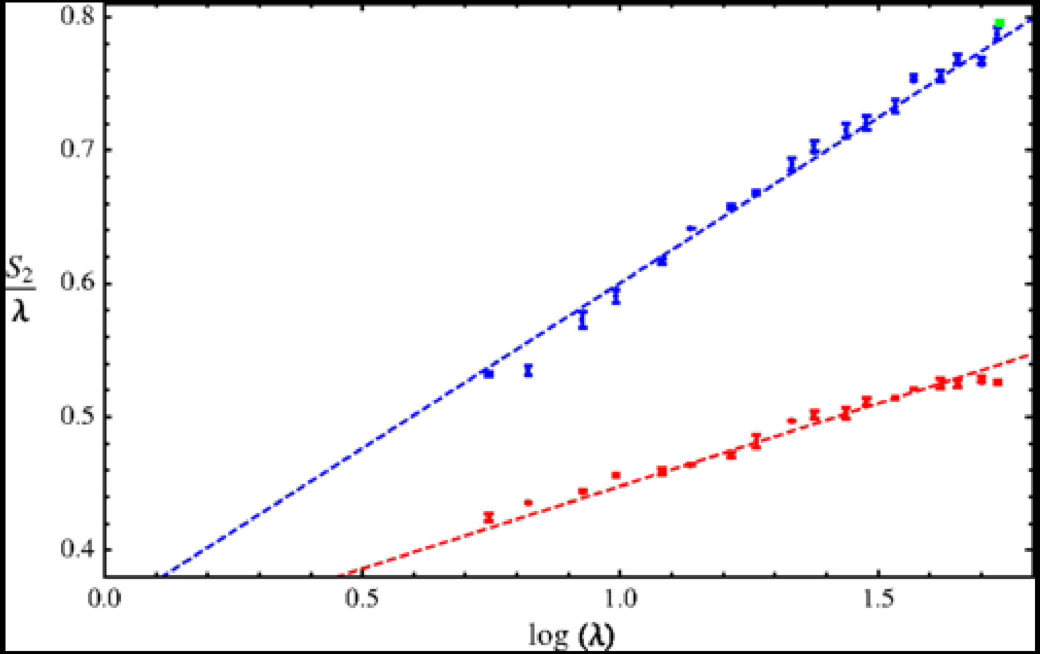
Entanglement Entropy of the Composite Fermion Fluid at $\nu=1/2$



Eun-Ah Kim
Cornell University

Shao, EAK, Haldane, Rezayi, PRL 114, 206402 (2015)

