s sourcery

Brian Swingle

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John McGreevy, Shenglong Xu

References

- s source framework, gapped phases w/ JM - 1407.8203
- entropy bound w/ JM - 1505.07106
- gapless “square root” states w/JM and SX - *coming soon*
- Important other work: White’s DMRG, MPS, Vidal’s MERA, other tensor networks, long history of real space RG, see 1407.8203 for an extensive list of references

The ground state problem

Given a local Hamiltonian H, "determine" its ground state and compute physical properties

Is there any way this problem could have a general solution? Very hard to imagine, provably false in some cases, yet I hope to convince you that for a very broad class of H the answer is YES!

[QMA hard: Gottesman et al., also glassy states, etc.]

Families of Hamiltonians/states

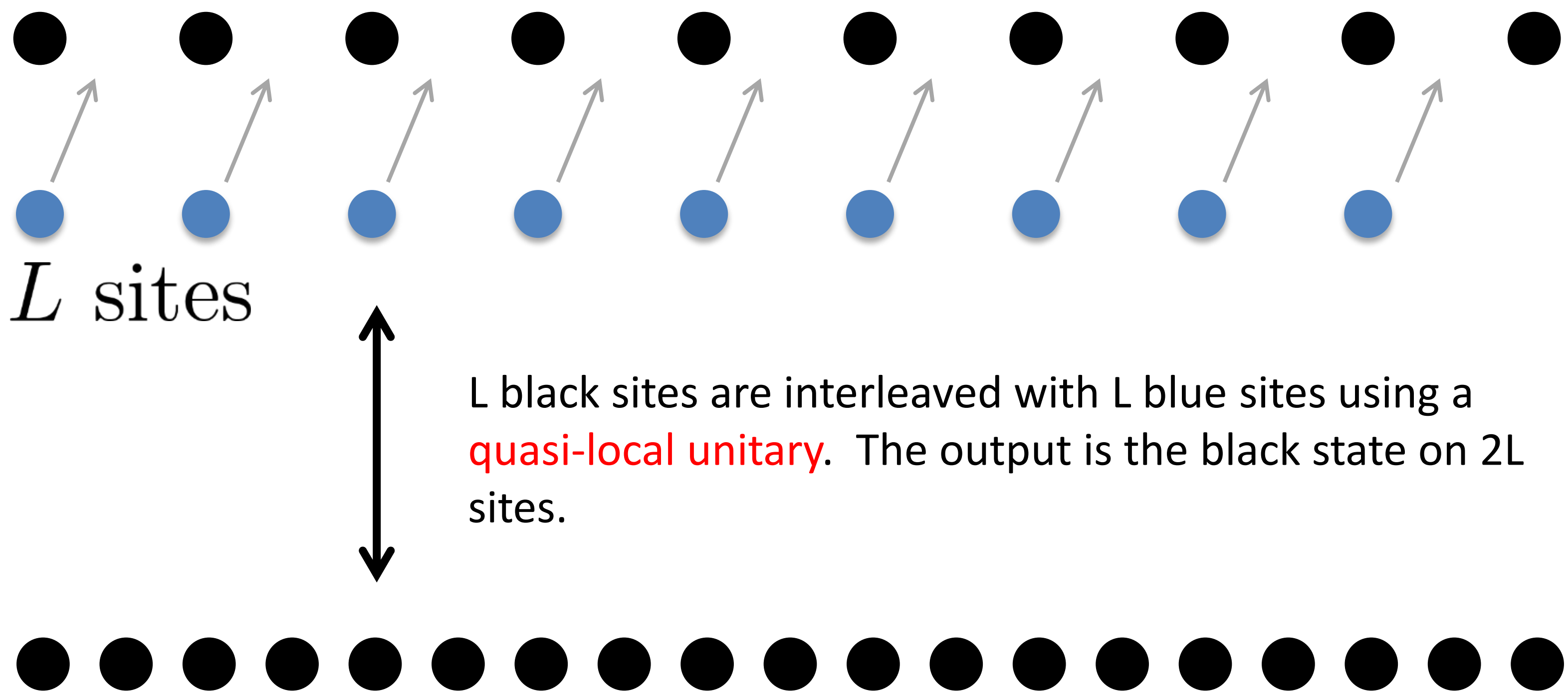
Throughout we consider families of Hamiltonians indexed by system size: $\{H_L\}$

