ER=EPR but entanglement is not enough

What's makes quantum information so different?

Two things:

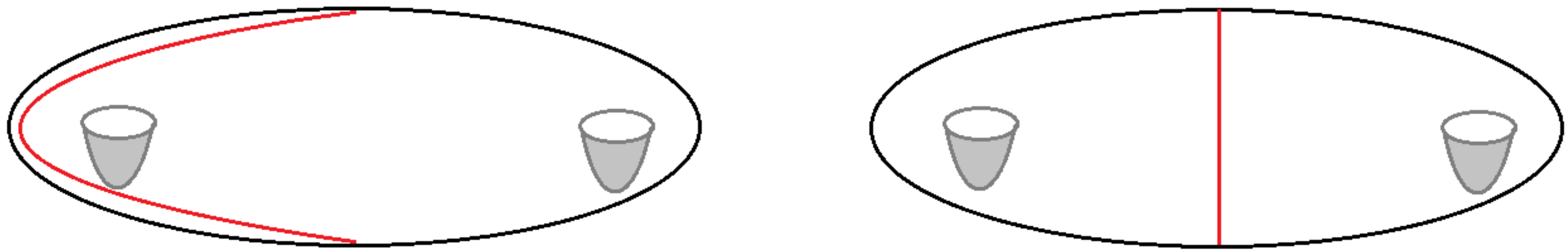
Entanglement: you can know everything about a system and know nothing about its parts. EPR

The capacity for exponentially large complexity. Feynman

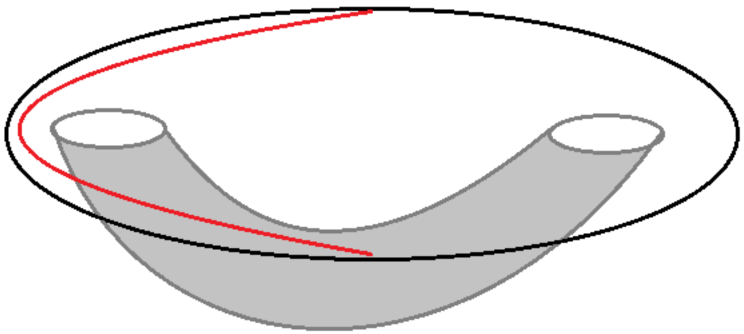
Leonard Susskind, Stanford

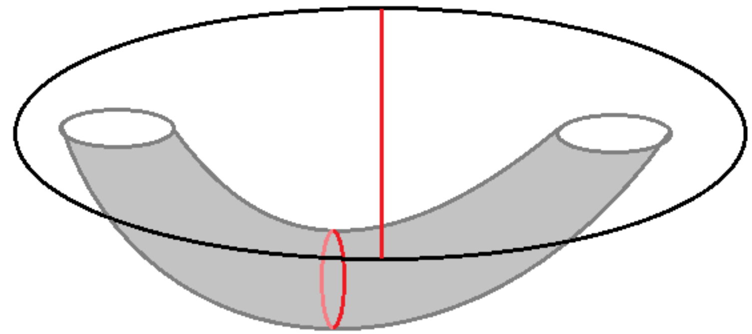
KITP Entangled15 Conference, Jun 4, 2015

Ryu-Takayanagi construction



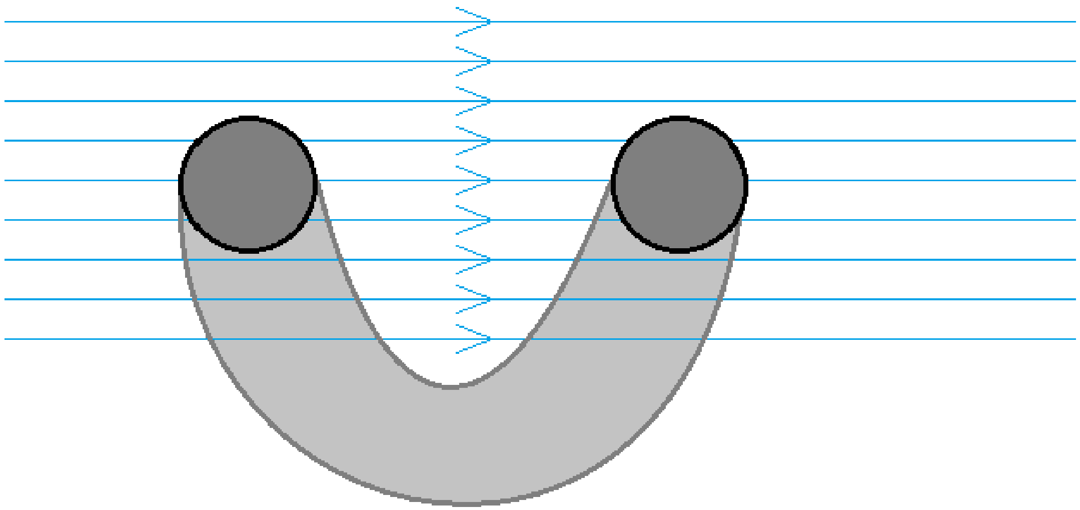
But what if the black holes are entangled?



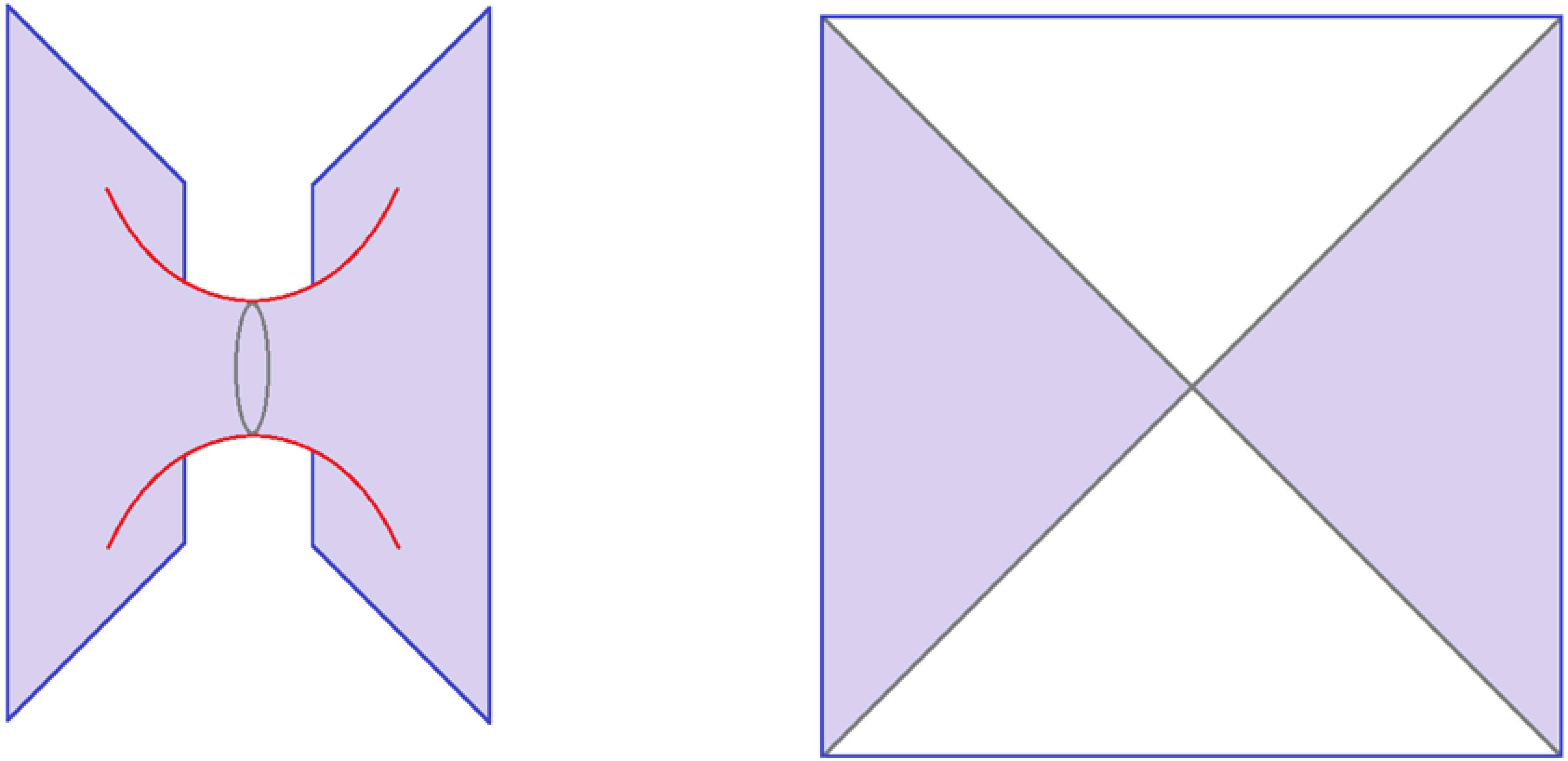


How can entangled black holes be created?

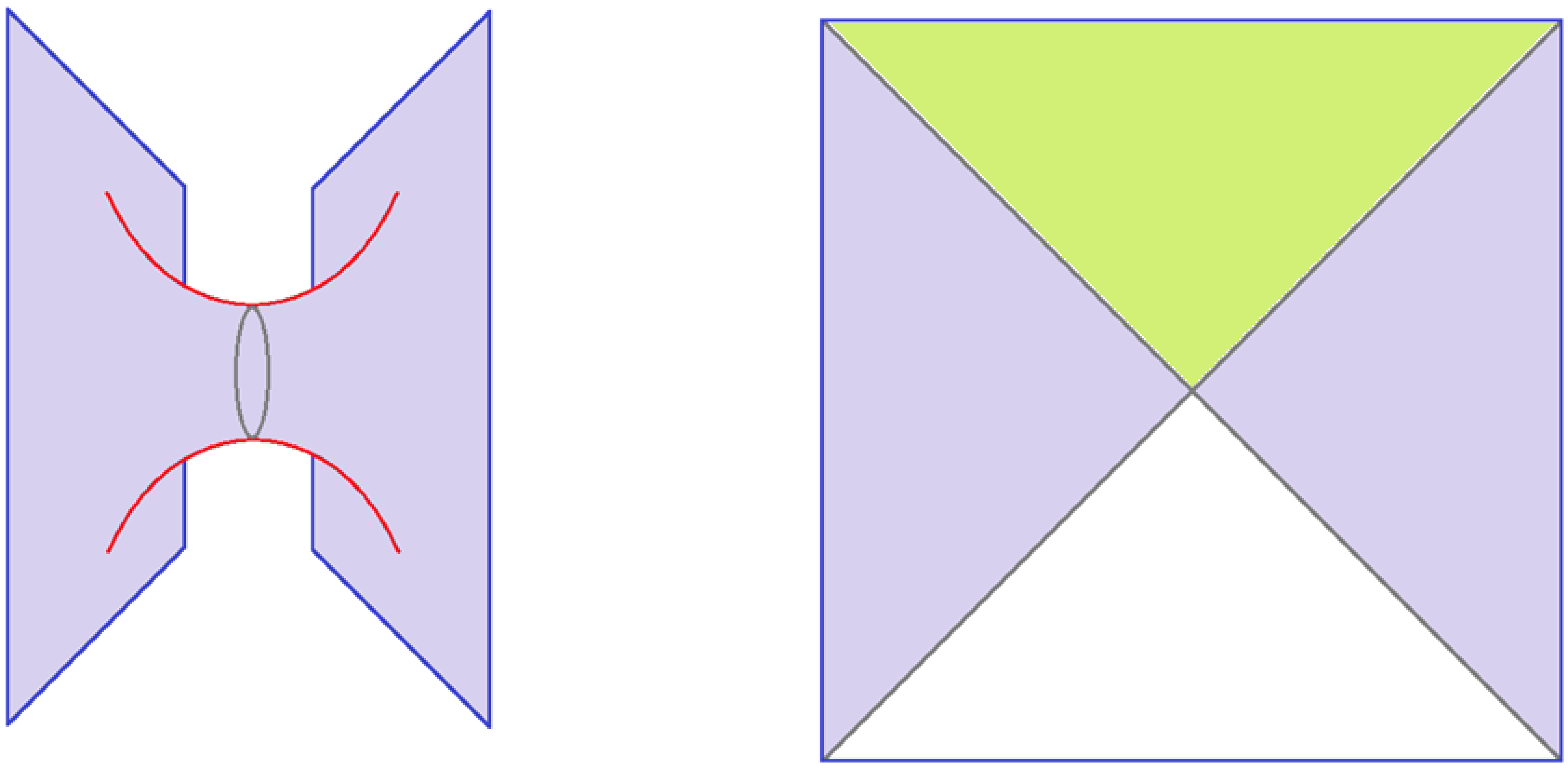
Pair creation in an electric field



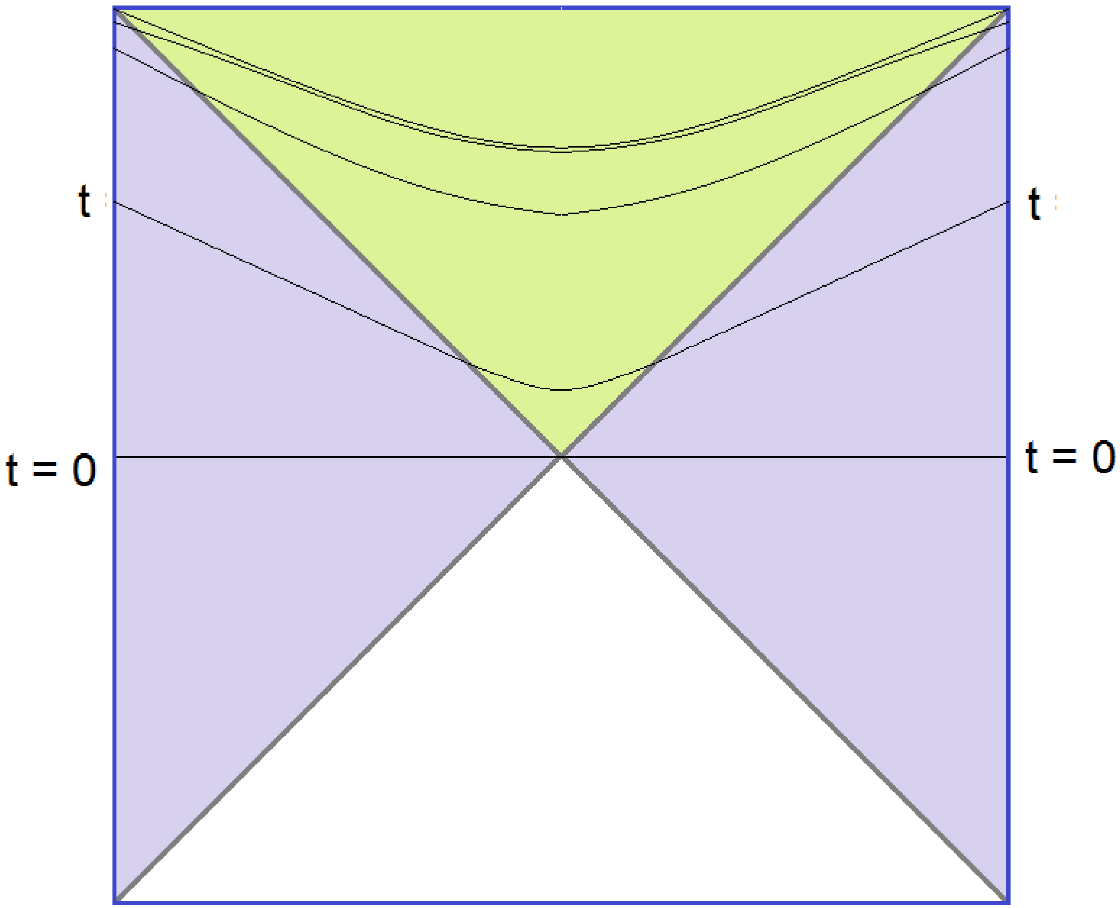
Collapse of entangled clouds

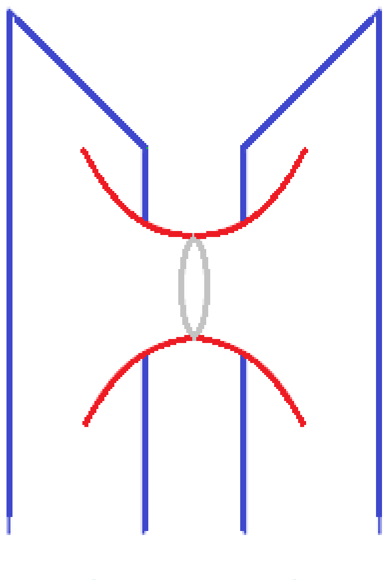


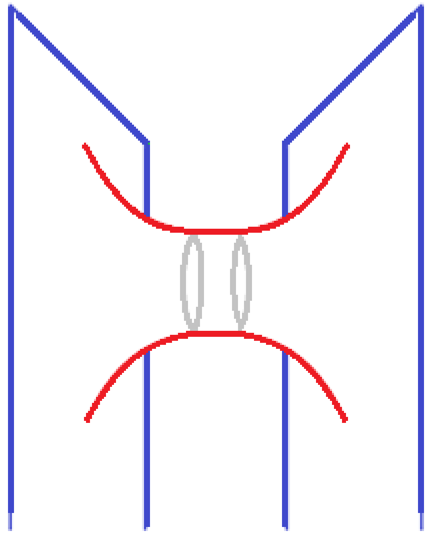
ER = EPR

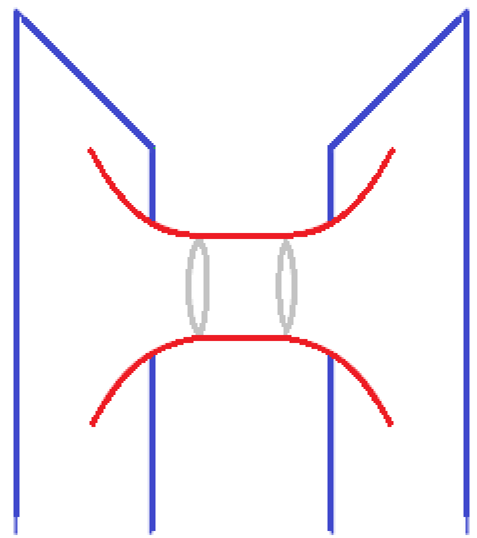


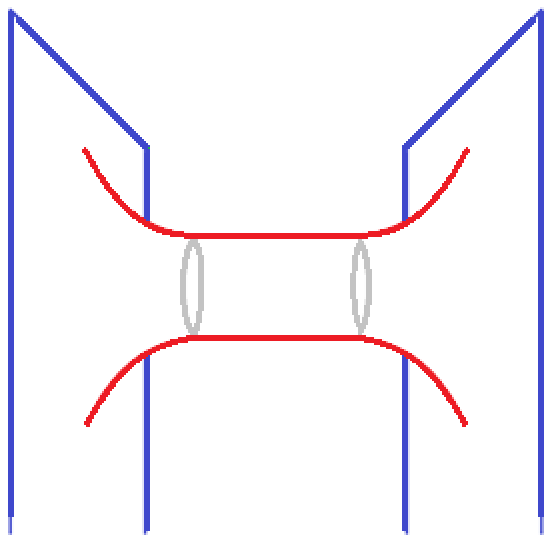
ER = EPR

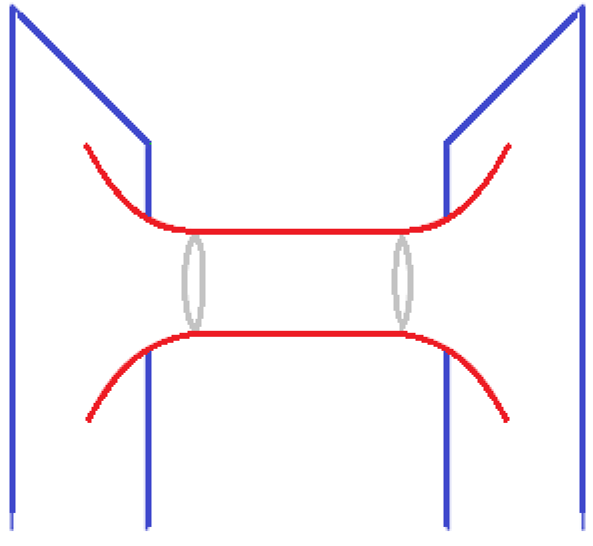

t
t
t = 0
t = 0

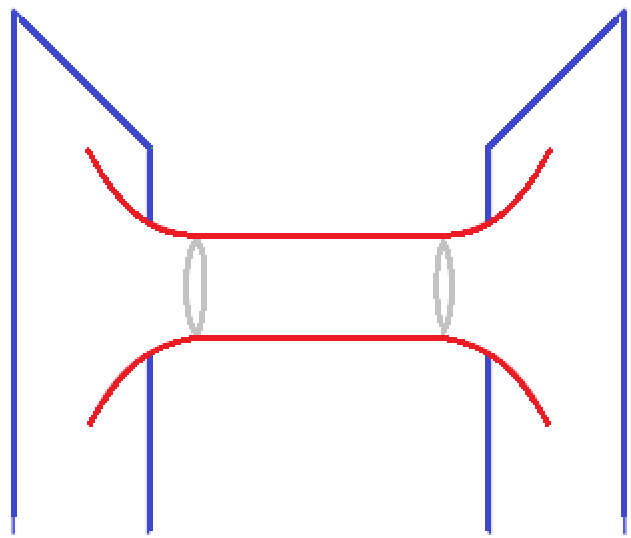


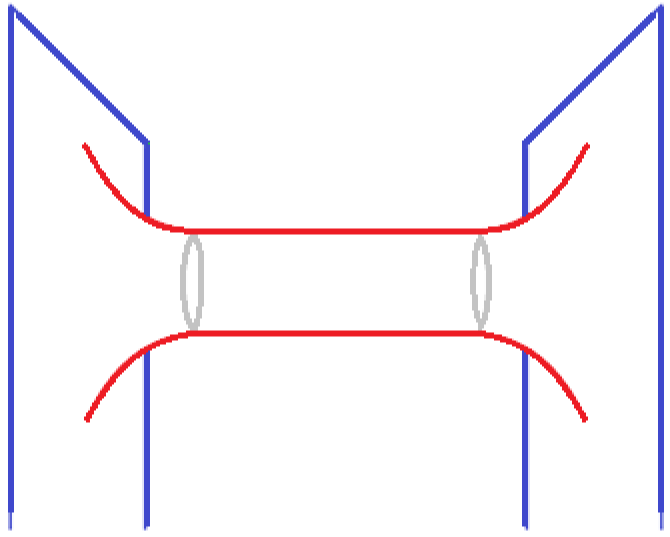


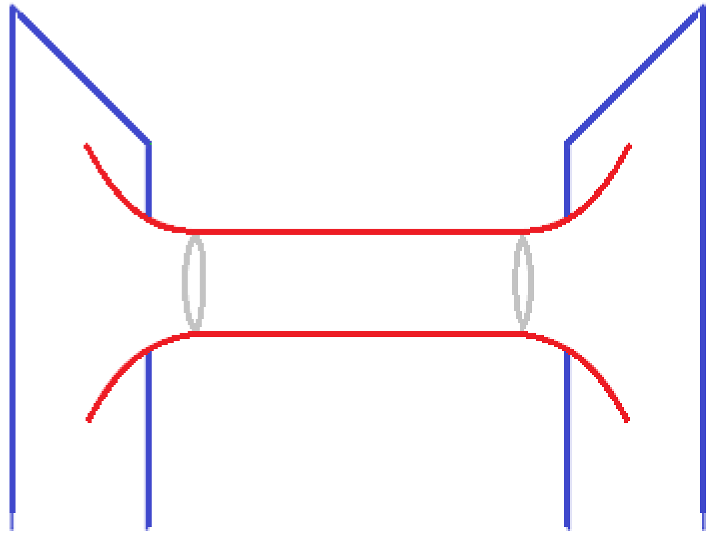


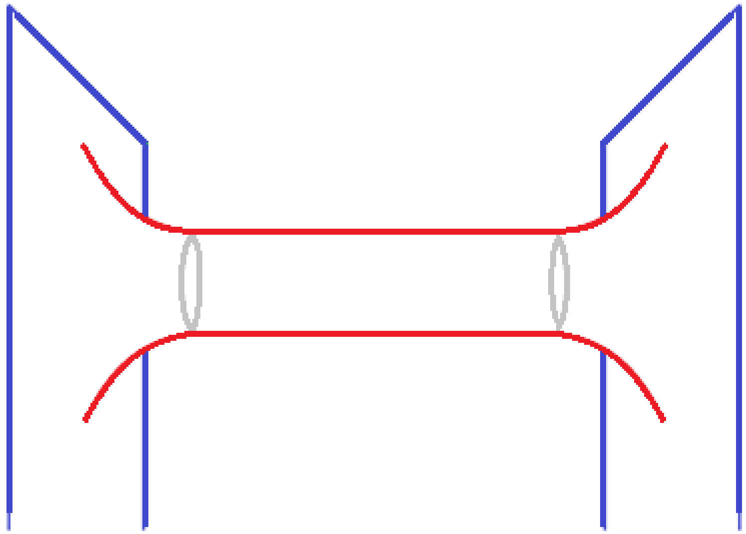






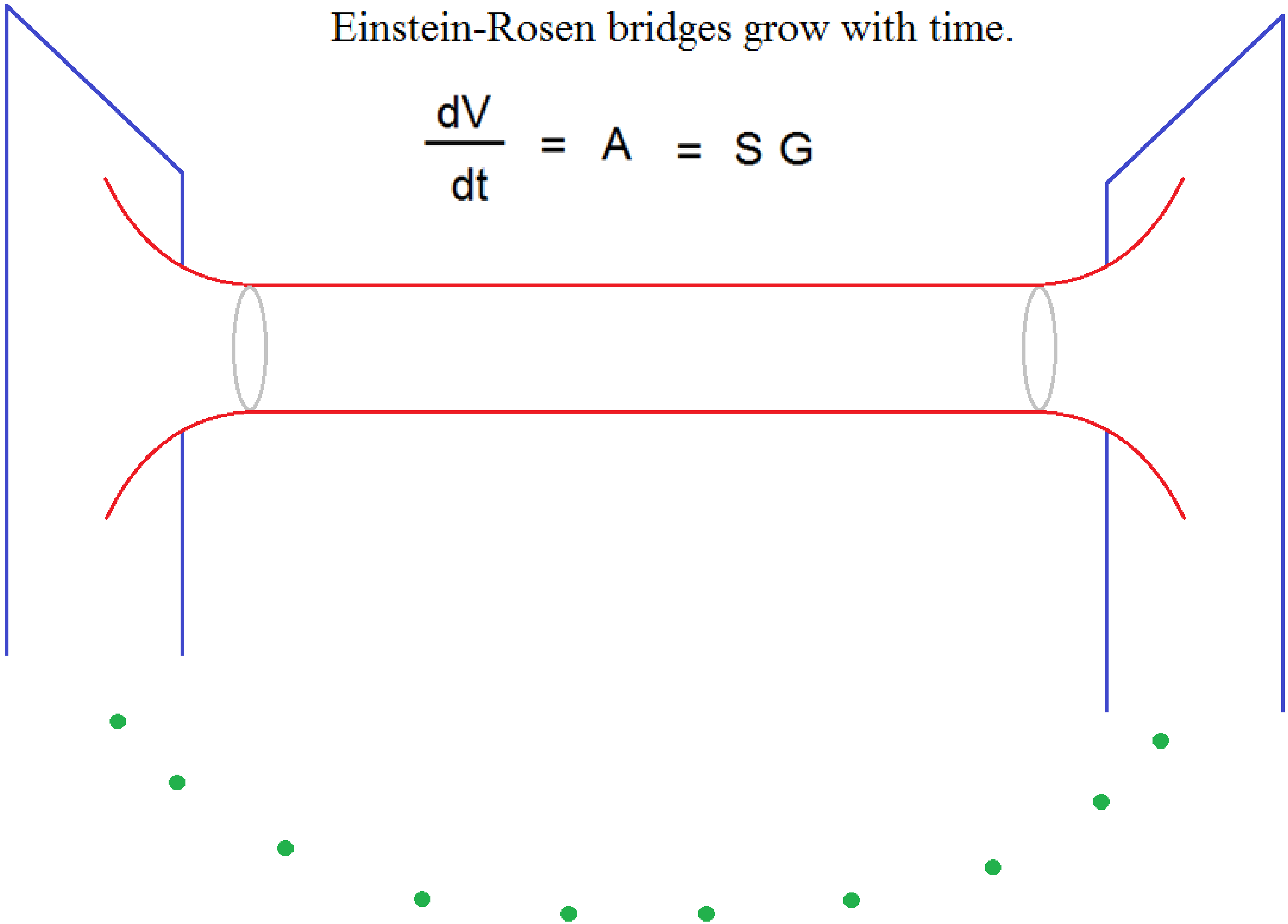






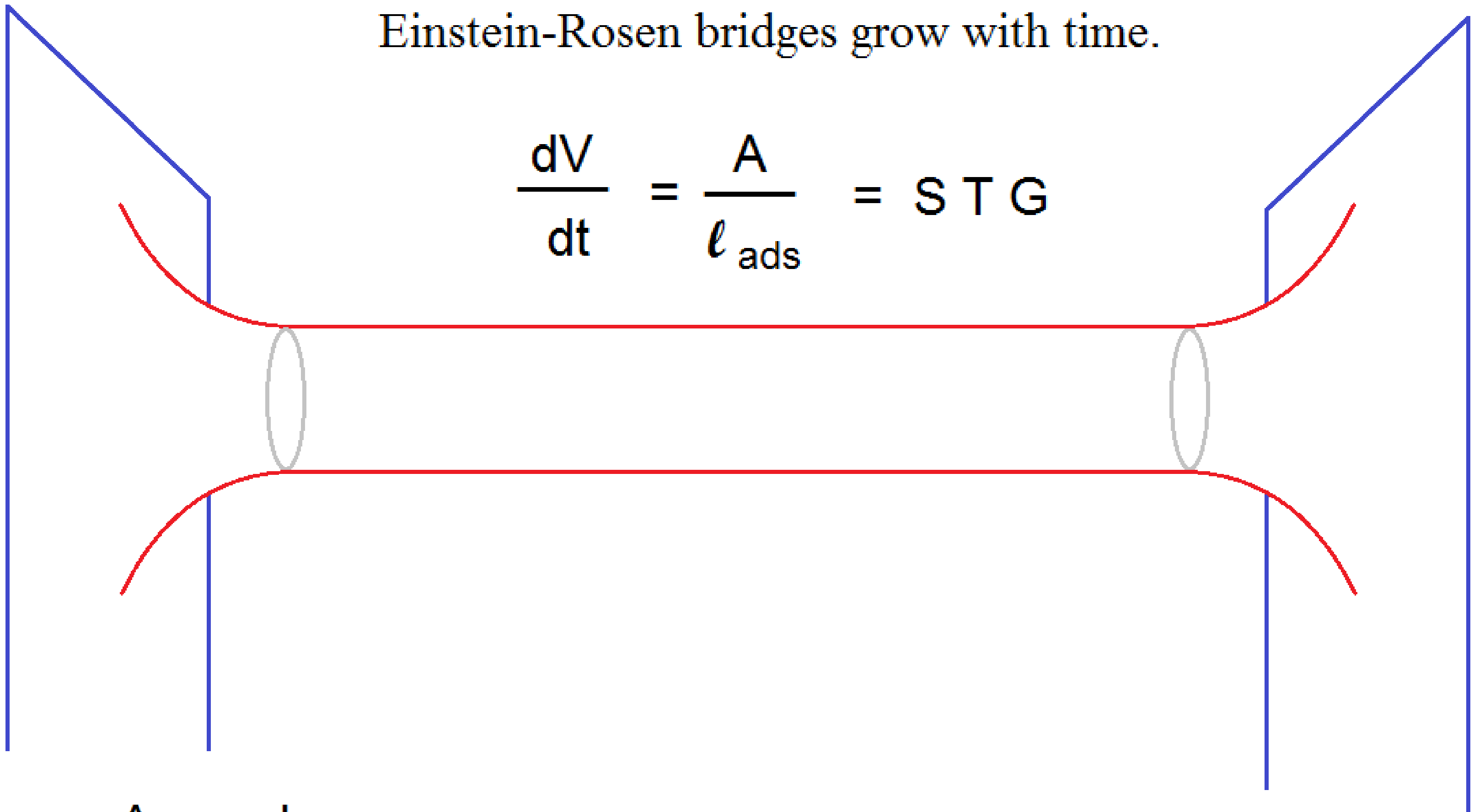
Einstein-Rosen bridges grow with time.

$$\frac{dV}{dt} = A = S\,G$$



Einstein-Rosen bridges grow with time.

$$\frac{dV}{dt} = \frac{A}{\ell_{ads}} = S\,T\,G$$



A paradox:

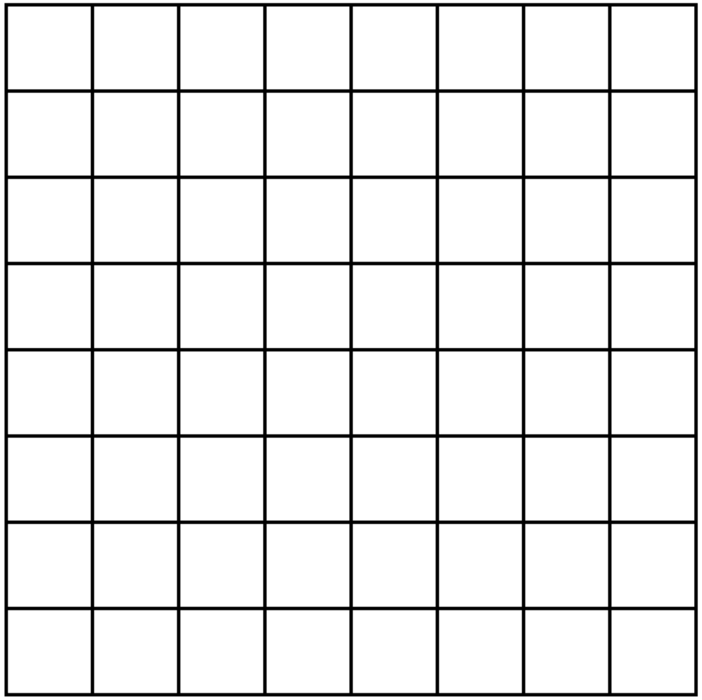
Something is growing but the black holes are in perfect thermal equilibrium? What quantity in the holographic dual continues to grow long after equilbrium?

