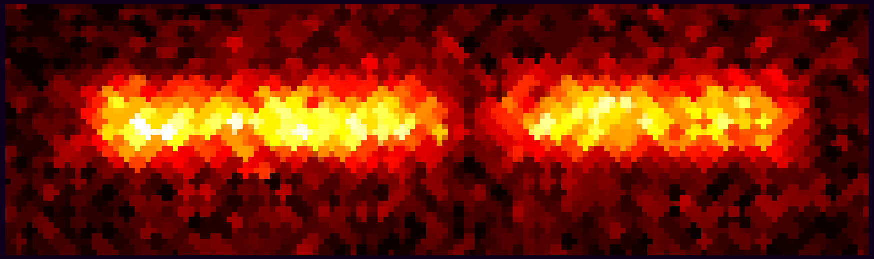
Spin-Entanglement detection in Bose-Hubbard chains



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Entanglement in strongly correlated quantum matter

May 19, 2015

Collaborations

Theory work with Pisa group:

- **Leonardo Mazza**
- Davide Rosini
- Rosario Fazio

L. Mazza, D. Rossini, R. Fazio, ME, New J. Phys. **17**, 013015 (2015)
arxiv:1408:4672

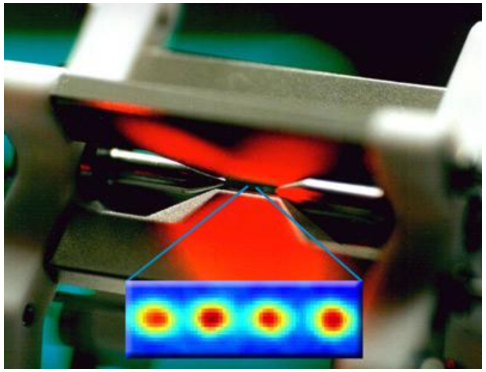
Experiments at MPQ Garching:

- **Takeshi Fukuhara**
- **Sebastian Hild**
- Johannes Zeiher
- Immanuel Bloch
- Christian Gross

T. Fukuhara, S. Hild, J. Zeiher, P. Schauß, I. Bloch, ME, C. Gross, PRL (accepted)
arxiv:1504.02582

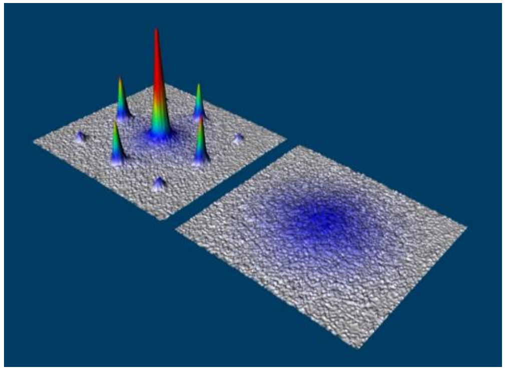
Single-particle control & detection

Few-atom systems (e.g. ion chain)



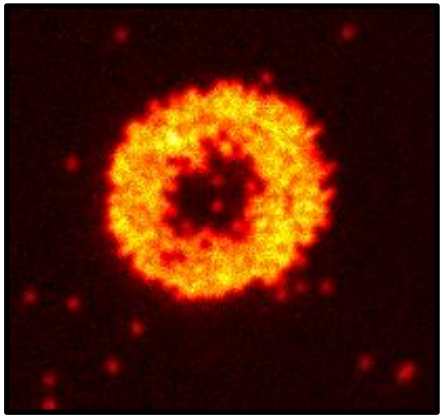
Coherent control over single particles

Many-body systems (e.g. ultracold atoms)



Large clouds of atoms in optical lattices

Single particle detection
+ control in many-body systems



Entanglement Detection

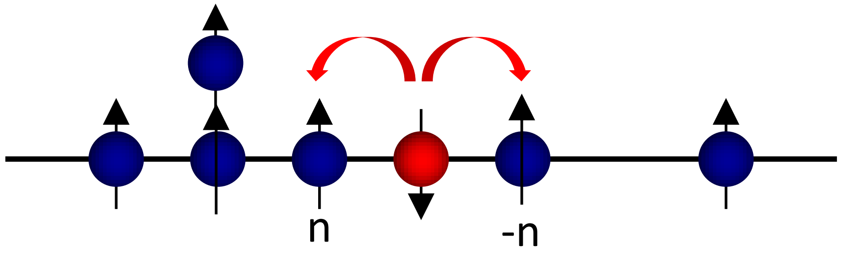
Can we probe entanglement in ultracold quantum gases in optical lattices?

Ion traps:

- Entanglement detection well established in **spin chains** of ~15 spins
- Quantum state state reconstruction using local rotations

Neutral atoms in optical lattices:

- So far, only global entanglement witnesses but no local detection
- Degrees of freedom:
  - Charge-degree of freedom (i.e., on-site occupation number)
  - Spin-degree of freedom (e.g., super-exchange)

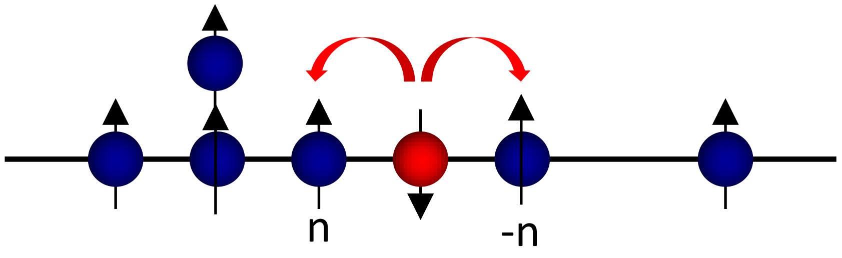


Entanglement Detection

Can we probe entanglement in ultracold quantum gases in optical lattices?

This talk: Entanglement detection in a **Bose-Hubbard chain**

- Entanglement in **spin-degree of freedom**
- Entanglement in subsystems of **two lattice sites**
- Generation and spreading of entanglement
- What‘s the influence of particle number fluctuations?



