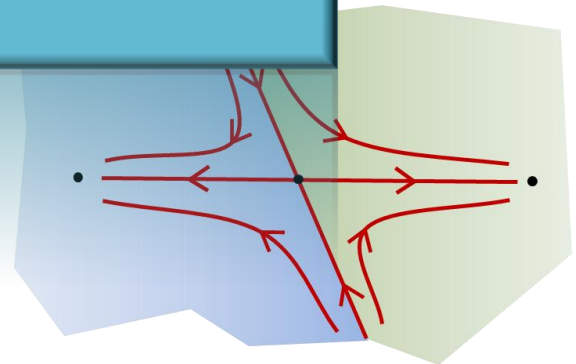
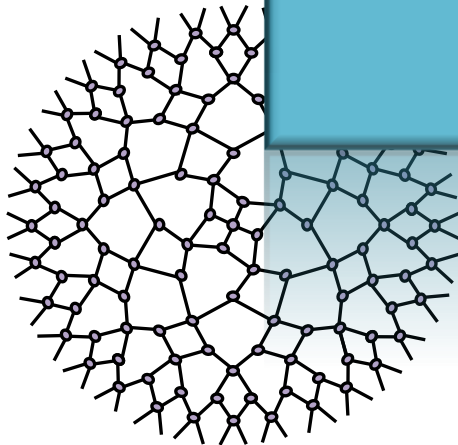


KITP Conference
Closing the entanglement gap:
Quantum information, quantum matter, and quantum fields
June 1st -5th 2015

Tensor Networks, Renormalization and Holography (overview)



Guifre Vidal

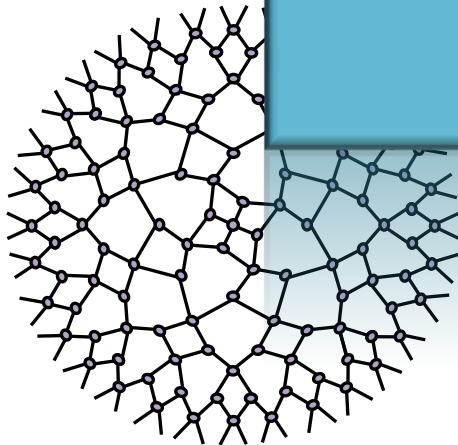
PERIMETER  INSTITUTE FOR THEORETICAL PHYSICS

SIMONS FOUNDATION

JOHN TEMPLETON
FOUNDATION

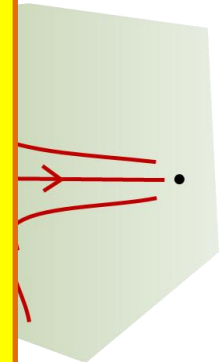
KITP Conference
Closing the entanglement gap:
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Tensor Networks, Renormalization and Holography (overview)



Glen Evenbly
UC Irvine

Guifre Vidal



PERIMETER  INSTITUTE FOR THEORETICAL PHYSICS

SIMONS FOUNDATION

JOHN TEMPLETON
FOUNDATION

outline

Part 1:
(old stuff)

Tensor networks

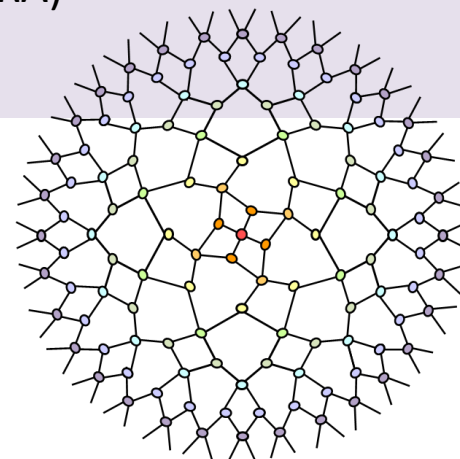
+

Renormalization group

=

Multi-scale entanglement
renormalization ansatz
(MERA)

- quantum circuit
- RG transformation



Part 2: MERA and holography (!?)

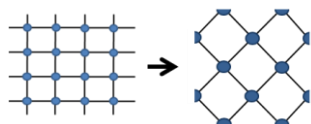
talks by → Bartek Czech (today)
→ John Preskill (tomorrow)
→ Tadashi Takayanagi (tomorrow)

Part 3:
(recent
developments)

Tensor network
renormalization
(TNR)

- MERA from Euclidean path integral
- thermal states (dual to a black hole?)

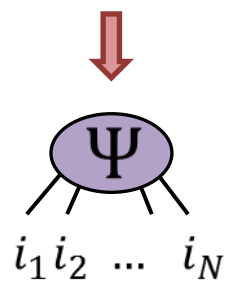
→ Glen Evenbly's talk (May 7th)



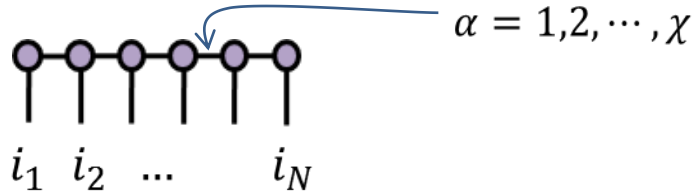
Many-body wave-function of N spins

$$|\Psi\rangle = \sum_{i_1, i_2, \dots, i_N} \Psi_{i_1 i_2 \dots i_N} |i_1 i_2 \dots i_N\rangle$$

2^N
parameters



tensor network



2^N
parameters

inefficient

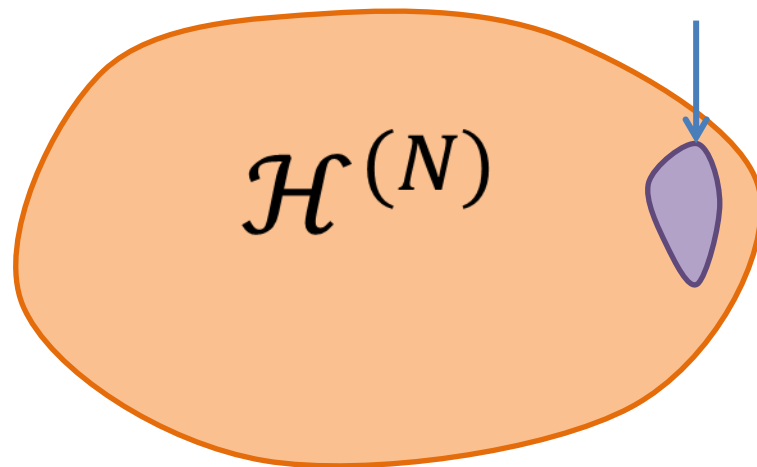
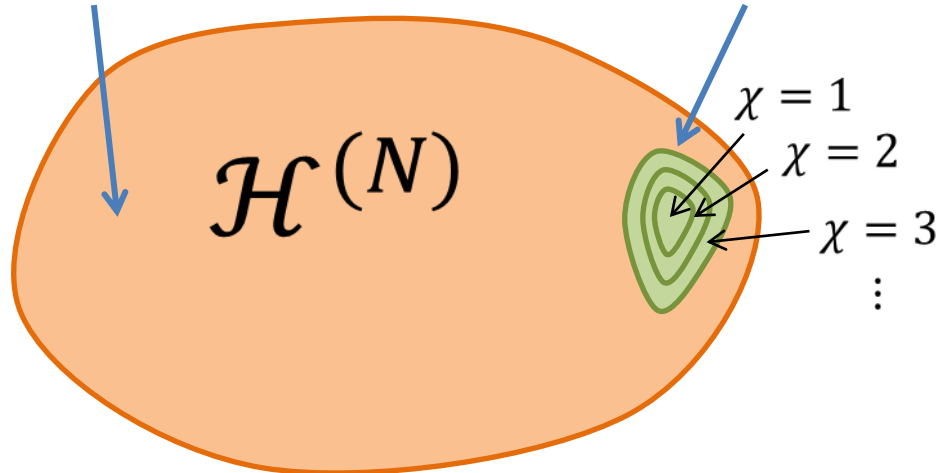
$O(N)$
parameters

efficient

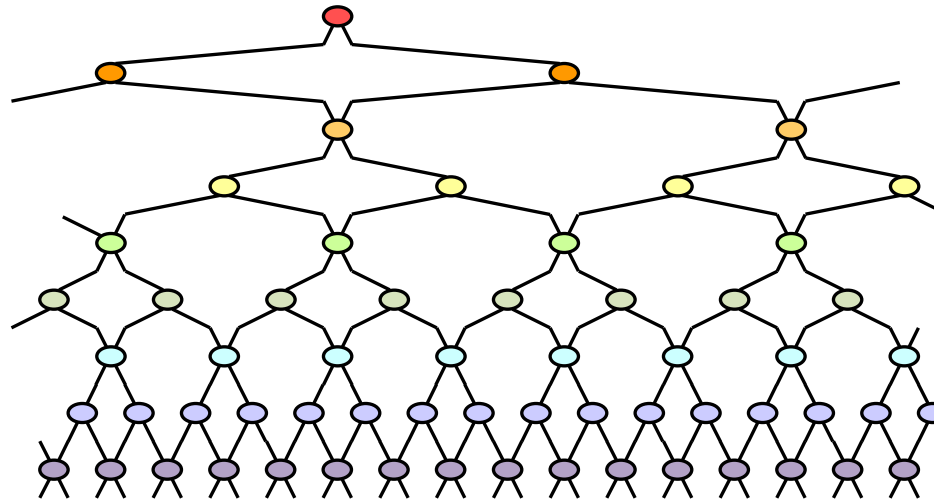
generic state

tensor network states

ground states of
local Hamiltonians

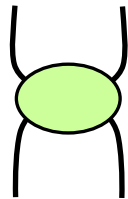
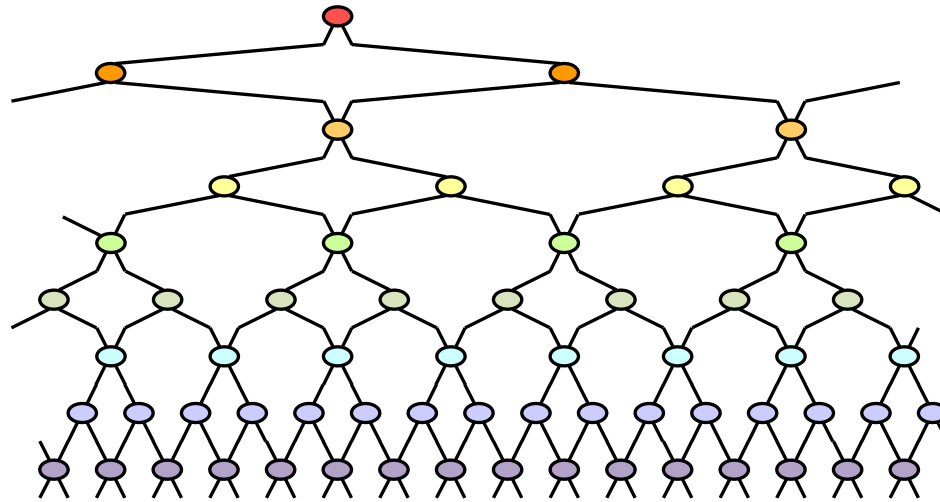


Multi-scale entanglement renormalization ansatz (MERA)

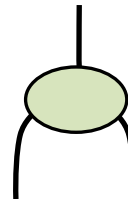


- Variational class of states for 1d systems, which extends in space and scale
- Variational parameters for different length scales stored in different tensors
- It is secretly a **quantum circuit** and an **RG transformation**

MERA as a quantum circuit

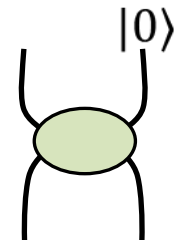


disentangler
two-body unitary gate



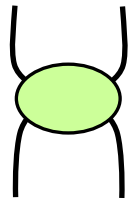
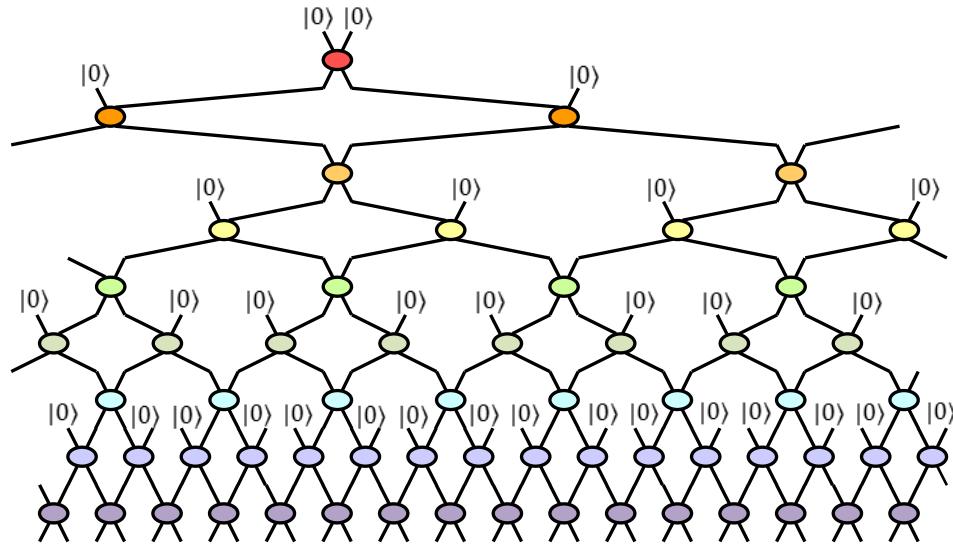
isometry

=

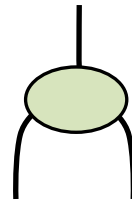


also a
two-body unitary gate

MERA as a quantum circuit

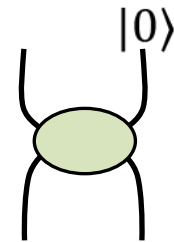


disentangler
two-body unitary gate



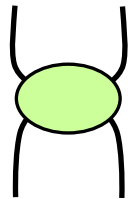
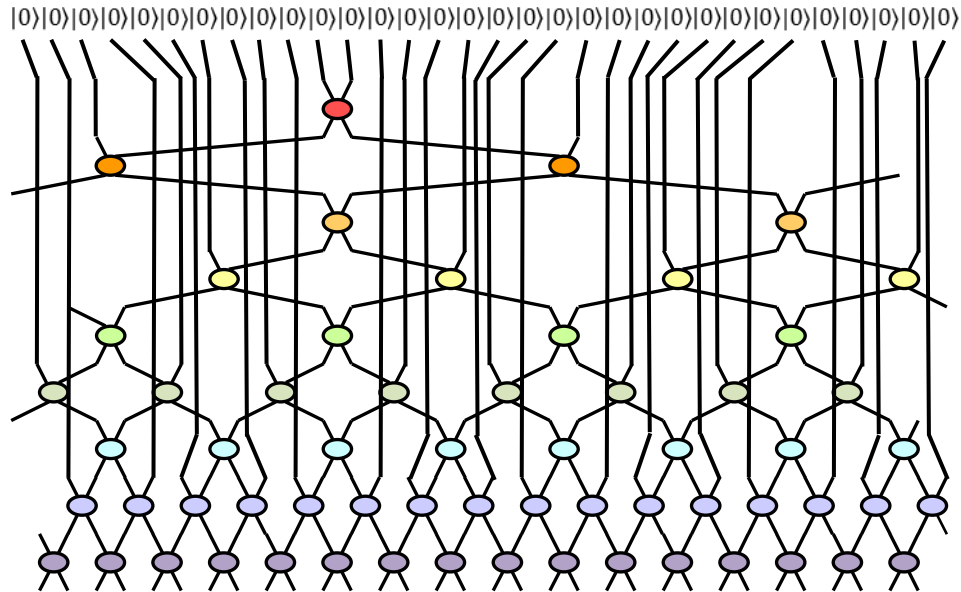
isometry

