

# Tunable Long-Range Spin Models With Trapped Ions

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INSTITUTE

**University of Maryland**

Boris  
2005

**T**rapped Ion Spin Hamiltonian Engineering

**G**round states and Adiabatic Protocols

**D**ynamics

Direct Many-Body Spectroscopy

Lieb-Robinson Bounds

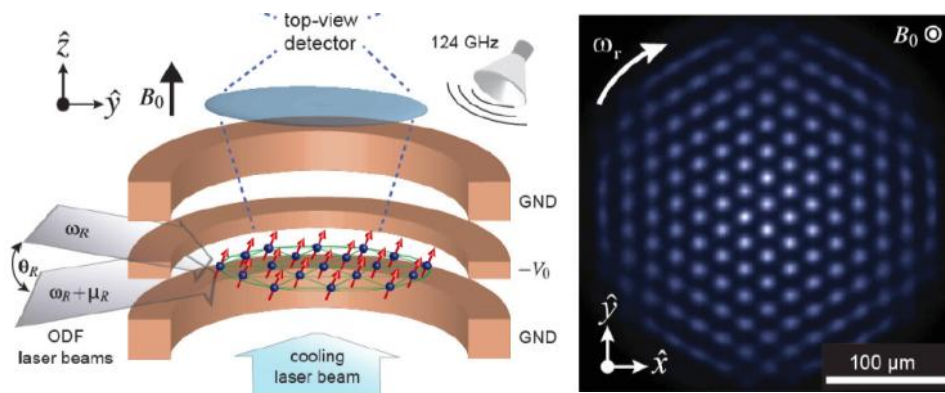
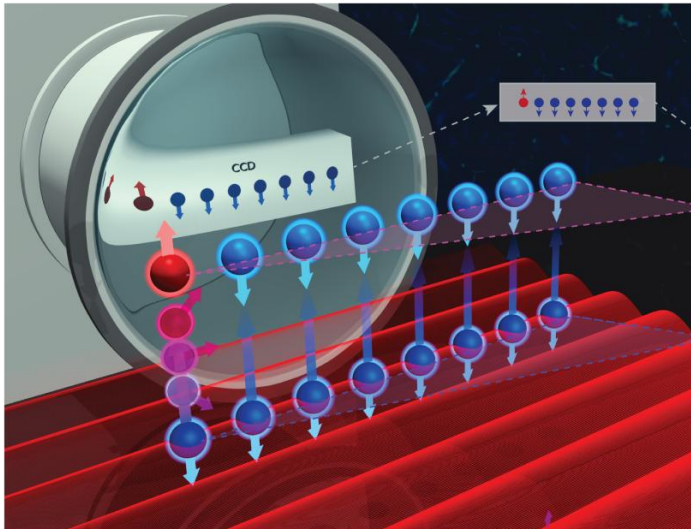
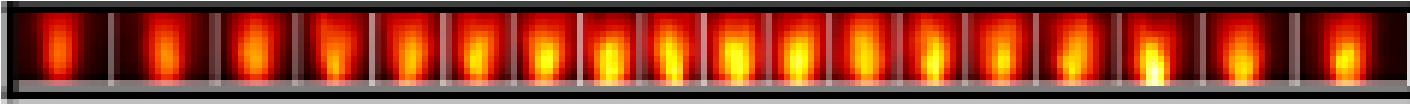
**M**any-Body Thermalization/Localization?

**S**pin-1

**F**uture



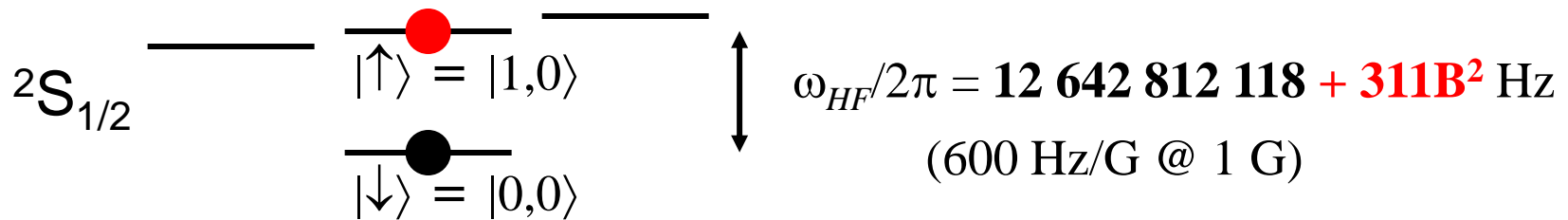
# Trapped Atomic Ions: Spin Models



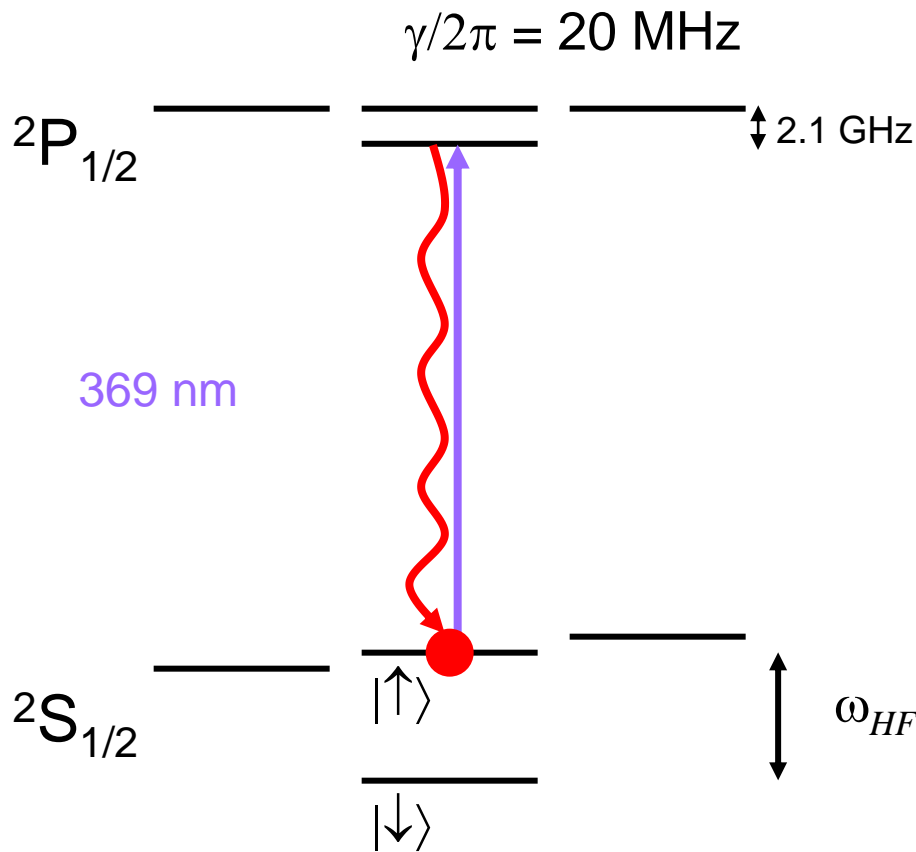
Porras and Cirac, PRL 92, 207901 (2004)  
 Deng, Porras, Cirac, PRA 72, 063407 (2005)  
 Taylor and Calarco, PRA 78, 062331 (2008)

- A. Friedenauer *et al.*, Nat. Phys. 4, 757 (2008)  
 K. Kim *et al.*, PRL 102, 250502 (2009)  
 K. Kim *et al.*, Nature 465, 590 (2010)  
 E. Edwards *et al.*, PRB 82, 060412 (2010)  
 J. Barreiro *et al.*, Nature 470, 486-491 (2011)  
 R. Islam *et al.*, Nature Comm. 2, 377 (2011)  
 B. Lanyon *et al.*, Science 334, 57 (2011)  
 J. Britton *et al.*, Nature 484, 489 (2012)  
 A. Khromova *et al.*, PRL 108, 220502 (2012)  
 R. Islam *et al.*, Science 340, 583 (2013)  
 P. Richerme *et al.*, PRL 111, 100506 (2013)  
 P. Richerme *et al.*, PRA 88, 012334 (2013)  
 P. Richerme *et al.*, Nature 511, 198 (2014)  
 P. Jurcevic *et al.*, Nature 511, 202 (2014)  
 C. Senko *et al.*, Science 345, 430 (2014)

# $^{171}\text{Yb}^+$ hyperfine spin

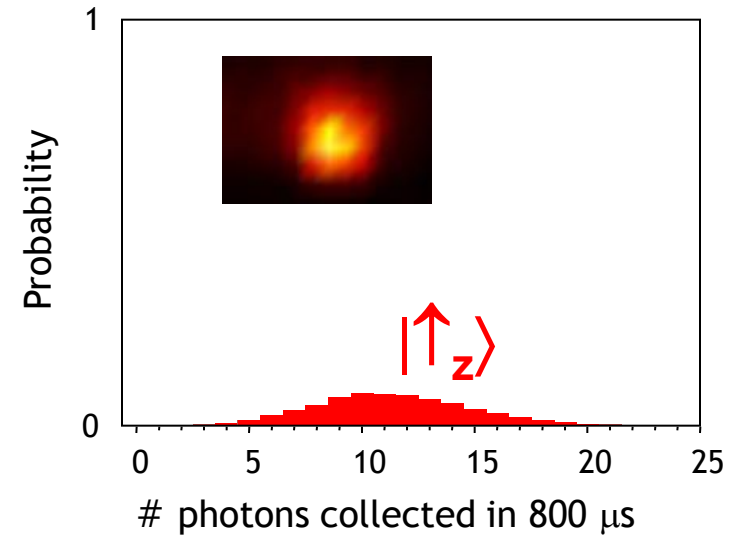


# $^{171}\text{Yb}^+$ spin detection

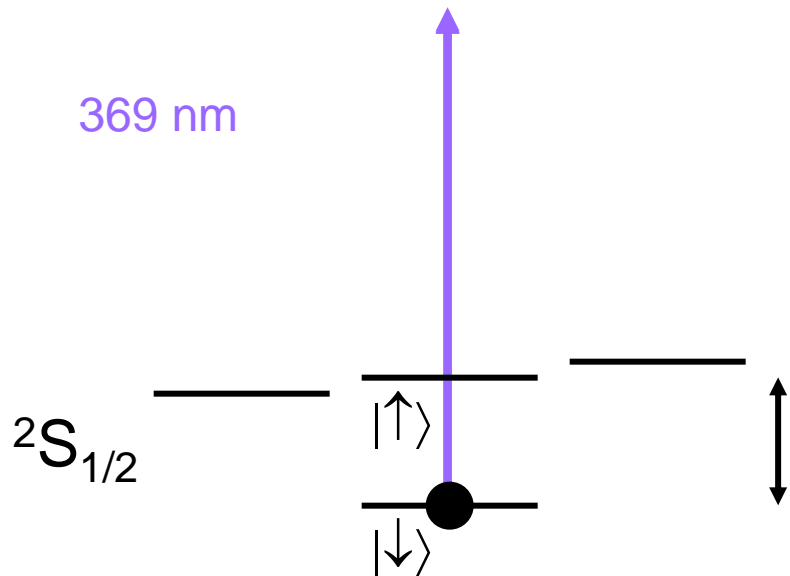
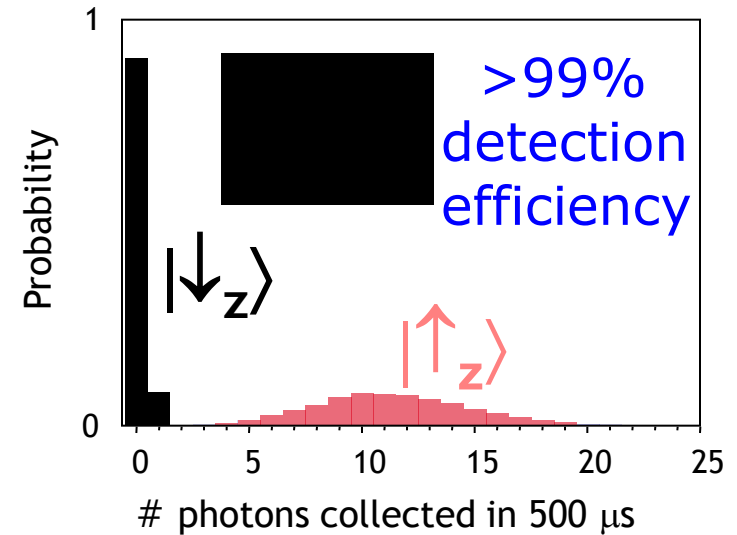
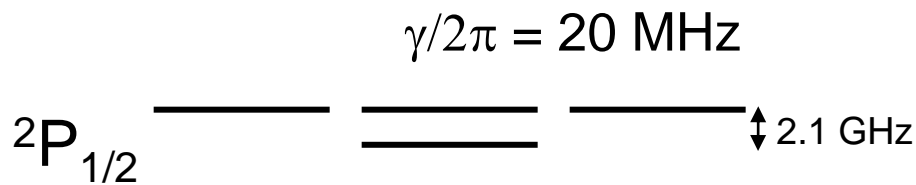


$$\omega_{HF}/2\pi = 12\,642\,812\,118 + 311B^2 \text{ Hz}$$

(600 Hz/G @ 1 G)