Outline

I. Introduction to single-site imaging

II. Observables

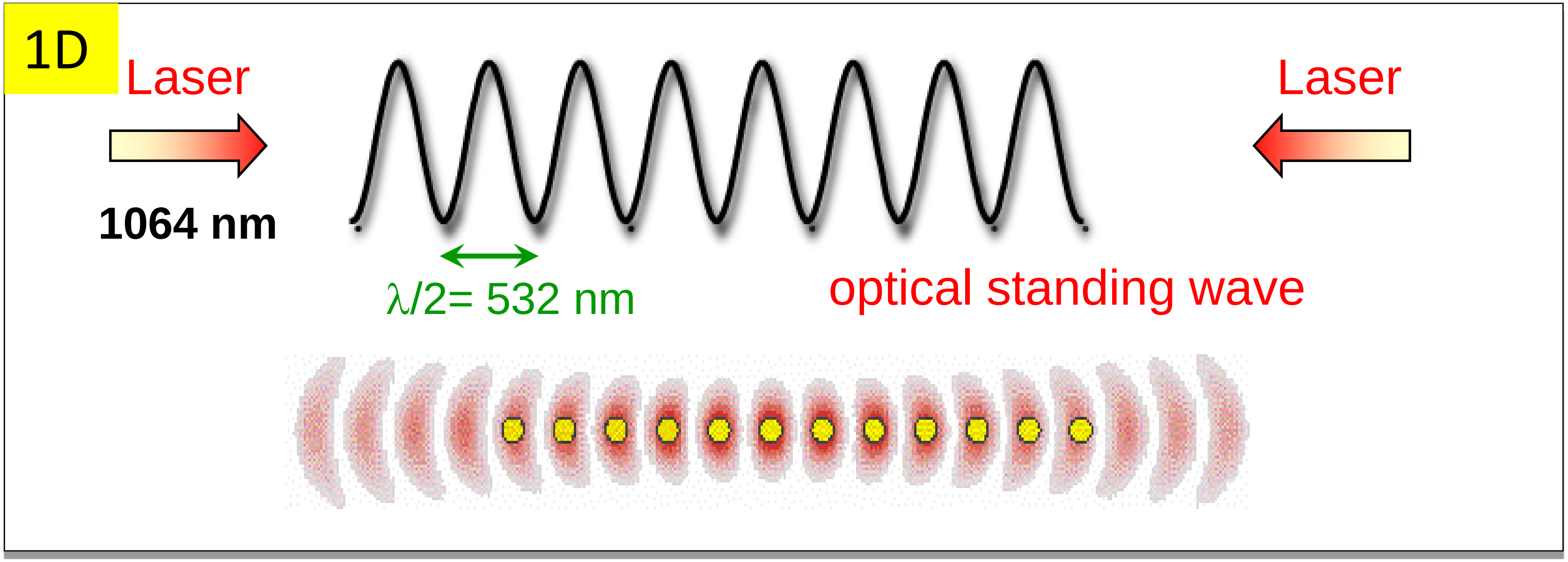
III. Spin-Impurity dynamics

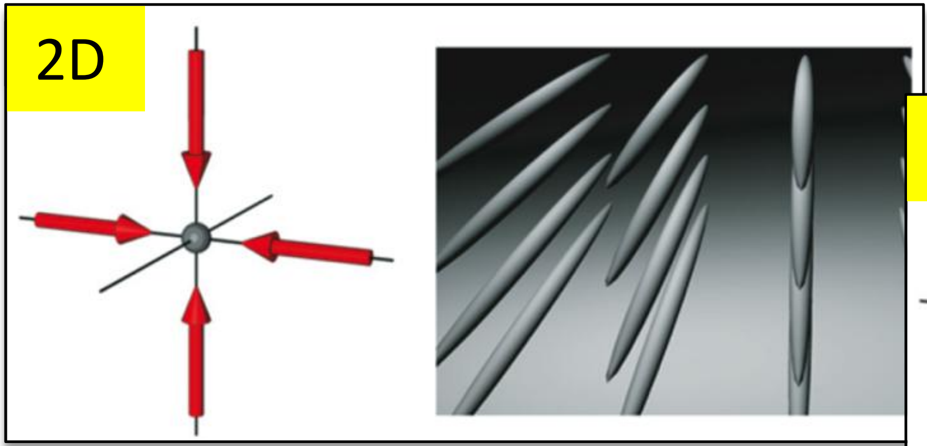
IV. Spin-Entanglement detection conceptually

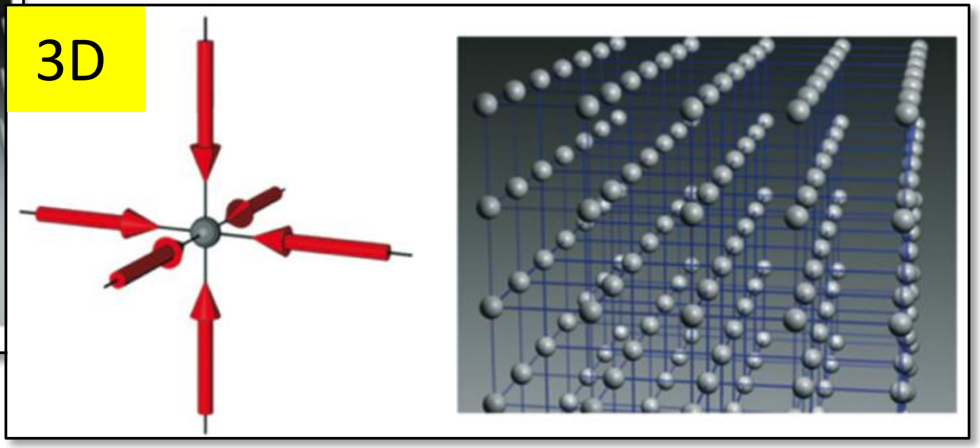
V. Spin-Entanglement detection in practice