=

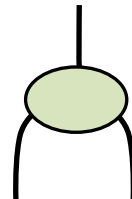


also a
two-body unitary gate

MERA as a quantum circuit

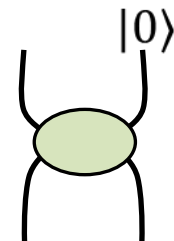


disentangler
two-body unitary gate



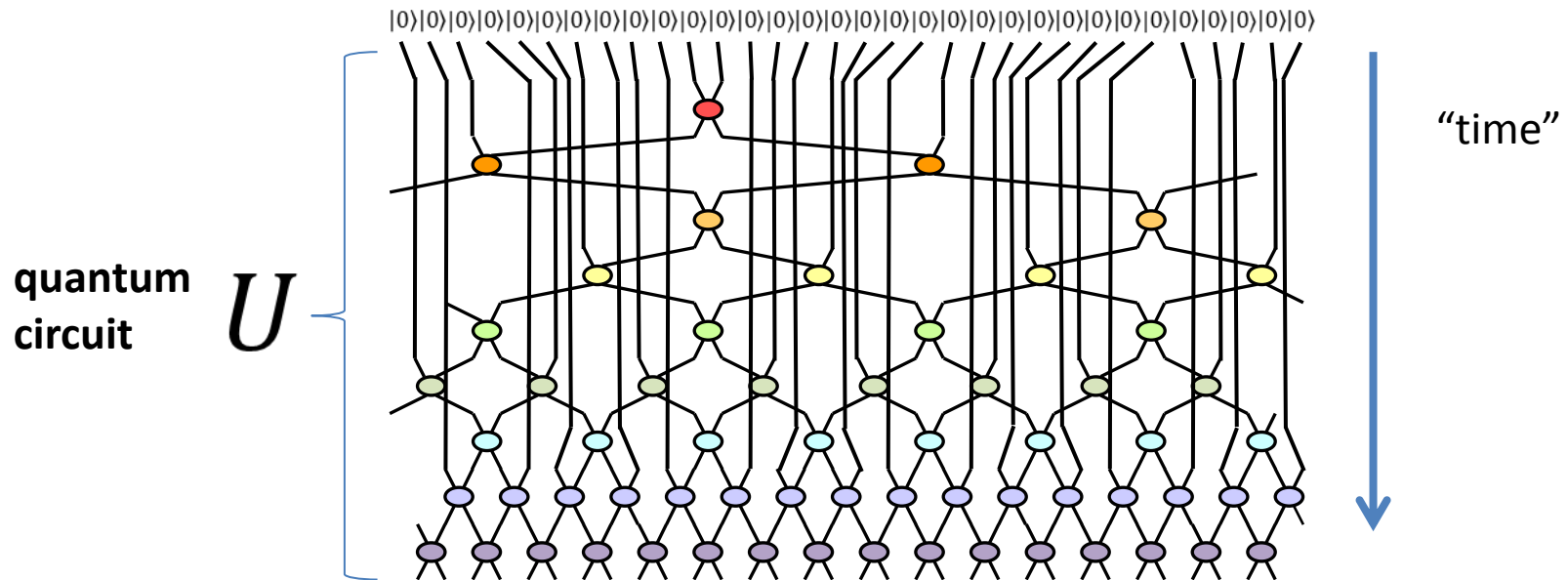
isometry

=



also a
two-body unitary gate

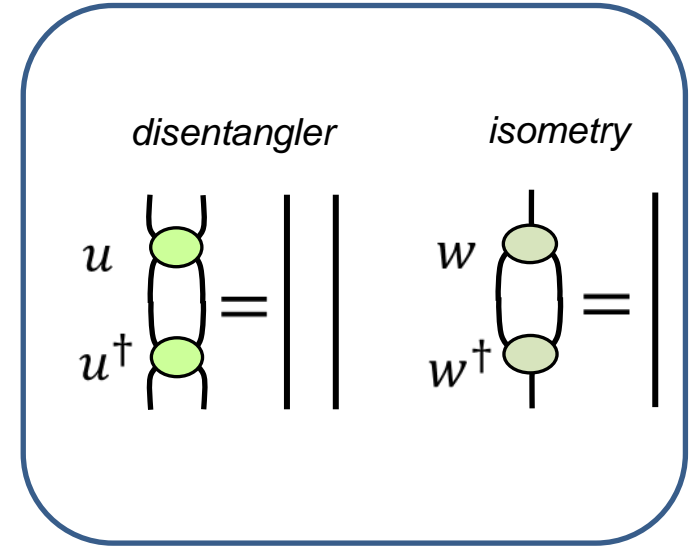
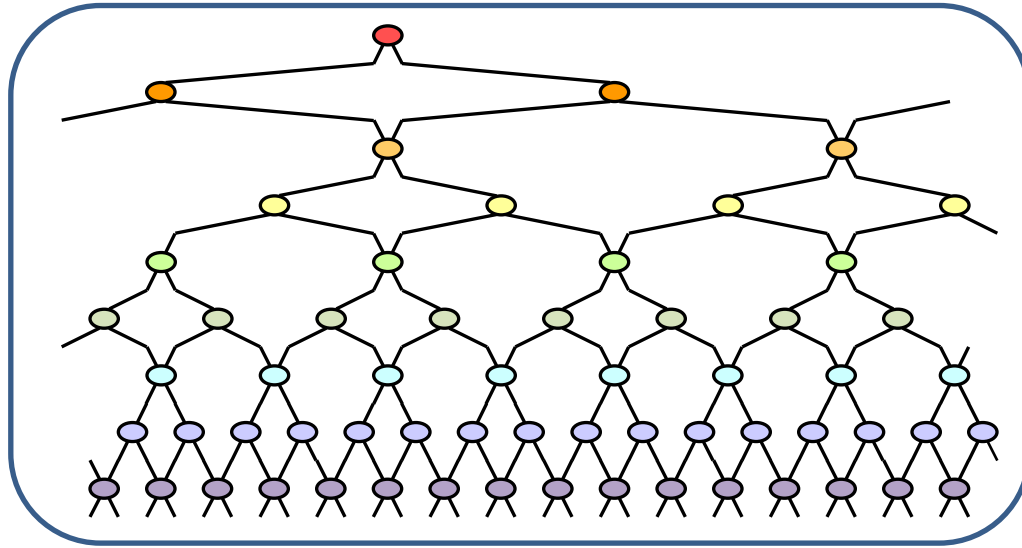
MERA as a quantum circuit



ground state $|\Psi\rangle = U|0^{\otimes N}\rangle$

Entanglement introduced by gates at different “times” (= length scales)

MERA = tensor network + isometric/unitary constraints

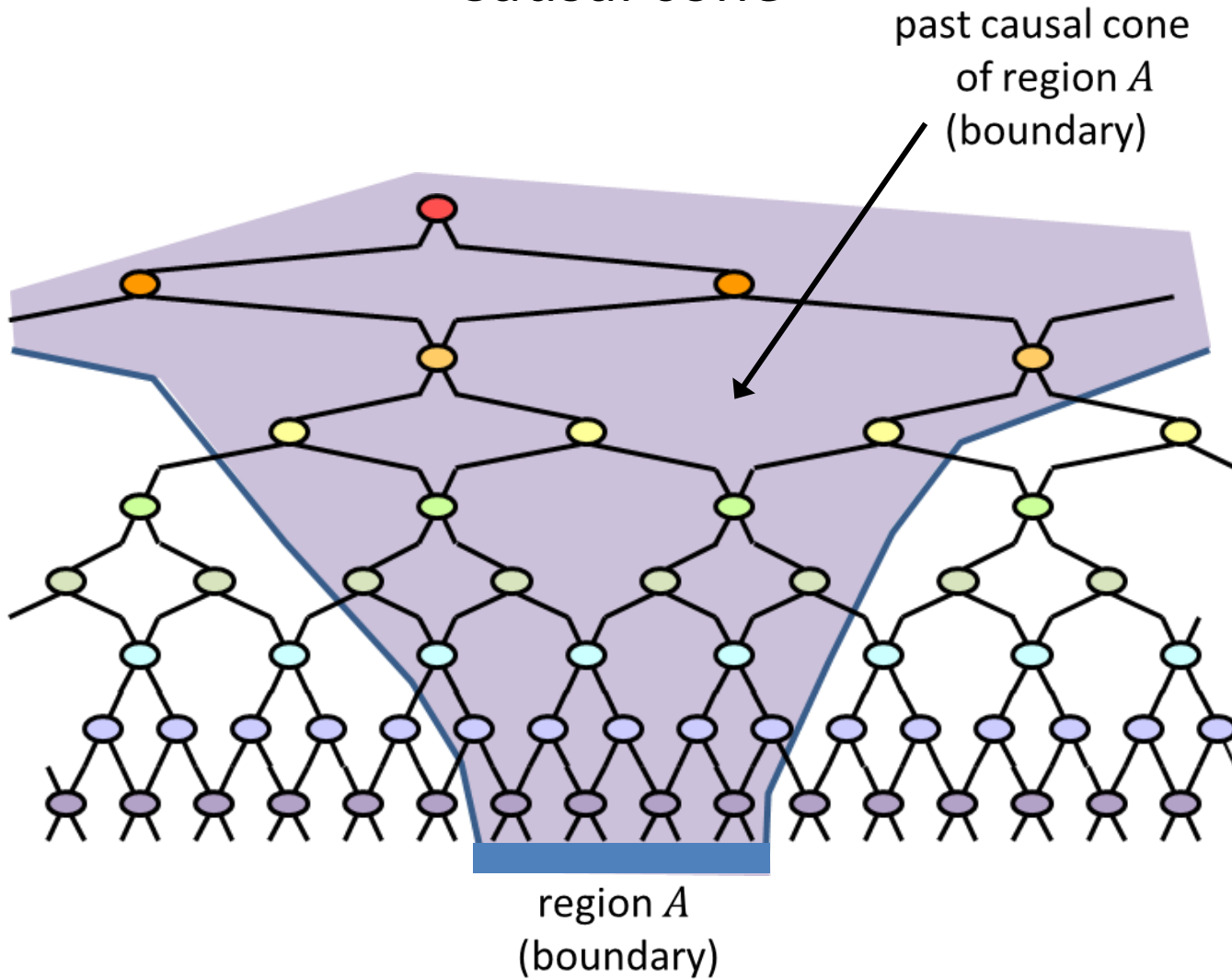


Causal structure

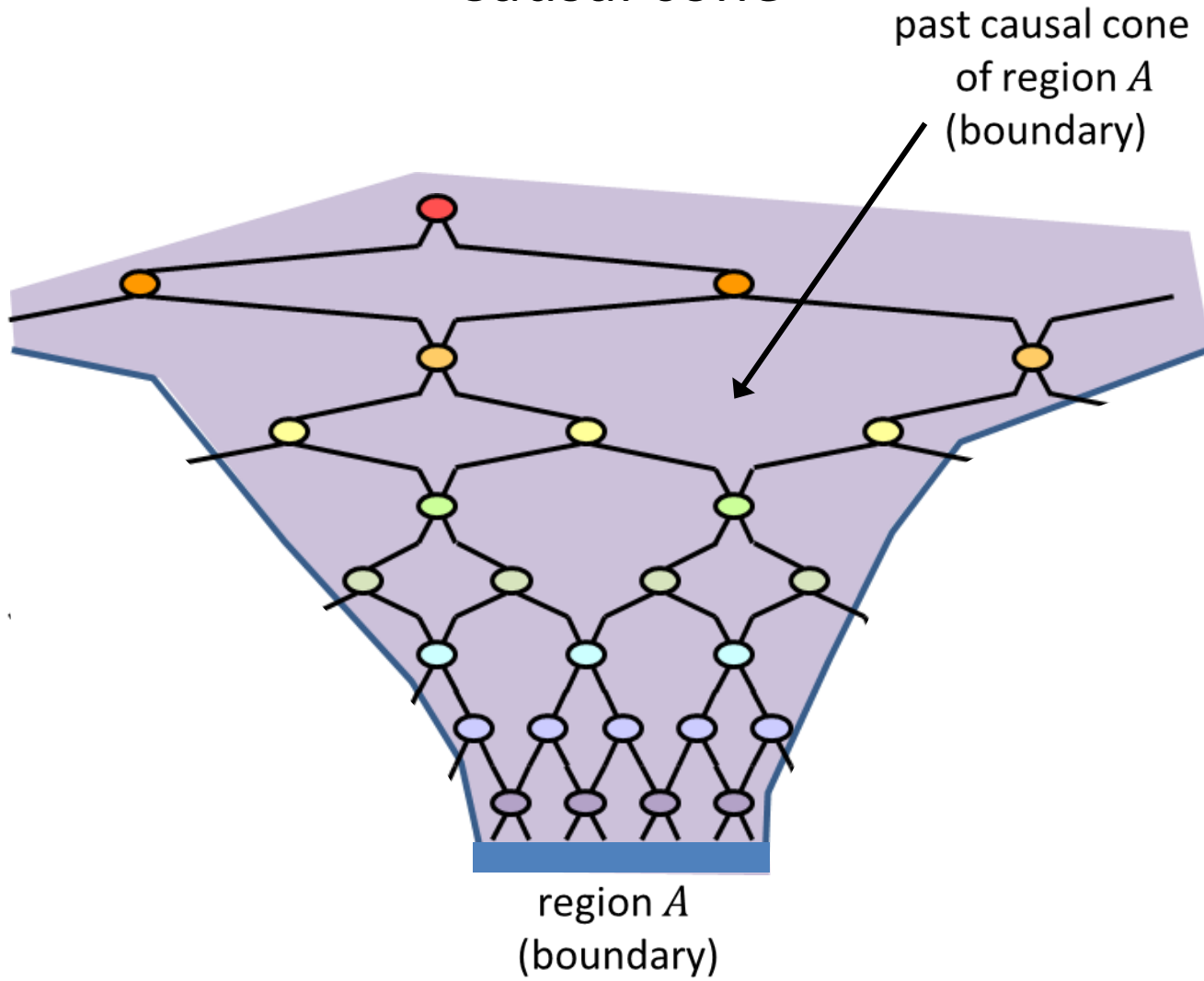
essential for many MERA properties
and computational efficiency

(Bartek Czech's talk) ~ de Sitter space

Causal cone



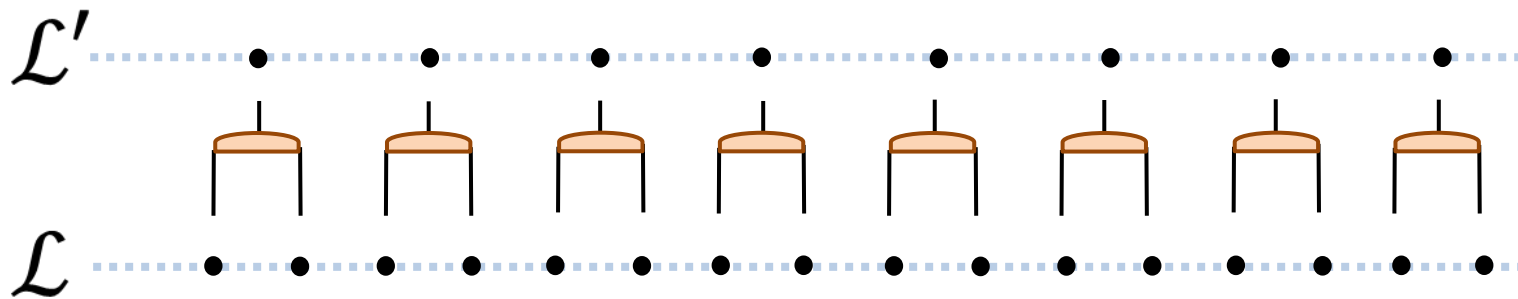
Causal cone



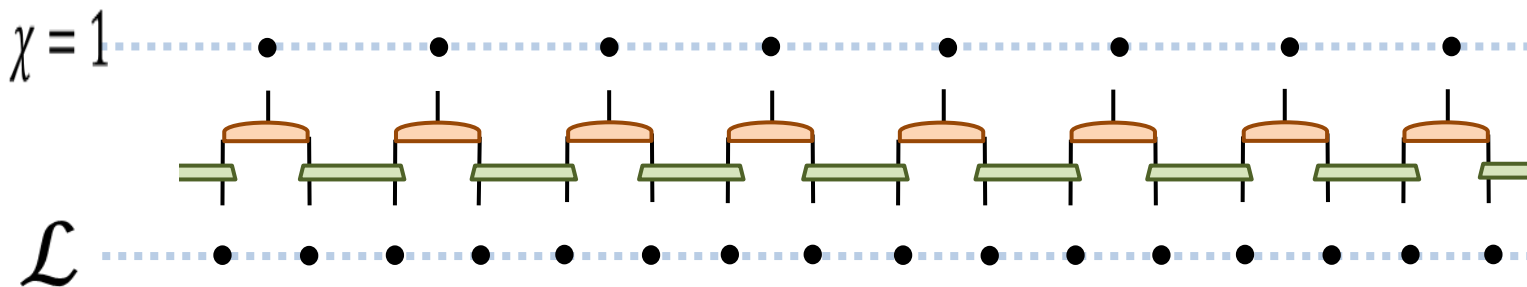
MERA as RG Transformation

Kadanoff (1966)
blocking

+ White (1992)
variational optimization



Entanglement renormalization (2005)

