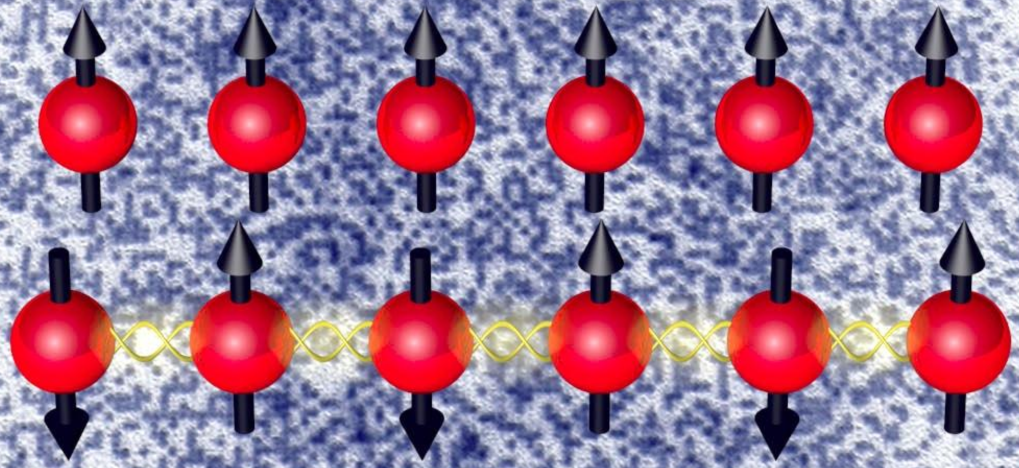
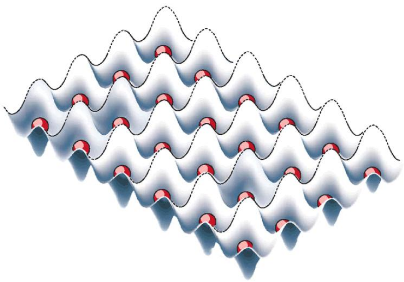
Measuring Entanglement Entropy in Synthetic Matter



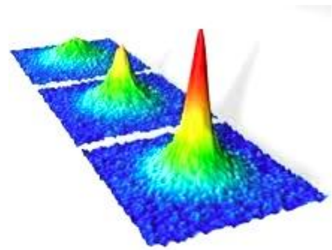
Markus Greiner
Harvard University

HARVARD UNIVERSITY · MIT
CENTER FOR ULTRACOLD ATOMS

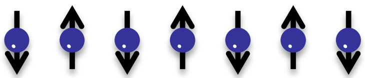
Ultracold atom synthetic quantum matter:
“First Principles” engineered materials



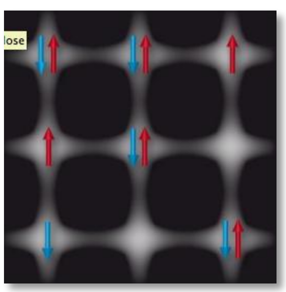
Bose Hubbard



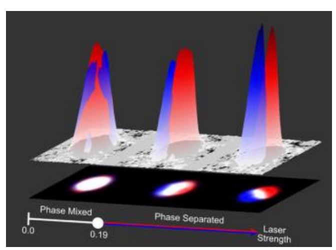
BEC-
BCS



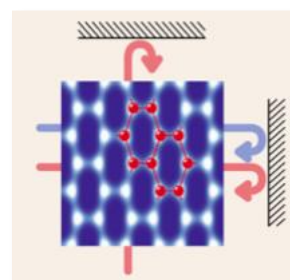
Ising spin



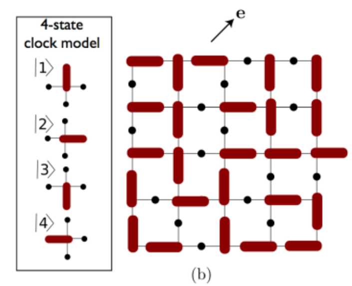
Fermi
Hubbard



spin-orbit



optical
lattice
graphene



spin
liquid

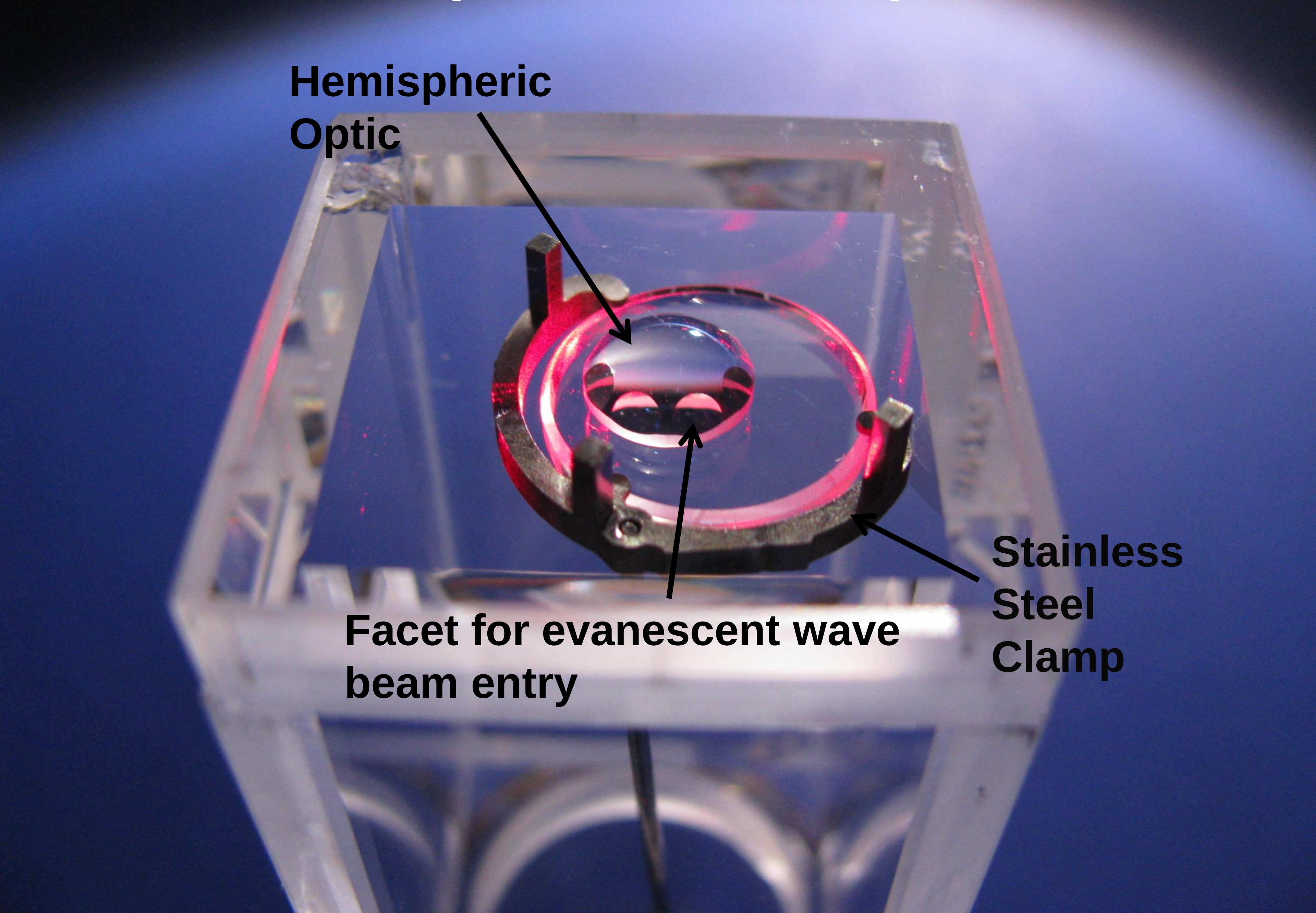
Quantum gas microscope

Bakr *et al*., Nature 462, 74 (2009), Bakr *et al*., Science.1192368 (June 2010)

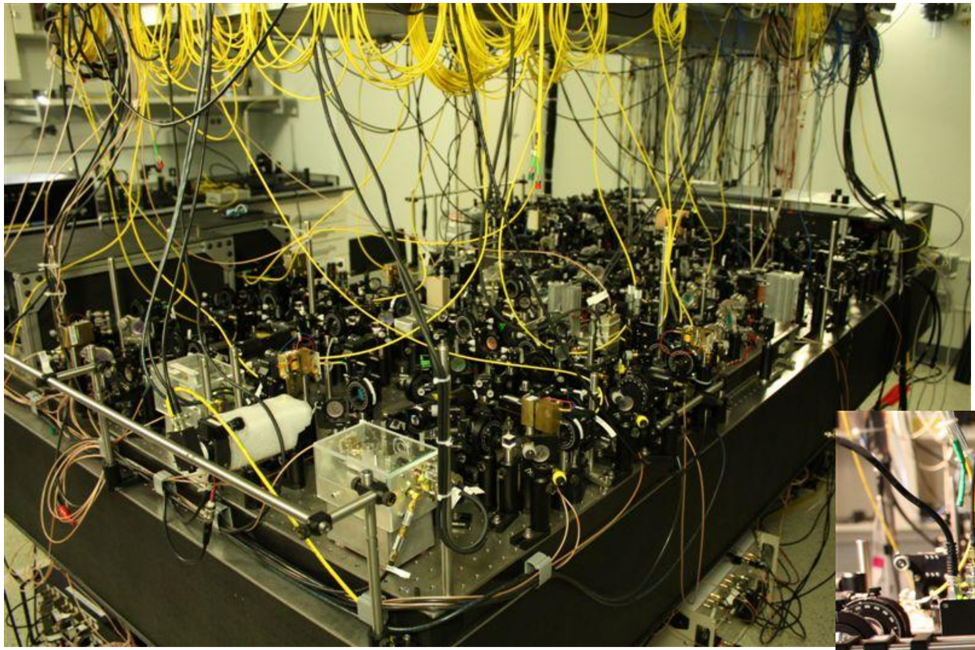
Previous work on single site addressability in lattices:

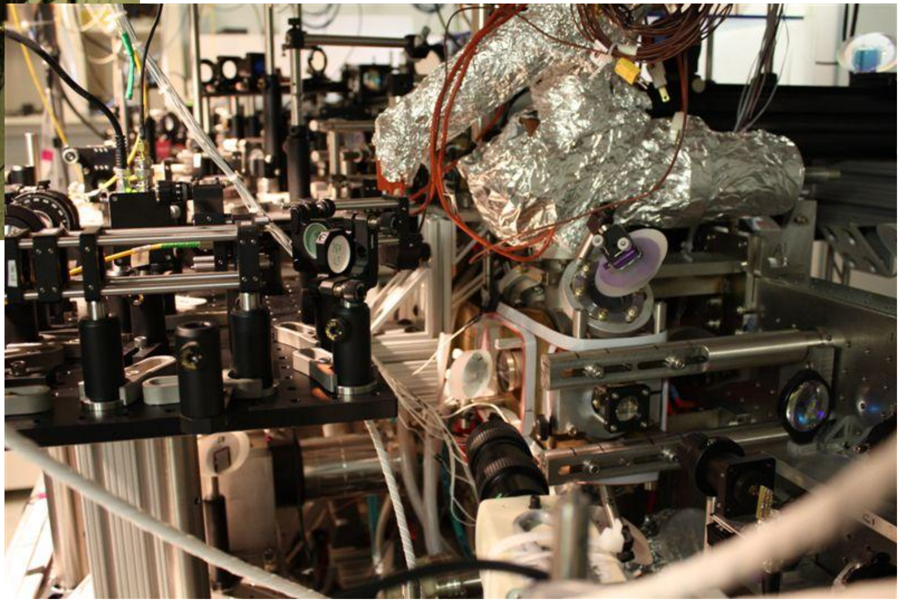
Detecting single atoms in large spacing lattices (**D. Weiss**) and 1D standing waves (D. Meschede), Electron Microscope (H. Ott), Absorption imaging (J. Steinhauer), single trap (P. Grangier, Weinfurter/Weber), **few site resolution (C. Chin),** See also: Sherson *et al*., Nature

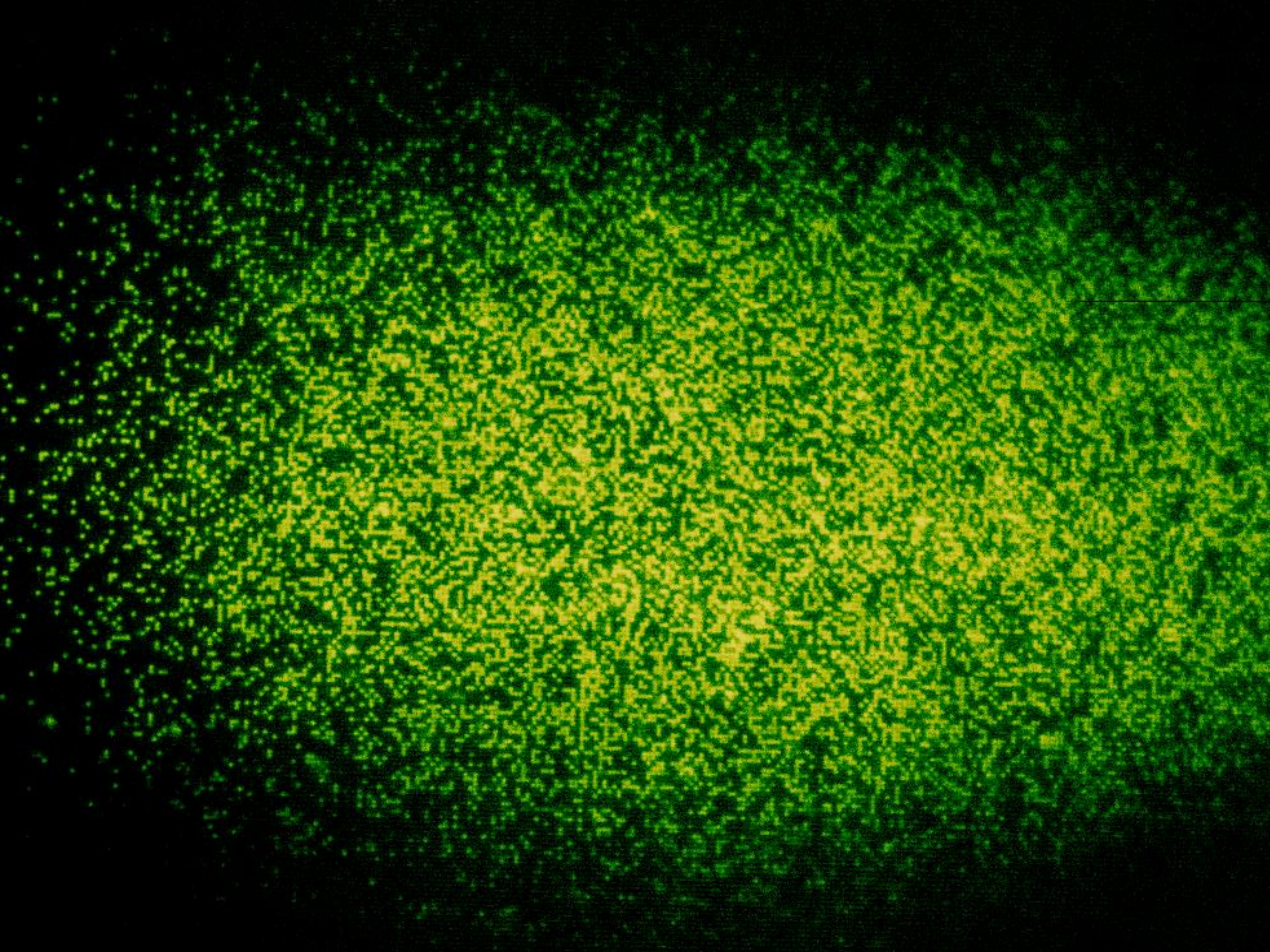
Experimental Setup



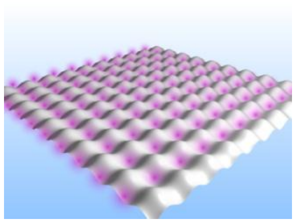
… and the whole apparatus







Bose-Hubbard Hamiltonian



$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Tunneling term:

