Ab Initio Holography

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Based on arXiv : 1503.06474

Peter Lunts, S. Bhattacharjee, J. Miller, E. Schnetter, Y.B. Kim, SL

Goal

- Construct a holographic dual for a concrete lattice model [U(N) vector mode] via quantum renormalization group
- Characterize different phases from the emergent geometries in the bulk

- Key message :

  (non-) locality as holographic order parameter

Vector model

$$\mathcal{S} = \int d^D x \left[|\nabla \vec{\phi}|^2 + m^2 |\vec{\phi}|^2 + \frac{\lambda}{N} (|\vec{\phi}|^2)^2 \right]$$

- Exact critical exponents in the large N limit
- But, still no exact Wilsonian RG
- Believed to be dual to Vasiliev's higher-spin gauge theory [Sezgin-Sundell, Polyakov-Klebanov, Giombi-Yin, ..]

Related works :

- S. R. Das and A. Jevicki, Phys. Rev. D 68 (2003) 044011.
- R. Koch, A. Jevicki, K. Jin and J. P. Rodrigues, arXiv:1008.0633.
- M. Douglas, L. Mazzucato, and S. Razamat, Phys. Rev. D 83 (2011) 071701.
- R. Leigh, O. Parrikar, A. Weiss, arXiv:1402.1430
- E. Mintun and J. Polchinski, arXiv:1411.3151

Lattice regularization

U(N) vector model for N complex bosons

$$\mathcal{S}_0 = m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2 - \sum_{ij} t_{ij}^{(0)} \left(\phi_i^* \cdot \phi_j\right)$$

i,j : 3-dimensional Euclidean lattice

$$\phi_i = \left(\phi_i^1, \phi_i^2, ..., \phi_i^N\right)$$

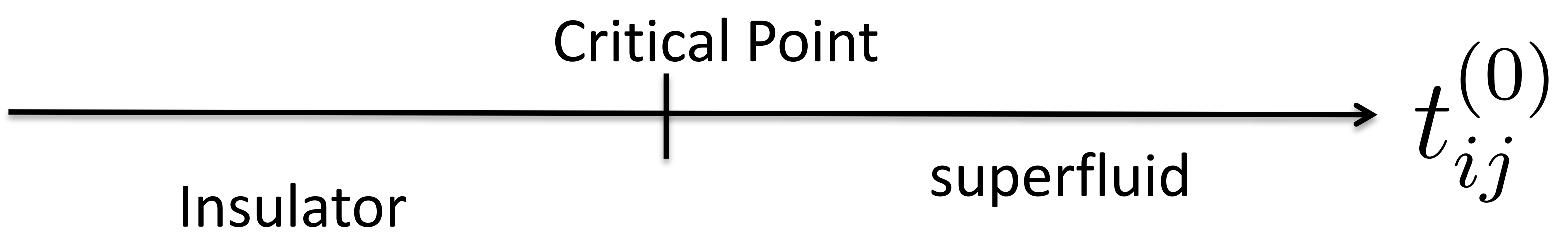
Two phases and one critical point

$$\mathcal{S}_0 = m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2 - \sum_{ij} t_{ij}^{(0)} \left(\phi_i^* \cdot \phi_j\right)$$

On-site

Hopping

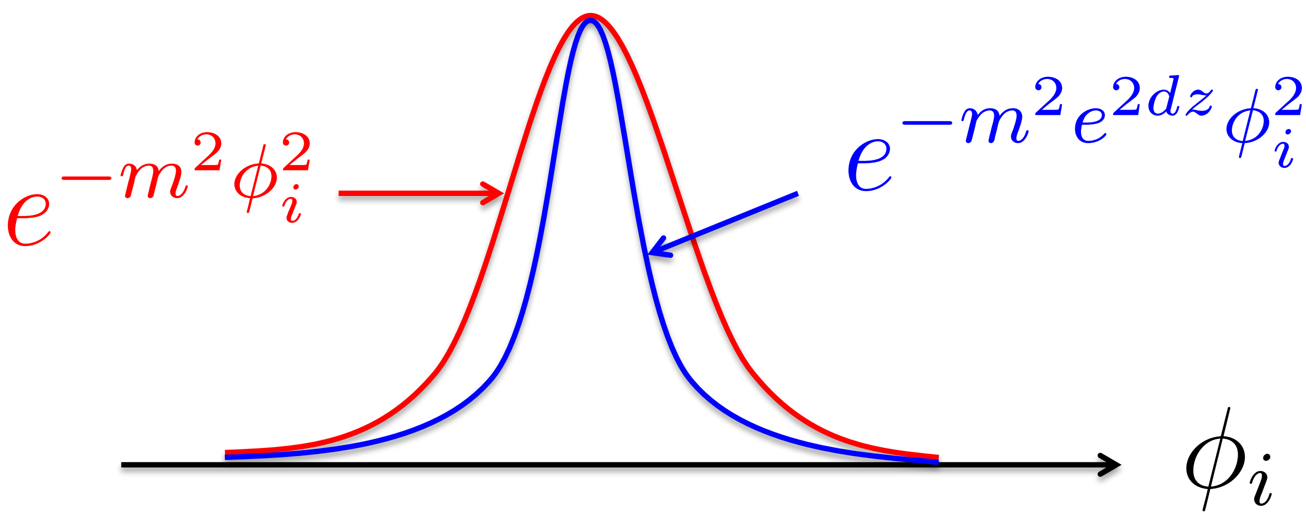
- The on-site term describes the insulating fixed point
- Hopping term is treated as deformation to the fixed point
- If the hopping is large, it flows to superfluid
- Although the quadratic term is enough to describe the Insulating fixed point, quartic term is needed to describe the phase transition



Coarse Graining in Real Space

$$Z = \int D\phi \; e^{-S[\phi;\; m^2, t_{ij}, \lambda]}$$

$$= \int D\phi \; e^{-S[\phi;\; m^2 e^{2dz}, t_{ij}+\delta t_{ij}, \lambda+\delta\lambda]-\delta S}$$



Renormalized Action

$$\tilde{\mathcal{S}}_1 = 2Ndz\left\{-\frac{1}{m^2}\sum_i t_{ii}^{(0)}\right\}$$

$$+2dz\left\{\frac{2\lambda\left(1+\frac{1}{N}\right)}{m^2}\sum_i(\boldsymbol{\phi}_i^*\cdot\boldsymbol{\phi}_i)-\frac{4\lambda^2}{m^2N^2}\sum_i(\boldsymbol{\phi}_i^*\cdot\boldsymbol{\phi}_i)^3\right\}$$

$$+2dz\left\{\frac{2\lambda}{m^2N}\sum_{ij}t_{ij}^{(0)}(\boldsymbol{\phi}_i^*\cdot\boldsymbol{\phi}_j)\left\{(\boldsymbol{\phi}_i^*\cdot\boldsymbol{\phi}_i)+(\boldsymbol{\phi}_j^*\cdot\boldsymbol{\phi}_j)\right\}\right\}$$

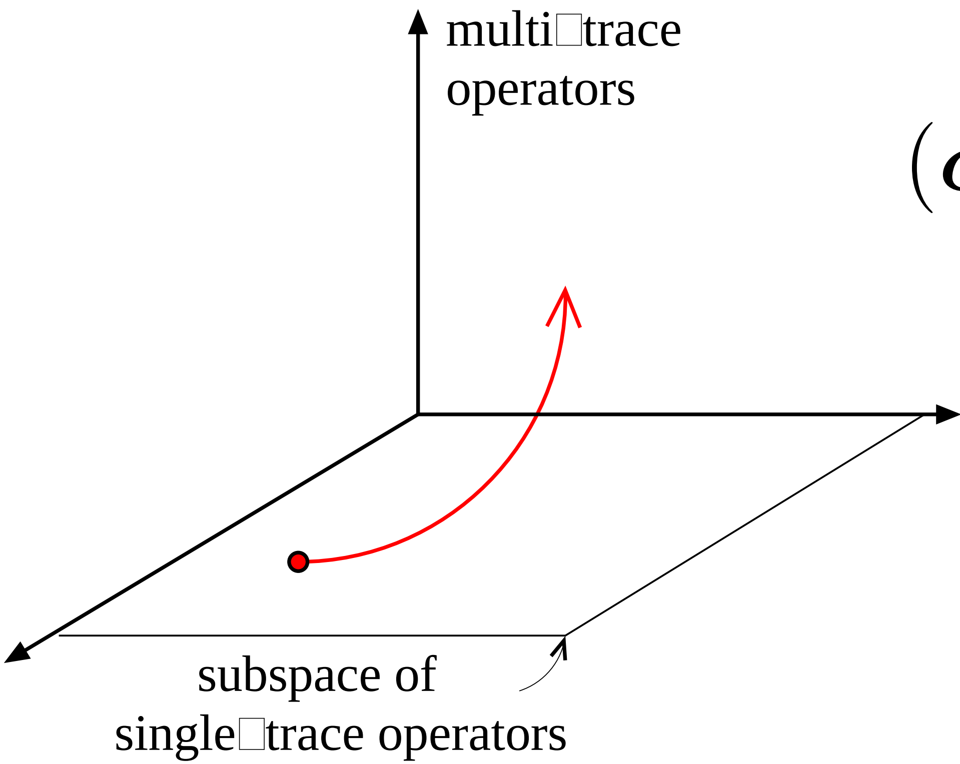
$$+2dz\left\{-\frac{1}{m^2}\sum_{ijk}t_{ik}^{(0)}t_{kj}^{(0)}(\boldsymbol{\phi}_i^*\cdot\boldsymbol{\phi}_j)-2\frac{\lambda}{N}\sum_i(\boldsymbol{\phi}_i^*\cdot\boldsymbol{\phi}_i)^2+\sum_{ij}t_{ij}^{(0)}\left(\boldsymbol{\phi}_i^*\cdot\boldsymbol{\phi}_j\right)\right\}$$

$$-\sum_{ij}t_{ij}^{(0)}\left(\boldsymbol{\phi}_i^*\cdot\boldsymbol{\phi}_j\right)+\boxed{m^2\sum_i(\boldsymbol{\phi}_i^*\cdot\boldsymbol{\phi}_i)+\frac{\lambda}{N}\sum_i(\boldsymbol{\phi}_i^*\cdot\boldsymbol{\phi}_i)^2}$$

deformation

On-site (reference action)

Conventional RG



$$\left(\phi_i^* \cdot \phi_j\right)$$

$$\left(\phi_i^* \cdot \phi_i\right)^n ,$$

$$\left(\phi_i^* \cdot \phi_i\right)^n \left(\phi_i^* \cdot \phi_j\right) , \ldots$$