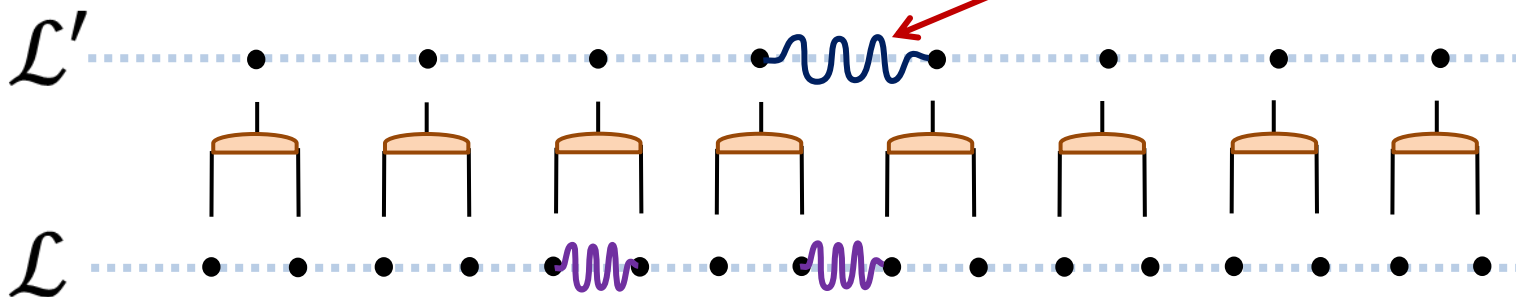


MERA as RG Transformation

Kadanoff (1966)
blocking

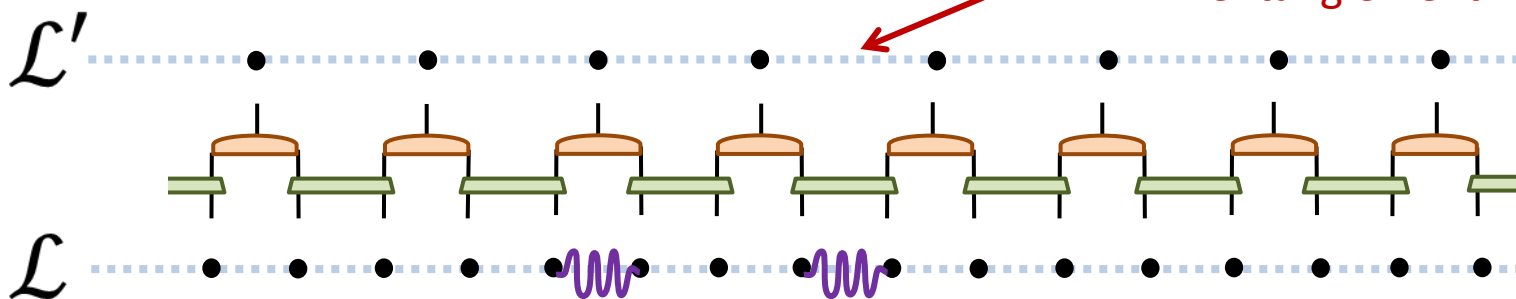
+ White (1992)
variational optimization

failure to remove
some short-range
entanglement !

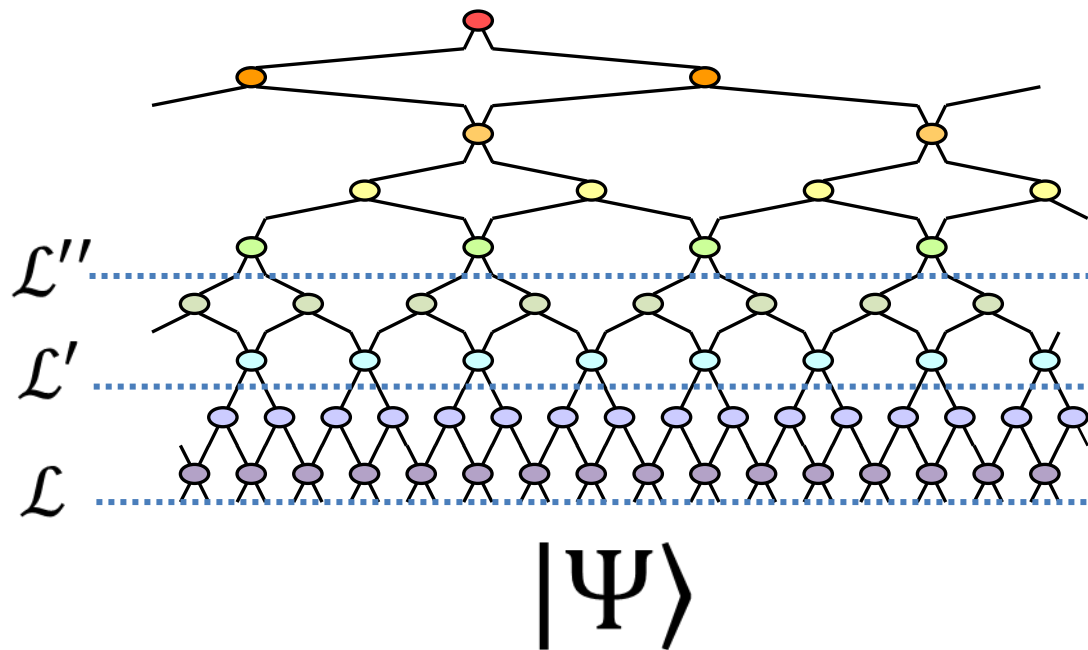


Entanglement renormalization (2005)

removal of *all*
short-range
entanglement

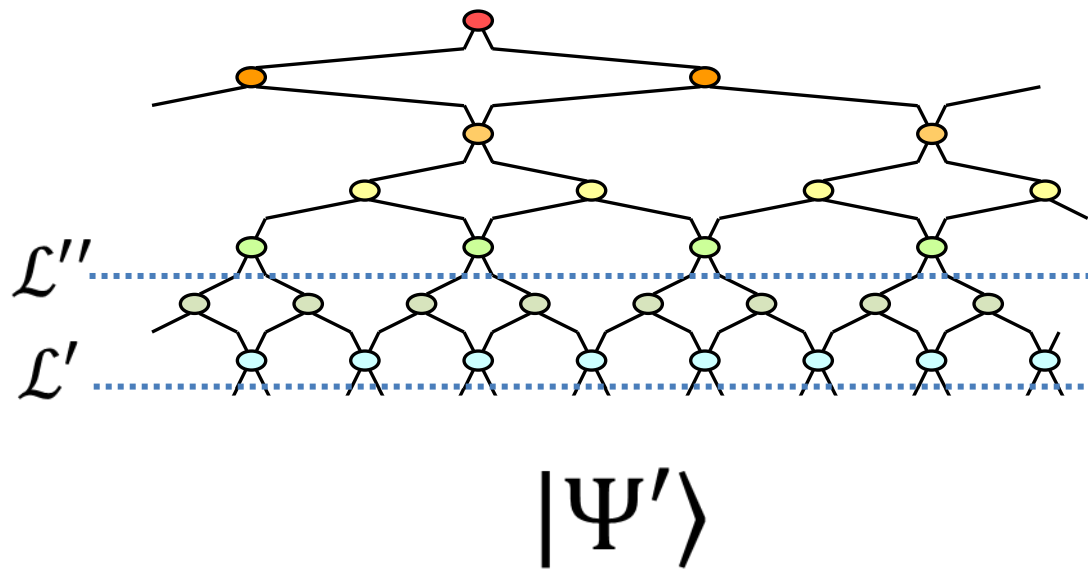


MERA as a sequence of ground state wave-functions



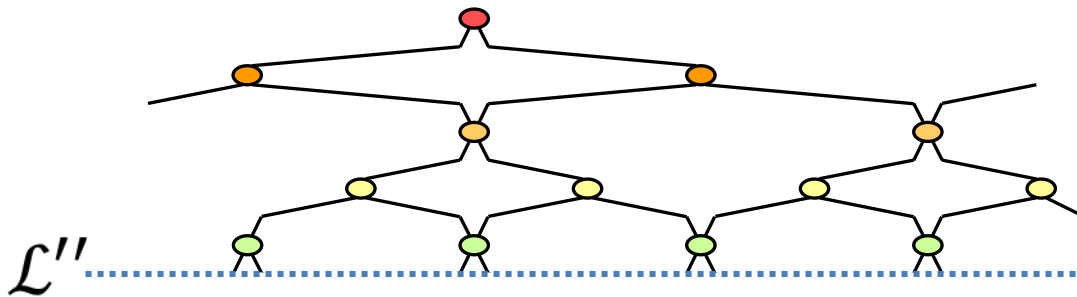
$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \dots$$

MERA as a sequence of ground state wave-functions



$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \dots$$

MERA as a sequence of ground state wave-functions

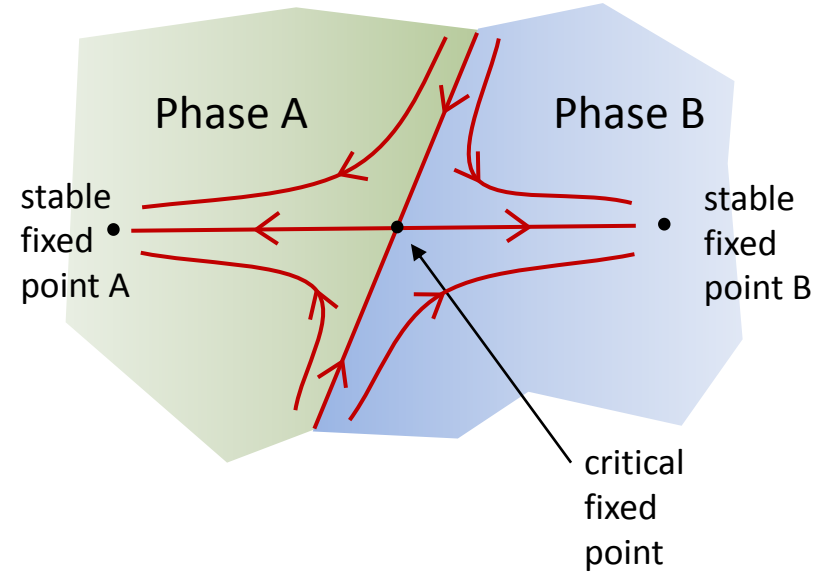
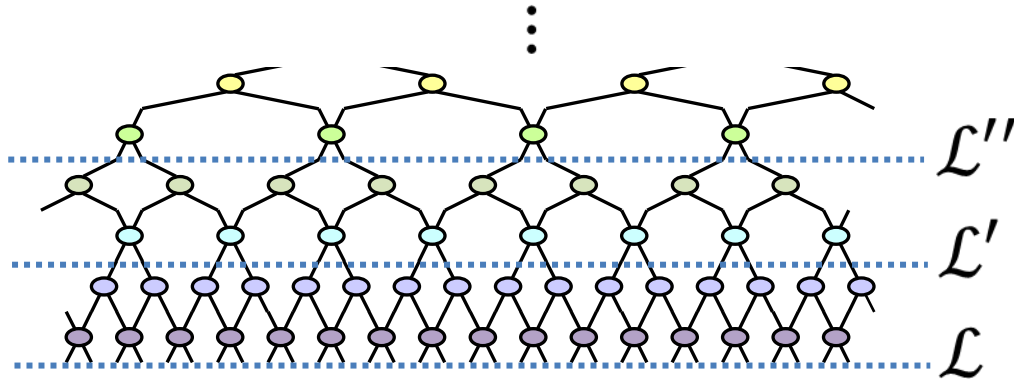


$$|\Psi''\rangle$$

$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \dots$$

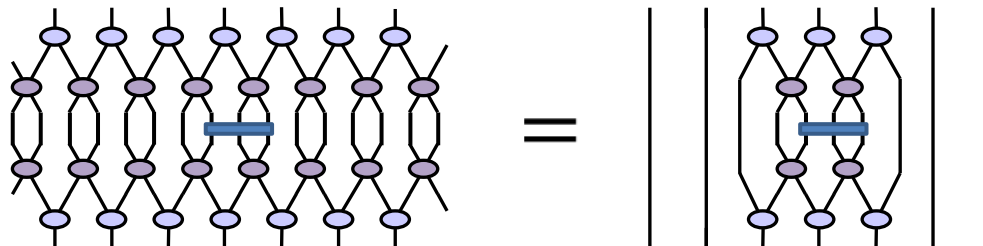
MERA defines an RG flow
in the space of wave-functions

$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \dots$$



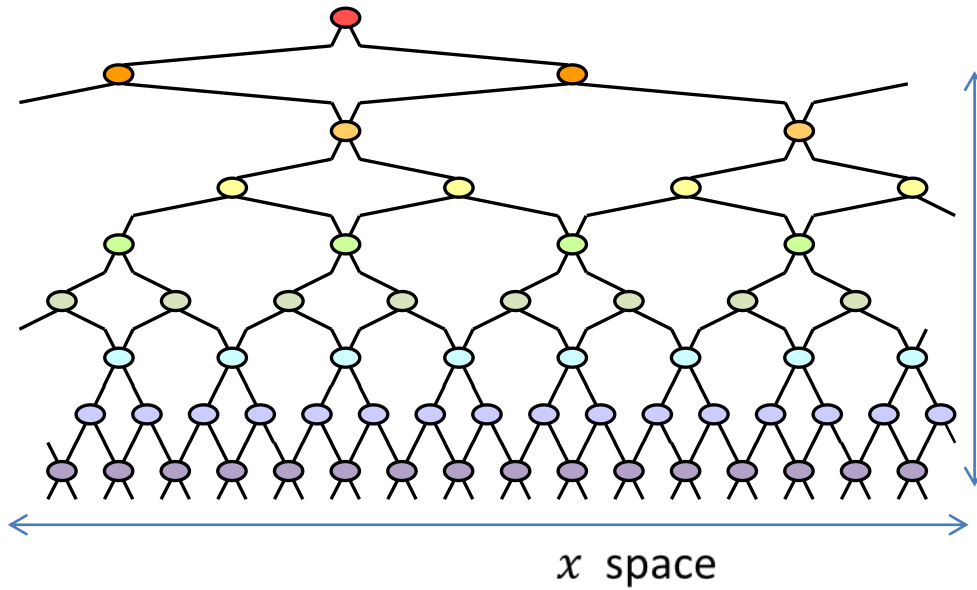
... and in the space of Hamiltonians

$$H \rightarrow H' \rightarrow H'' \rightarrow \dots$$



local operators
are mapped into
local operators !