Fermions coupled to Chern-Simons

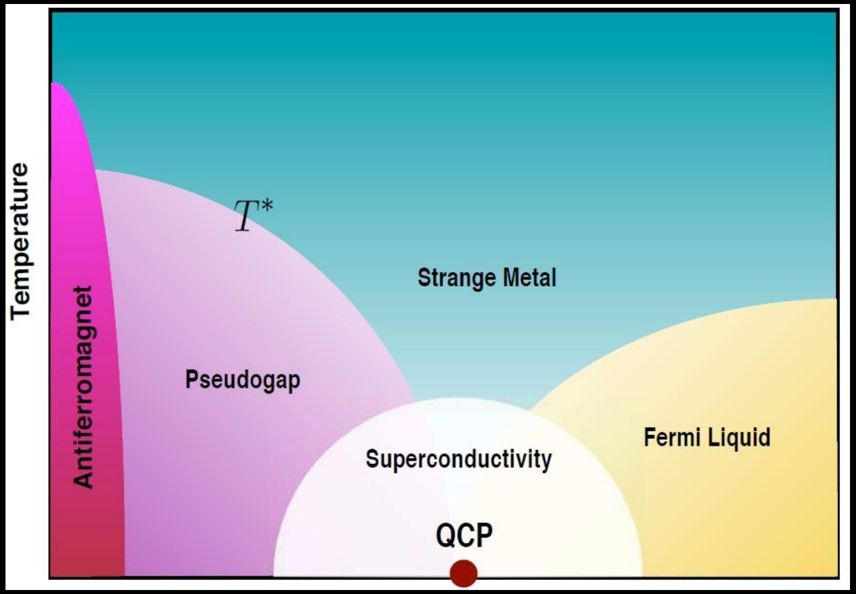
ν=1/2 Fermi Fluid

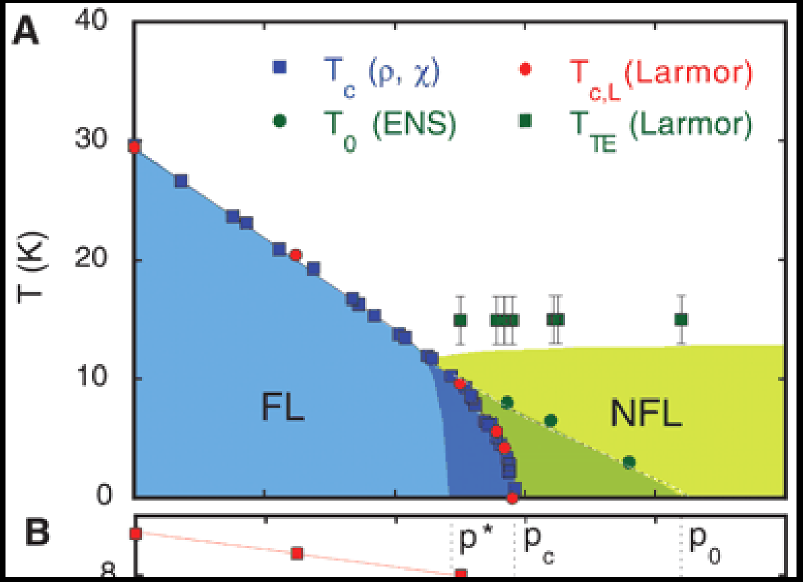
Composite Fermion Wave function

Reny Entanglement Entropy

"Variational" Monte Carlo

Ubiquitous Non-Fermi liquids





.....

Cuperates

MnSi

Heavy fermions etc...

vanishing quasi-particle weight, power-law transport, power-law specfic heat, diverging effective mass...