I:
Introduction to single-site imaging

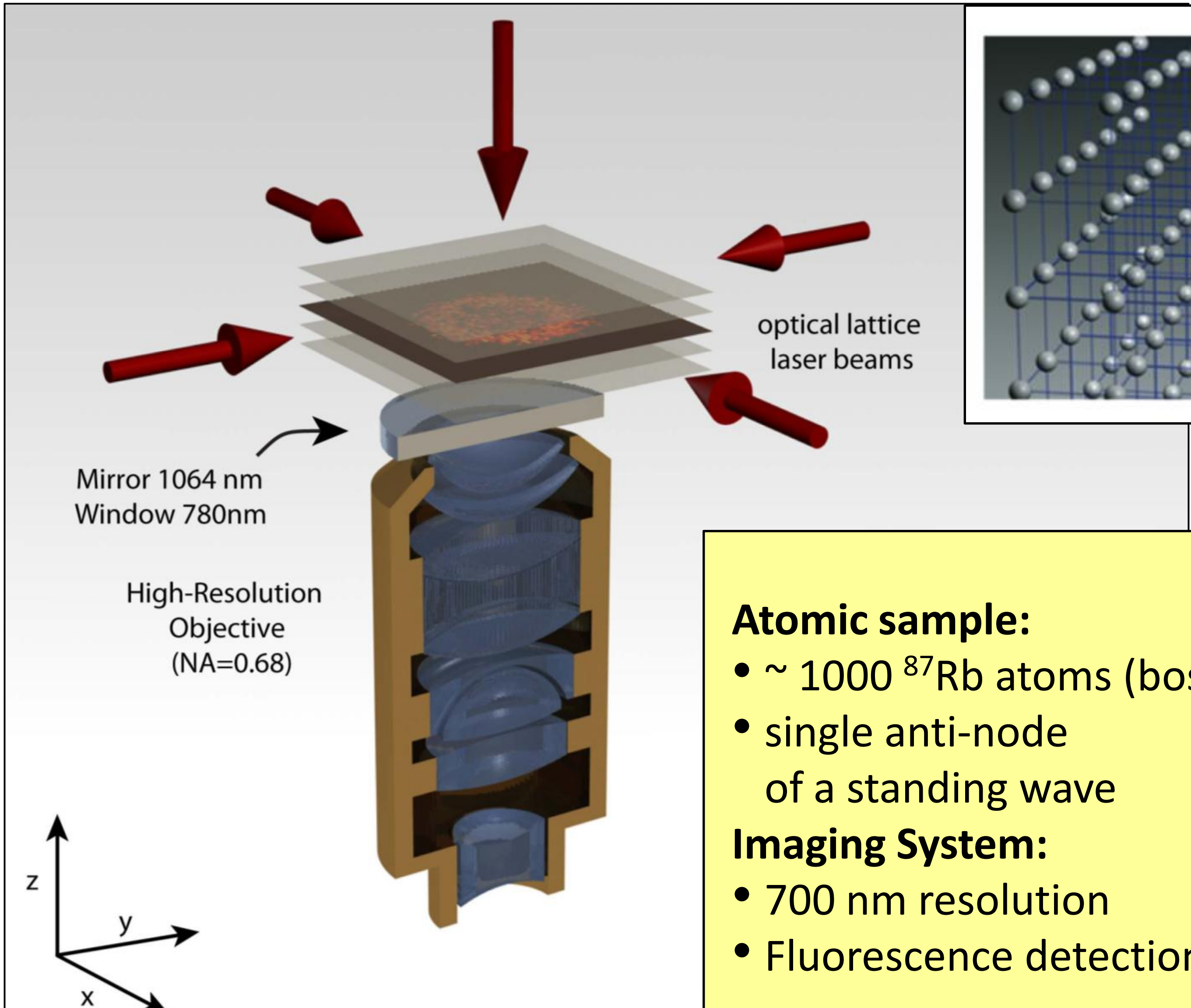
Optical lattices



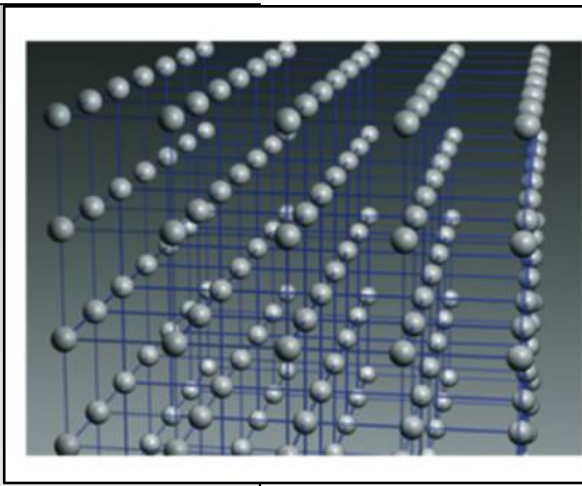




Experimental Setup



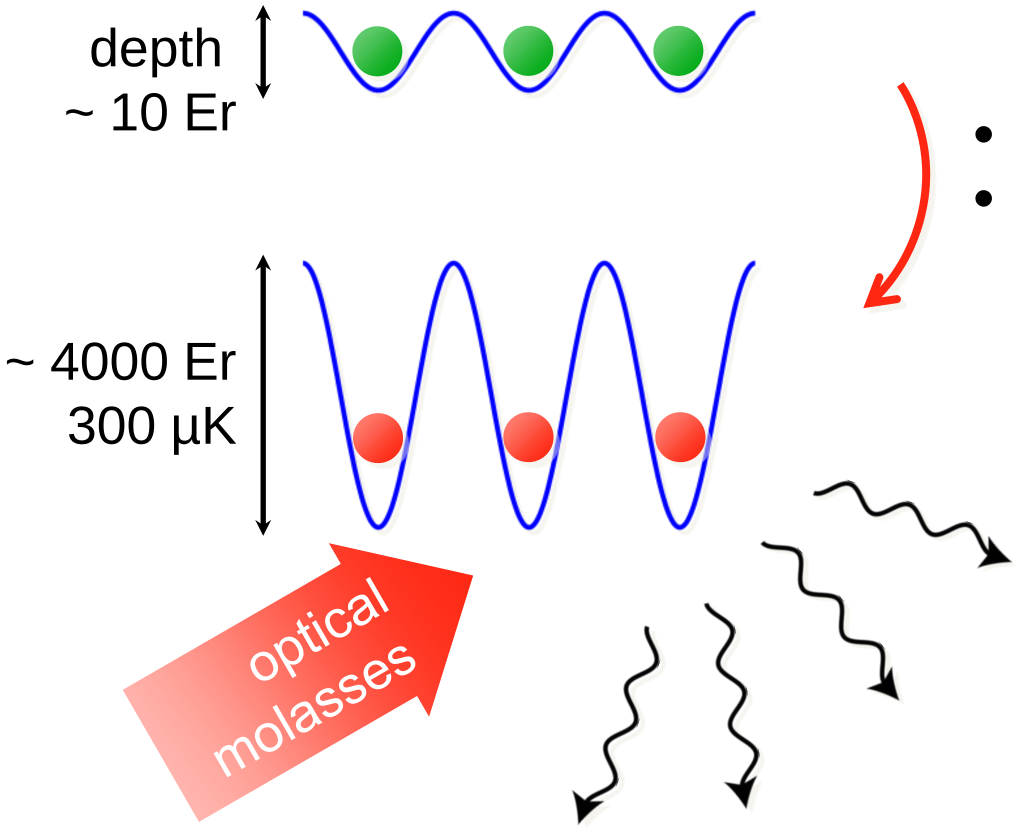
J.F. Sherson *et al.*, Nature **467**, 68 (2010).



Atomic sample:
- ~ 1000 ^{87}Rb atoms (bosons)
- single anti-node of a standing wave

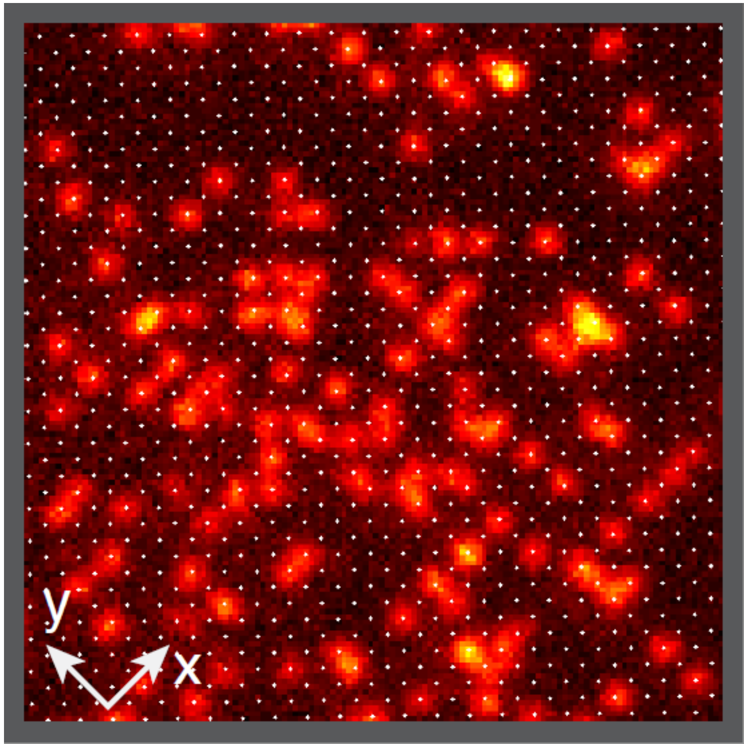
Imaging System:
- 700 nm resolution
- Fluorescence detection

Fluorescence imaging



- freeze density distribution
- lose phase coherence

~ 5000 photons/atom collected in ~1s



J.F. Sherson *et al.*, Nature **467**, 68 (2010)
see also: W. Bakr *et al.*, Nature **462**, 74 (2009)

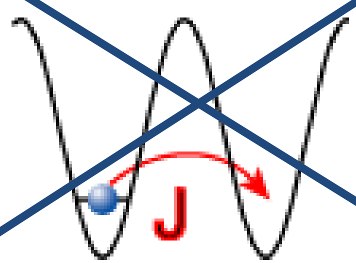
Bose-Hubbard

Atomic Limit:

Bose-Hubbard Hamiltonian

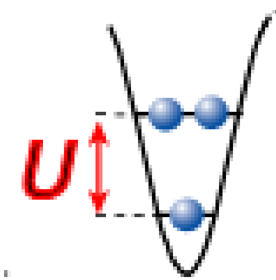
$$H = -J\sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \varepsilon_i \hat{n}_i + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Tunnelmatrix element



J/U<<1

Onsite interaction



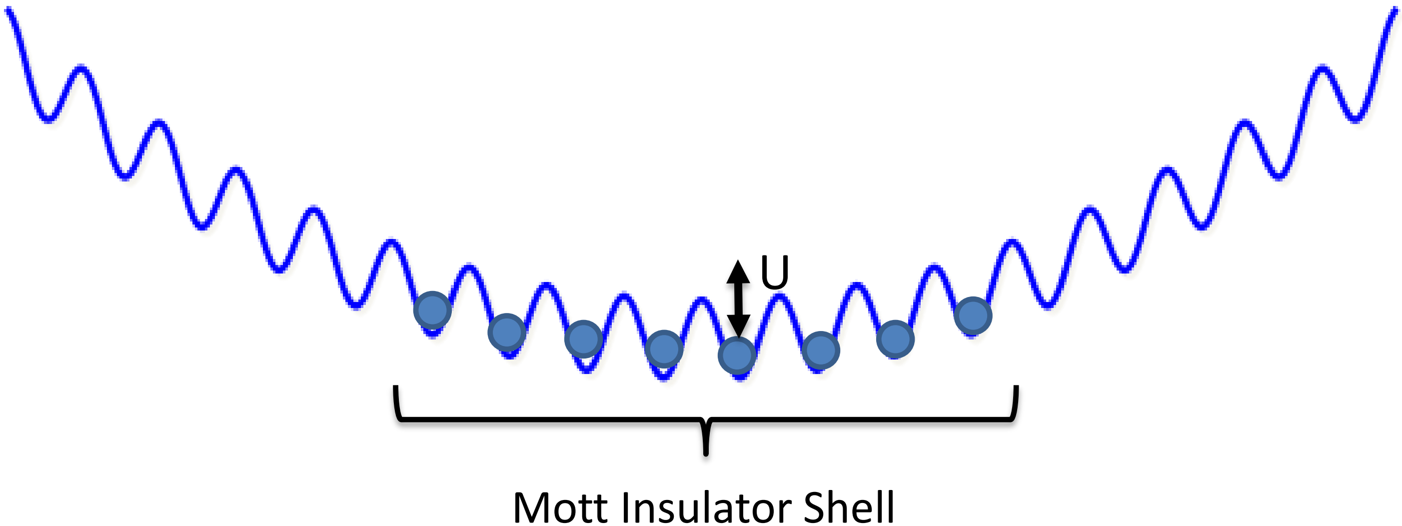
$\hat{a}_i^\dagger, \hat{a}_i$: creation and anihilation operator for Boson on i^{th} lattice site

$\hat{n}_i$: number operator

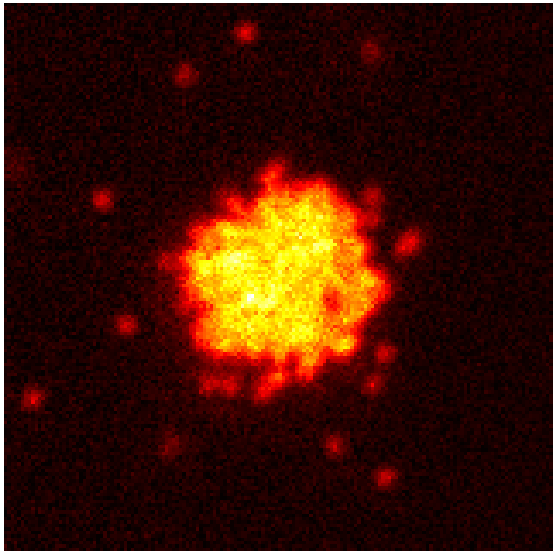
M.P.A. Fisher et al., Phys. Rev. B **40**, 546 (1989);