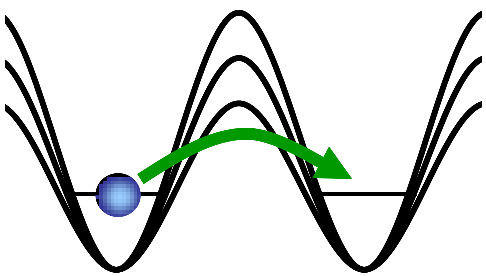
J: tunneling matrix element

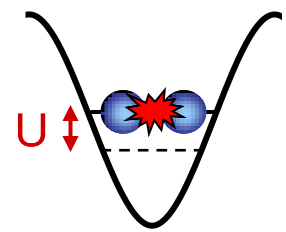
$\hat{a}_i^\dagger \hat{a}_j$: tunneling from site j to site i

Interaction term:

U : on-site interaction matrix element

$\hat{n}_i (\hat{n}_i - 1)$: n atoms collide with n-1 atoms on same site



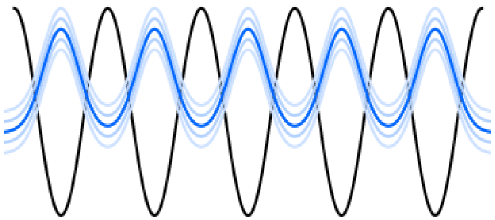


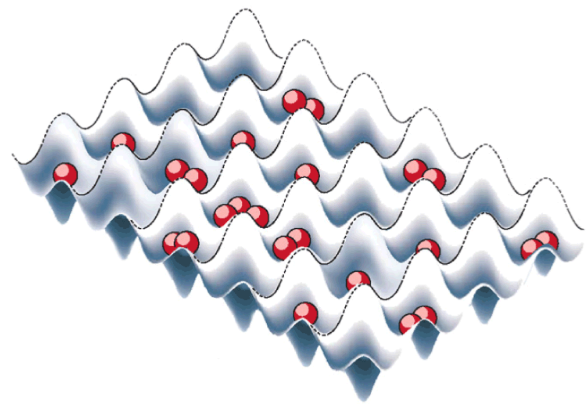
Ratio between tunneling J and interaction U can be widely varied
by changing depth of 3D lattice potential!

M.P.A. Fisher et al, PRB 40, 546 (1989), D. Jaksch et al., PRL 81, 3108 (1998)

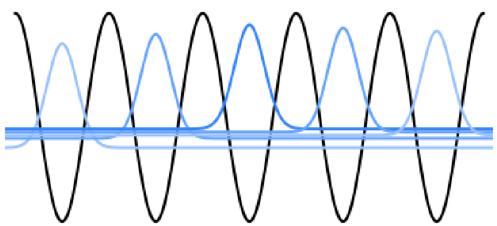
Superfluid – Mott insulator quantum phase transition

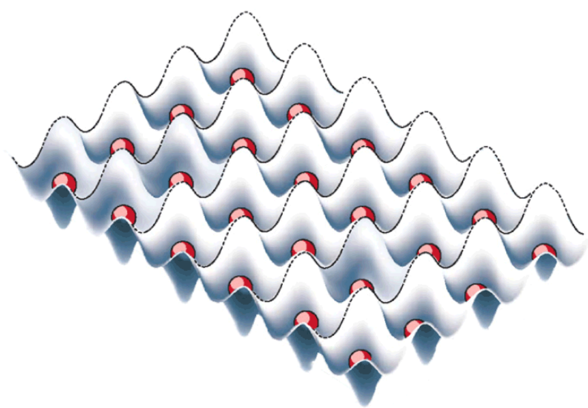
Superfluid





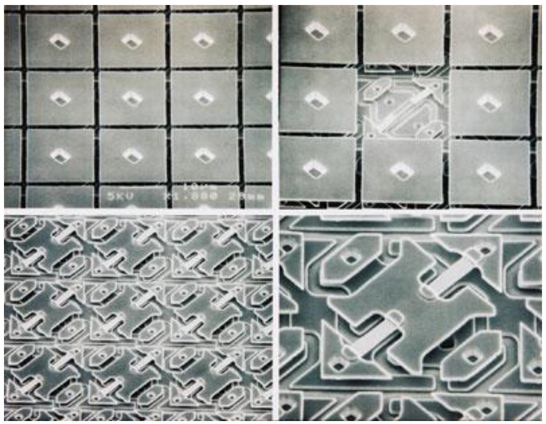
Mott insulator



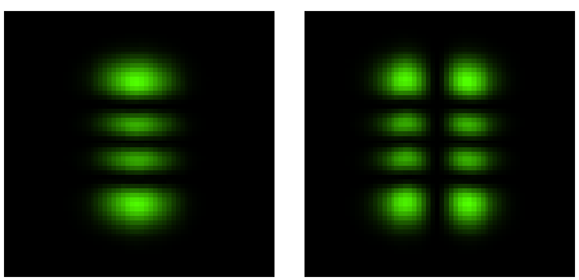


Bakr *et al*., Science.1192368 (June 2010)

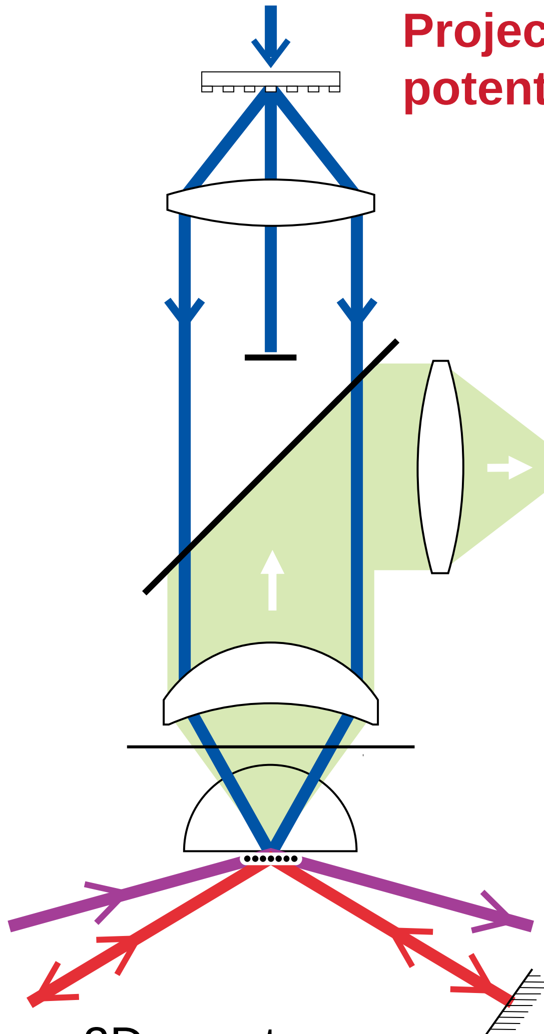
Projecting arbitrary potential landscapes



Digital Mirror Device (DMD)



Projecting high resolution potential landscapes



High resolution imaging

High aperture objective NA=0.8

2D quantum gas in optical lattice

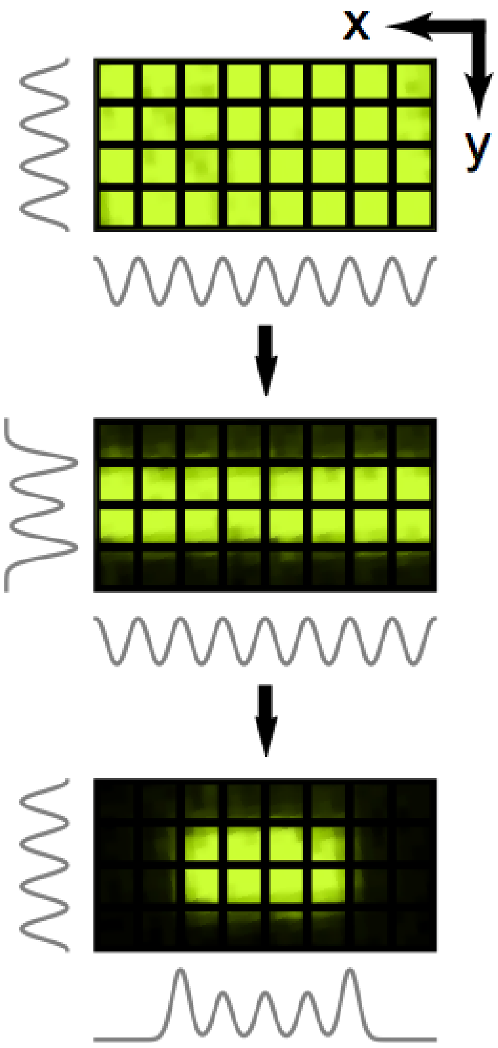
Projecting arbitrary potential landscapes

Prepare low entropy

Mott insulator state

Modify potential landscape
to create desired system

e.g. system with 2x4 lattice
sites



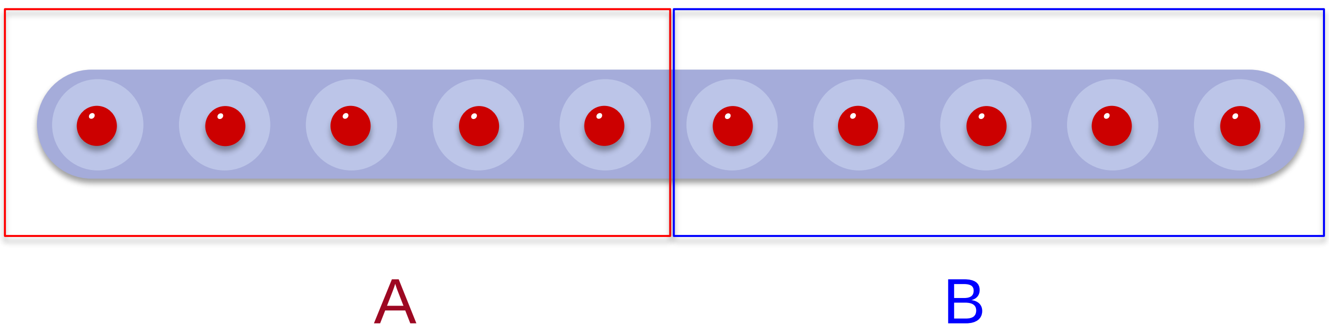
Entanglement In many-body systems

Simplest case: two spins

Bell state

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle\otimes|\downarrow\rangle+|\downarrow\rangle\otimes|\uparrow\rangle\right)$$

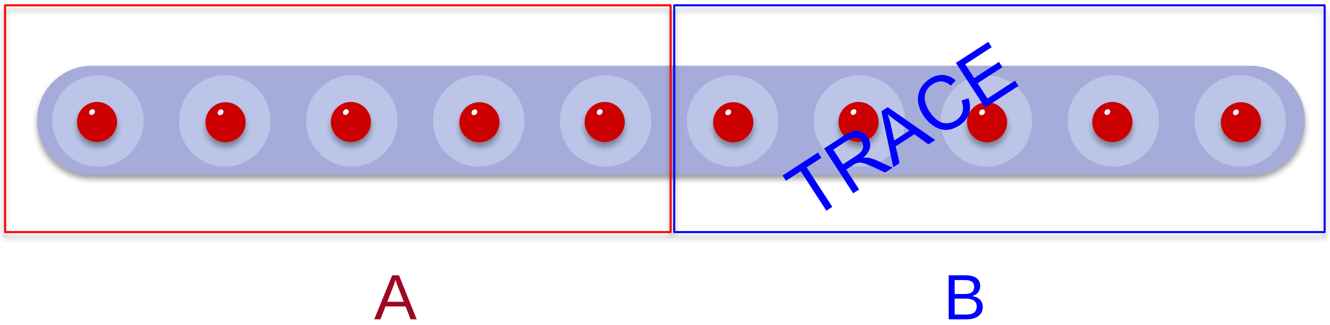
Many-body system: Bipartite entanglement



Product state: $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$ e.g. Mott insulator

Entangled state: $|\Psi\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$ e.g. Superfluid

