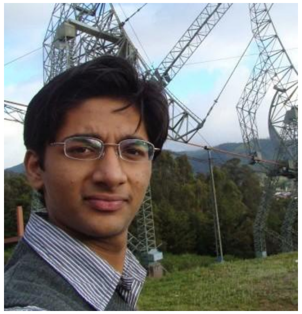
More to do in experiment

- True random disorder.
- Study the two dimensional case.
- Controlled coupling to a bath.
- Coupling to external noise or periodic drive.


I. Bloch


U. Schneider


S. Hodgman


P. Bordia


M. Schreiber

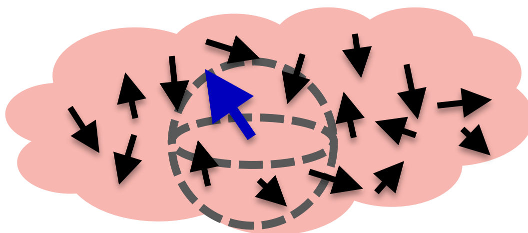
Theory:


Mark Fischer


Ronen Vosk

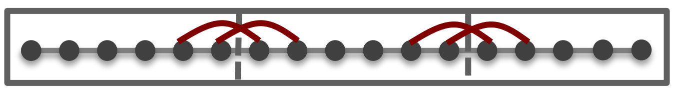
Summary

Many-body localized



Quantum coherent dynamics

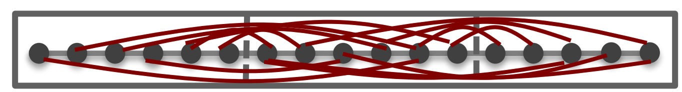
Area law entanglement

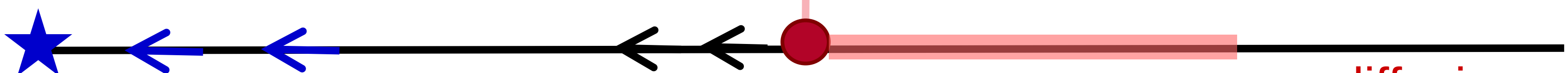


Thermalizing

"Classical" dynamics

Volume law entanglement





Dynamical RG

Random matrix RG

sub-diffusive

diffusive

Localized fixed-point