Computational Complexity

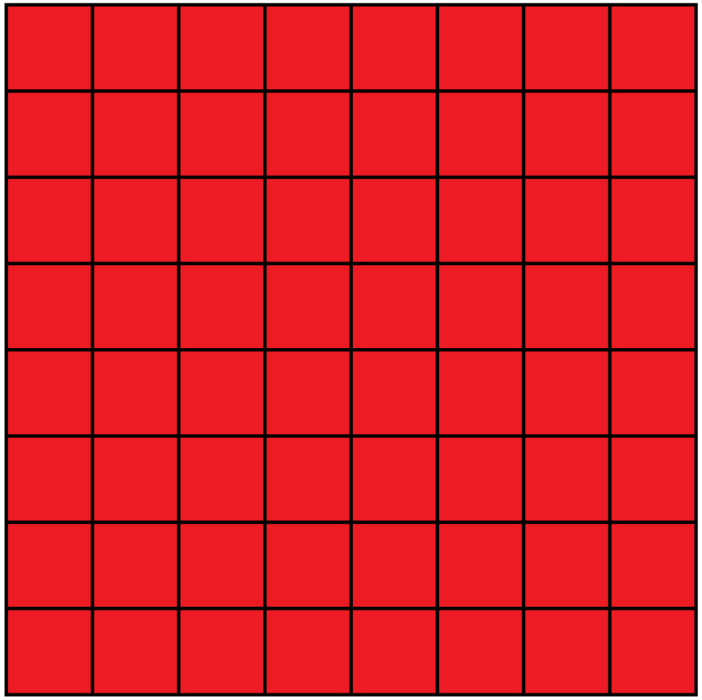
A classical example

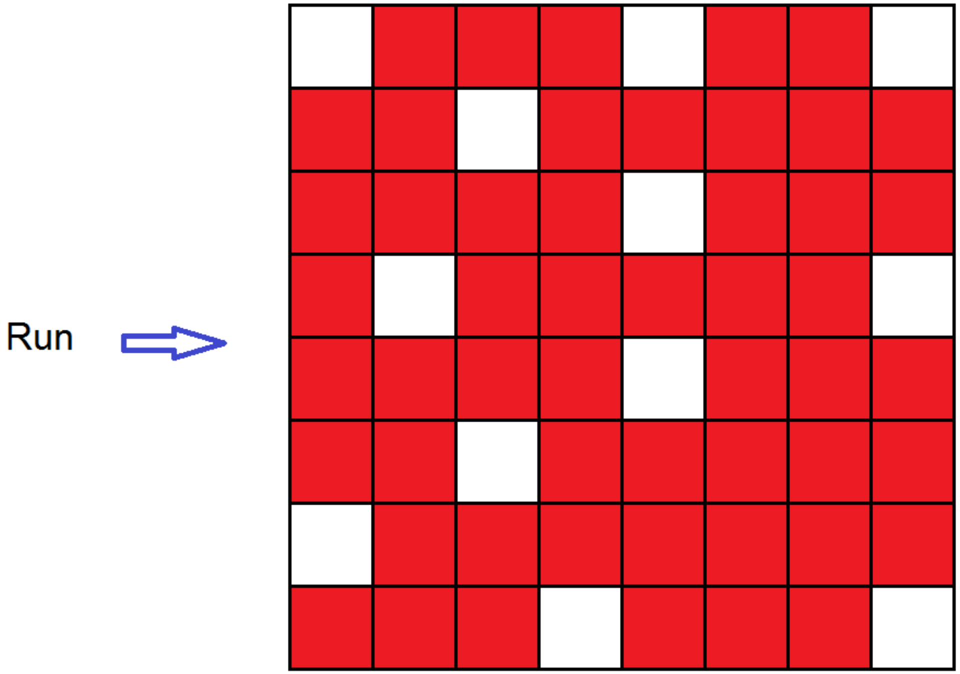
Cellular Automaton

Simple state

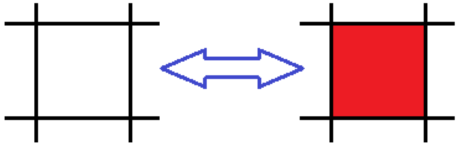


Simple state





Simple process



Complexity = <u>minimum</u> number of simple steps needed to get to configuration.

Not generally number taken by the CA.

$$C_{max} = N/2$$

$$S_{max} = N \log 2$$

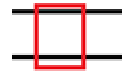
Classical

(0010111001001010.......01) N binary digits

Quantum

Basis $|\, i \rangle = |0010111001001010.......01\rangle$

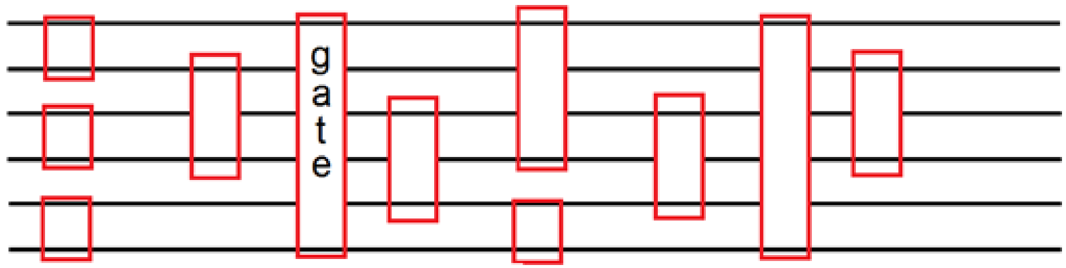
General state $|\Psi\rangle = \sum_{i=1}^{2^N} f(i)\, |\, i \rangle$ 2^N complex numbers simple state $|000......0\rangle$

simple process  gate

Cellular automaton $\longrightarrow$ quantum circuit

Simple state

$|000.......0\rangle$



Output

$|\Psi\rangle$

Complexity = minimum number of gates

to go from $|000......0\rangle$ to $|\Psi\rangle$.

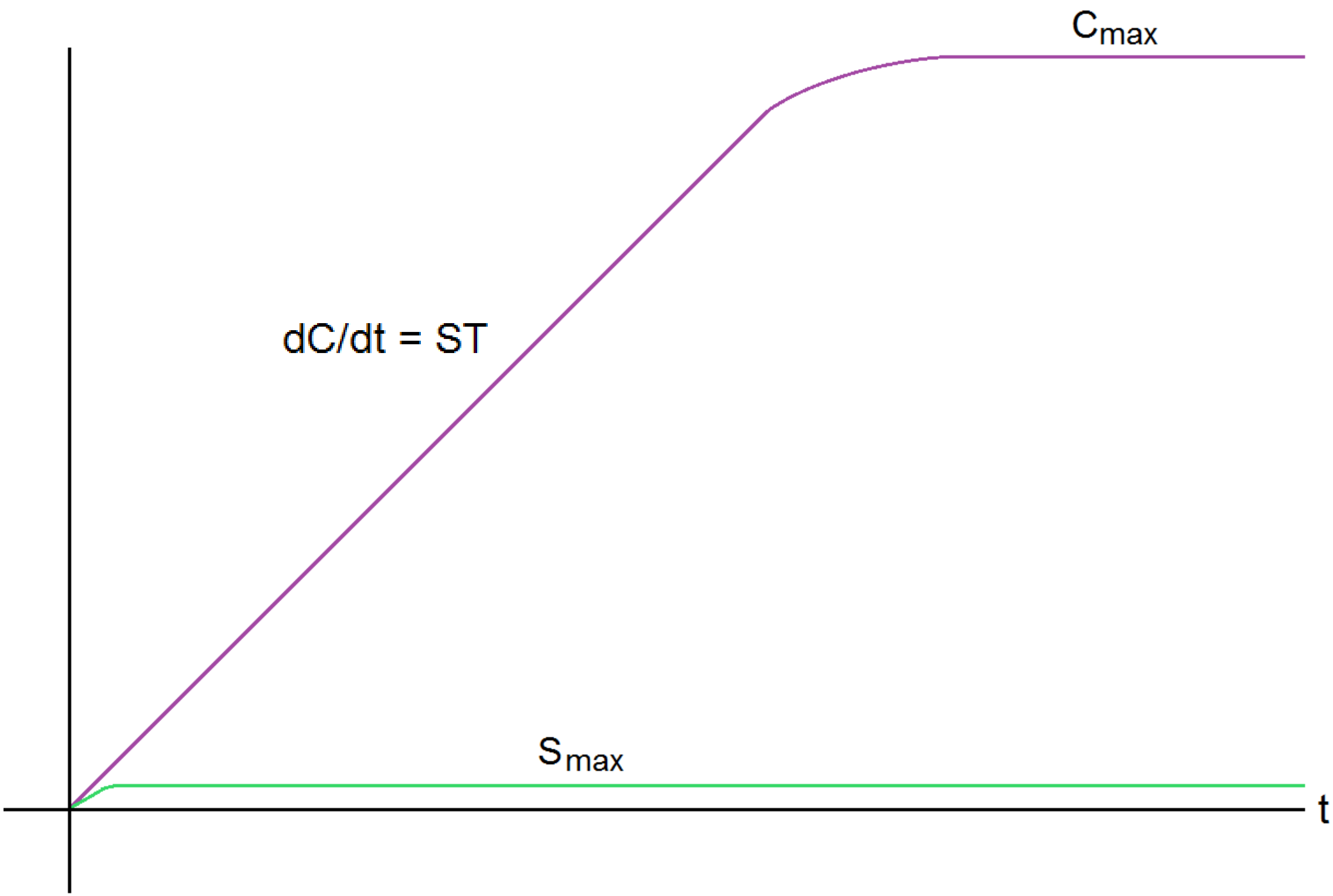
$$C_{max} = \exp N$$

$$S_{max} = N \log 2$$

Time to achieve $S_{max} \sim \log N$

Time to achieve $C_{max} \sim \exp N$

What goes on between S_{max} and C_{max} ?



Compare

$$dC/dt = ST = \frac{A}{G\,\ell_{ads}}$$

$$dV/dt = A$$

$$\boxed{C = \frac{V}{G\,\ell_{ads}}}$$

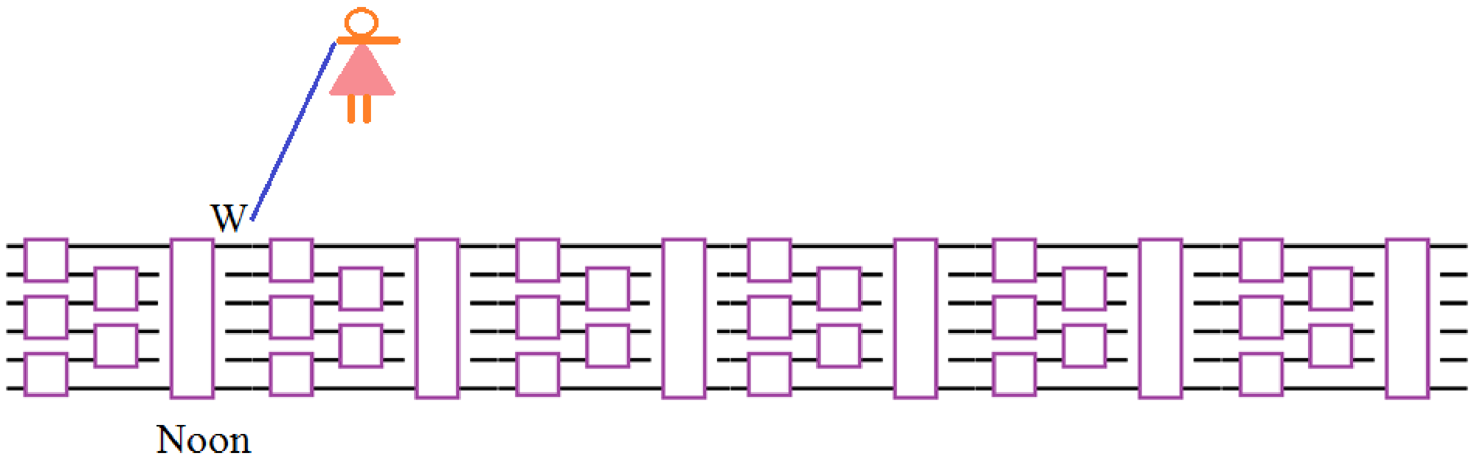
Coincidence?

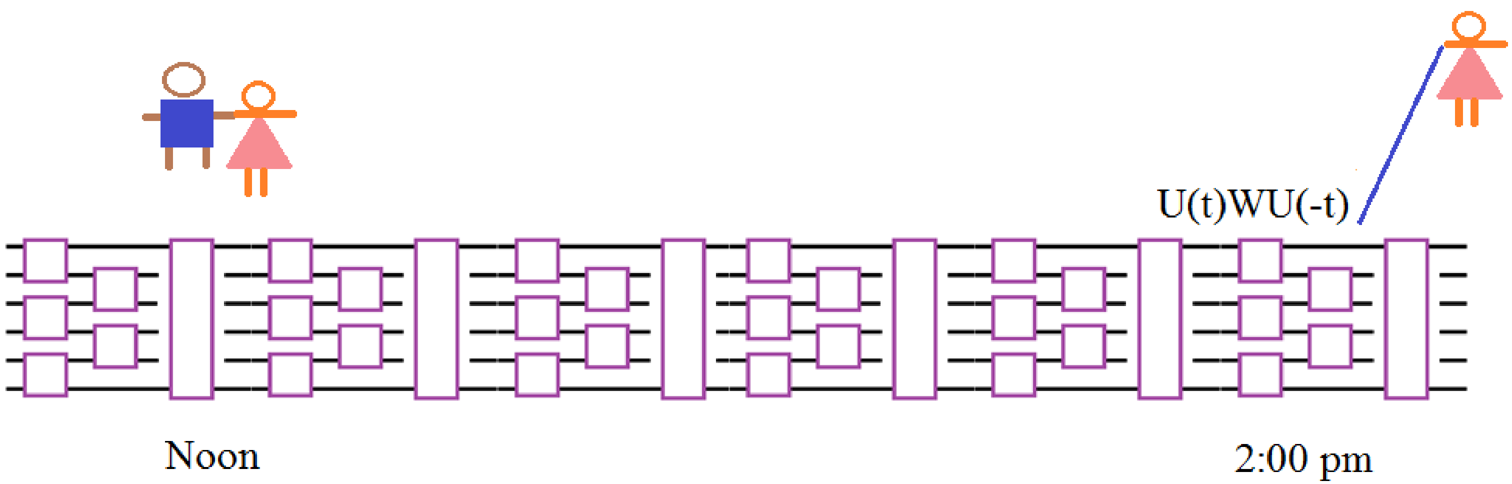
Precursors geometries provide a strong test.

Shenker Stanford

Stanford LS

Alice’s Lunch Date

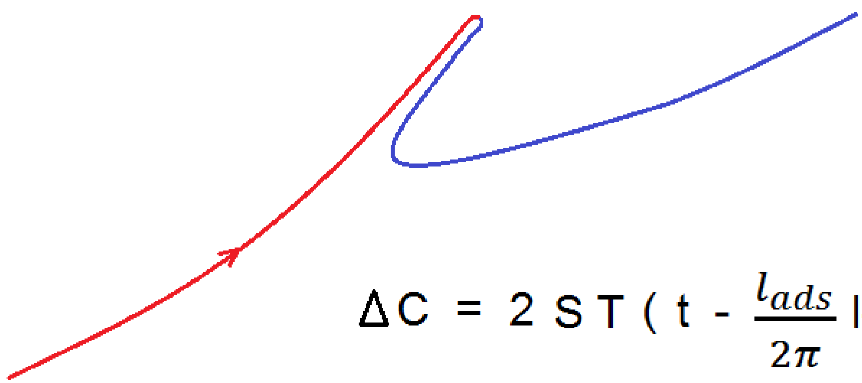




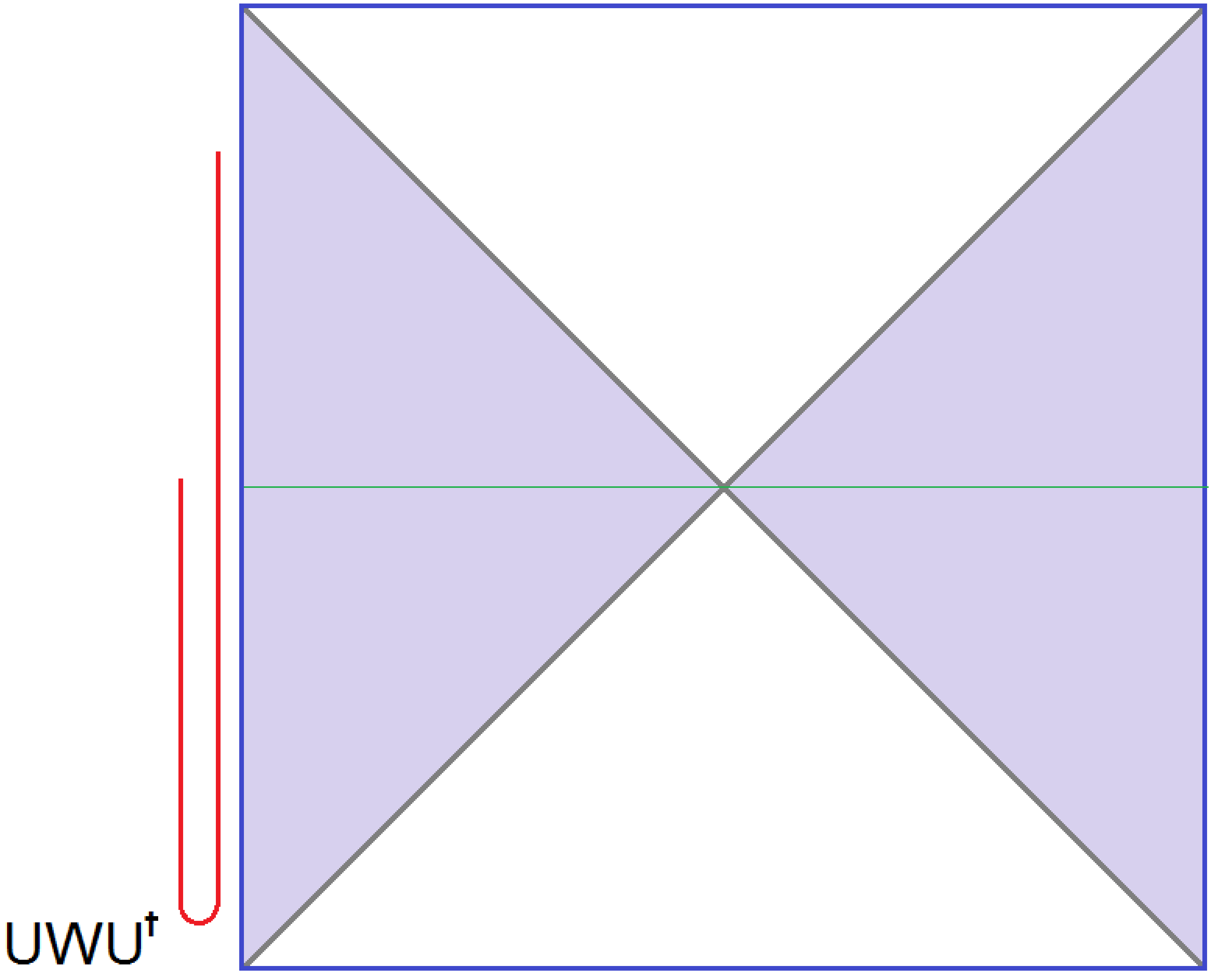
Additional complexity due to U(t) W U(-t)

$\Delta C = 2\, S\, T\, t$ naively

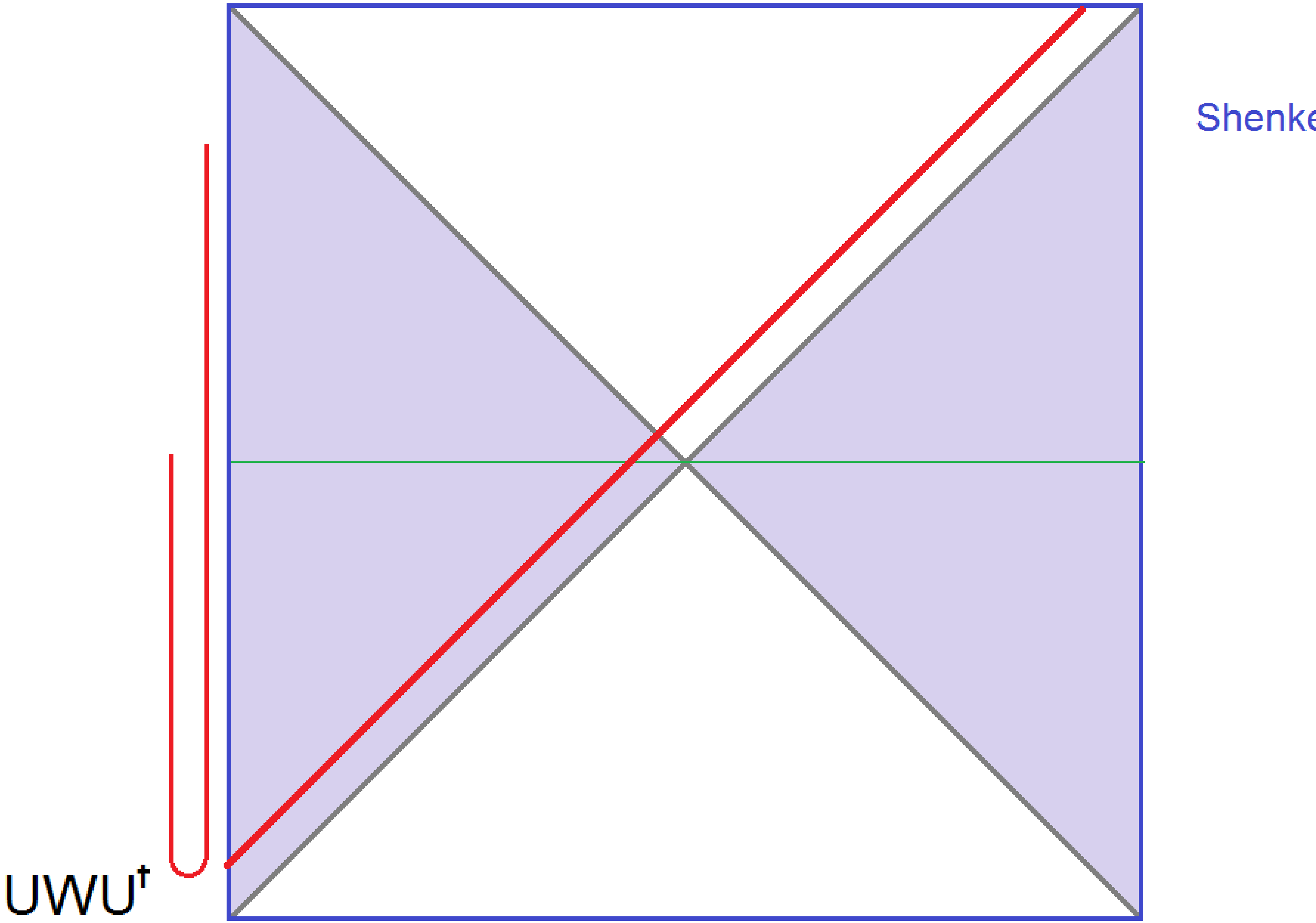
But what if W = 1? Then there is cancelation and $\Delta C = 0$.



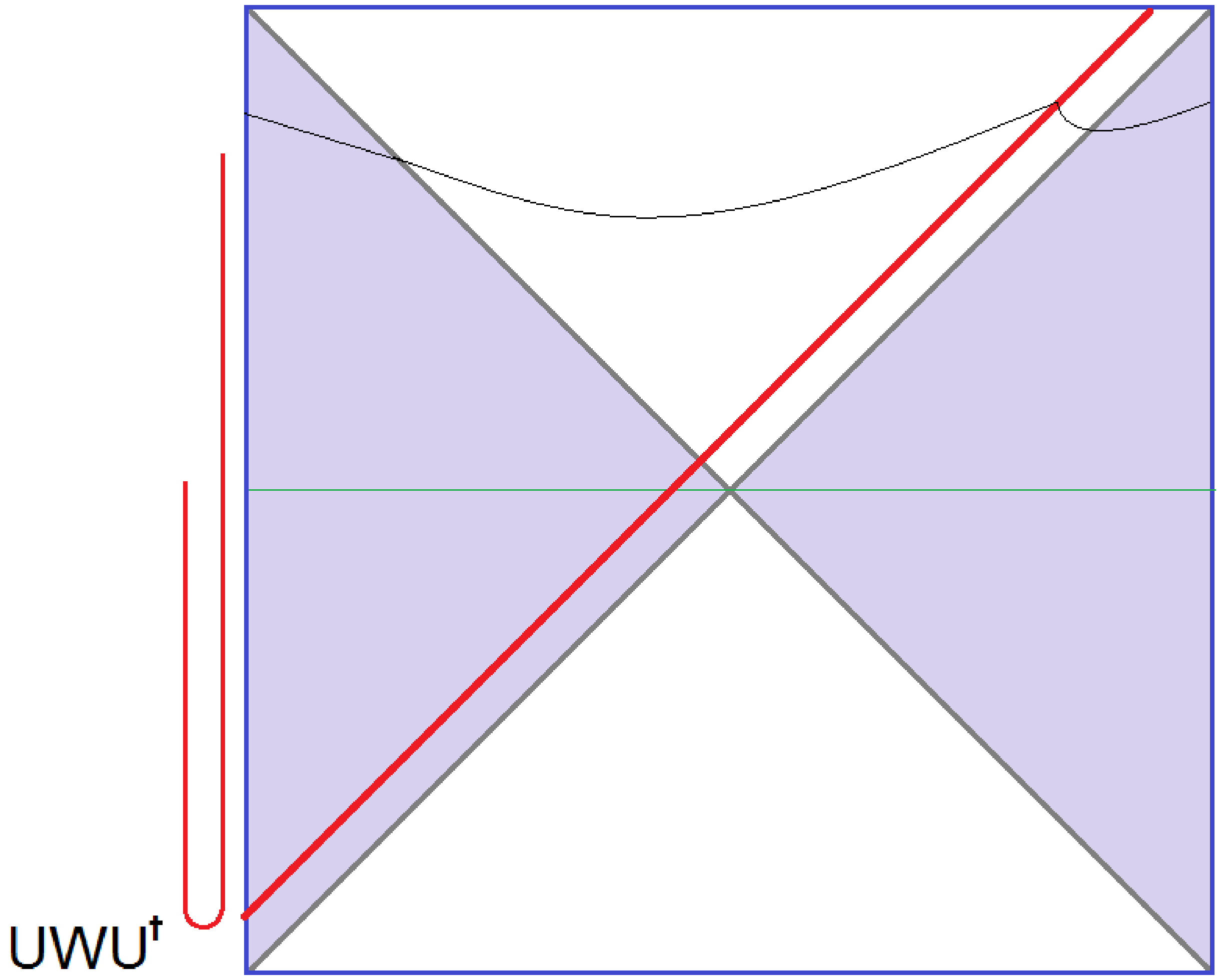
$$\Delta C = 2\, S\, T\, \left(t - \frac{l_{ads}}{2\pi} \log S \right)$$



$UWU^\dagger$



Shenke
UWU†



$$\Delta V = 2SG(t_w - t_*)$$

$$t_* = \frac{l_{ads}}{2\pi} \log S$$

D. Stanford, LS
D. Roberts, D. Stanford, LS
LS, Y. Zhao

Implications

The holographic encoding of space behind the horizon is different. It is built out of the extremely subtle correlations that define quantum complexity, not the ordinary correlations that define entanglement and entropy.

Complexity and geometry are dual to one another: complexity of the gauge theory state, and geometry of the interior.

The classical evolution of geometry cannot persist for all time. Complexity and therefore volume must stop growing at times of order exp S.

Questions

Does complexity actually grow for an exponential time or does it stall? What are the implications one way or another? (Scott Aaronson, LS to appear some day)

Do we need new non-linear rules for QM behind the horizon or is standard QM ok?

Is the growth of space always connected with the growth of complexity. Cosmological growth?

Different tools are needed to understand the emergent space behind horizons. Entanglement is not enough.