Entanglement entropy



Reduced density matrix:

$$\rho_A = \mathrm{tr}_B\{\rho\} = |\Psi_A\rangle \otimes \langle\Psi_A|$$

Product state

→ Pure state

Entangled state

→ Mixed state

Von Neuman entropy

$$S_{VN}(\rho_A) = -\mathrm{tr}\{\rho_A \log \rho_A\}$$

Renyi Entropy

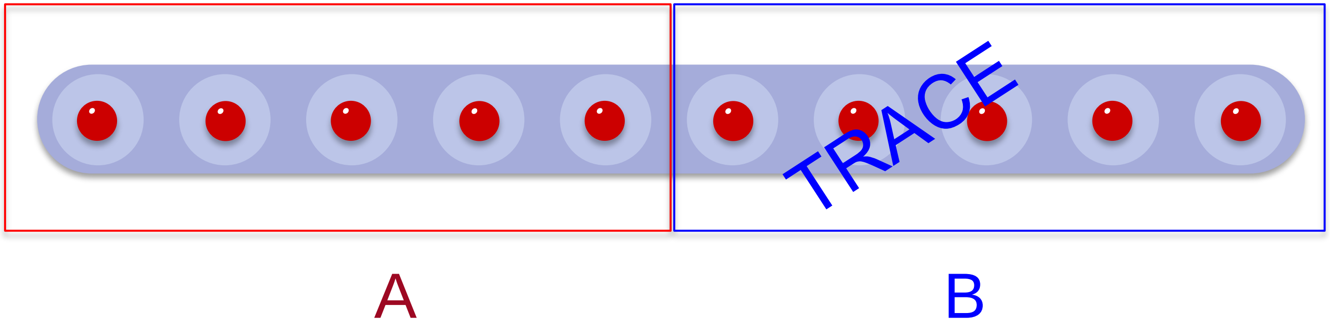
$$S_n(\rho_\alpha) = \frac{1}{1-n} \log \mathrm{Tr}\{\rho_\alpha^n\}$$

$= 0$

> 0

→ Entanglement entropy

Idea: Measure State purity in many-body systems



Reduced density matrix:

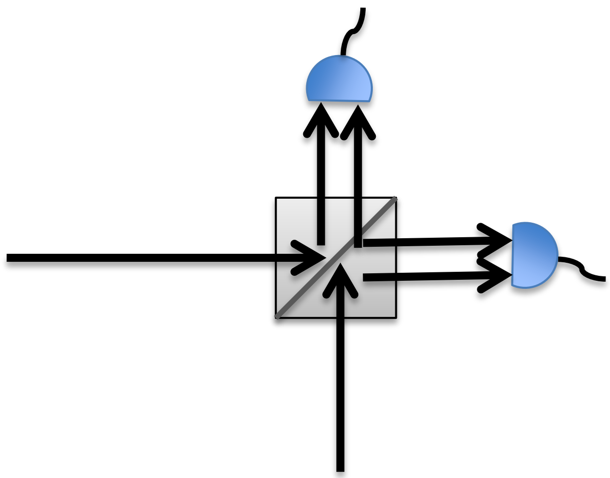
$$\rho_A = \mathrm{tr}_B\{\rho\} = |\Psi_A\rangle \otimes \langle\Psi_A|$$

Product state	**Entangled state**
→ Pure state	**→ Mixed state**

Many-body Hong-Ou-Mandel interferometry

Alves and Jaksch, PRL 93, 110501 (2004)
Mintert et al., PRL 95, 260502 (2005)
Daley et al., PRL 109, 020505 (2012)

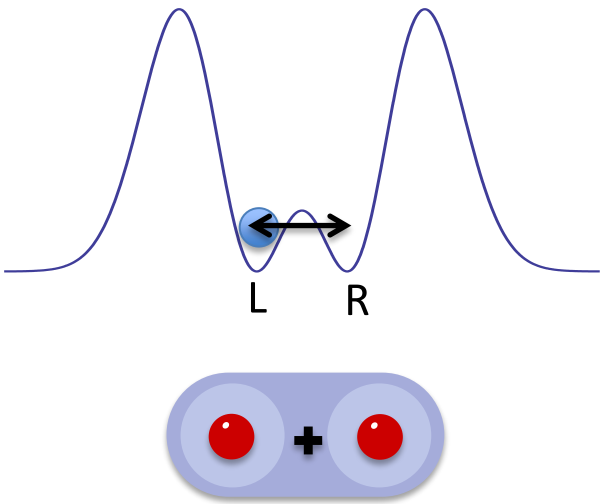
Hong-Ou-Mandel interference

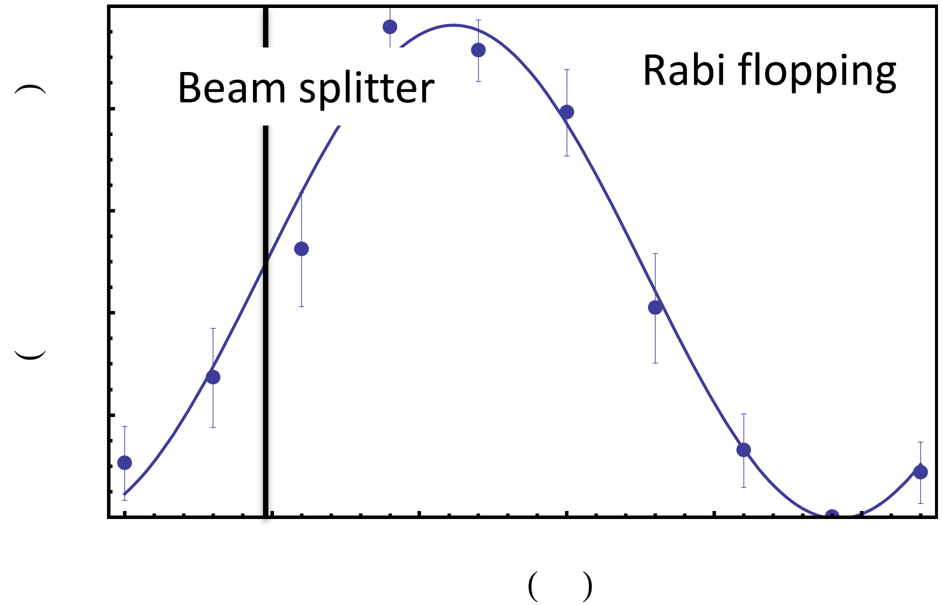


No coincidence detection
for identical photons

Hong C K, Ou Z Y and Mandel L Phys. Rev. Lett. 59 2044 (1987)

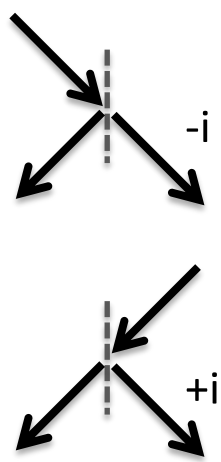
Beam splitter operation: Rabi flopping in a double well





$$a_L^\dagger \to a_L^\dagger - i a_R^\dagger$$

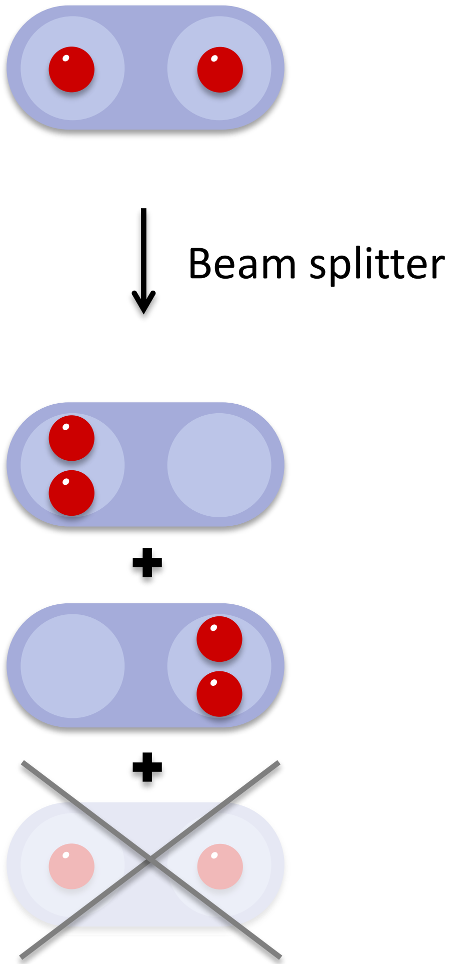
$$a_R^\dagger \to a_L^\dagger + i a_R^\dagger$$



Also see: Kaufman A M *et al.*,
Science 345, 306 (2014)
Without single atom detection:
Trotzky et al., PRL 105, 265303 (2010)
also Esslinger group

Two bosons on a beam splitter

Hong-Ou-Mandel interference



$$a_L^\dagger a_R^\dagger$$

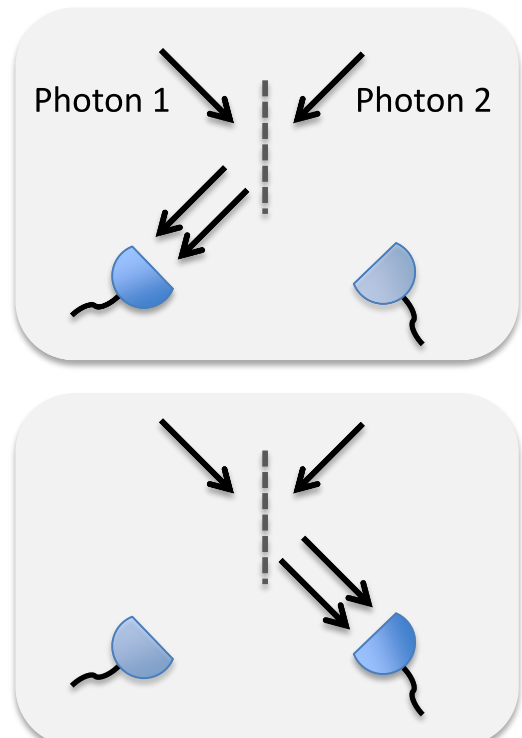
$$a_L^\dagger \rightarrow a_L^\dagger - i a_R^\dagger$$

$$a_R^\dagger \rightarrow a_L^\dagger + i a_R^\dagger$$

$$a_L^\dagger a_L^\dagger$$

$$+$$

$$a_R^\dagger a_R^\dagger$$



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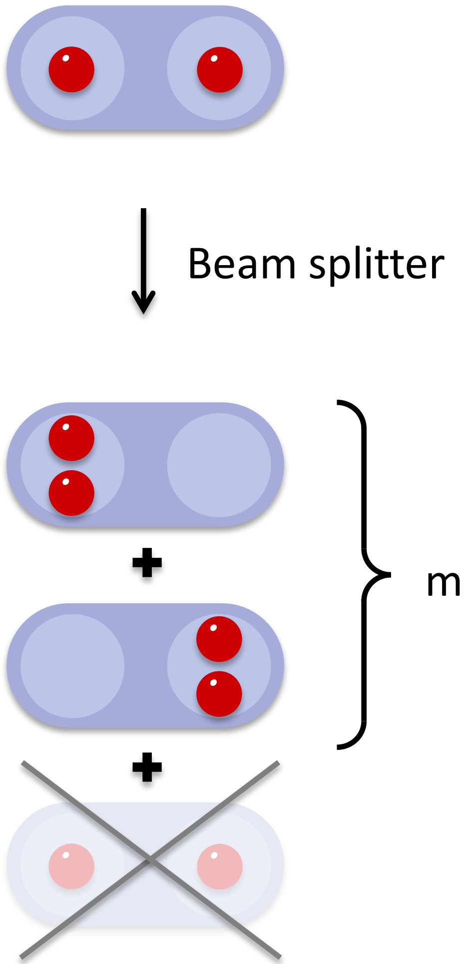
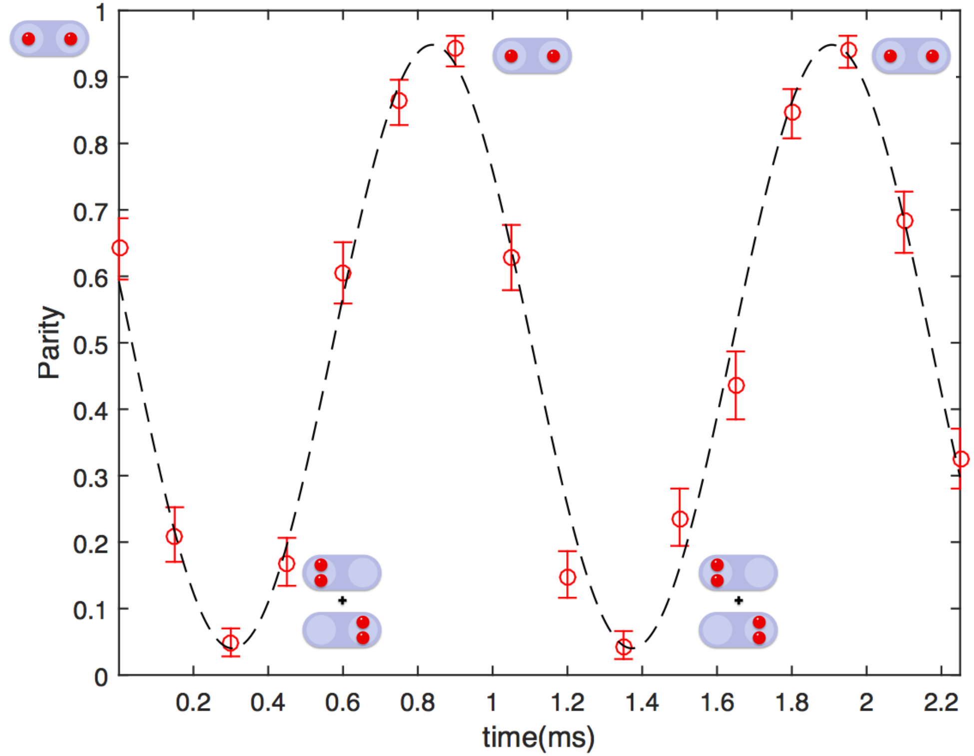
Two bosons on a beam splitter

Hong-Ou-Mandel interference



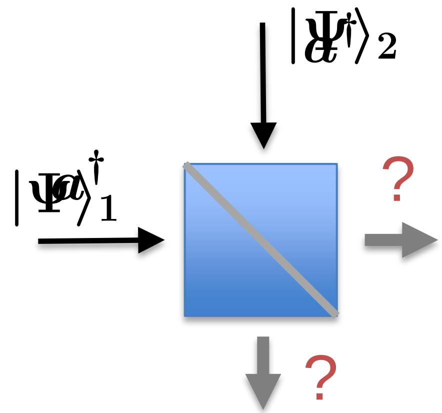


measured fidelity:	first revival:
96(4)%	9(6)%
4(4)%	91(6)%

limited by interaction

Also see: Kaufman A M *et al.*, Science 345, 306 (2014)
Without single atom detection: Trotzky et al., PRL 105, 265303 (2010), also Esslinger group

HOM-Interference of Many-Body States

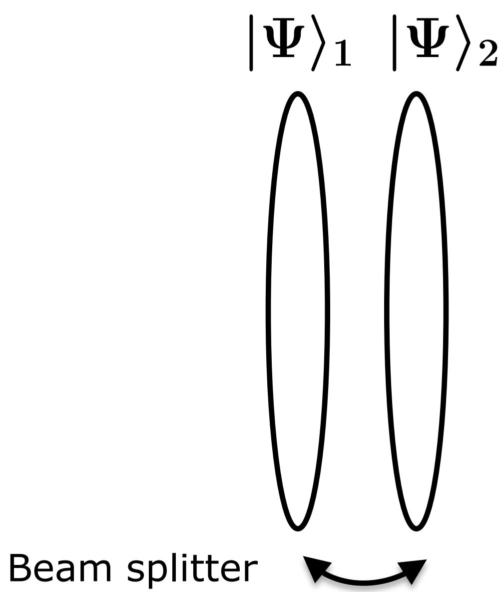


How "identical" are the **particles**?

vs.