These Hamiltonians have corresponding ground states: $\{|\psi_L\rangle\}$

We will study transformations between states at different L; product states can always be subtracted or added at will: $|0\rangle^M, \forall M$

DEFINITION AND PROPERTIES

“RG” construction of wavefunction



L black sites are interleaved with L blue sites using a quasi-local unitary. The output is the black state on 2L sites.

$$|\psi_{2L}\rangle = U\left(|\psi_L\rangle|0\rangle^L\right)$$

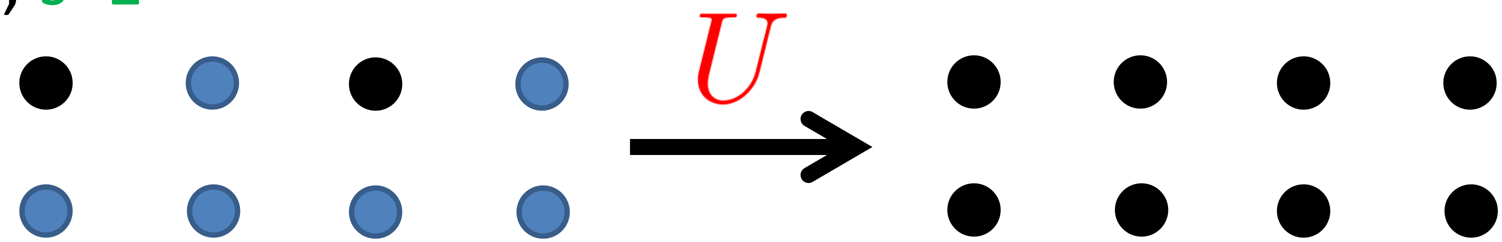
slight abuse of notation: “fixed point”

[BGS-McGreevy ‘14]

s source RG fixed point

A d-dimensional s source RG fixed point is a system where a ground state on $(2L)^d$ sites can be constructed from s copies of ground states on L^d sites times some unentangled degrees of freedom by acting with a quasi-local unitary

d=2, s=1



s source RG definition again

A family of states is s source fixed point if (for large enough L):

$$\underbrace{L \times .... \times L}_{d} \quad \text{(d-torus)} \quad \Longleftrightarrow \quad \underbrace{2L \times .... \times 2L}_{d} \quad \text{(d-torus)}$$

$$U_L \left(\underbrace{|\psi_L\rangle \ldots |\psi_L\rangle}_{s} |0\rangle^{(2^d - s)L^d} \right) = |\psi_{2L}\rangle$$

Some properties

$$S(A) = -\mathrm{tr}(\rho_A \log \rho_A)$$

Recursive entropy bounds:

$$S(2R) \leq s S(R) + k R^{d-1}$$

$$S(2R) \geq s S(R) - k' R^{d-1}$$

result uses [Van Acoleyen-Marien-Verstraete]

$$G(L) = \text{ground state degeneracy}$$

Ground state degeneracy lemma (for gapped case):

$$G(2L) = G(L)^s$$

Local operators → local operators

$$|\psi_{2L}\rangle = U\left(|\psi_L\rangle|0\rangle^L\right)$$

$$\langle\psi_{2L}|O_{loc}|\psi_{2L}\rangle = \langle\psi_L|\tilde{O}_{loc}|\psi_L\rangle$$