Entanglement entropy and correlations



- entanglement entropy

$$S_L \approx \log(L)$$

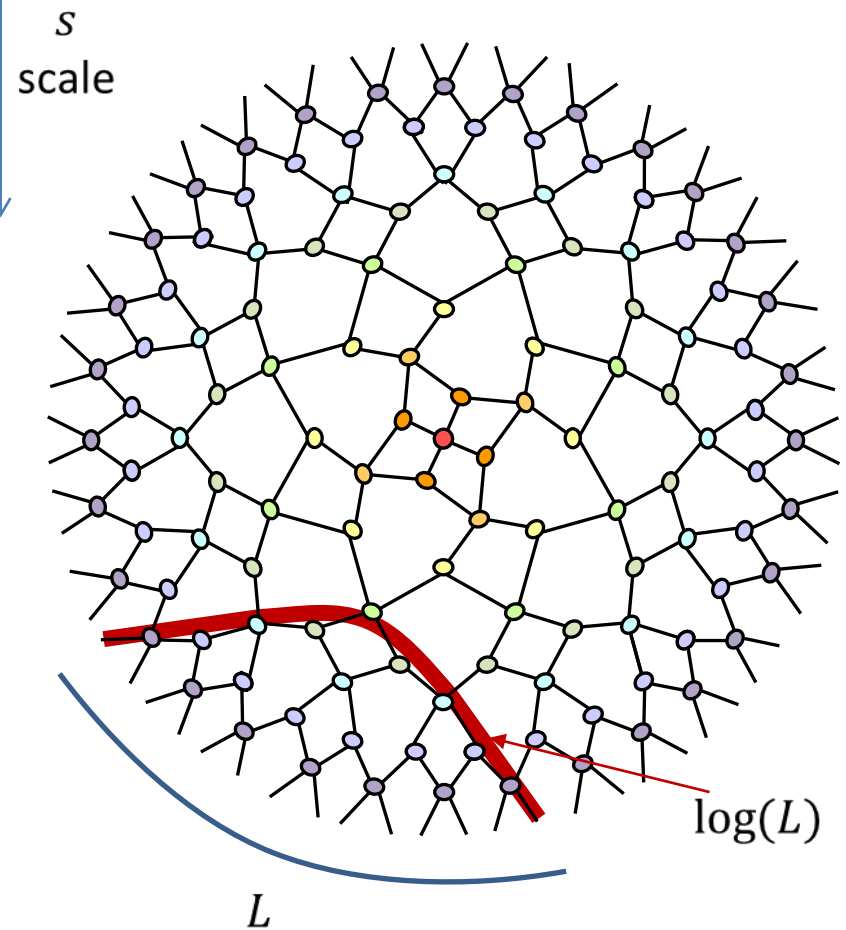
Computation of density matrix requires tracing out $\sim \log(L)$ indices

- two-point correlations

$$C(L) \approx L^{-2\Delta}$$

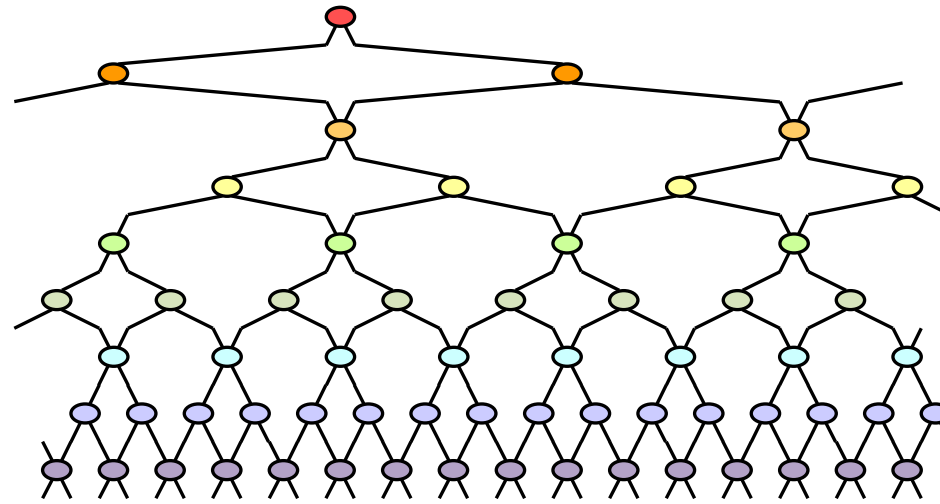
Geodesic distance $D \approx \log(L)$ (as in hyperbolic space*)

$$C(L) \approx e^{-D} = e^{-2\Delta \log(L)} = L^{-2\Delta}$$



Summary so far

MERA



- Variational parameters for different length scales

- It is secretly a **quantum circuit**



“entanglement at
different length scales”

and an **RG transformation**

“removes
short-range
entanglement”

$$|\Psi\rangle \rightarrow |\Psi'\rangle$$

$$H \rightarrow H'$$

“preservation
of locality”

- Entanglement entropy and correlations as in 1+1 critical ground states

$$S_L \approx \log(L)$$

$$C(L) \approx L^{-2\Delta}$$

blah, blah, blah... However, does it work?

[Given lattice Hamiltonian H ,

optimize variational parameters by energy minimization]

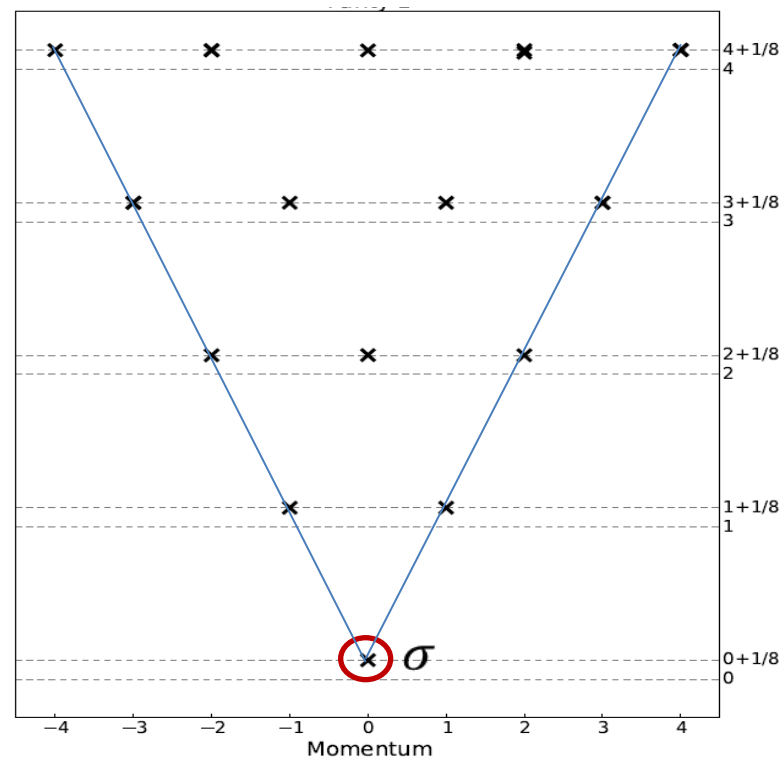
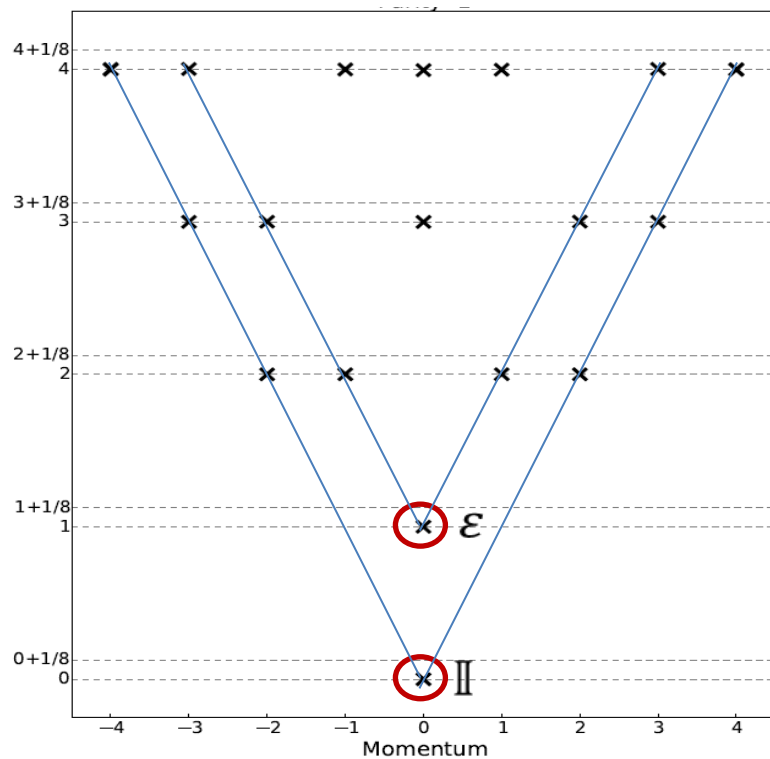
Critical Ising model

(from partition function)



Markus Hauru
Perimeter Institute

central charge $c = 0.500091$



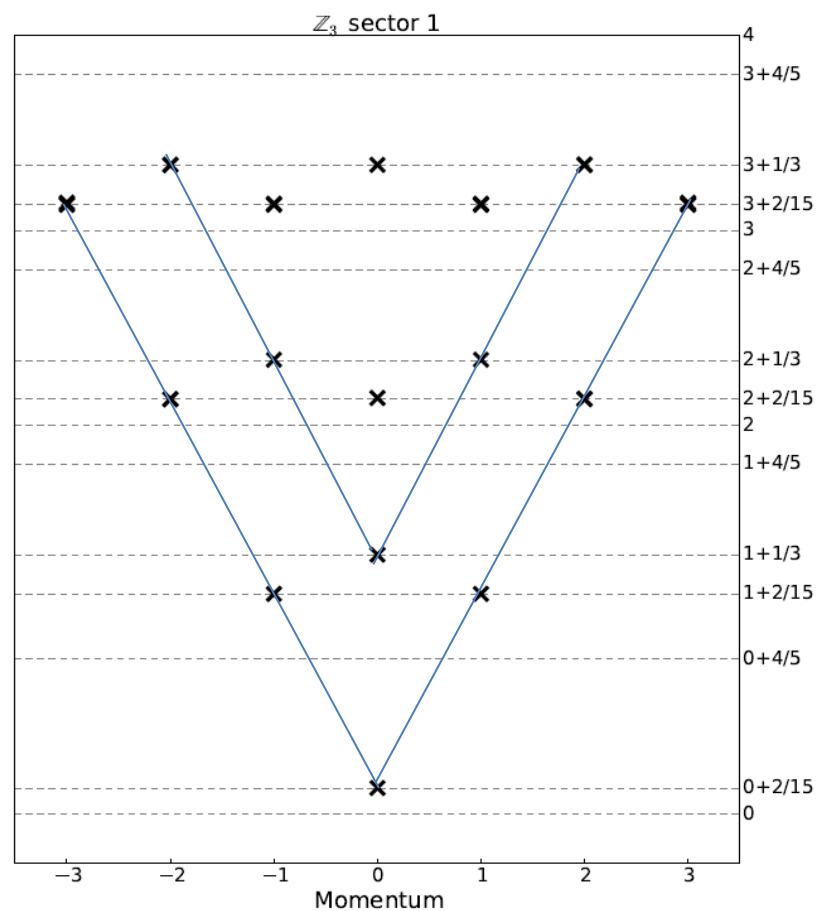
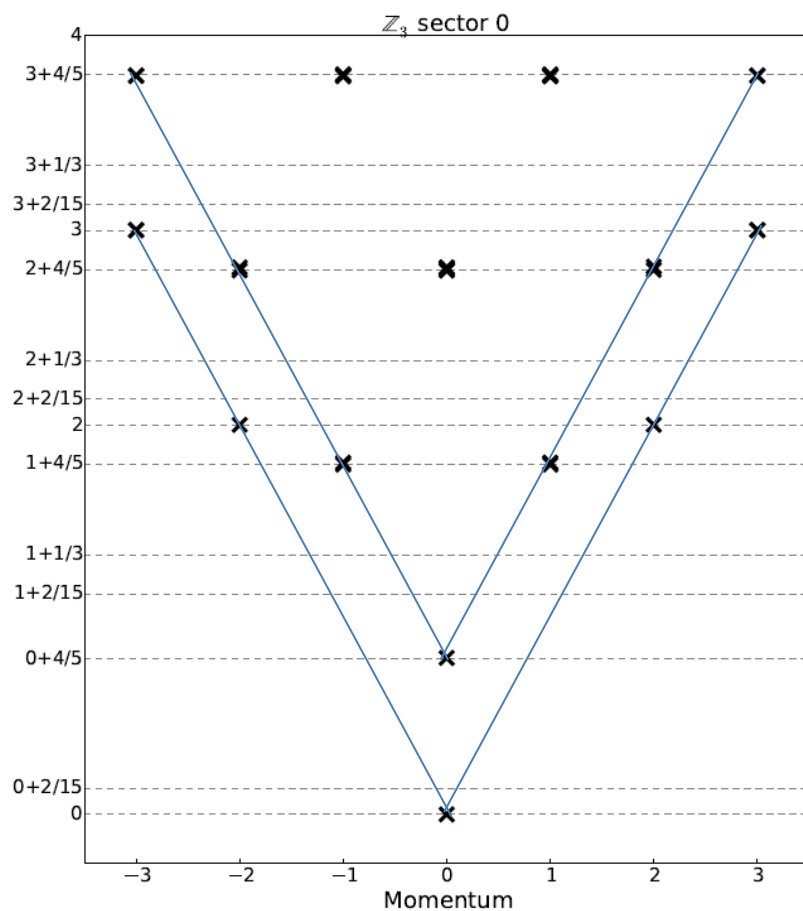
Critical 3-level Potts model

(from partition function)



Markus Hauru
Perimeter Institute

central charge $c = 0.802303$



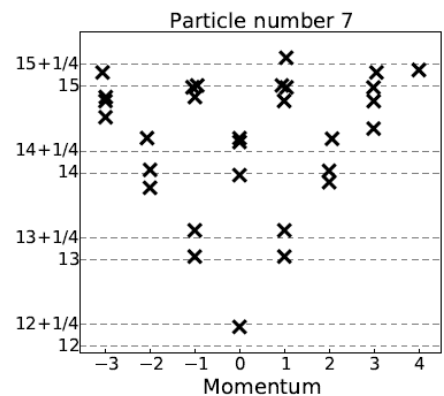
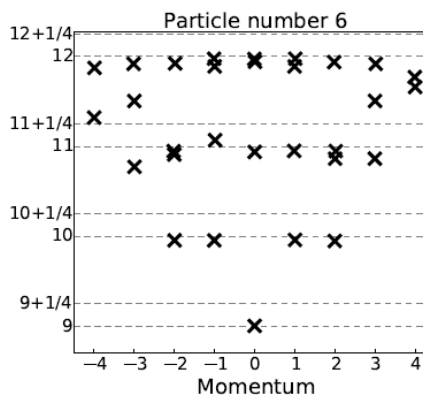
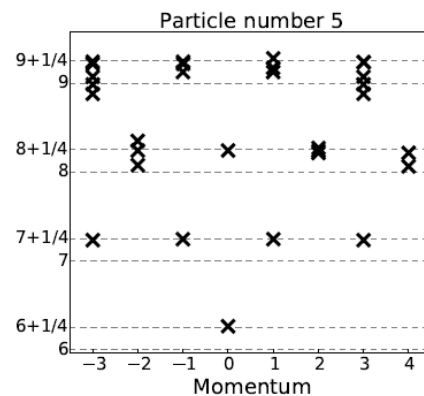
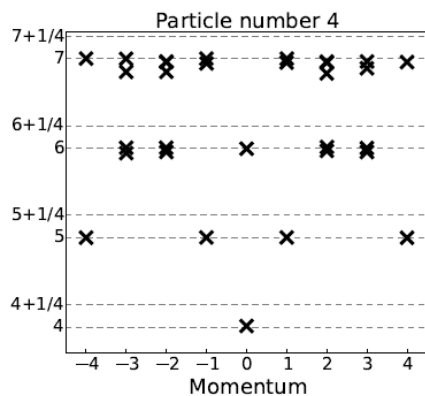
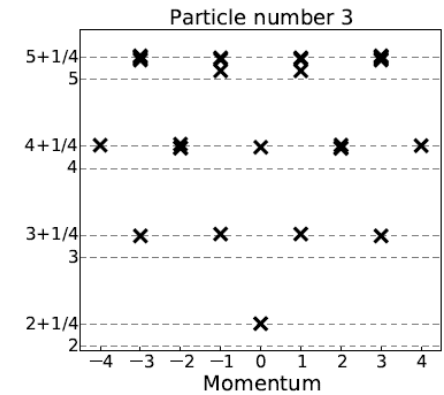
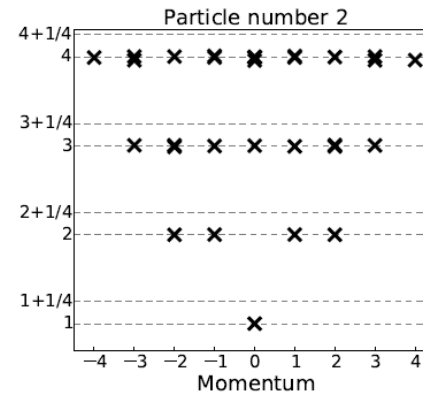
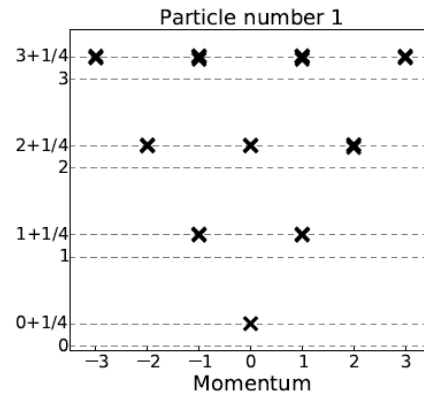
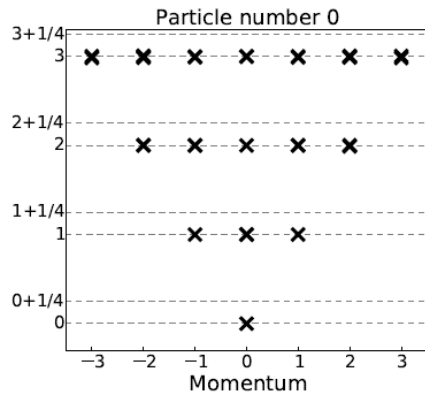
Six vertex model (critical XX quantum spin model)