How "identical" are the **states**?

Interference of many-body states



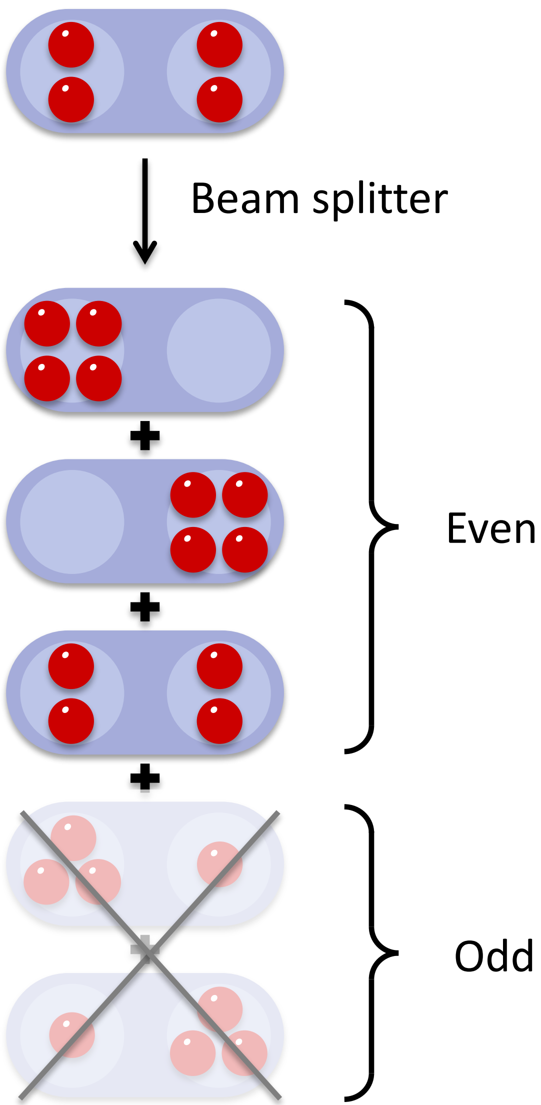
If $|\Psi\rangle_1 = |\Psi\rangle_2$, **deterministic parity** after beam splitter

Measure purity $Tr(\rho^2)$

Alves and Jaksch, PRL **93** (2004)
Daley et al., PRL **109** (2012)

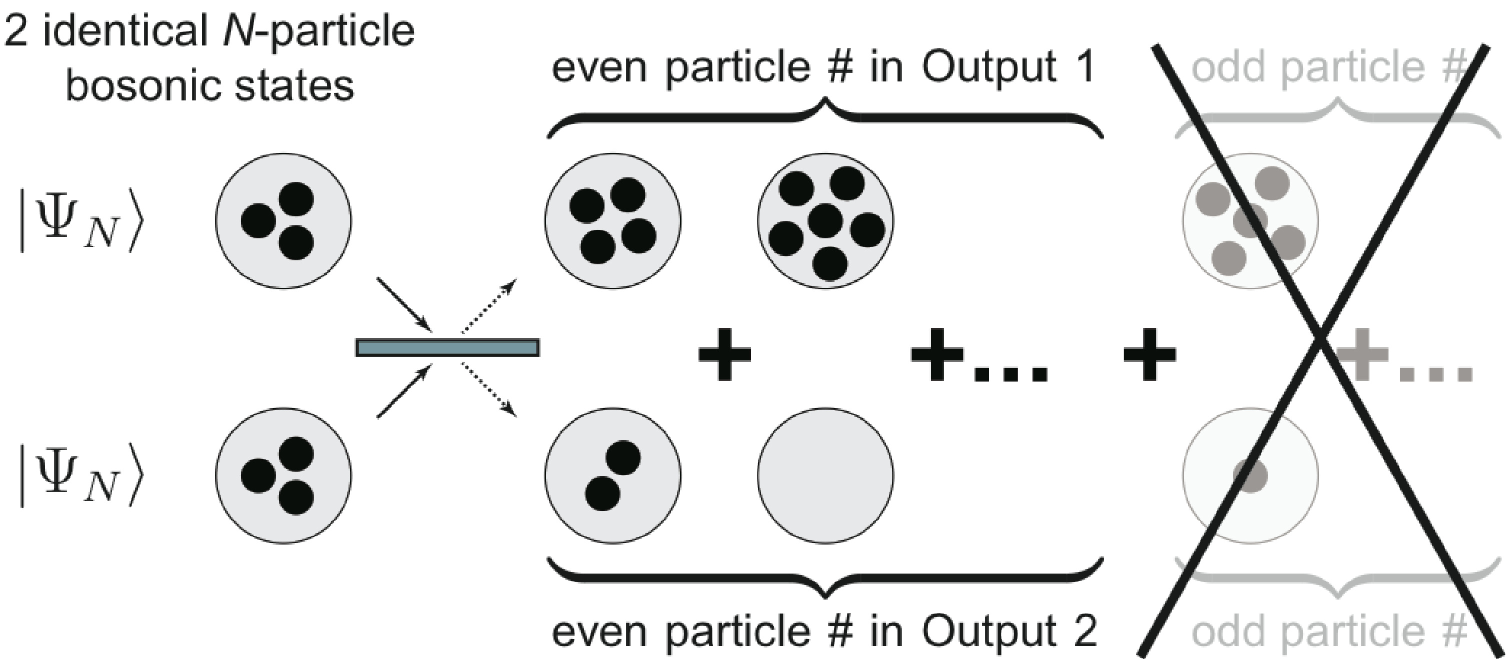
Generalized Hong-Ou-Mandel interference: 2x2 particles

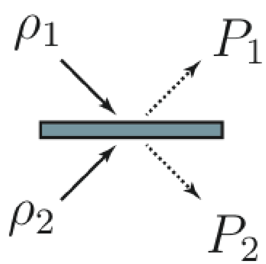
preliminary



	measured probability:		first revival:
Even	75%	50%	6%
		25%	68%
Odd	25%		26%

Quantum interference of bosonic many body systems





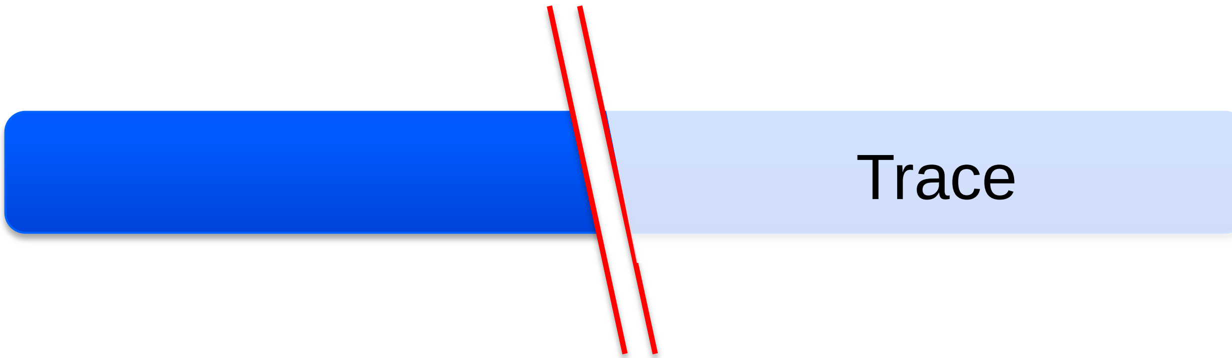
$$\langle P_i \rangle = \mathrm{Tr}\,(\rho_1 \rho_2) \overset{\rho_1 = \rho_2}{=\!=} \mathrm{Tr}\,(\rho^2)$$

average parity — quantum state overlap — purity

$$P_i = \prod_k p_i^{(k)}$$

Entanglement entropy

Many body quantum system



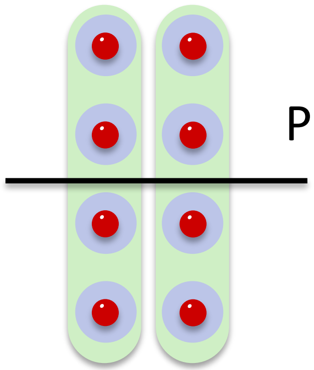
Initially: System in pure state

Cut: Entangled ?