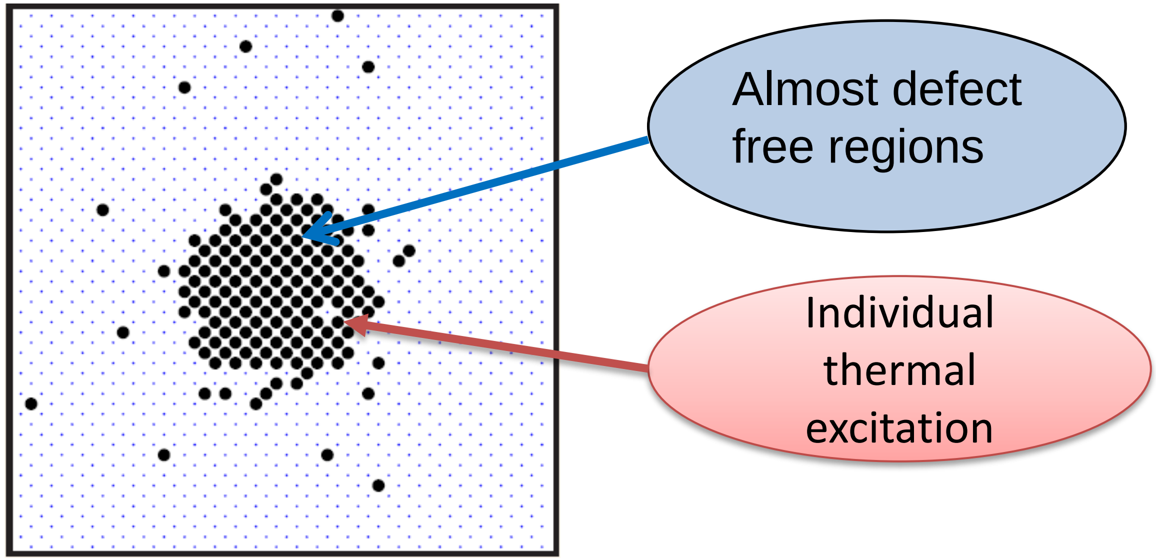
Ground state atomic limit

$$H = -J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j + \sum_i \varepsilon_i\hat{n}_i + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$



Atomic Limit Mott Insulators





J.F. Sherson, C. Weitenberg *et al.*, Nature **467**, 68 (2010).

II:
Observables

Generic observable spinless

Pure state:

$$|\Psi\rangle = \sum_{\{n_i\}} \alpha_{n_1,\ldots,n_N} |n_1,\ldots,n_N\rangle,$$

Most general observable:

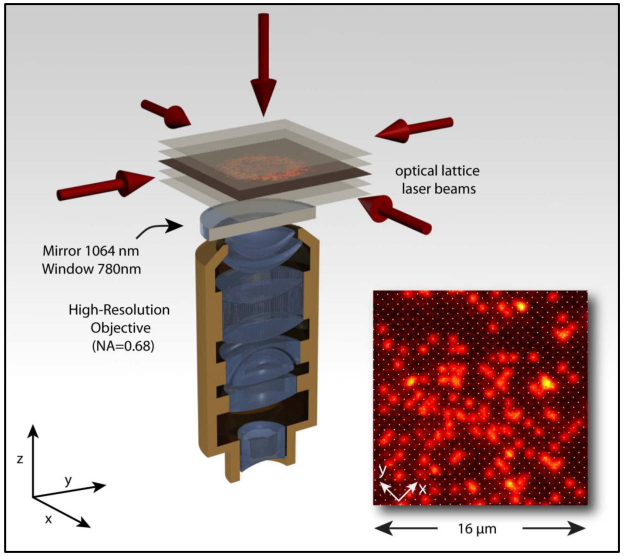
$$p(n_1,\ldots,n_N) = |\alpha_{n_1,\ldots,n_N}|^2$$

(includes all density-density correlations)

Mixed state:

$$\hat{\rho} = \sum_{\{n_i\}} |\alpha_{n_1,\ldots,n_N}|^2 |n_1,\ldots,n_N\rangle\langle n_1,\ldots,n_N|$$

Measurement is limited to diagonal elements
of the density operator in occupation number basis!



M. Endres et al., Appl. Phys. B **113**, 27 (2013)

Summary of experiments

For entanglment detection we will need access to off-diagonal elements:
Option 1: off-diagonal with respect to **on-site occupation number**
Option 2: off-diagonal with respect to **spin state**

- Detection of various correlations functions in equilibrium across SF-Mott transition

  M. Endres *et al.*, Science **334**, 200 (2011)

- Light-cone-like spreading of correlations after quantum quench

  M. Cheneau *et al.*, Nature **481**, 484 (2012)

- Dynamical response close to SF-Mott in 2d: ‚Higgs amplitude mode'

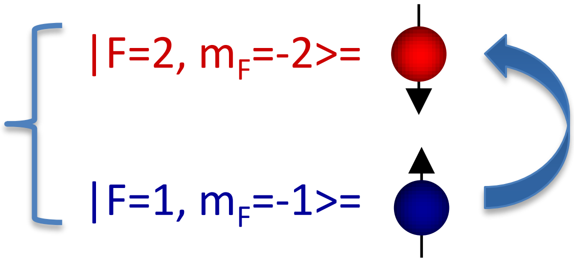
  M. Endres *et al.,* Nature **487**, 454 (2012)

- Spin-impurity dynamics

  T. Fukuhara et al., Nature **502**, 76–79 (2013)
  T. Fukuhara *et al.*, Nature Phys. **9**, 235 (2013)

Spin-Degree

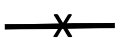
Use two different hyperfine levels:

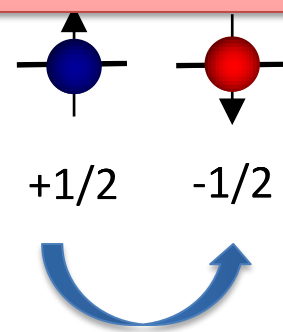


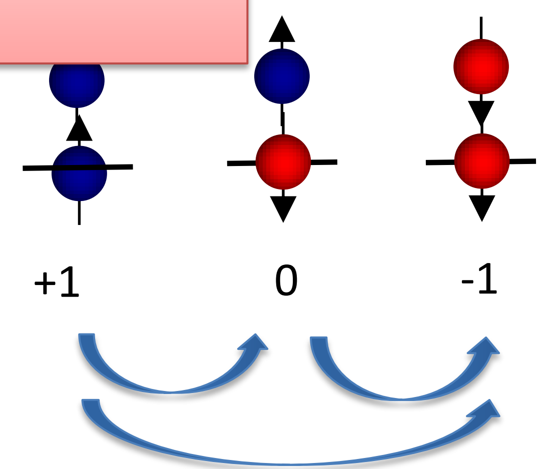
Map difference

Spin degree realized with hyperfine levels.
Rotations in spin-space possible (at fixed local total spin)
->apply rotation before imaging
->off-diagonal elements

Spin 0:



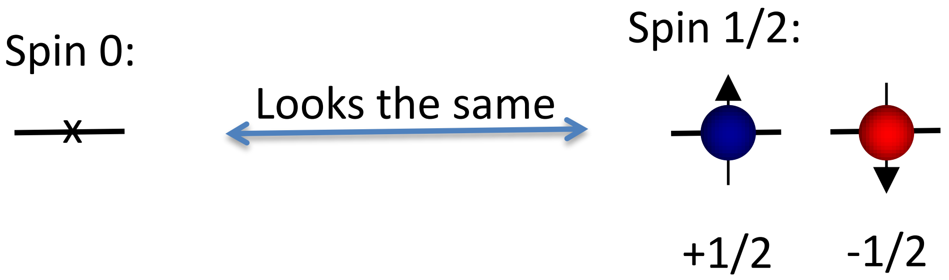




Spin-Imaging

Ideally we could image both states in one shot -> not possible

Eliminate one component with a ‚push-out pulse‘:



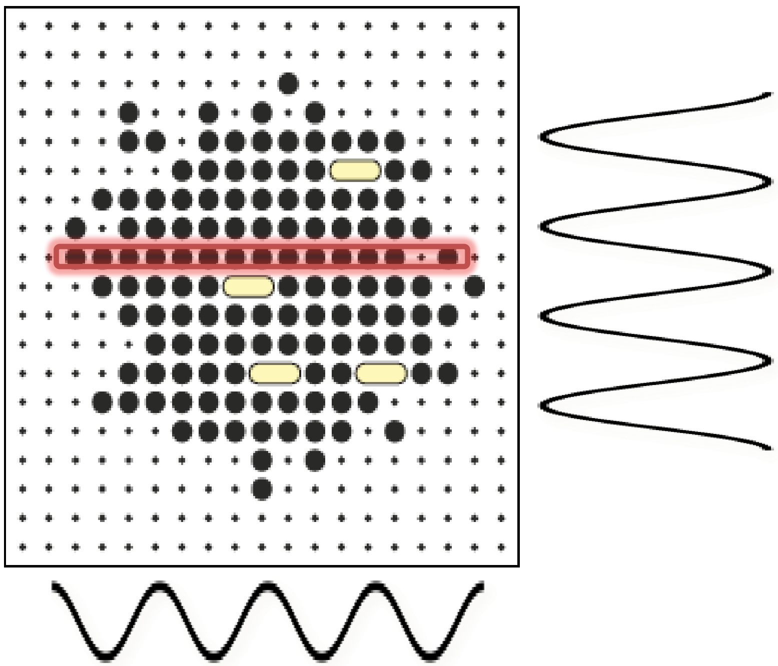
Spin resolved imaging possible
but it cannot distinguish holes from one of the spin states