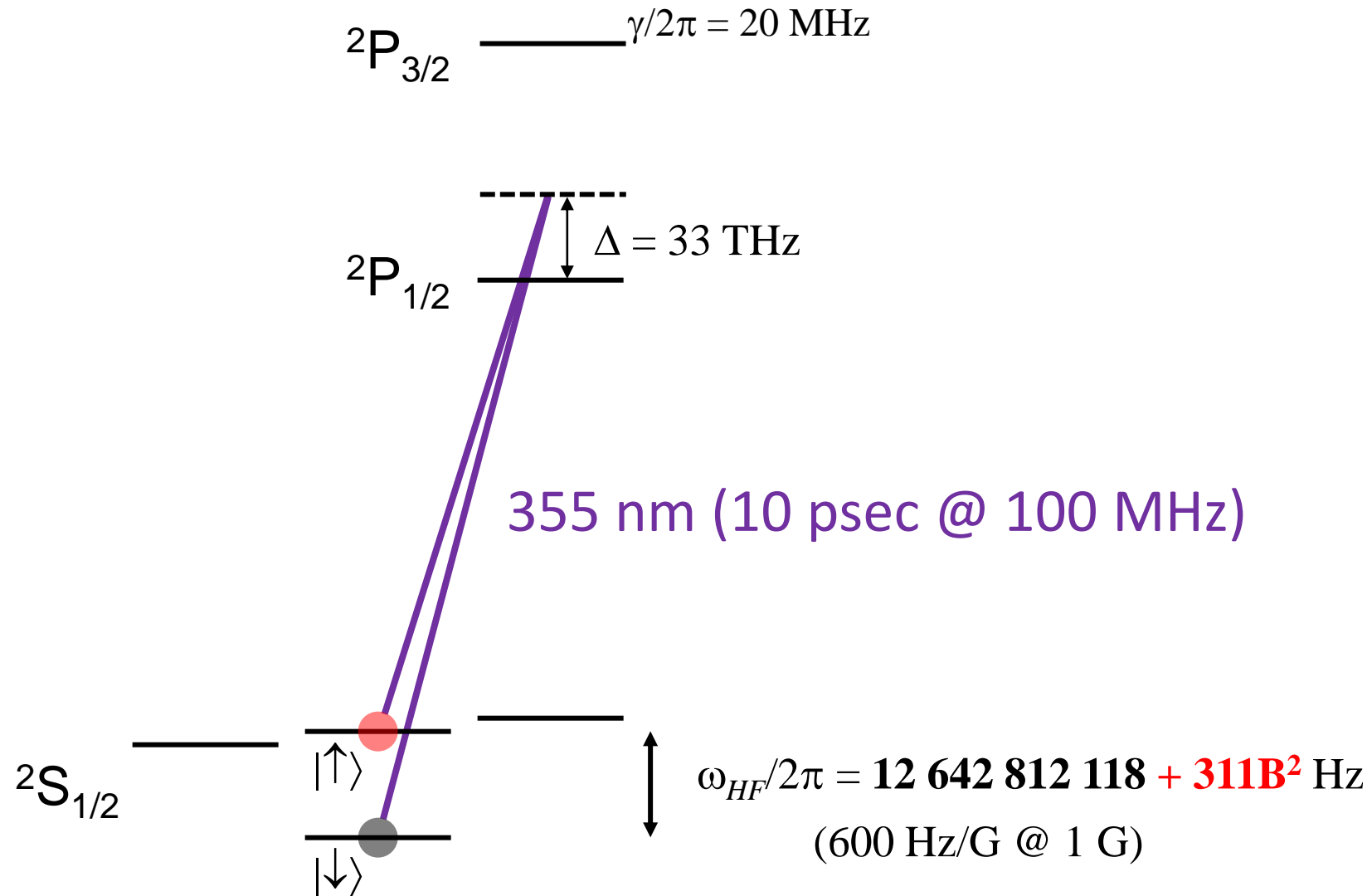
# $^{171}\text{Yb}^+$ spin detection



$$\omega_{HF}/2\pi = 12\,642\,812\,118 + 311B^2\text{ Hz}$$

( $600\text{ Hz/G}$  @  $1\text{ G}$ )

# $^{171}\text{Yb}^+$ spin manipulation



# Entangling Trapped Ion Spins



“dipole-dipole coupling”

$$\Delta E = \frac{e^2}{\sqrt{r^2 + \delta^2}} - \frac{e^2}{r} \approx -\frac{(e\delta)^2}{2r^3}$$

$\delta \sim 10 \text{ nm}$   
 $e\delta \sim 500 \text{ Debye}$

$$\begin{aligned} |\downarrow\downarrow\rangle &\rightarrow |\downarrow\downarrow\rangle \\ |\downarrow\uparrow\rangle &\rightarrow e^{-i\varphi} |\downarrow\uparrow\rangle \\ |\uparrow\downarrow\rangle &\rightarrow e^{-i\varphi} |\uparrow\downarrow\rangle \\ |\uparrow\uparrow\rangle &\rightarrow |\uparrow\uparrow\rangle \end{aligned}$$

$$\varphi = \frac{\Delta E t}{\hbar} = \frac{e^2 \delta^2 t}{2\hbar r^3} = \frac{\pi}{2} \quad \text{for full entanglement}$$

Cirac and Zoller (1995)  
 Mølmer & Sørensen (1999)  
 Solano, de Matos Filho, Zagury (1999)  
 Milburn, Schneider, James (2000)

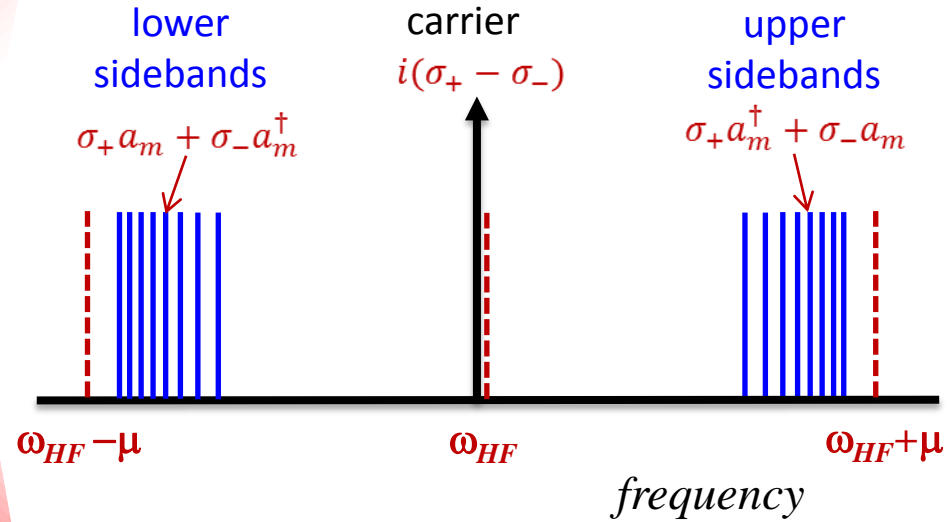
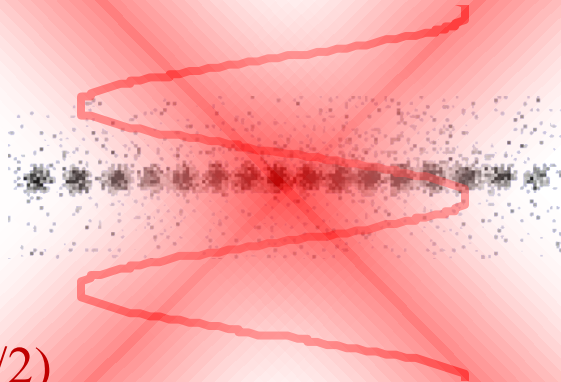


# Resonant spin-dependent force

Raman  
beatnotes:

$$\omega_{HF} \pm \mu$$

$$\omega_{HF} (\Delta\phi = \pi/2)$$



$$H = \Delta k \Omega \sum_{i,m} \hat{\sigma}_x^{(i)} x_0^m b_i^m [a_m^\dagger e^{i(\mu - \omega_m)t} + a_m e^{-i(\mu - \omega_m)t}]$$

normal mode eigenvectors  
(ion  $i$  mode  $m$ )

$$H_{eff} = \sum_{i < j} J_{i,j} \hat{\sigma}_x^{(i)} \hat{\sigma}_x^{(j)} + B \sum_i \hat{\sigma}_y^{(i)}$$

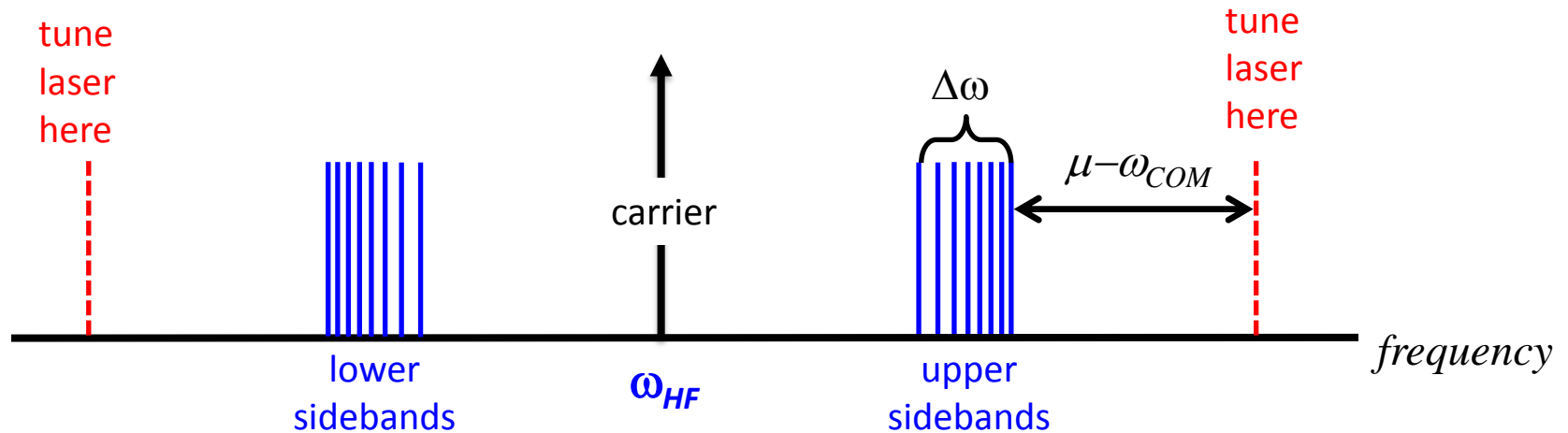
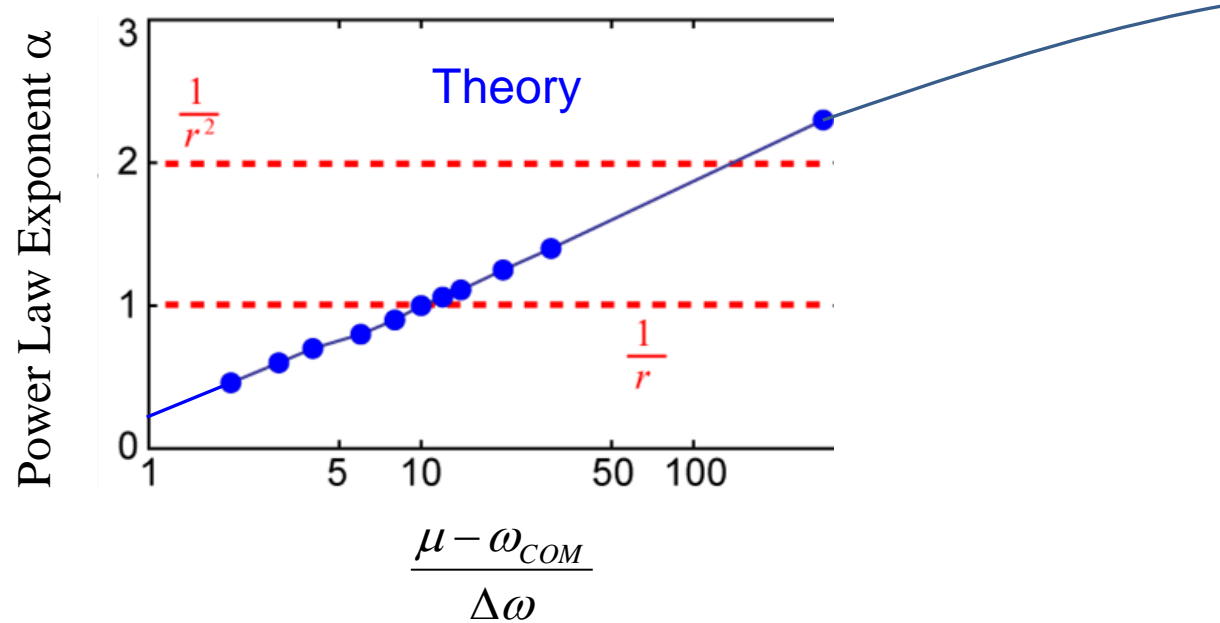
$$J_{i,j} = \Omega^2 \left( \frac{\hbar \Delta k^2}{2m} \right) \sum_m \frac{b_i^m b_j^m}{\mu^2 - \omega_m^2}$$

control

# Control of interaction range

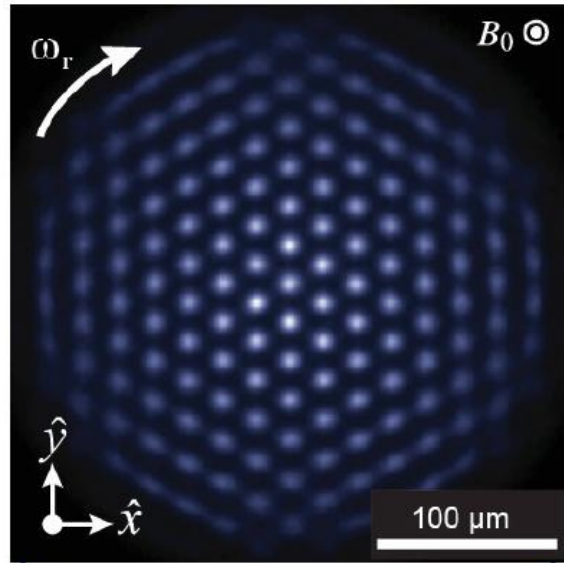
$$J_{i,j} = \Omega^2 \frac{\hbar(\Delta k)^2}{2m} \sum_k \frac{b_i^k b_j^k}{\mu^2 - \omega_k^2}$$

$$\sim + \frac{1}{r^\alpha}$$

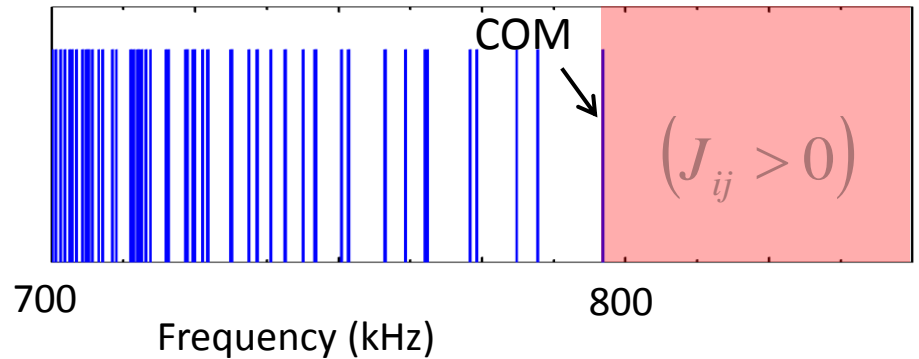
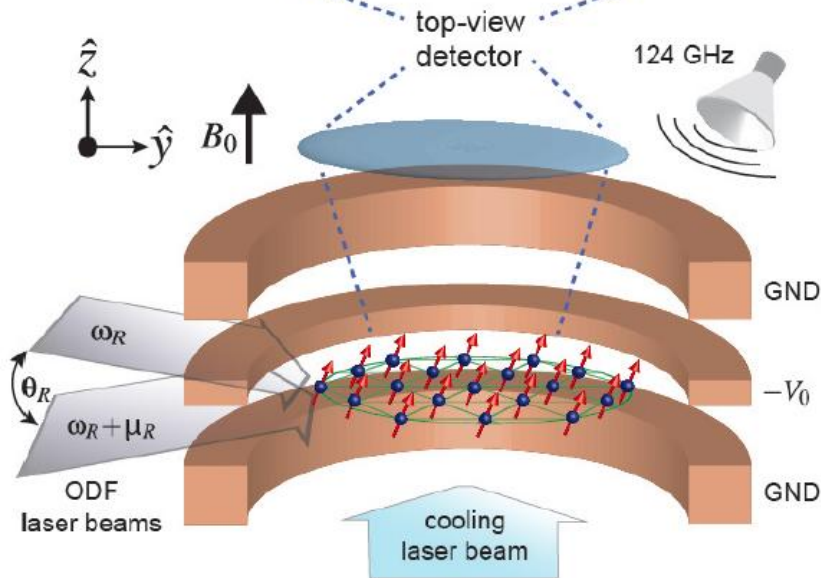
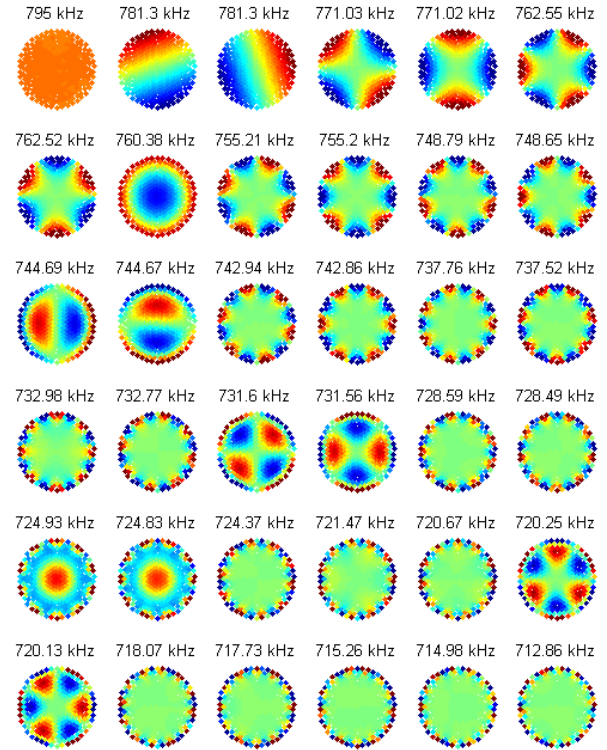


