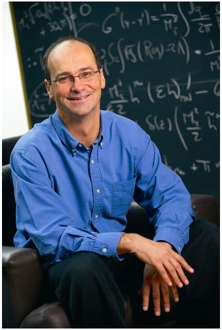
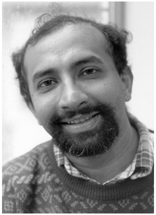
Entanglement and emergence
– a second quantum revolution

Xiao-Gang Wen MIT/Perimeter (June 5, 2015, KITP UCSB)





Thanks!

The cycles of the development of physics theory

Discovery → **Unification** →
More discovery → **More unification** → **...**

- There are a few big unifications = revolutions in physics

- Each revolution leads us into

\- a new **world view**
\- a new **language** (new mathematics) to describe the new view
\- a **unification** of seemingly total unrelated phenomena

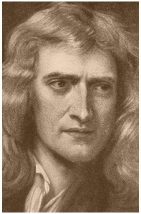
Mechanical revolution

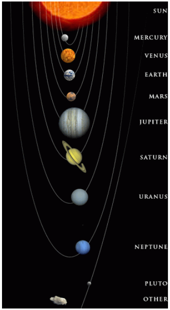
Newton (1687)

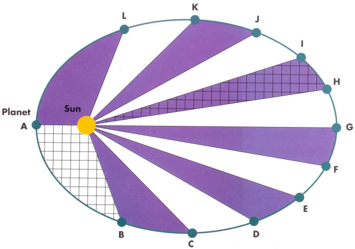
- **Unified** falling apple on earth and the planet motion in sky
- **New world view:**

- **matter** = **particles**
- The particle motion is described by $\boldsymbol{F} = \boldsymbol{ma}$.

- **New math**: **Calculus**





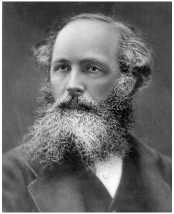


Electromagnetic revolution

Maxwell (1861)

- **Unified** electricity, magnetism, and light
- **New world view:**

- a new form of matter **wave-like matter**,
- which causes **electromagnetic interaction** between the **particle-like matter**.
- The motion of **wave-like matter** is described by the Maxwell equation $\dot{\boldsymbol{E}} - c\boldsymbol{\partial} \times \boldsymbol{B} = \dot{\boldsymbol{B}} + c\boldsymbol{\partial} \times \boldsymbol{E} = 0$.

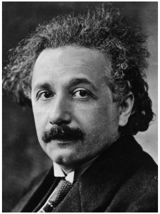


- **New math**: **Fiber bundle (gauge theory)**

The first compass

Relativity revolution

Einstein (1905,1916)

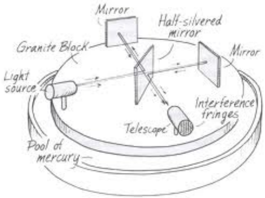


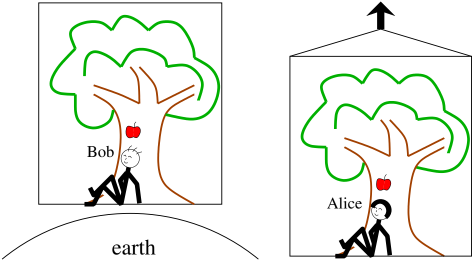
- **Unified** space, time, and gravitional interaction
- **New world view:**

- Space-time is dynamical = **wave-like matter**.
- space-time wave-like matter causes gravitational interaction between the particle-like matter
- The new wave-like matter satisfies the Einstein equation $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = -\frac{8\pi}{c^4} T_{\mu\nu}$

- **New math**: **Riemannian geometry (curved space)**

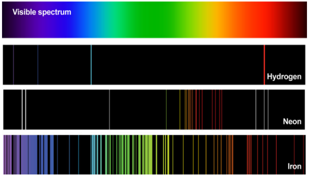
Michelson-Morley (1887)

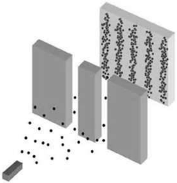


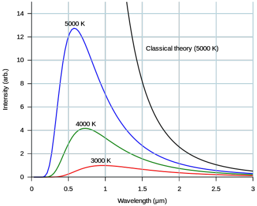


Quantum revolution

- **Unified**:Hydrogen spectra, blackbody radiation, interference
- **New world view:**
  particle-like matter = wave-like matter
  → A new form of matter – **particle-wave-like matter**.
- **New math**: **linear algebra and tensor product**





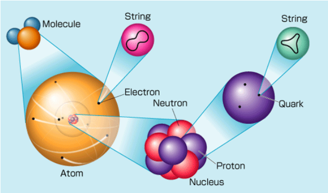


The essence of quantum theory is a unification between **matter** and **information**

We used to think information and matter are two very different things: information is the attribute carried by matter.