- Multi-trace deformations are generated

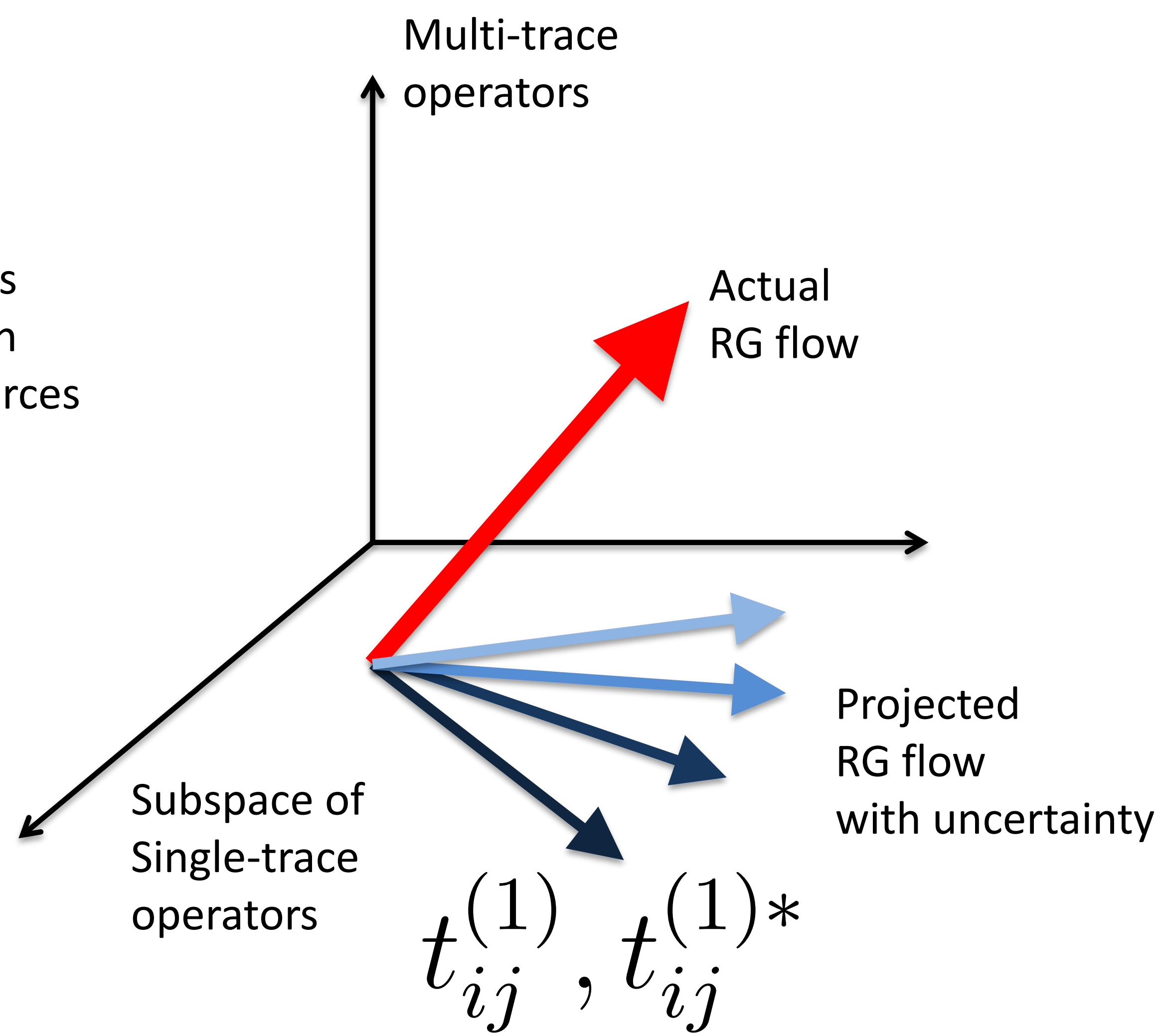
Multi-trace operator = Single-trace operator with dynamical sources

$$\mathcal{Z} = \int \mathcal{D}\phi\, \mathcal{D}\phi^* \boxed{\mathcal{D}t_{ij}^{(1)}\, \mathcal{D}t_{ij}^{*(1)}}\, e^{-\mathcal{S}_2},$$

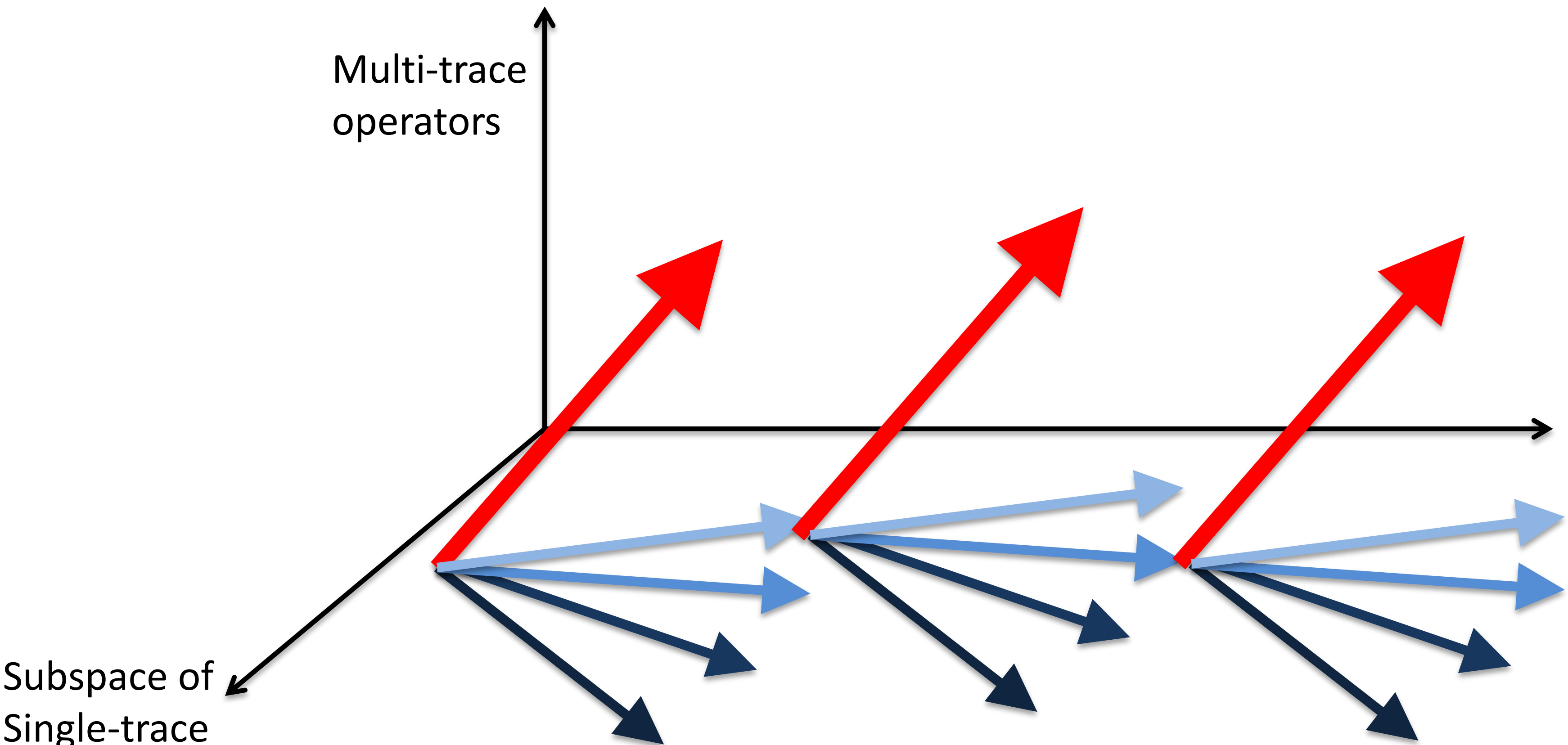
$$\begin{aligned}
\mathcal{S}_2 = & \; N \left\{ -\sum_{ij} \left[(t_{ij}^{(0)} - t_{ij}^{(1)})\, t_{ij}^{*(1)} \right] \right\} \\
& + 2N\alpha dz \left\{ -\frac{1}{m^2} \sum_i t_{ii}^{(0)} + \frac{2\lambda\left(1+\frac{1}{N}\right)}{m^2} \sum_i t_{ii}^{*(1)} - \frac{4\lambda^2}{m^2} \sum_i \left(t_{ii}^{*(1)}\right)^3 \right\} \\
& + 2N\alpha dz \left\{ \frac{2\lambda}{m^2} \sum_{ij} t_{ij}^{(0)} t_{ij}^{*(1)} \left[t_{ii}^{*(1)} + t_{jj}^{*(1)} \right] - \frac{1}{m^2} \sum_{ijk} t_{ik}^{(0)} t_{kj}^{(0)} t_{ij}^{*(1)} - 2\lambda \sum_i \left(t_{ii}^{*(1)}\right)^2 + \sum_{ij} t_{ij}^{(0)}\, t_{ij}^{*(1)} \right\} \\
& \boxed{-\sum_{ij} t_{ij}^{(1)} (\phi_i^* \cdot \phi_j) + m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2}
\end{aligned}$$

Projection creates uncertainty

- Theory with multi-trace operators can be mapped into a theory with single-trace operators whose sources are dynamical



Quantum RG

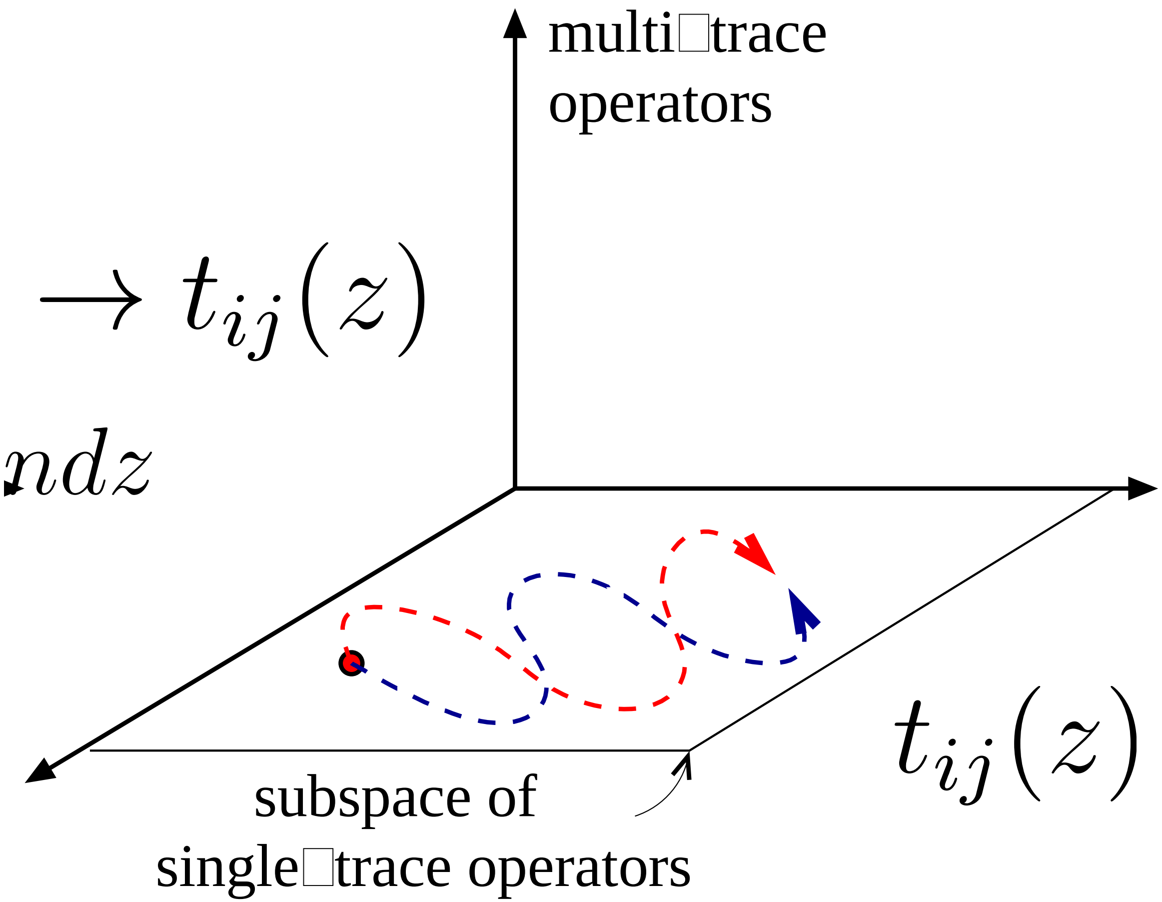


$$t_{ij}^{(1)}, t_{ij}^{(1)*} \quad t_{ij}^{(2)}, t_{ij}^{(2)*} \quad t_{ij}^{(3)}, t_{ij}^{(3)*}$$