iterate: can compute local expectation value in O(log(L)) steps

$$\tilde{O}_{loc} = \langle 0|^L U^\dagger O_{loc} U |0\rangle^L$$

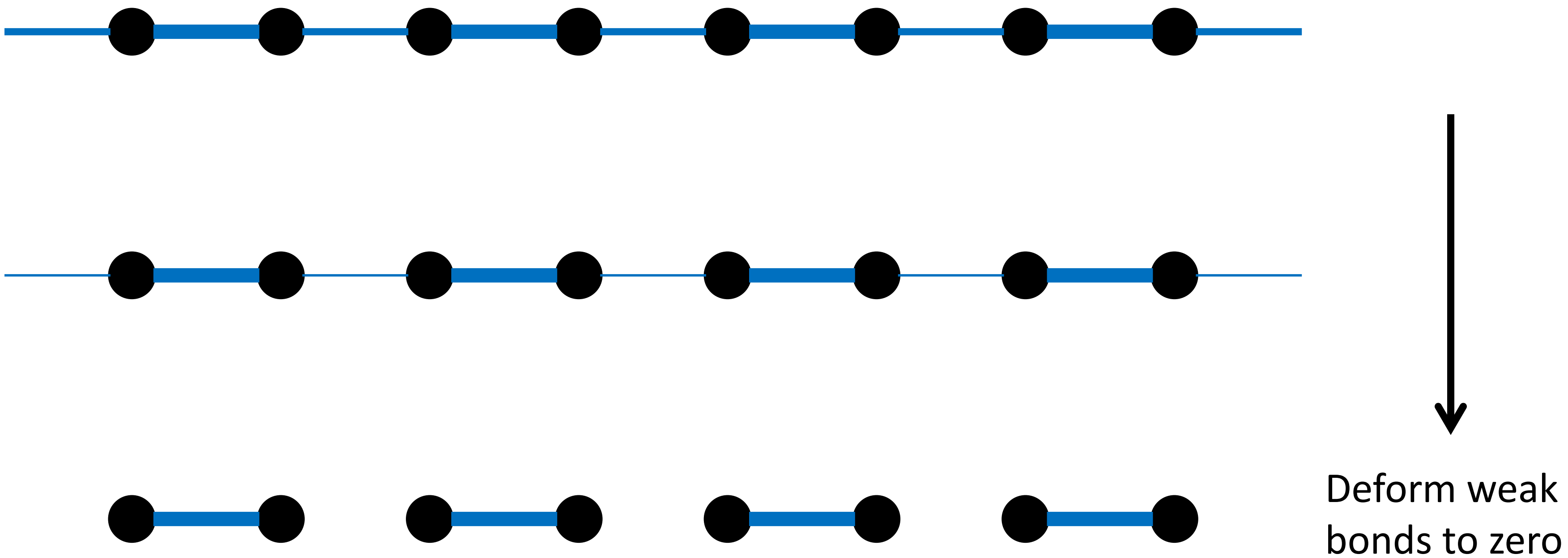
O remains local because:

1. U spreads O by at most the speed of light times a time of order one
2. The number of sites is halved at every step

EXISTENCE RESULTS

Example: trivial insulator, s=0

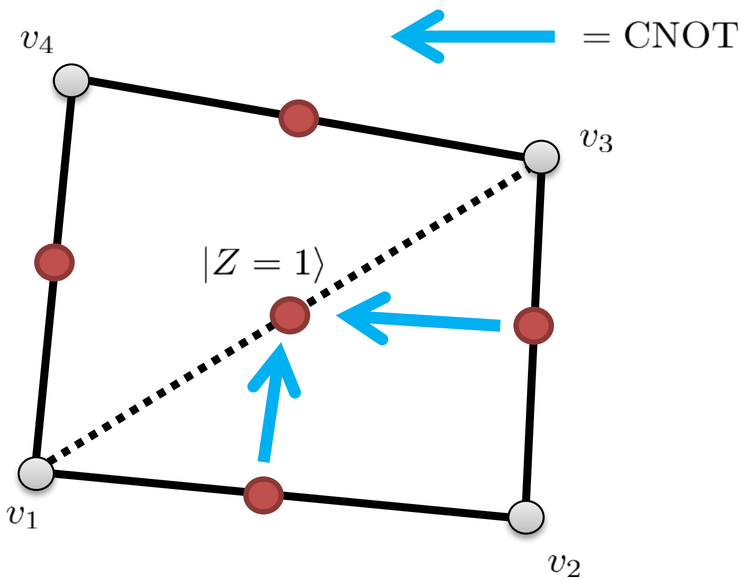
One particle per unit cell, alternating weak bonds:



Example: (gapped) gauge theory, s=1

Toric code, discrete gauge theory, d>2, …

[Aguado-Vidal, Gu-Levin-BGS-Wen]



Topological quantum liquid: insensitive to arbitrary smooth deformations of space

Tool: adiabatic expansion

Given:

$$H(\eta)$$ gapped

$$H(0) \longrightarrow H(1)$$

$$|\psi(\eta)\rangle$$

ground state on size L (and product states)

$$|\psi(0)\rangle = |\psi_L\rangle|0\rangle^{(2^d-1)L^d}$$

ground state on size 2L

$$|\psi(1)\rangle = |\psi_{2L}\rangle$$

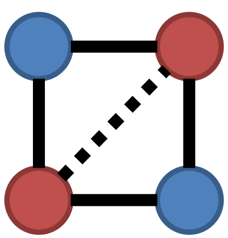
→ Exact: use Hastings-Wen quasi-adiabatic tech

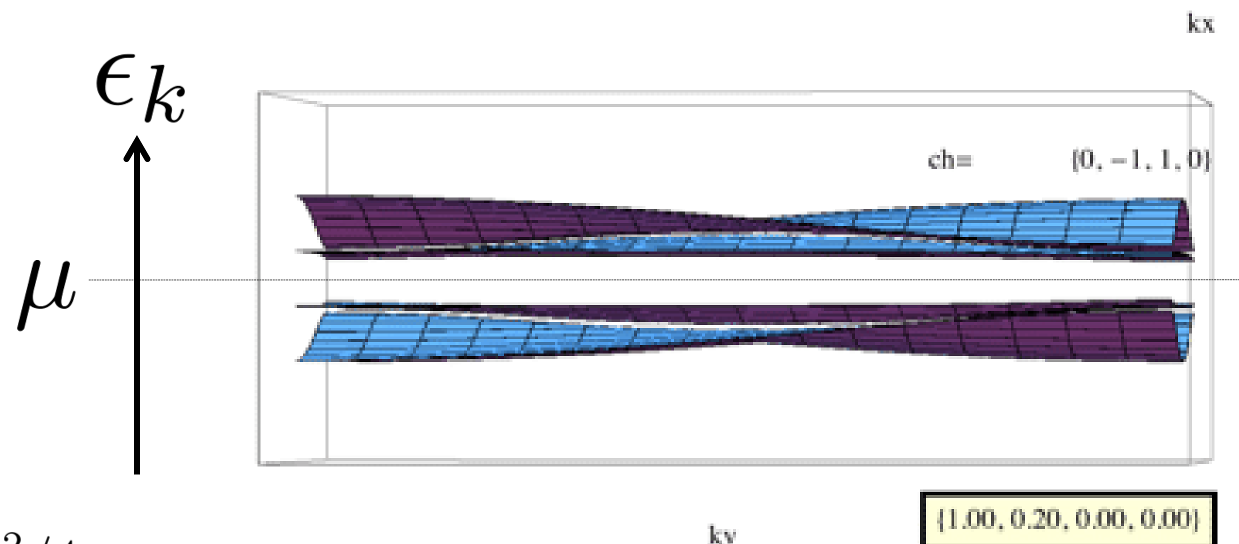
Note: this may not be the most efficient U. However, this is a non-variational way to construct the ground state!