(from partition function)



Markus Hauru
Perimeter Institute

central charge $c = 1.00204$



outline

Part 1:
(old stuff)

Tensor networks

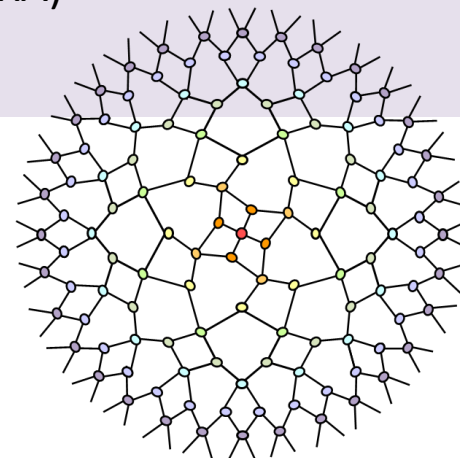
+

Renormalization group

=

Multi-scale entanglement
renormalization ansatz
(MERA)

- quantum circuit
- RG transformation

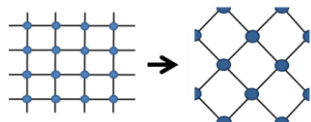


Part 2: MERA and holography (!?)

Part 3:
(recent
developments)

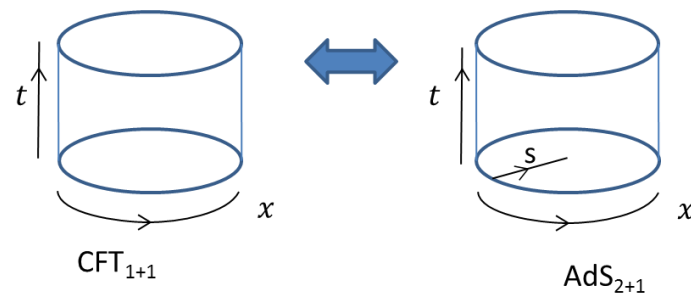
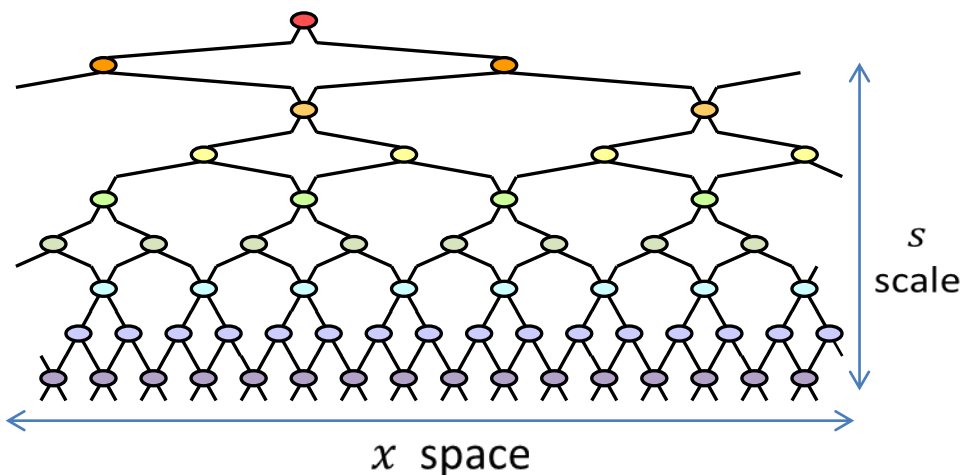
Tensor network
renormalization
(TNR)

- MERA from Euclidean path integral
- thermal states (dual to a black hole?)



MERA and HOLOGRAPHY

[Swingle, 2009]



- entanglement entropy

$$S_L \approx \log(L)$$

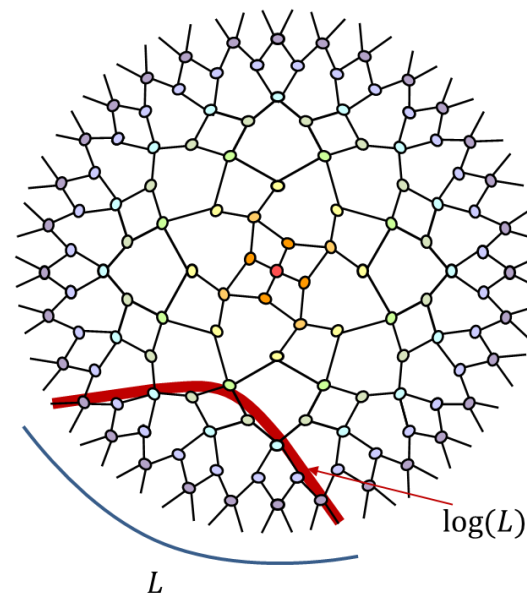
parallel to area of minimal surface in Ryu-Takayanagi

- two-point correlations

$$C(L) \approx L^{-2\Delta}$$

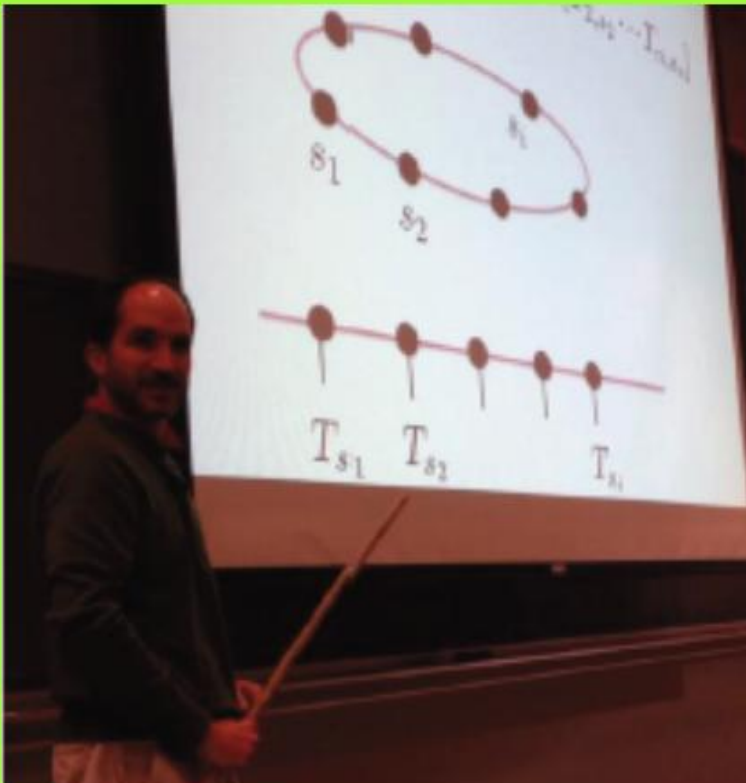
geodesic distance $D \approx \log(L)$ as in hyperbolic space

$$C(L) \approx e^{-D} = e^{-2\Delta \log(L)} = L^{-2\Delta}$$



Poincare disk

August 20th 2013

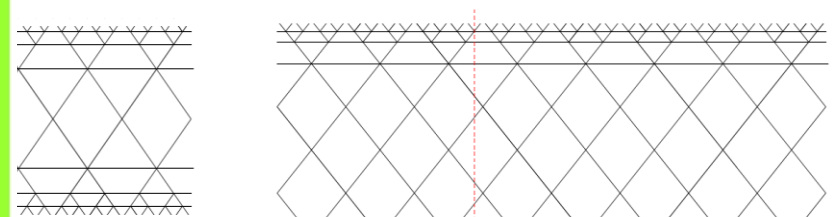


and Holography”

... using entanglement renormalization”

... spacetime,
... spectrum of scaling dimensions, etc

... “Black Hole Interiors” [Hartman, Maldacena 13]



... hole

... thermal state + time evolution

“Entanglement renormalization for quantum fields”

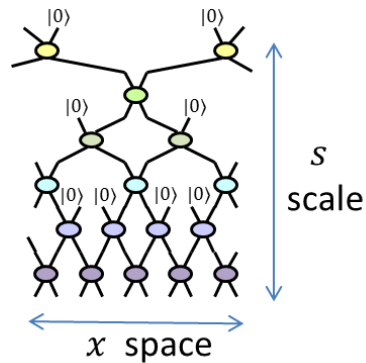
[Haegeman, Osborne, Verschelde, Verstraete 11]

ground state

$$|\Psi\rangle = U |0\rangle^{\otimes N}$$

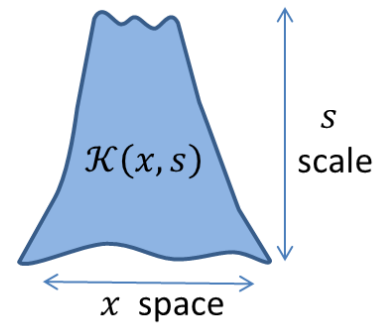
MERA

$$U = \prod_{x,s} u_{x,s}$$



continuous MERA
(cMERA)

$$U = \mathcal{P} e^{-i \int (\mathcal{K}(x,s) + \mathcal{L}) ds}$$



“Holographic Geometry of Entanglement Renormalization in Quantum Field Theories”

[Nozaki, Ryu, Takayanagi, 12]

“Holographic Geometry of cMERA for Quantum Quenches and Finite Temperature”

[Mollabashi, Nozaki, Ryu, Takayanagi 13]

“Boundary States as Holographic Duals of Trivial Spacetimes”

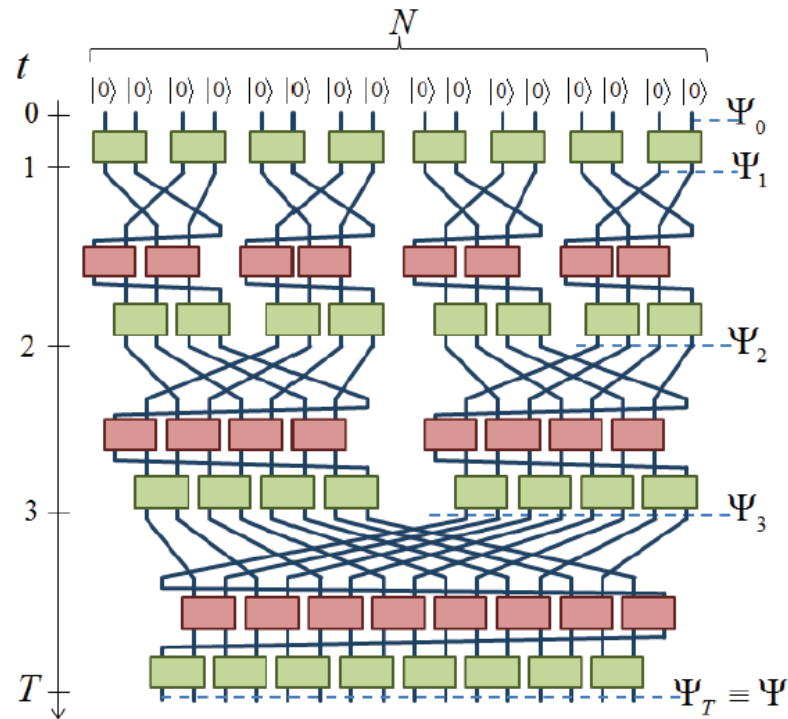
[Miyaji, Ryu, Takayanagi, Wen, 14]

“Information Geometry of Entanglement Renormalization for free Quantum Fields”

[Molina-Vilaplana, 15]

Branching MERA

[Evenbly, Vidal13]



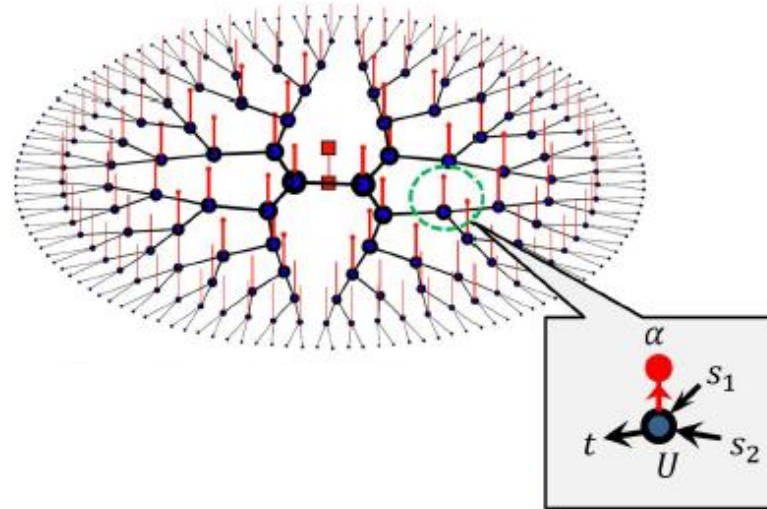
→ Brian Swingle's talk (yesterday)

→ Jeongwan Haah's talk (today, after Bartek Czech's)

"Many-body entanglement witness and branching MERA"

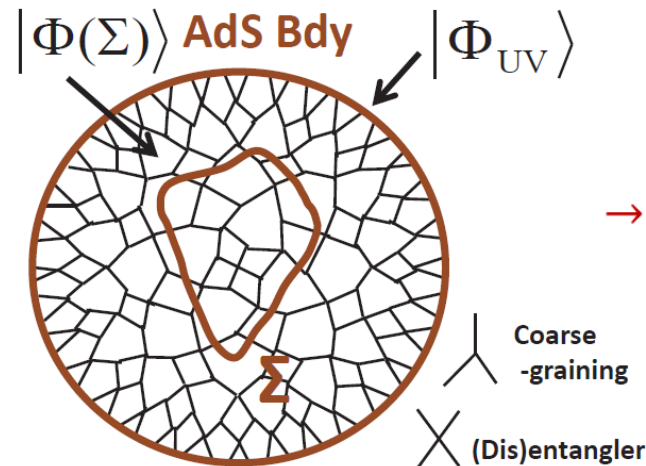
“Exact holographic mapping and emergent space-time geometry”

[Xiaoliang Qi, 13]

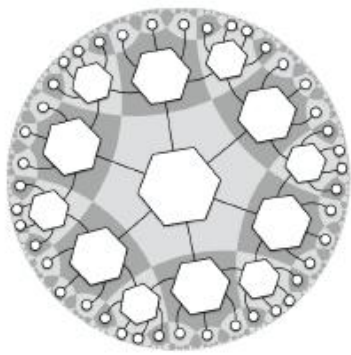


“Surface/State Correspondence as a Generalized Holography”

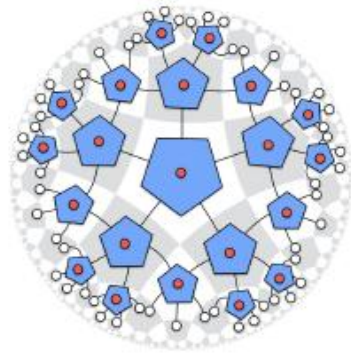
[Miyaji, Takayanagi, 15]



→ Tadashi Takayanagi's talk (tomorrow)



Holographic hexagon state



Holographic pentagon code

“Holographic quantum error-correcting codes:
Toy models for the bulk/boundary correspondence”

[Pastawki, Yoshida, Harlow, Preskill, 15]

→ John Preskill’s talk (tomorrow):

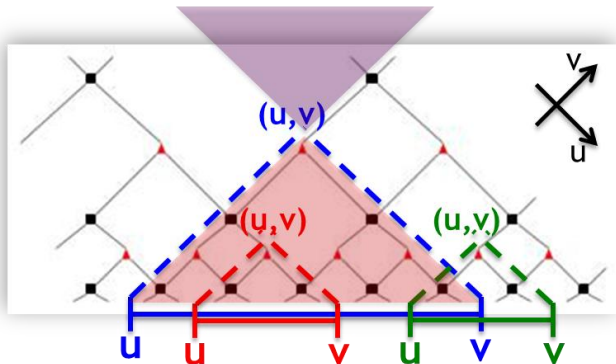
“Is spacetime a quantum error-correcting code?”