# 2D Penning trap

John Bollinger (NIST Boulder)



Axial mode spectrum



# **T**rapped Ion Spin Hamiltonian Engineering

## **G**round states and Adiabatic Protocols

### **D**ynamics

Direct Many-Body Spectroscopy

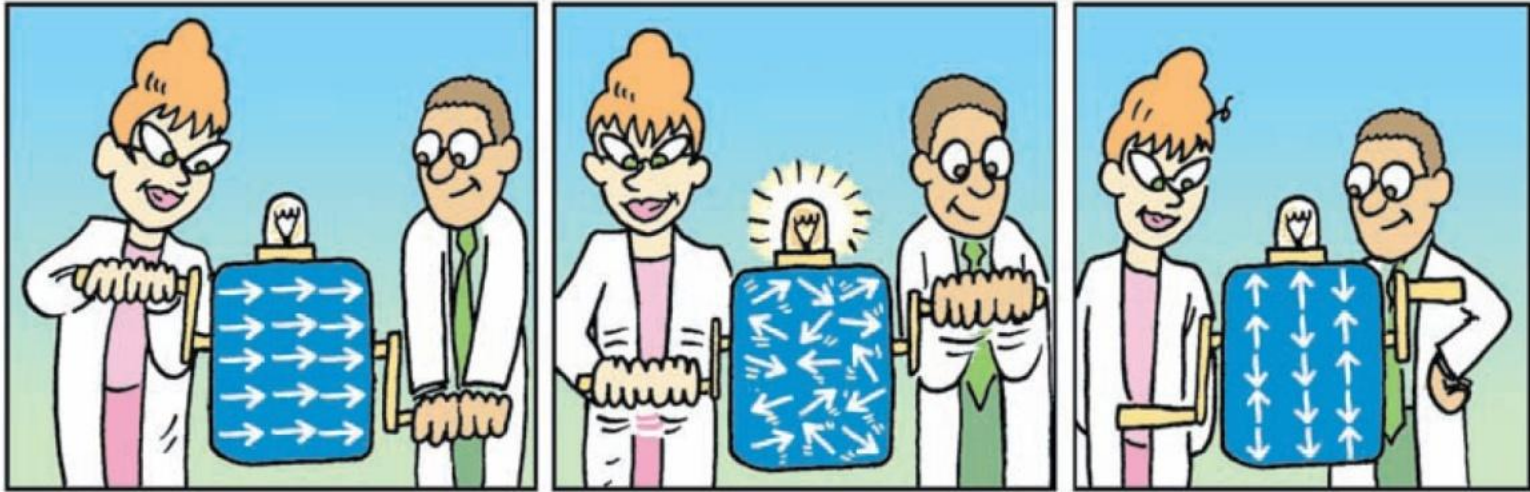
Lieb-Robinson Bounds

### **M**any-Body Thermalization/Localization?

### **S**pin-1

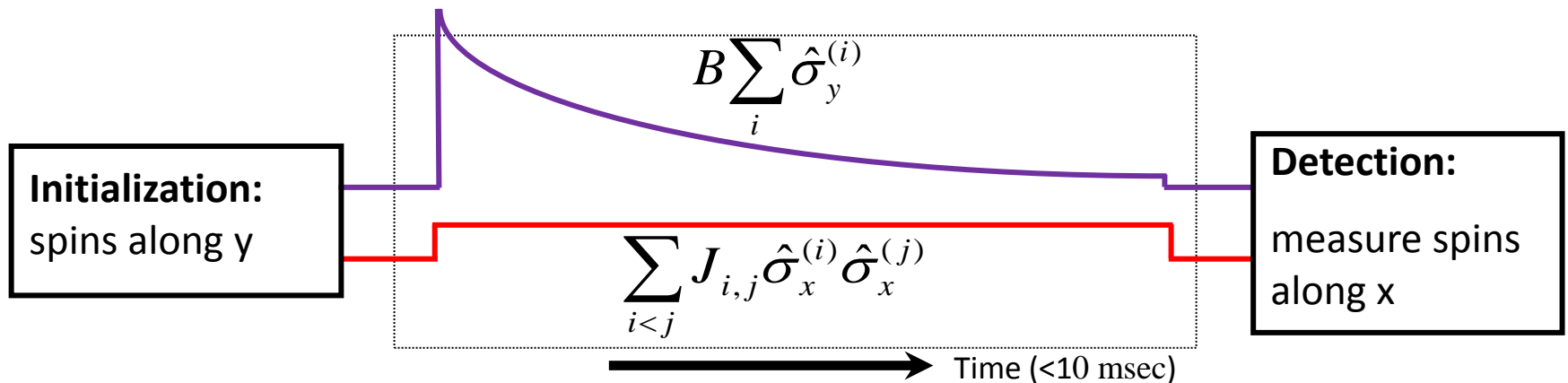
### **F**uture

# Equilibrium: Adiabatic Quantum Simulation



from S. Lloyd, Science **319**, 1209 (2008)

$$H_{eff} = \sum_{i < j} J_{i,j} \hat{\sigma}_x^{(i)} \hat{\sigma}_x^{(j)} + B \sum_i \hat{\sigma}_y^{(i)}$$

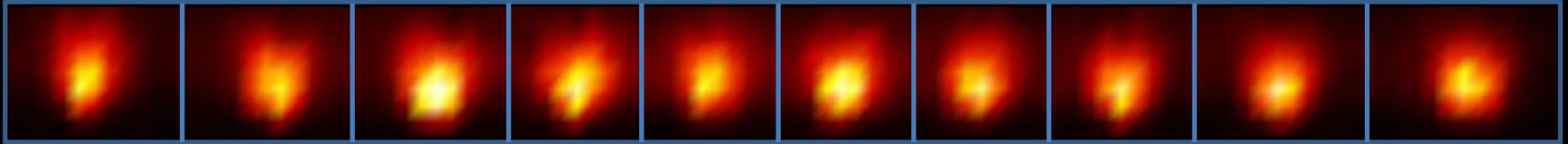




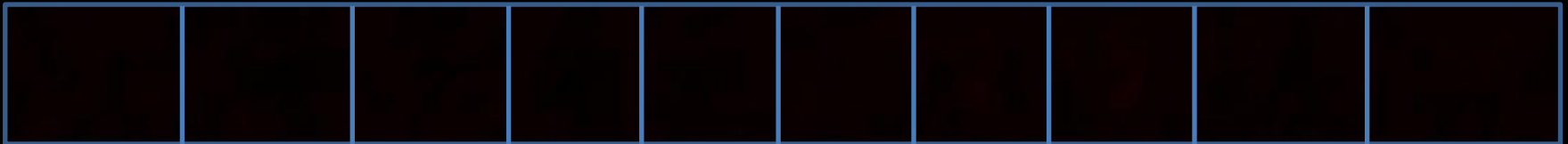
# Antiferromagnetic Néel order of N=10 spins

2600 runs,  $\alpha=1.12$

All in state  $\uparrow$

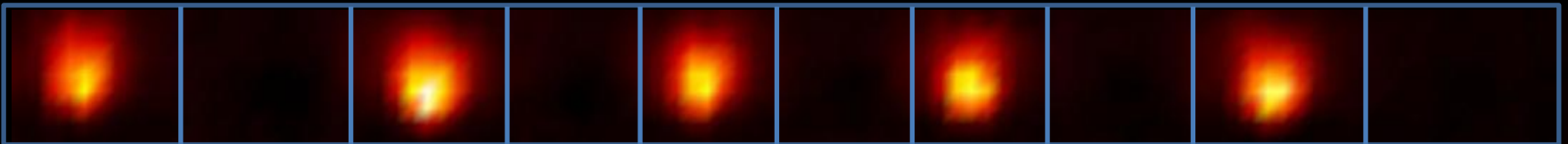


All in state  $\downarrow$

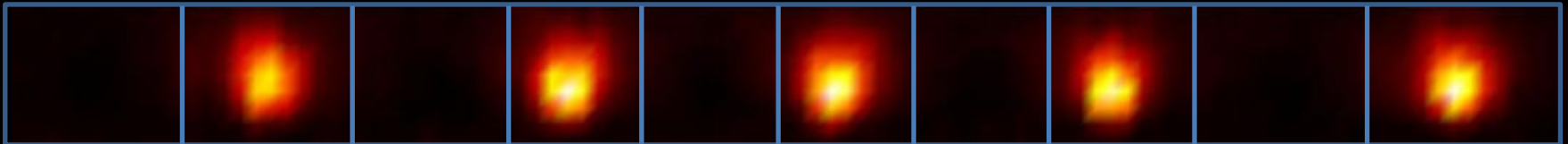


AFM ground state order

222 events



219 events



441 events out of 2600 = 17%

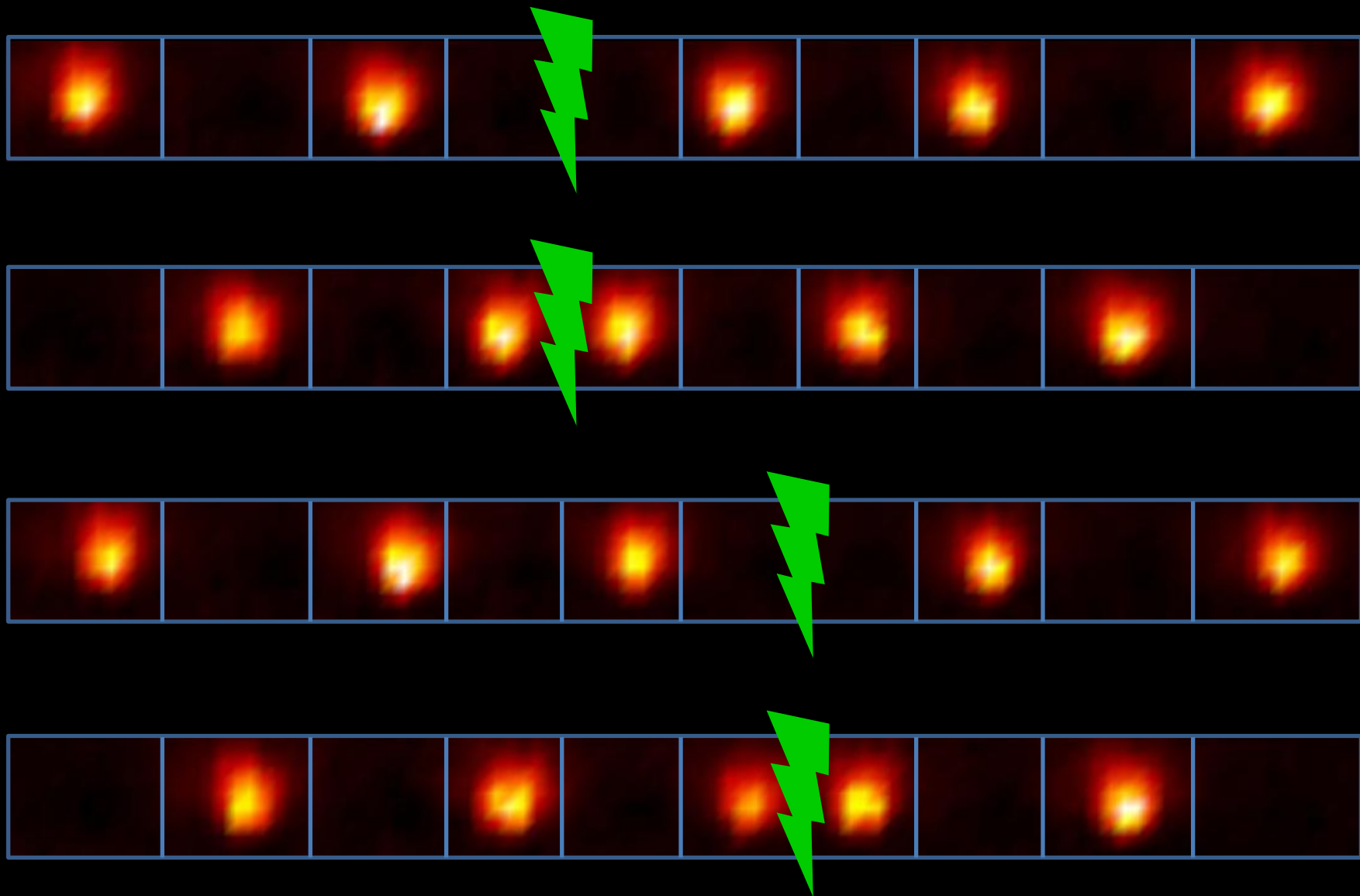
R. Islam et al., Science

Prob of any state at random =  $2 \times (1/2^{10}) = 0.2\%$

340, 583 (2013)