order parameter &
symmetry breaking

Theoretical Understanding of NFL?

- Holographic constructions
- Diagrammatic studies of Fermions coupled to

  emergent gauge boson...

  quantum critical boson...
- Many-body wave functions

  d-wave bose metal, Jiang et al (2013)
- ν=1/2 Fermi Fluid

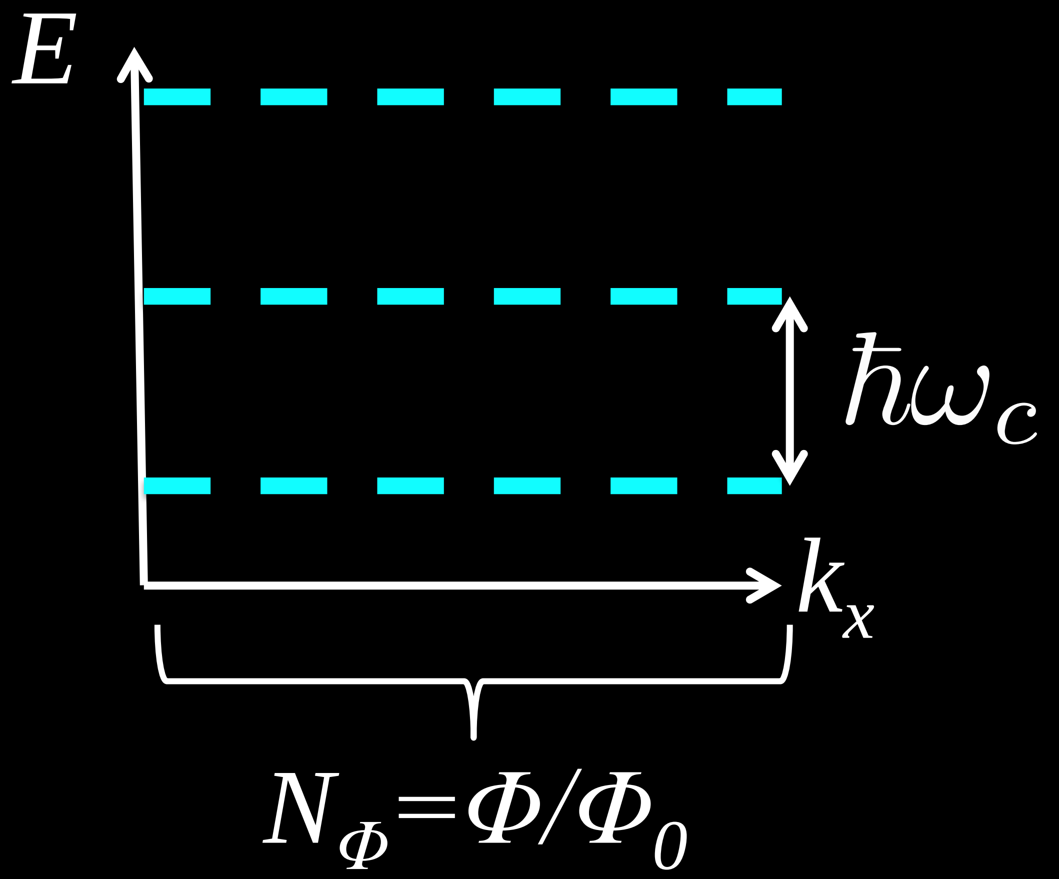
WF, QFT, and experiment!!!