III:
Single Spin-Impurity Dynamics in 1d

T. Fukuhara *et al.*, Nature Phys. **9**, 235 (2013)

1d limit

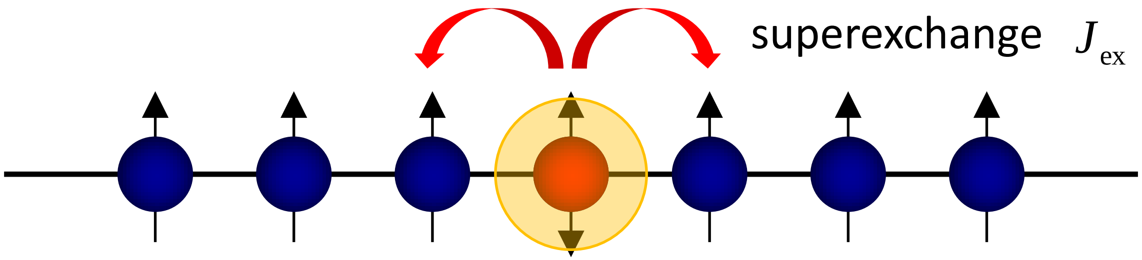
How to get 1d systems?



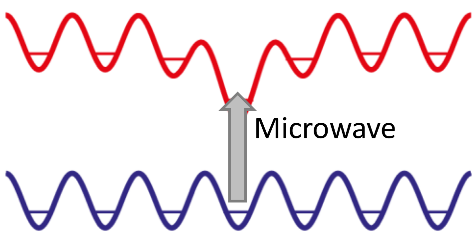
Keep one lattice axis deep
->no dynamics

Lower other axis
->dynamics

Preparation of single spin impurity



superexchange $J_{ex} = 4\frac{J^2}{U}$

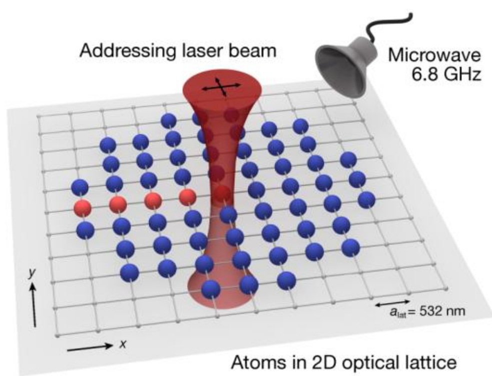


↓: |F=2, m_F=-2>

↑: |F=1, m_F=-1>

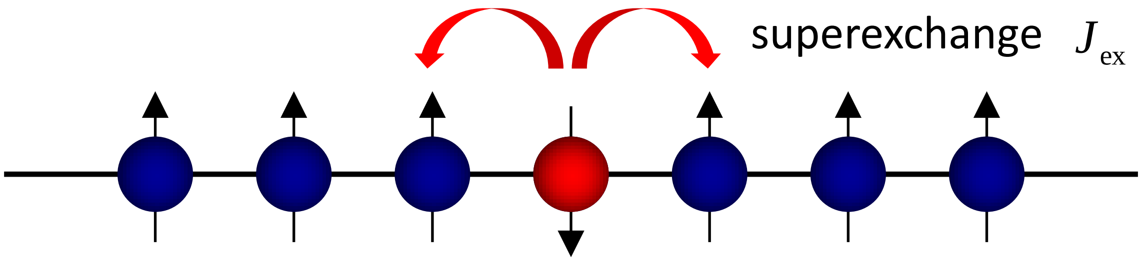
Single-spin addressing scheme

C. Weitenberg *et al.*, Nature **471**, 319 (2011)



T. Fukuhara *et al.*, Nature Phys. **9**, 235 (2013)

Spin impurity dynamics



superexchange $J_{\text{ex}} = 4\frac{J^2}{U}$

Heisenberg Hamiltonian (U>>J):

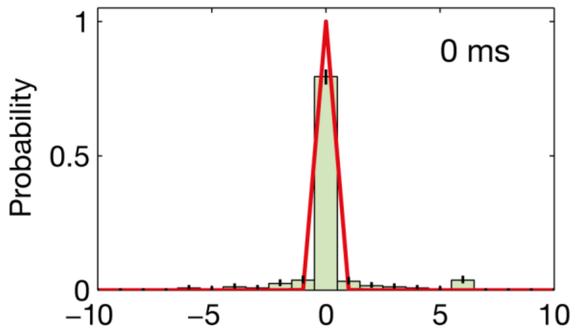
$$\hat{H} = -J_{\text{ex}} \sum_{\langle j,k \rangle} \hat{\boldsymbol{S}}_j \cdot \hat{\boldsymbol{S}}_k$$
$$= -\frac{J_{\text{ex}}}{2} \sum_{\langle j,k \rangle} \left(\hat{S}_j^+ \hat{S}_k^- + \hat{S}_j^- \hat{S}_k^+ \right) - J_{\text{ex}} \sum_{\langle j,k \rangle} \hat{S}_j^z \hat{S}_k^z$$

Free propagation

Spin attraction

PRL 90.100401 (2003), PRL 91, 090402 (2003),
NJP 5, 76 (2003), NJP 5, 113 (2003).

Coherent quantum dynamics



V = 10 *Er*
$J_{ex}/\hbar$=65(1) Hz

Observation of a spin wave consisting of only a single spin!

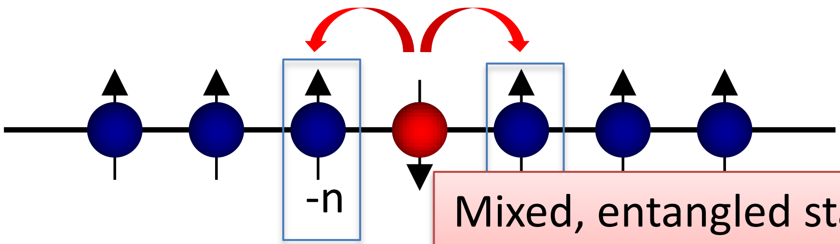
$$P_j\left(t\right) = \left[\mathcal{J}_j\left(\frac{J_{\text{ex}}t}{\hbar}\right)\right]^2$$

$\mathcal{J}_j$: Bessel function of the first kind

IV:
Spin-Entanglement Detection
Conceptually

Quantum state

Impurity is in superposition over several sites:



$$|\psi(t)\rangle = \sum \phi_j(t)|\uparrow, \ldots, \uparrow, \downarrow, \uparrow, \ldots, \uparrow\rangle$$

Mixed, entangled state
-> entanglement measure for mixed states

Pick two sites n and –n and look at reduced density operator:

$$\rho_{n,-n}(t) = 2|\phi_n|^2|\Psi^+\rangle\langle\Psi^+|$$

Bell state $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle)$

Concurrence

- Entanglement measure: Concurrence C

Can we circumvent a full state tomography?
-> detect lower bound for concurrence

- Pure states: $C(|\psi_{1,2}\rangle) = \sqrt{2(\langle\psi_{1,2}|\psi_{1,2}\rangle - \mathrm{Tr}(\rho_1^2))}$,
- Mixed states: $C(\hat{\rho}_{1,2}) = \inf \sum_i p_i C(|\phi_i\rangle)$ $\quad \hat{\rho}_{1,2} = \sum_i p_i |\phi_i\rangle\langle\phi_i|$
- Can be analytically calculated for two spins if density matrix is completely known

Wootters, PRL 80, 2245–2248 (1998)

Concurrence from X-Matrix

- Concurrence is bounded by concurrence of X-Matrix part:

$$\hat{\rho}_{A,B} = X \quad \begin{pmatrix} P_{\uparrow,\uparrow} & 0 & 0 & \rho_{\uparrow\uparrow} \\ \rho^*_{\uparrow\uparrow} & 0 & 0 & P_{\downarrow,\downarrow} \end{pmatrix}$$

Experimentally detect lower bound for two-site concurrence using diagonal elements and only one off-diagonal element

$$\mathcal{C}(\hat{X}) \leq \mathcal{C}(\hat{\rho}_{A,B})$$

- Concurrence of X-Matrix part:

$$\mathcal{C}(\hat{X}) = 2\max(0, |\rho_{\uparrow\uparrow}| - \sqrt{P_{\uparrow,\downarrow}P_{\downarrow,\uparrow}}, |\rho_{\uparrow\downarrow}| - \sqrt{P_{\uparrow,\uparrow}P_{\downarrow,\downarrow}})$$

Rafsanjani & Agarwal, arXiv:1204.3912 (2012)

Lower bound for concurrence

$$2(|\rho_{\uparrow,\downarrow}| - \sqrt{P_{\uparrow,\uparrow}P_{\downarrow,\downarrow}}) \leq \mathcal{C}(\hat{\rho}_{i,j})$$

Measure for coherent superposition

Measure for unintented double spin-preparation

How to measure the indvidual elements?