Sum over all RG paths in the space of hoppings

$$t_{ij}^{(n)} \to t_{ij}(z)$$

$$z = n dz$$

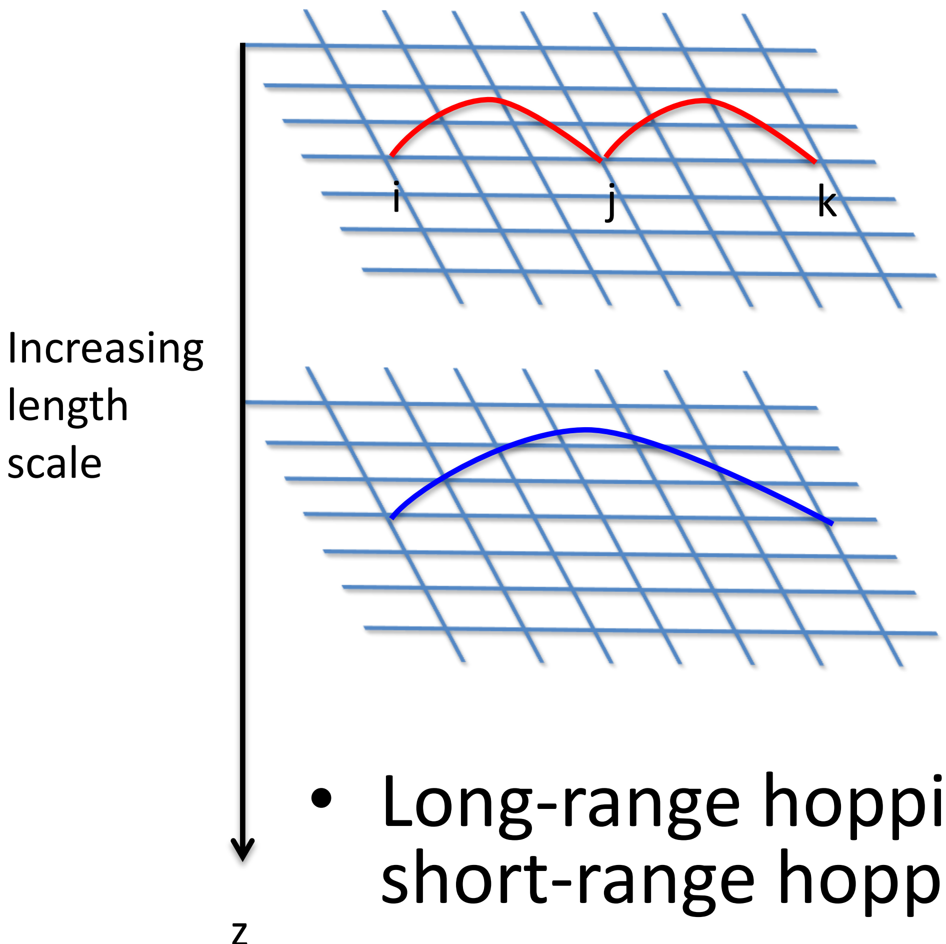


Weight for each RG path : Bulk Action

$$Z = \int Dt_{ij}(z) Dt^*_{ij}(z) e^{-NS_{bulk}[t,t^*]}$$

$$\begin{aligned}\mathcal{S}_{Bulk} \;=\; & \int_0^\infty dz \left\{ \sum_{ij} t^*_{ij}(z)\,\partial_z t_{ij}(z) \right. \\ & + \sum_i \left[-\frac{2}{m^2} t_{ii}(z) + \frac{4\lambda\left(1+\frac{1}{N}\right)}{m^2} t^*_{ii}(z) - 4\lambda\left(t^*_{ii}(z)\right)^2 - \frac{8\lambda^2}{m^2}\left(t^*_{ii}(z)\right)^3 \right] \\ & \left. + \sum_{ij}\left[2t_{ij}(z)\; t^*_{ij}(z) + \frac{4\lambda}{m^2} t_{ij}(z) t^*_{ij}(z)\left(t^*_{ii}(z) + t^*_{jj}(z)\right)\right] - \frac{2}{m^2}\sum_{ijk}\left[t_{ik}(z) t_{kj}(z) t^*_{ij}(z)\right] \right\}\end{aligned}$$

Hopping amplitudes are promoted to quantum operators



$$t^{\dagger}_{ik}t_{ij}t_{jk}$$

- Long-range hoppings are generated as short-range hoppings merge
- Hopping fields determine the connectivity (geometry) of the bulk

Saddle point approximation

$$\partial_z t_{ij} = -2\left\{\frac{2\lambda\,\delta_{ij}}{m^2} - \delta_{ij}\left[4\lambda + \frac{12\lambda^2}{m^2}t^*_{ii}\right]t^*_{ii} + \frac{2\lambda\,\delta_{ij}}{m^2}\sum_k\left(t_{ik}t^*_{ik} + t_{ki}t^*_{ki}\right)\right.$$
$$\left. + \left[1 + \frac{2\lambda}{m^2}\left(t^*_{ii} + t^*_{jj}\right)\right]t_{ij} - \frac{1}{m^2}\sum_k t_{ik}t_{kj}\right\}$$

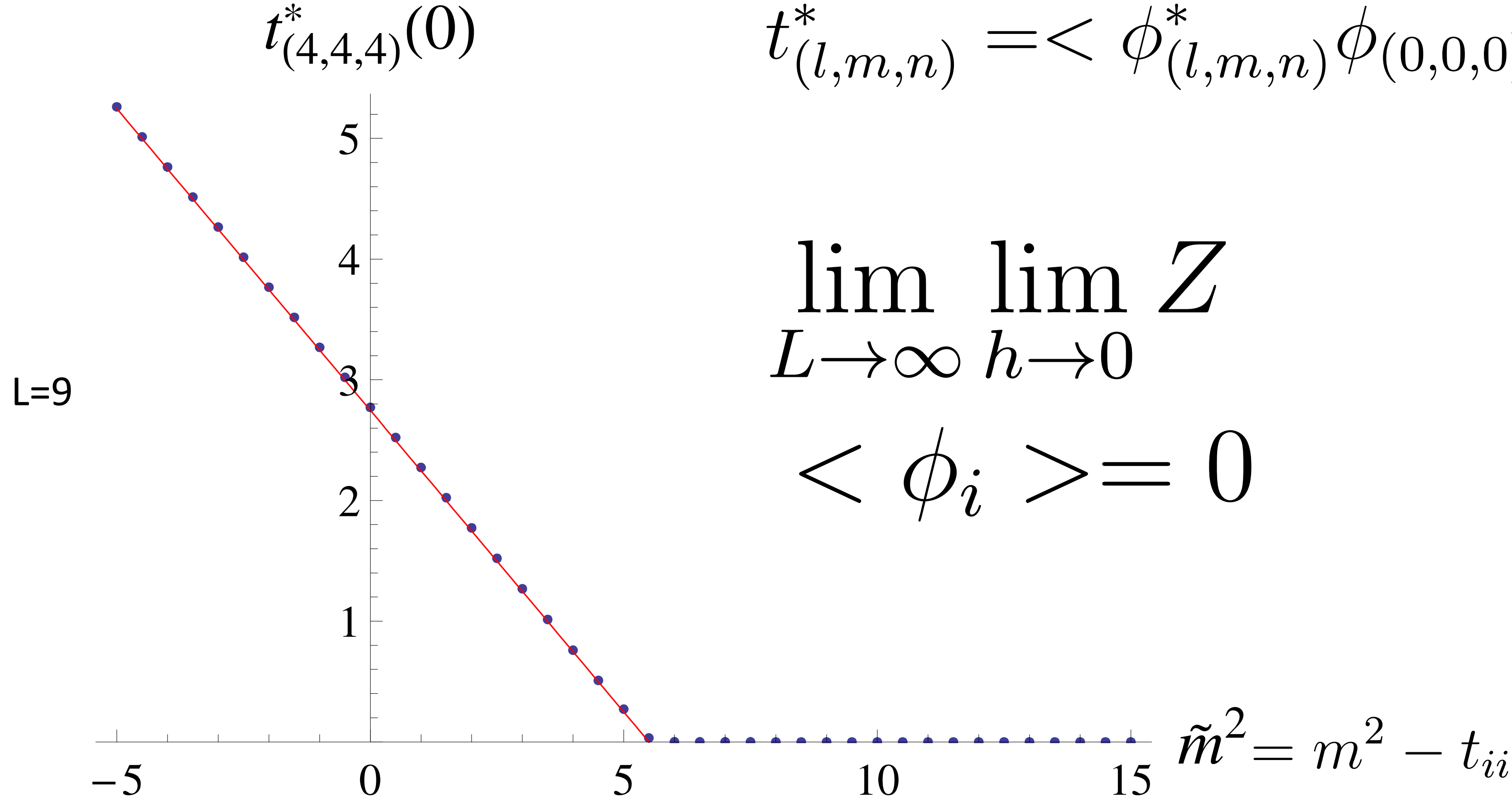
$$\partial_z t^*_{ij} = 2\left\{-\frac{\delta_{ij}}{m^2} + \left[1 + \frac{2\lambda}{m^2}\left(t^*_{ii} + t^*_{jj}\right)\right]t^*_{ij} - \frac{1}{m^2}\sum_k\left(t^*_{ik}t_{jk} + t_{ki}t^*_{kj}\right)\right\}$$

- In the large N limit, semi-classical RG path dominates the partition function
- At the saddle point t_{ij}, t^*_{ij} take independent values

$$t^*_{ij} = \frac{1}{N} < \phi^*_i \phi_j >$$

Field theory observable at the UV boundary

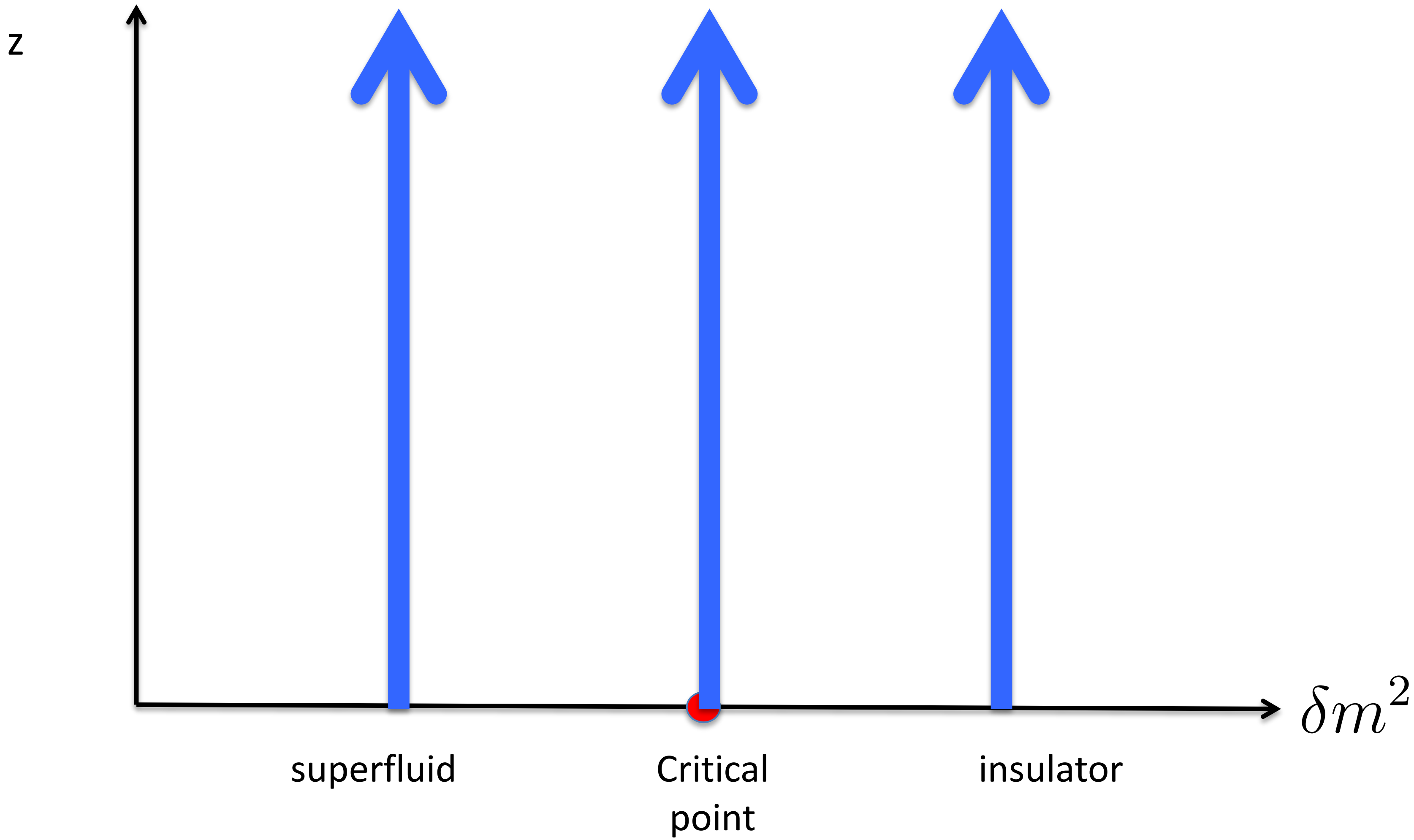
$$t^{*}_{(l,m,n)} = < \phi^{*}_{(l,m,n)} \phi_{(0,0,0)} >$$