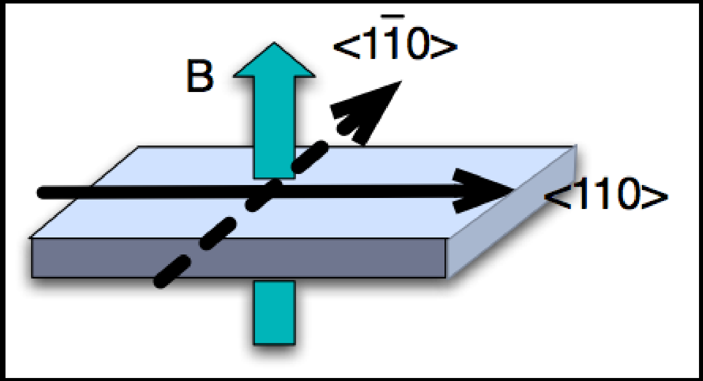
EE of the poster-child NFL: ν=1/2 Composite Fermion Fluid

Larger Context: EE vs Correlation

2D Free Fermions under B

$$H = \frac{1}{2m}(i\vec{\nabla} + \vec{A})^2$$

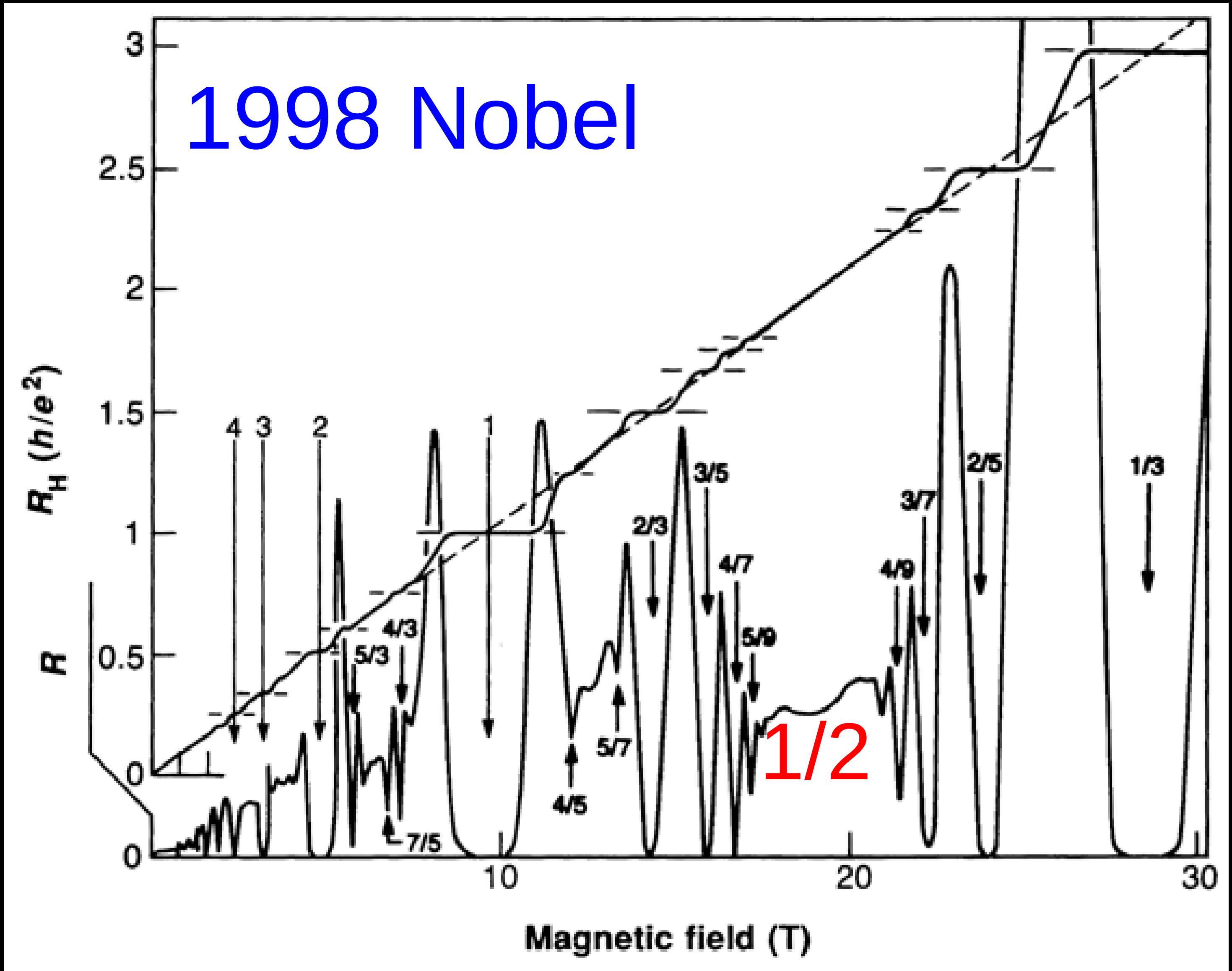




Filling factor:

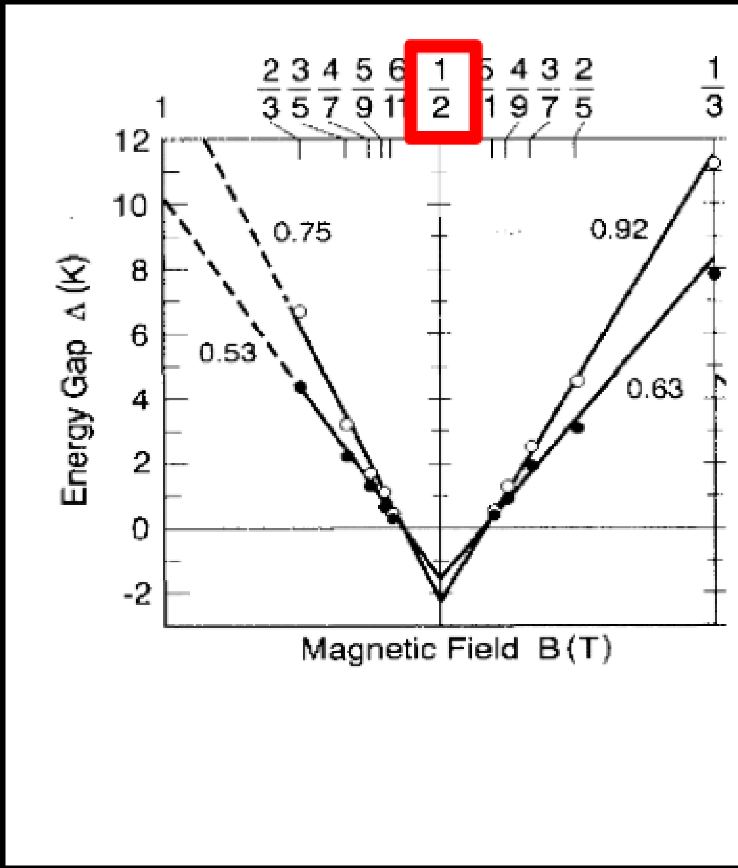
$$\nu = N_e/N_\Phi$$

Interacting Electrons under B



Gapless ν=1/2 Phenomenology

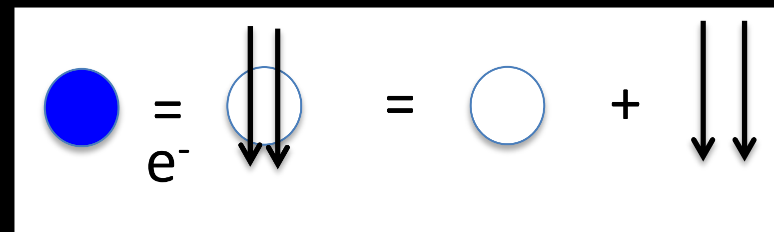
- "Mother" of FQH



Susceptible to pairing and nematic

Composite Fermion

- Flux attachment

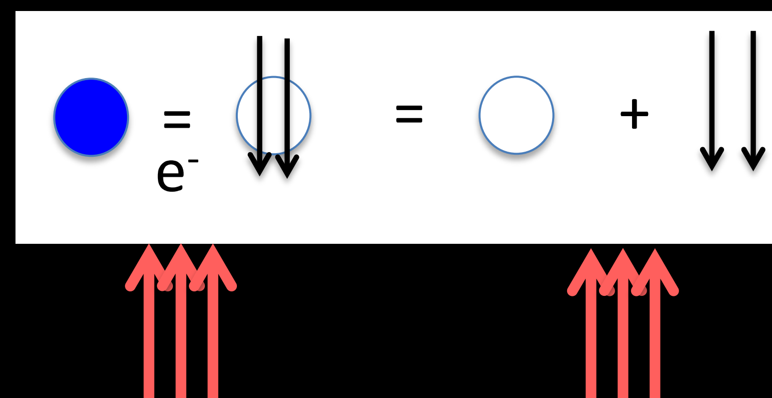


Composite Fermion

- FQH of e = IQH of ψ

(Jain, 1989)

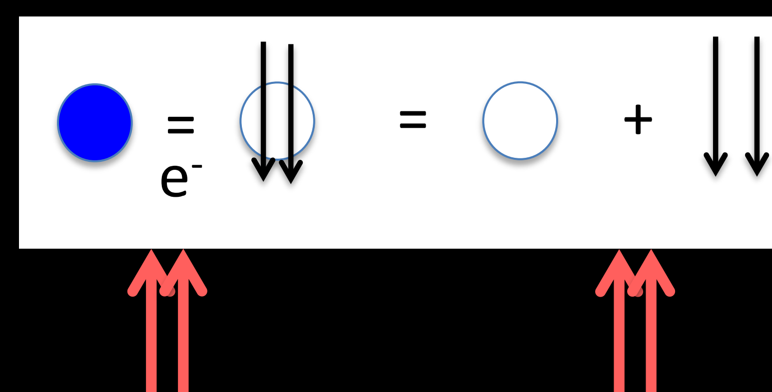
$\nu=1/3$



$\nu_{eff}=1$

- $\nu=1/2$ of e = ψ without field

(Halperin, Lee, Read, 1993)



Chern Simons - Fermion

- Field theoretic flux attachment (HLR 1993)

$$\mathcal{L}_{\mathrm{cf}} = \frac{1}{2}\frac{1}{4\pi} a\partial a +$$

$$a\partial a \equiv \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \qquad \psi(\vec{r}) = c(\vec{r}) e^{-i\Phi \int d\vec{r}' \arg(\vec{r}-\vec{r}')\rho(\vec{r}')}$$

constraint: $$\frac{\delta \mathcal{L}_{\mathrm{cf}}}{\delta a_t} = 0 \longrightarrow \frac{1}{4\pi}\epsilon_{ij}\partial_i a_j + \;^{\dagger} = 0$$

mean-field: $$\langle a \rangle = -\langle A^E \rangle$$

Composite Fermion WF

$$\det_{ij} e^{i\boldsymbol{k}_i\cdot\boldsymbol{R}_j}|\Psi_L^{1/2}\rangle = \det_{ij} t_i(\boldsymbol{d}_j)|\Psi_L^{1/2}\rangle$$