Trace: If entangled, trace creates mixed state,
→ entropy is increased

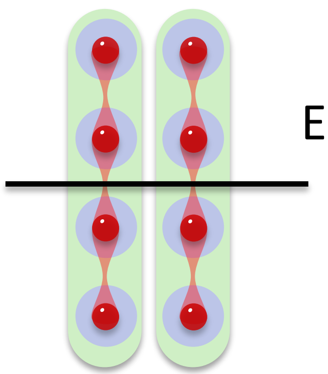
Measuring many-body entanglement

Mott Insulator



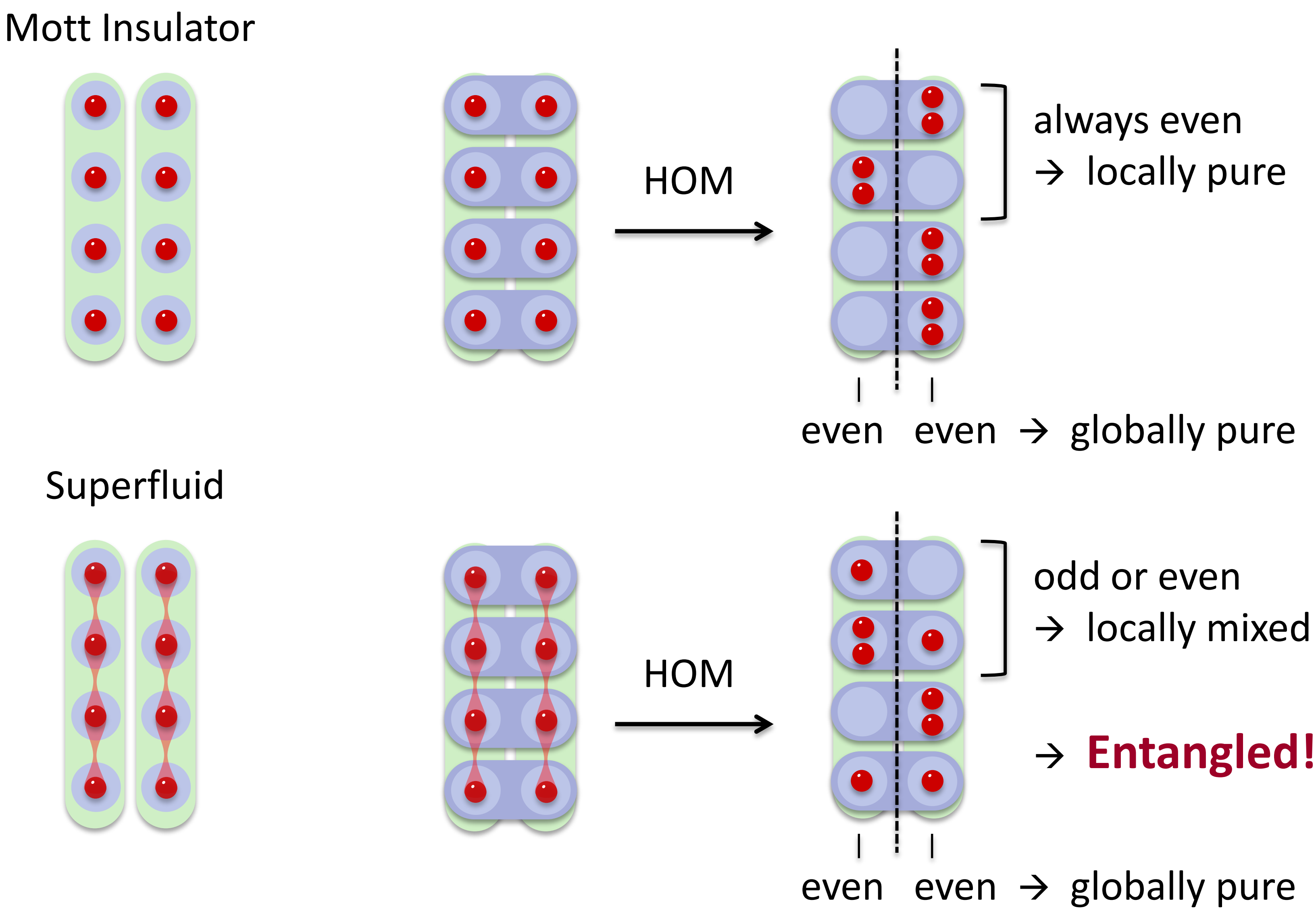
Product State

Superfluid



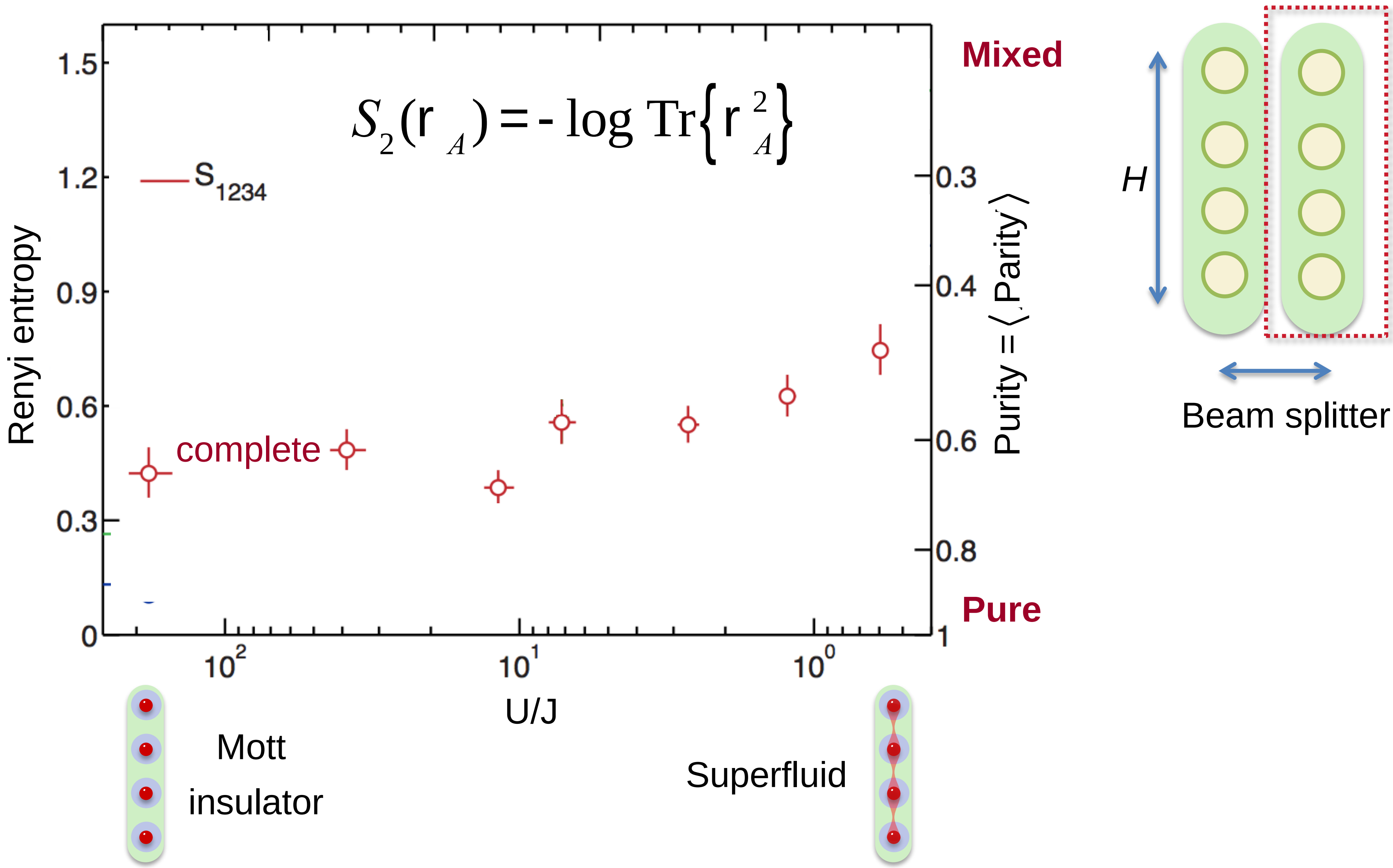
Entangled

Measuring many-body entanglement

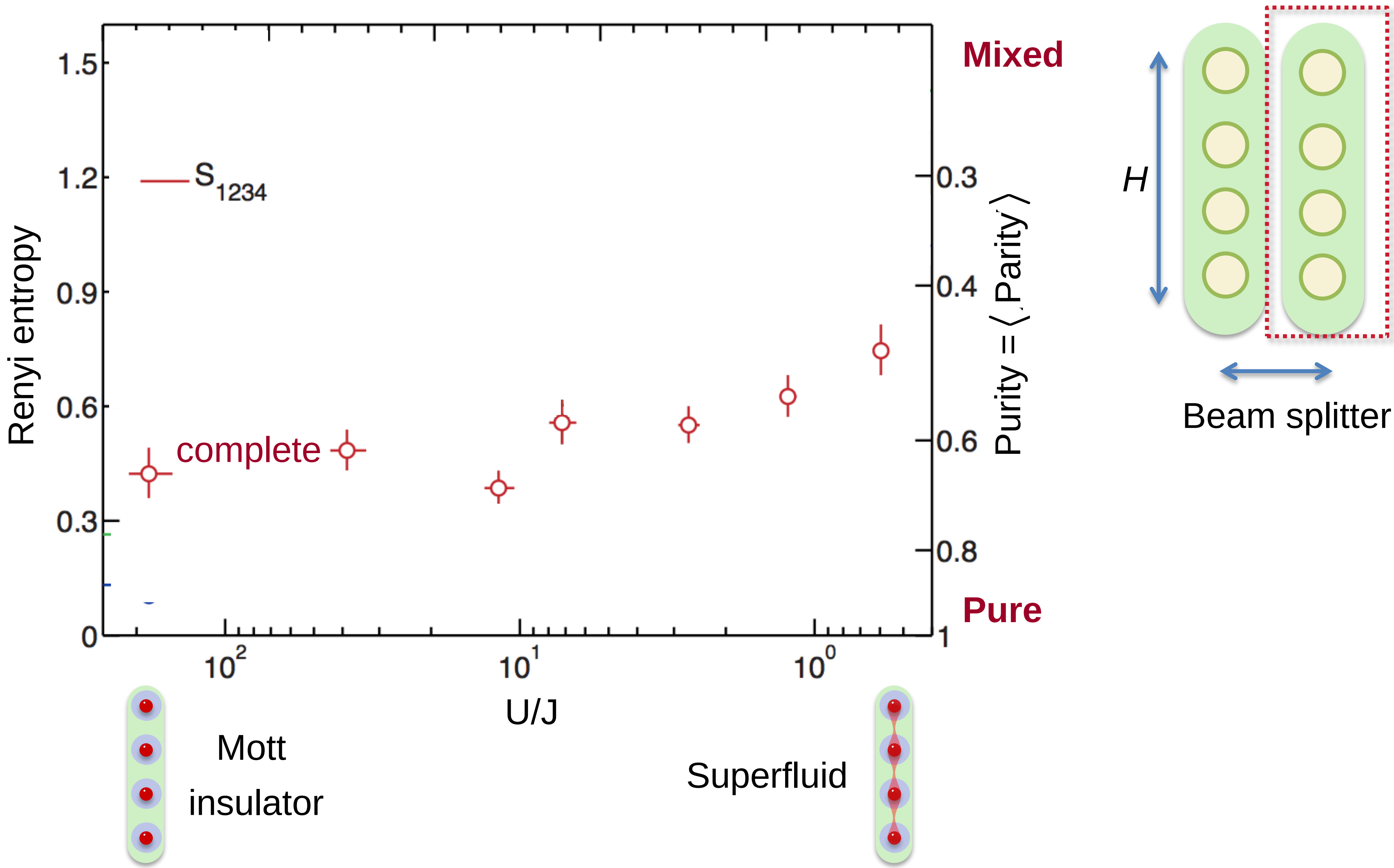


Ref: Alves C M, Jaksch D, PRL 93, 110501 (2004), Daley A J et al, PRL 109, 020505 (2012)

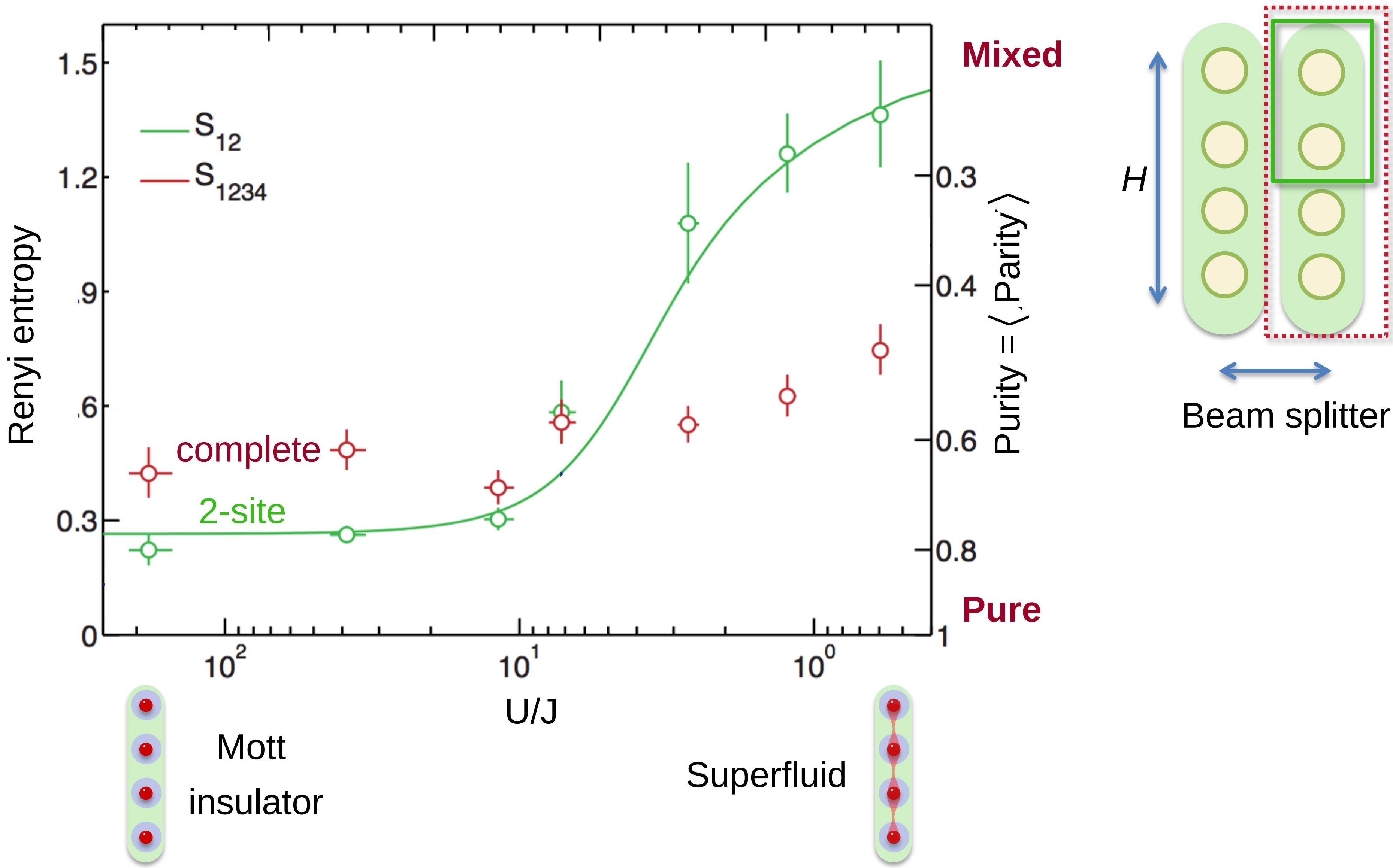
Entanglement Entropy for 2 copies of 4-site systems



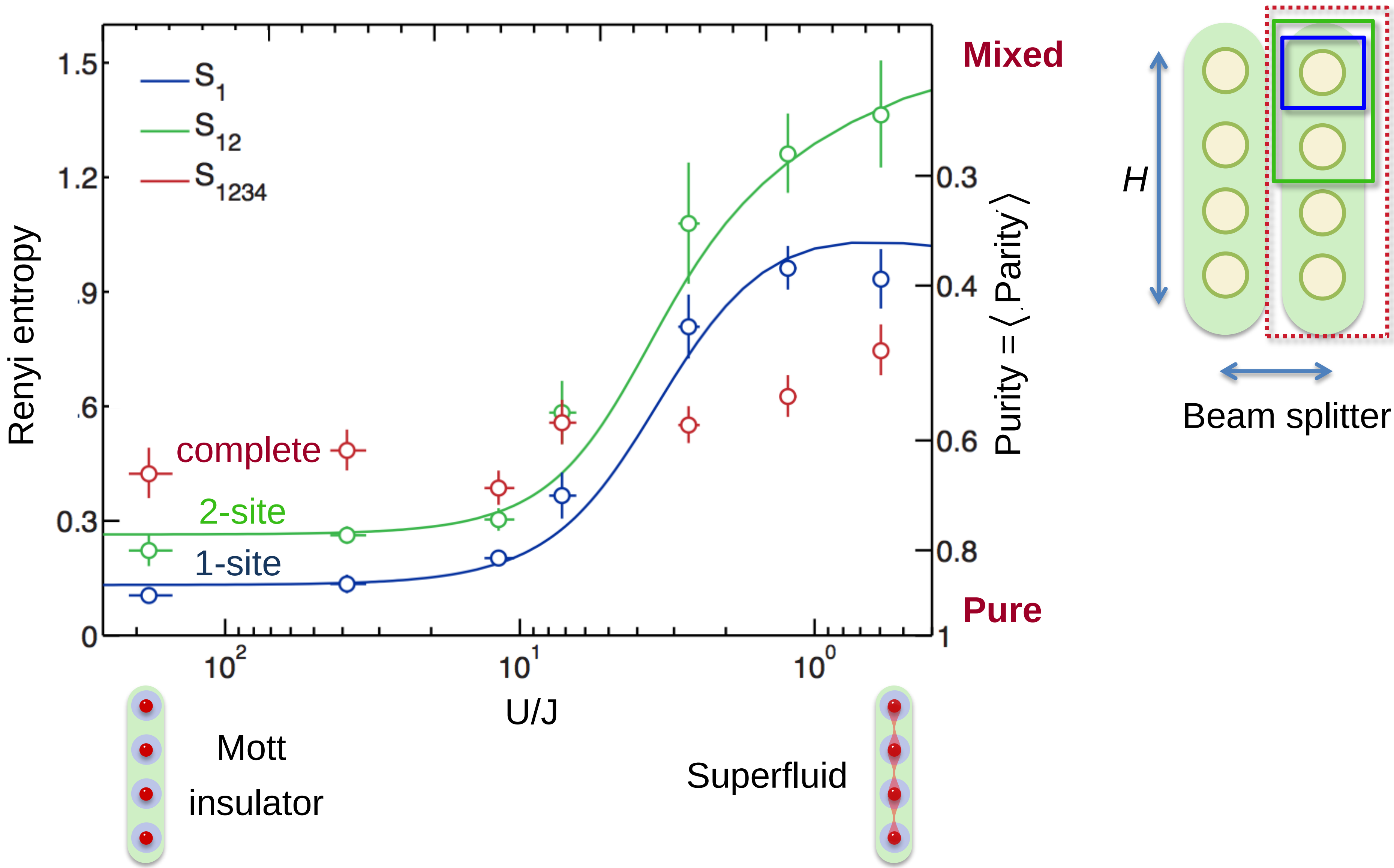
Entanglement Entropy for 2 copies of 4-site systems