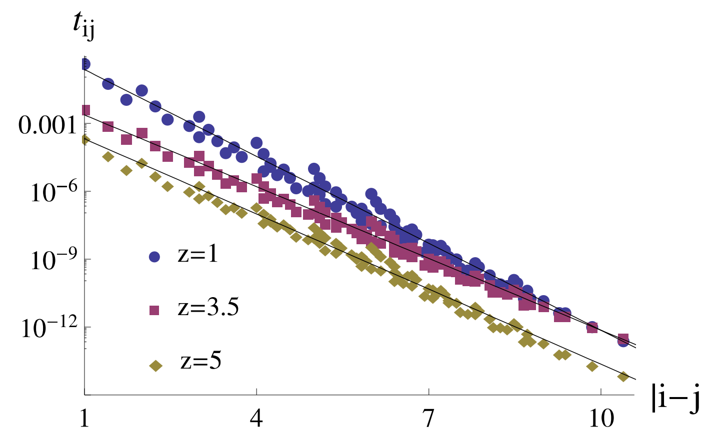
$$\lim_{L\to\infty} \lim_{h\to 0} Z$$

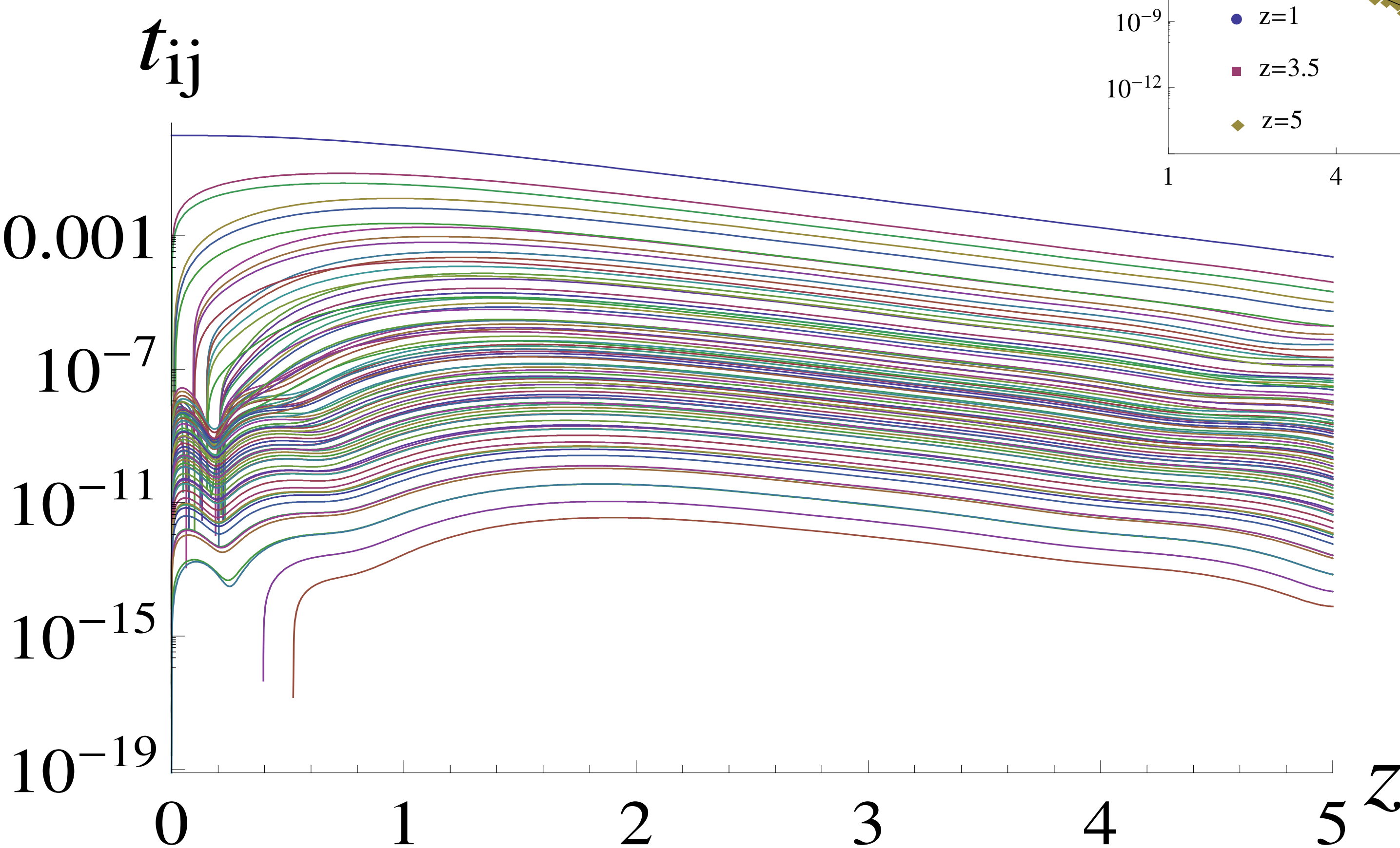
$$< \phi_i > = 0$$

Bulk fields



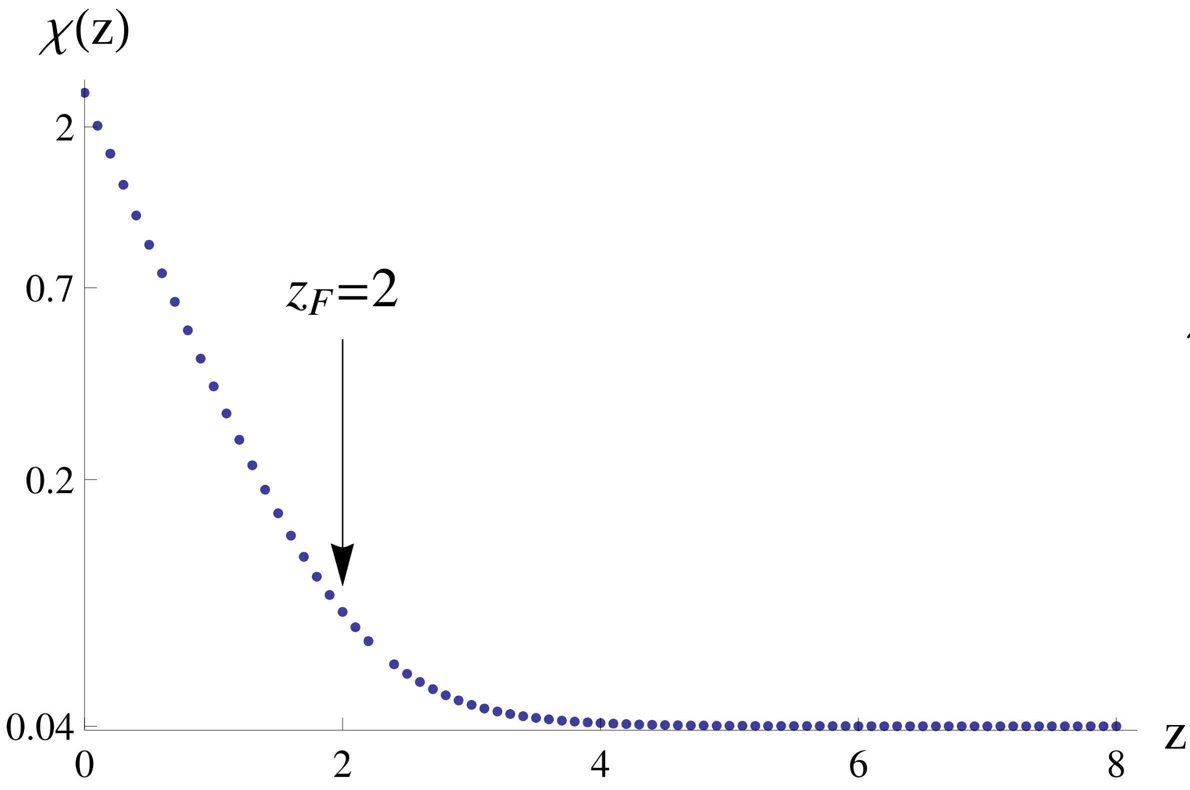
Insulating Phase





- $t_{ij}(z)$ decay exponentially both in |i-j| and z

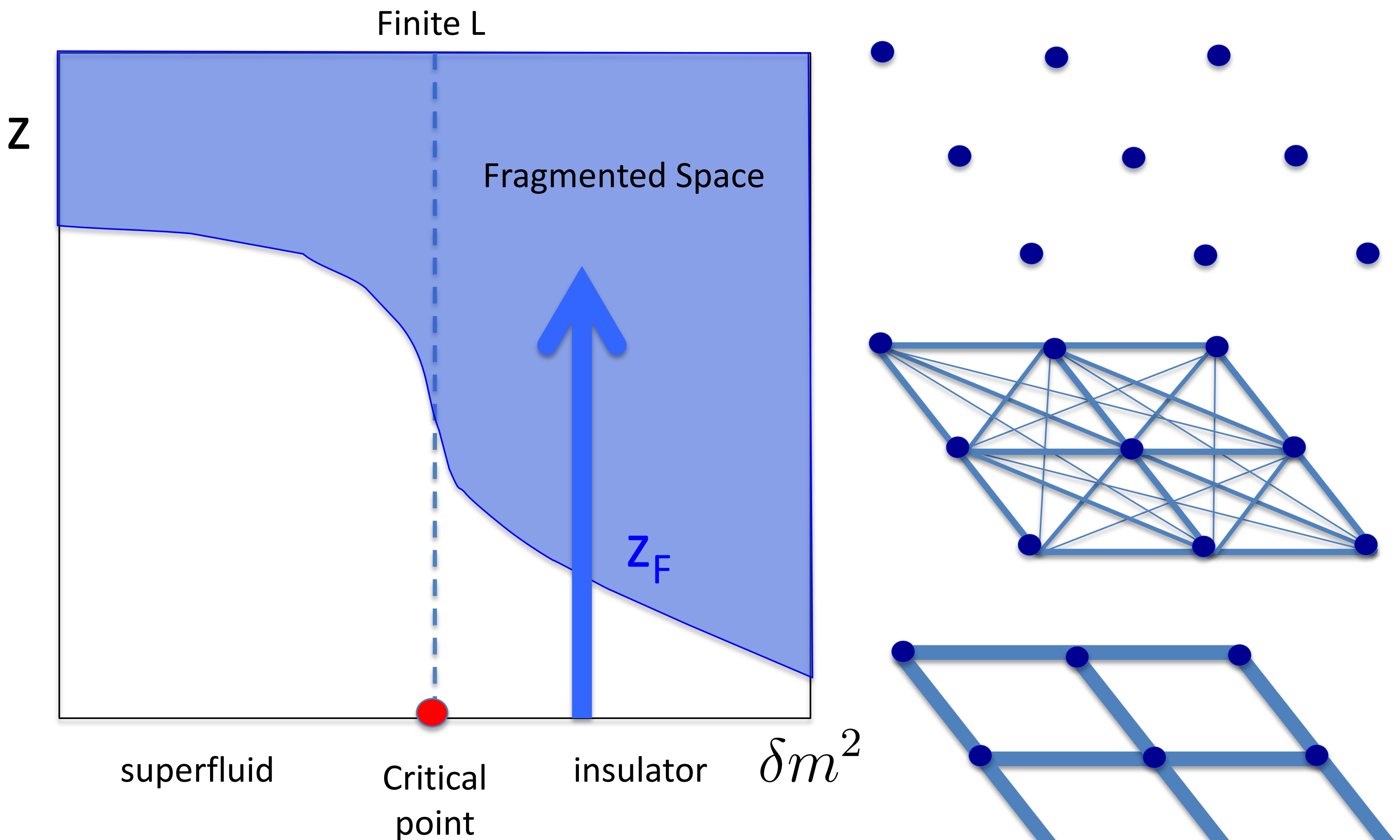
Fragmentation



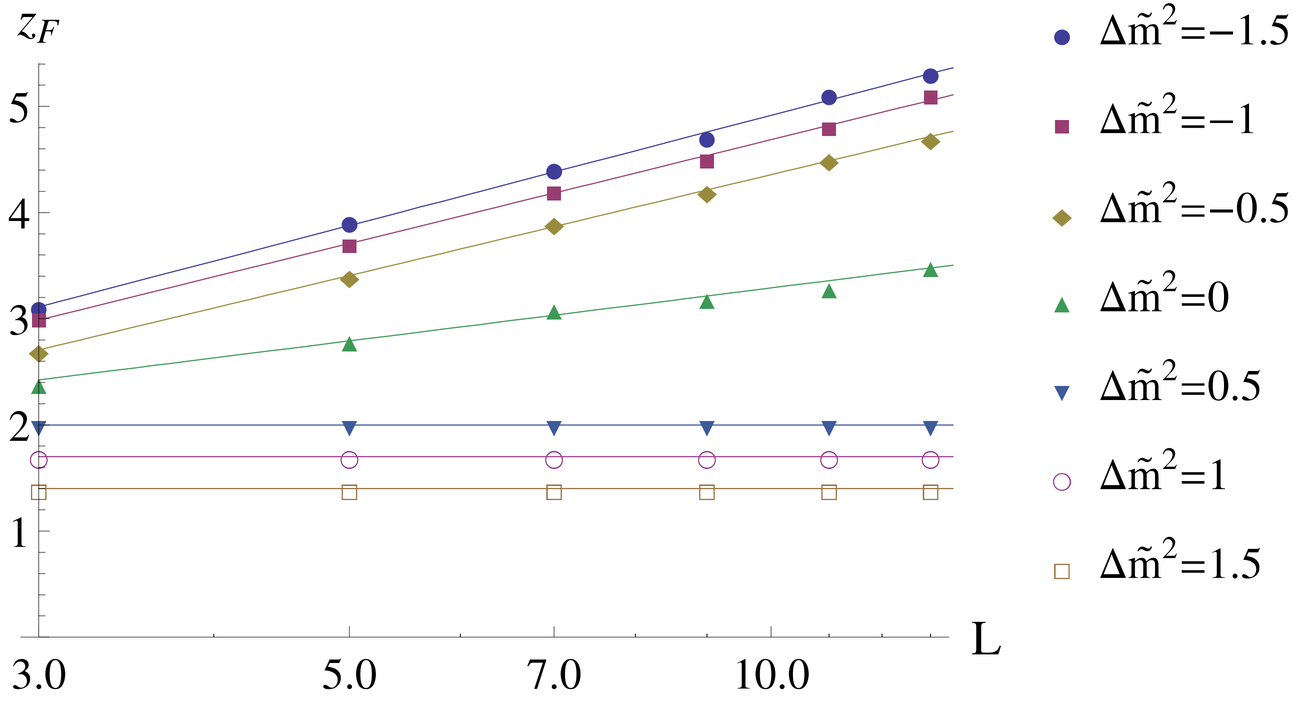
$$\chi = \sum_j t^*_{ij}$$