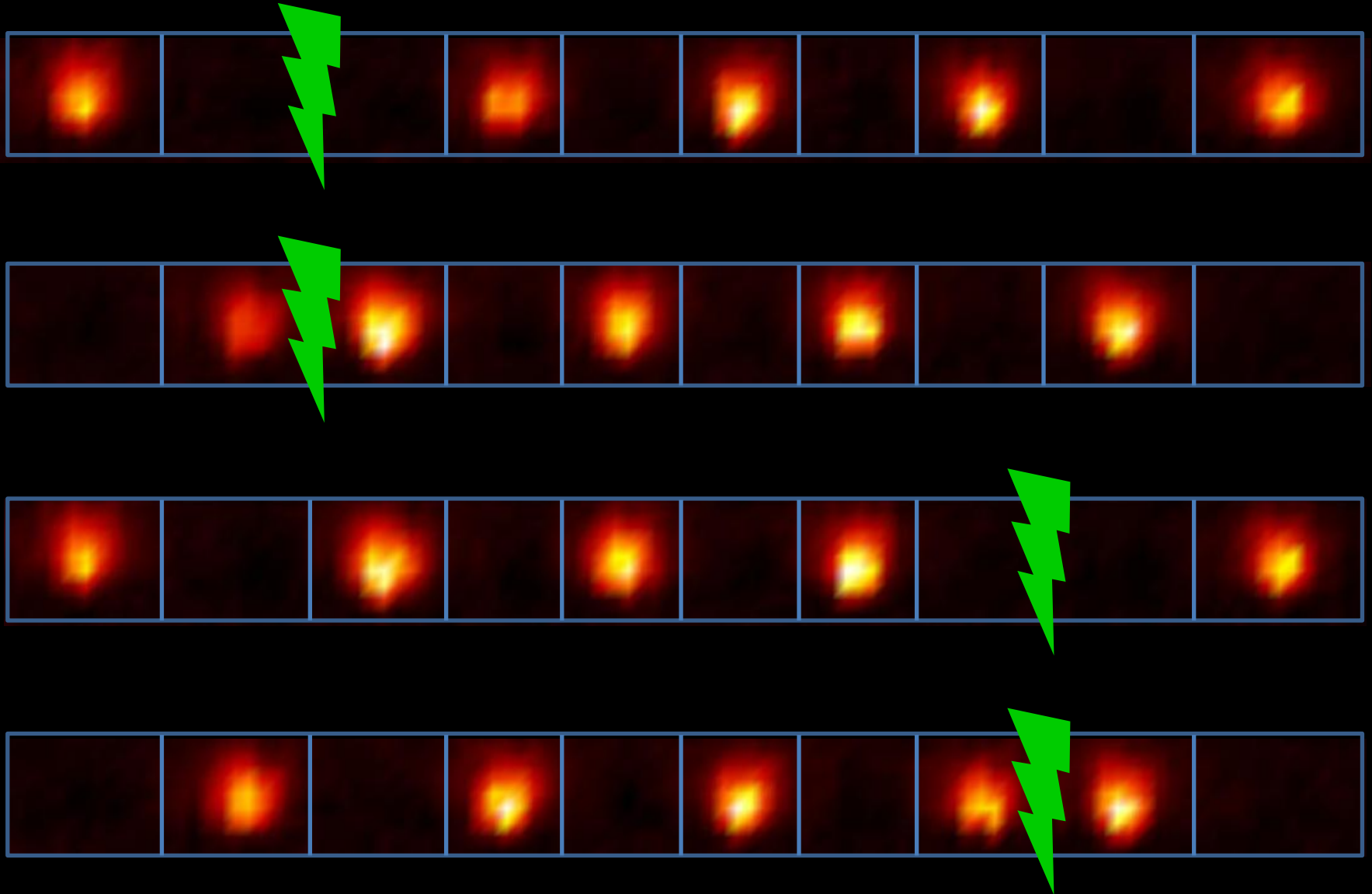
# First Excited States

(Pop. ~2% each)

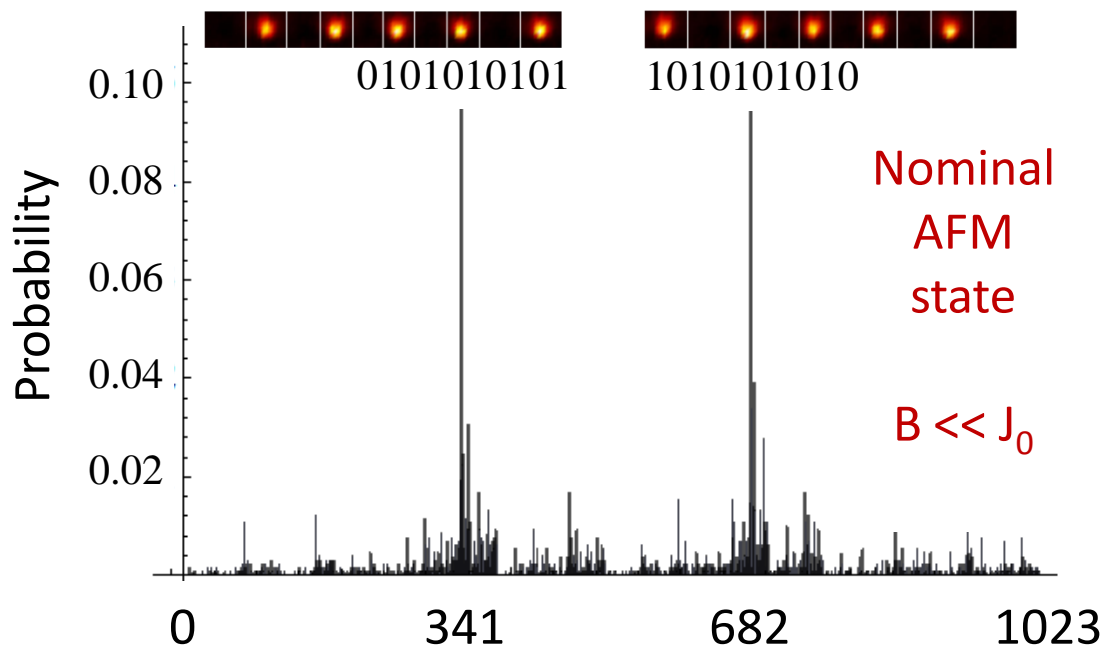
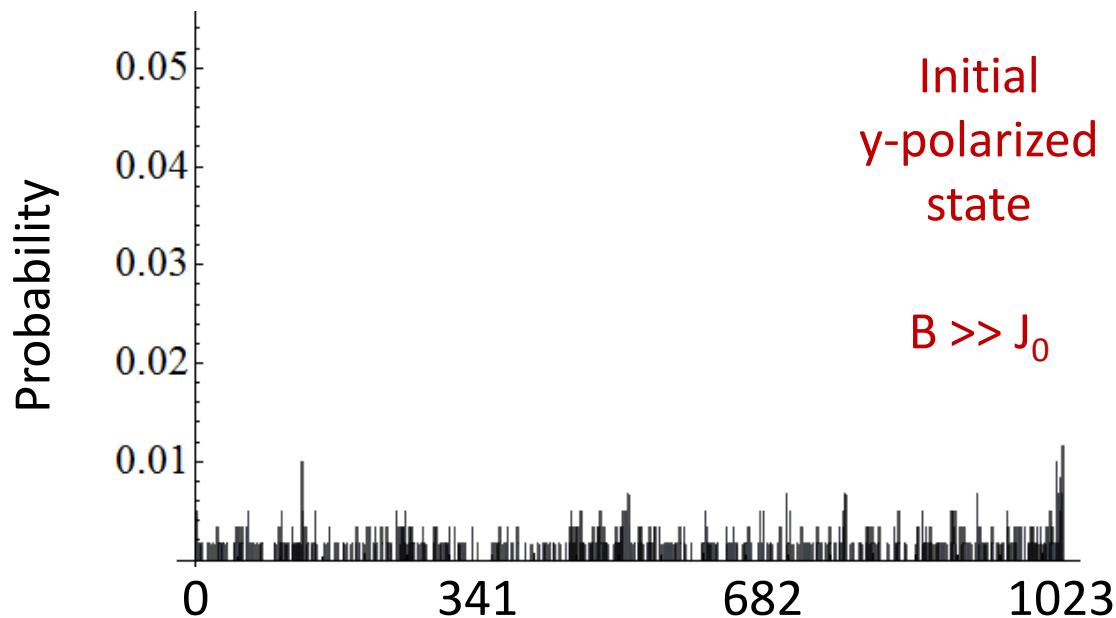


# Second Excited States

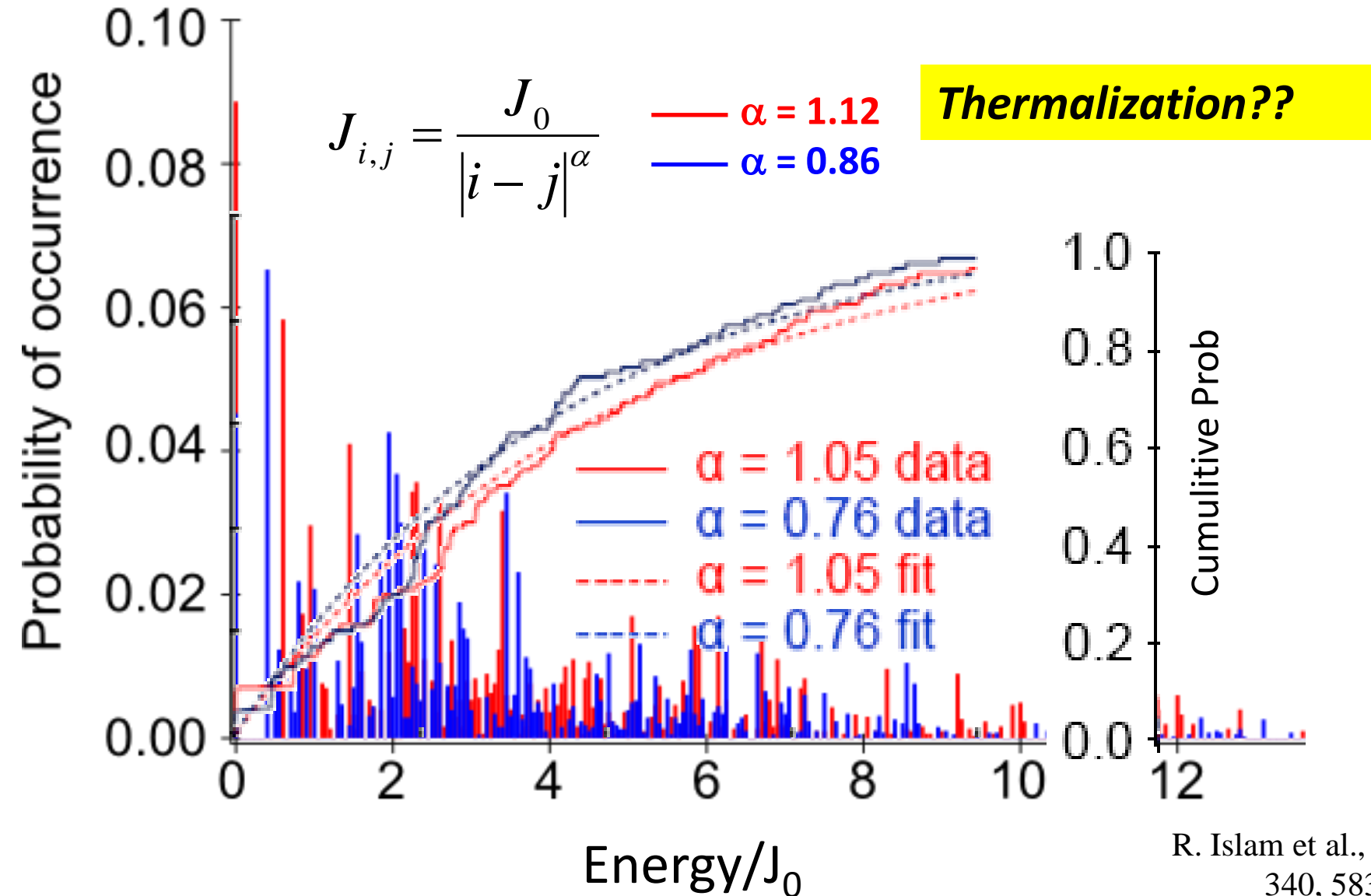
(Pop. ~1% each)



# Distribution of all $2^{10} = 1024$ states

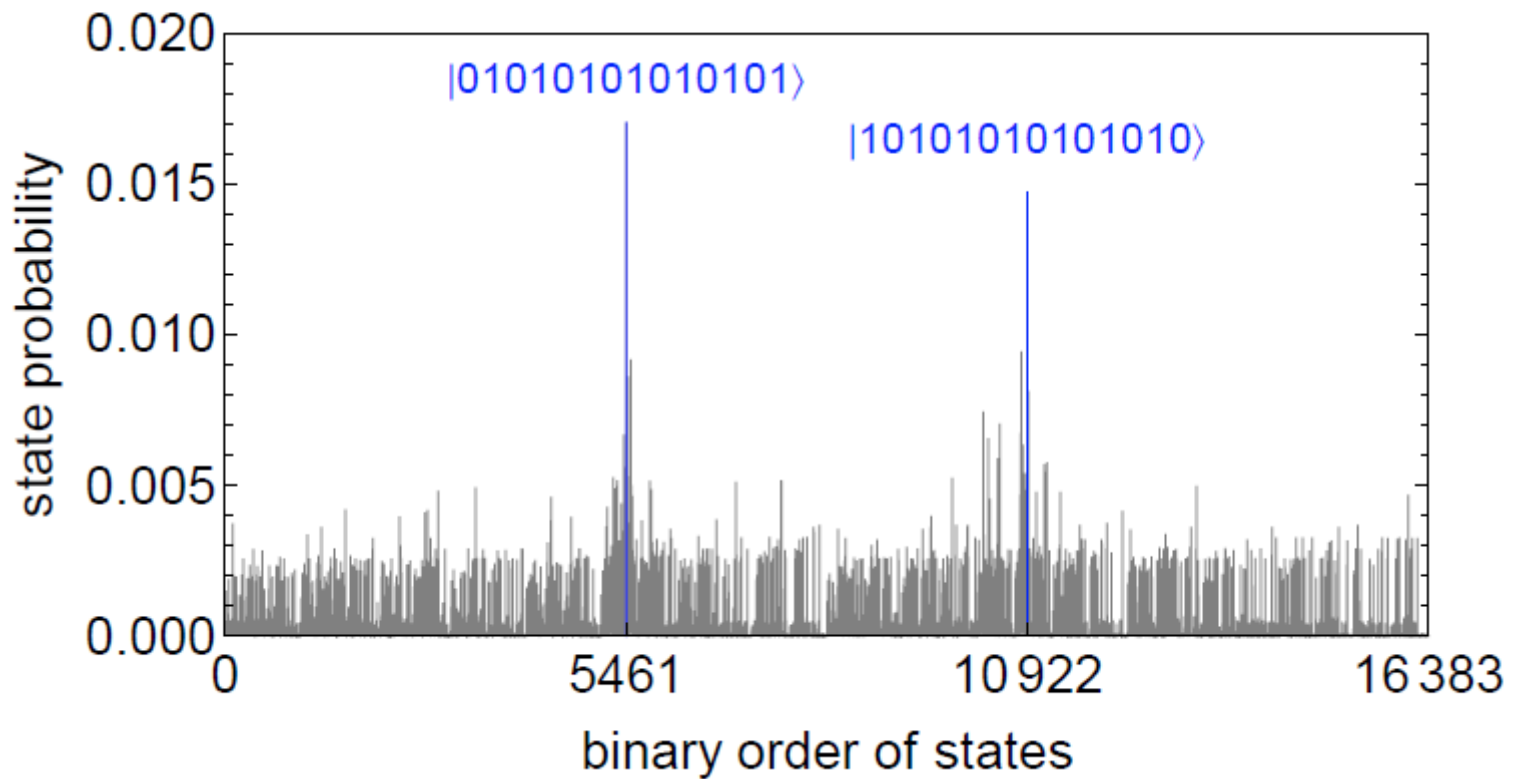
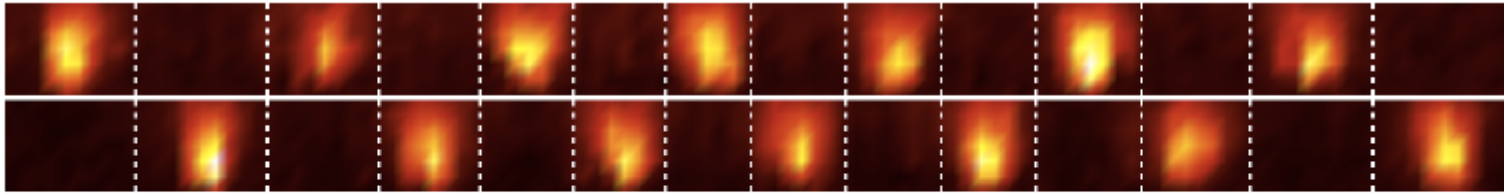