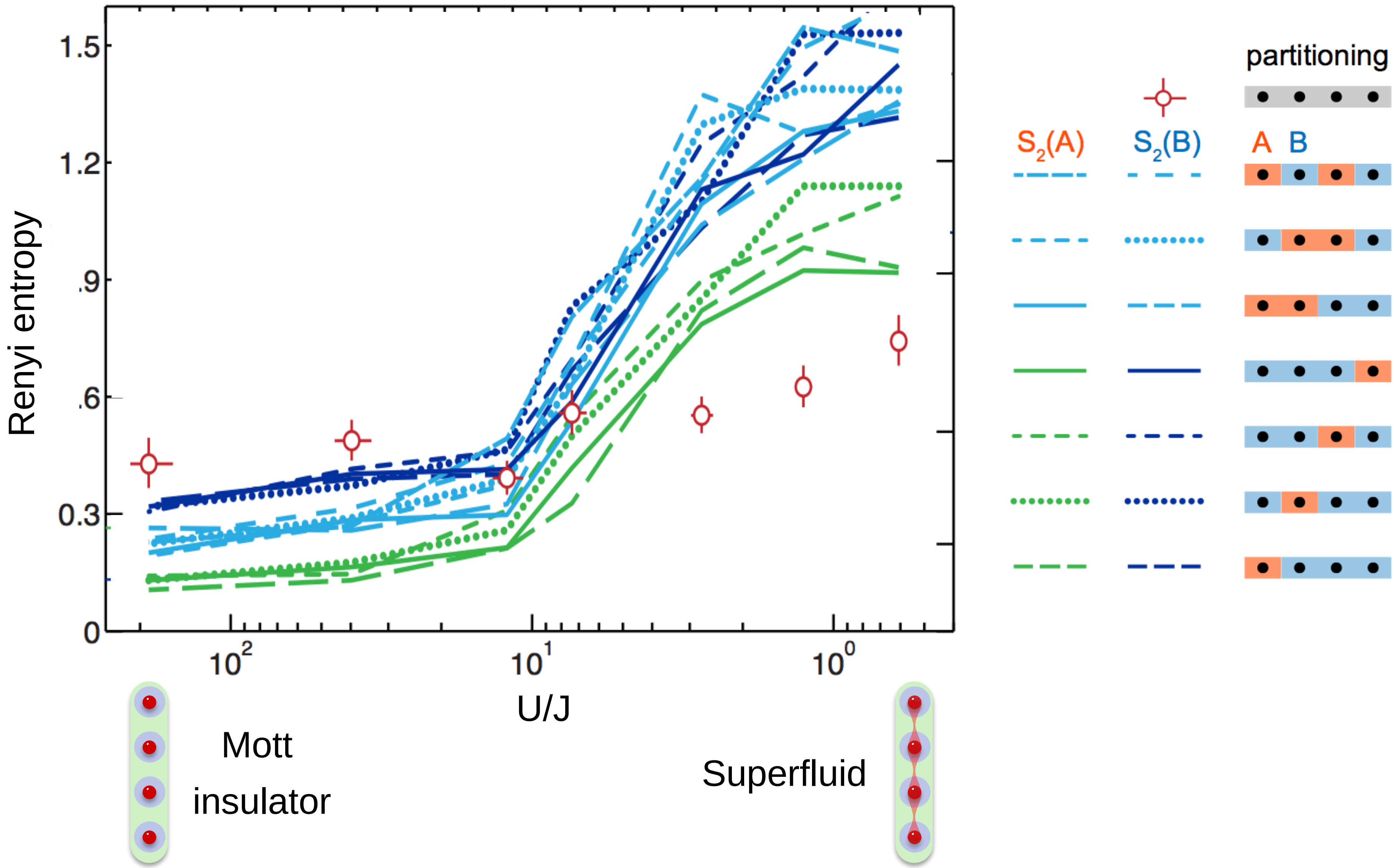
Entanglement Entropy for 2 copies of 4-site systems



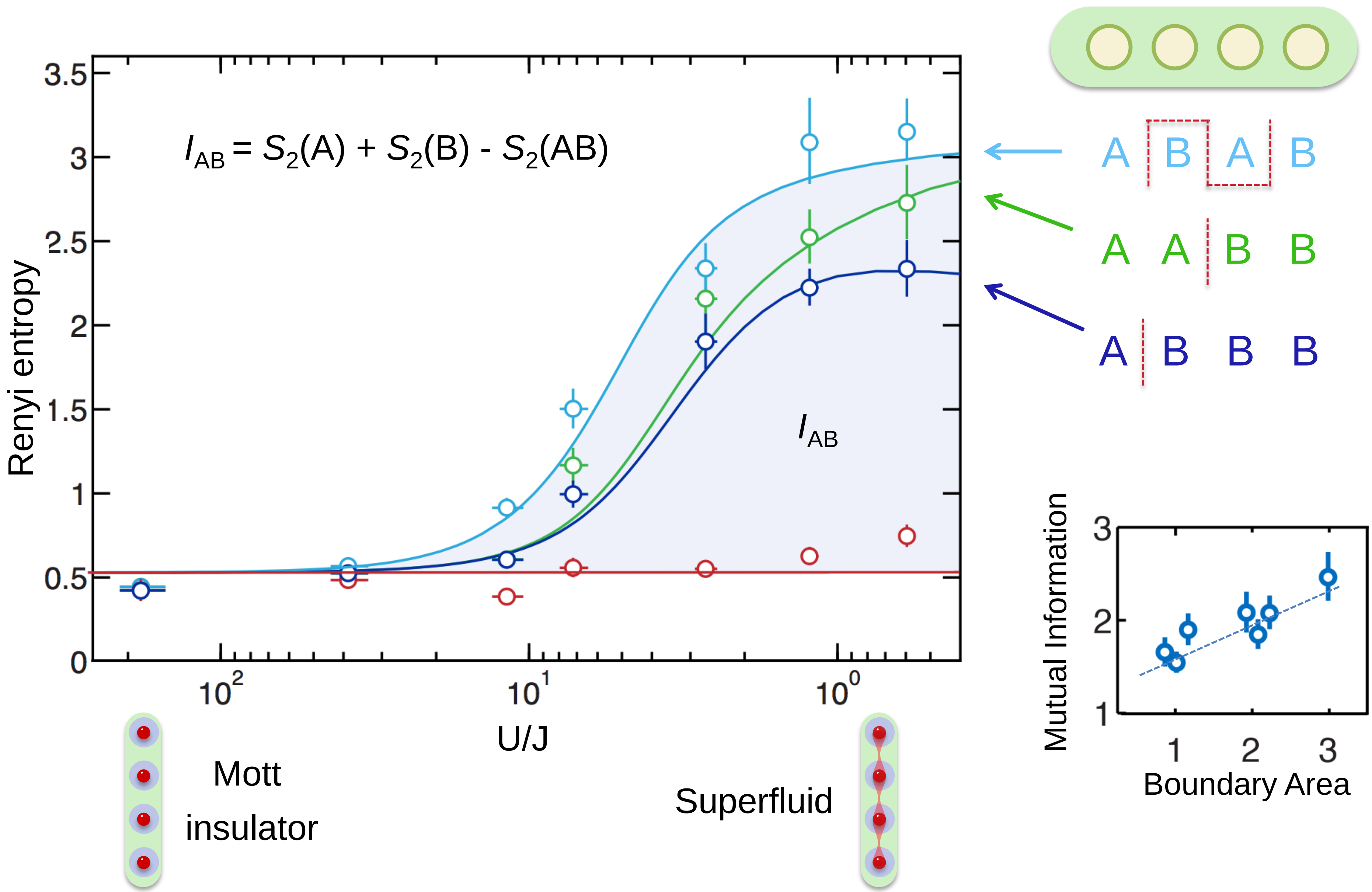
Entanglement Entropy for 2 copies of 4-site systems



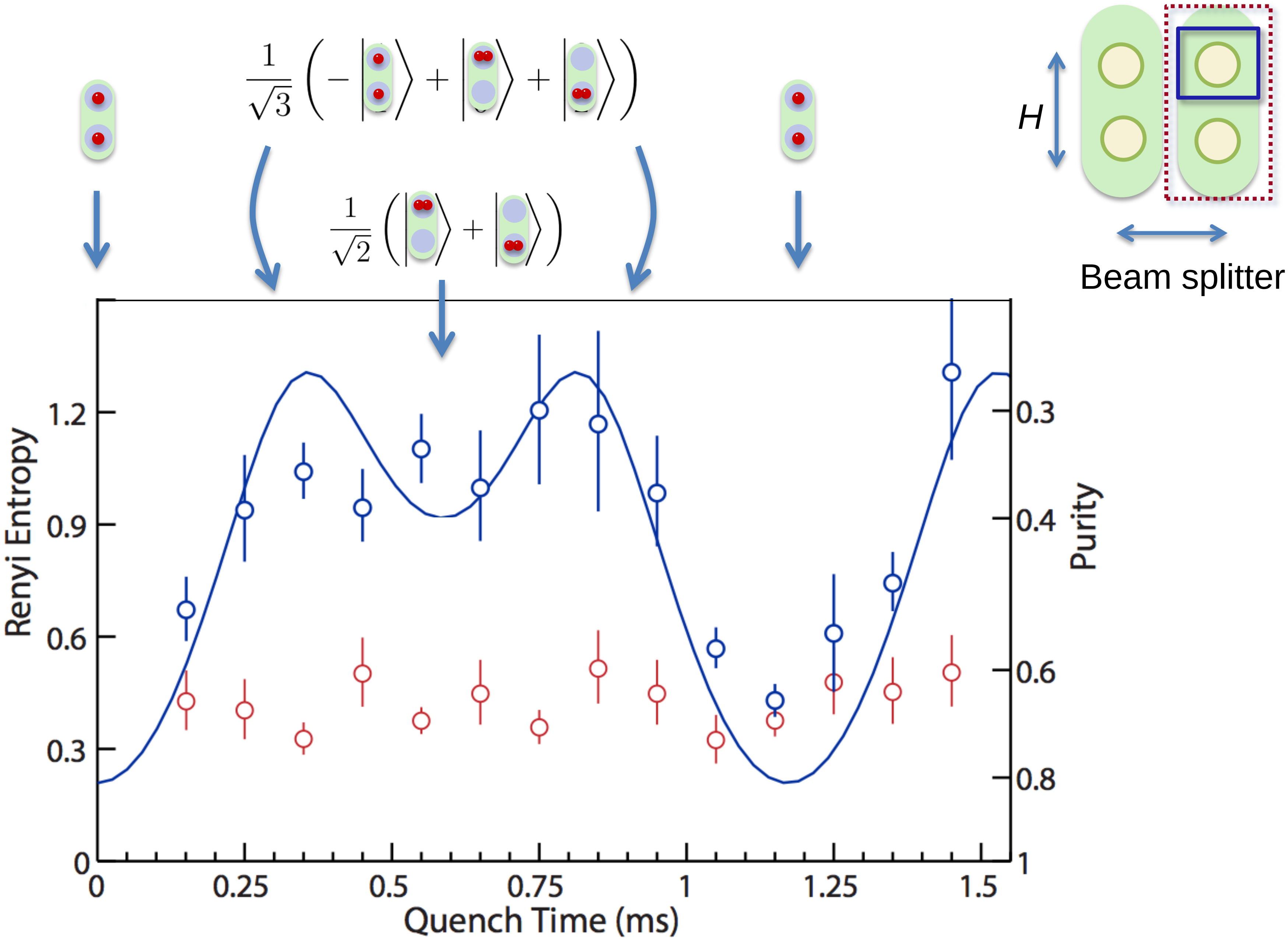
Entanglement Entropy for 2 copies of 4-site systems



Mutual Information I_{AB}

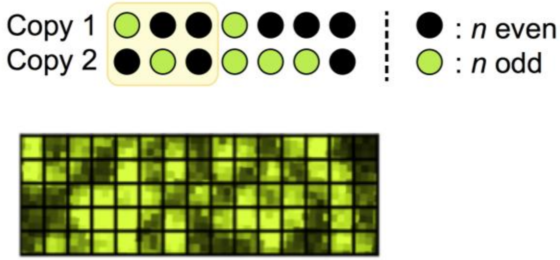


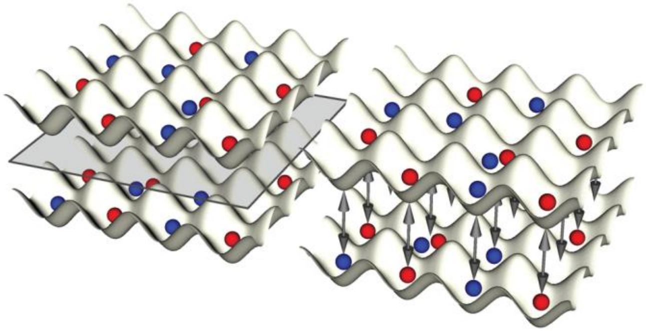
Non equilibrium: Quench dynamics



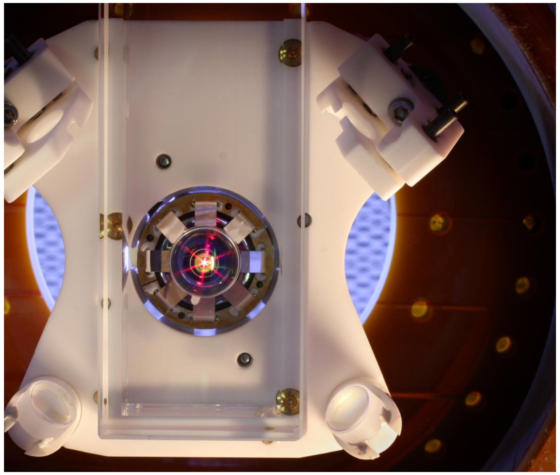
Entanglement Entropy

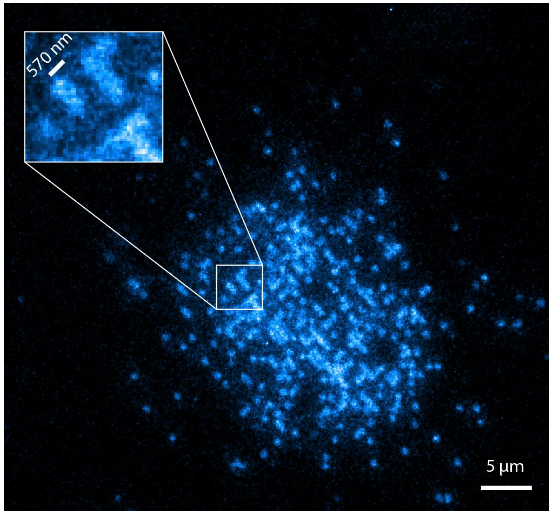
- Measure entanglement entropy in Bose-Hubbard lattice