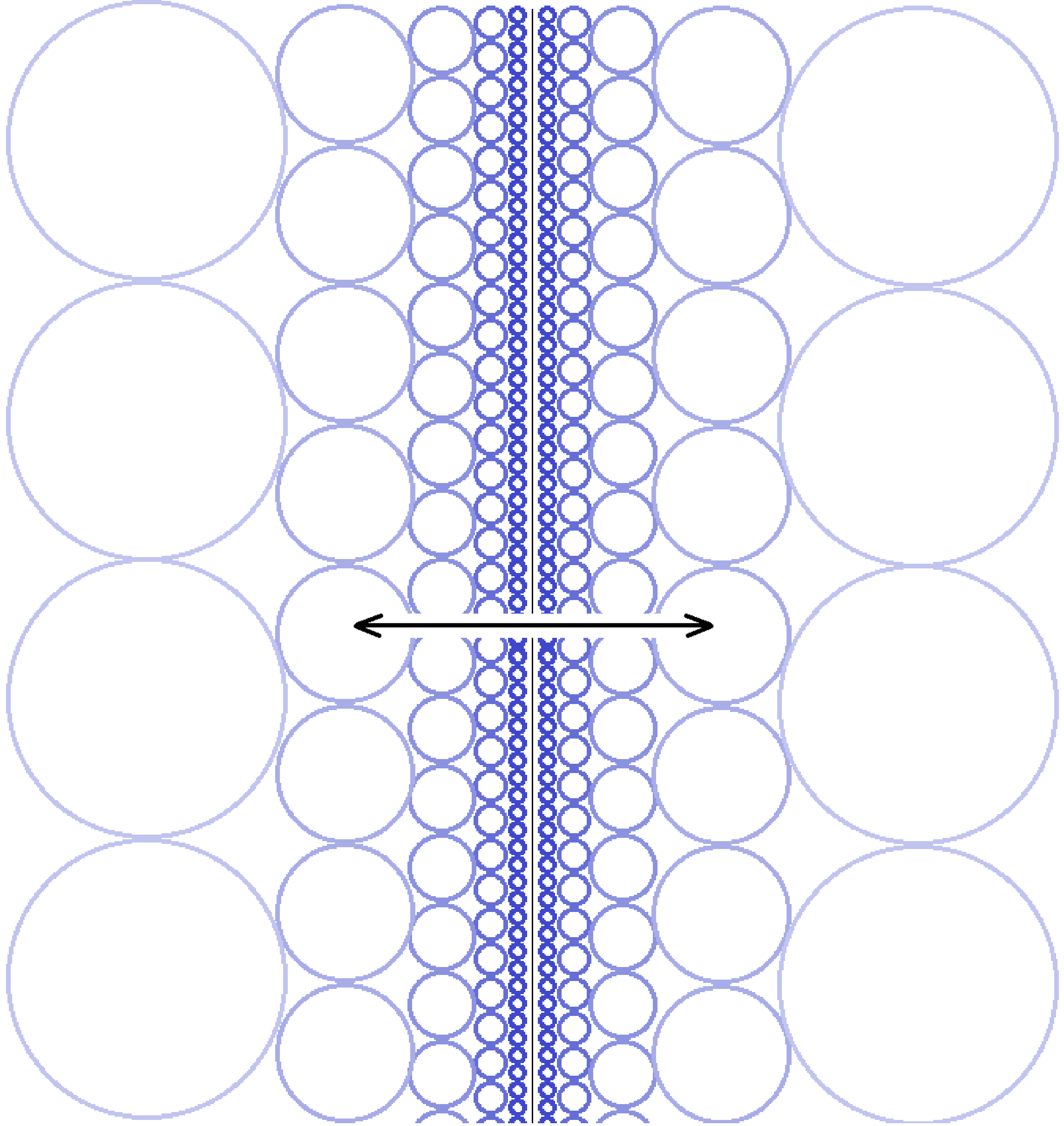
Complexity theory will be one of these tools.

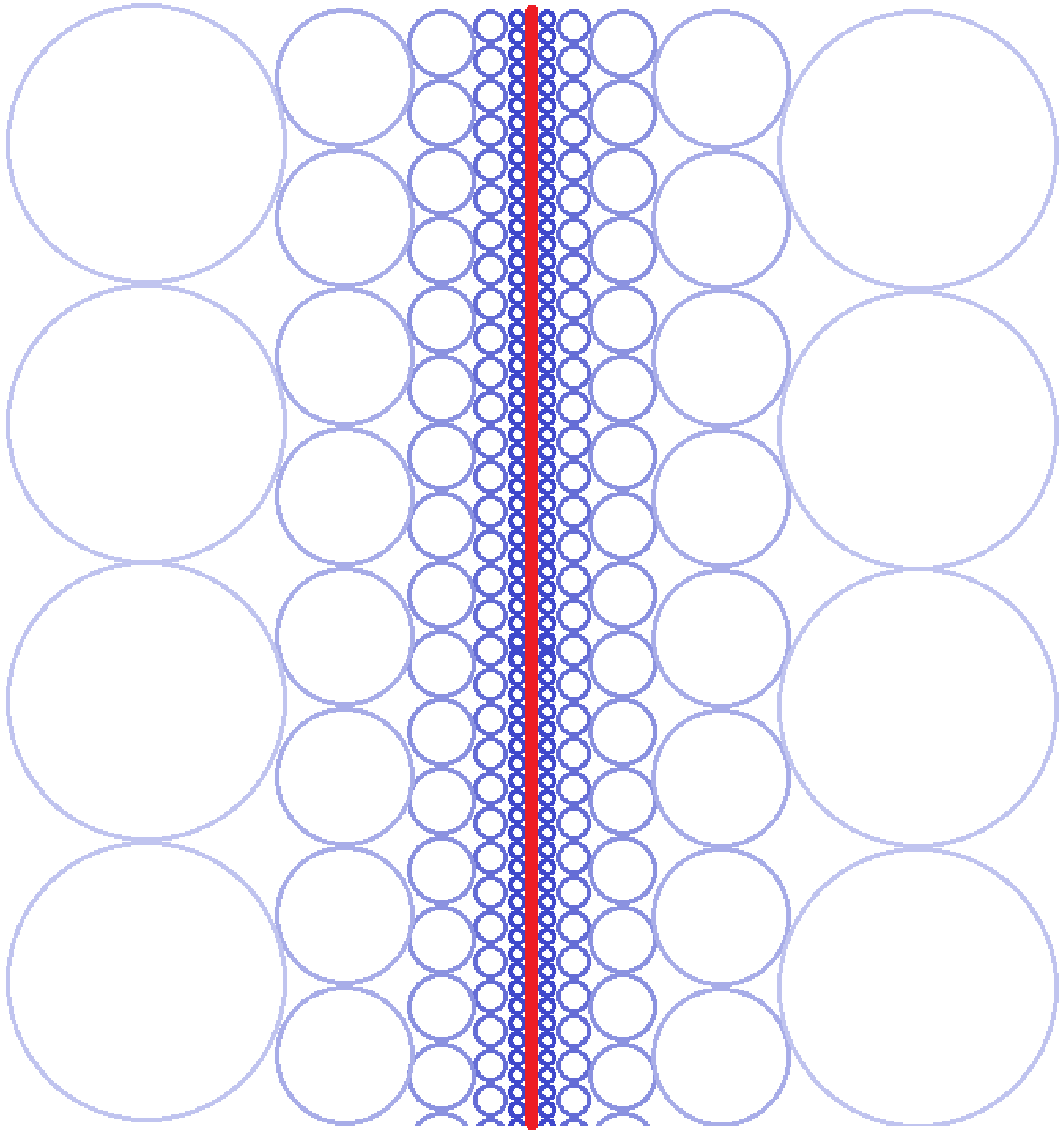
Questions:

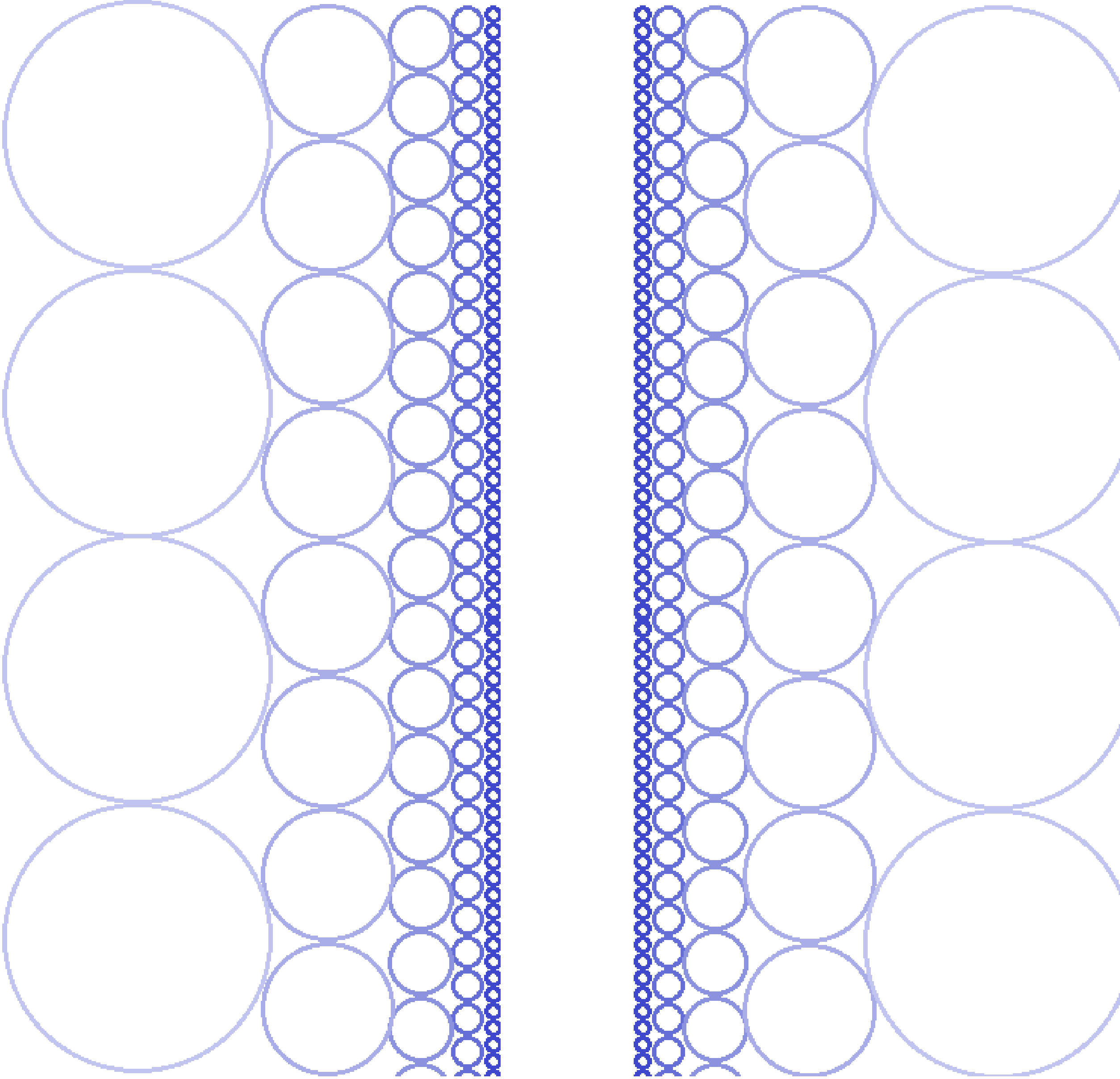
Do V and C grow like t for exponential time? What are the implications for CS?
PSPACE $\not\subset$ BQP/poly   Scott Aaronson, LS

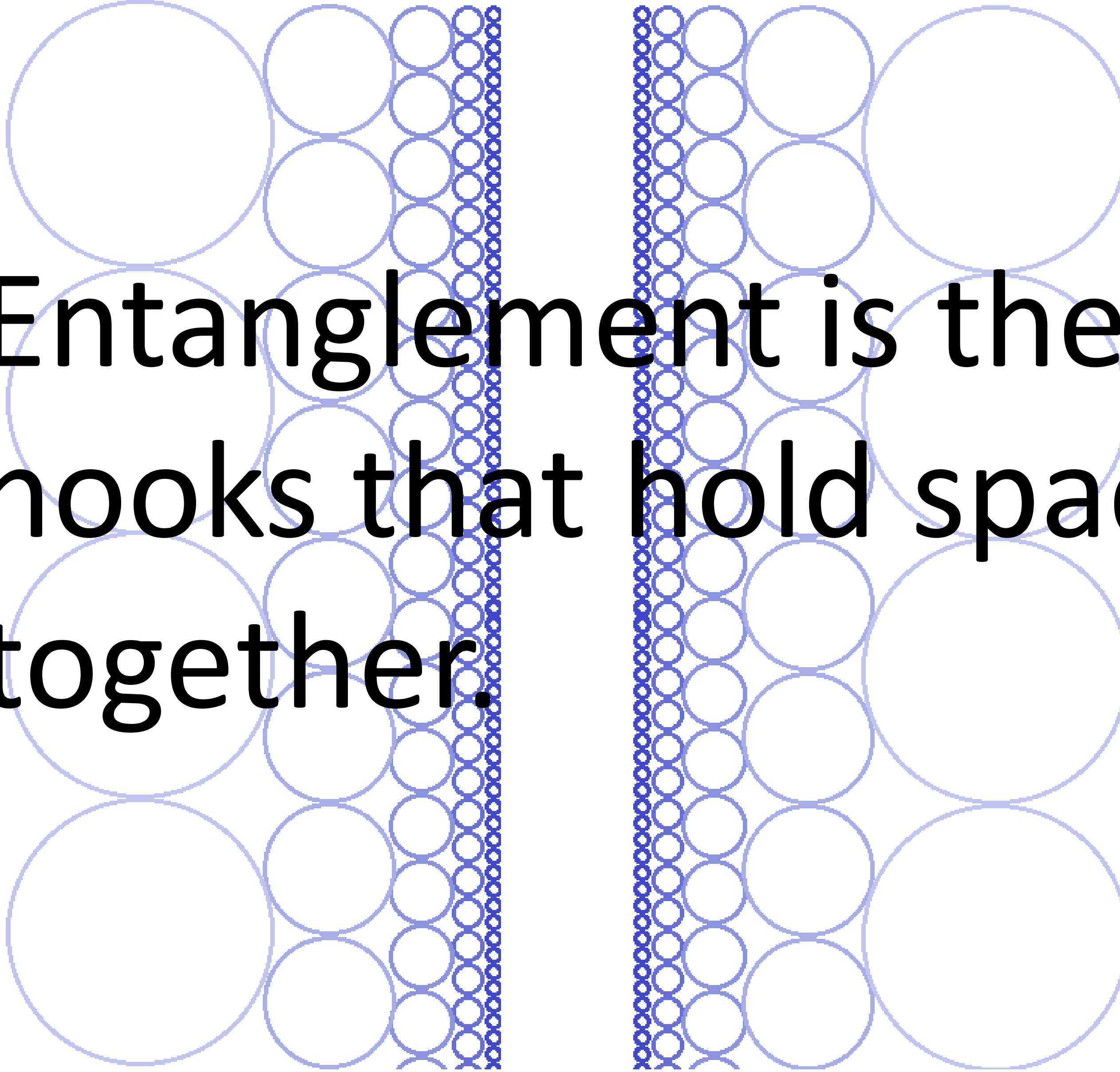
Is the bulk-boundary dictionary behind the horizon non-linear (state dependent)?

Does the growth of ERB's and its relation to complexity tell us anything about the growth of cosmological space?



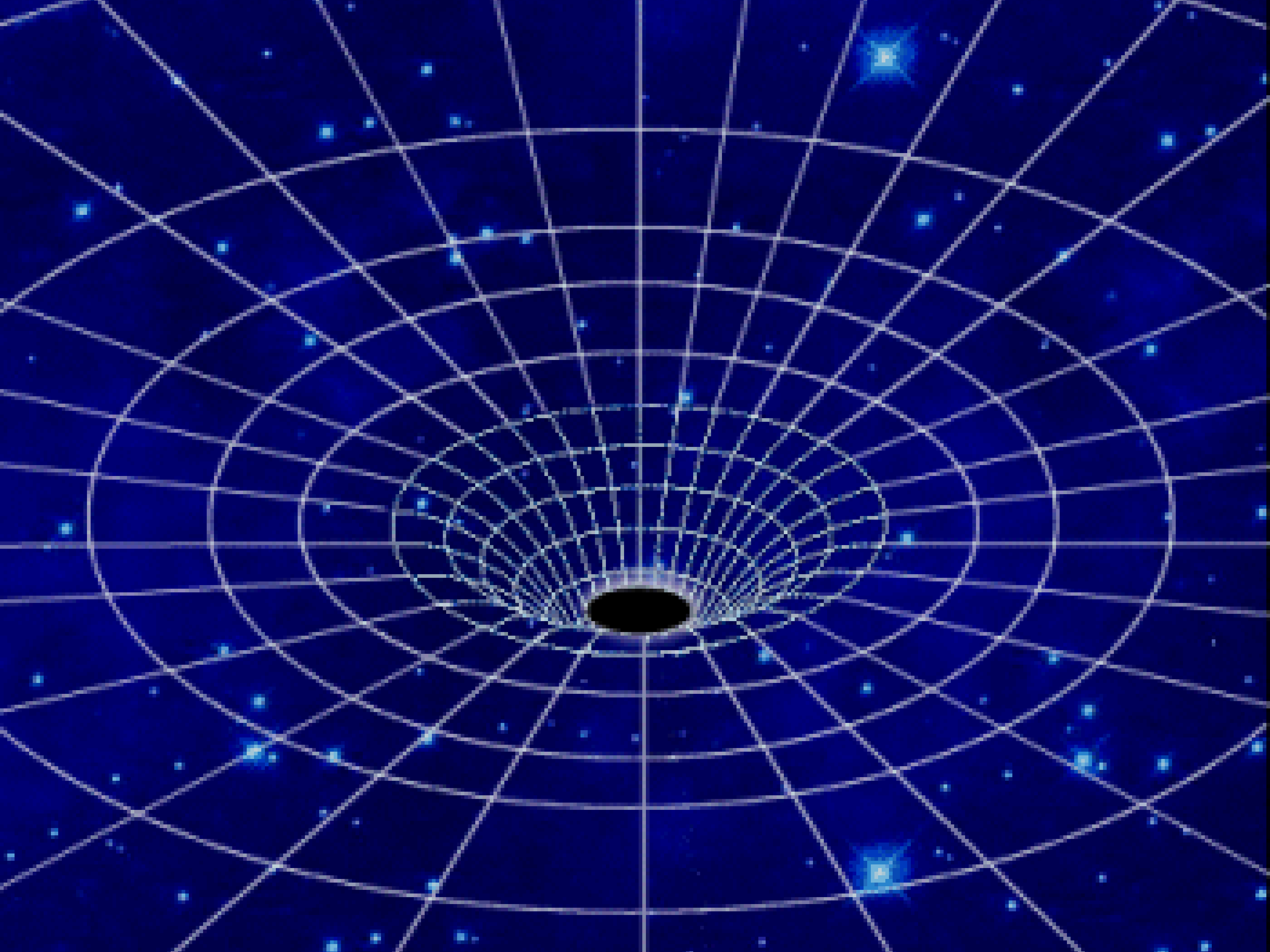




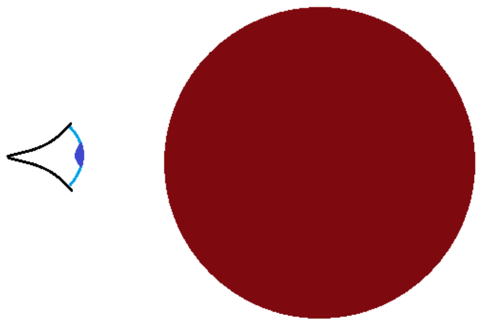


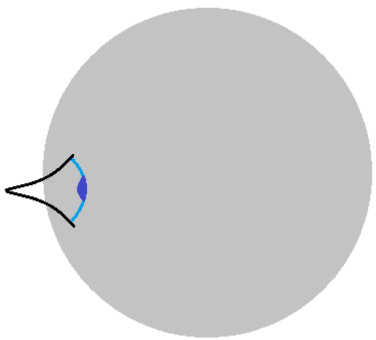
Entanglement is the hooks that hold space together.

Mark Van Raamsdonk



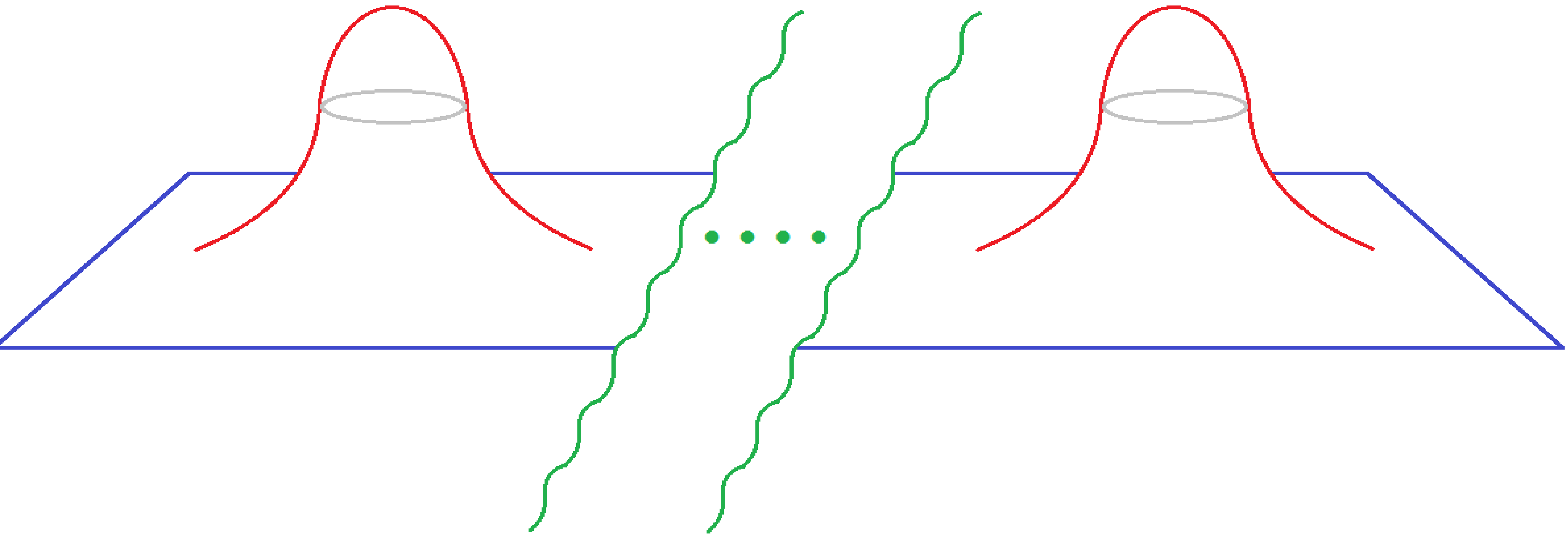
From the outside black Holes are thermodynamic systems. They quickly come to almost perfect thermal equilibrium, maximizing their entropy. Their entropy is proportional to the area of their horizons. But you can fall through the horizon into an interior region.





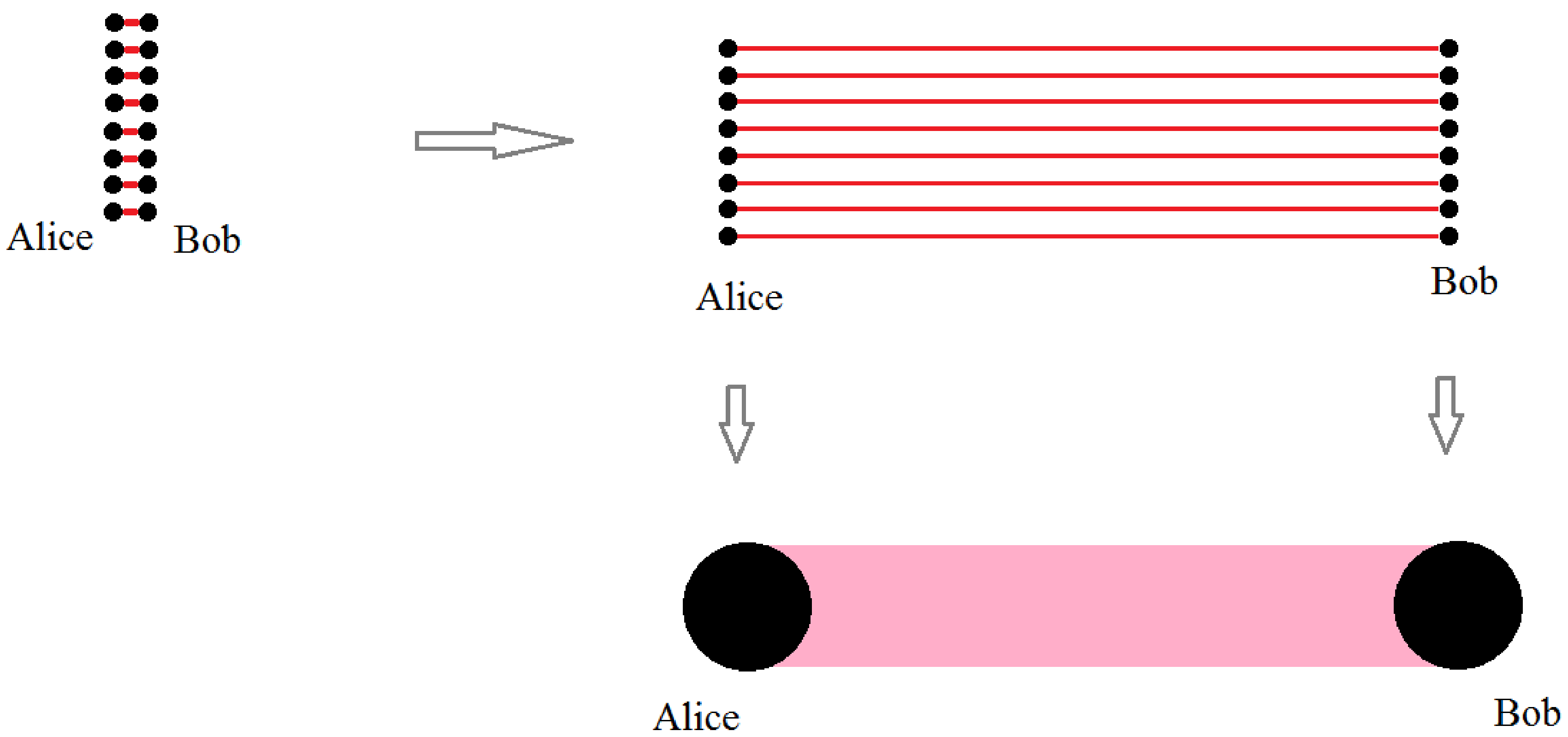
Version 1

Alice and Bob independently
create black holes.



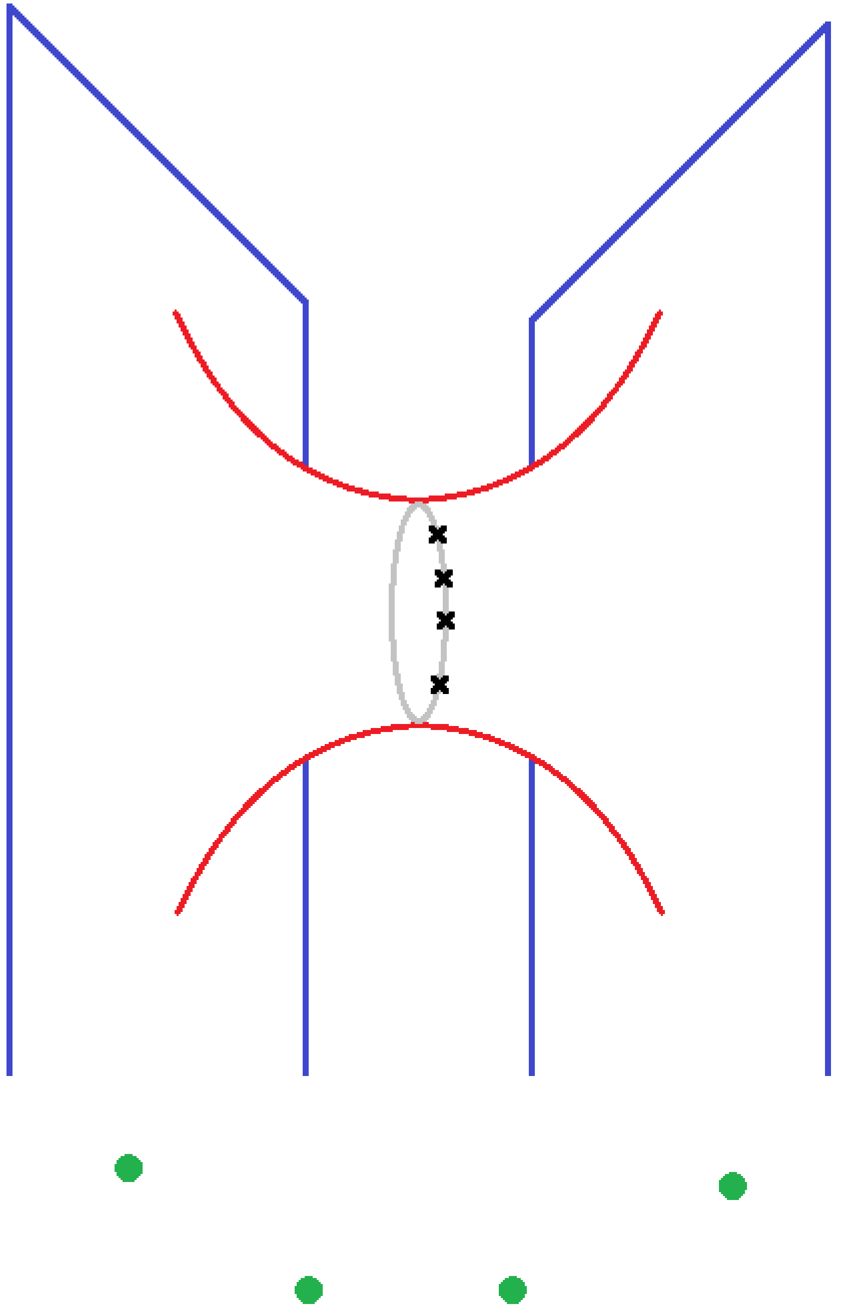
V 2

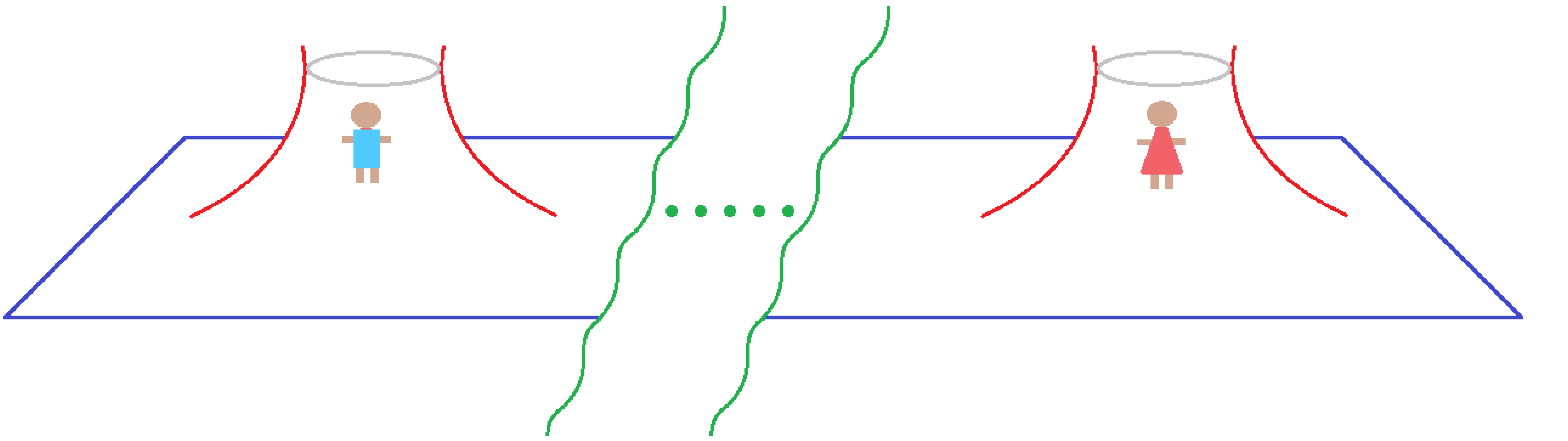
Alice and Bob create black holes
out of entangled matter.