# Distribution of states ordered by energy (N=10)

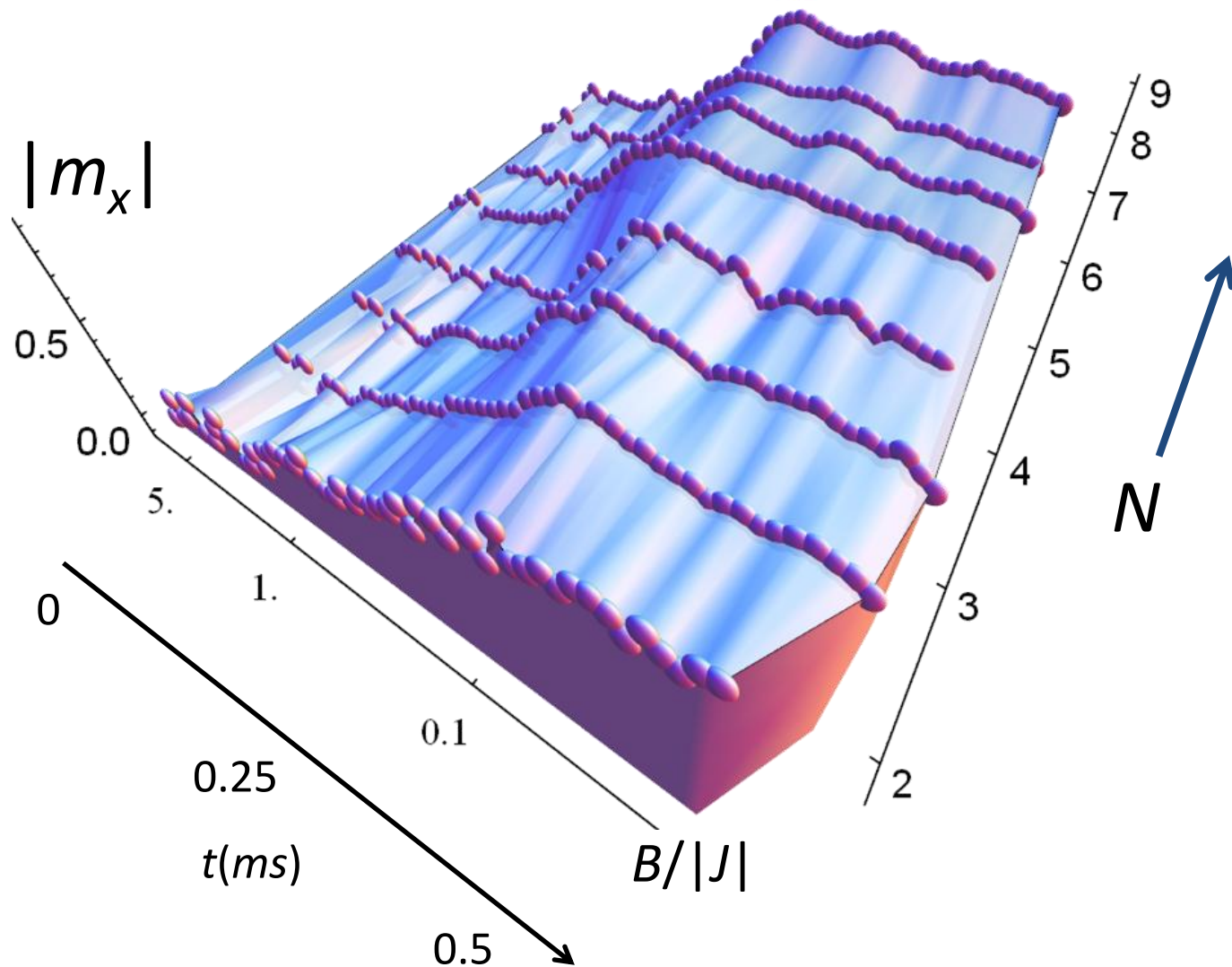




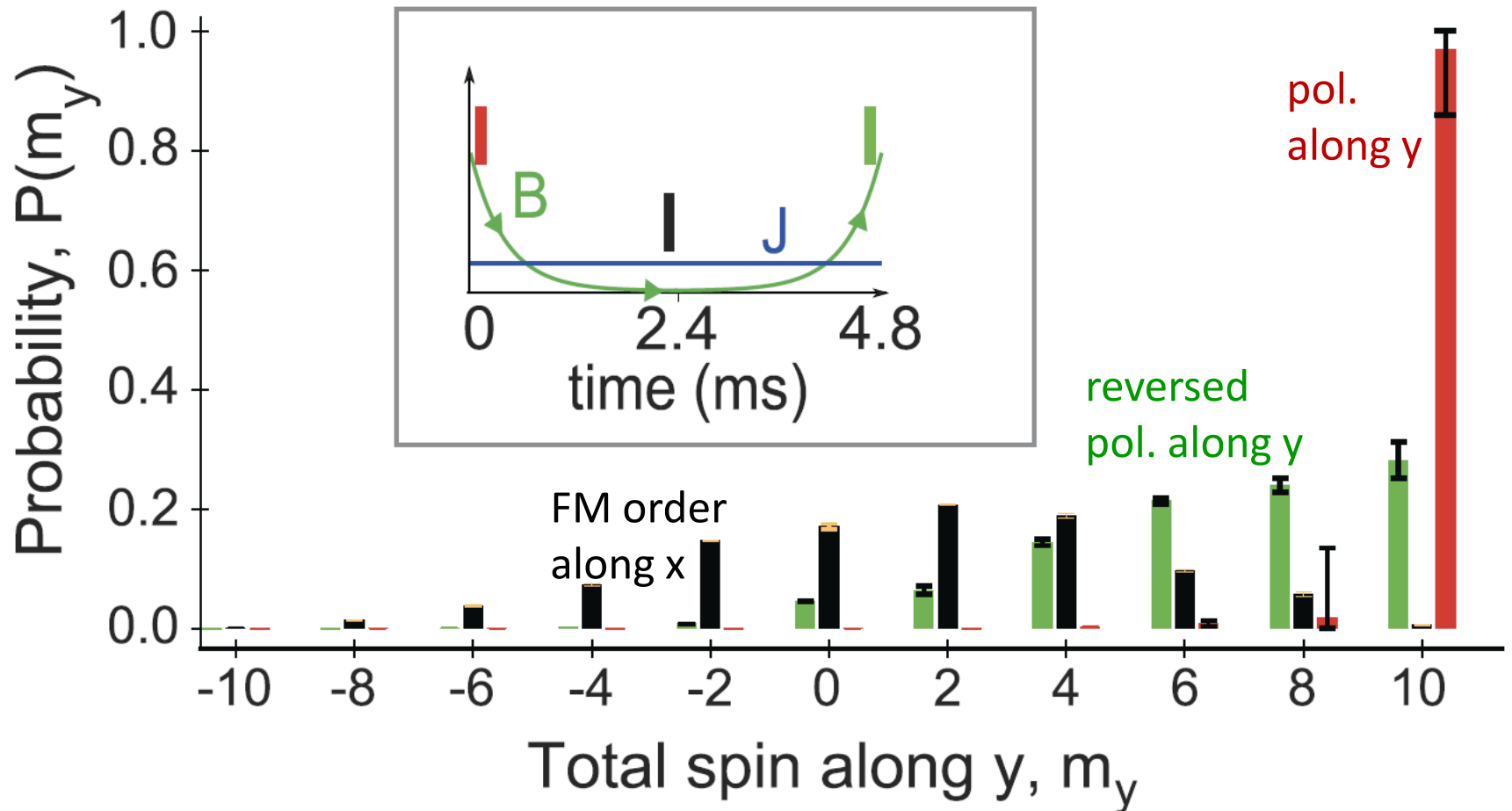
# AFM order of N=14 spins (16,384 configurations)



# Ferromagnet order vs. # spins N



# Ferromagnetic reversal of evolution (N=10)

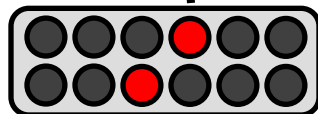
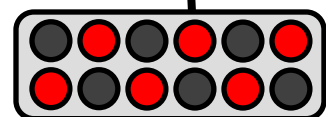
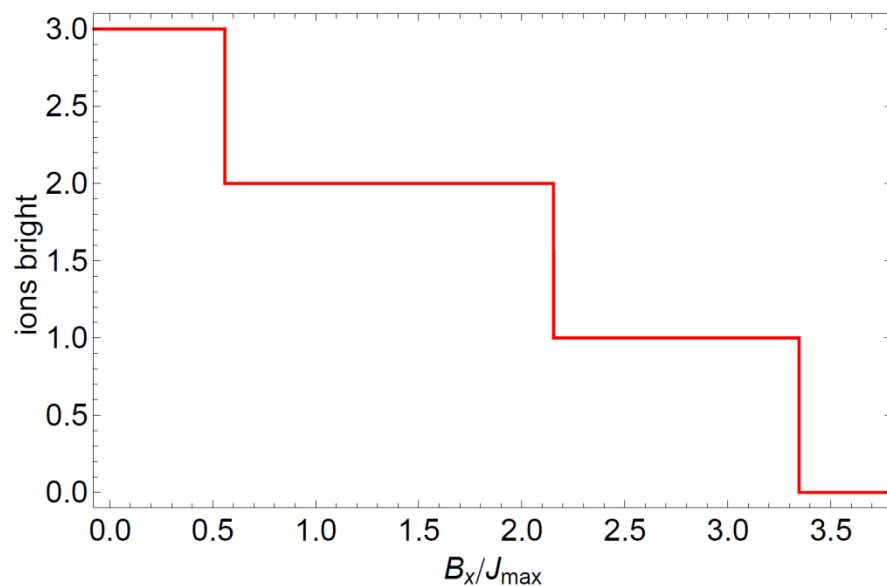
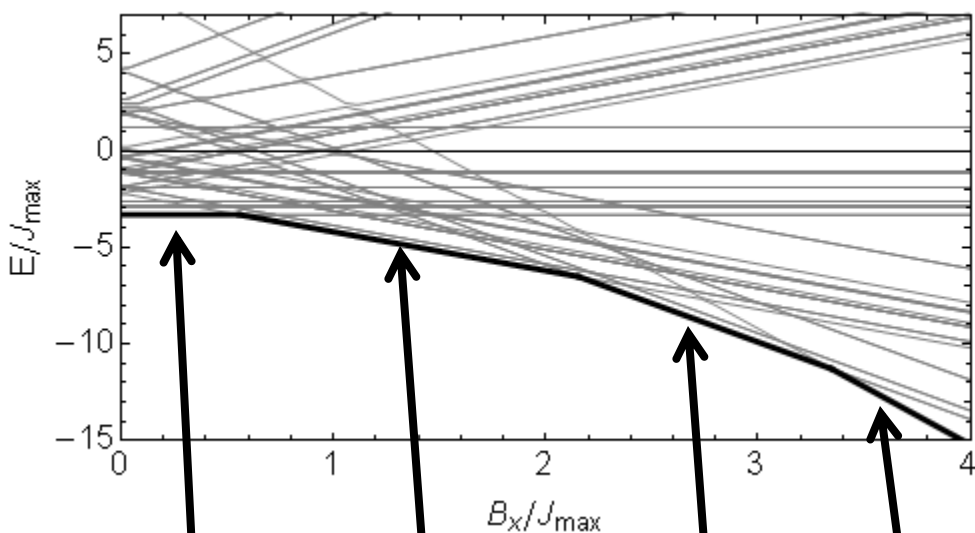




$\langle m_y \rangle$  returns to 70% of its initial value

# AFM Ising Model with axial Field

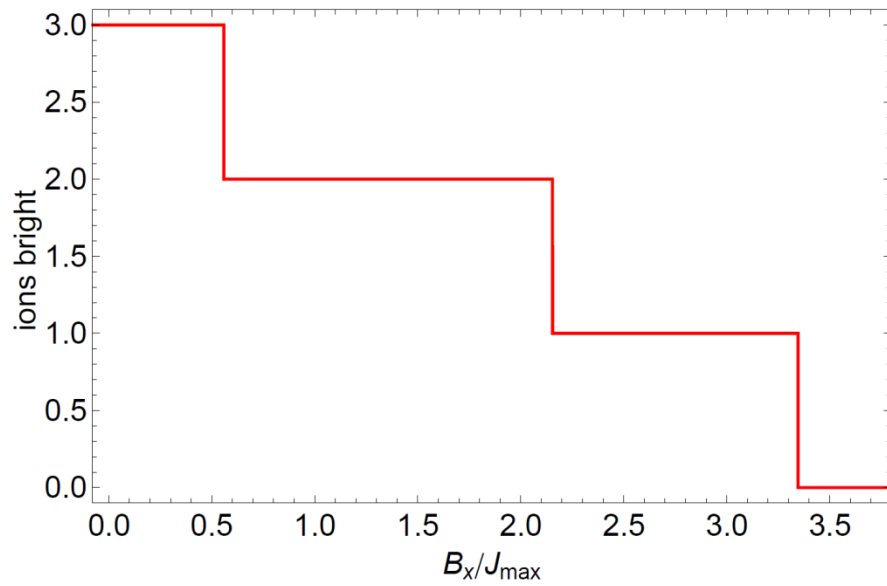
At  $B_y = 0$ :

$$H = \sum_{i \neq j} J_x^{i,j} \hat{\sigma}_x^{(i)} \hat{\sigma}_x^{(j)} + \cancel{B_y(t) \sum_i \hat{\sigma}_y^{(i)}} + B_x \sum_i \hat{\sigma}_x^{(i)}$$