Information: Changing information (qubits) $\to$ frequency
According to quantum physics: frequency $\to$ energy
According relativity: energy $\to$ mass $\to$ **Matter**

=



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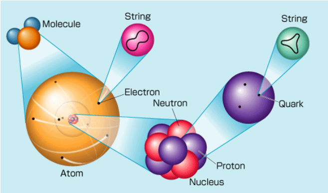
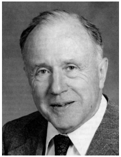
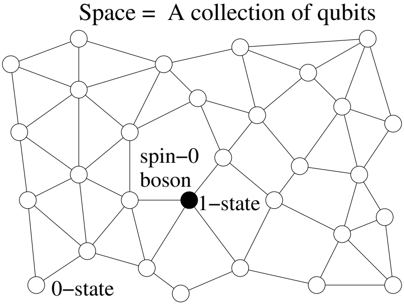


- But can simple qubits (quantum information) really produce all kinds of matter (and all the elementary particles)?

If matter was formed by spin-0 bosons, then **It from bit**

- The space = a collection of qubits.
- The 0-state = the vacuum.
- The 1-state = a spin-0 boson.





- *Ground state of the space-forming qubits = vacuum*
  Excitations above the ground state = elementary particles
  Massless Excitations from continuous symmetry breaking

But our elementary particles are very complicated

Eight strange properties of elementray particles:

1. Locality
2. Identical particles
3. Spin-1 bosons with only two-components (gauge bosons)
4. Particles with Fermi statistics
5. Fractional angular momentum (spin-1/2)
6. Only left-hand fermions couple the $SU(2)$-gauge-bosons
7. Lorentz symmetry
8. Spin-2 bosons with only two-components (gravitons?)

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Can simple qubits produce the above eight strange properties?

- **Yes**, 1-7 is possible (*ie* the standard model can emerge) if the space-forming qubits are **Long-range entangled** Chen-Gu-Wen 10 (also referred as **topologically ordered** Wen 89)

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- **A new unification**: Qubits unify gauge boson and fermion
- **A new world view**: Our world is made of quantum information!
- Yet to be done: *The emergence of gapless gravitons from qubits*

A second quantum revolution

Quantum information = Matter

Entanglement $\rightarrow$ Gauge interation

fiber bundle

Entanglement $\rightarrow$ Fermi statistics

Entangled qubits $\rightarrow$ Standard model

Entanglement $-?\!\rightarrow$ Geometry

tangent bundle

It from qubit, not bit

Long-range entanglement → Gauge interactions

- Chiral-spin/FQH liquid → Chern-Simons gauge theory $\mathcal{L} = \frac{k}{4\pi} ada$

  Wen-Wilczek-Zee PRB **39** 11413 (89); Zhang-Hansson-Kivelson PRL **62** 82 (89)
- RVB spin liquid → low energy effective Z_2 gauge theory

  Read-Sachdev PRL **66** 1773 (91); Wen PRB **44** 2664 (91)
- Wave in closed-string liquid Wen cond-mat/0210040

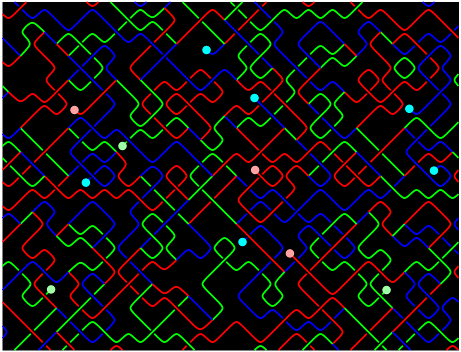
String density $\boldsymbol{E}(\boldsymbol{x})$ wave → 3+1D $U(1)$-Maxwell equation .

Baskaran-Anderson PRL **37** 580 (88); Wen hep-th/01090120
Senthil-Motrunich cond-mat/0201320; Hermele-Fisher-Balents cond-mat/0305401

- Low energy Maxwell theory is a stable IR fixed point. Massless photons are robust against any local perturbations (even the string breaking ones). Hastings-Wen cond-mat/0503554

Long-range entanglement → Gauge interactions

- If the strings have several types and can join in certain way → String-net liquid → Non-Abelian gauge theory.



A picture of our vacuum — A string–net theory of light and electrons

Levin-Wen cond-mat/0404617

- The gauge group comes from (1) the number of string types and (2) the way how strings join.
- The non-Abelian gauge phase and the emergent gauge theory are robust against any perturbations (even the string breaking ones).