V:
Spin-Entanglement Detection in Practice

Coherence detection

- Let's neglect holes and doubly occupied sites for the moment
- Detection using push-out scheme: 
- Only use **probability that both sites are occupied after push-out pulse** P^{11}
- Apply rotations on both sites **before push-out:**

1. No rotation: $P^{11} = P_{\downarrow,\downarrow}$
2. Pi-rotation: $P^{11} = P_{\uparrow,\uparrow}$
3. Pi/2-rotation: $P^{11}_{\perp} = \frac{1}{2}\Re[\rho_{\uparrow,\downarrow}] + \frac{1}{4}$

$$\longrightarrow (P^{11}_{\perp} - \frac{1}{4}) \leq \frac{1}{2}|\rho_{\uparrow,\downarrow}|$$

Lower bound for concurrence

$$2(|\rho_{\uparrow,\downarrow}| - \sqrt{P_{\uparrow,\uparrow}P_{\downarrow,\downarrow}}) \leq \mathcal{C}(\hat{\rho}_{i,j}) \qquad P_{\perp}^{11} - \frac{1}{4} \leq \frac{1}{2}|\rho_{\uparrow,\downarrow}|$$

$$2(2(\underbrace{P_{\perp}^{11} - \frac{1}{4}}_{C_{i,j}}) - \sqrt{P_{\uparrow,\uparrow}P_{\downarrow,\downarrow}}) \leq \mathcal{C}(\hat{\rho}_{i,j})$$

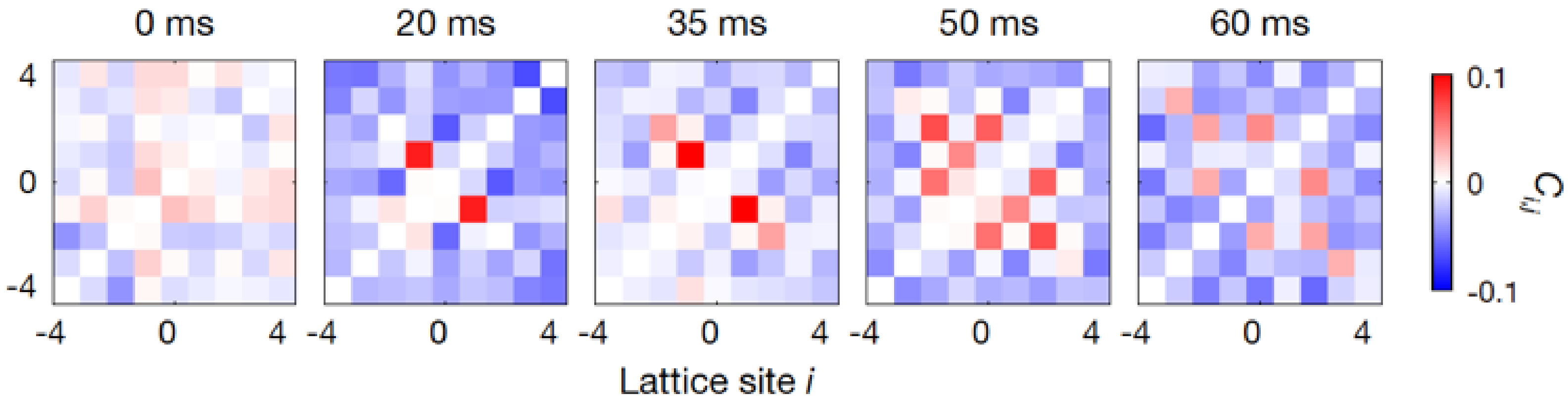
Transverse correlations: $C_{i,j}$

$$C_{i,j} = \frac{1}{2}(\langle \hat{S}_i^x \hat{S}_j^x \rangle + \langle \hat{S}_i^y \hat{S}_j^y \rangle)$$

Transverse correlation data

1st row: $C_{i,j} = P^{1,1}_{i,j,\perp} - \frac{1}{4}$

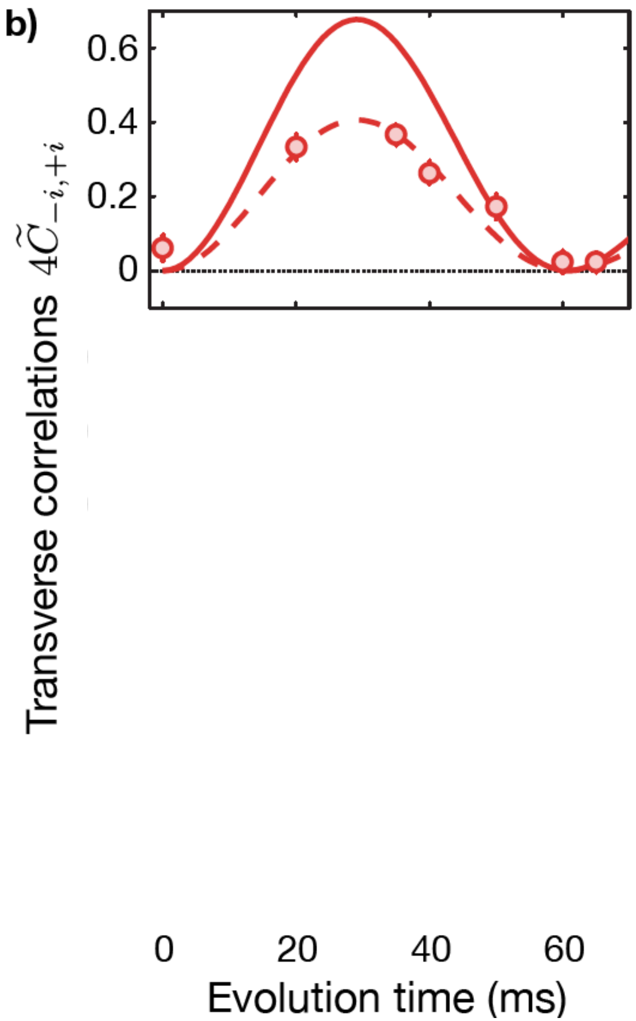
2nd row: $\tilde{C}_{i,j} = P^{11}_{i,j,\perp} - P^{1}_{i,\perp} P^{1}_{j,\perp}$



Transverse correlation data

Transverse correlations reduced due to number fluctuations

$$\tilde{C}_{i,j} = P^{11}_{i,j,\perp} - P^{1}_{i,\perp} P^{1}_{j,\perp}$$



For sites -1 and 1

Concurrence bound

$$2(|\rho_{\uparrow,\downarrow}| - \sqrt{P_{\uparrow,\uparrow}P_{\downarrow,\downarrow}}) \leq \mathcal{C}(\hat{\rho}_{i,j})$$

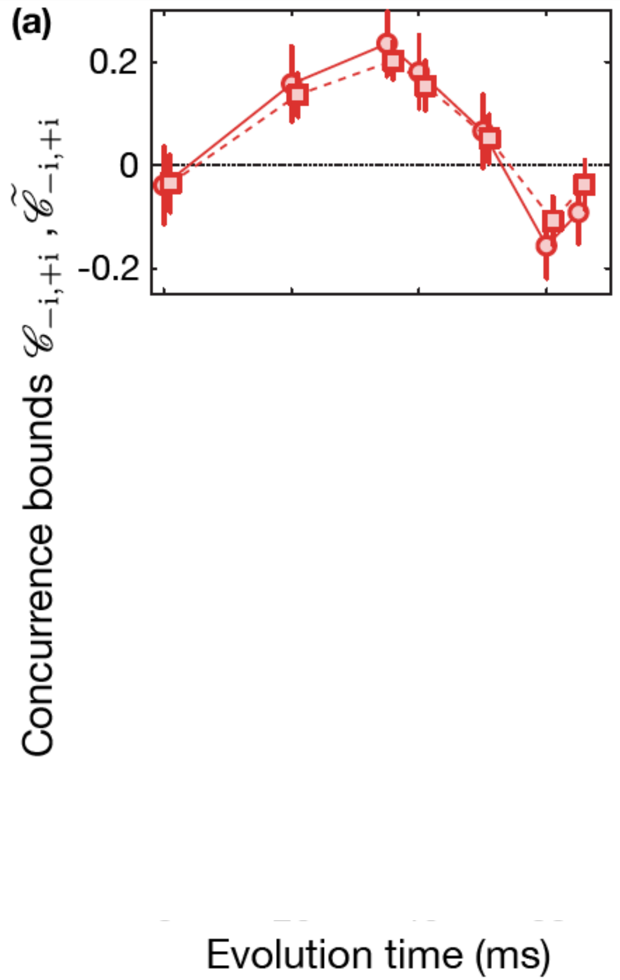
Entanglement spreading observed up to 7 sites distance