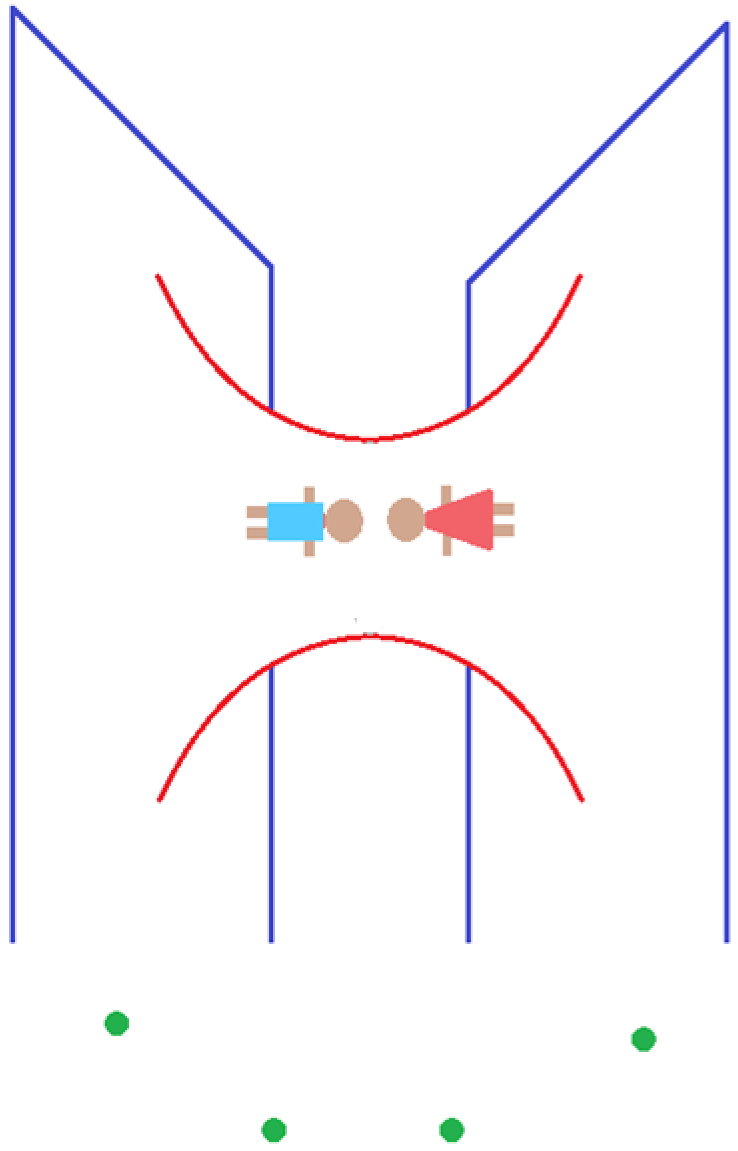


ER=EPR
EPR=ER

J. Maldacena, LS





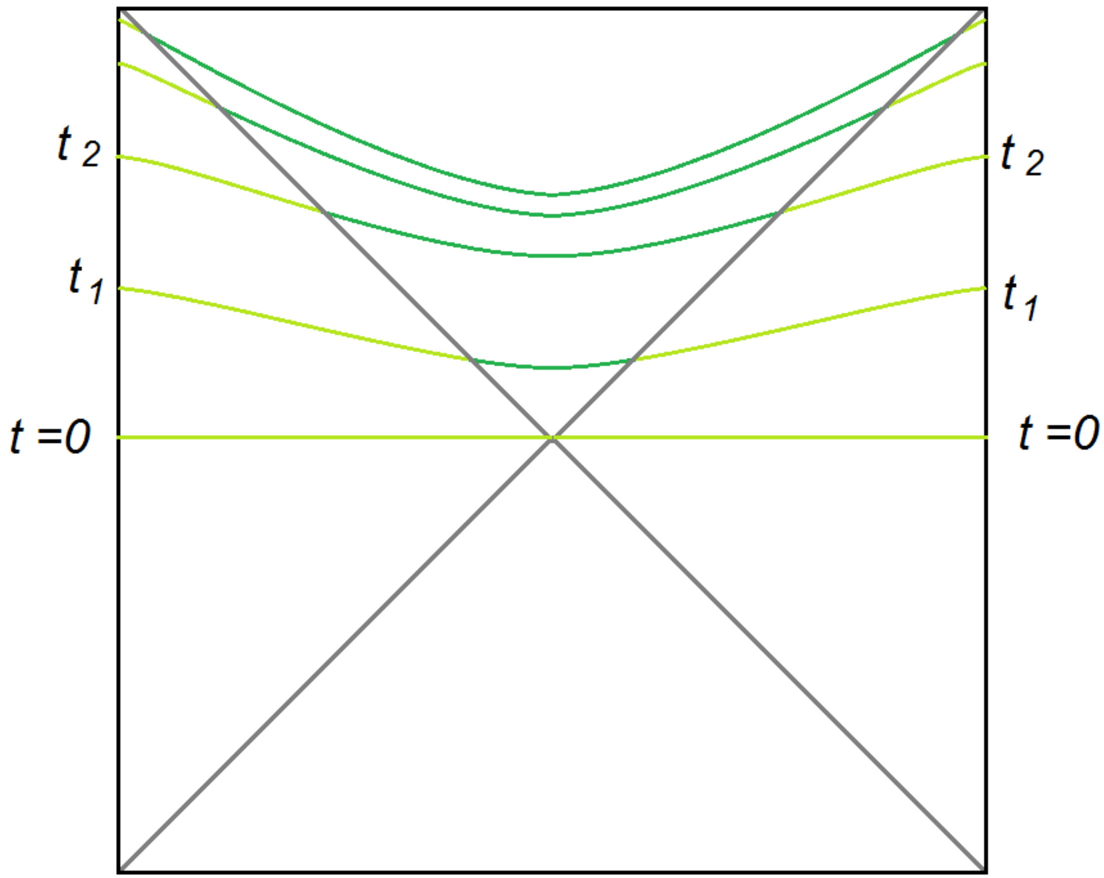


Both Entanglement and ERBs are forms of non-local connectivity but they don't violate any physical laws or principles

Entanglement cannot be used to send signals by local operations.

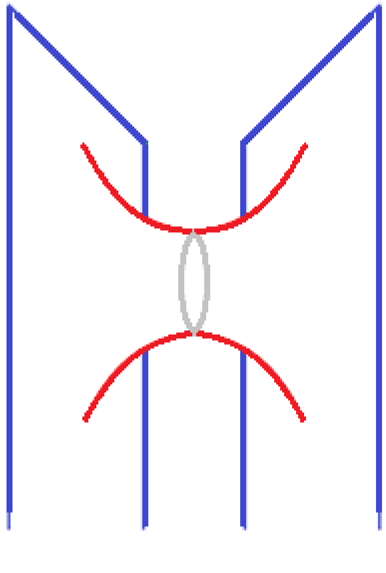
Signals cannot be sent through an ERB by local operations.

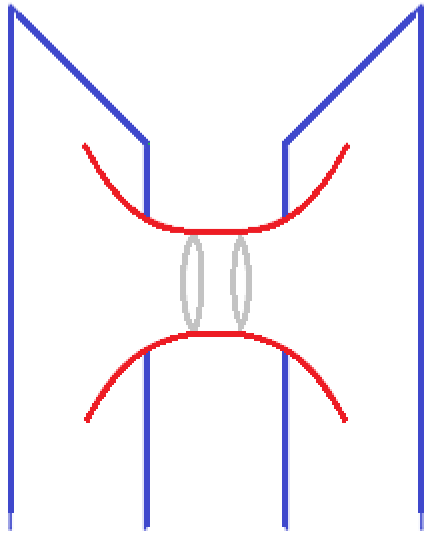
(ERBs are non-traversable)

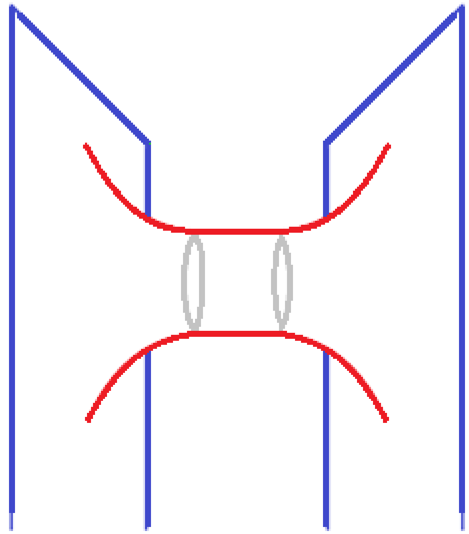


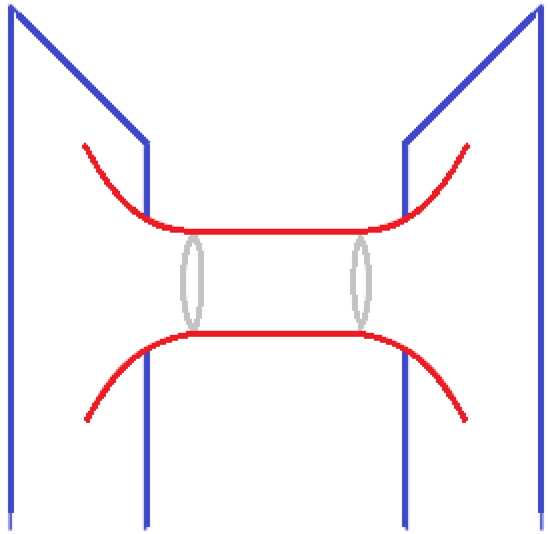
Volume of an ERB grows linearly

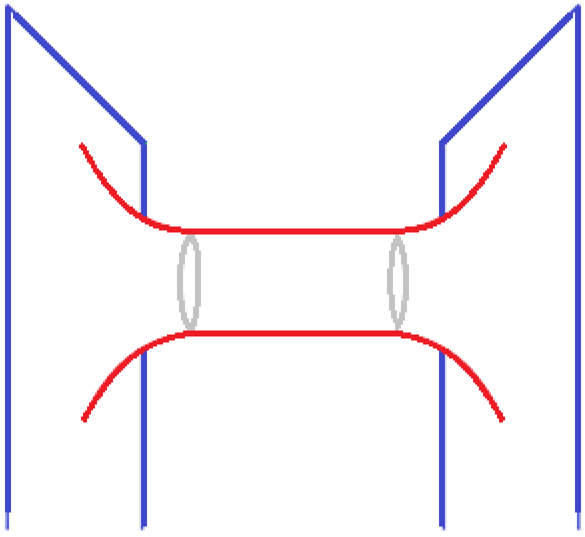
$$\frac{dV}{dt} = G_N\ S$$

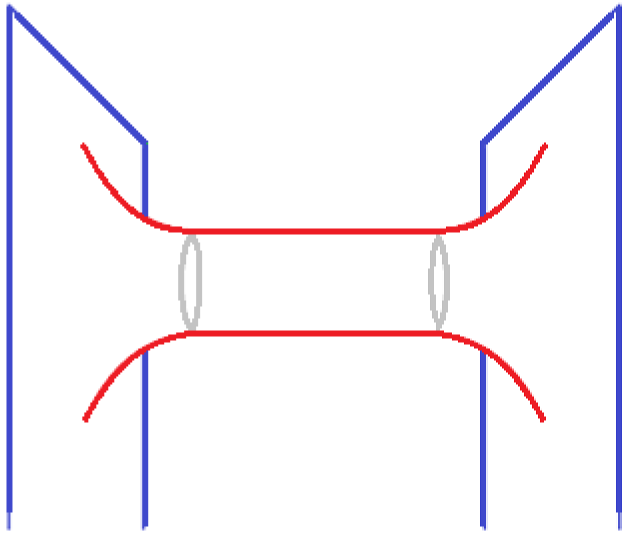


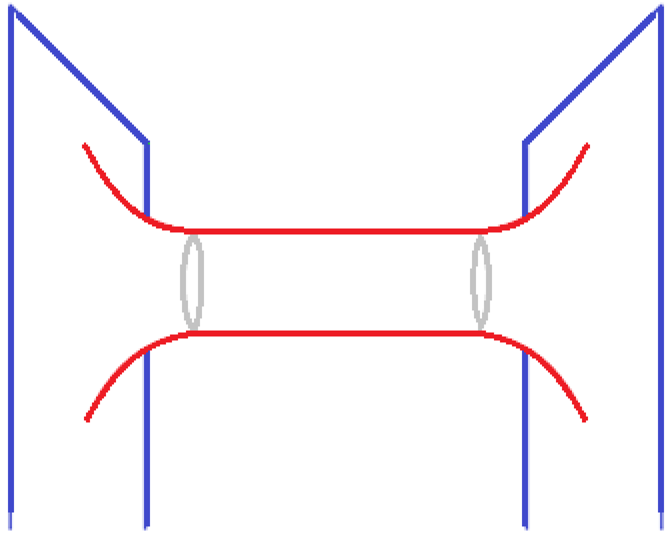


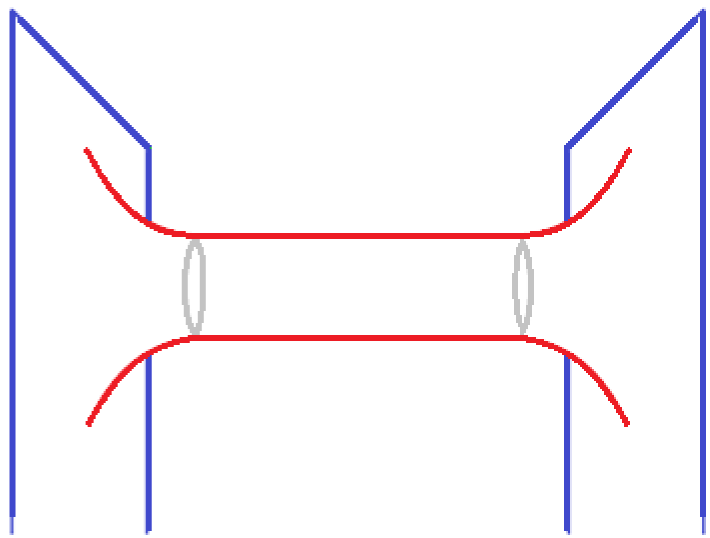


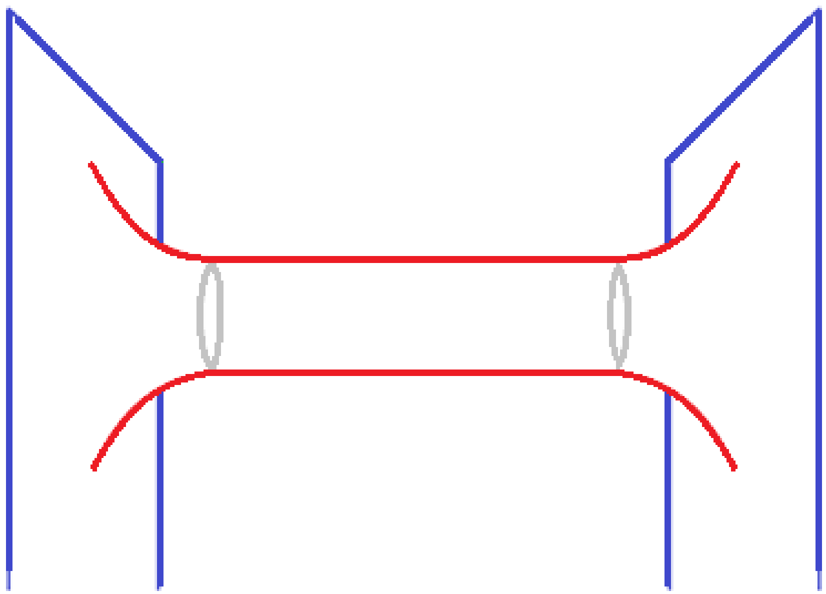


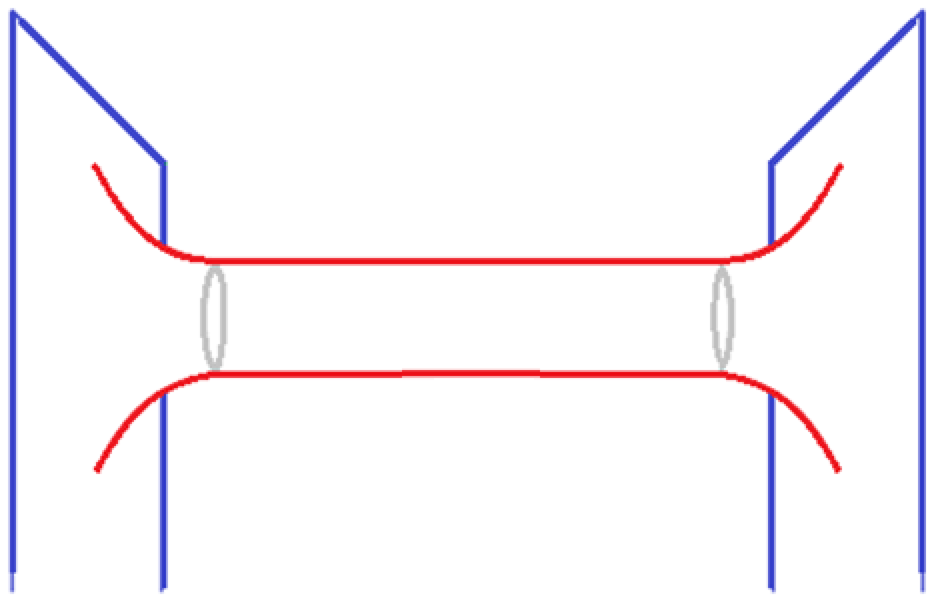


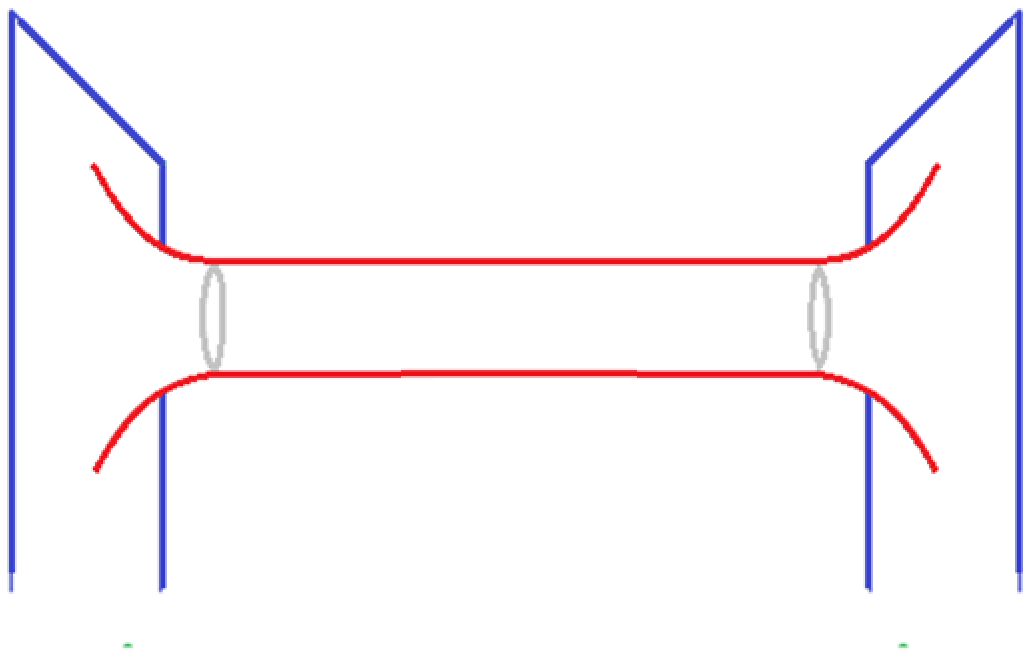








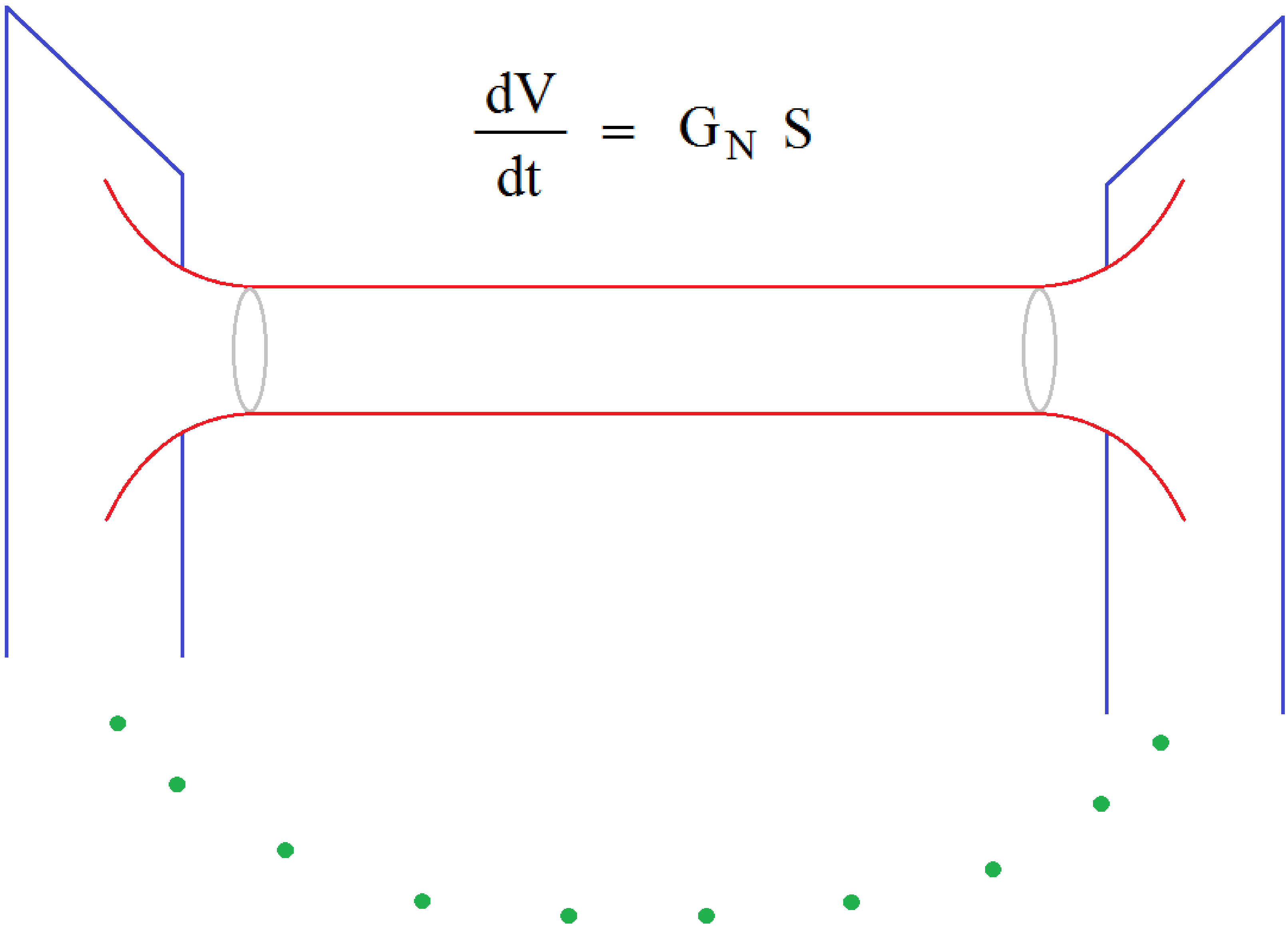




Signals from outside to outside have to go the long way.

Einstein-Rosen bridges grow with time.

$$\frac{dV}{dt} = G_N S$$



Paradox:

Black holes come to thermal equilibrium extremely rapidly (a millisecond for a solar mass black hole).

But the interior geometry grows for an extremely long time.

Q: What property of the quantum state continues to evolve long after thermal equilibrium?

Why is there an "arrow of time in the ERB?

Might entanglement continue to spread among the degrees of freedom of the black holes?

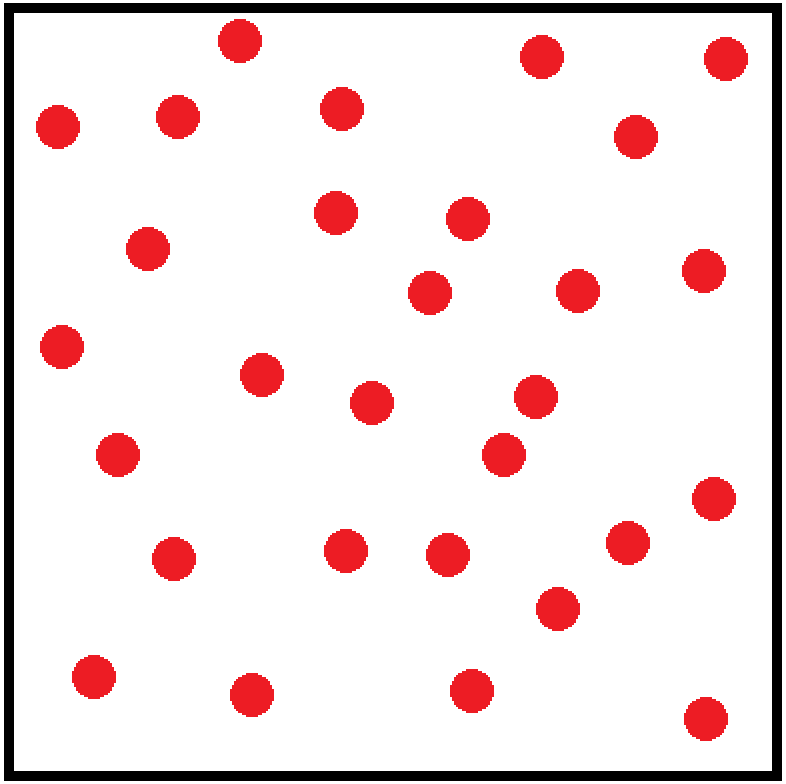
No, that's part of coming to thermal equilibrium and it's over very quickly.

Entanglement is not enough.

(Quantum) Computational Complexity

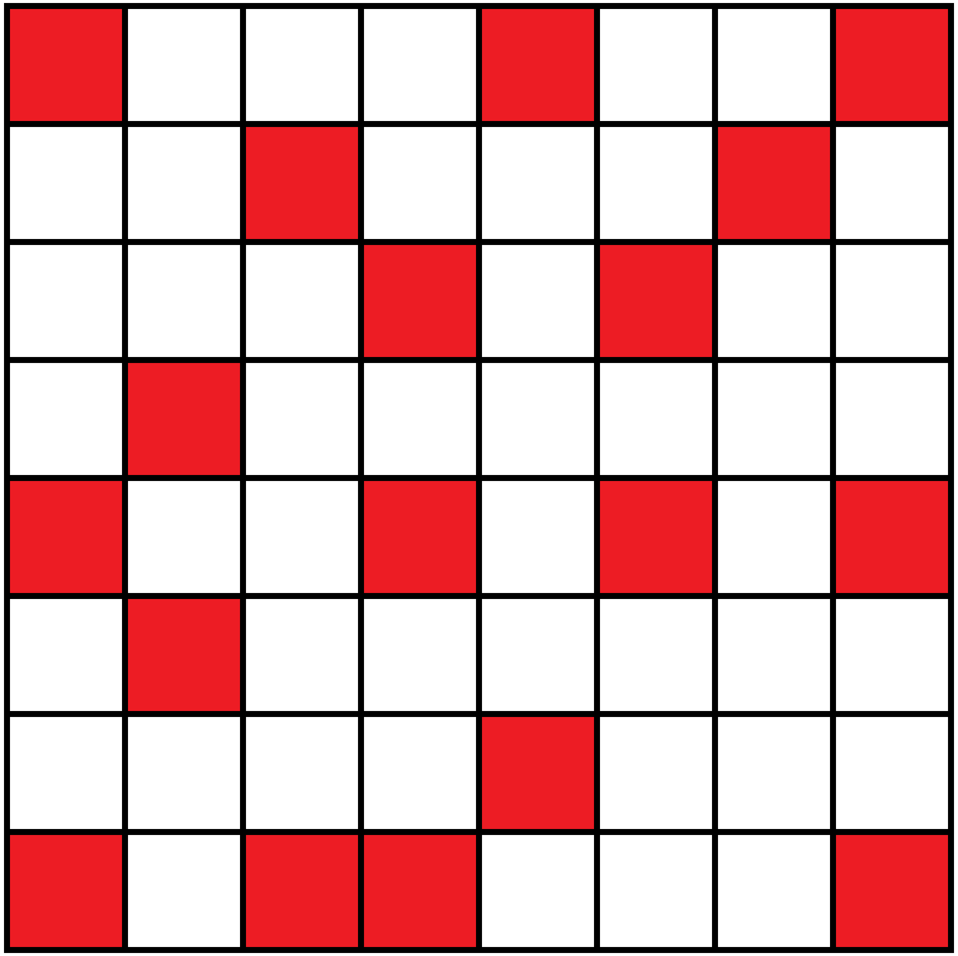
D. Harlow and P. Hayden
LS
D. Stanford and LS

a "computer"



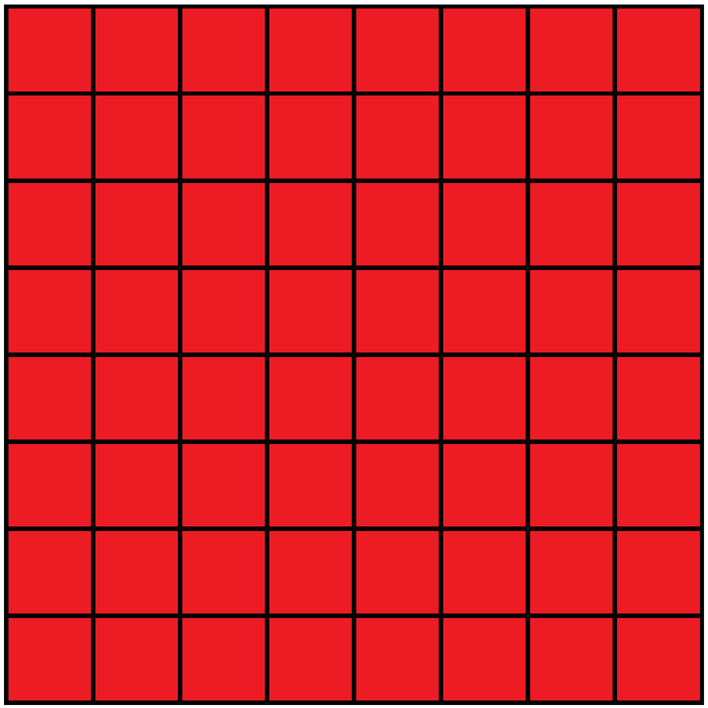
The program is the laws of motion.
The system can be initialized.
It updates itself.
One can measure the outcome.

a cellular automaton

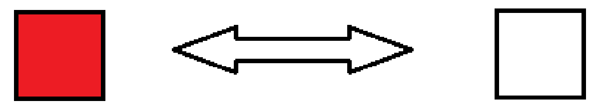


What is complexity?

Simple state



Simple process



Complexity is the minimal number of simple processes required to produce a given state from a simple state.

Maximum complexity and maximum entropy

$$C_{\max} = N/2$$ almost all

$$S_{\max} = N \log 2$$

When a classical system of bits reaches maximal entropy it also reaches maximal complexity.