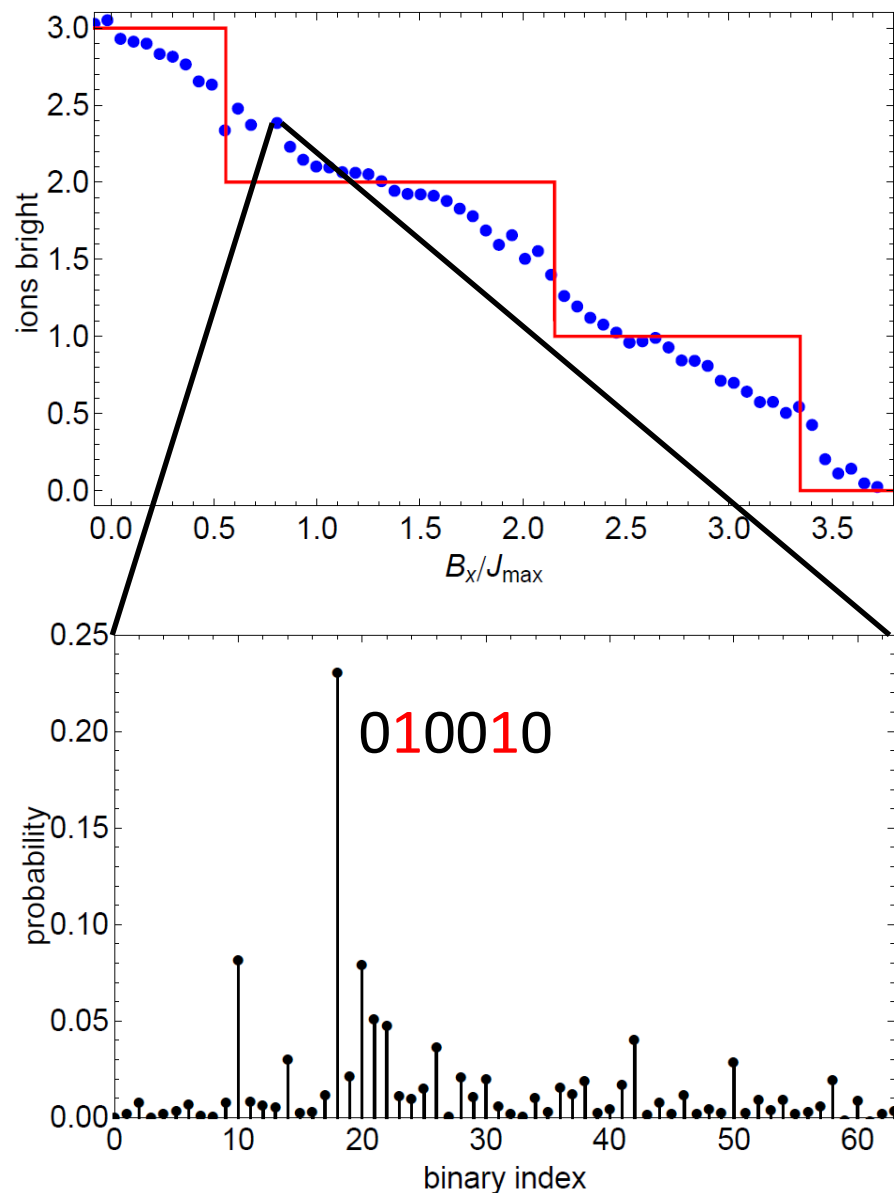
 =  $\uparrow$   
 =  $\downarrow$

# AFM Ising Model with axial field + tranverse field ramp: 6 ions

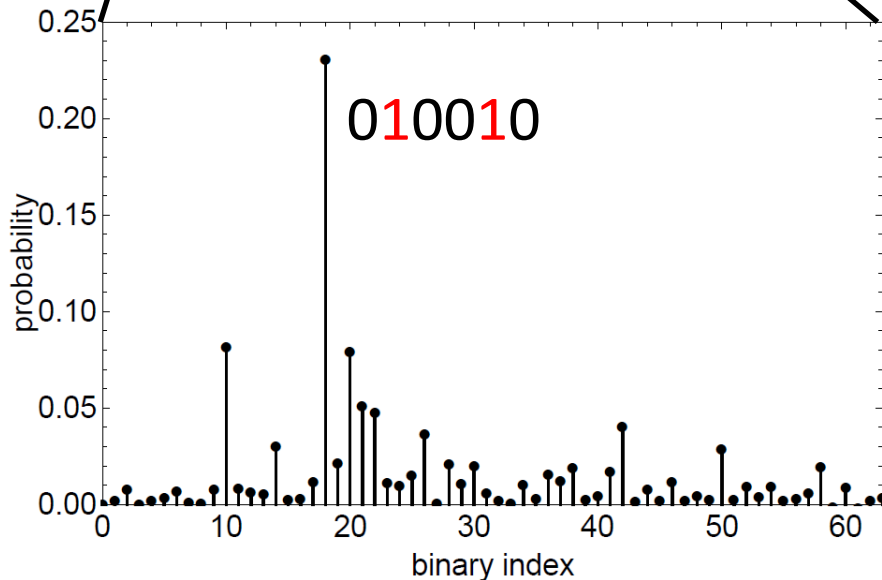
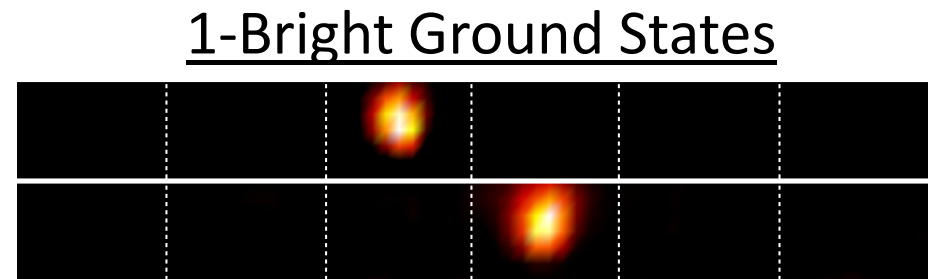
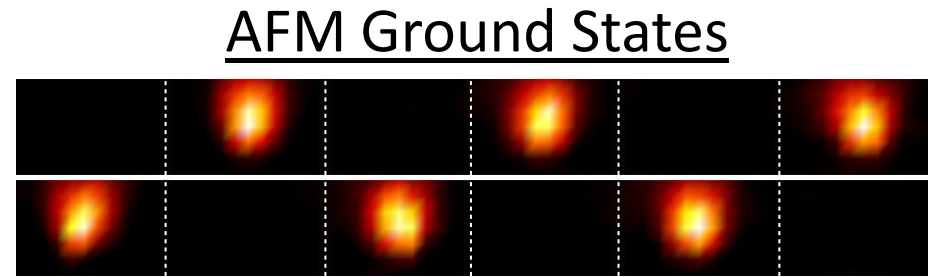
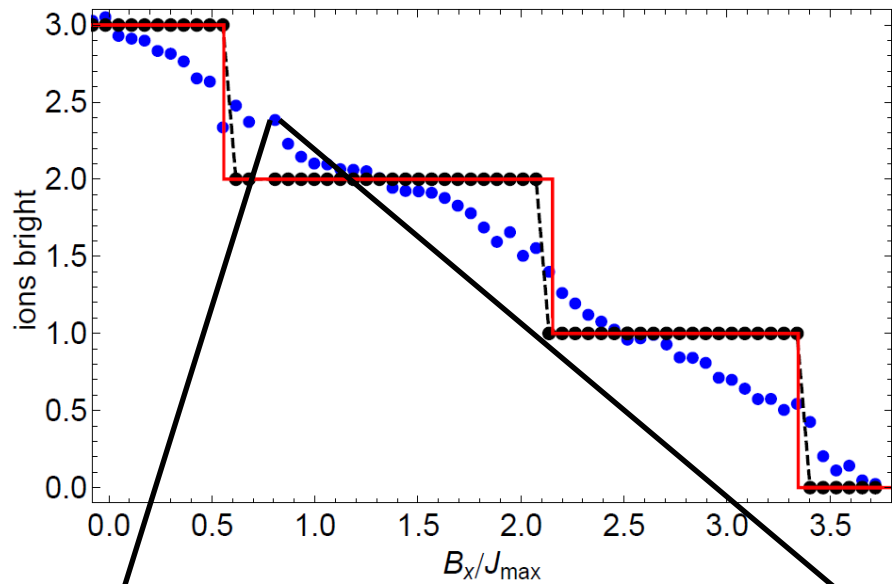




# AFM Ising Model with axial field + tranverse field ramp: 6 ions

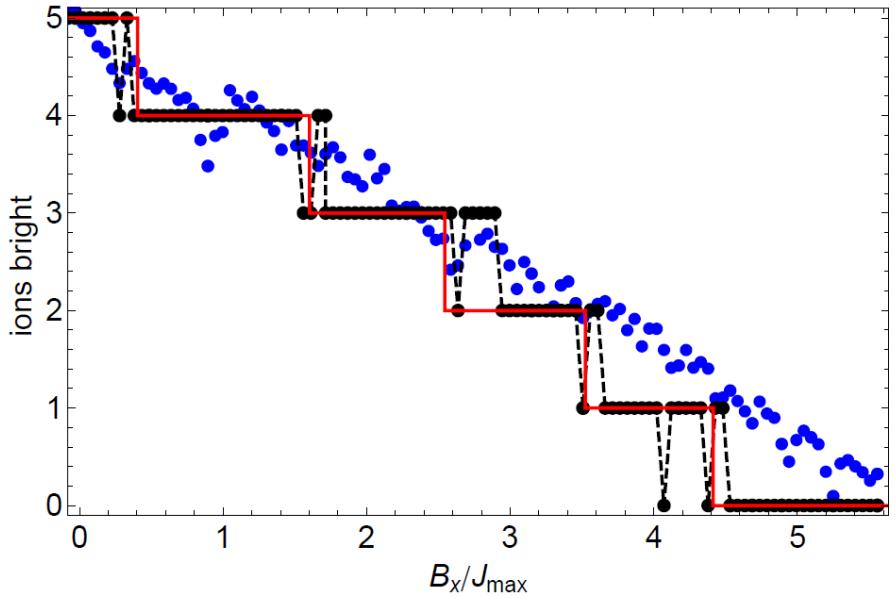


# AFM Ising Model with axial field + tranverse field ramp: 6 ions

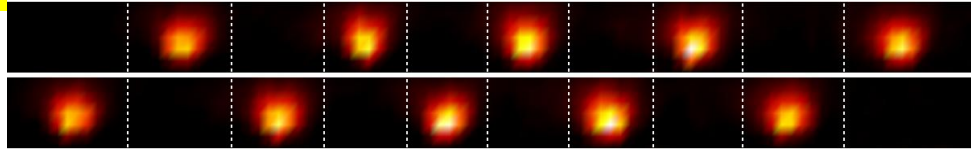


# AFM Ising Model with axial field + tranverse field ramp: 10 ions

$$H = \sum_{i \neq j} J_x^{i,j} \hat{\sigma}_x^{(i)} \hat{\sigma}_x^{(j)} + B_y(t) \sum_i \hat{\sigma}_y^{(i)} + B_x \sum_i \hat{\sigma}_x^{(i)}$$



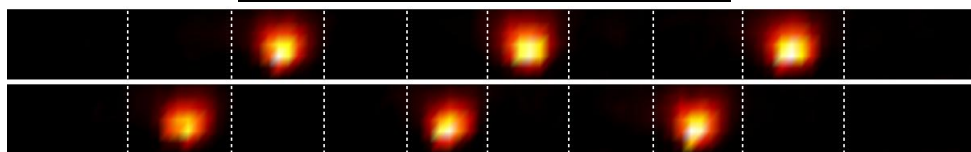
5-Bright (AFM) Ground States



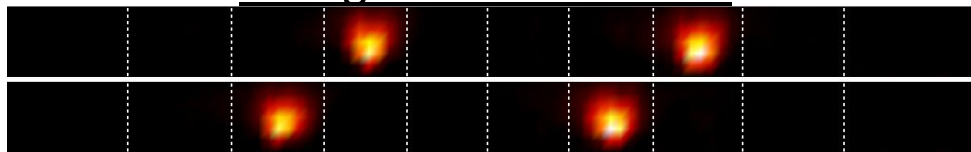
4-Bright Ground States



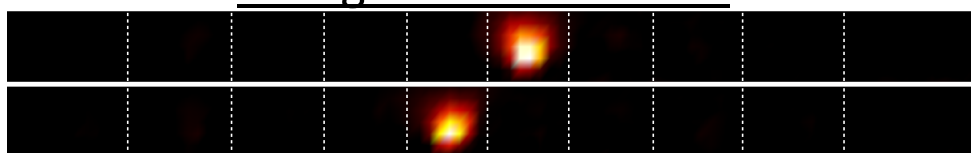
3-Bright Ground States



2-Bright Ground States



1-Bright Ground States



0-Bright Ground State



System exhibits a complete devil's staircase for  $N \rightarrow \infty$

**T**rapped Ion Spin Hamiltonian Engineering

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Direct Many-Body Spectroscopy