Solid: using

$$C_{i,j} = P^{1,1}_{i,j,\perp} - \frac{1}{4}$$

Dashed: using

$$\tilde{C}_{i,j} = P^{11}_{i,j,\perp} - P^{1}_{i,\perp}P^{1}_{j,\perp}$$



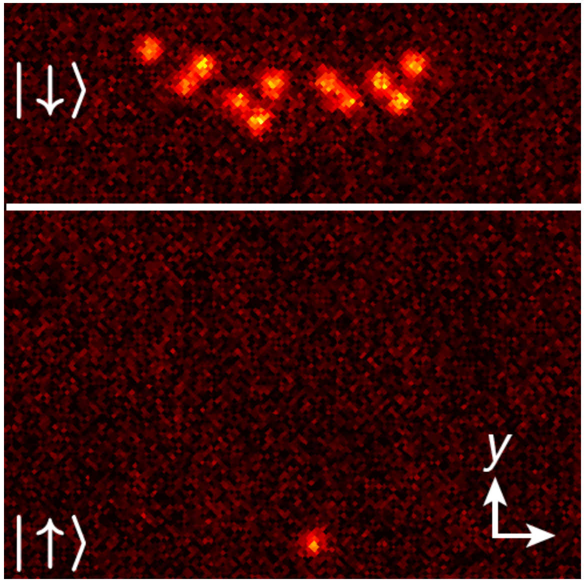
For sites -1 and 1

VI:
Influence of holes: In-situ Stern Gerlach

In-situ Stern-Gerlach

Influence of holes hard to access using current imaging technique

In-situ Stern-Gerlach imaging with full spatial resolution
-> On-site occupation number and spin at once!

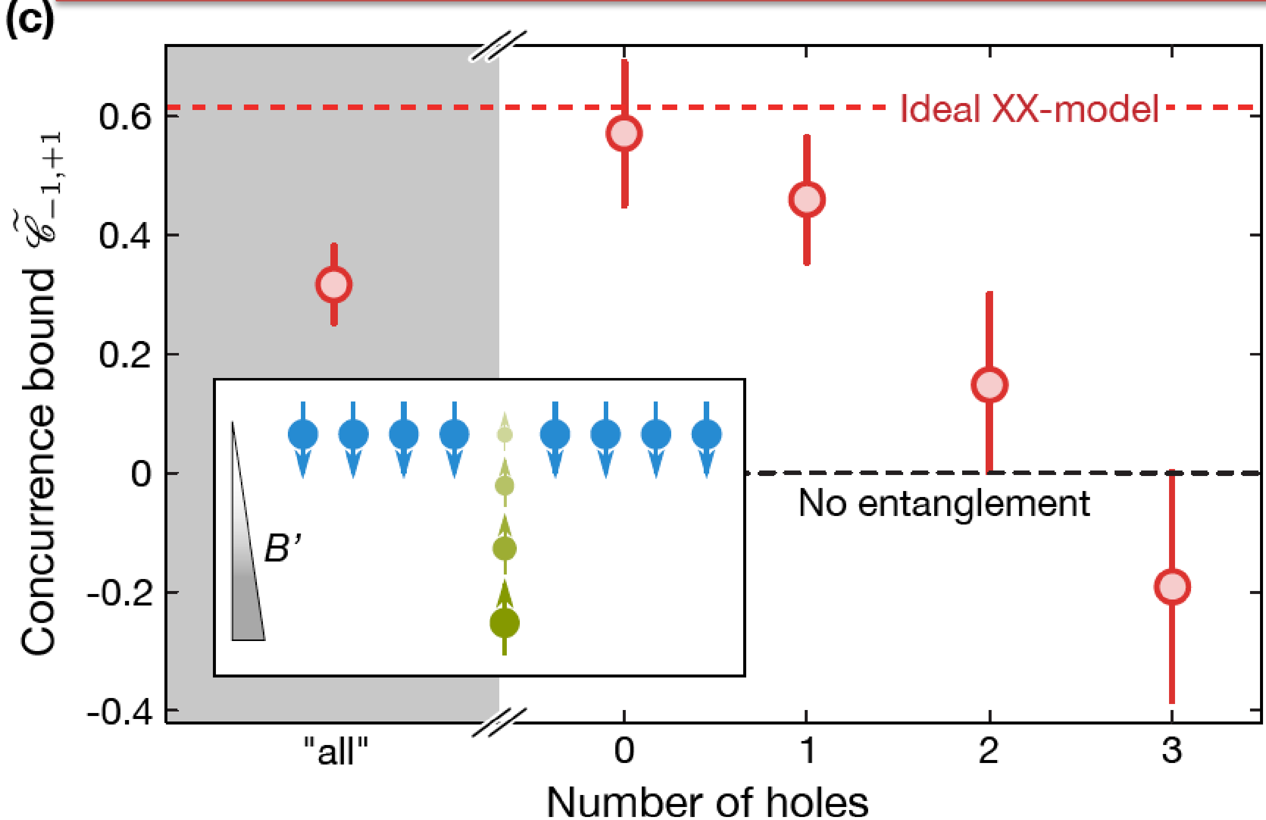


Influence of holes

How does the entanglement evolution depend on the number of holes in the chain?

-> Post-selection of entanglement data on number of holes!

Holes have a strong influence on Spin-Entanglement dynamics

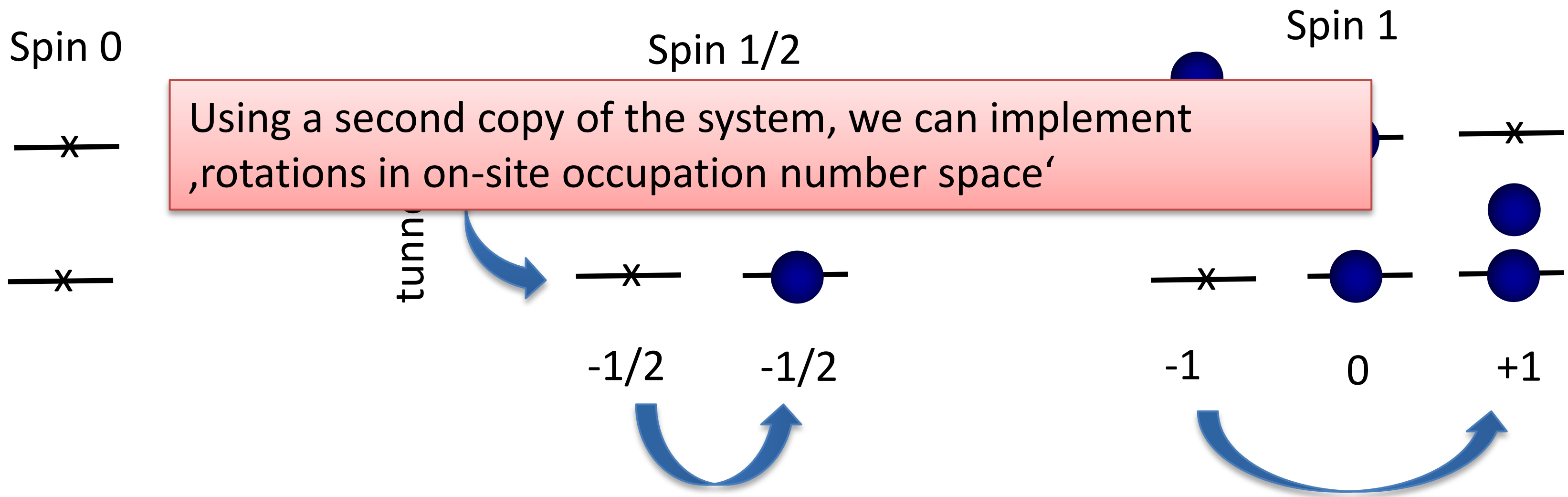


Outlook

Particle-number Entanglement

Can we detect entanglement in the on-site occupation number in a similar way?