“Consistency Conditions for an AdS/MERA Correspondence”

[Bao, Cao, Carroll, Chatwin-Davies, Hunter-Jones, Pollack, Remmen, 15]

“Integral Geometry and Holography”

[Czech, Lamprou, McCandlish, Sully 15]



→ Bartek Czech’s talk (later today) :

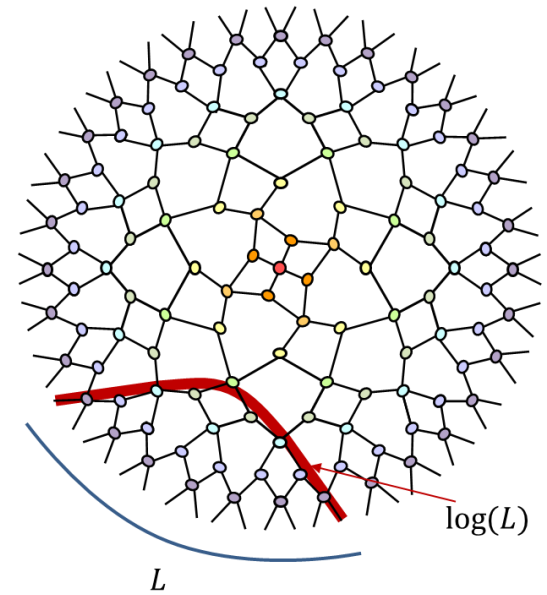
“From tensor networks to holography via integral geometry”

MERA = RG

e.g. for Ising model
(non-interacting, no large N)

Tensor network for ground state of CFT
in extra dimension corresponding to scale

boundary	bulk
state of CFT (space of CFT)	tensor network (?) (EHM)
Scaling dimension Δ	mass $\sim \Delta$
entanglement entropy	“minimal connecting surface”
global on-site symmetry (e.g. Z_2)	local/gauge symmetry (e.g. Z_2)



MERA operates at scale of AdS radius
smaller scale? \rightarrow cMERA

What happens with
large N, strong interactions?

outline

Part 1:
(old stuff)

Tensor networks

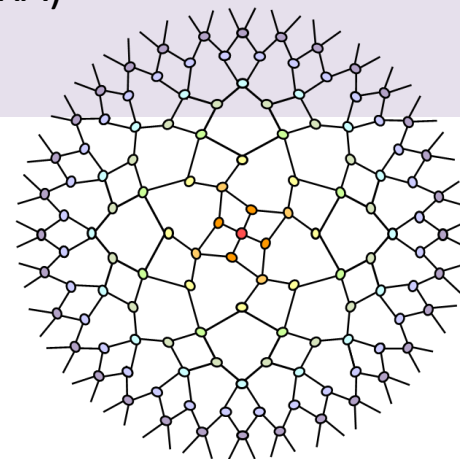
+

Renormalization group

=

Multi-scale entanglement
renormalization ansatz
(MERA)

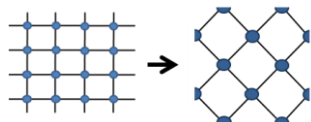
- quantum circuit
- RG transformation



Part 2: MERA and holography (!?)

Part 3:
(recent
developments)

Tensor network
renormalization
(TNR)

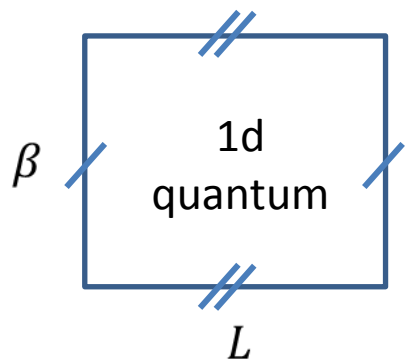


- MERA from Euclidean path integral
- thermal states (dual to a black hole?)

→ Glen Evenbly's talk (May 7th)

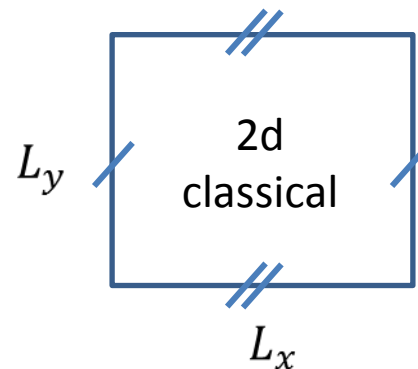
Euclidean path integral

$$Z(\lambda) = \text{tr} e^{-\beta H_q^{1d}}$$



Statistical partition function

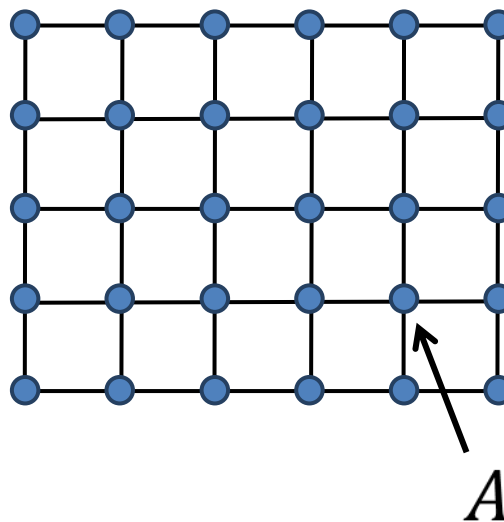
$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$



\sim

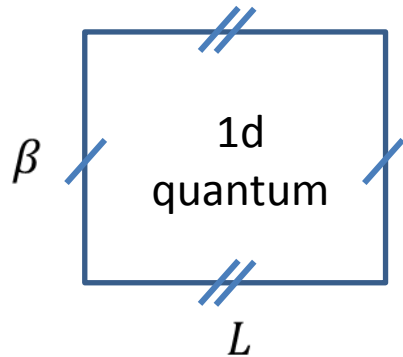
as a tensor network

$$Z =$$



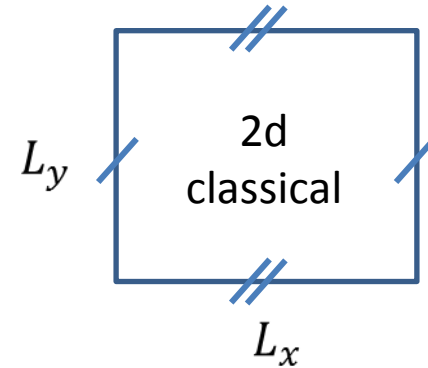
Euclidean path integral

$$Z(\lambda) = \text{tr} e^{-\beta H_q^{1d}}$$



Statistical partition function

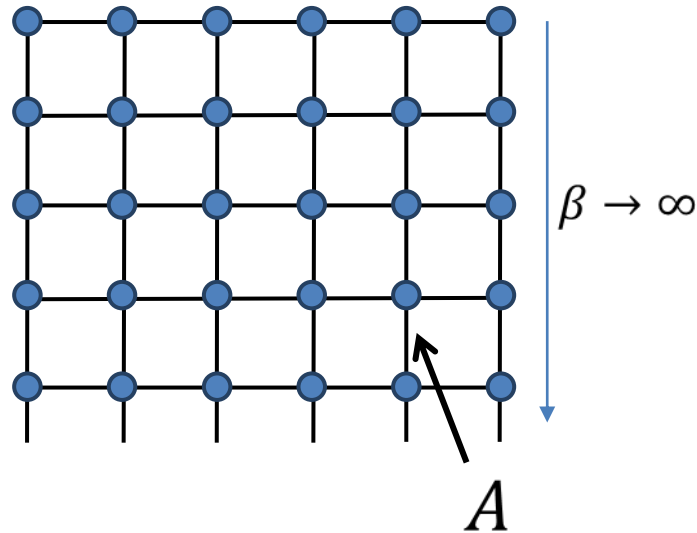
$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$

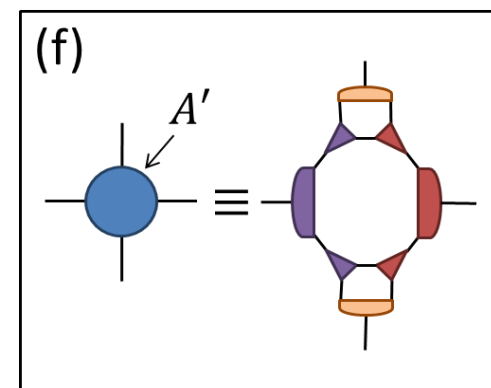
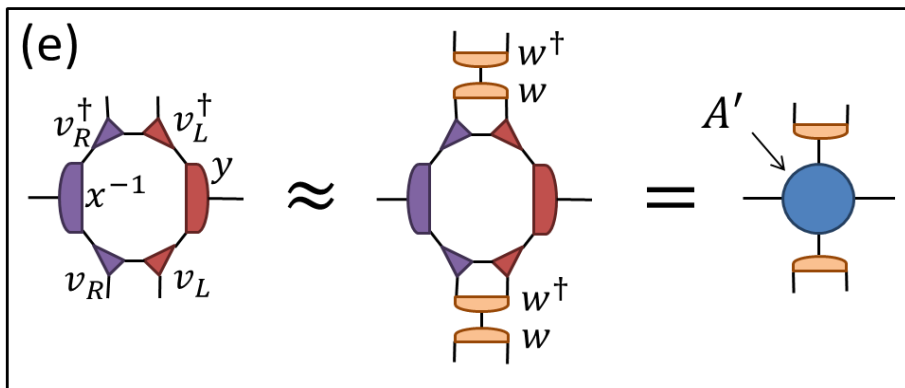
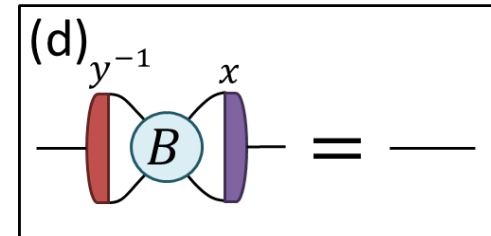
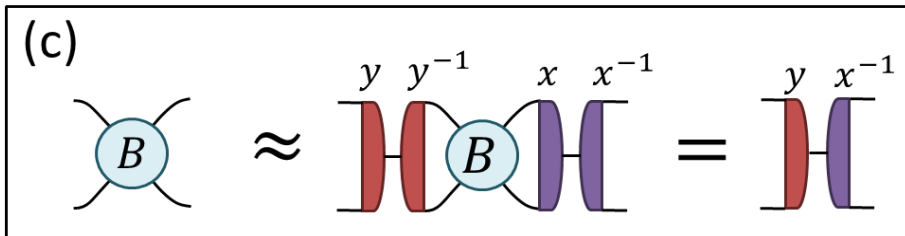
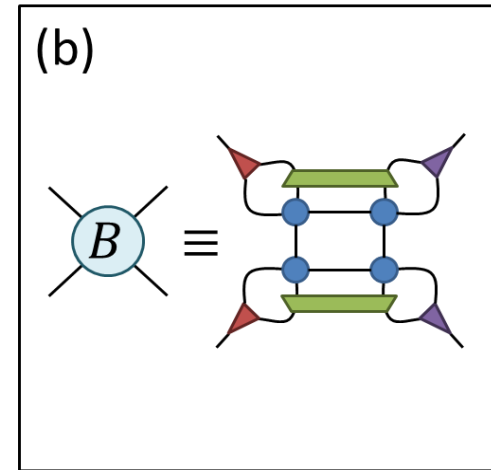
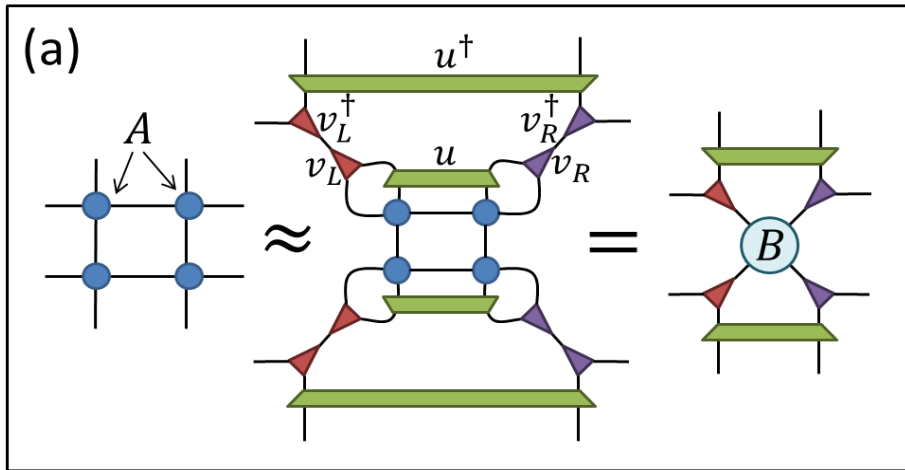