- For $z>z_F$, correlation length of $t_{ij}^*(z) \sim e^{-|i-j|/\xi}$ becomes much less than the lattice spacing (not a sharp transition)

Fragmentation in insulating phase

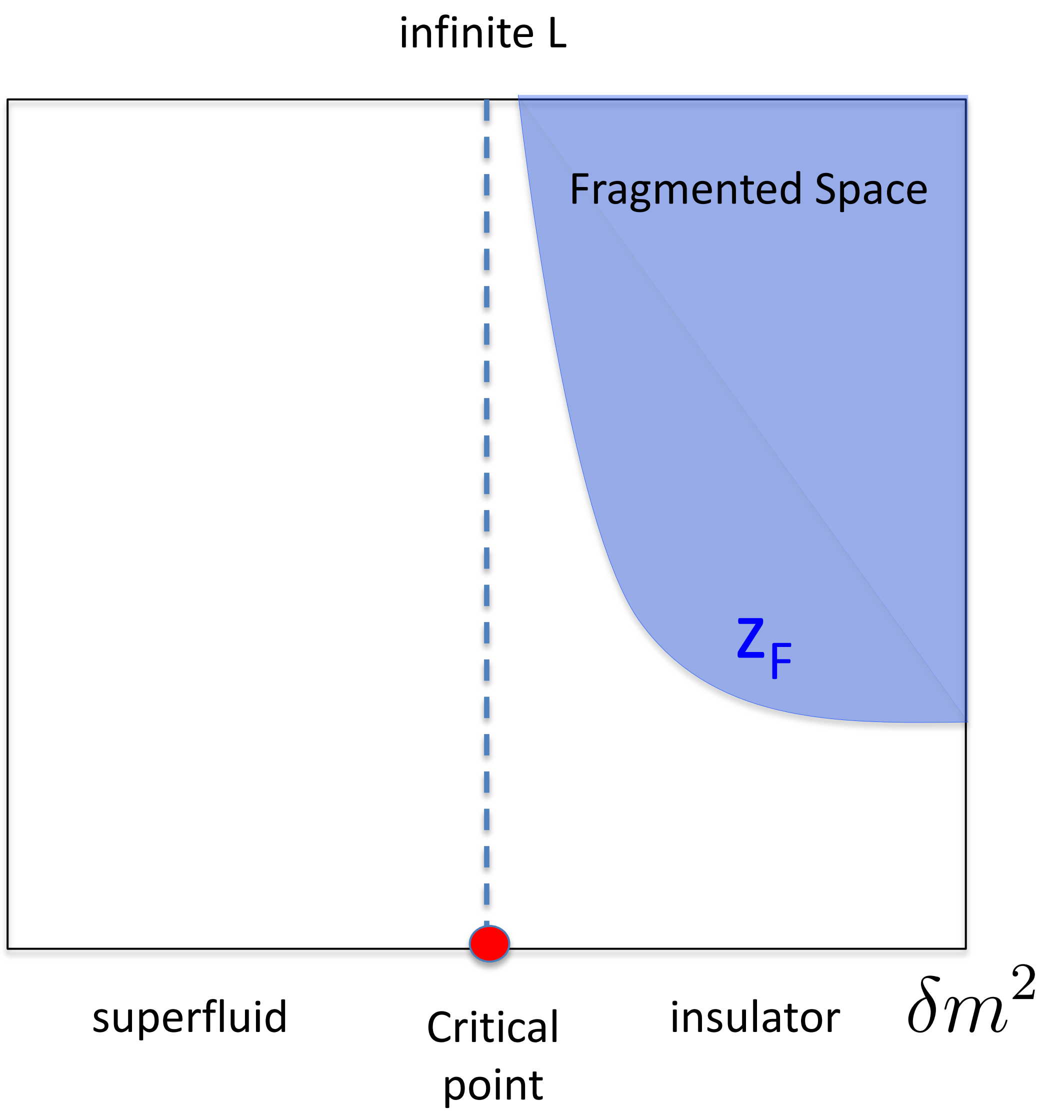


Fragmentation in superfluid phase is a finite size effect

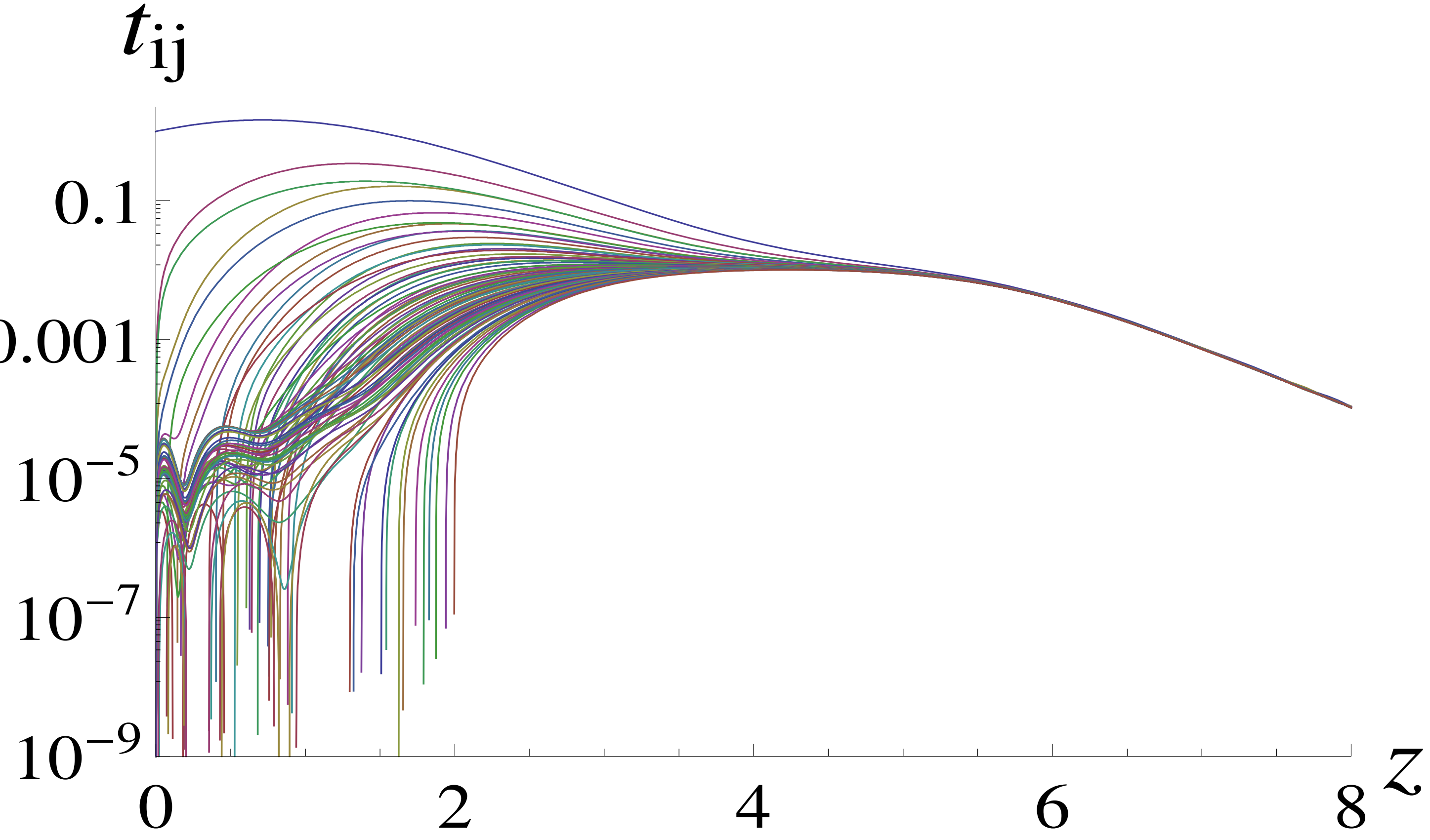


- In the insulating phase, z_F is independent of system size
- In the superfluid phase and at the critical point, z_F diverges in the thermodynamic limit