Lieb-Robinson Bounds

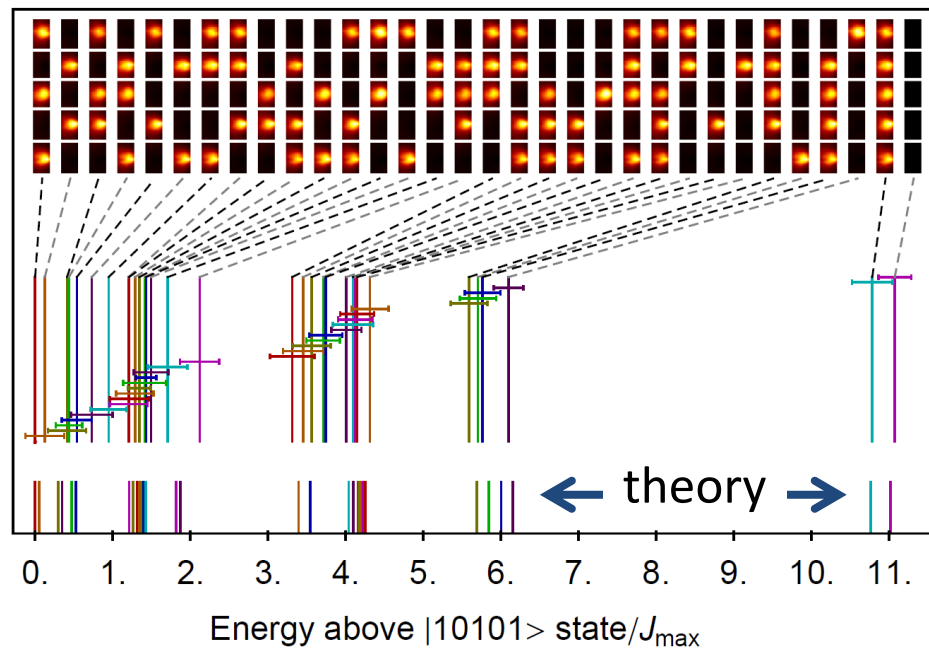
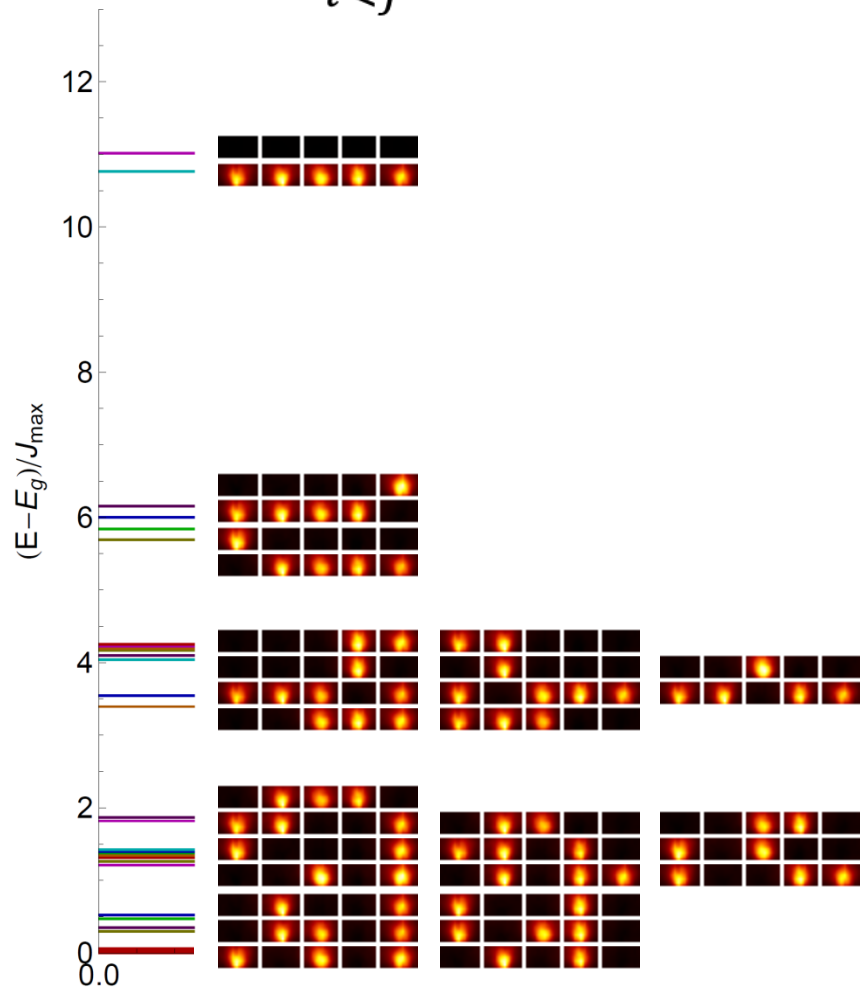
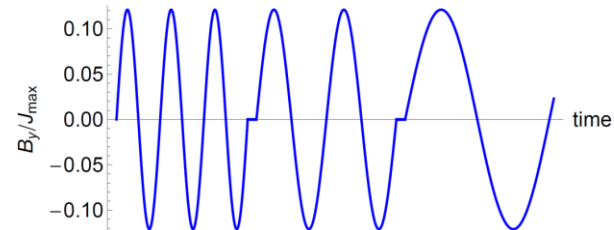
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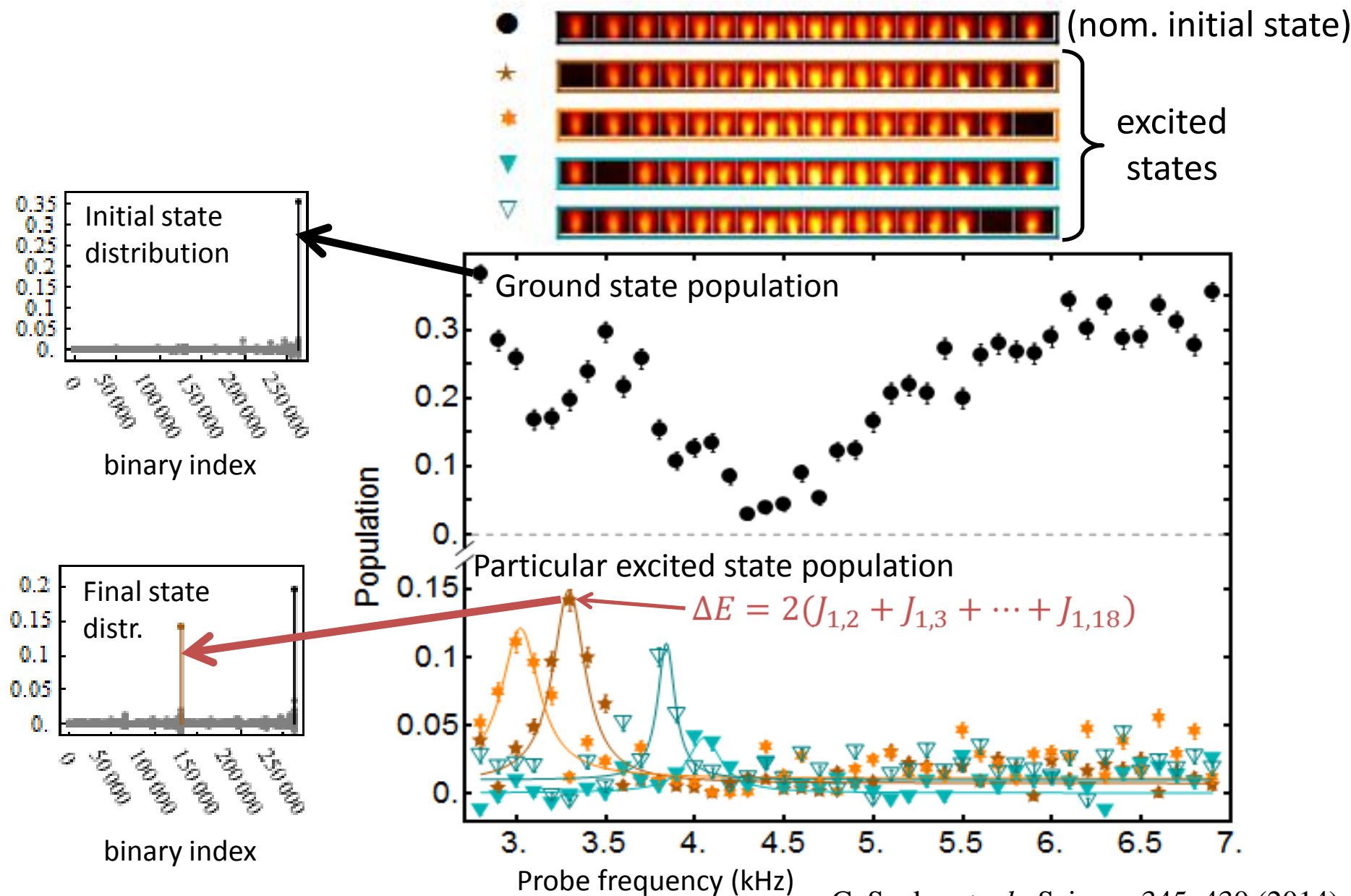
# Coherent Imaging Spectroscopy (N=5 spins)

$$H = \sum_{i < j} J_{ij} \sigma_x^i \sigma_x^j$$



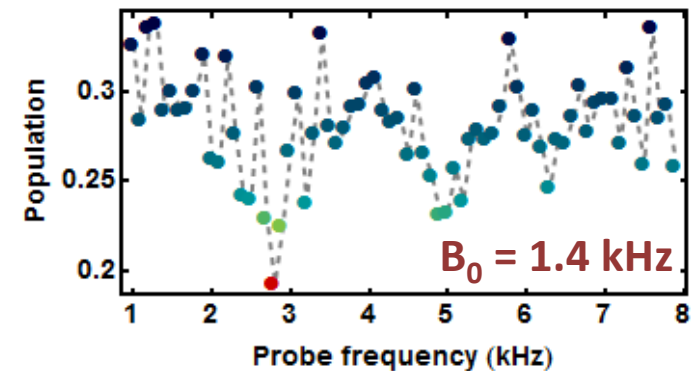
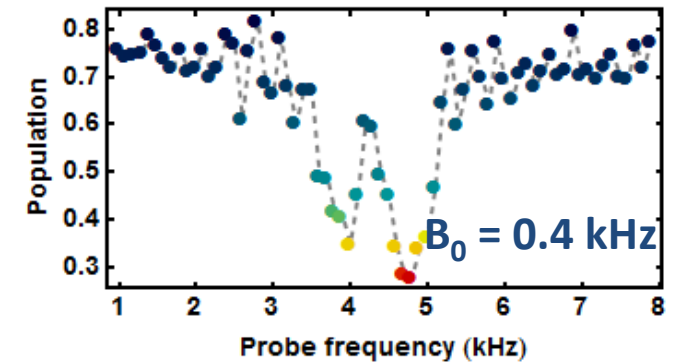
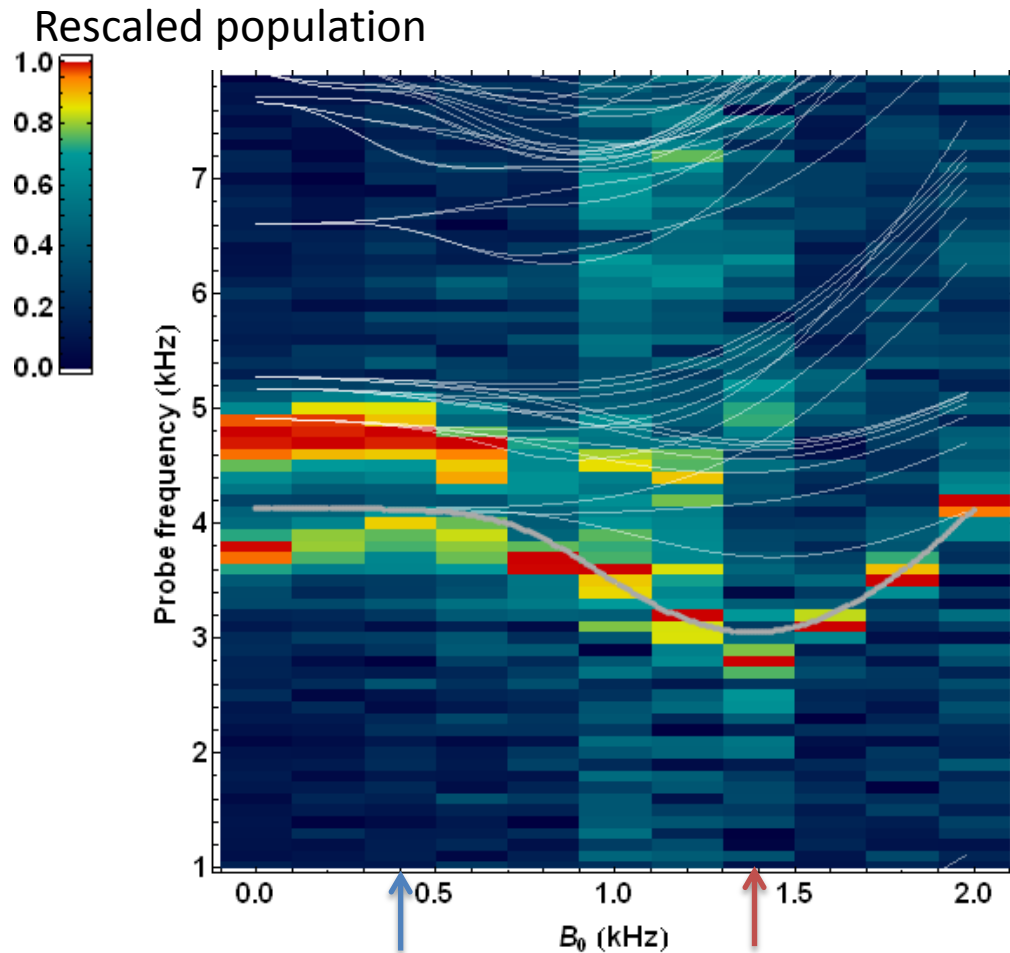


# Coherent Imaging Spectroscopy (N=18 spins)



# Coherent Imaging Spectroscopy: Critical Gap (N=8 spins)

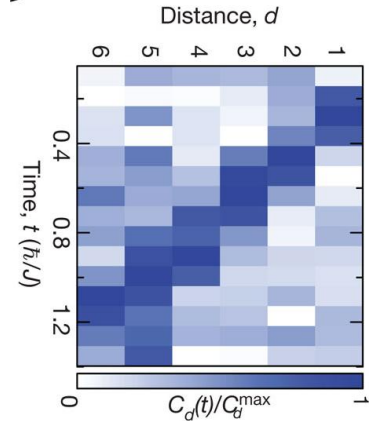
$$H = \sum_{i < j} J_{ij} \sigma_x^i \sigma_x^j + [B_0 + B_y \sin(\omega t)] \sum_i \sigma_y^i$$



# Dynamics: quantum quench

E.H. Lieb and D.W. Robinson, “*The finite group velocity of quantum spin systems,*”  
Commun. Math. Phys. 28, 251–257 (1972).