Quantum States are far more complex. Feynman

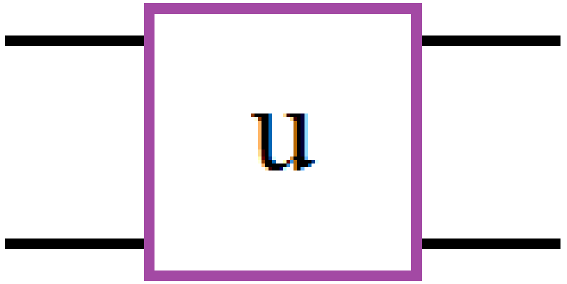
$$|\psi\rangle = \sum_{1}^{2^N} C_i |i\rangle$$

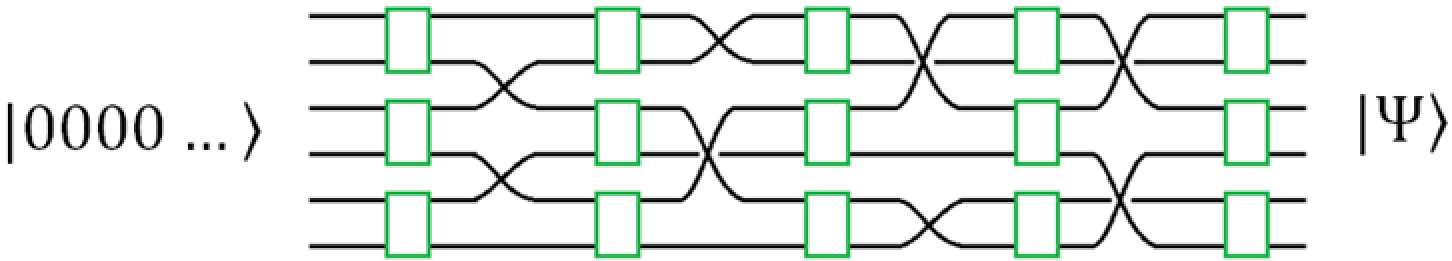
Specify 2^N complex coefficients.

Simple state: Same as before.

Simple process:

Unitary quantum gate





Complexity is the minimum number of gates in any circuit that can simulate $|\Psi\rangle$

$$C_{\max} \sim e^{N}$$ almost all

$$S_{\max} = N \log 2$$

Chaotic Systems

Initially quantum complexity grows linearly with time, but it continues to grow long past the time that the entropy reaches its maximum.

It is much to subtle to affect the thermal properties of a system in an observable way.

Volume of an ERB grows linearly

$$\frac{dV}{dt} = G_N \, S$$

Complexity of a chaotic quantum system grows linearly

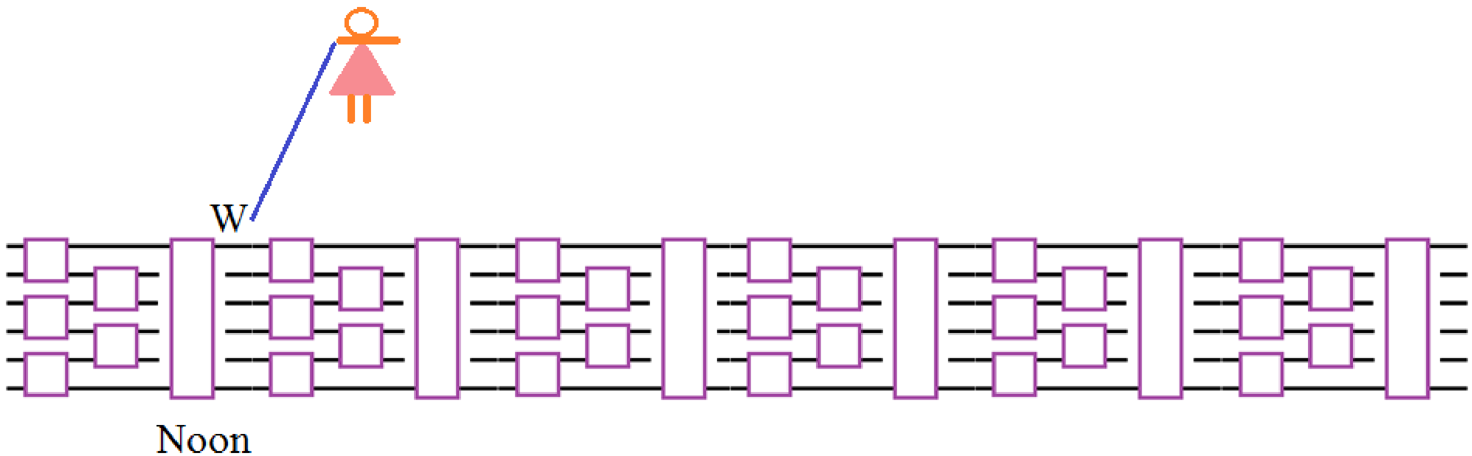
$$\frac{dC}{dt} = S$$

Volume – Complexity duality

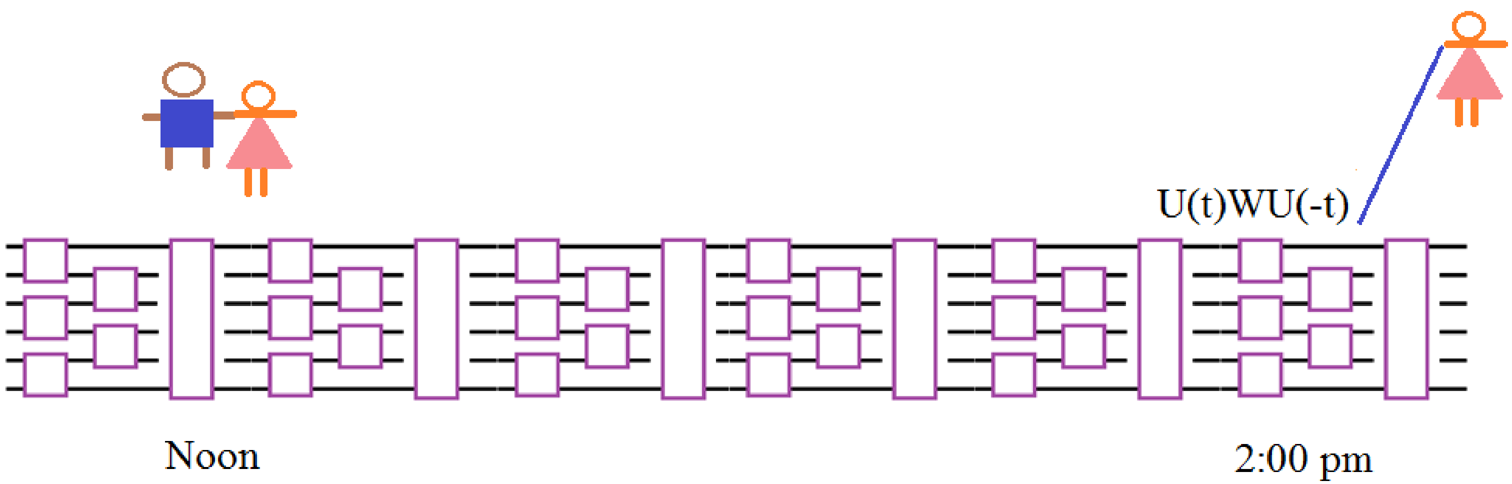
$$C = \frac{V}{G_N}$$

What evidence do we have for C/V duality?
Lots. Precursors are good probes.

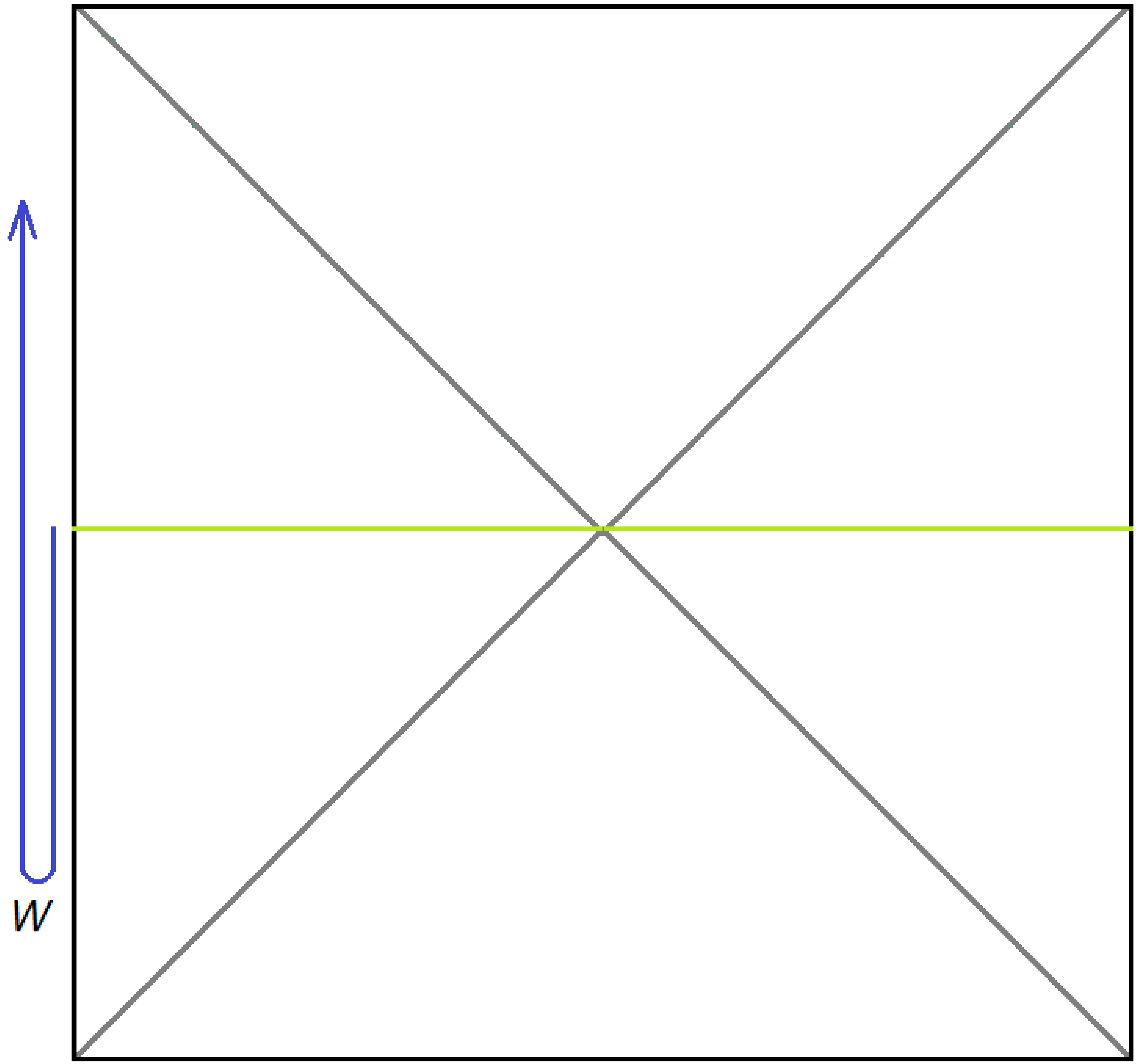
Consider Alice's lunch date problem



W
Noon

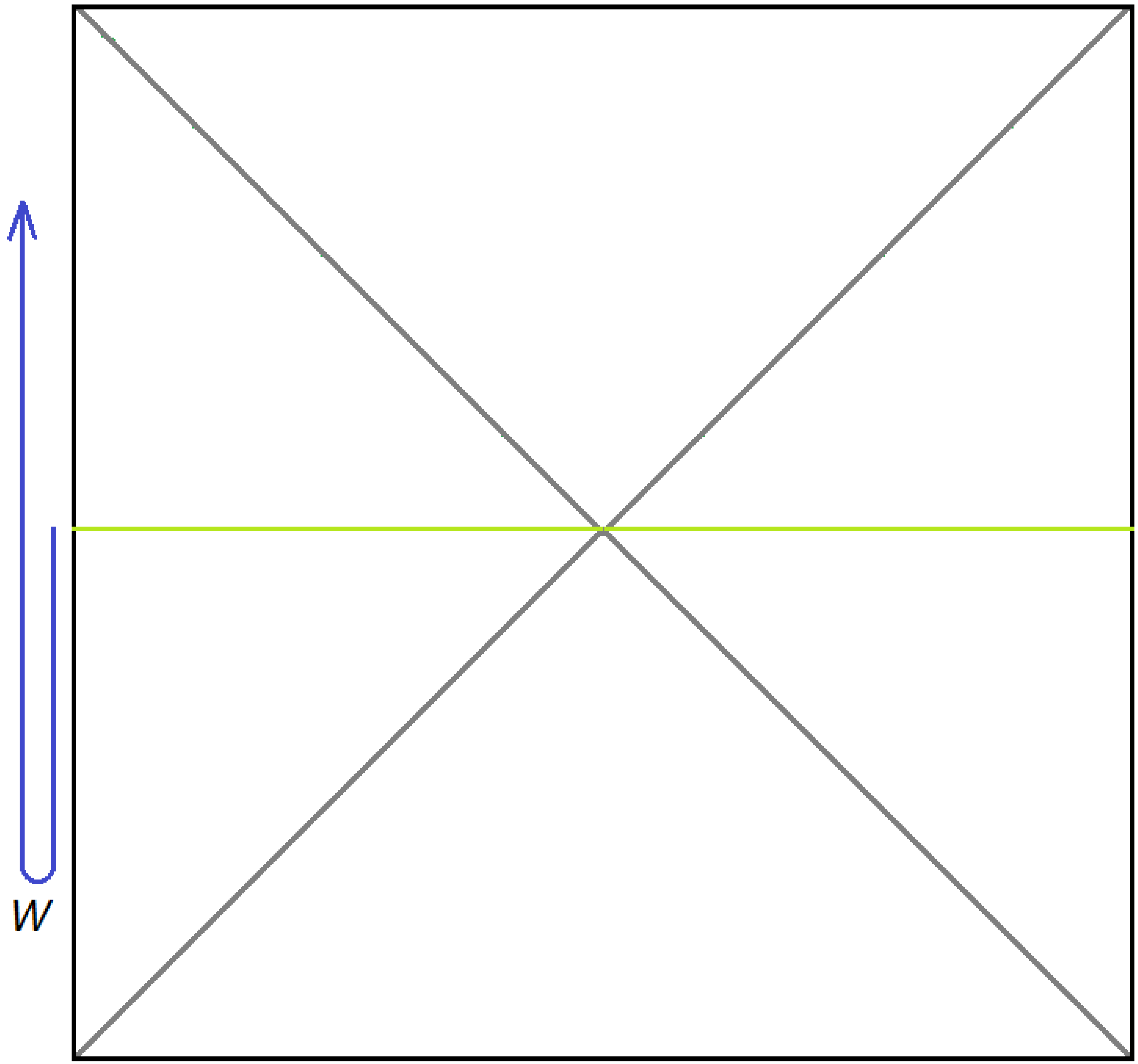


U(t)WU(-t)
Noon
2:00 pm



Added Complexity

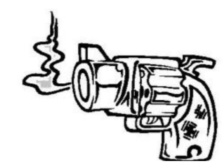
$= 2 S t_W$

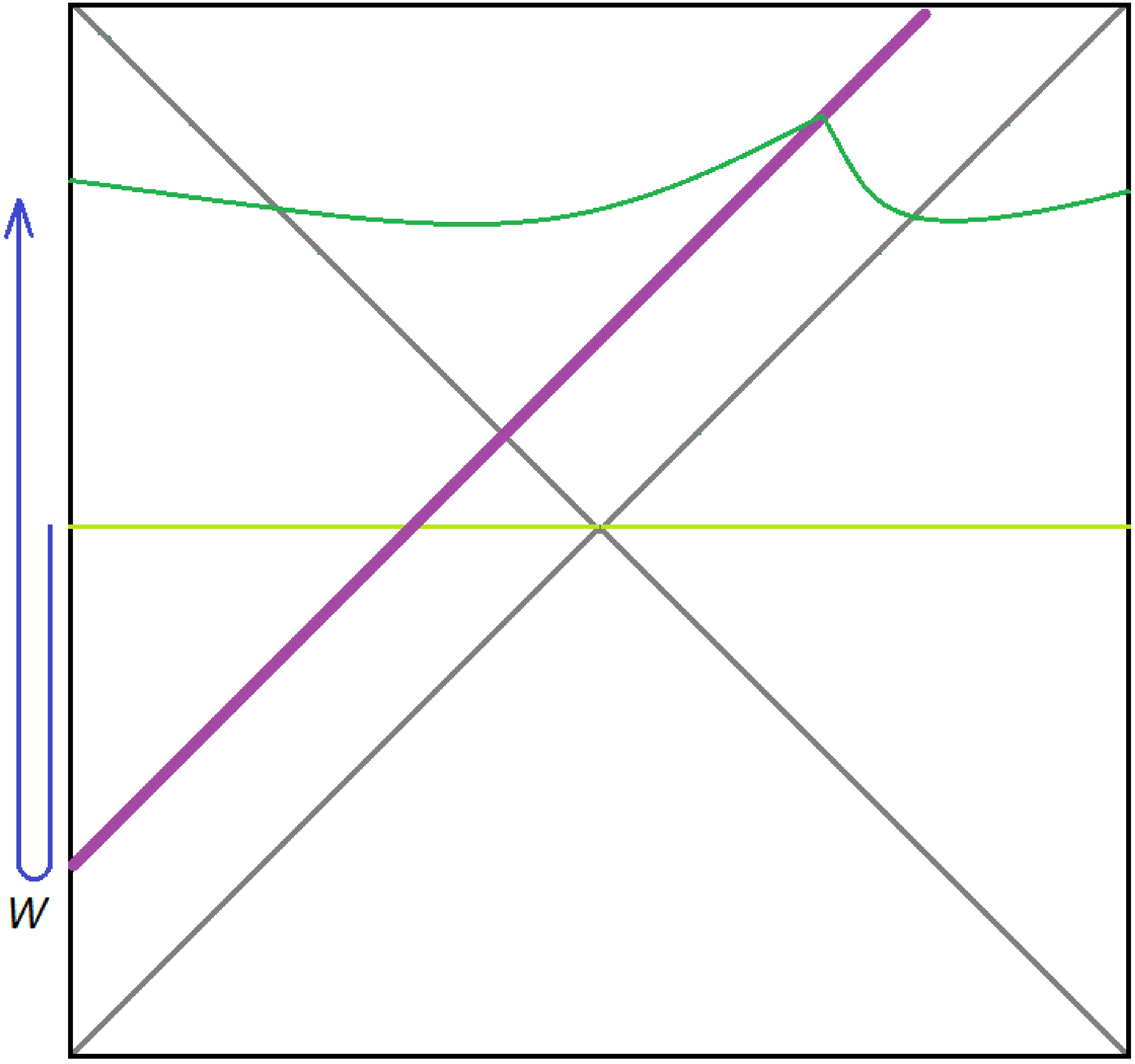


Added Complexity

$$= 2S(t_W - t_*)$$

$$t_* = \log S$$





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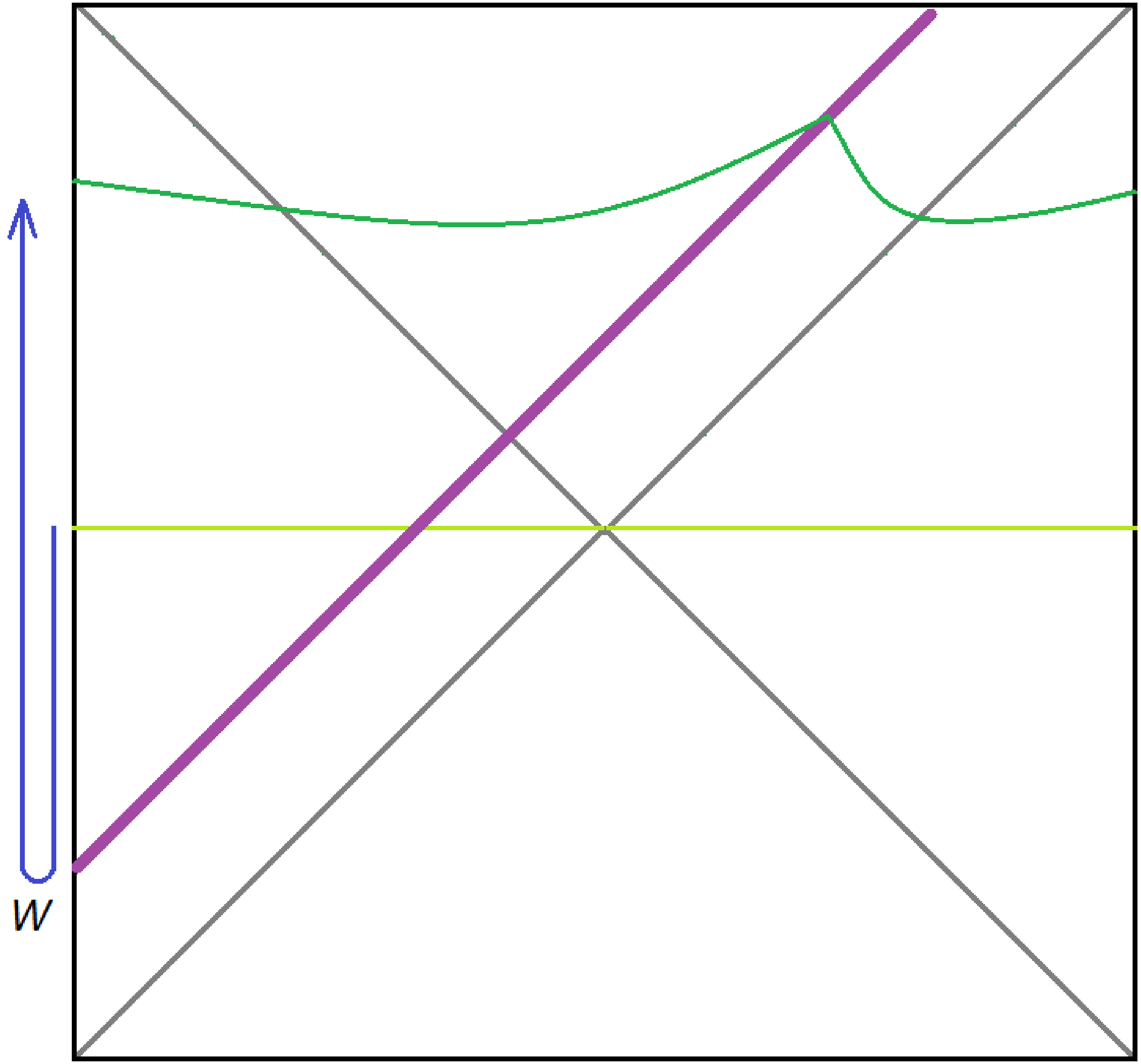


Added Volume

$$= 2S(t_W - t_*)$$

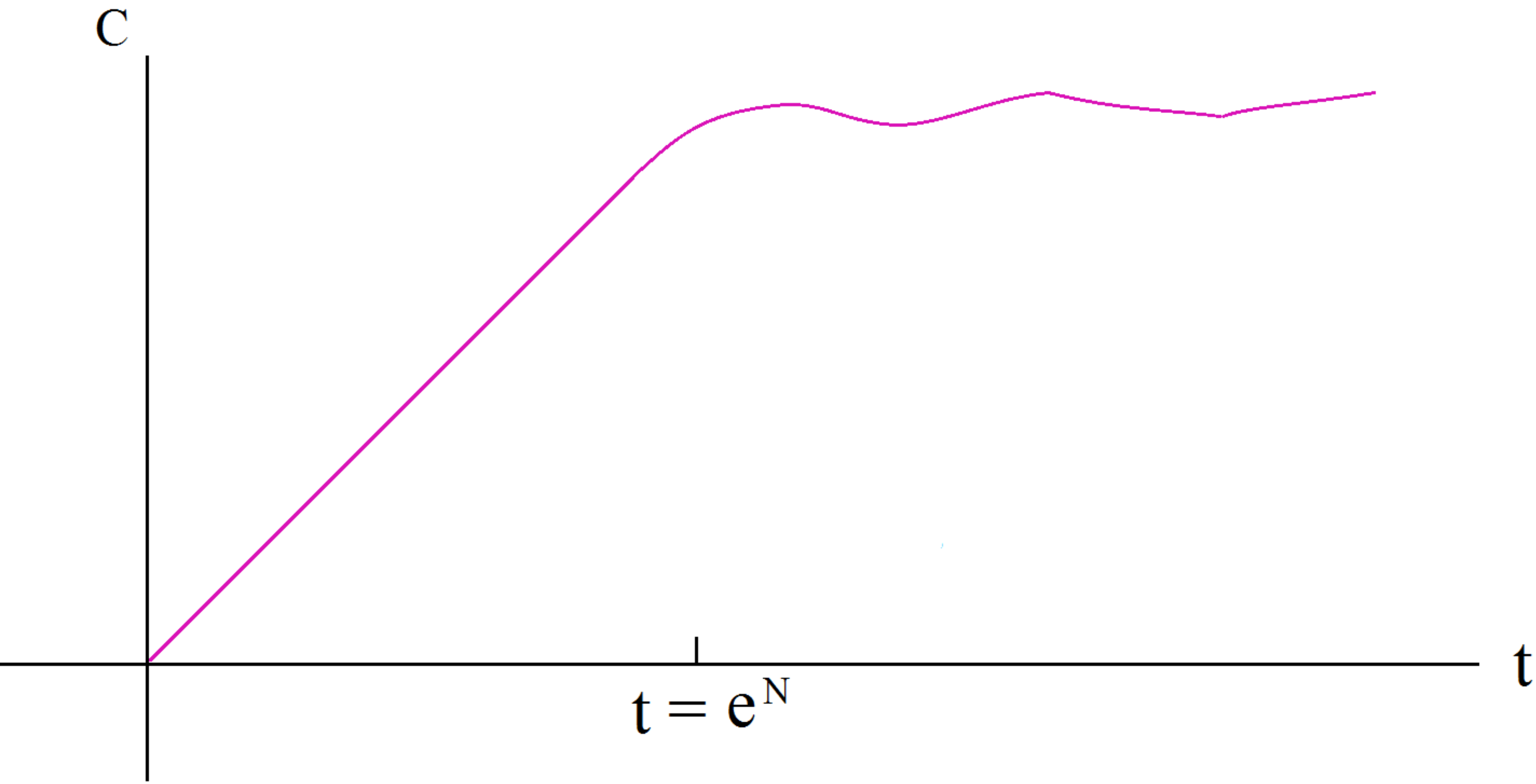
$$t_* = \log S$$

D. Stanford and LS

The two essential differences that make quantum information quantum.

1. Entanglement: the hooks that hold space together.

2. The enormous capacity for complexity: It allows volume to grow almost indefinitely.


C
t
t = e^N

Potential for deep connections with theoretical computer science and complexity theory.

Black Holes and Complexity Classes

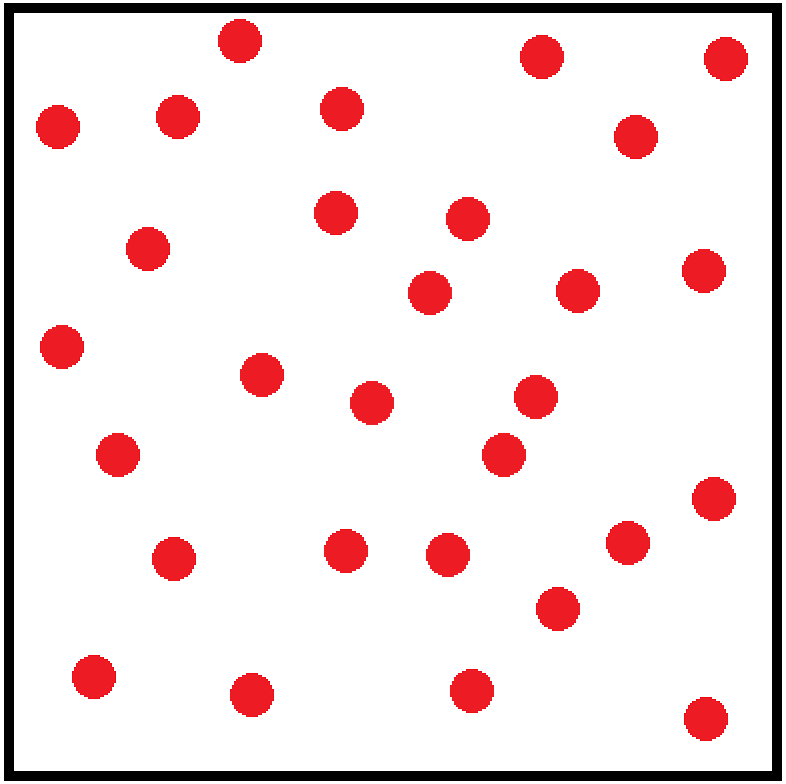
Scott Aaronson[1] and Leonard Susskind[2]

[1] Massachusetts Institute of Technology, Cambridge, MA USA
[2] Stanford Institute for Theoretical Physics and Department of Physics, Stanford University, Stanford, CA 94305-4060, USA

Abstract

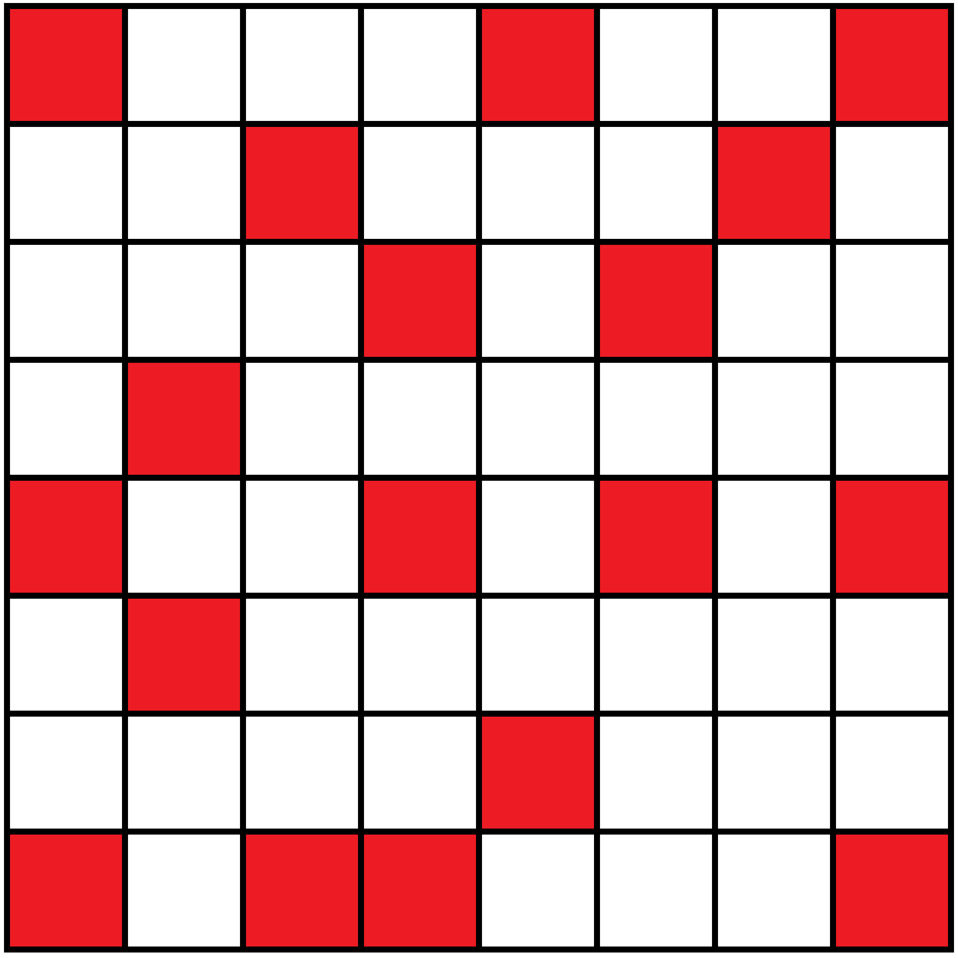
The purpose of this paper is to prove a theorem connecting the growth of complexity in certain types of quantum circuits, with plausible but unproved relations between complexity classes; for example that PSPACE is not contained in BQX/X, where X could stand either for polynomial or sub-exponential. The theorems are suggested by two conjectures about black holes. The first is that the volume of an Einstein-Rosen bridge is dual to the complexity of the quantum state of the black hole. The second is the the growth of an ERB follows the classical behavior as long as possible, namely for a time exponential in the entropy of the black hole. Showing that PSPACE is not contained in BQX/X would give evidence for the latter conjecture, and conversely.

a "computer"



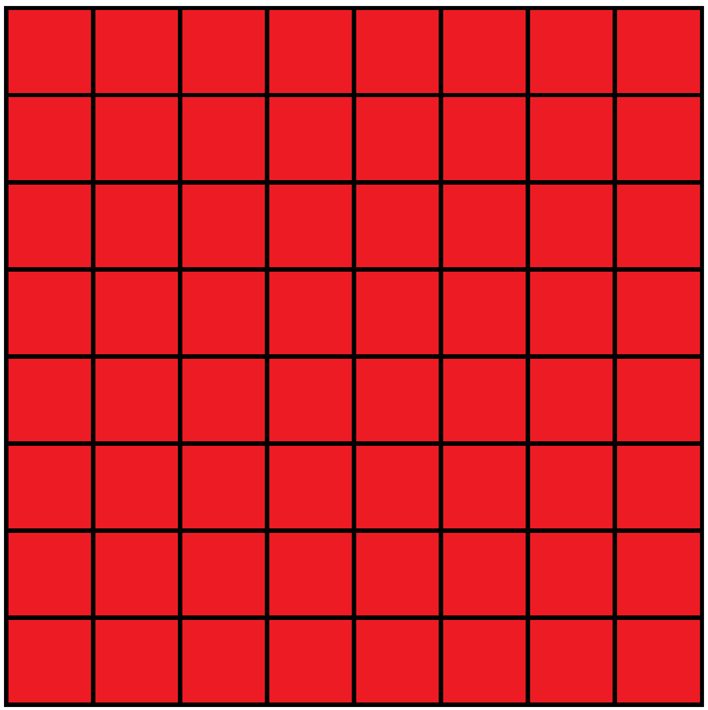
The program is the laws of motion.
The system can be initialized.
It updates intelf.
One can measure the outcome.

a cellular automaton

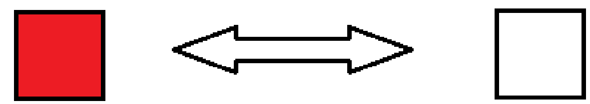


What is complexity?