Fragmentation in the insulating phase

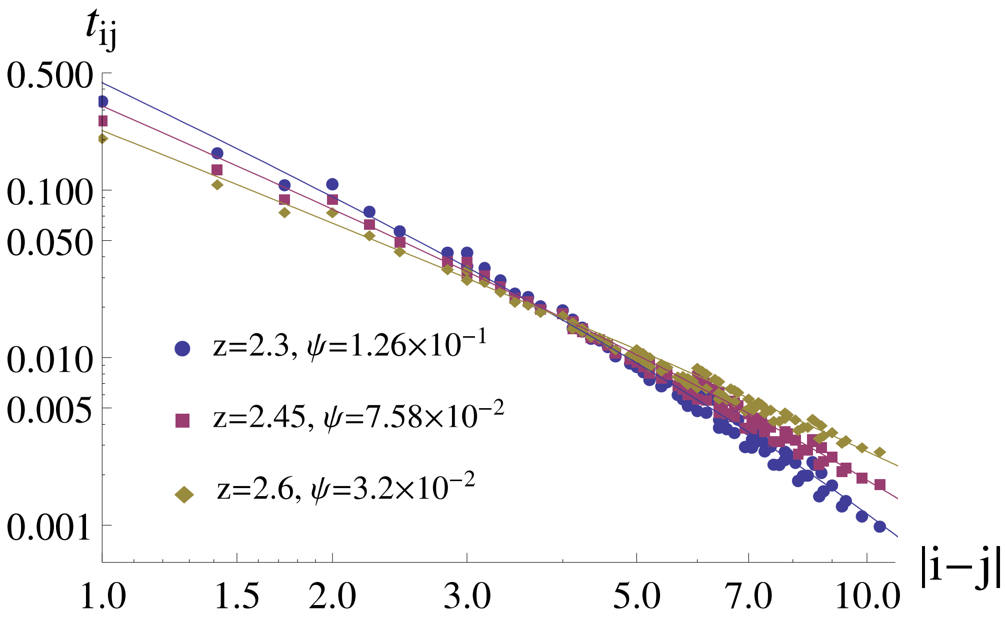


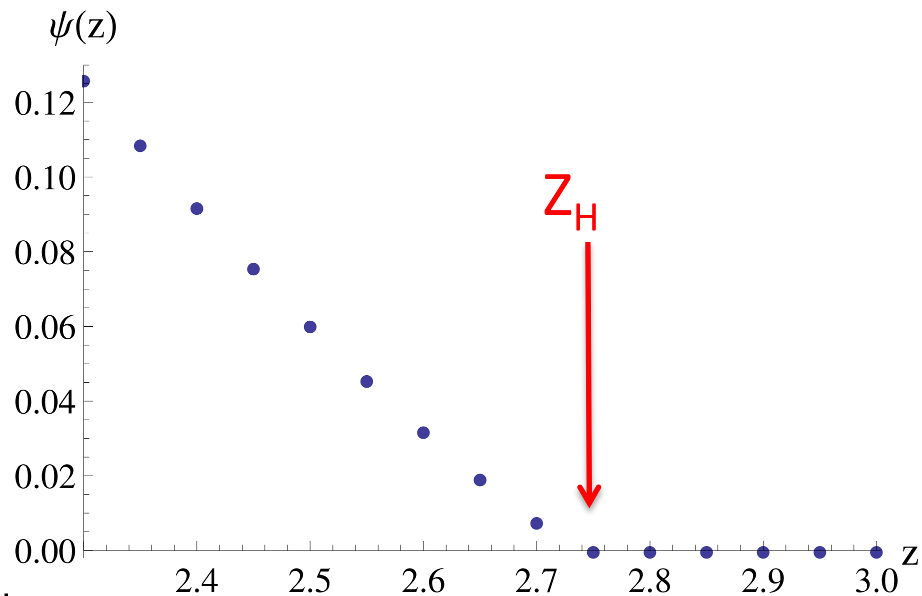
Superfluid Phase



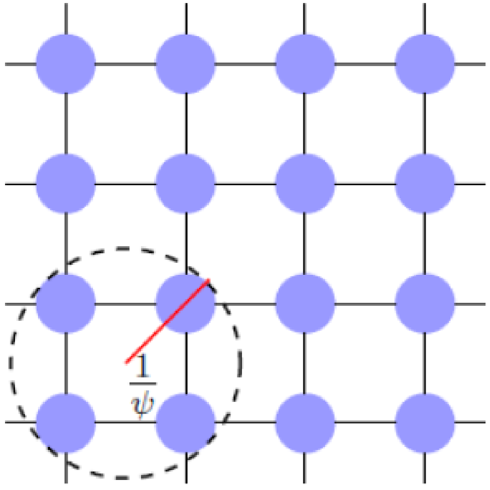
- In the supefluid phase, locality is lost in the bulk

Horizon in the superfluid phase





$$t_{ij}(z) \sim \frac{e^{-\psi(z)|i-j|}}{|i-j|^{\kappa}}$$

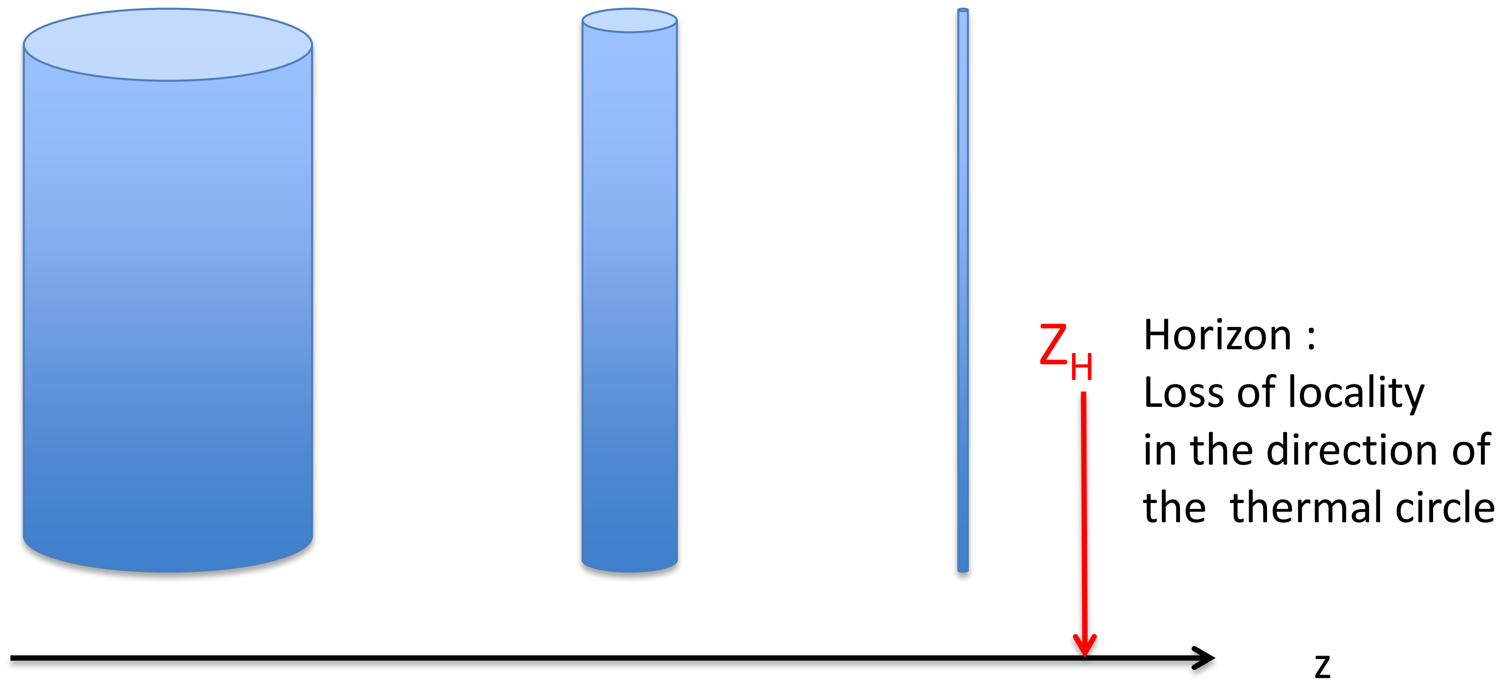


Range of hopping diverges at horizon : Volume shrinks to zero

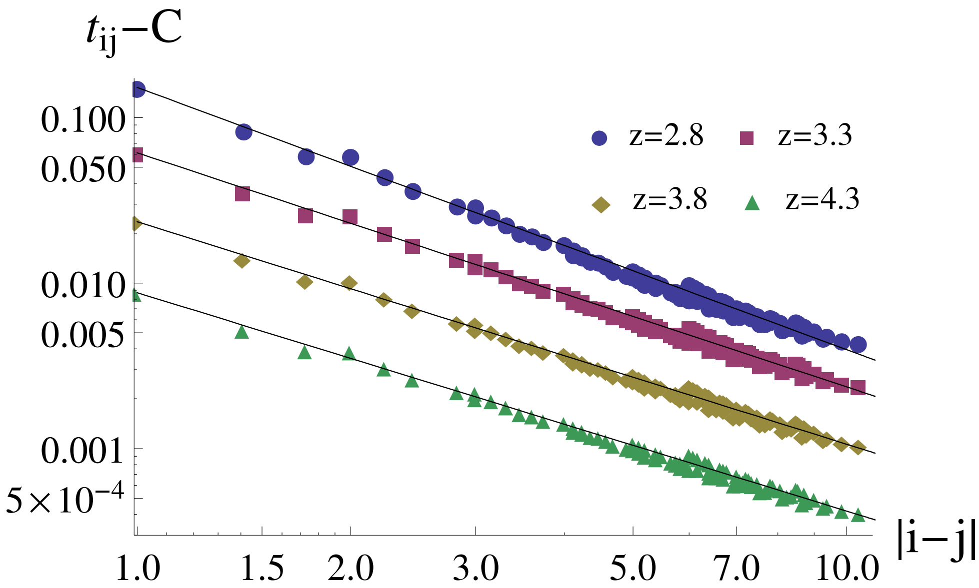
Why called horizon?