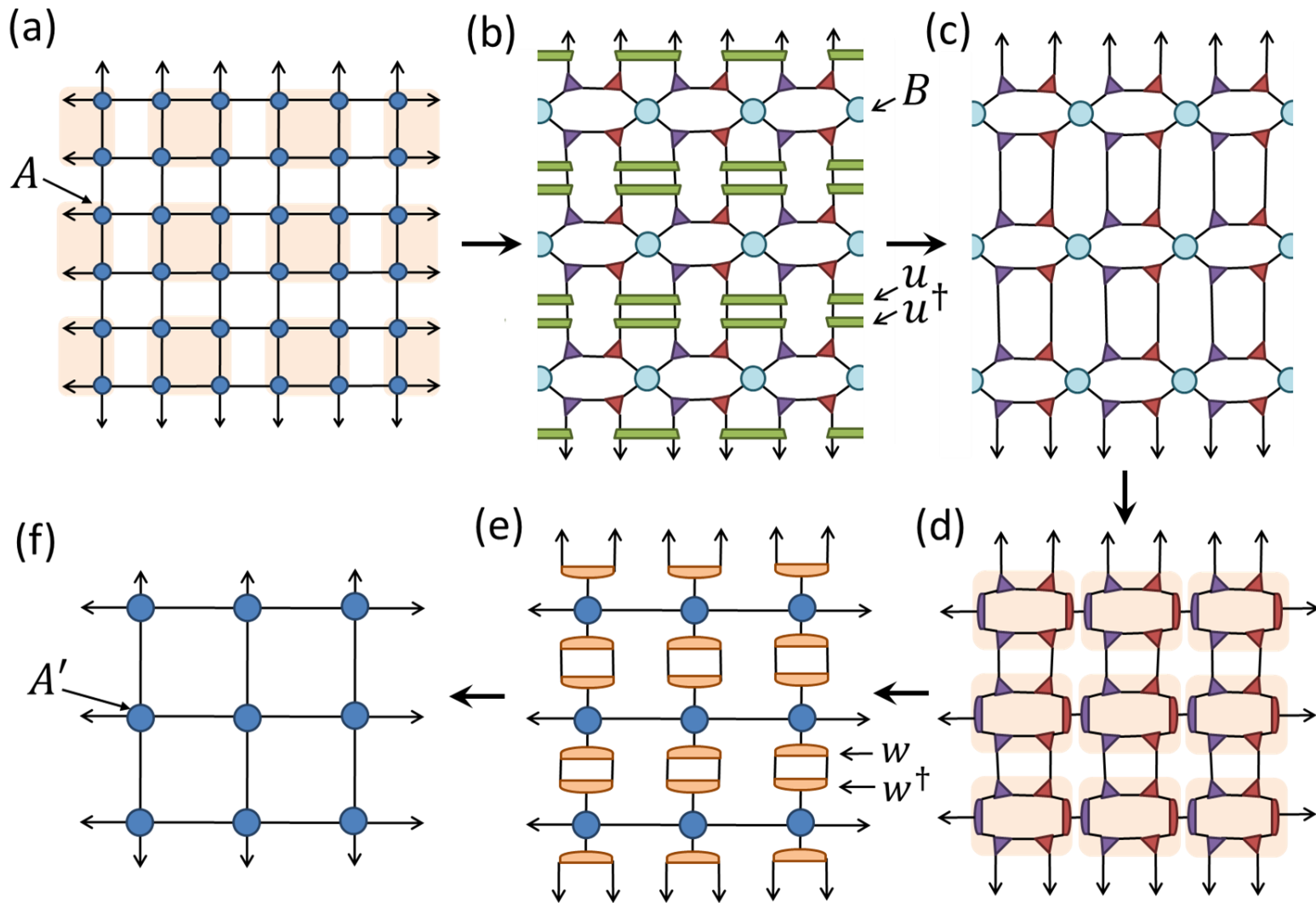
\sim

$$|\Psi\rangle \sim e^{-\beta H} |\phi_0\rangle$$

$$|\Psi\rangle \sim$$



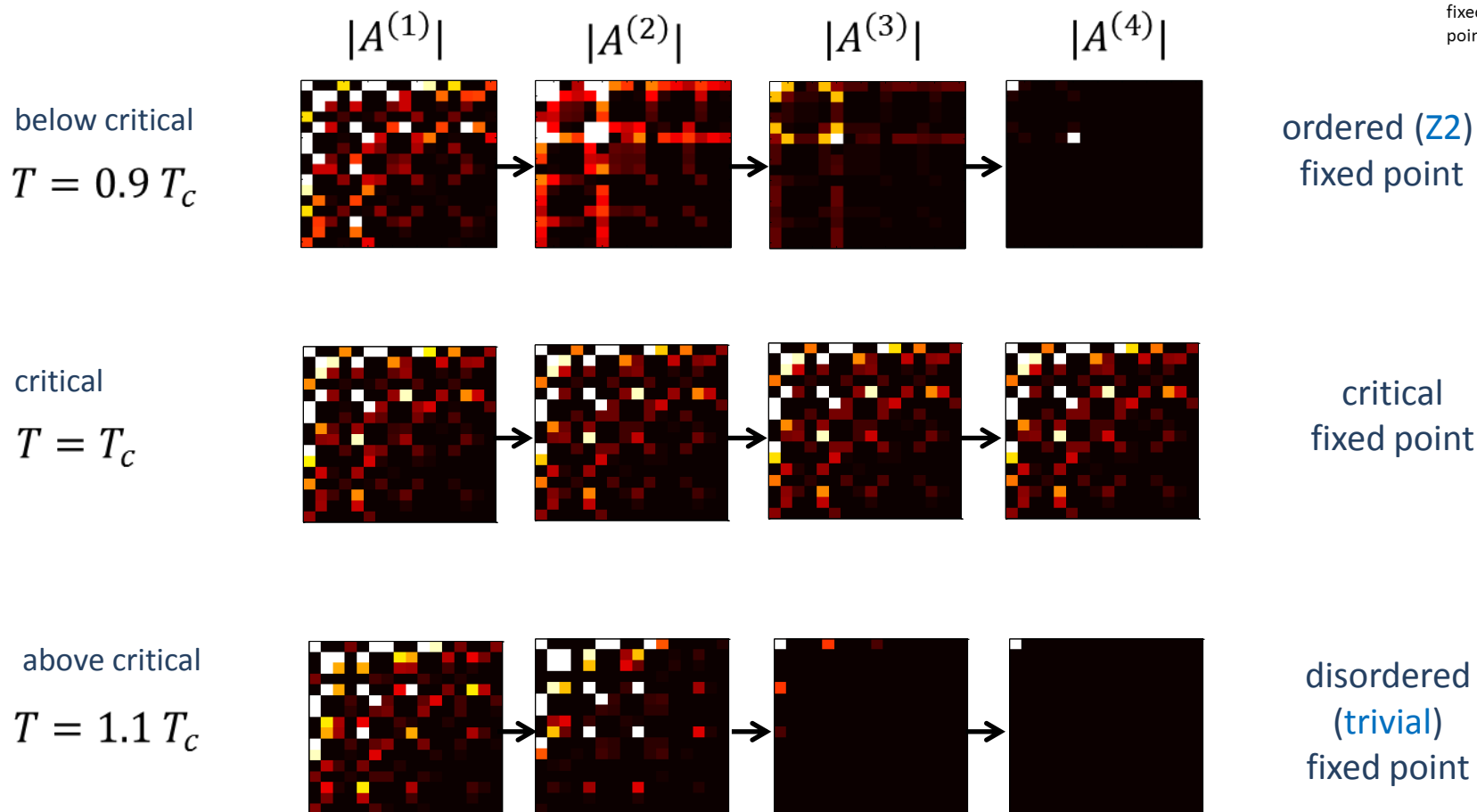
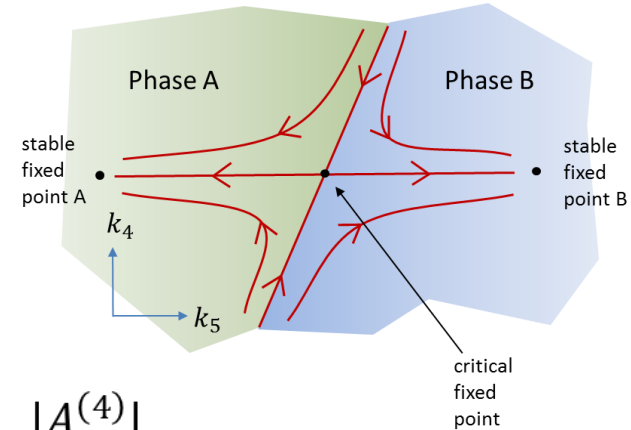




TNR -> proper RG flow

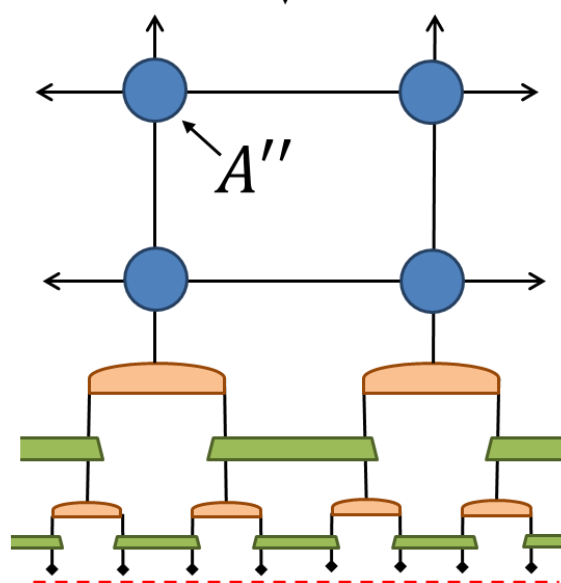
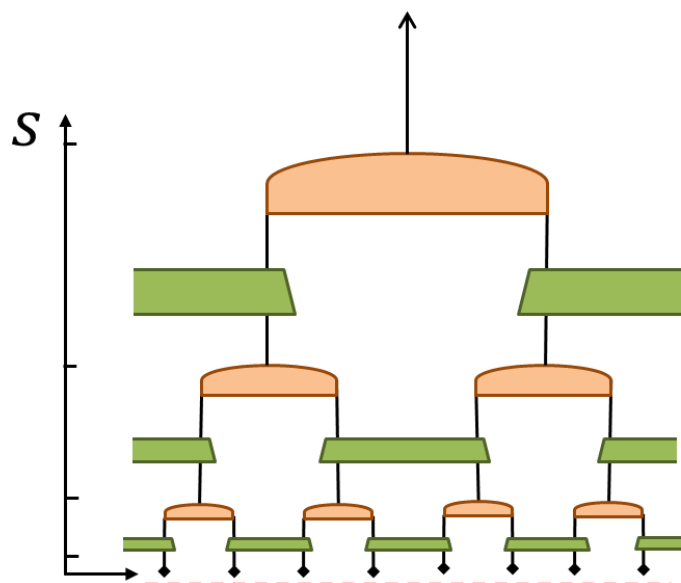
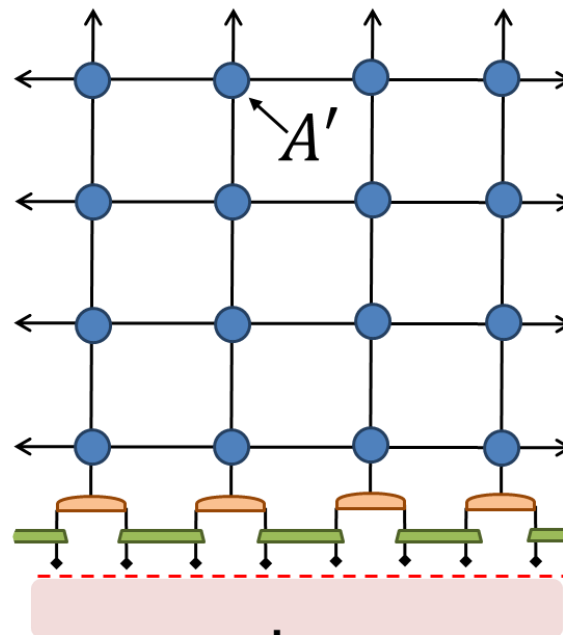
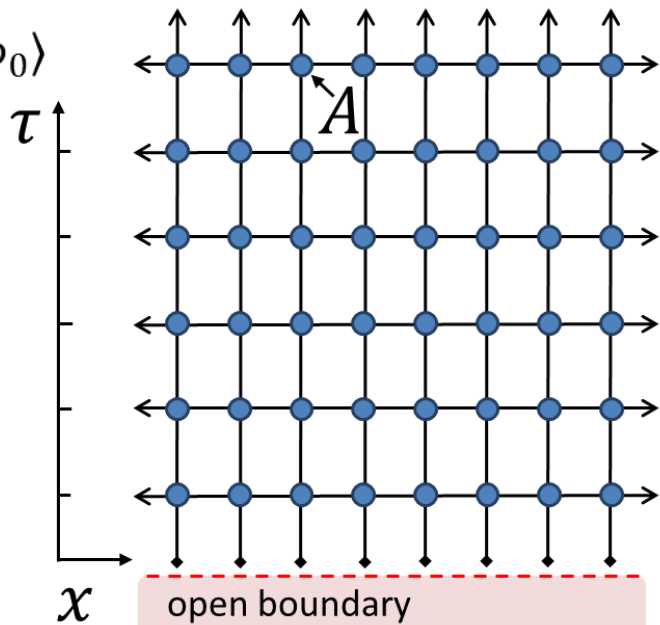
Example: 2D classical Ising

$$A \rightarrow A' \rightarrow A'' \rightarrow \dots \rightarrow A^{fp}$$



TNR yields MERA

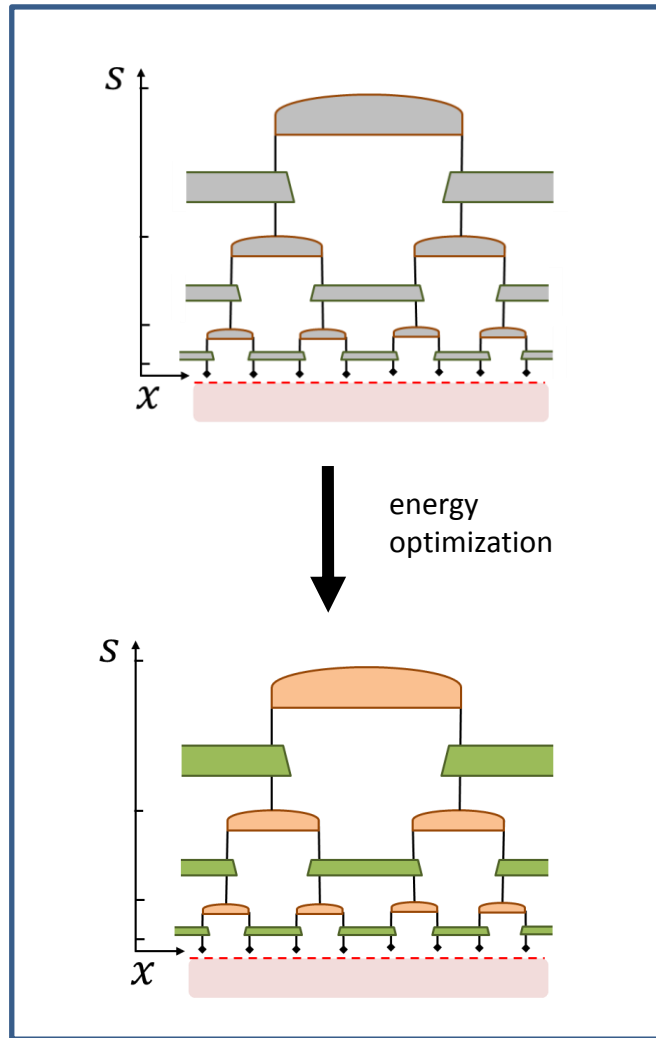
$$|\Psi\rangle \sim e^{-\tau H} |\phi_0\rangle$$



MERA = variational ansatz

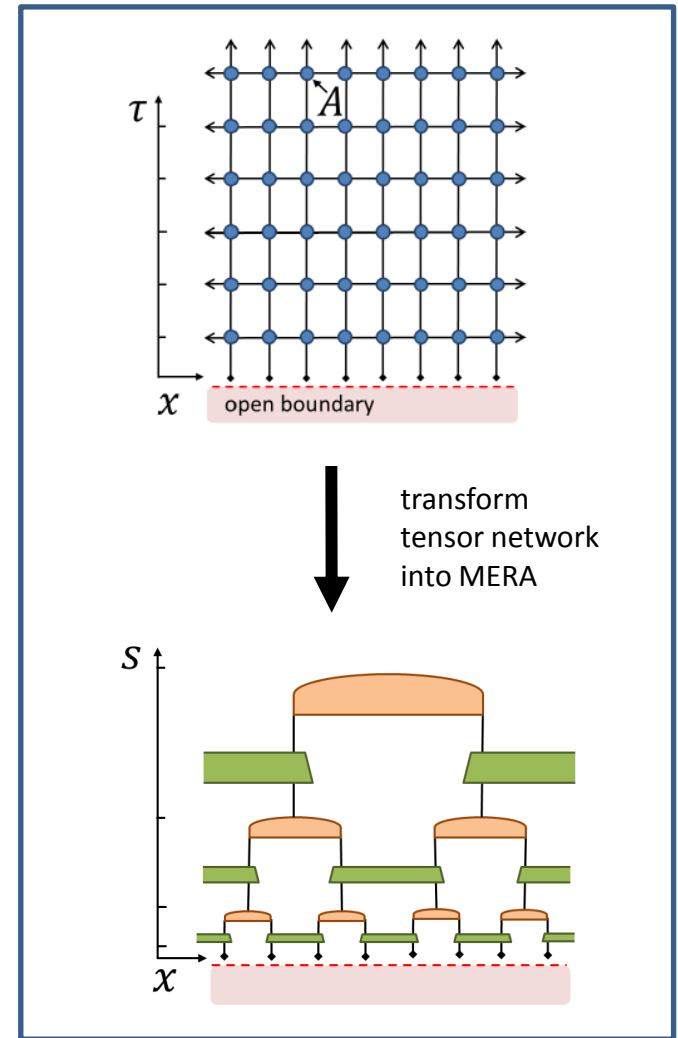


MERA = by-product of TNR



energy minimization

- 1000s of iterations over scale
- local minima
- correct ground ?