non-commutative guiding center within LL

translation by $d^a = \epsilon^{ab}k_b l_B^2$

Bosonic Laughlin

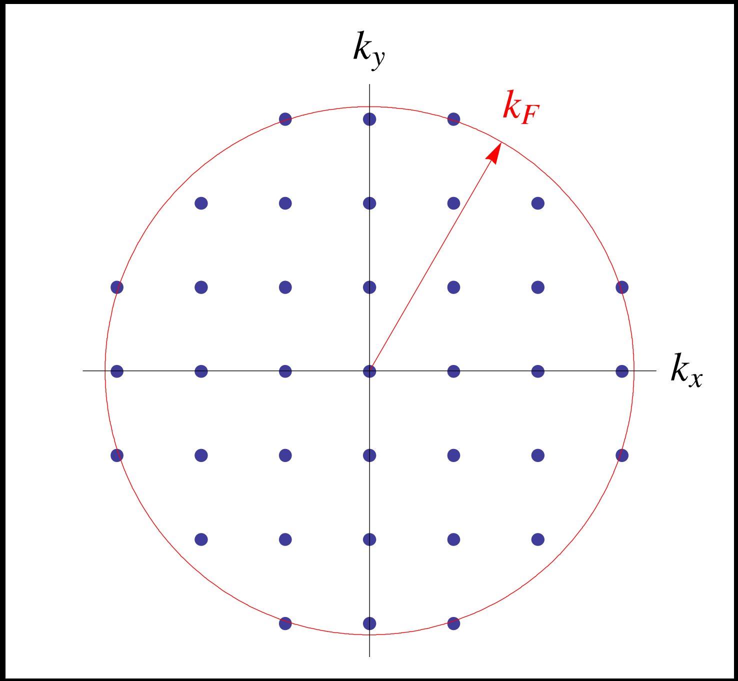
$(z_i-z_j)^2$ with periodic boundary

: elliptic functions

Rezayi, Read (1994), Haldane, Rezayi (1985), Rezayi, Haldane (2000)

Filling the Fermi Sea

- Challenge: choosing { $\boldsymbol{k_i}$ } in the absence of kinetic energy, i.e. flat band
  - Minimize $$H = \frac{\hbar^2}{2mN} \sum_{i<j} |\mathbf{k}_i - \mathbf{k}_j|^2$$
- For total K=0, { $\boldsymbol{k_i}$ } minimizes usual K.E.
- N=37, 137,...

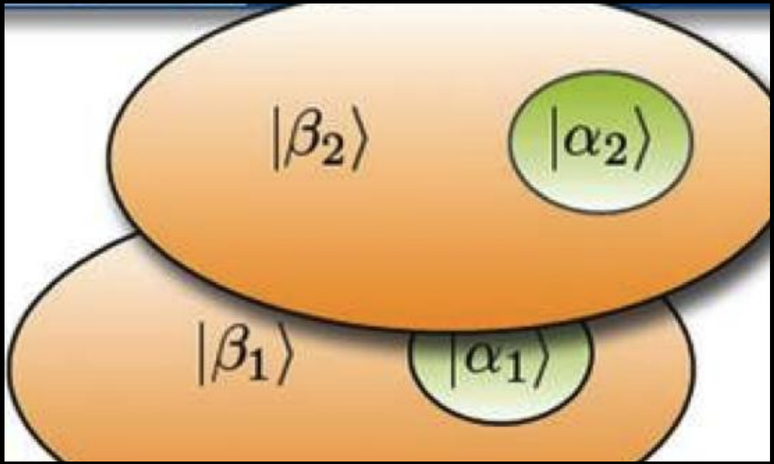


$$S_2(\rho_A) = -\ln(\mathrm{Tr}(\rho_A^2))$$ from <SWAP$_A$>

Hastings, Gonzales, Kallin, Melko (2010)

$$\langle \Psi_0 \otimes \Psi_0 | Swap_A | \Psi_0 \otimes \Psi_0 \rangle$$

$$= \sum_{\alpha_1, \alpha_2} \langle \alpha_1 | \rho_A | \alpha_2 \rangle \langle \alpha_2 | \rho_A | \alpha_1 \rangle = \mathrm{Tr}(\rho_A^2)$$



- Converngence issues

➢ Factorize <SWAP$_A$>: sign trick (Zhang, Grover, Vishwanath, 2011)

particle number trick (Shao, EAK, Haldane, REzayi, 2015)

Lauchli (2013), McMinis (2013)


INTERMISSION

Logarithmic enhancement in Fermi Gas

- Consistent with Widom conjecture

$$S_\alpha = \frac{1+\alpha}{24\alpha} c(\mu) L_A^{d-1} \ln L_A$$

Widom (1982), Gioev and Klich (2006), Leschke, Sobolev, Spitzer (2015)

- Purely geometric coefficient

$$c(\mu) = (2\pi)^{1-d} \int_{\partial\Omega} dS_x \int_{\partial\Gamma} dS_k |\mathbf{n}_k \cdot \mathbf{n}_x|$$

Boundary of subregion A ($\partial\Omega$)   Fermi surface ($\partial\Gamma$)

Insight into the logarithmic enhancement

Swingle (2010)