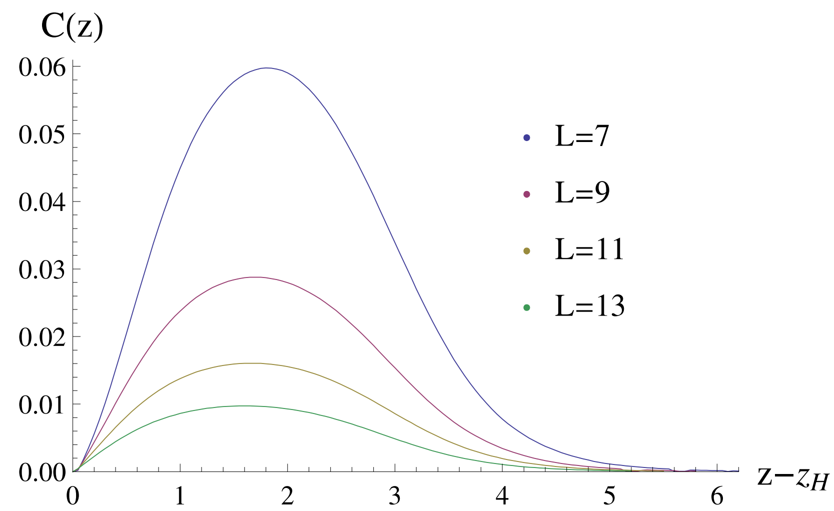
At finite temperature

$$\beta = \frac{1}{T}$$



Beyond horizon





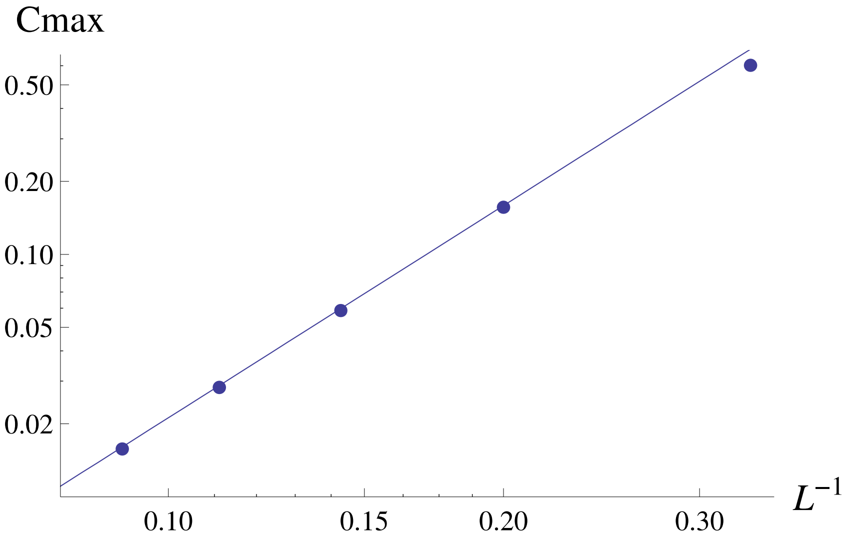
$z \geq z_H$

$$t_{ij}(z) = C(z) + \frac{B(z)}{|i-j|^{\kappa}}$$

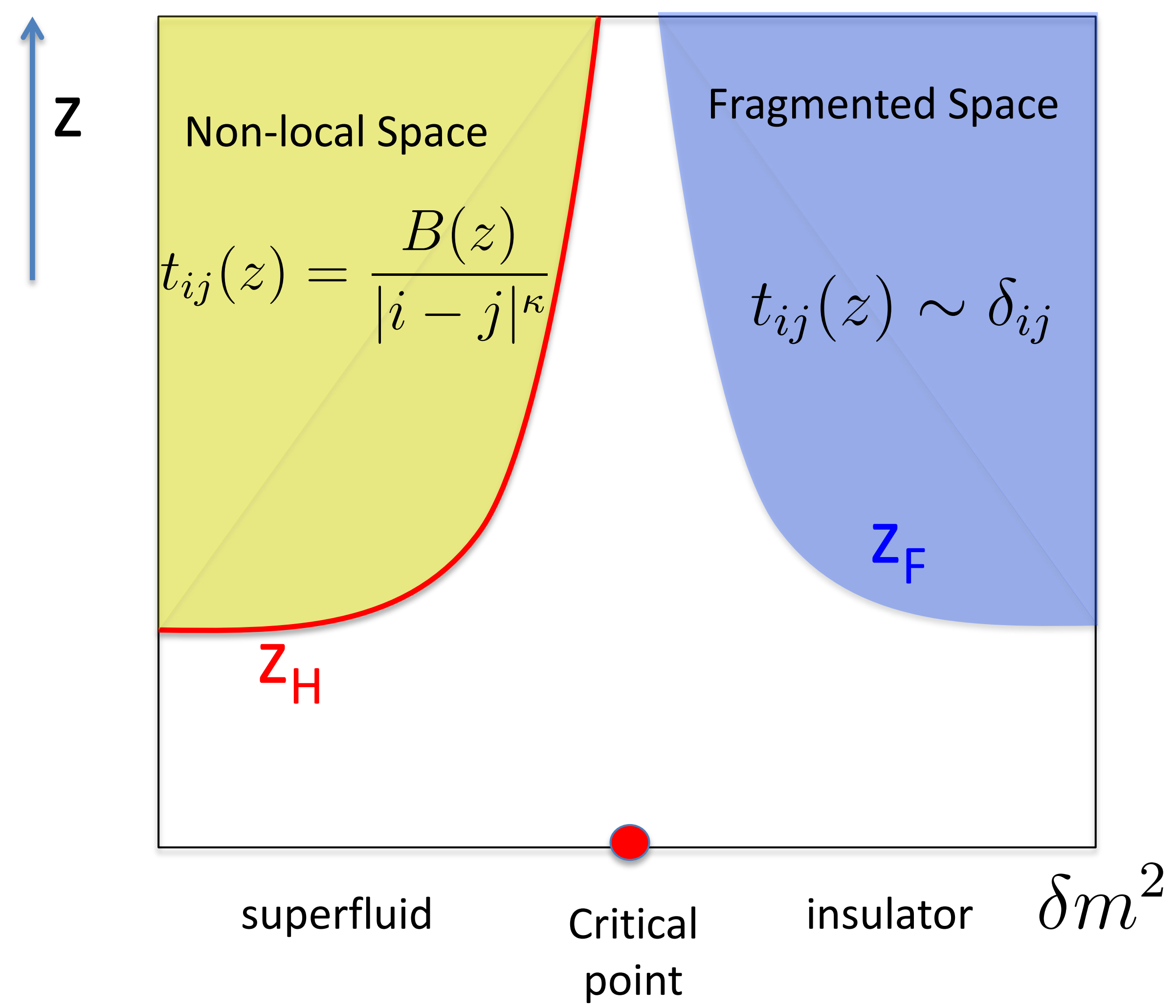
Beyond horizon in the thermodynamic limit

$$\lim_{L\to\infty} \quad t_{ij}(z) = C(z) + \frac{B(z)}{|i-j|^{\kappa}}$$

$$0$$

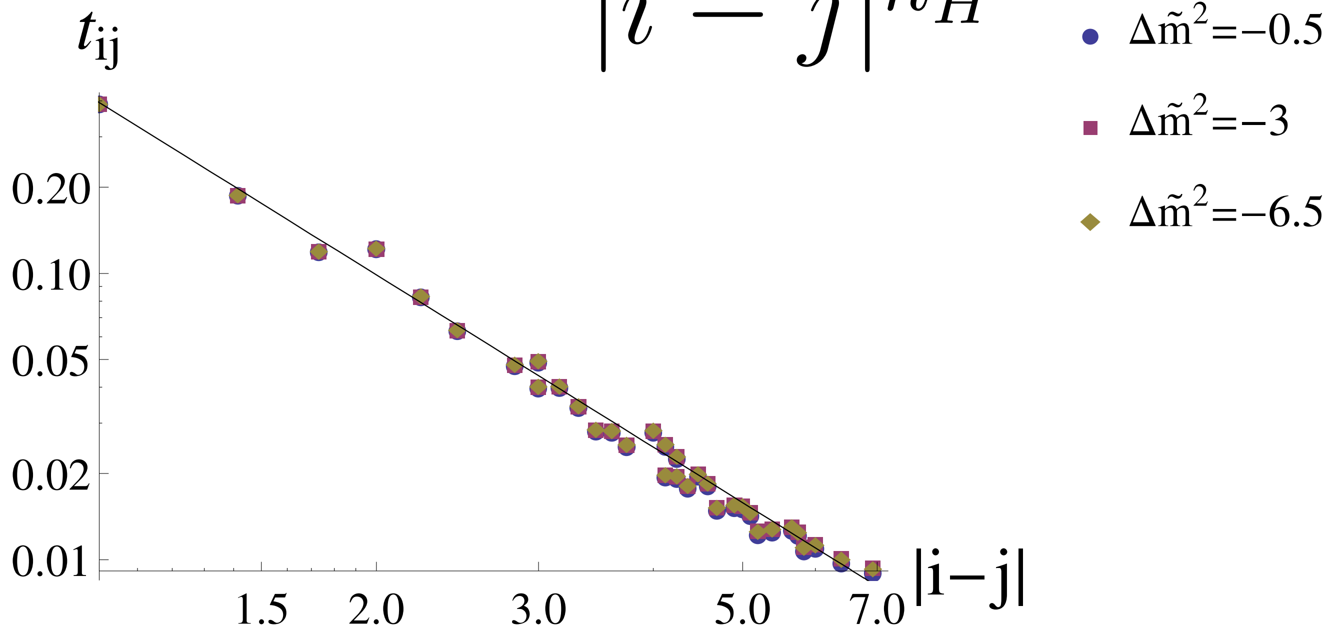


Holographic phase diagram



Universal power-law at the horizon

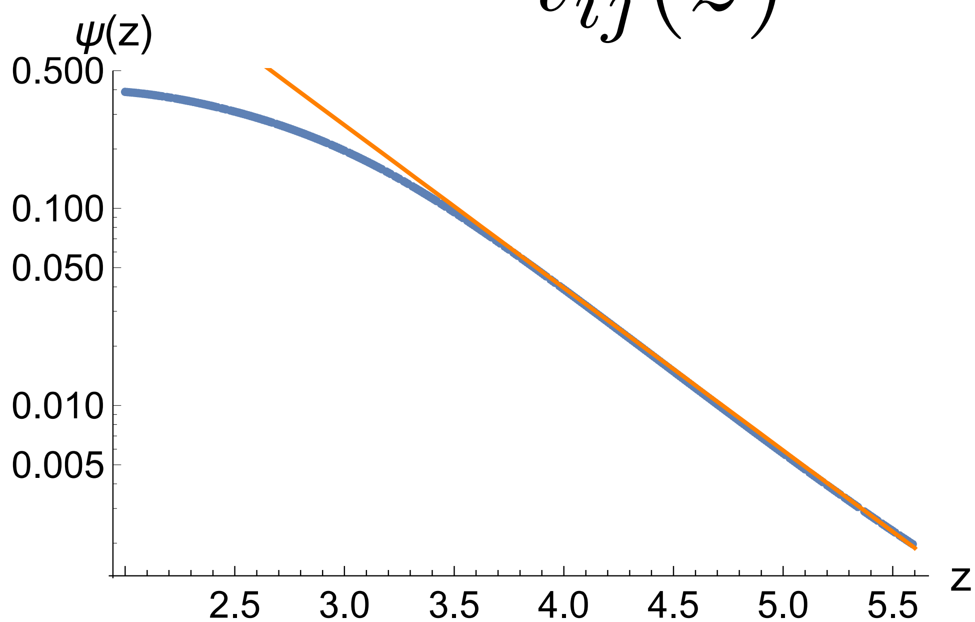
$$t_{ij}(z) \sim \frac{1}{|i-j|^{\kappa_H}}$$



- At the horizon, the hopping fields decay with a universal exponent independent of δm^2