Simple state



Simple process



Complexity is the minimal number of simple processes required to produce a given state from a simple state.

Maximum complexity and maximum entropy

$$C_{\max} = N/2$$ almost all

$$S_{\max} = N \log 2$$

When a classical system of bits reaches maximal entropy it also reaches maximal complexity.

Quantum States are far more complex.

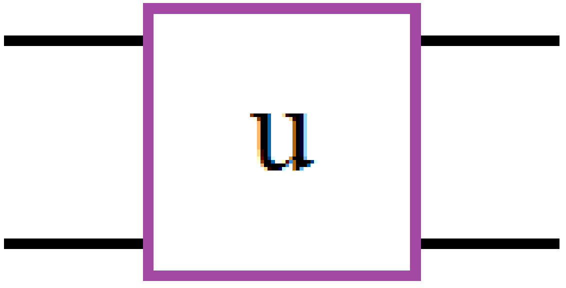
$$|\psi\rangle = \sum_{1}^{2^N} C_i |i\rangle$$

Specify 2^N complex coefficients.

Simple state: Same as before.

Simple process:

Unitary quantum gate



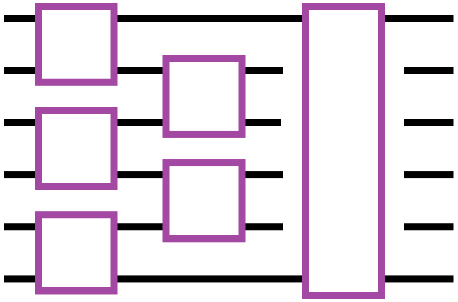
Complexity is the minimum number of gates required.....

$$C_{\max} \sim e^{N}$$ almost all

$$S_{\max} = N \log 2$$

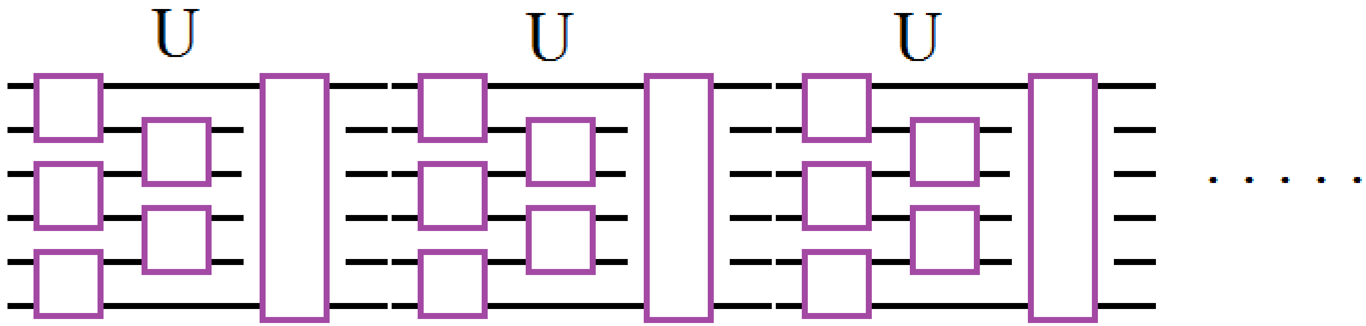
Quantum complexity may continue to grow long after entropy reaches its maximum.

U



(universal and unitary)

$$U^t =$$



$$|\Psi(t)\rangle = U^t |\Psi\rangle$$

The complexity of such circuits grows linearly in time.

$$\frac{dC}{dt} = S\,T$$

for some period.

Volume of ERB grows linearly

$$\frac{dV}{dt} = S\,G$$

Complexity grows linearly

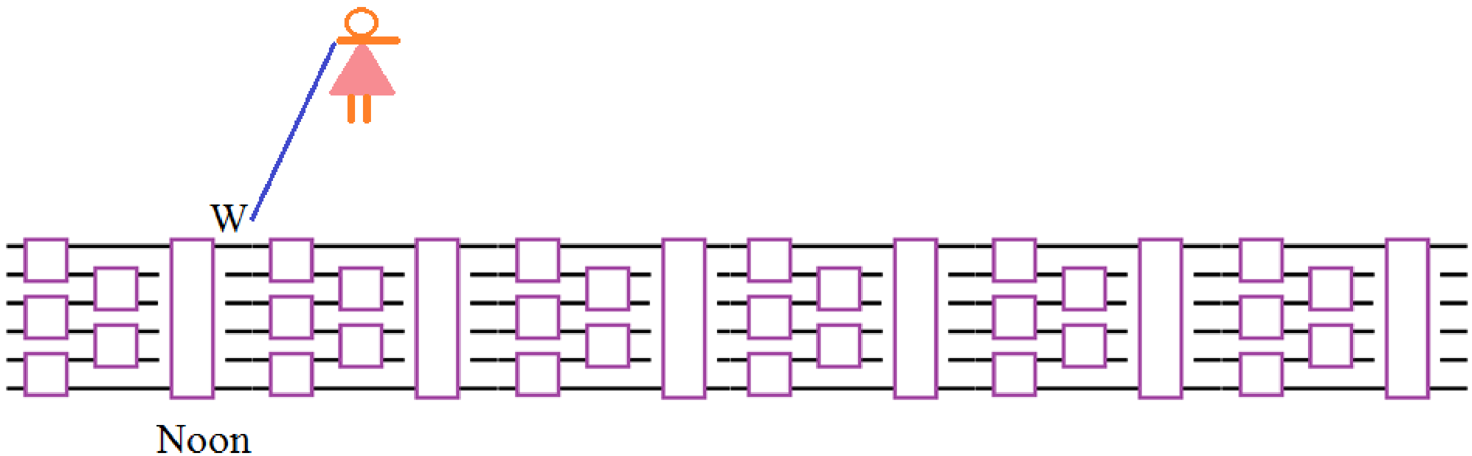
$$\frac{dC}{dt} = S$$

Volume of Einstein-Rosen bridge ~ complexity of quantum state

$$V = C\,G$$

What evidence do we have for C/V duality?
Lots. Precursors are good probes.

Consider Alice's lunch date problem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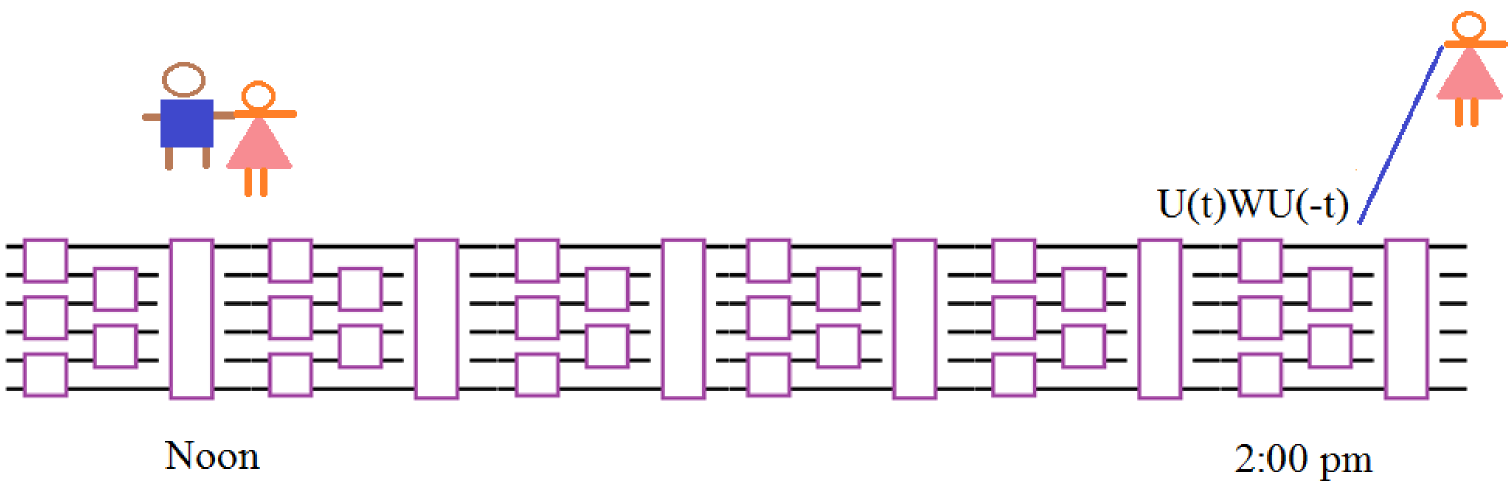
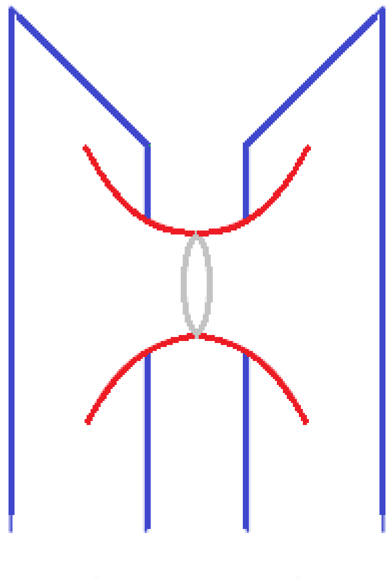

U(t)WU(-t)
Noon
2:00 pm

Precursor operators

U(t) W U(-t)



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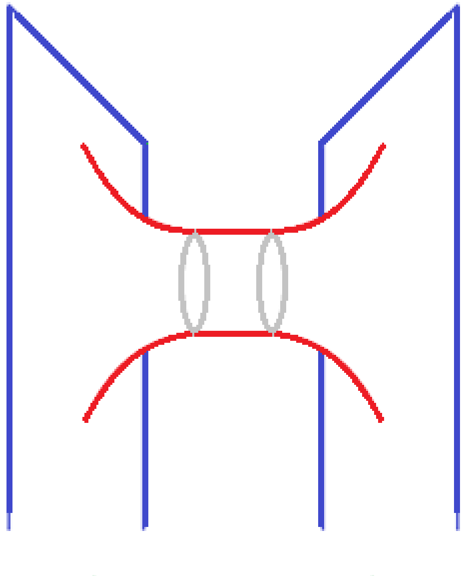
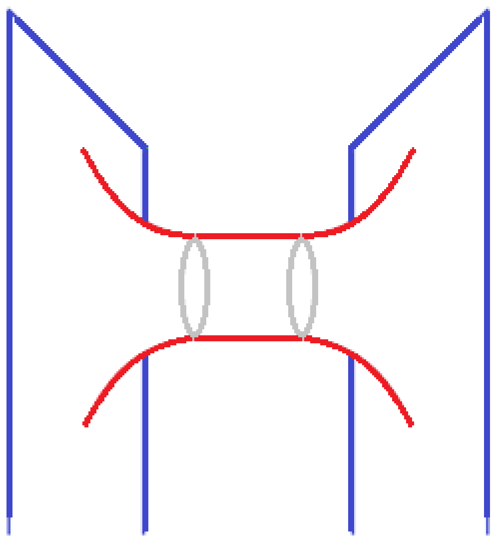
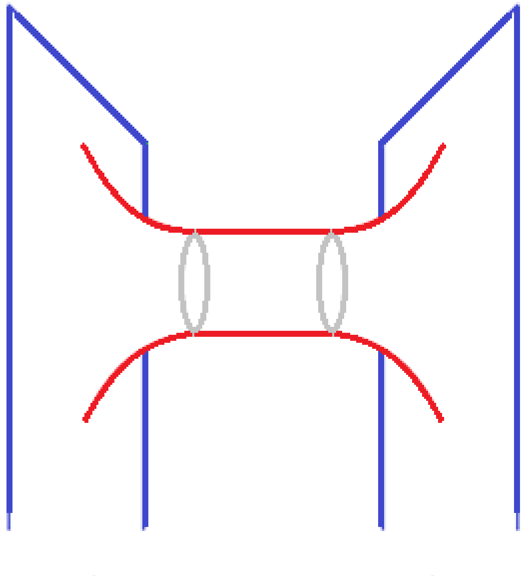
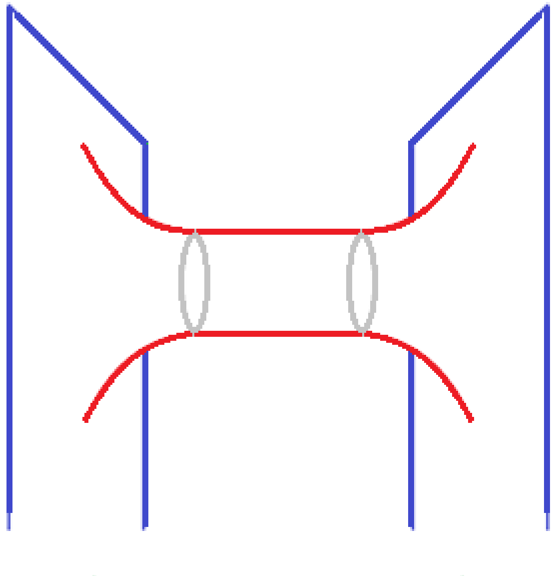
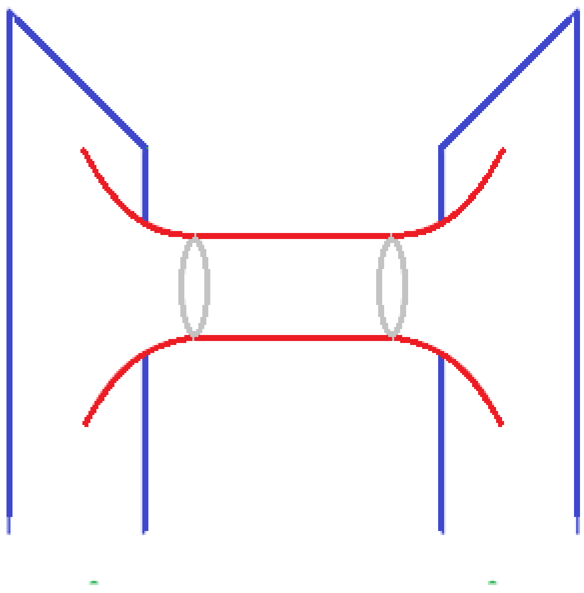
D. Stanford, LS
D. Roberts, D. Stanford, LS

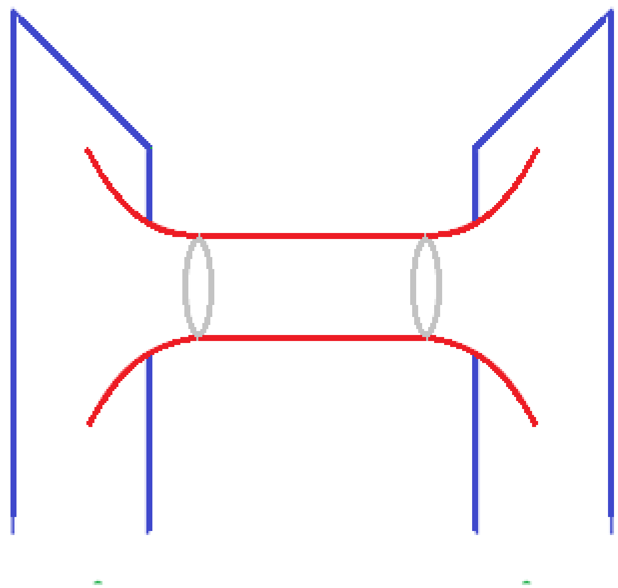


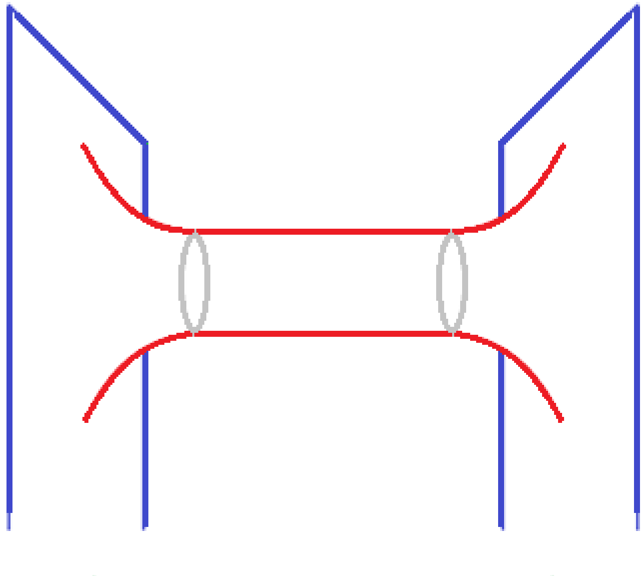


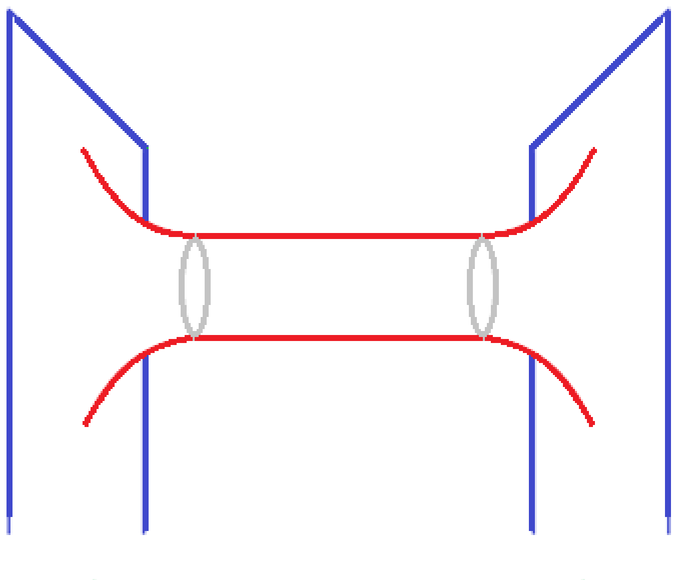


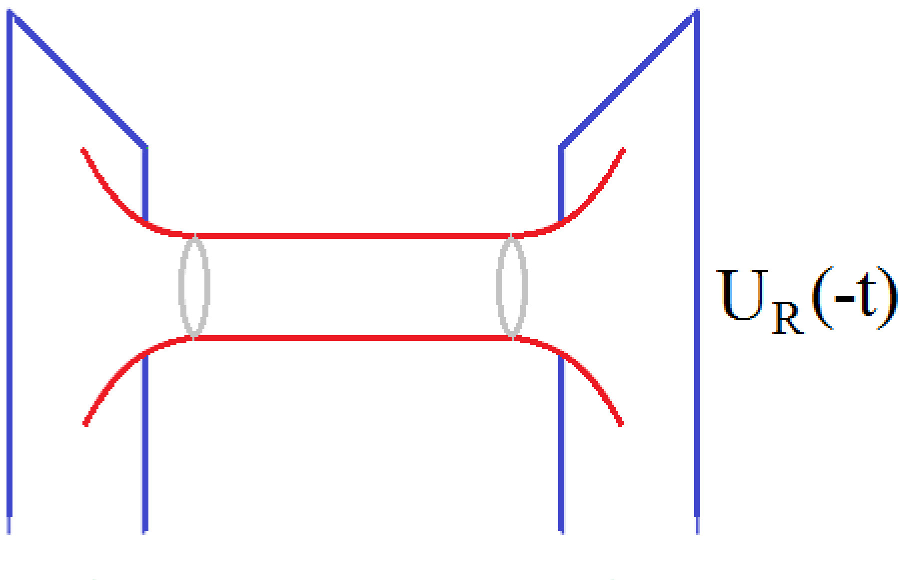


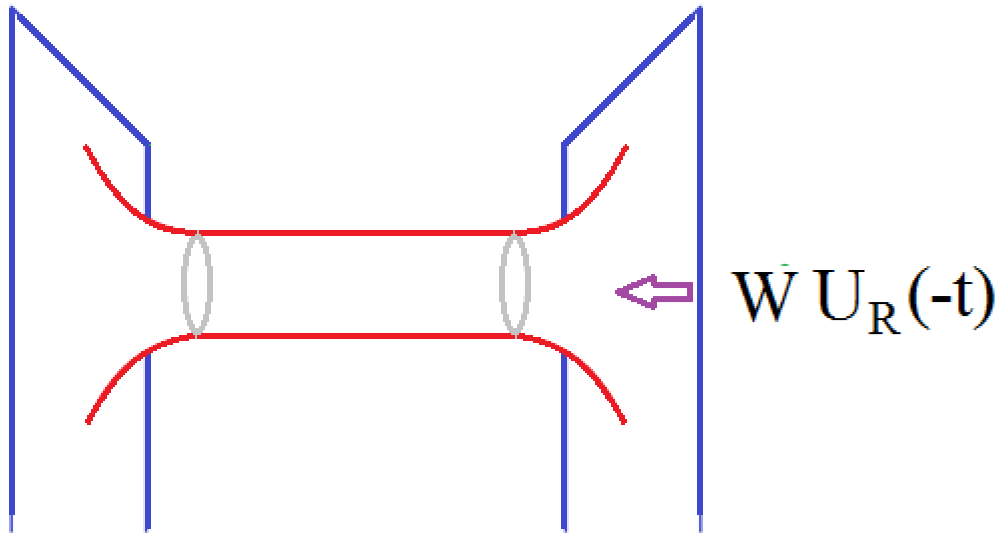


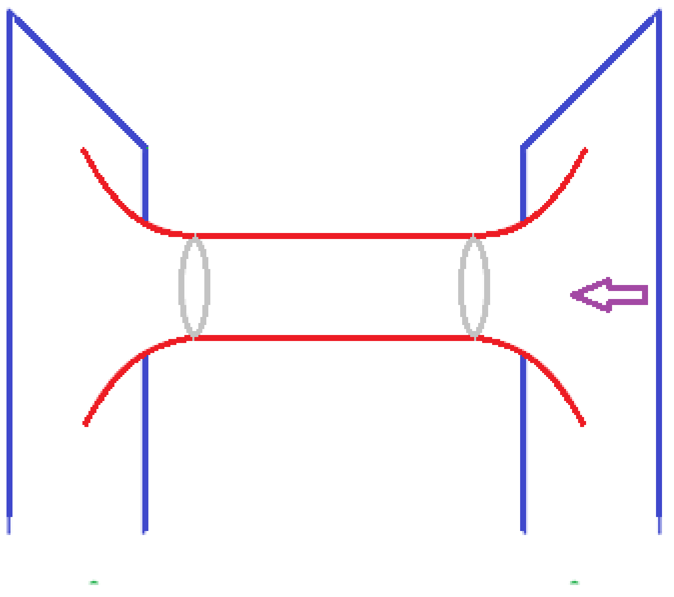


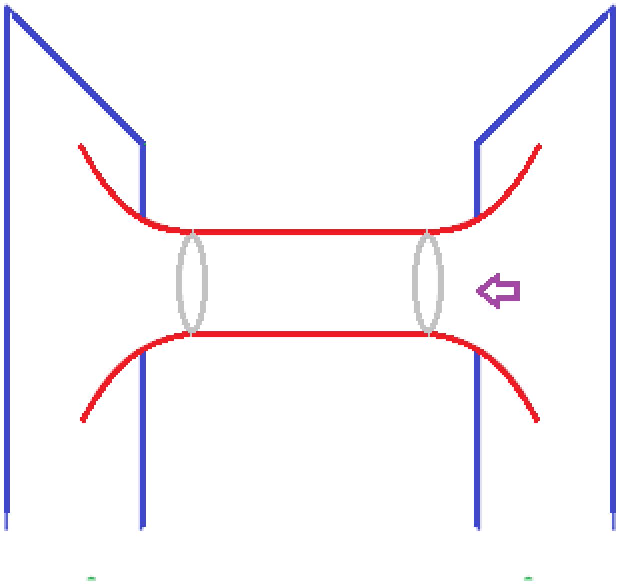


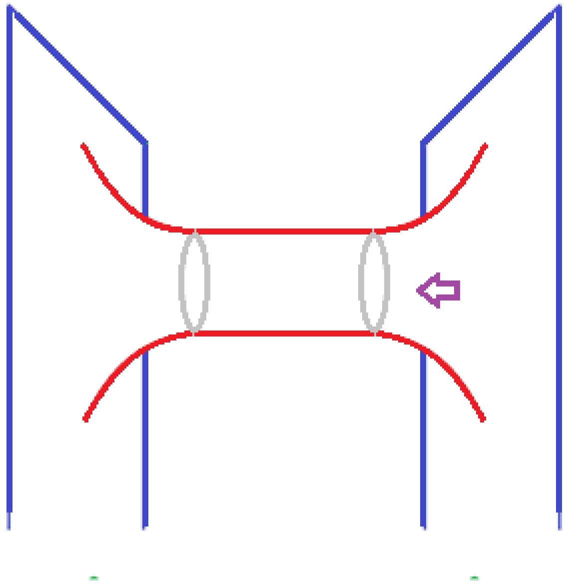


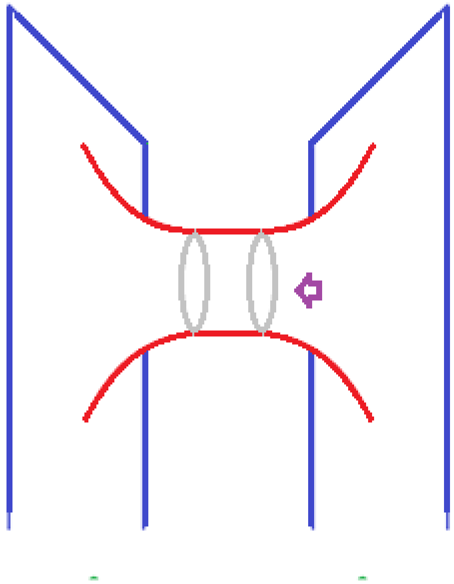

$U_R(-t)$

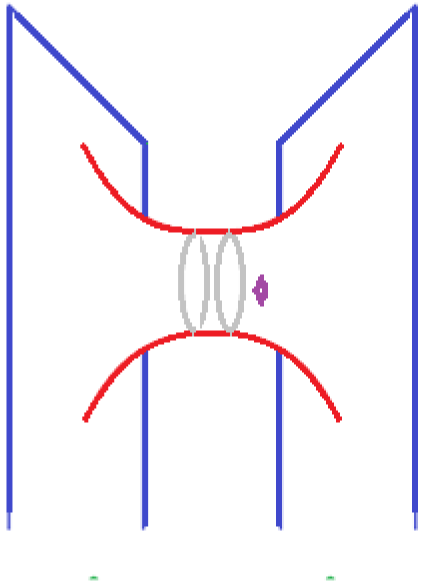

W U_R(-t)