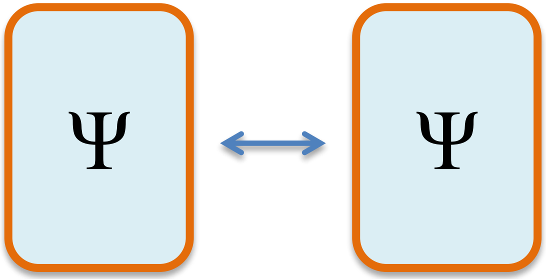


- Advanced systems: Fermions, Fractional quantum Hall states, …





Generalized HOM



Quantum-compare two sytems

- what else can I learn? what correlation functions would be interesting?
- validate quantum simulation
- n-systems: extract quantities that are polynomial in ρ^n (here: purity trace of ρ^2)

Thank you

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CENTER FOR ULTRACOLD ATOMS

Rubidium lab:
Eric Tai
Ruichao Ma
Philipp Preiss
Matthew Rispoli
Alex Lukin
Rajibul Islam

Lithium lab:
Maxwell Parsons
Anton Mazurenko
Sebastian Blatt
Christie Chiu

Erbium lab:
Susannah Dickerson
Anne Hebert
Aaron Krahn

Recent group members
Florian Huber
Jon Simon
Waseem Bakr
Philip Zupancic

Thanks to Theory: Peter Zoller, Andrew Daley, Hannes Pichler, Manuel Endres …

Moore Foundation

MURI Program in Optical Lattices

Many-body Hong-Ou Mandel

- Bosonic states under a beamsplitter operation:

$$T = \pi/(4J)$$

symmetric states basis (+1):

$$\{(a_1^\dagger - a_2^\dagger)^{2n}(a_1^\dagger + a_2^\dagger)^m|\text{vac}\rangle\}$$

$$a_1^\dagger|\text{vac}\rangle \to (a_1^\dagger + a_2^\dagger)/\sqrt{2}|\text{vac}\rangle$$
$$a_2^\dagger|\text{vac}\rangle \to (a_1^\dagger - a_2^\dagger)/\sqrt{2}|\text{vac}\rangle$$

→ even total number in copy 2 after beamsplitter

→ even total number in copy 1 as total number of particles is even

- For many sites, the symmetry under exchange can be taken by multiplying the results from each individual sites (it is more subtle for fermions)
- Thus, total even numbers in each copy are directly related to symmetry of the state under exchange

Relationship back to many-body inner product of states

- Why are the states symmetric if they are identical? Consider the swap operation on two copies of a state

$$V_2|\psi_1\rangle \otimes |\psi_2\rangle = |\psi_2\rangle \otimes |\psi_1\rangle$$

$$\begin{aligned}
\mathrm{tr}\{V_2\rho_1 \otimes \rho_2\} &= \mathrm{tr}\left\{V_2 \sum_{ijkl} \rho_{ij}^{(1)} \rho_{kl}^{(2)} |i\rangle\langle j| \otimes |k\rangle\langle l|\right\} \\
&= \mathrm{tr}\left\{\sum_{ijkl} \rho_{ij}^{(1)} \rho_{kl}^{(2)} |k\rangle\langle j| \otimes |i\rangle\langle l|\right\} \\
&= \sum_{ijkl} \rho_{ij}^{(1)} \rho_{kl}^{(2)} \delta_{kj}\delta_{il} = \sum_{ik} \rho_{ik}^{(1)} \rho_{ki}^{(2)} = \mathrm{tr}\{\rho_1\rho_2\}
\end{aligned}$$

A. K. Ekert et al., Phys. Rev. Lett. 88, 217901 (2002).

- The swap operation can be split into symmetric and anti-symmetric subspaces

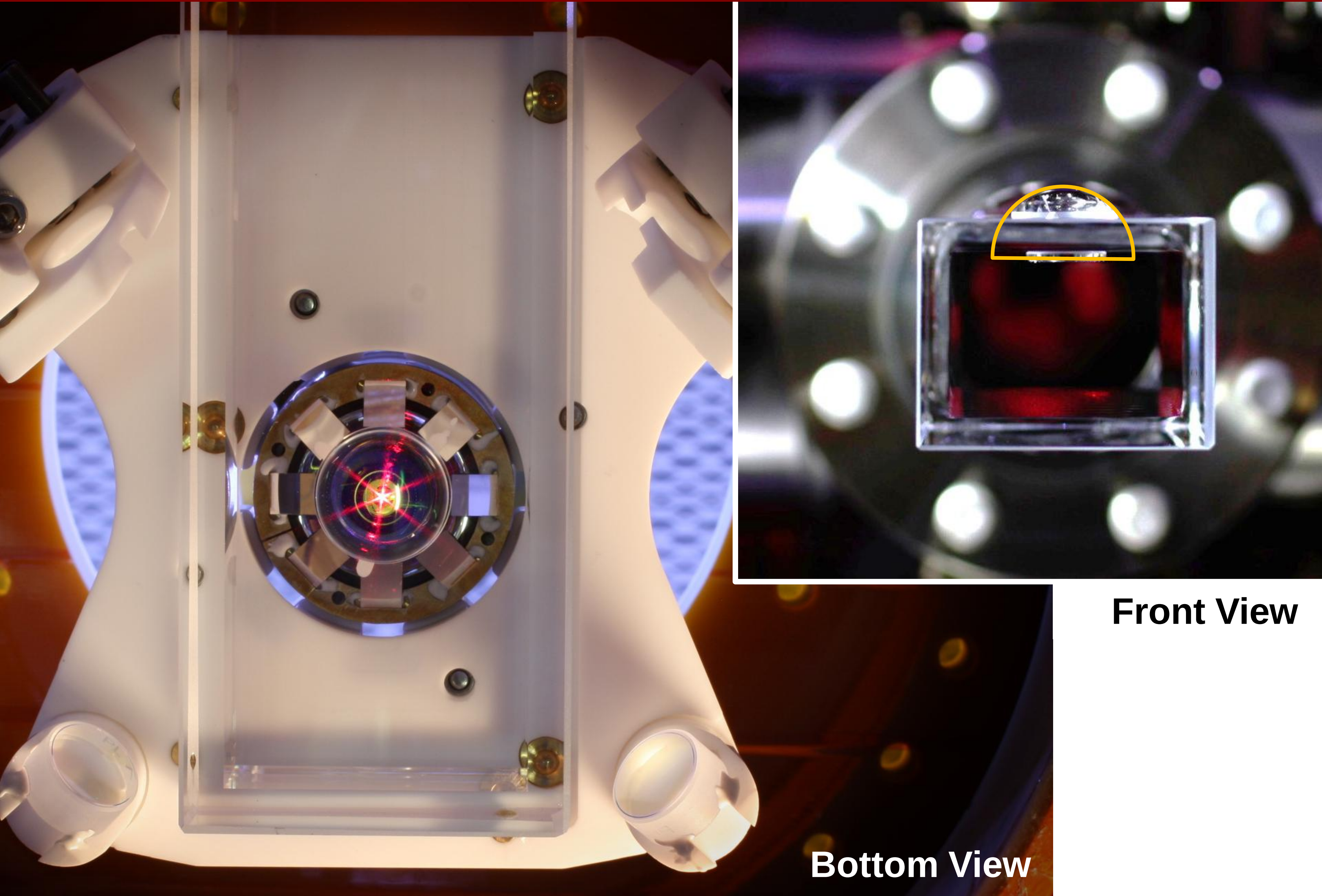
$$\mathrm{Tr}\{\rho^2\} = \mathrm{Tr}\{V\rho \otimes \rho\}$$

$$V = P^{(+)} - P^{(-)}$$

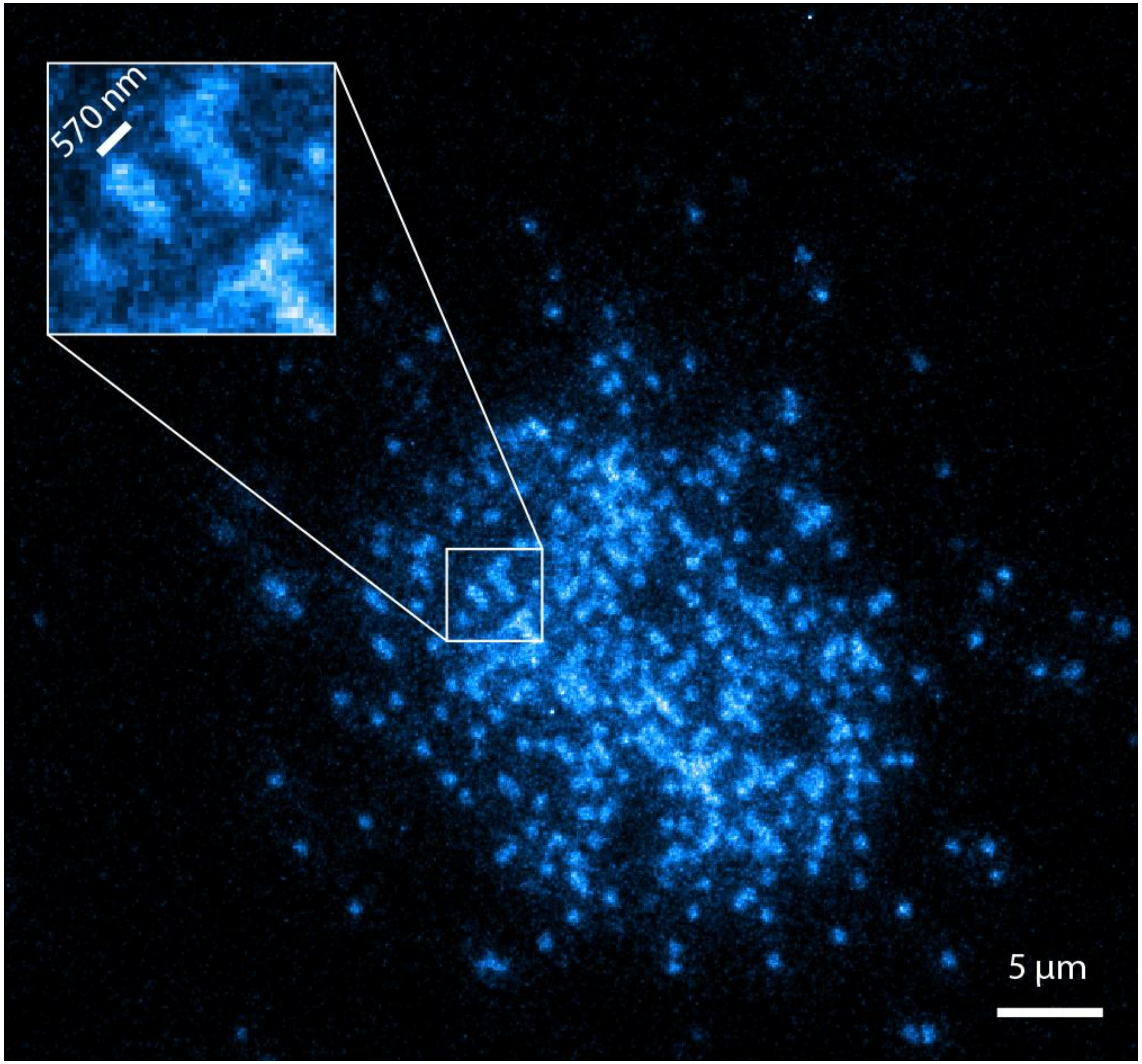
C. Moura Alves et al., Phys. Rev. Lett. 93, 110501 (2004)
F. Mintert et al., Phys. Rev. Lett. 95, 260502 (2005)

- The beamsplitter identifies these subspaces, we we saw on the previous slide
- For identical initial states (and for bosons, where there are no complications with exchange signs between different lattice sites), we then obtain even number of particles after the beamsplitter in each copy.
- For fermions this applies for a single site, but is more subtle with multiple sites.