Example: chiral insulators, s=1

Examples:

1. Integer quantum Hall, Chern insulators
2. Massive Dirac fermion, d=2





Sites: $L^2 \to L^2/2 \to L^2/4$

[BGS-McGreevy '14]

Example: CFTs, s=1 [CONJECTURE]

Some evidence:

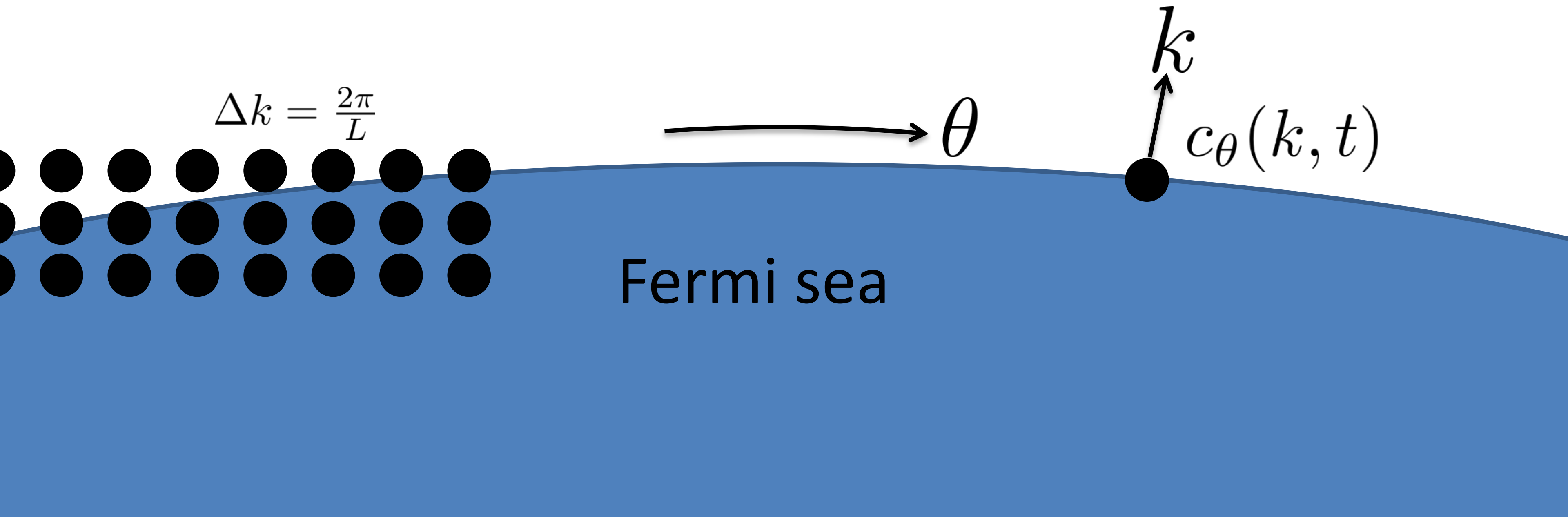
1. Consistent with structure of entanglement and correlations
2. Correlations easy to include
3. MPS approximation results in 1d [Verstraete-Cirac]

Later: provably true for some other gapless (but non-relativistic) scale invariant states …

Example: FS, $s=2^{d-1}$ [CONJECTURE]

- Conjecture: Metals (Fermi liquids) in d dimensions are fixed points but require multiple copies of size L to make size 2L