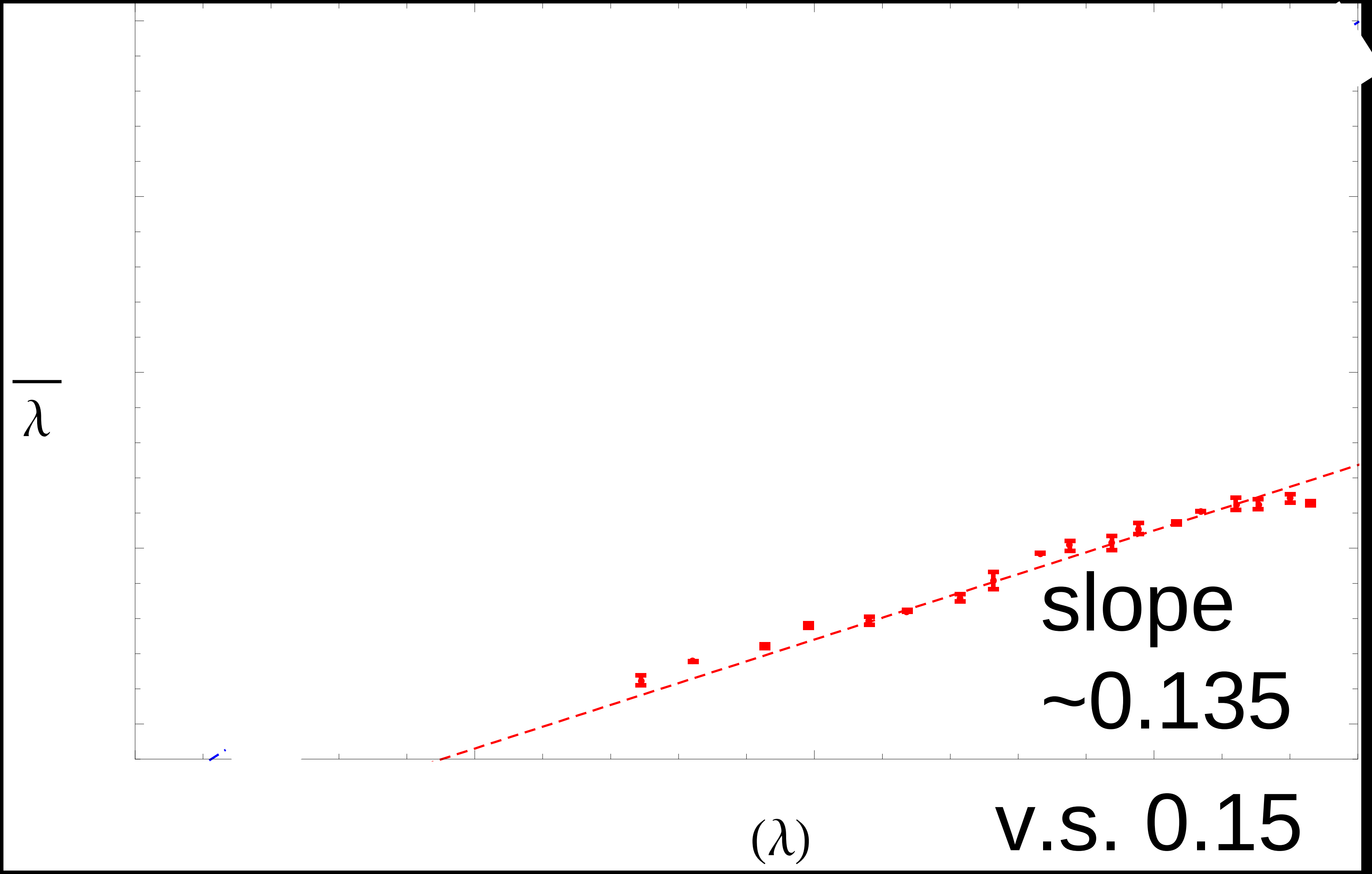
- EE of 1D CFT: $\frac{(c_R + c_L)}{6} \log(L/\epsilon)$

Calabrese & Cardy (2004), Holzhey et al (1994)