Long-range entanglement → Fermi (fractional) statistics

- 2+1D FQH states → emgergent fractional statistics

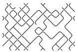
- Binding flux to charge in 2+1D → fractional statistics

Halperin PRL **52** 1583 (84); Arovas-Schrieffer-Wilczek PRL **53** 722 (84)

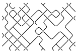
- In 3+1D $U(1)$ gauge theory, monople + charge → Fermi statistics.

Witten Phys. Lett. B **86** 283 (79)

- A general mechanism for any dimensions (with or without $U(1)$): **end of string in string-net liquid → Fermi statistics**

- For string liquid state $|\Phi\rangle = \sum_{\text{all conf.}} \Big|$  $\Big\rangle$,  $= +1$

→ End of strings = boson (Higgs boson).

- For string liquid state $|\Phi\rangle = \sum_{\text{all conf.}} \pm \Big|$  $\Big\rangle$,  $= -1$

→ End of string = fermion

Levin-Wen cond-mat/0302460

- **End of string** = gauge charge → A prediction: **All (composite) fermions carry gauge charges → The standard model must contain extra gauge "symmetry" → new cosmic string.**

- **A unification of gauge interactions and Fermi statistics**

Quantum entanglement → New mathematics

- Pattern of many-body entanglement = phase of quantum matter

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  SPT orders → "classified" by $\mathcal{H}^{d+1}(G \times SO_\infty, \mathbb{R}/\mathbb{Z})$

Chen-Gu-Liu-Wen 1106.4772; Vishwanath-Senthil 1209.3058

Kapustin 1404.6659; Wen 1410.8477

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Kitaev cond-mat/0506438; Rowell-Stong-Wang 0712.1377

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Levin-Wen cond-mat/0404617

- in $2+1$D with only Abelian statistics, by integer K-matrix.

Wen-Zee PRB **46** 2290 (92)

Quantum entanglement → New mathematics

- Long range-entanglement in $2+1$D with symmetry G
  → classified by G-crossed modular tensor category.

  Barkeshli-Bonderson-Cheng-Wang 1410.4540

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  Barkeshli-Bonderson-Cheng-Wang 1410.4540

- For fermion systems, the super-version of the above math theories

- Fermionic unitary fusion category — Gu-Wang-Wen 1010.1517
- Group super-cohomology — Gu-Wen 1201.2648; Gaiotto-Kapustin 1505.05856
- Spin-cobordism — Kapustin-Thorngren-Turzillo-Wang 1406.7329

2+1D topological orders with N type of topo. excitations

c = central charge, d_i = quantum dimensions, s_i = spins, $\zeta_n^m = \frac{\sin[\pi(m+1)/(n+2)]}{\sin[\pi/(n+2)]}$