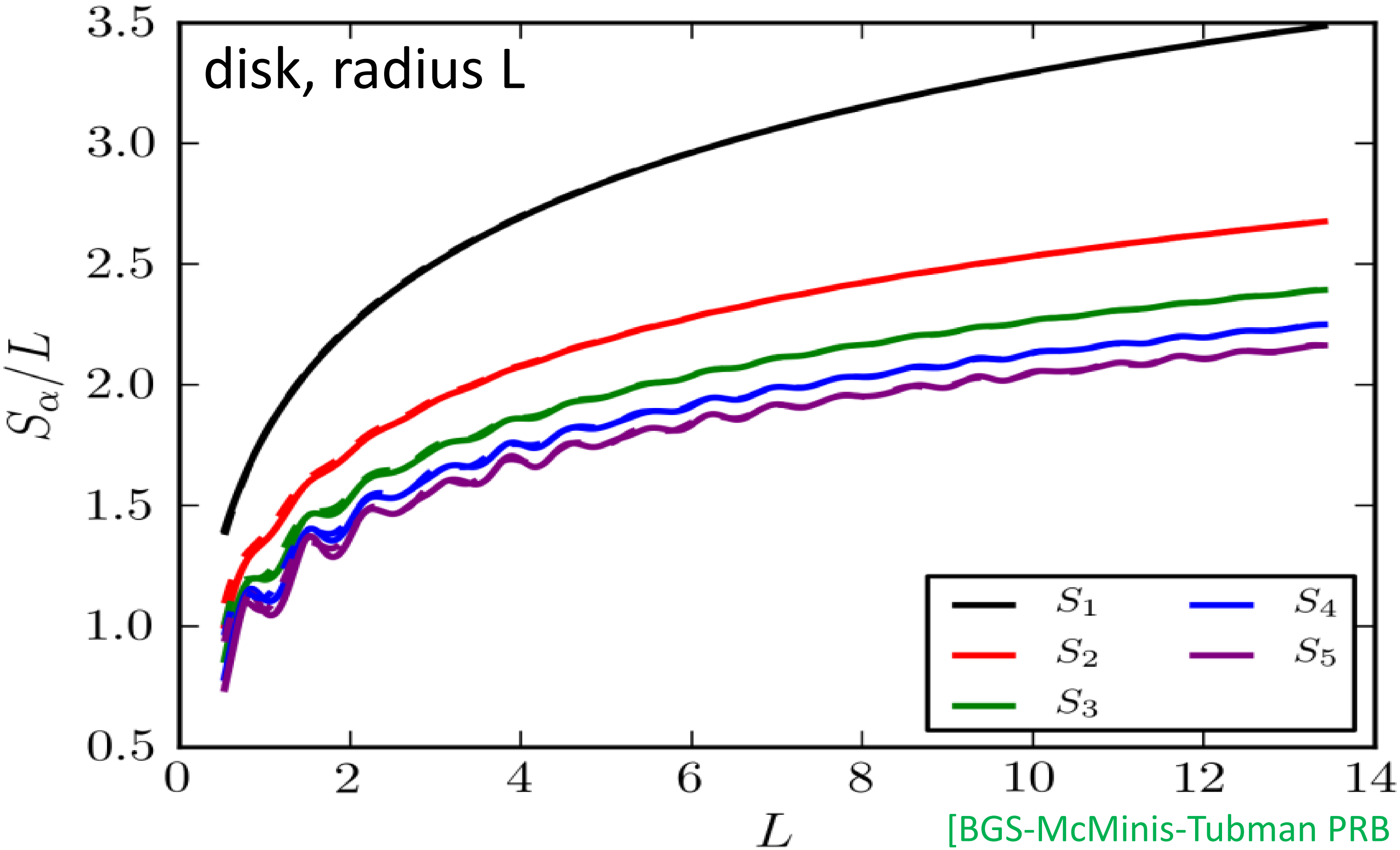
$$\mathcal{S} = \int d\theta \int dt\, dk\, c^\dagger_\theta(k,t)(i\partial_t - v_F k) c_\theta(k,t)$$



$$S_\alpha = \frac{1}{1-\alpha}\log(\mathrm{tr}(\rho^\alpha))$$ general definition, probes spectrum

$$S_\alpha = a_1 L \ln(L) + a_2 L + a_3 \frac{1}{L^{a_4}} \cos(a_5 L)$$ form of S for a metal, d=2