Problem: no local rotations possible

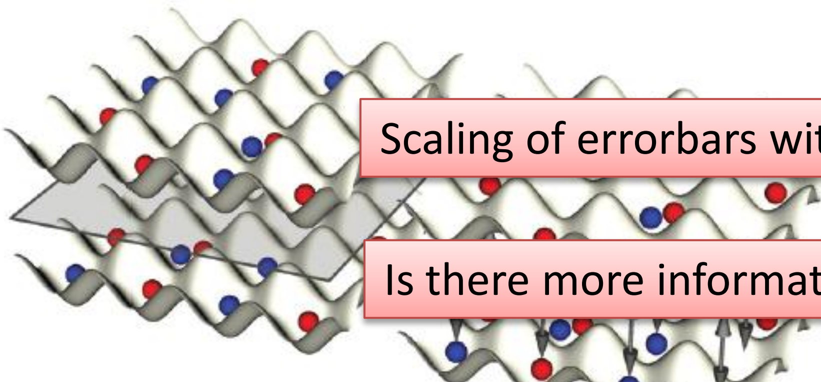


Using a second copy of the system, we can implement ‚rotations in on-site occupation number space‘

General entanglement detection

Zoller: Daley et. al, PRL **109**, 020505 (2012), Pichler et. al, New J. Phys. **15** 063003 (2013)
Jacksch: Alves et. al,PRL **93**, 11 (2004)

Measurement of the purity $tr(\hat{\rho}^2)$ of subsystems:



1. Create two copies of the system
   them
2. Count atoms in copy A and B
   ll possible subsystems

Scaling of errorbars with subsystem size?

Is there more information than the purity?

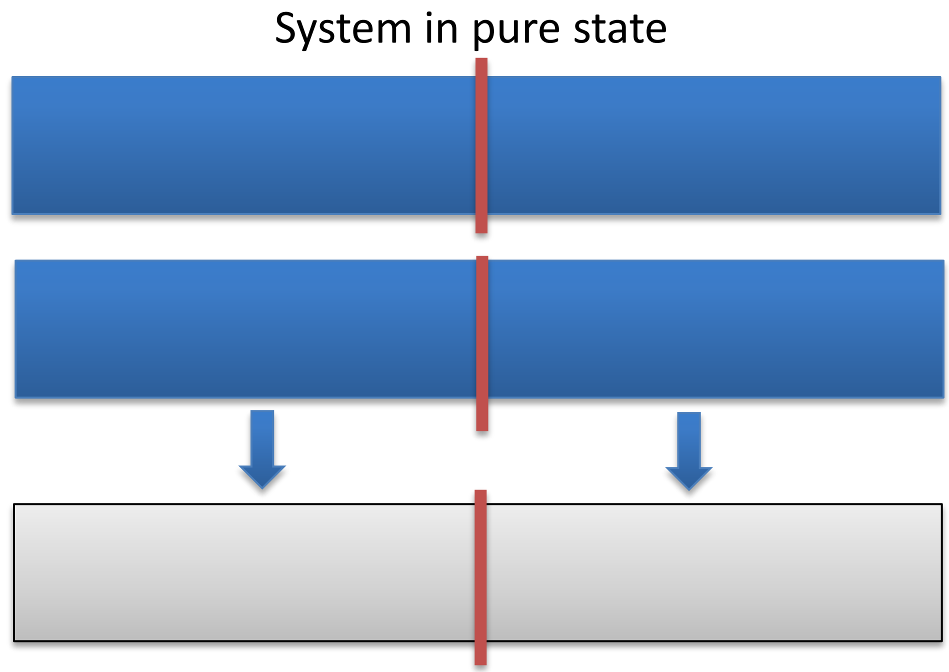
Rényi entro
->Entangle

On-going work with Michael Knap:

- Off-diagonal correlation functions: $\langle \delta\hat{\hat{n}}_j \delta\hat{\hat{n}}_k \rangle = 2|\langle \hat{a}_j \hat{a}_k \rangle|^2$
  (similar to transverse correlations)
- Dynamical correlators (Green function like)

Related work: Abanin, Demler, PRL **109**, 020504 (2012)

Naive entanglement detection



1. Cut it in two

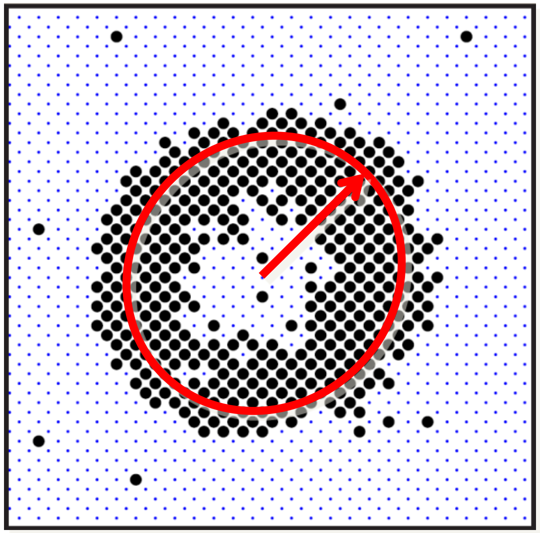
2. Let it equilibrate

3. Adiabatic change of Hamiltonian
in both subsystems to ‚simple Hamiltonian‘

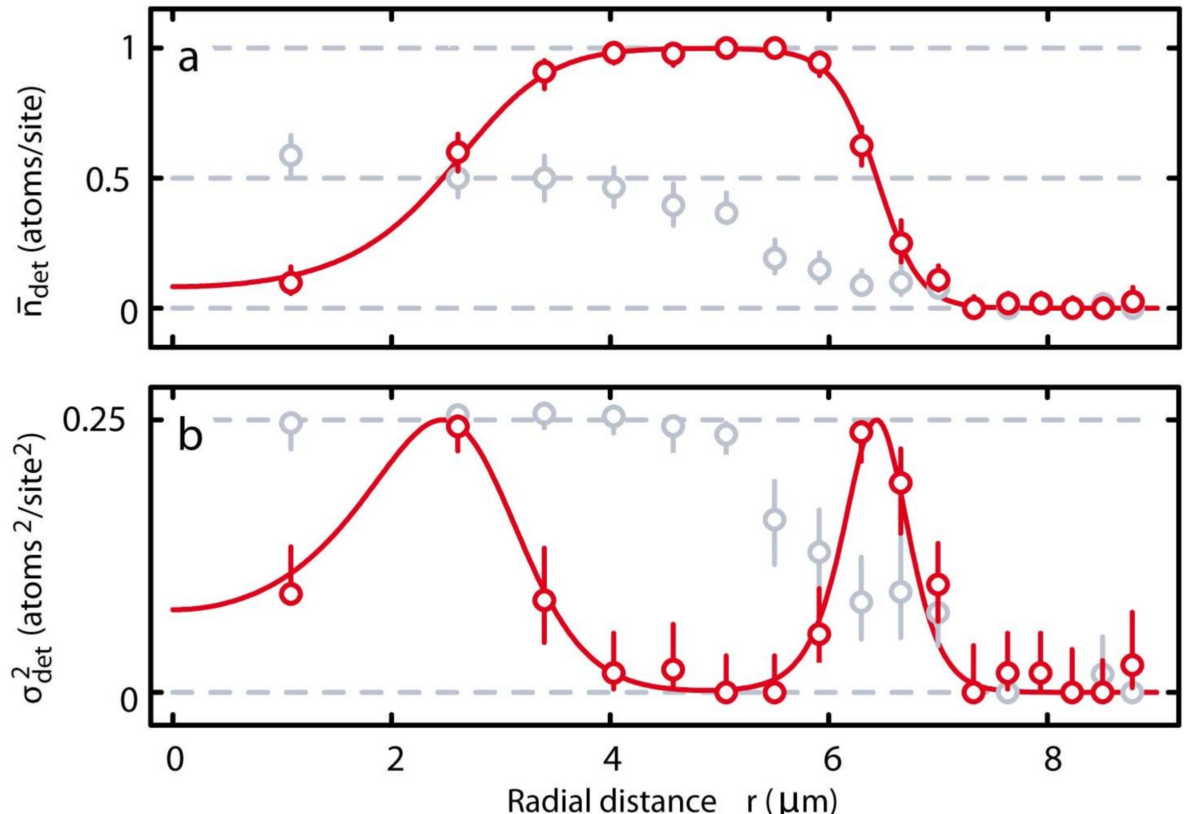
‚simple Hamiltonian‘=diagonal in measurement basis

Equilibrium states will be diagonal in measurment basis
-> read off-entropy of subsystems

Thermometry



T = 0.074(5) U/k_B



Single shot thermometry of MIs (atomic limit)

zero-tunneling approximation

$$P_r(n) = e^{\beta[\mu_{\rm loc}(r)n - E_n]}/Z(r)$$

fit parameters: T/U; µ/U; U/ω^2

J.F. Sherson *et al.*, Nature **467**, 68 (2010).

Summary

Introduction to single-site resolved imaging in optical lattices

Observables

Thanks!

Spin-Entanglement detection during single Spin-Impurity dynamics

Influence of on-site particle number fluctuations on Spin-Entanglement dynamics

Generalization to entanglement detection in particle-number sector

L. Mazza, D. Rossini, R. Fazio, ME, **New J. Phys.** 17, 013015 (2015)
arxiv:1408:4672

T. Fukuhara, S. Hild, J. Zeiher, P. Schauß, I. Bloch, ME, C. Gross, **PRL** (accepted)
arxiv:1504.02582