$N\|c$	$d_1, d_2, \cdots$	$s_1, s_2, \cdots$	$N\|c$	$d_1, d_2, \cdots$	$s_1, s_2, \cdots$
$1\|1$	1	0			
$2\|1$	$1, 1$	$0, \frac{1}{4}$	$2\|-1$	$1, 1$	$0, -\frac{1}{4}$
$2\|\frac{14}{5}$	$1, \zeta_3^1$	$0, \frac{2}{5}$	$2\|-\frac{14}{5}$	$1, \zeta_3^1$	$0, -\frac{2}{5}$
$3\|2$	$1, 1, 1$	$0, \frac{1}{3}, \frac{1}{3}$	$3\|-2$	$1, 1, 1$	$0, -\frac{1}{3}, -\frac{1}{3}$
$3\|\frac{1}{2}$	$1, 1, \zeta_2^1$	$0, \frac{1}{2}, \frac{1}{16}$	$3\|-\frac{1}{2}$	$1, 1, \zeta_2^1$	$0, \frac{1}{2}, -\frac{1}{16}$
$3\|\frac{3}{2}$	$1, 1, \zeta_2^1$	$0, \frac{1}{2}, \frac{3}{16}$	$3\|-\frac{3}{2}$	$1, 1, \zeta_2^1$	$0, \frac{1}{2}, -\frac{3}{16}$
$3\|\frac{5}{2}$	$1, 1, \zeta_2^1$	$0, \frac{1}{2}, \frac{5}{16}$	$3\|-\frac{5}{2}$	$1, 1, \zeta_2^1$	$0, \frac{1}{2}, -\frac{5}{16}$
$3\|\frac{7}{2}$	$1, 1, \zeta_2^1$	$0, \frac{1}{2}, \frac{7}{16}$	$3\|-\frac{7}{2}$	$1, 1, \zeta_2^1$	$0, \frac{1}{2}, -\frac{7}{16}$
$3\|\frac{8}{7}$	$1, \zeta_5^1, \zeta_5^2$	$0, -\frac{1}{7}, \frac{2}{7}$	$3\|-\frac{8}{7}$	$1, \zeta_5^1, \zeta_5^2$	$0, \frac{1}{7}, -\frac{2}{7}$
$4\|0$	$1, 1, 1, 1$	$0, 0, 0, \frac{1}{2}$	$4\|0$	$1, 1, 1, 1$	$0, 0, \frac{1}{4}, -\frac{1}{4}$
$4\|1$	$1, 1, 1, 1$	$0, \frac{1}{8}, \frac{1}{8}, \frac{1}{2}$	$4\|-1$	$1, 1, 1, 1$	$0, -\frac{1}{8}, -\frac{1}{8}, \frac{1}{2}$
$4\|2$	$1, 1, 1, 1$	$0, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}$	$4\|-2$	$1, 1, 1, 1$	$0, -\frac{1}{4}, -\frac{1}{4}, \frac{1}{2}$
$4\|3$	$1, 1, 1, 1$	$0, \frac{3}{8}, \frac{3}{8}, \frac{1}{2}$	$4\|-3$	$1, 1, 1, 1$	$0, -\frac{3}{8}, -\frac{3}{8}, \frac{1}{2}$
$4\|4$	$1, 1, 1, 1$	$0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$4\|\frac{9}{5}$	$1, 1, \zeta_3^1, \zeta_3^1$	$0, -\frac{1}{4}, \frac{3}{20}, \frac{2}{5}$
$4\|-\frac{9}{5}$	$1, 1, \zeta_3^1, \zeta_3^1$	$0, \frac{1}{4}, -\frac{3}{20}, -\frac{2}{5}$	$4\|\frac{19}{5}$	$1, 1, \zeta_3^1, \zeta_3^1$	$0, \frac{1}{4}, -\frac{7}{20}, \frac{2}{5}$
$4\|-\frac{19}{5}$	$1, 1, \zeta_3^1, \zeta_3^1$	$0, -\frac{1}{4}, \frac{7}{20}, -\frac{2}{5}$	$4\|0$	$1, \zeta_3^1, \zeta_3^1, \zeta_3^1\zeta_3^1$	$0, \frac{2}{5}, -\frac{2}{5}, 0$
$4\|\frac{12}{5}$	$1, \zeta_3^1, \zeta_3^1, \zeta_3^1\zeta_3^1$	$0, -\frac{2}{5}, -\frac{2}{5}, \frac{1}{5}$	$4\|-\frac{12}{5}$	$1, \zeta_3^1, \zeta_3^1, \zeta_3^1\zeta_3^1$	$0, \frac{2}{5}, \frac{2}{5}, -\frac{1}{5}$
$4\|\frac{10}{3}$	$1, \zeta_7^1, \zeta_7^2, \zeta_7^3$	$0, \frac{1}{3}, \frac{2}{9}, -\frac{1}{3}$	$4\|-\frac{10}{3}$	$1, \zeta_7^1, \zeta_7^2, \zeta_7^3$	$0, -\frac{1}{3}, -\frac{2}{9}, \frac{1}{3}$
$5\|0$	$1, 1, 1, 1, 1$	$0, \frac{1}{5}, \frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}$	$5\|4$	$1, 1, 1, 1, 1$	$0, \frac{2}{5}, \frac{2}{5}, -\frac{2}{5}, -\frac{2}{5}$
$5\|2$	$1, 1, \zeta_4^1, \zeta_4^1, 2$	$0, 0, \frac{1}{8}, -\frac{3}{8}, \frac{1}{3}$	$5\|2$	$1, 1, \zeta_4^1, \zeta_4^1, 2$	$0, 0, -\frac{1}{8}, \frac{3}{8}, \frac{1}{3}$
$5\|-2$	$1, 1, \zeta_4^1, \zeta_4^1, 2$	$0, 0, \frac{1}{8}, -\frac{3}{8}, -\frac{1}{3}$	$5\|-2$	$1, 1, \zeta_4^1, \zeta_4^1, 2$	$0, 0, -\frac{1}{8}, \frac{3}{8}, -\frac{1}{3}$
$5\|\frac{16}{11}$	$1, \zeta_9^1, \zeta_9^2, \zeta_9^3, \zeta_9^4$	$0, -\frac{2}{11}, \frac{2}{11}, \frac{1}{11}, -\frac{5}{11}$	$5\|-\frac{16}{11}$	$1, \zeta_9^1, \zeta_9^2, \zeta_9^3, \zeta_9^4$	$0, \frac{2}{11}, -\frac{2}{11}, -\frac{1}{11}, \frac{5}{11}$
$5\|\frac{18}{7}$	$1, \zeta_5^2, \zeta_5^2, \zeta_{12}^2, \zeta_{12}^4$	$0, -\frac{1}{7}, -\frac{1}{7}, \frac{1}{7}, \frac{3}{7}$	$5\|-\frac{18}{7}$	$1, \zeta_5^2, \zeta_5^2, \zeta_{12}^2, \zeta_{12}^4$	$0, \frac{1}{7}, \frac{1}{7}, -\frac{1}{7}, -\frac{3}{7}$