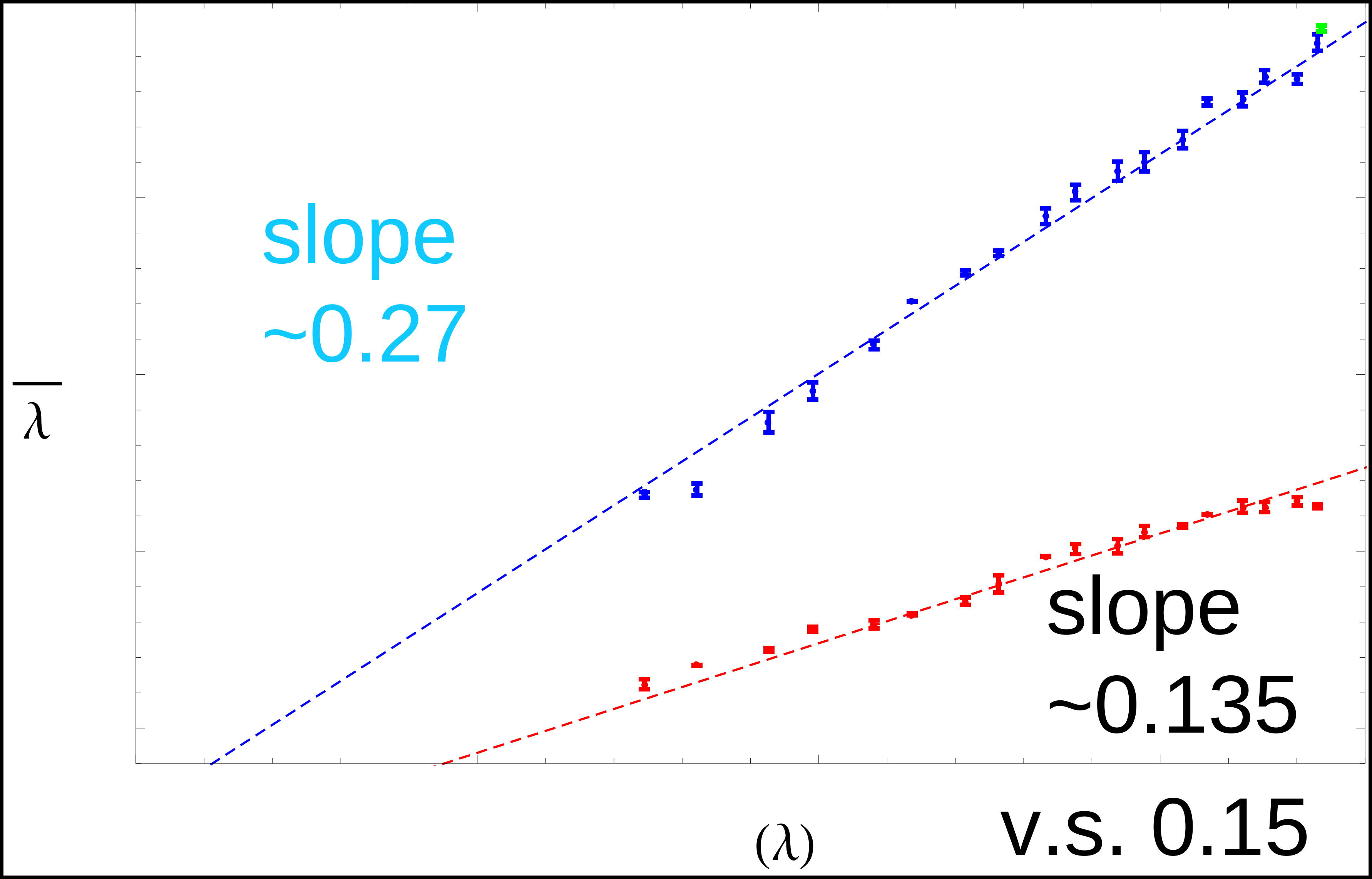
- 2D free Fermion:
  - patches of 1D chiral fermion
  - Number of patches ~ L/a:
    density of modes L, length of FS 1/a

End of

Result I: Free Fermion



Result II: Composite Fermion



Some Previous Results

- Critical spin-liquid (Gutzwiller projected Fermi surface): $S_2 = S_{2,\text{ free}}$ [Zhang, Grover, Vishwanath, (2011)]
- Fermi liquid via 2D bosonization: S_2=$S_{2,\text{ free}}$
  [Ding, Seidel, Yang (2012)]
- Slater-Jastrow wave functions: slight enhancement
  [McMinis & Tubman (2013)]
- Scaling conjecture for some 2d NFL: $S_2 \propto L \log L$
  [Swingle & Senthil (2013)]

Summary and Questions

- Numerical evaluation of the scaling of S_2

Free fermion

$$S_2 = 0.135\ \lambda \log \lambda$$

Composite fermion

$$S_2 = 0.27\ \lambda \log \lambda$$

- 2= 2x1?, 2=1+1?, 2=???
- General conditions for correlations to affect Entanglement?

Speculations

- Could it be simply the $(z_i - z_j)^2$?
  - No
- Could it be the over-damped collective mode?

1-loop gauge fluctuation effects:

$S(T) \sim T \log T$ with Coulomb interaction...

Halperin, Lee, Read (1993)

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