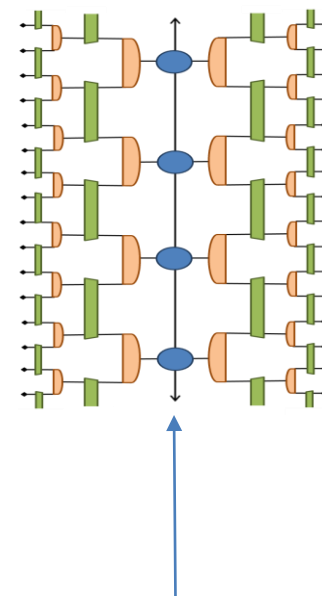
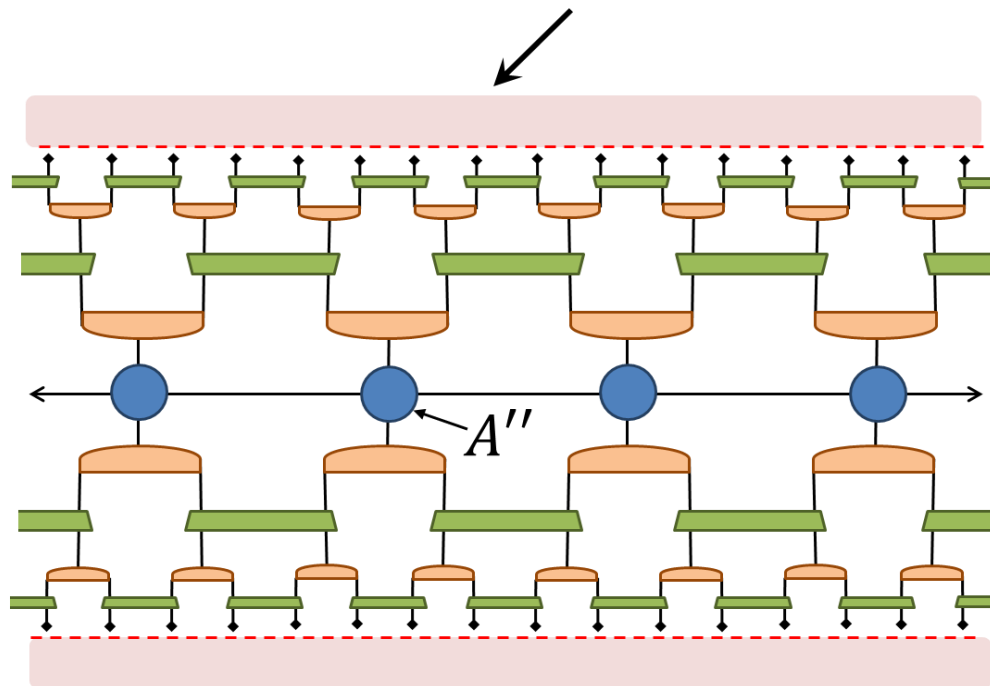
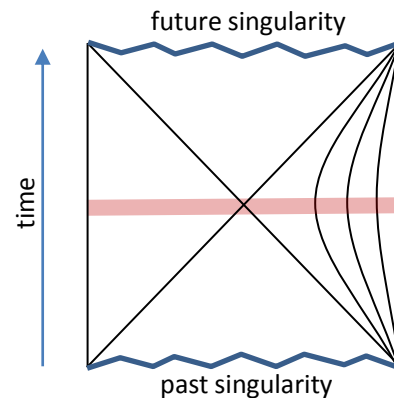
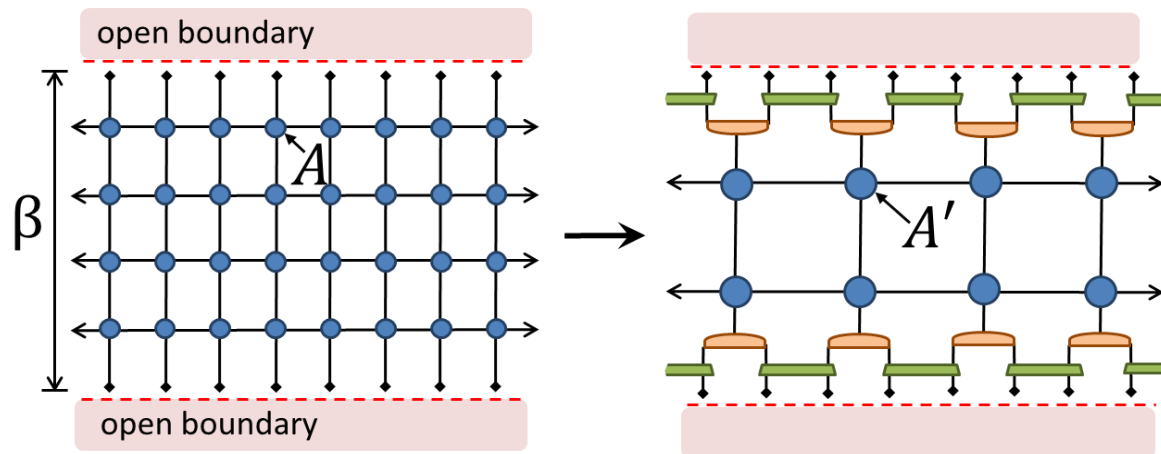


TNR -> MERA

- single iteration over scale
- rewrite tensor network for ground state
- certificate of accuracy

extra bonus: MERA for a thermal state (or black hole in holography)

$$\rho_\beta \sim e^{-\beta H}$$



Einstein-Rosen bridge

Extra bonus: conformal transformations

[Evenbly, Vidal, 15]

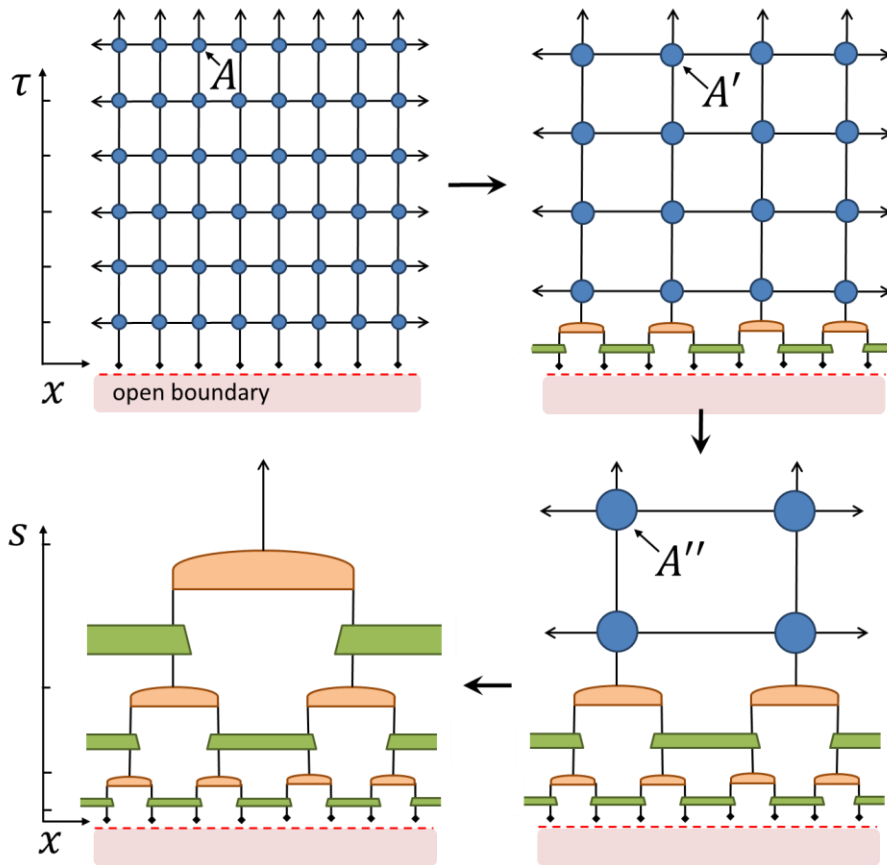
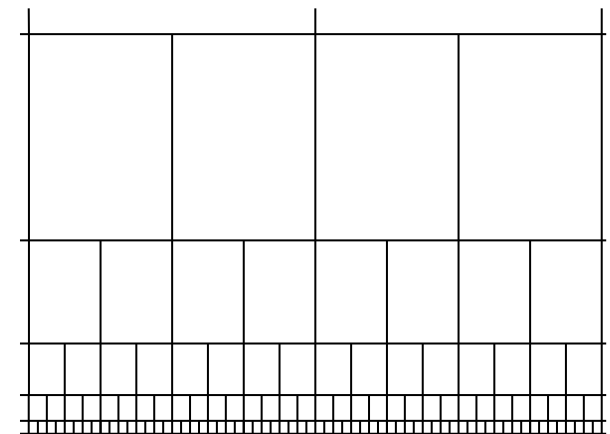
Conformal transformation 1:

Upper half-plane to H^2

$$dx^2 + d\tau^2 = \tau^2 \left(\frac{dx^2}{\tau^2} + \frac{d\tau^2}{\tau^2} \right)$$

$$\rightarrow \left(\frac{dx^2}{\tau^2} + \frac{d\tau^2}{\tau^2} \right) = \frac{dx^2}{2^{2s}} + ds^2$$

$$s = \log_2(\tau)$$

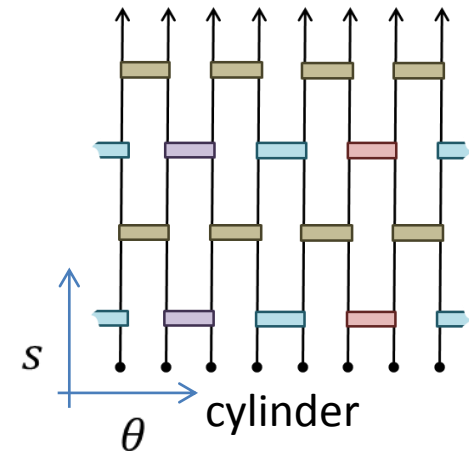
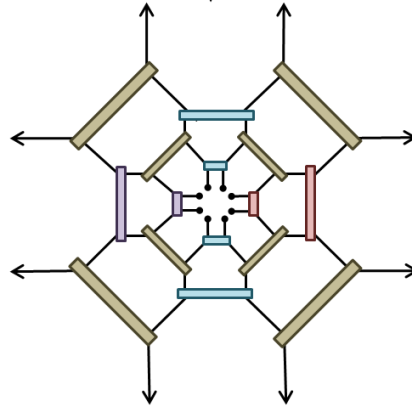
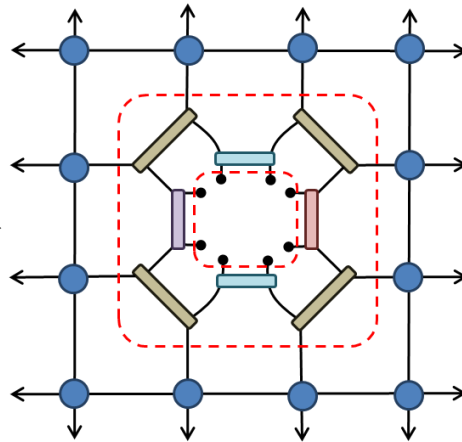
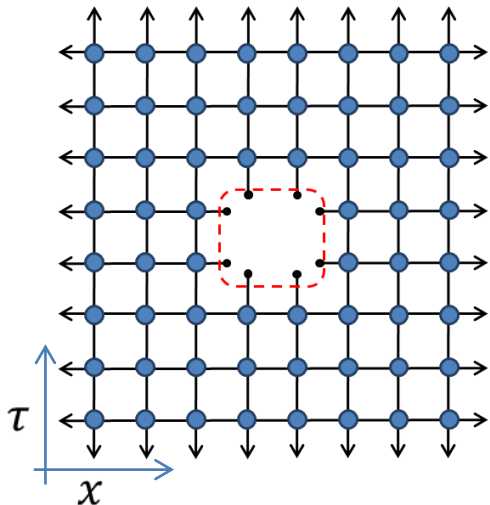


Holographic description

Extra bonus: conformal transformations

[Evenbly et al, in preparation]

plane



- Extraction of scaling dimensions

Conformal transformation 2:

Plane to cylinder

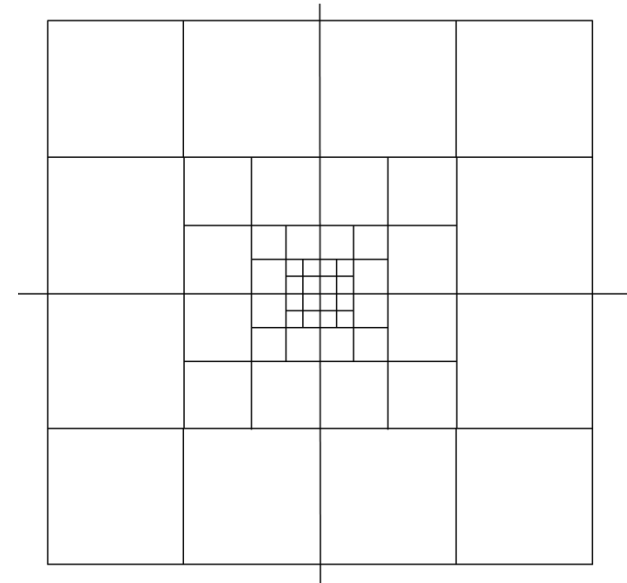
(radial quantization in CFT)

$$z \equiv x + i\tau$$

$$z = 2^w$$

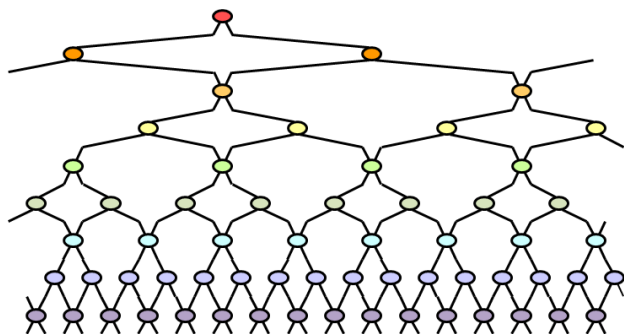
$$w \equiv s + i\theta$$

$$s \equiv \log_2(x^2 + \tau^2)$$



summary

Tensor Networks, Renormalization and Holography (overview)



- MERA as variational ansatz for ground states of CFTs (on the lattice)
 - e.g. critical Ising model
- + branching MERA
 - Jeongwan Haah's talk (today)
- MERA as a lattice realization of AdS/CFT
 - Bartek Czech's talk (next)
 - Tadashi Takayanagi's talk (tomorrow)
 - John Preskill's talk (tomorrow)
- MERA as a by-product of coarse-graining the Euclidean path integral

THANK YOU!