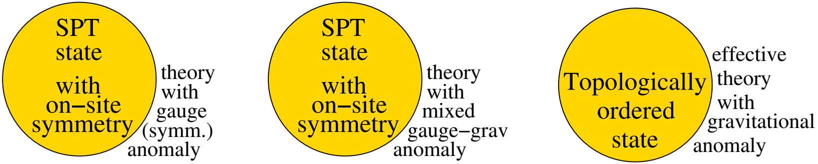
Rowell-Stong-Wang 0712.1377; Wen to appear

Quantum entanglement classifies gauge/gravity anomalies

- Anomaly-free = Has UV completion (can be put on lattice).

Wen 1303.1803; Kapustin-Thorngren 1404.3230; Kong-Wen 1405.5858

SPT order by $\mathcal{H}^d(G, \mathbb{R}/\mathbb{Z})$ Topological order by n-category



SPT order by $\mathcal{H}^d(G \times SO_\infty, \mathbb{R}/\mathbb{Z})$

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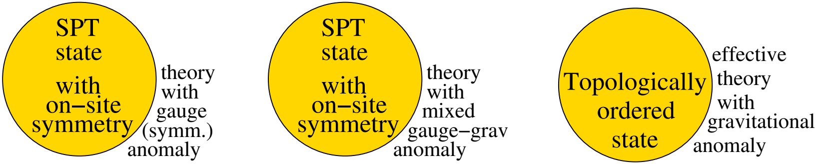

SPT order by $\mathcal{H}^d(G \times SO_\infty, \mathbb{R}/\mathbb{Z})$

A solution of chiral fermion problem Eichten-Preskill NPB **268** 179 (86)

(standard model on lattice) Wen 1305.1045; You-Xu 1412.4784

$SO(10)$ or $SU(4) \times SU(2) \times SU(2)$ w/ 16 Weyl fermions

---

trivial SPT state and trivial topological order
in $4+1$D with finite width in the 4th dimension

---

mirror of $SO(10)$ or $SU(4) \times SU(2) \times SU(2)$ chiral GUT, which can be fully gapped **without breaking the gauge symmetries**

Quantum entanglement $\to$ gapless "gravitons"

From local $3+1$D lattice models of qubits

$\to$ gapless pseudo gravitons with helicity $0, \pm 2$ and $\omega \sim k^2$

Xu cond-mat/0609595

$\to$ gapless gravitons with helicity ± 2 and $\omega \sim k$ (???)
The Lorentz symmetry is emergent and non-exact at finite energies to evade Witten-Weinberg theorem (???) Gu-Wen gr-qc/0606100

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Low energy Lifshitz gravity as a stable IR fixed point (against any local perturbations) Gu-Wen 0907.1203; Xu-Horava 1003.0009

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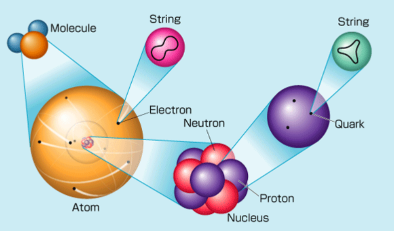
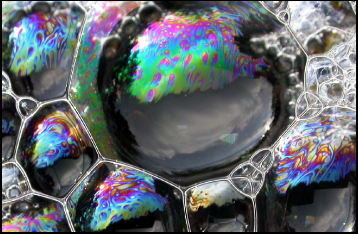
- From holographer's point of view, here we assume that the "bulk" observer can choose a proper local bluk time such that the gauge fixed bulk theory has an emergent **locality**: $\mathcal{V}_{\text{tot}} = \otimes_i \mathcal{V}_i$ (*ie* anomaly-free). In contrast, in holographic approach, the bulk theory as the holographic dual of boundary CFT do not have to have such a strict locality, and can be anomalous (?)





matter = quantum information

Gauge int. and Fermi statistics come from Long-range entanglement