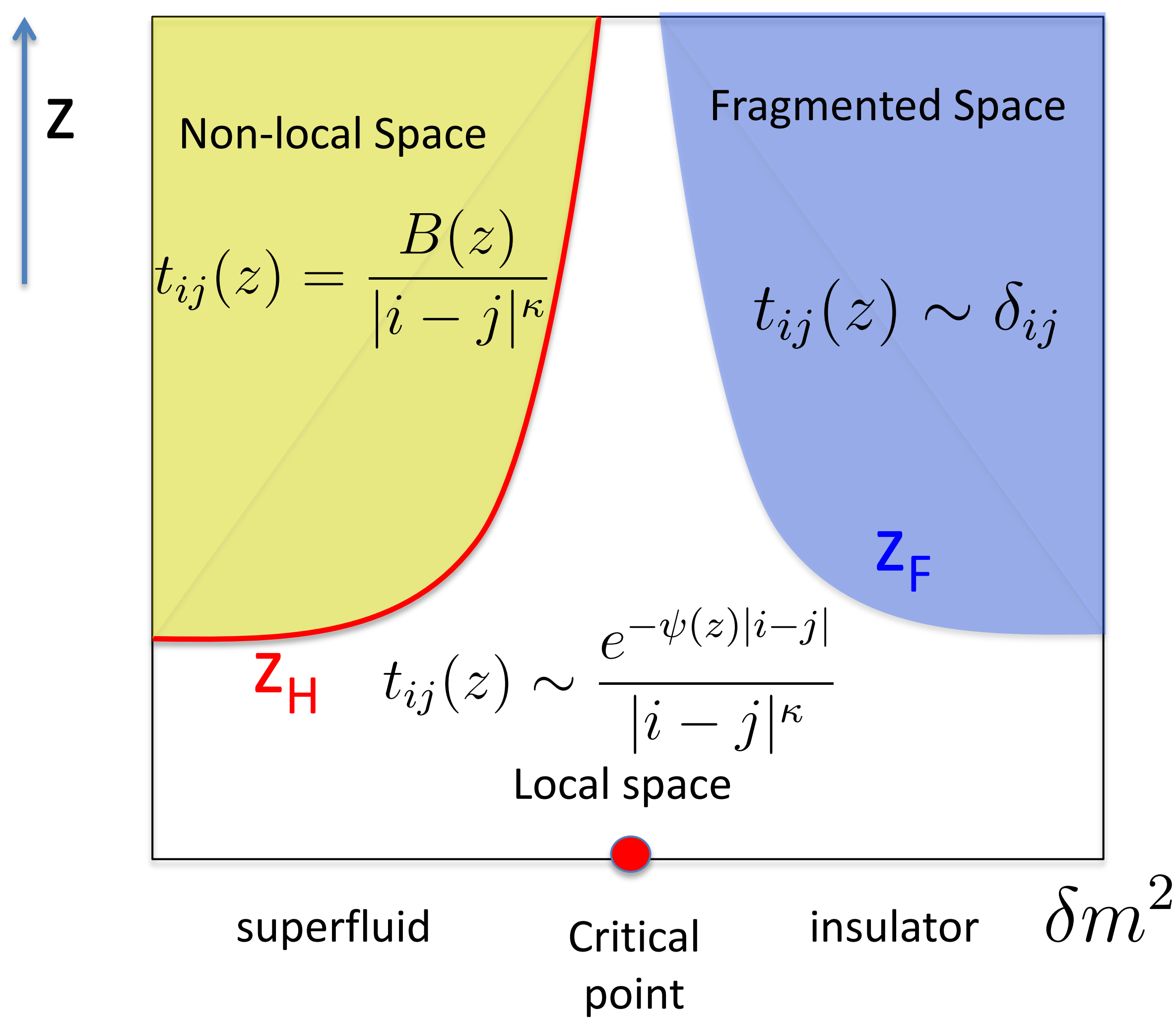
Critical Point

$$t_{ij}(z) \sim \frac{e^{-\psi(z)|i-j|}}{|i-j|^{\kappa}}$$

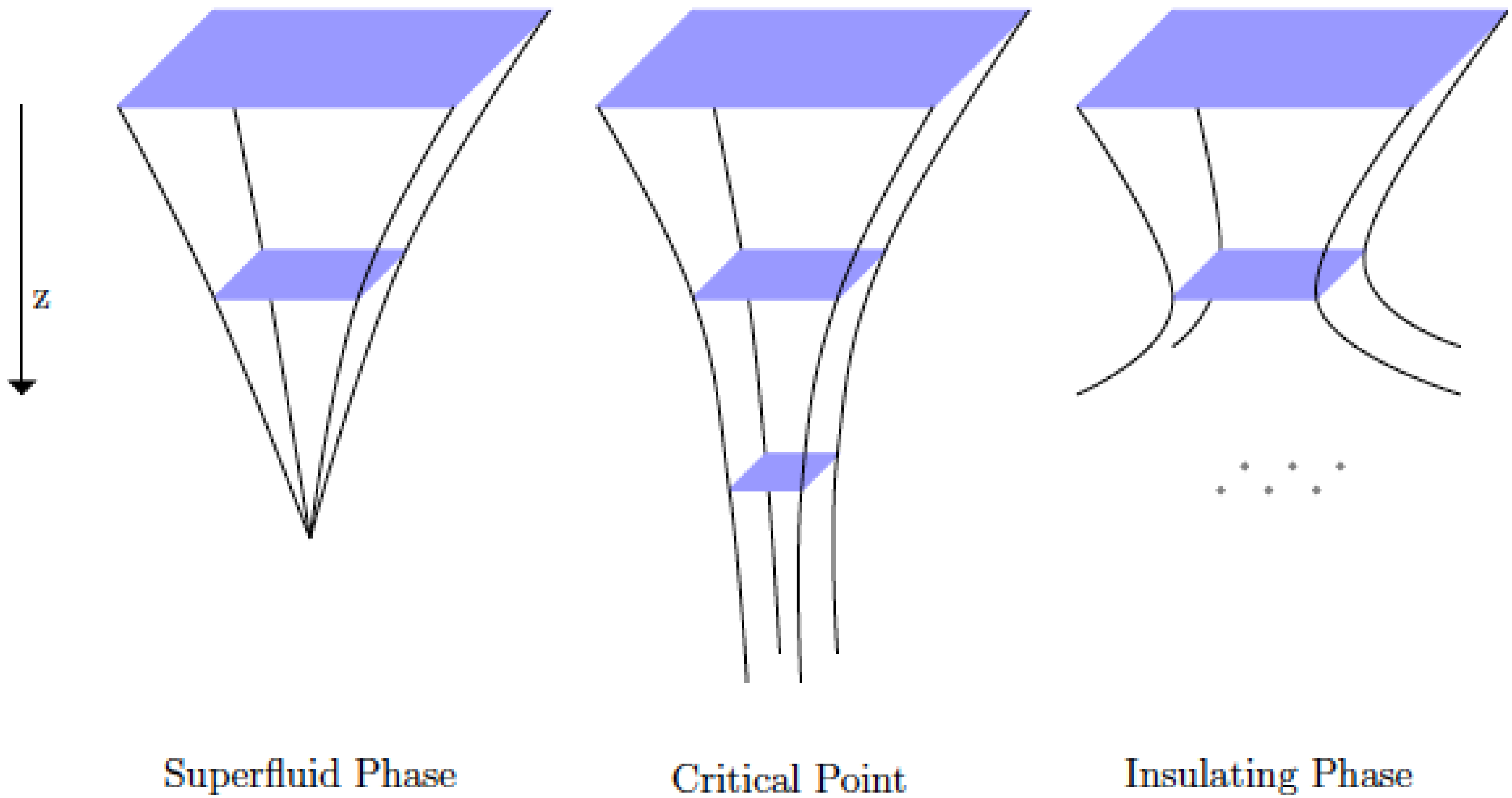


- At the critical point, the length scale associated with the hopping diverges exponentially in z

Holographic phase diagram



Locality as order parameter



$f(z) = 0 \quad \text{for} \quad z \geq z_H$

$f(z) \sim e^{-2z}$

$f(z) \to \infty, \quad z \to \infty$

$$ds^2 = dz^2 + f(z)dx_i^2$$

Figure from MSc thesis of Mao Tian Tan

Summary

- Holographic solution of U(N) vector model via quantum RG
- (Non-)Locality serves as an order parameter
  - Insulator : Fragmented geometry
  - Superfluid : Non-locality geometry behind horizon
  - Critical point : Local geometry

Gauge symmetry in the bulk

$$\mathcal{S}_{Bulk} = \int_0^\infty dz \left\{ \sum_{ij} t^*_{ij}(z)\, \partial_z t_{ij}(z) \right.$$

$$+ \sum_i \left[-\frac{2}{m^2} t_{ii}(z) + \frac{4\lambda\left(1+\frac{1}{N}\right)}{m^2} t^*_{ii}(z) - 4\lambda\left(t^*_{ii}(z)\right)^2 - \frac{8\lambda^2}{m^2}\left(t^*_{ii}(z)\right)^3 \right]$$

$$\left. + \sum_{ij} \left[2t_{ij}(z)\ t^*_{ij}(z) + \frac{4\lambda}{m^2} t_{ij}(z) t^*_{ij}(z)\left(t^*_{ii}(z) + t^*_{jj}(z)\right) \right] - \frac{2}{m^2} \sum_{ijk} \left[t_{ik}(z) t_{kj}(z) t^*_{ij}(z) \right] \right\}$$

in the radial gauge

Higher spin symmetry :
(λ=0)

$$t_{ij}(z) \to U^\dagger_{ii'} t_{i'j'}(z) U_{j'j}$$

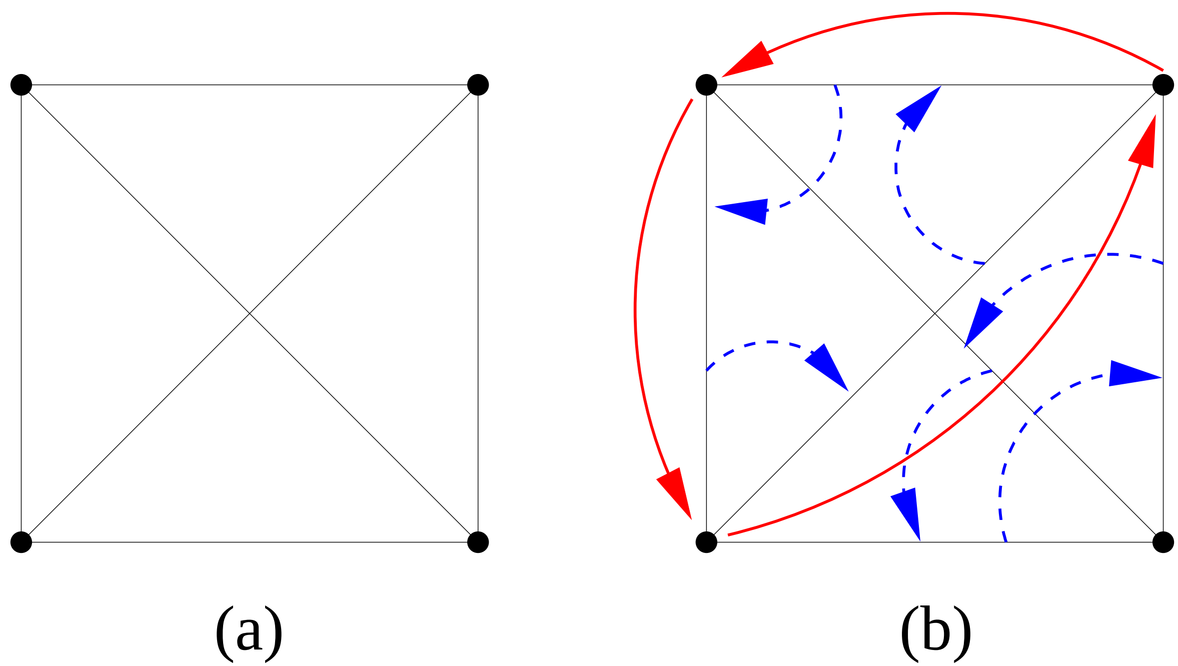
[R. Leigh, O. Parrikar, A. Weiss, arXiv:1402.1430]

Only discrete diffeomorphism survives :
for λ≠0

$$U_{ij} = \delta_{ij+N_i}$$

Origin of D-dim discrete diffeomorphism inv.

$$\mathcal{S}_0 = m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2 - \sum_{ij} t_{ij}^{(0)} \left(\phi_i^* \cdot \phi_j\right)$$



Permutation (event symmetry) in D-dimensional network