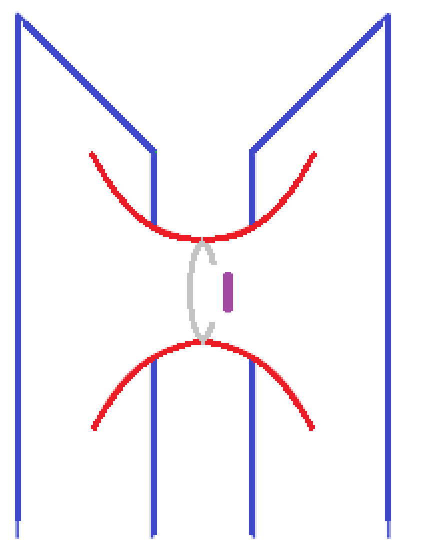




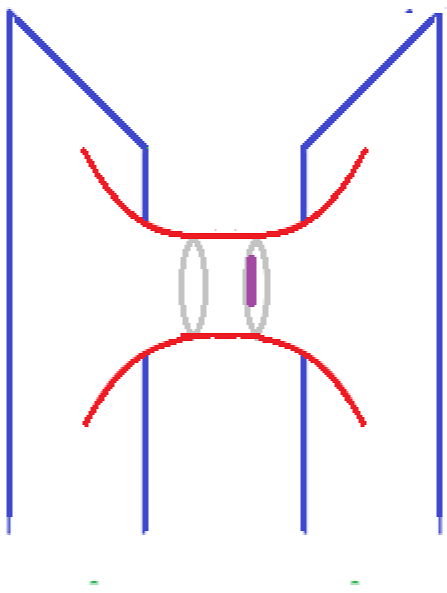


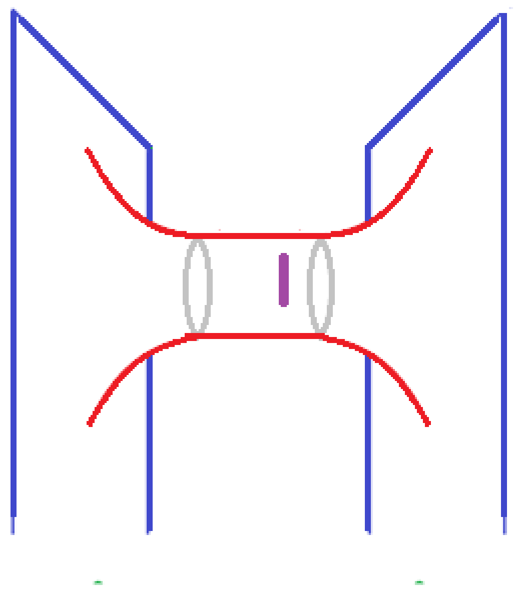


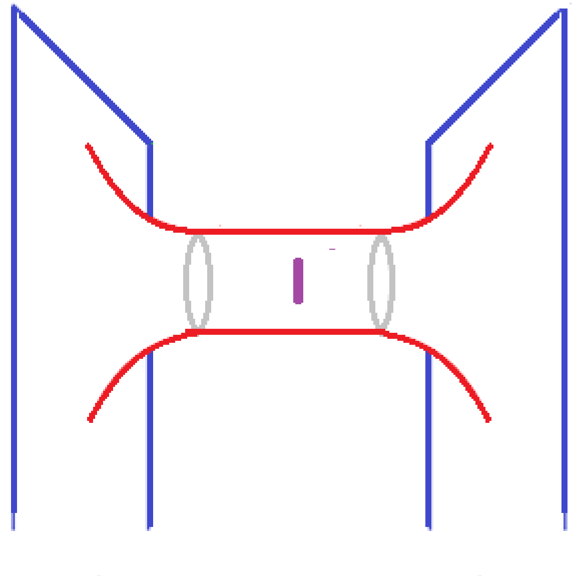


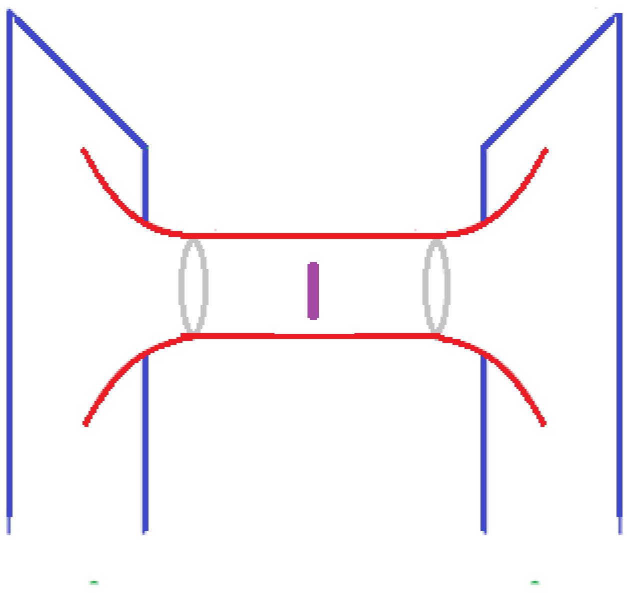


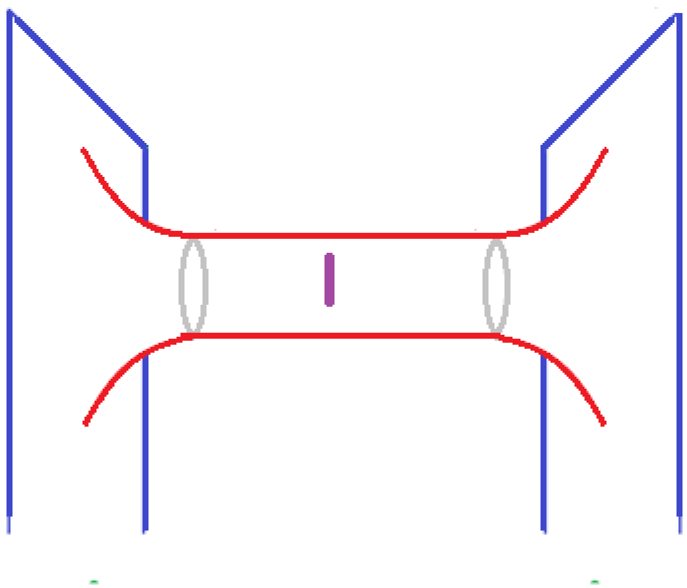
$U_R(t)WU_R(-t)$

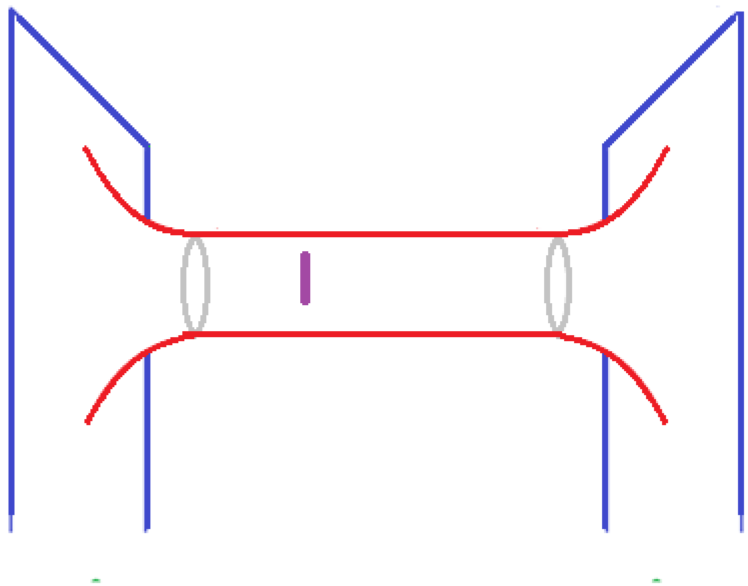






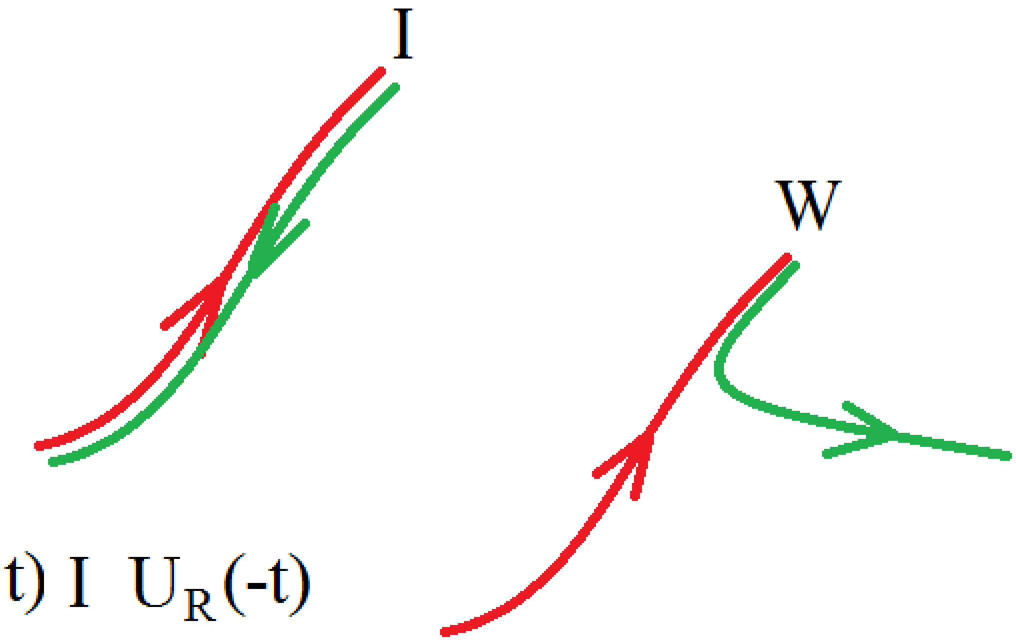






Additional Complexity due to precursor

$U_R(t)\,W\,U_R(-t)$:



$U_R(t)\,I\;U_R(-t)$

$U_R(t)\,W\,U_R(-t)$

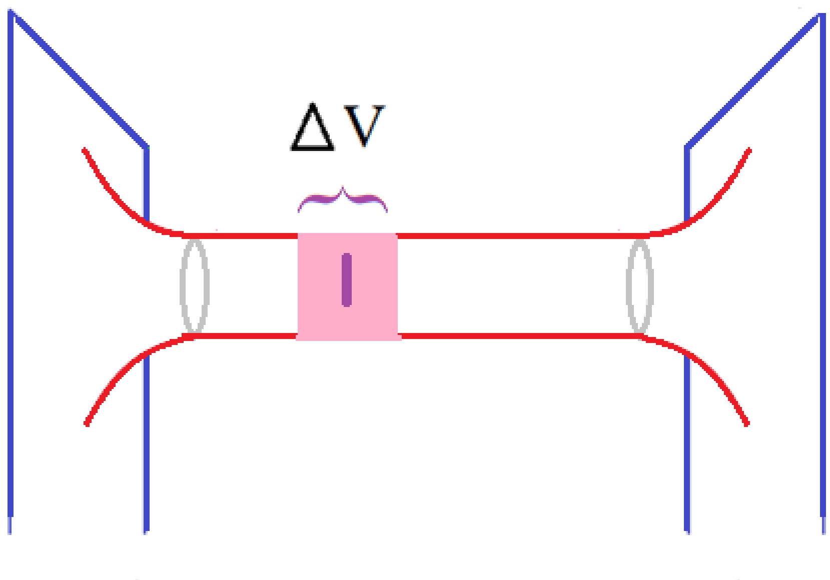
$$\boxed{\Delta C = 2\,S\,(t - t^*)}$$

All this was pure quantum circuitry and counting the minimum number of gates needed to make U(t)WU(-t).

On the other hand the Einstein field equations of GR can be solved including the reaction of the geometry to the shock wave.

S. Shenker, D. Stanford

The effect of the shockwave is to add a segment of volume ΔV to the bridge.



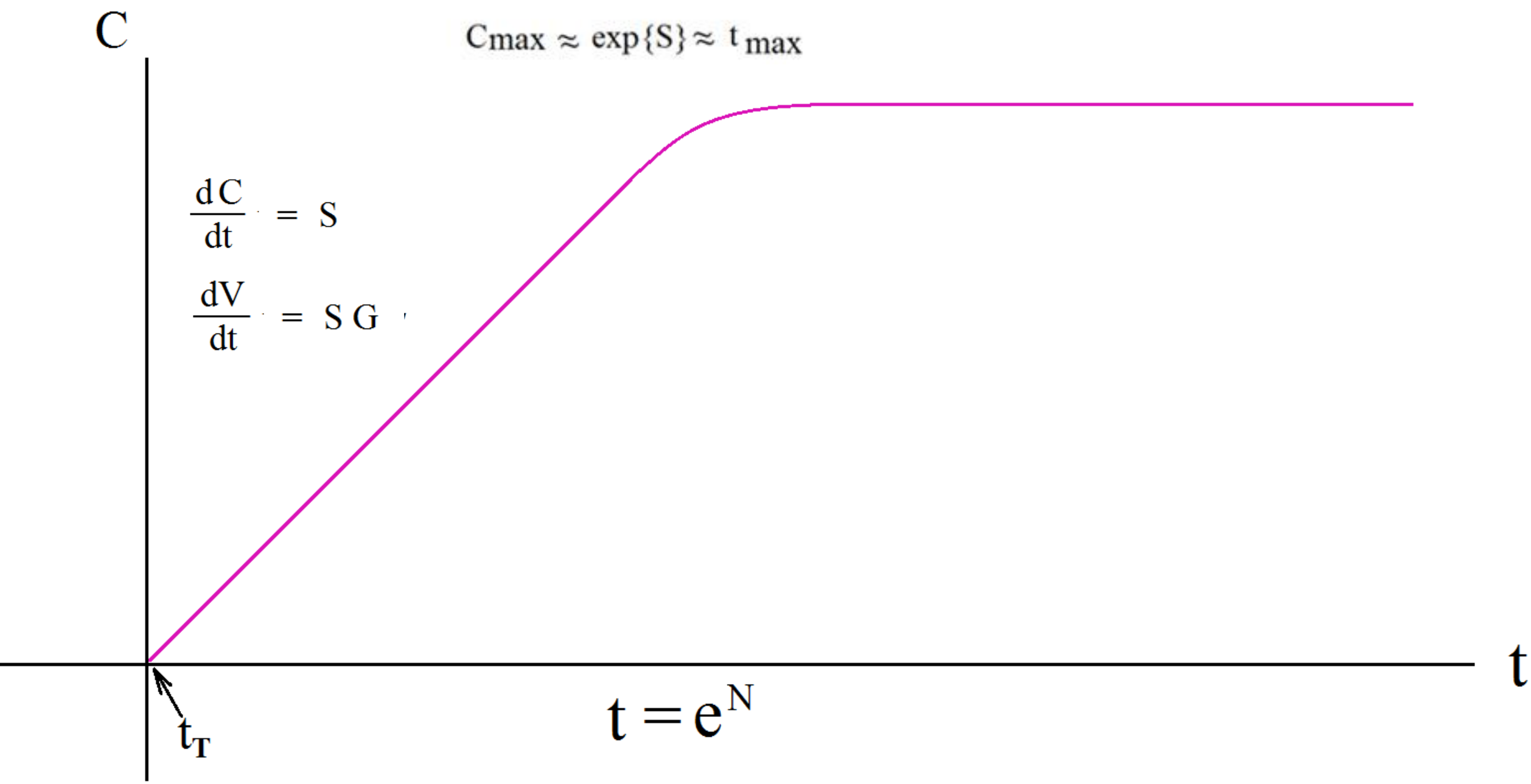
ΔV is calculated by solving the Einstein GR equations with the shockwave as source.

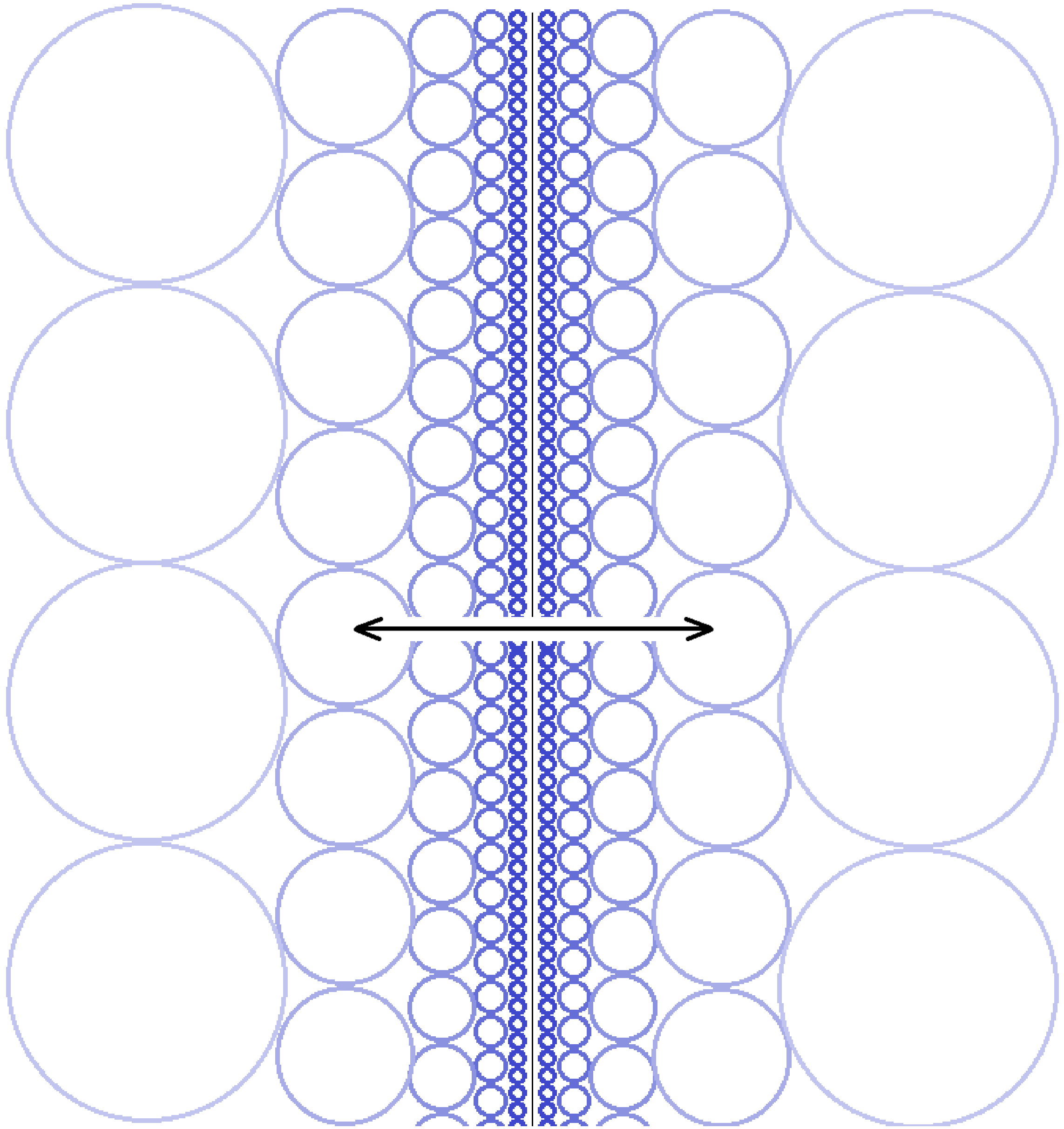
$$\boxed{\Delta V = G_N \, \Delta C}$$

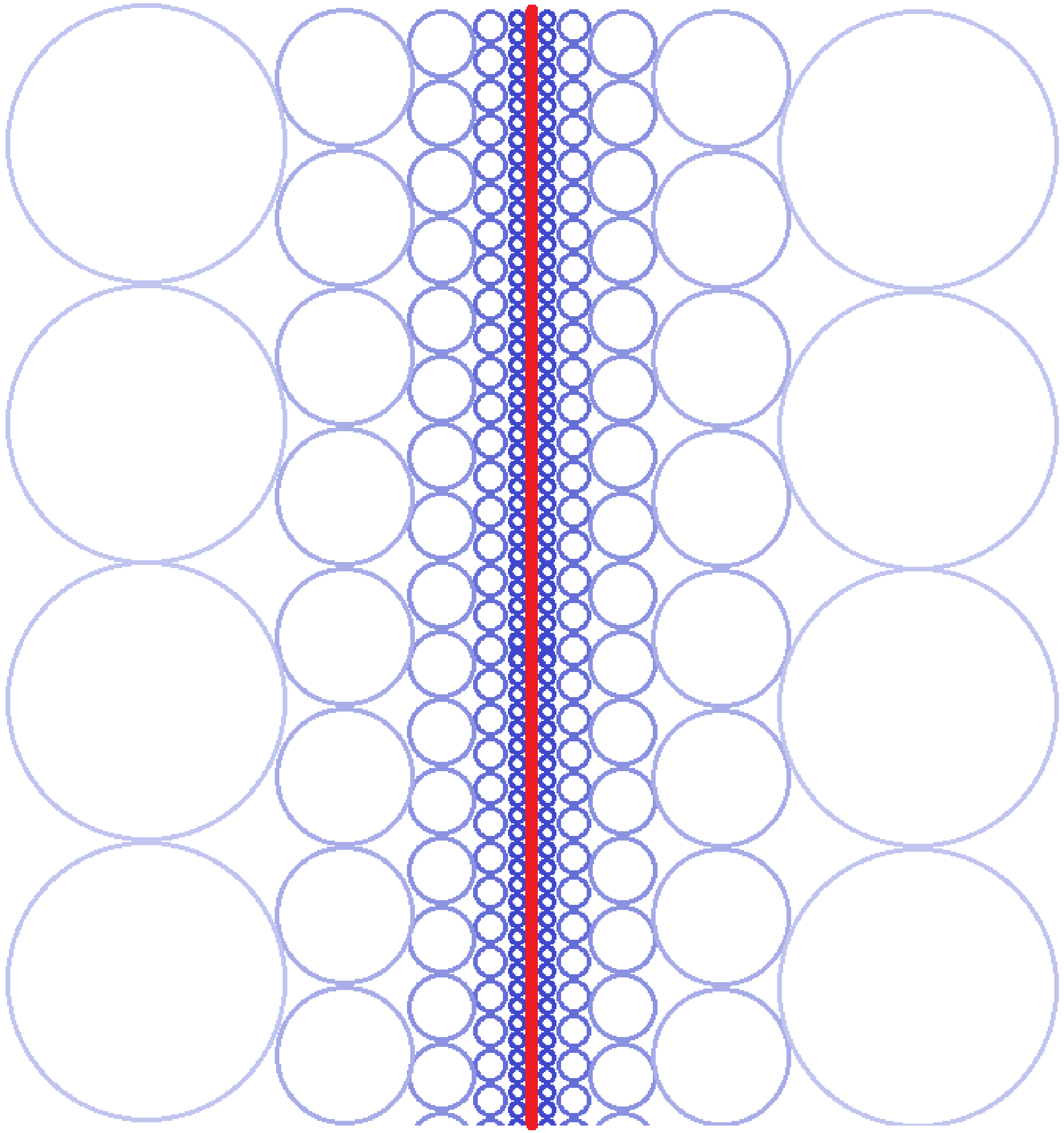
D. Stanford, LS
D. Roberts, D. Stanford, LS

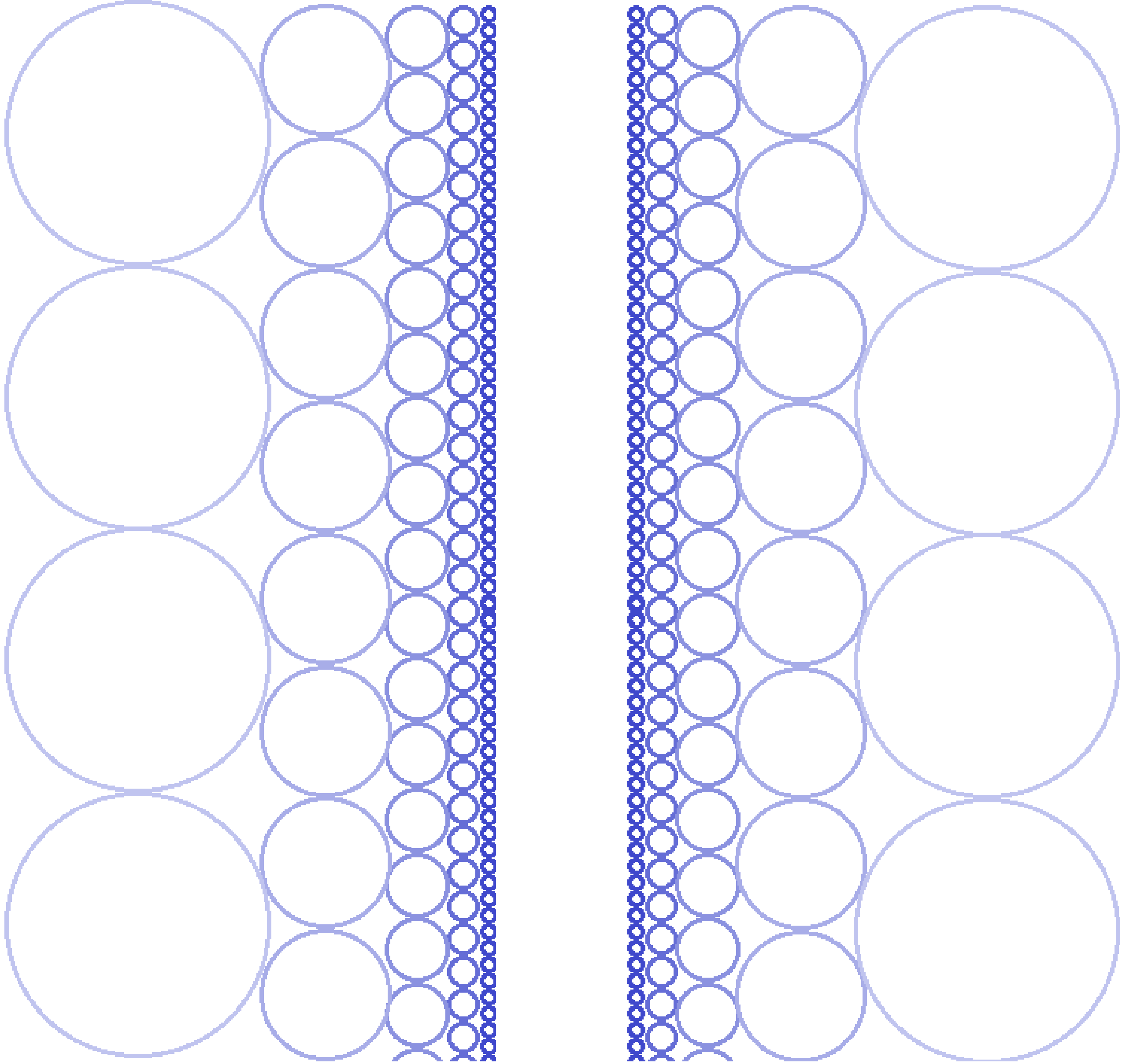
Assumimg V ~ C there is an upper bound on how long classical GR can describe the ERB.

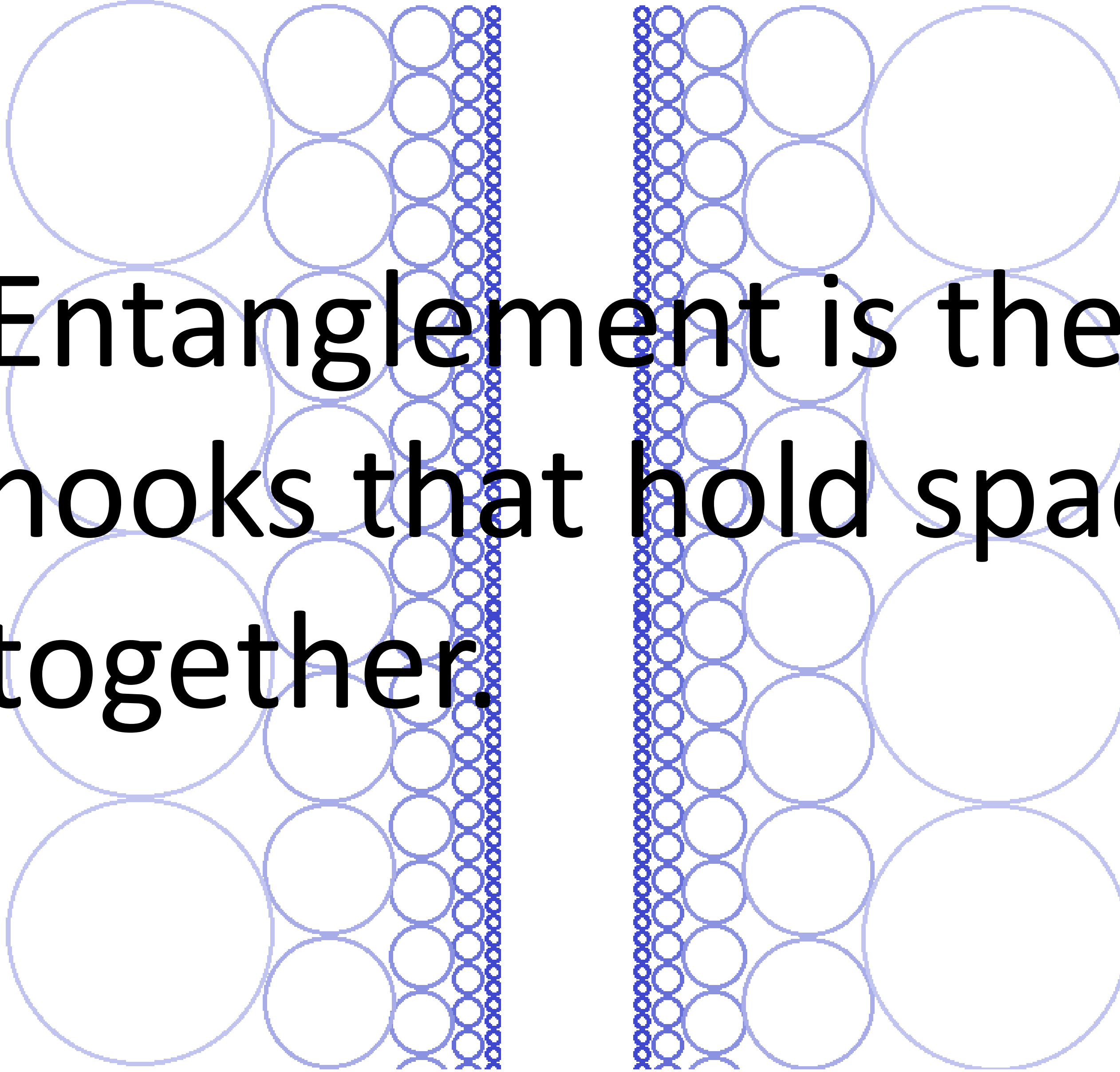
$$C_{max} \approx \exp\{S\} \approx t_{max}$$









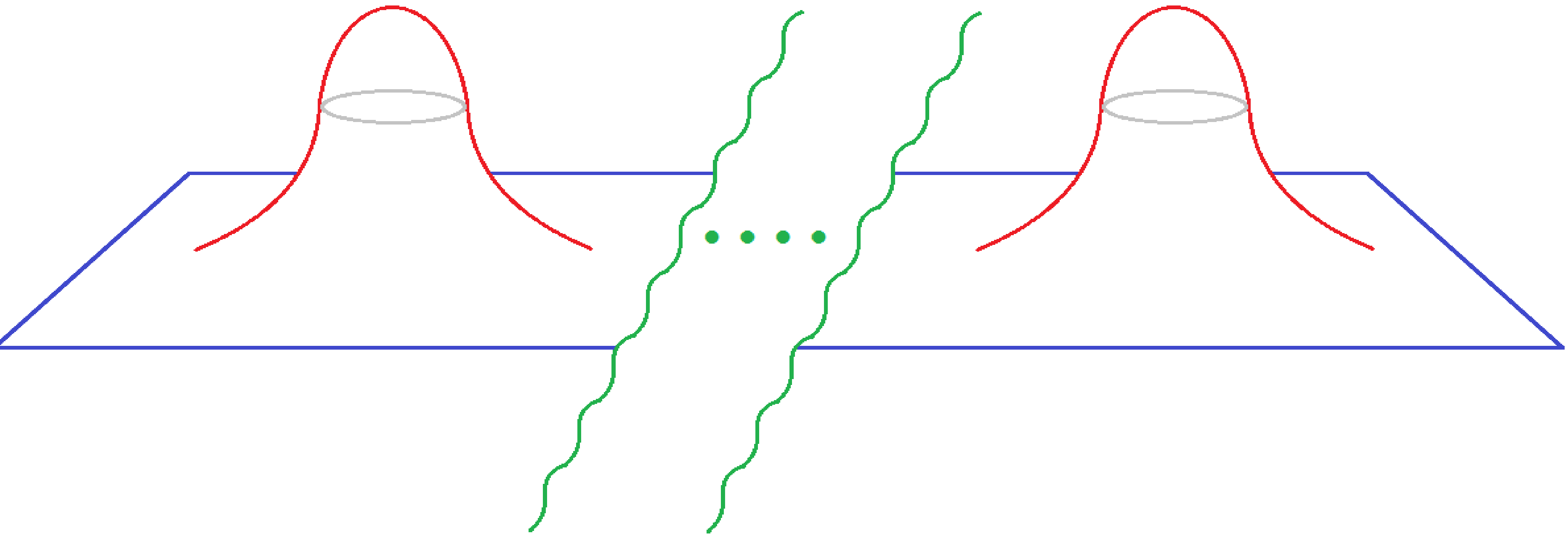


Entanglement is the hooks that hold space together.