Fermi quantum gas microscopes

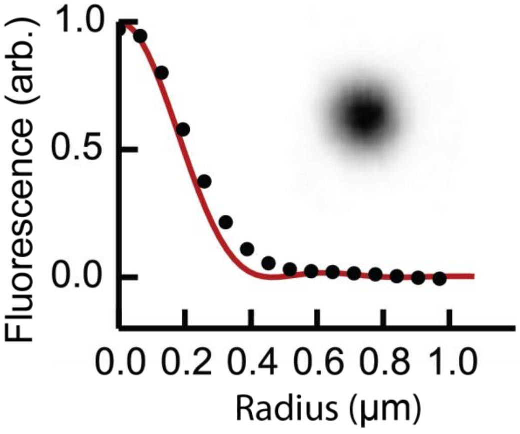


Sample Image

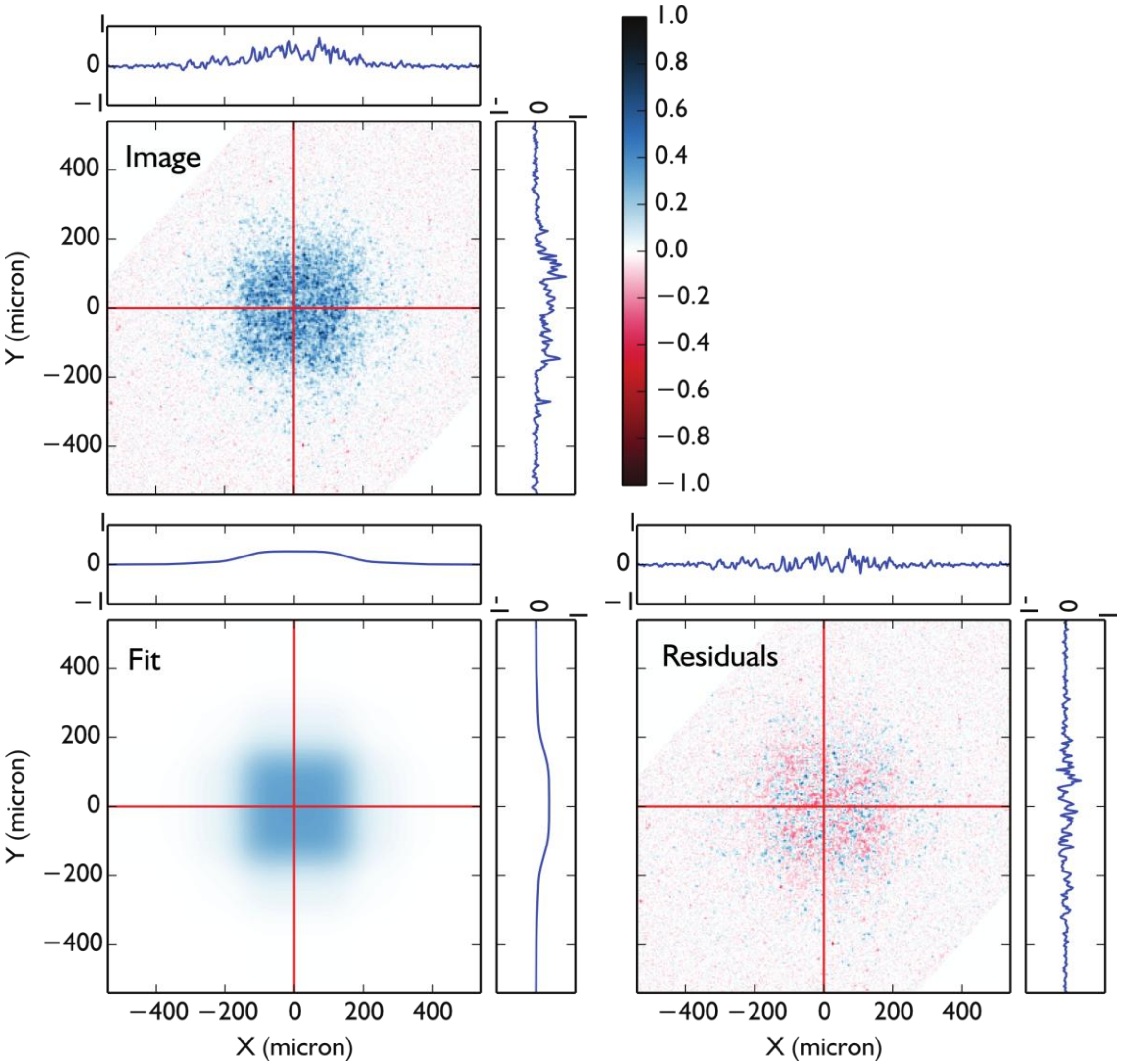


- 44,000 imaging pulses
- Collect ~1000 photons/atom

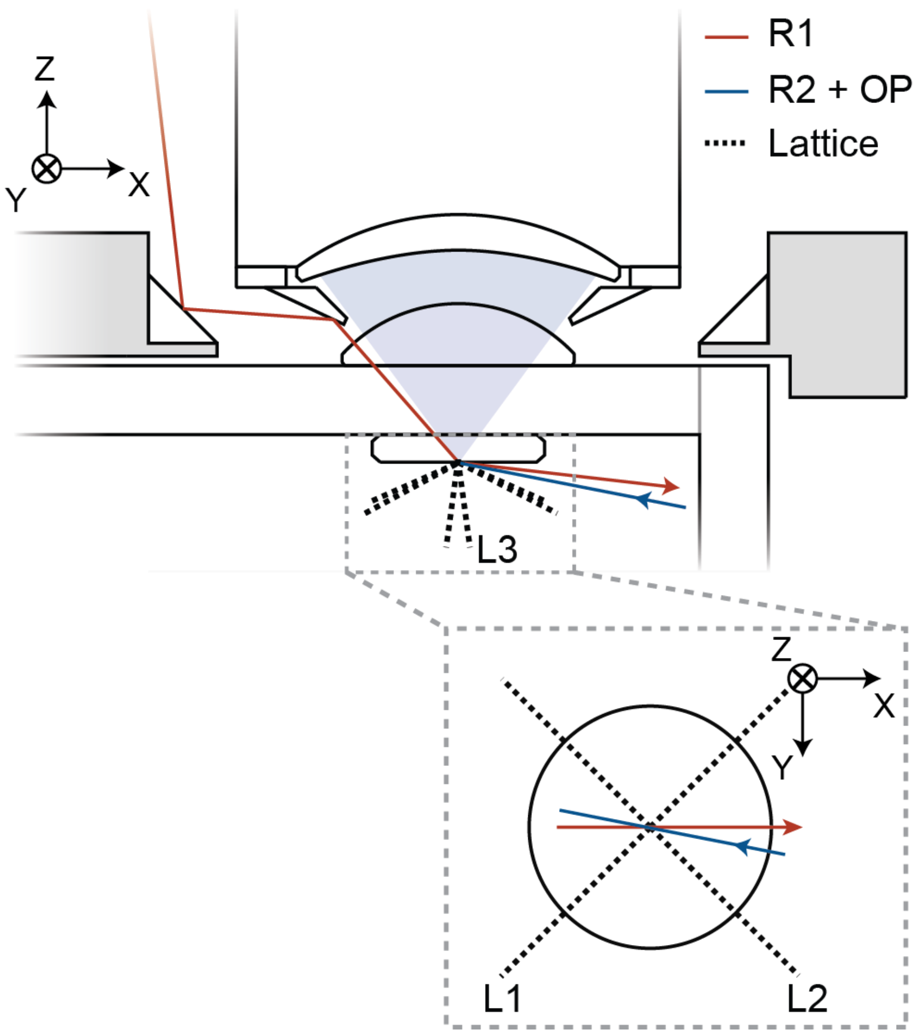
Point-spread Function



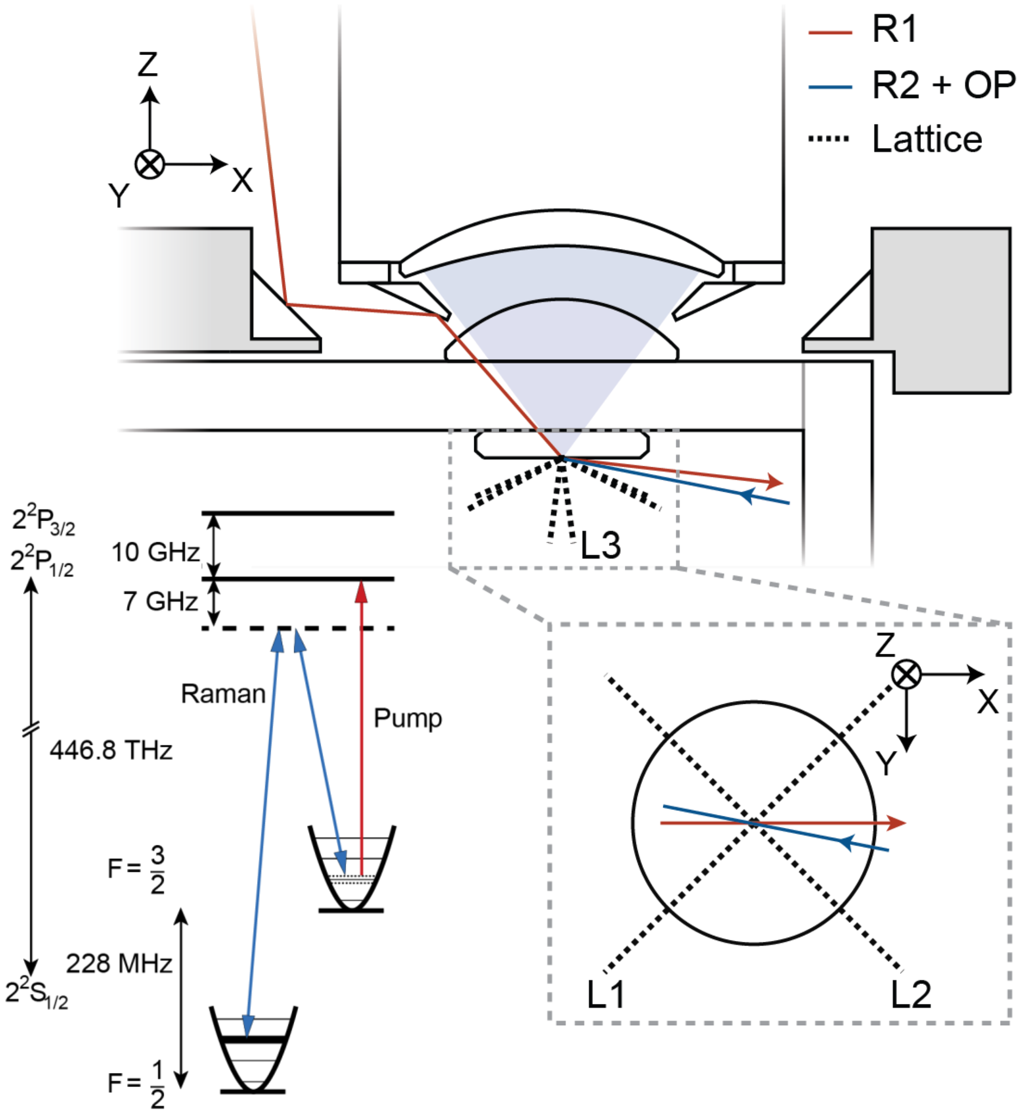
Band mapping with 3000 atoms



Raman Imaging Scheme

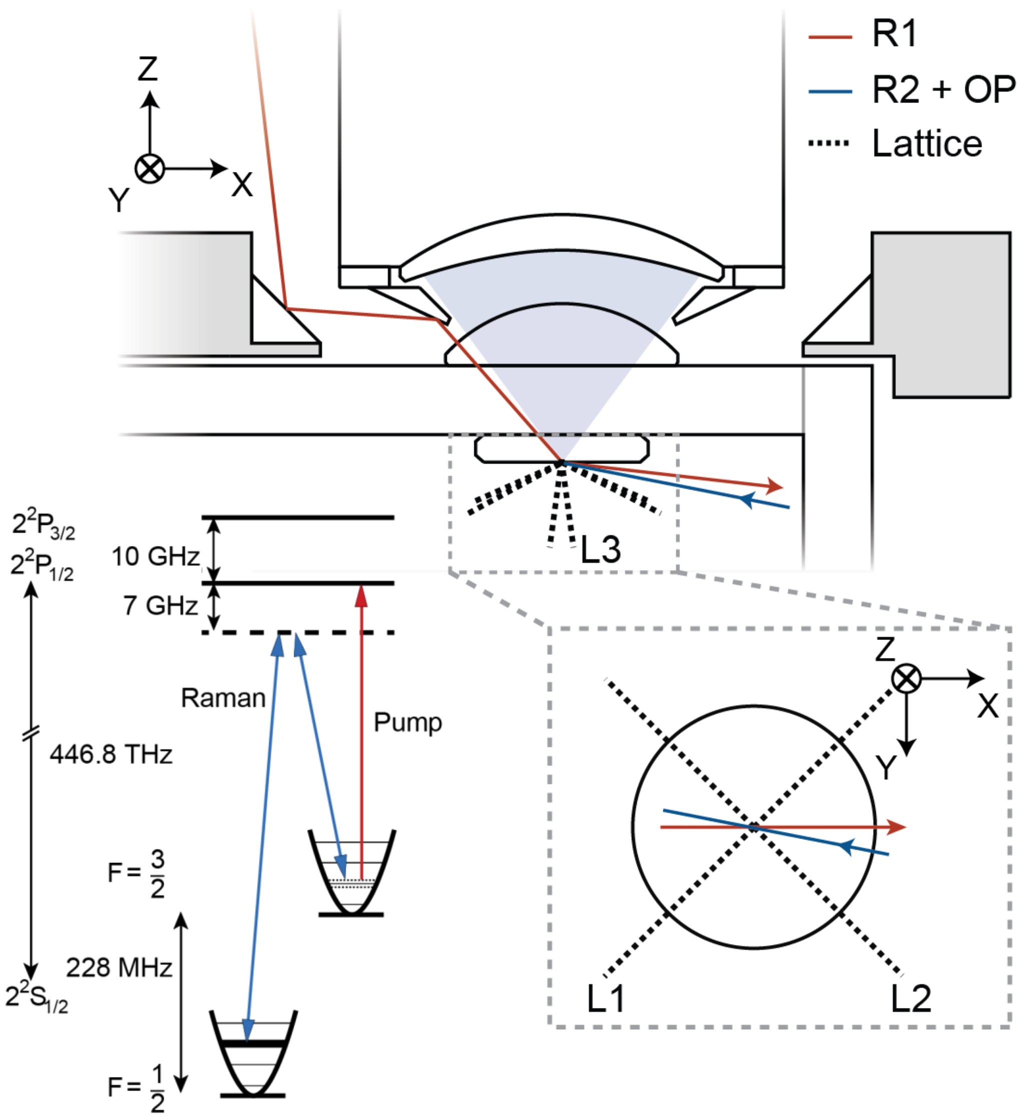


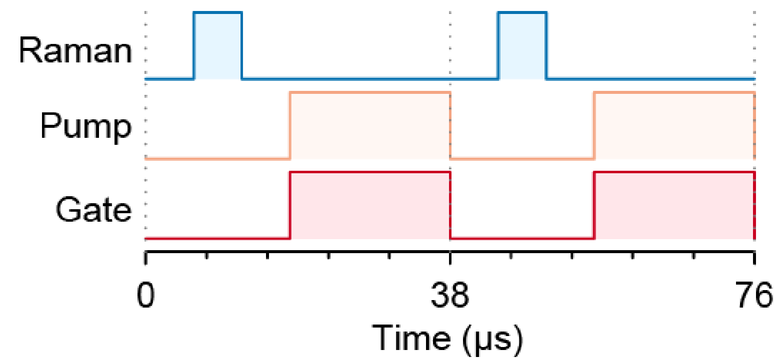
Raman Imaging Scheme



- Single pair of beams: momentum transfer along all axes with degenerate trap frequencies (1.4 MHz)
- Pulsed cooling necessary to eliminate background from Raman light

Raman Imaging Scheme





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