[Klich-Gioev PRL '06, BGS PRL '09]



[BGS-McMinis-Tubman PRB '13]

Example: “square root states”, s=1

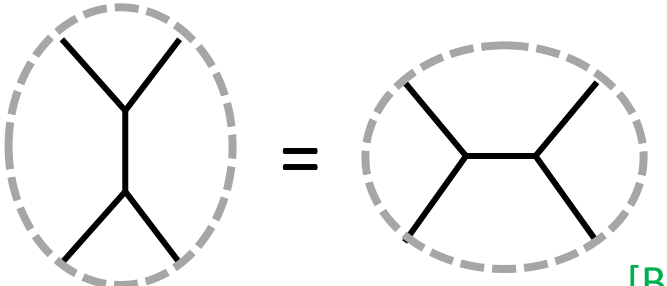
$$H = \sum_r \left(-X_r + e^{-\beta J Z_r \sum_{r' \in nn(r)} Z_{r'}} \right)$$

$$|\mathrm{Ising}\rangle = \frac{1}{\sqrt{Z}} \sum_\sigma e^{\frac{\beta J}{2} \sum_{rr'} \sigma_r \sigma_{r'}} |\sigma\rangle$$

Hamiltonian is positive $\quad H = \sum_r Q_r, \; Q_r \geq 0$

Use invariance of statistical partition function:

[Levin-Nave]



→ s=1

[BGS-Xu-McGreevy soon]

EXTENSIONS, COMMENTS, AND WRAP-UP

What is s?

- Gapped systems: usually $s=0$ or $s=1$
- Gapless systems, scaling to a point: likely $s=1$
- Metals and NFLs: likely $s=2^{d-1}$

Is there a principled way of determining s?

Intuition: lots of entanglement → lots of low lying states

Scaling theory of critical states

$$\xi(T) \sim T^{-1/z}$$ correlation length

$$s(T) \sim \left(\frac{1}{\xi}\right)^{d-\theta} \sim T^{\frac{d-\theta}{z}}$$ thermal entropy density

Key idea: $-\log(\rho_{A,\mathrm{gs}}) \sim \sum_x \frac{H_x}{T(x)}$ half space gs density matrix (max ent)

$$\theta < d-1 \rightarrow S_{EE} \sim \text{area}$$
$$\theta = d-1 \rightarrow S_{EE} \sim \text{area*log}$$

$$s = 2^{\theta}$$

[BGS-McGreevy '15]

Evidence for generality of s sourcery

- Rigorous constructions
- Sufficient to capture correlations and entanglement entropy
- Some numerical evidence
- Even sufficient to capture effective Schmidt rank …

Effective Schmidt rank

Q: What is the physical meaning of entanglement entropy?

A1: $nS(\rho)$ is the cost to compress $\rho^{\otimes n}$ in the limit of a large n number of copies

A2: For a single copy the answer is

$$H^{\epsilon}_{\max}(\rho) = \min_{\|\sigma-\rho\|_1<\epsilon} \log(\text{rank}(\sigma))$$

$$H^{\epsilon}_{\max}(A, \text{CFT gs}) = S(A)\left(1+\mathcal{O}\left(\sqrt{\frac{\log\left(\frac{1}{\epsilon}\right)}{S(A)}}\right)\right)$$

[Czech-Lashkari-Hayden-Swingle '14, BGS-Hayden soon]

Summary

- An RG inspired framework for the exact description of ground states – "efficient" calculation of physical properties, mounting evidence of generality
- Stay tuned for extensions to thermal states, etc. and results about CFTs, etc.
- A possibly hard case is FS+gauge field …
- Foundation for holography via tensor networks?