Mark Van Raamsdonk

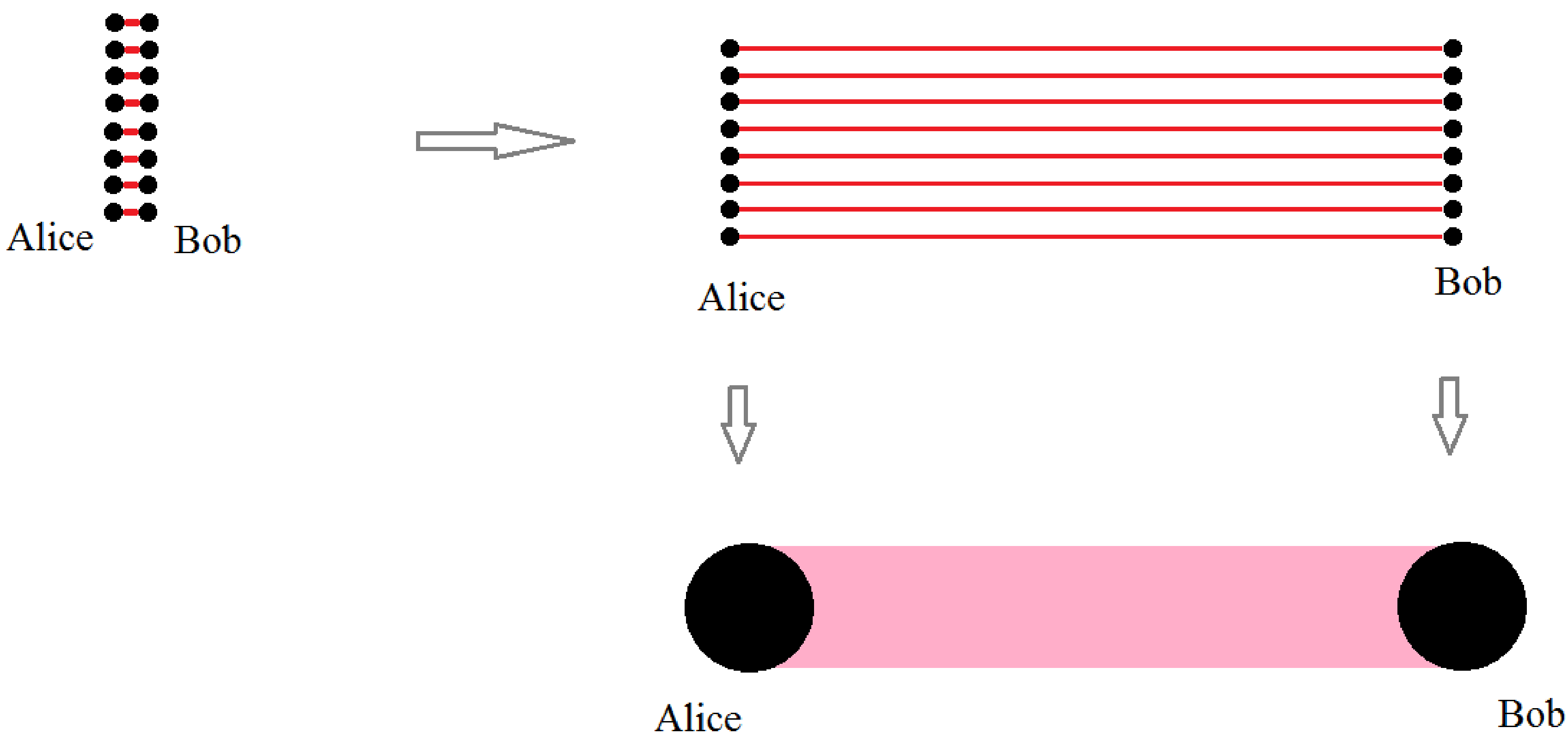
Version 1

Alice and Bob independently
create black holes.



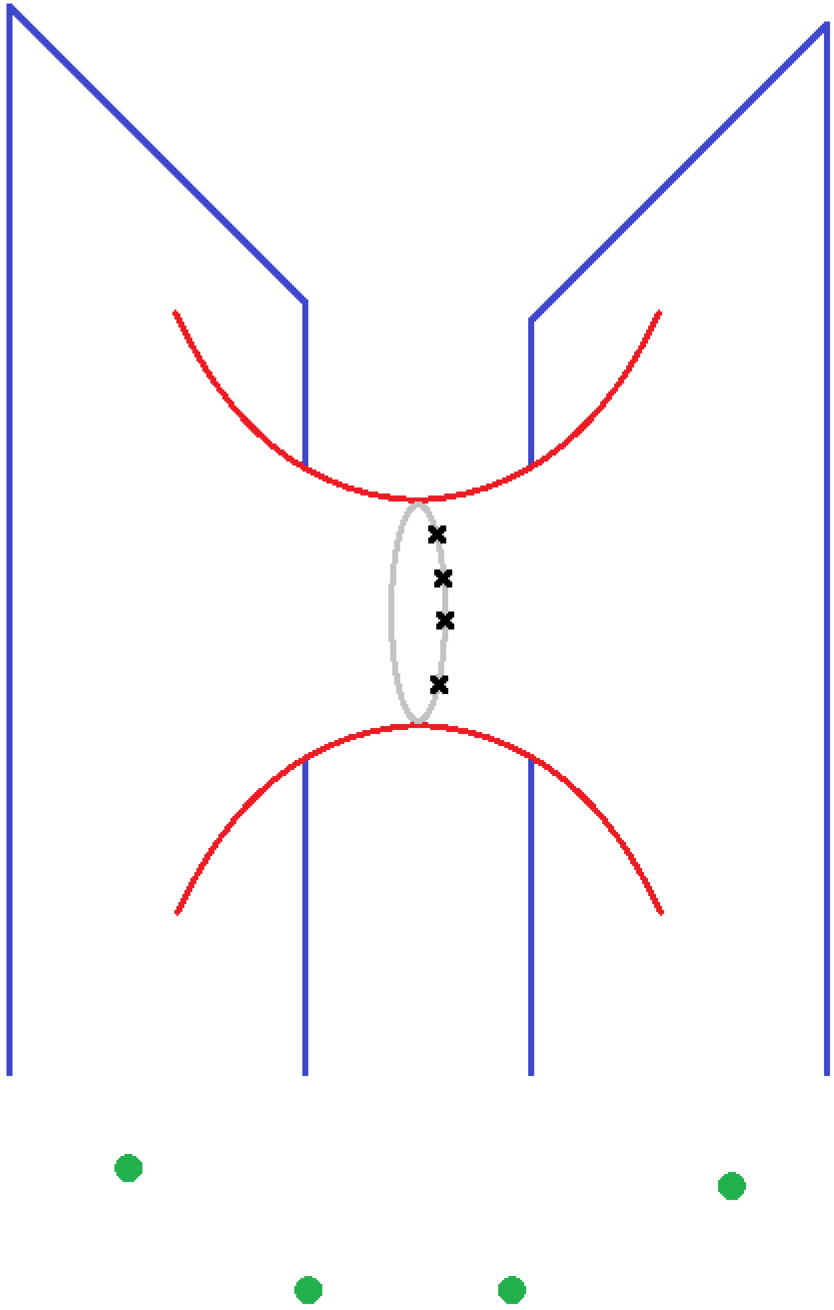
V 2

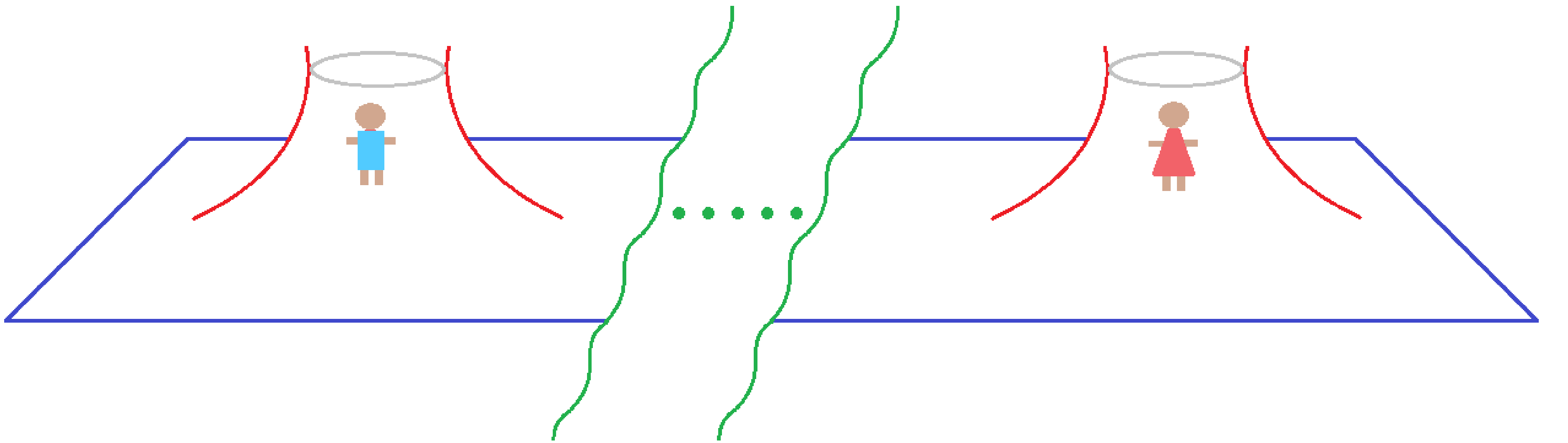
Alice and Bob create black holes out of entangled matter.

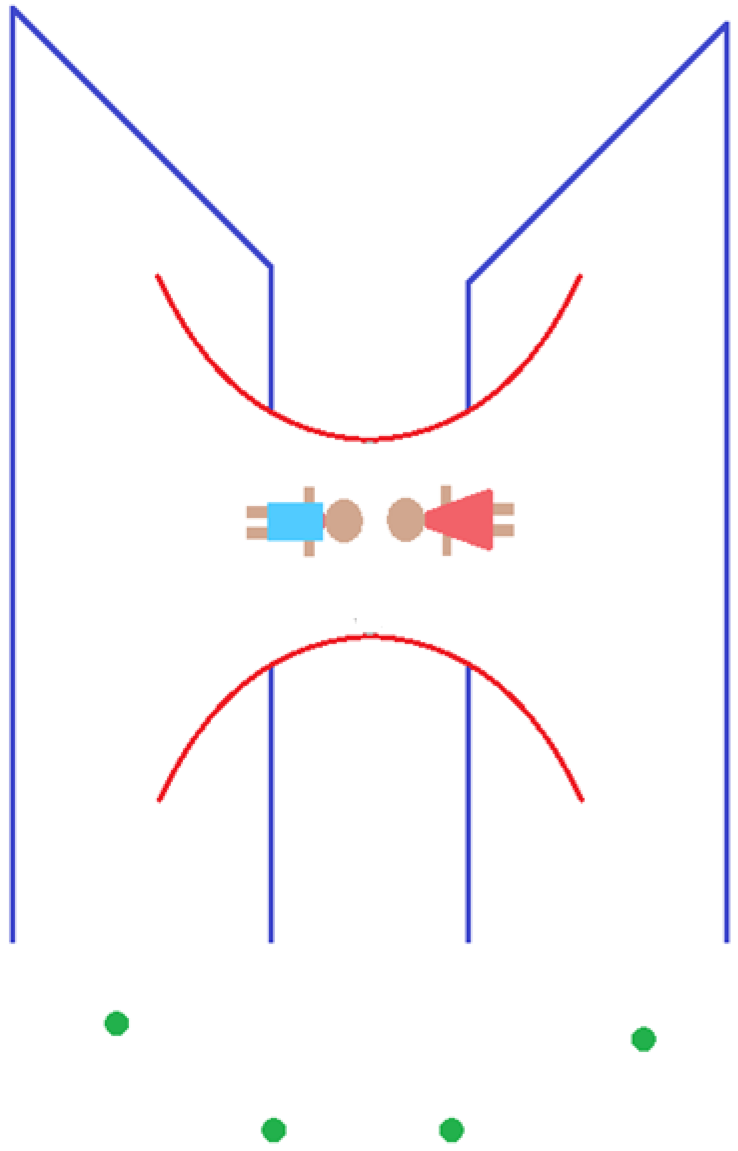


ER=EPR
EPR=ER

J. Maldacena, LS







Complexity increases. But is it interesting?

It depends who asks.

No for a condensed matter physicst interested in thermodynmics, transport, corrlation functions....

Computer scientist? The system could be a quantum computer solving harder and harder problems or even a theorm-proving machine. The growth of complexity is closely related to fundamenal questions about how powerful QC are: questions such as: Is PSPACE contained in BQP/poly?

Complexity increases. But is it interesting?

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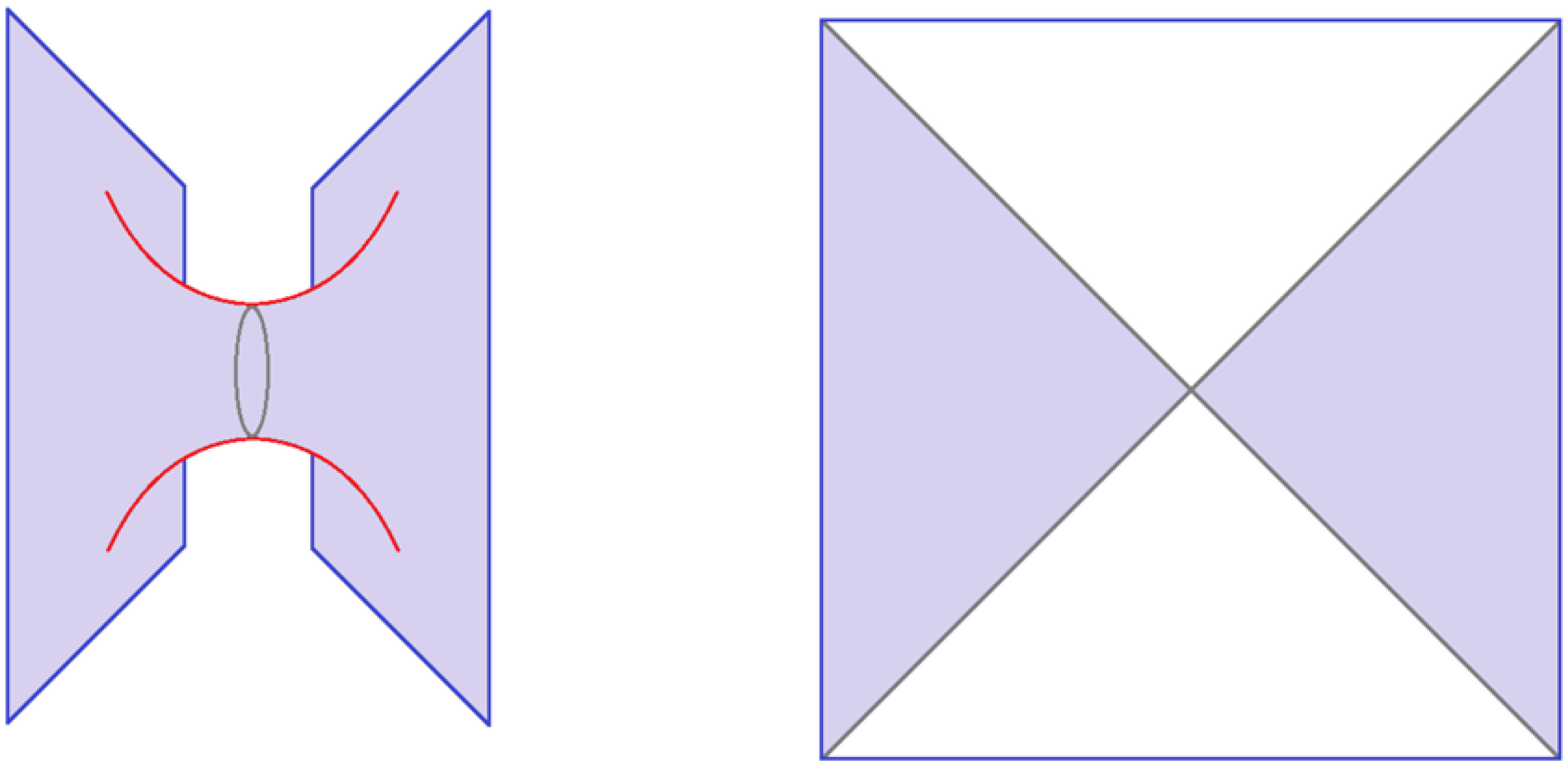
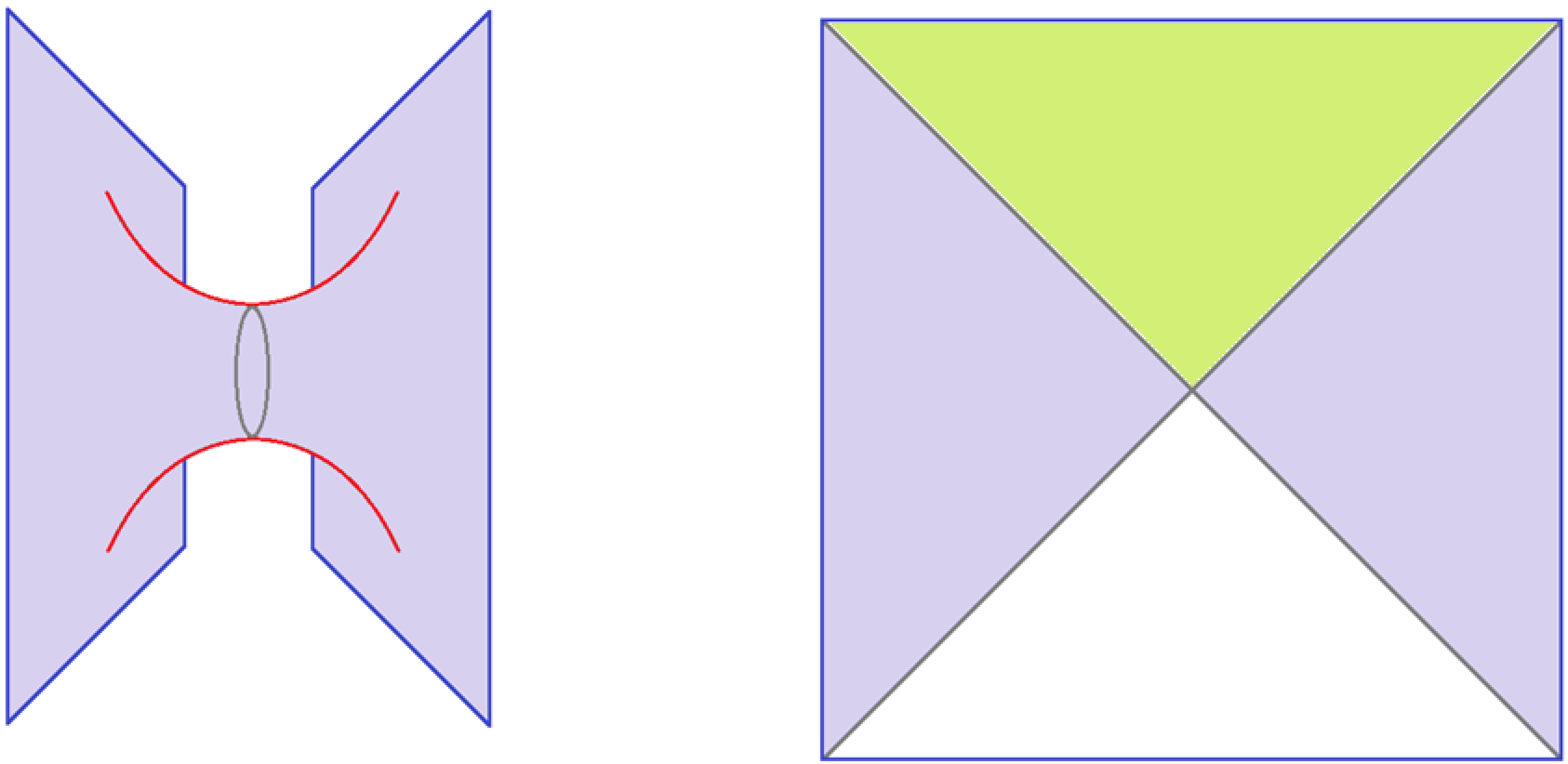
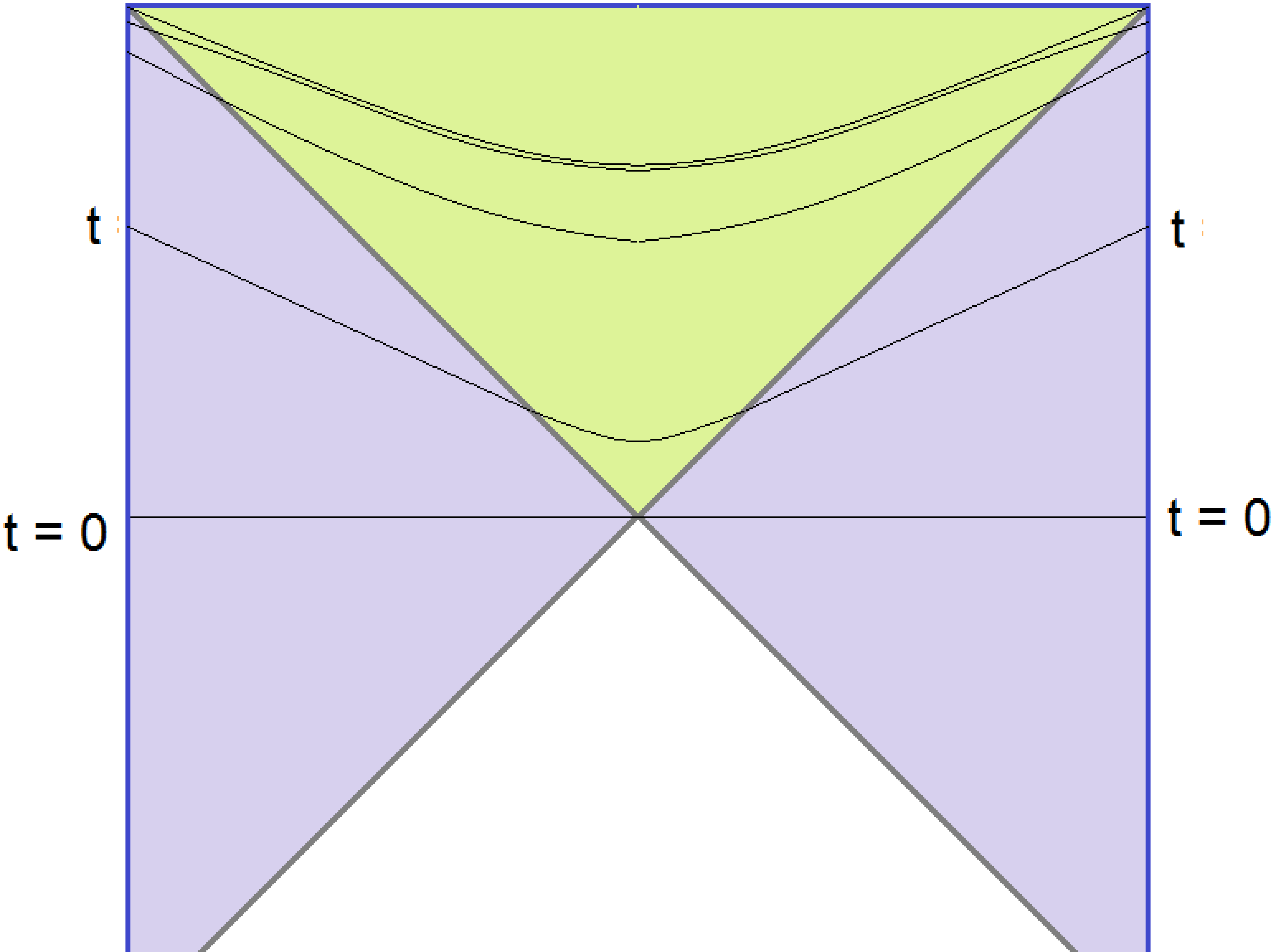
Black hole theorist? The growth of complexity governs the emergence of the space behind the horizon.

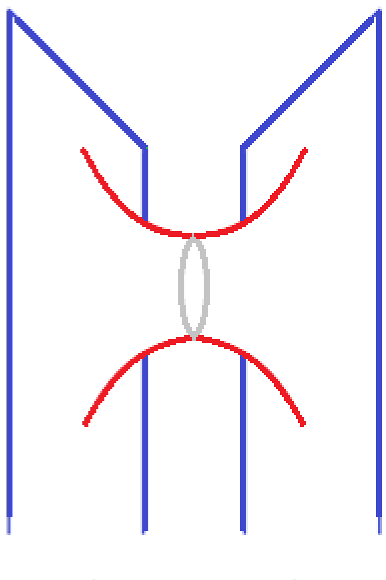


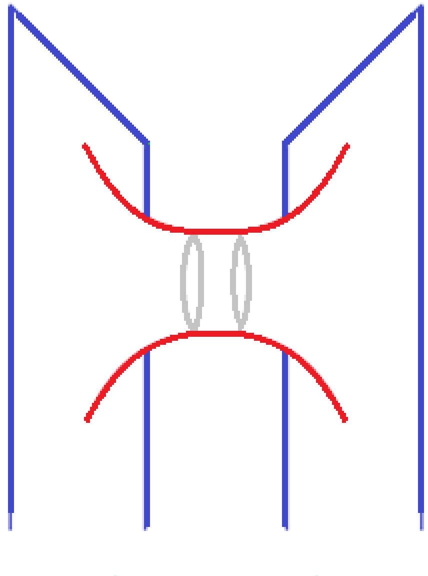
ER = EPR

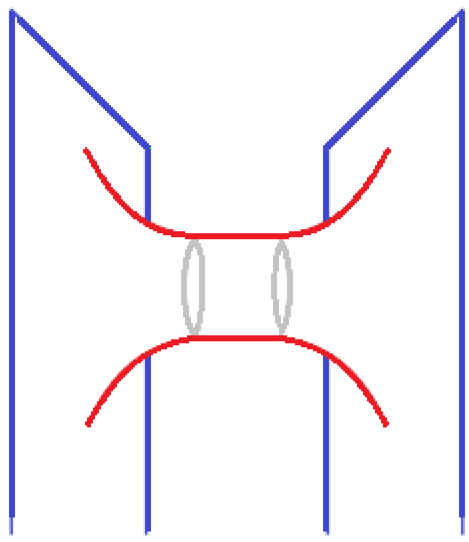


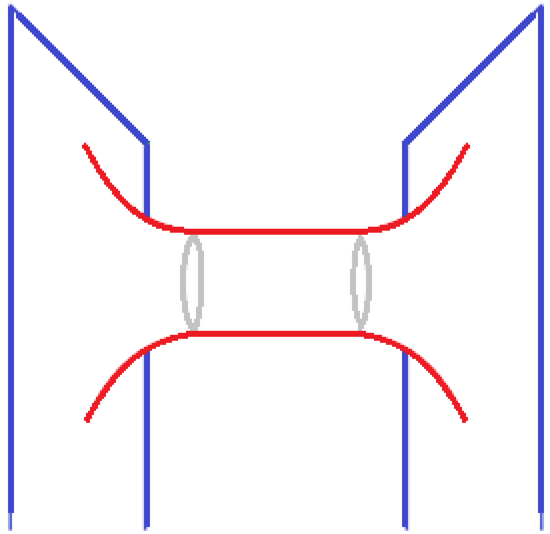
ER = EPR

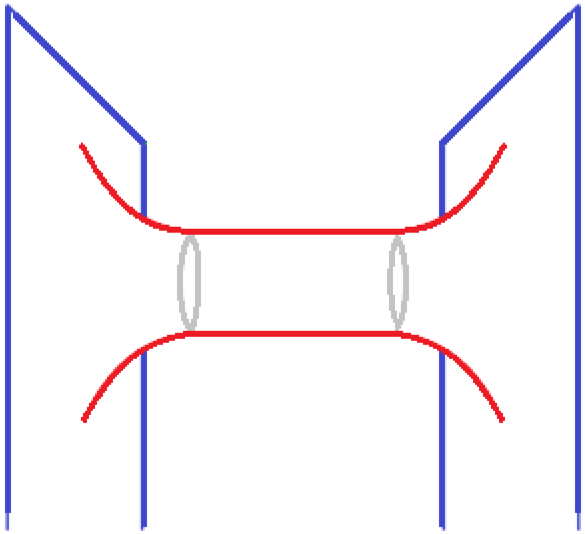

t
t
t = 0
t = 0

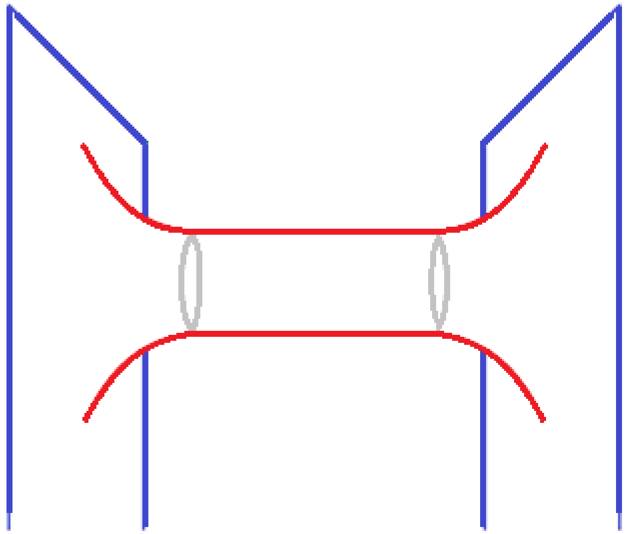


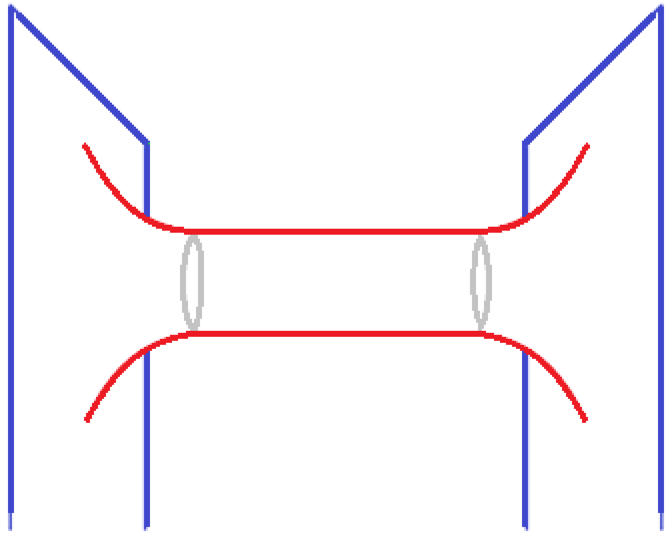


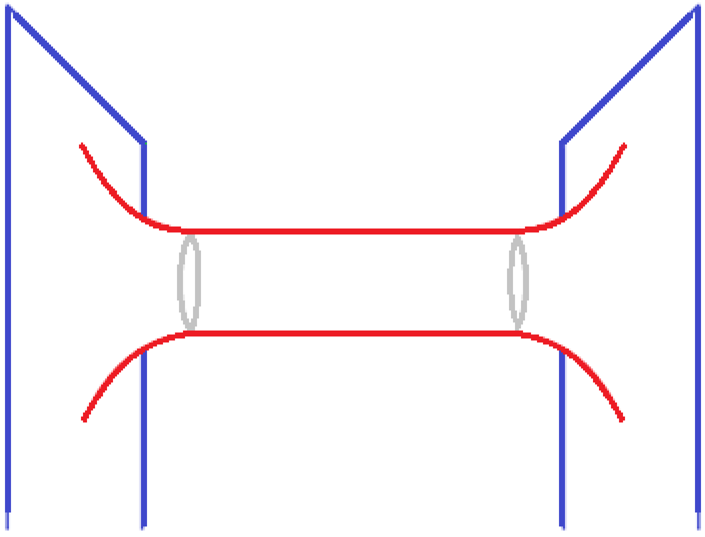


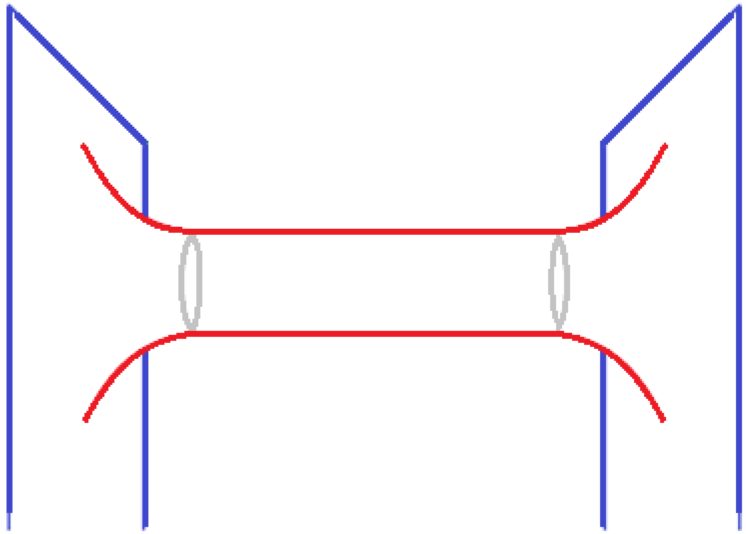












Einstein-Rosen bridges grow with time.

$$\frac{dV}{dt} = \frac{A}{\ell_{ads}} = S\,T\,G$$