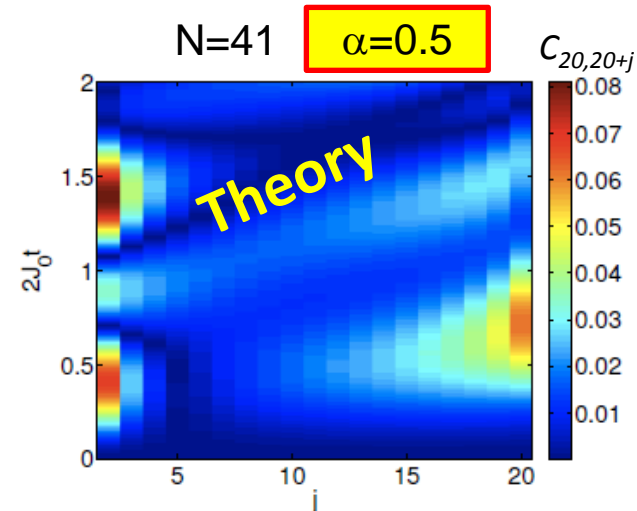
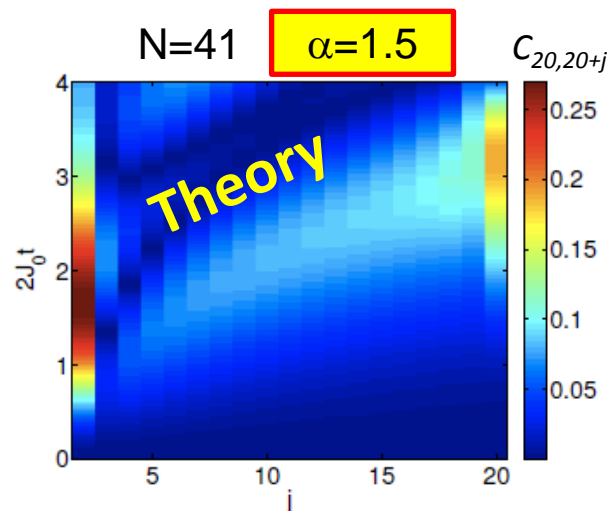
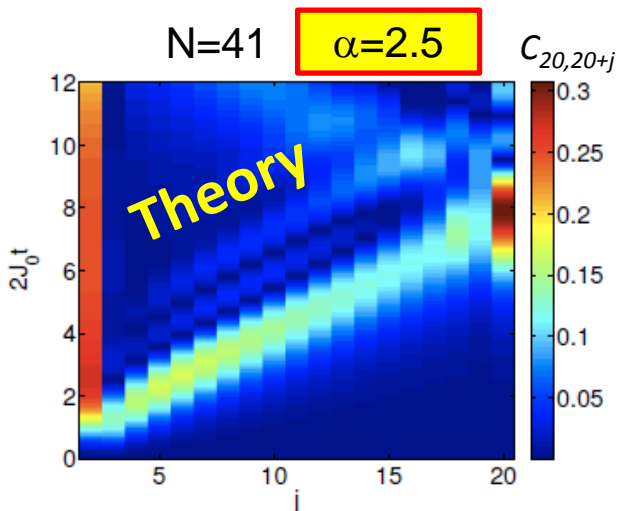
- M. Cheneau, et al., Nature 481, 484 (2012) →
- P. Hauke, et al., PRL 111, 207202 (2013)
- M. Knap, et al., PRL 111, 147205 (2013)
- Z.-X. Gong, et al., PRL 113, 030602 (2014)



## “Global Quench”

- (a) Prepare  $(\downarrow_x + \uparrow_x)^{\otimes N}$  “ $kT = \infty$ ”
- (b) Turn on XX Ising interactions
- (c) Measure correlations  $C_{ij}(t) = \langle \sigma_z^{(i)} \sigma_z^{(j)} \rangle - \langle \sigma_z^{(i)} \rangle \langle \sigma_z^{(j)} \rangle$

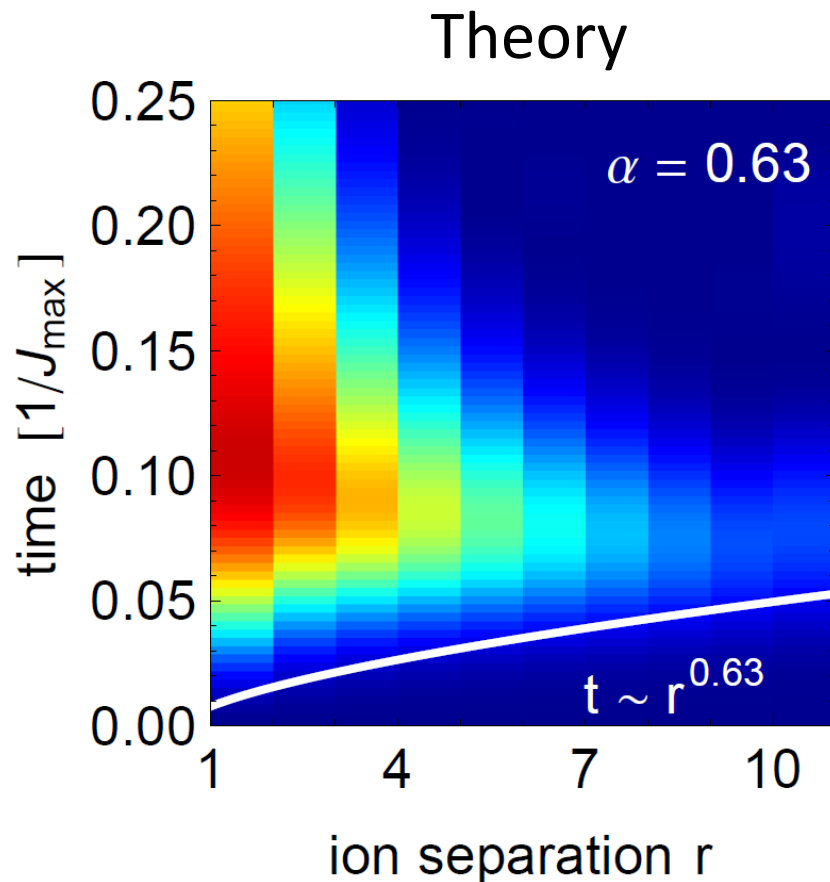
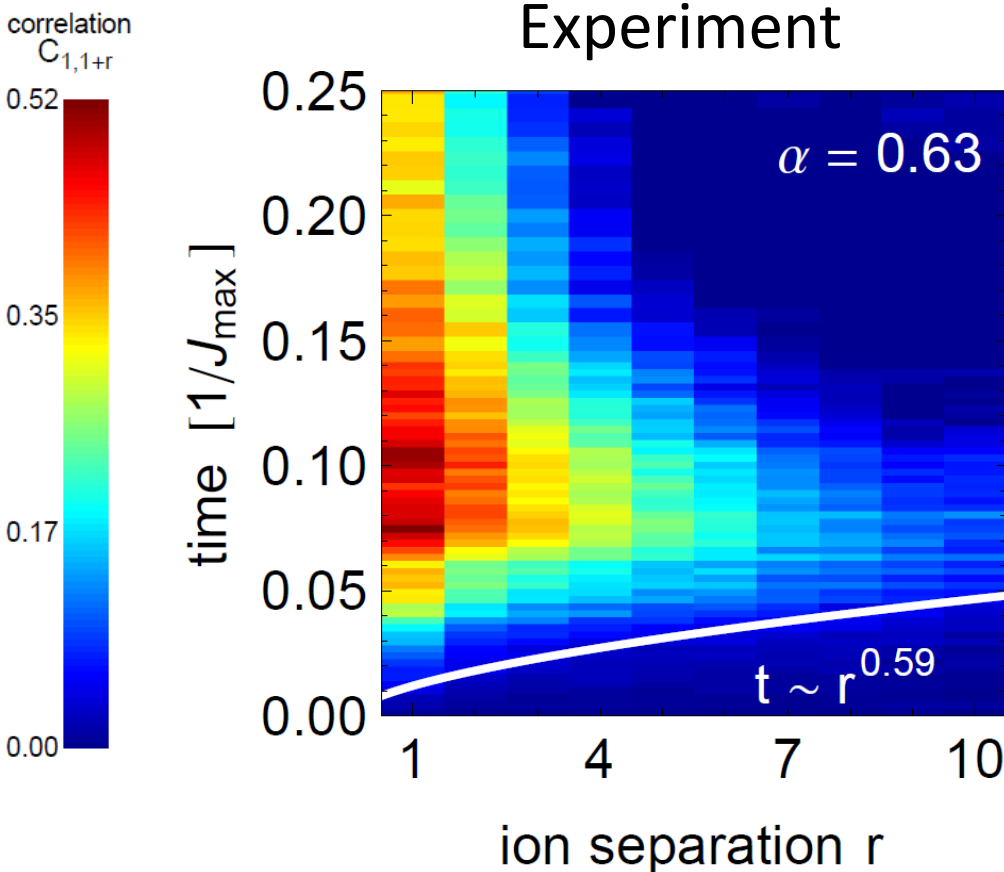
$$J_{i,j} = \frac{J_0}{|i - j|^\alpha}$$



# Long range “Light Cones” (Ising: N=11 spins)

Ising Model  $H_{Ising} = \sum_{i<j} J_{ij} \sigma_x^i \sigma_x^j$

$$J_{ij} \approx \frac{J_0}{r^\alpha}$$



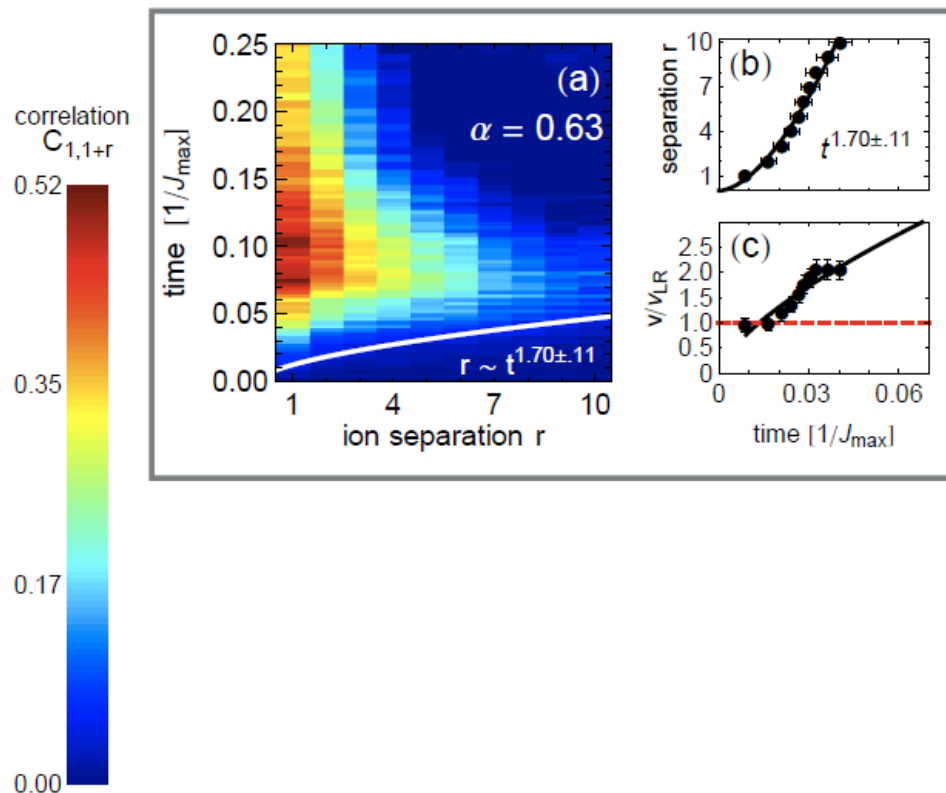
P. Richerme et al., *Nature* **511**, 198 (2014)

P. Jurcevic et al., *Nature* **511**, 202 (2014)

# Long range “Light Cones” (Ising: N=11 spins)

Ising Model  $H_{Ising} = \sum_{i<j} J_{ij} \sigma_x^i \sigma_x^j$

$$J_{ij} \approx \frac{J_0}{r^\alpha}$$



# Long range “Light Cones” (XY: N=11 spins)

XY Mode  $H_{XY} = \sum_{i<j} J_{ij} (\sigma_x^i \sigma_x^j + \sigma_z^i \sigma_z^j) + B_y \sum_i \sigma_y^i$   $B_y \gg J_{ij}$

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**S**pin-1

**F**uture



# Thermalization/Localization

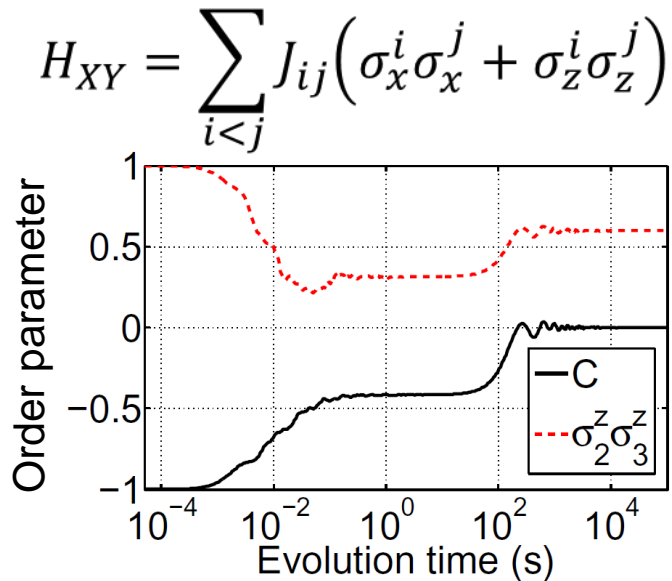
How can quantum systems thermalize?

→ Eigenstate Thermalization Hypothesis (Rigol et al., *Nature* 2008)

How can quantum systems **fail** to thermalize?

## Prethermalization

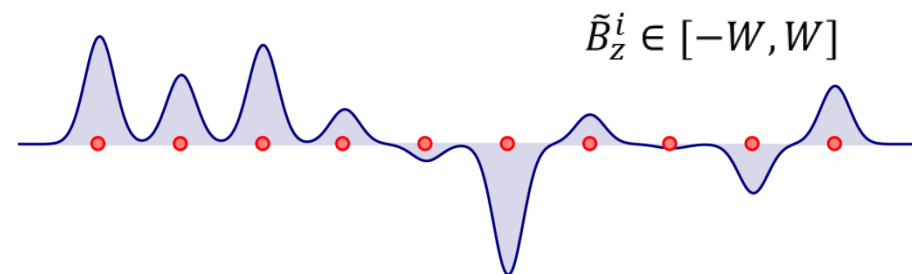
- XY Model with inhomogeneous couplings



## Many-Body Localization

- Ising Model with random disorder

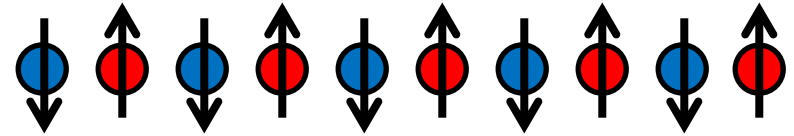
$$H_{MBL} = \sum_{i<j} J_{ij} \sigma_x^i \sigma_x^j + \sum_i \tilde{B}_Z^i \sigma_z^i$$



- A. Polkovnikov et al., *Rev. Mod. Phys.* **83**, 863 (2011)  
P. Hauke and M. Heyl, arXiv:1410.1491 (2014)  
R. Nandkishore & D. Huse, *Annual Review of Condensed Matter Physics* **6**, 15 (2015)

# Thermalization/Localization

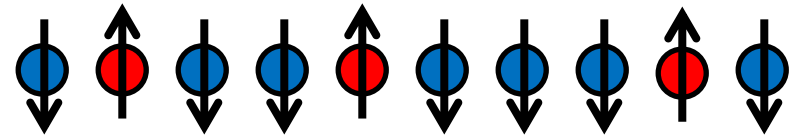
**Step 1:** Initialize spins AFM Neel ordered along z (“kT=∞”)



**Step 2:** Quench to transverse Ising model with random disorder

$$H_{MBL} = \sum_{i<j} J_{ij} \sigma_x^i \sigma_x^j + B_0 \sum_i \sigma_z^i + \sum_i \tilde{B}_z^i \sigma_z^i \quad \tilde{B}_z^i \in [-W, W]$$

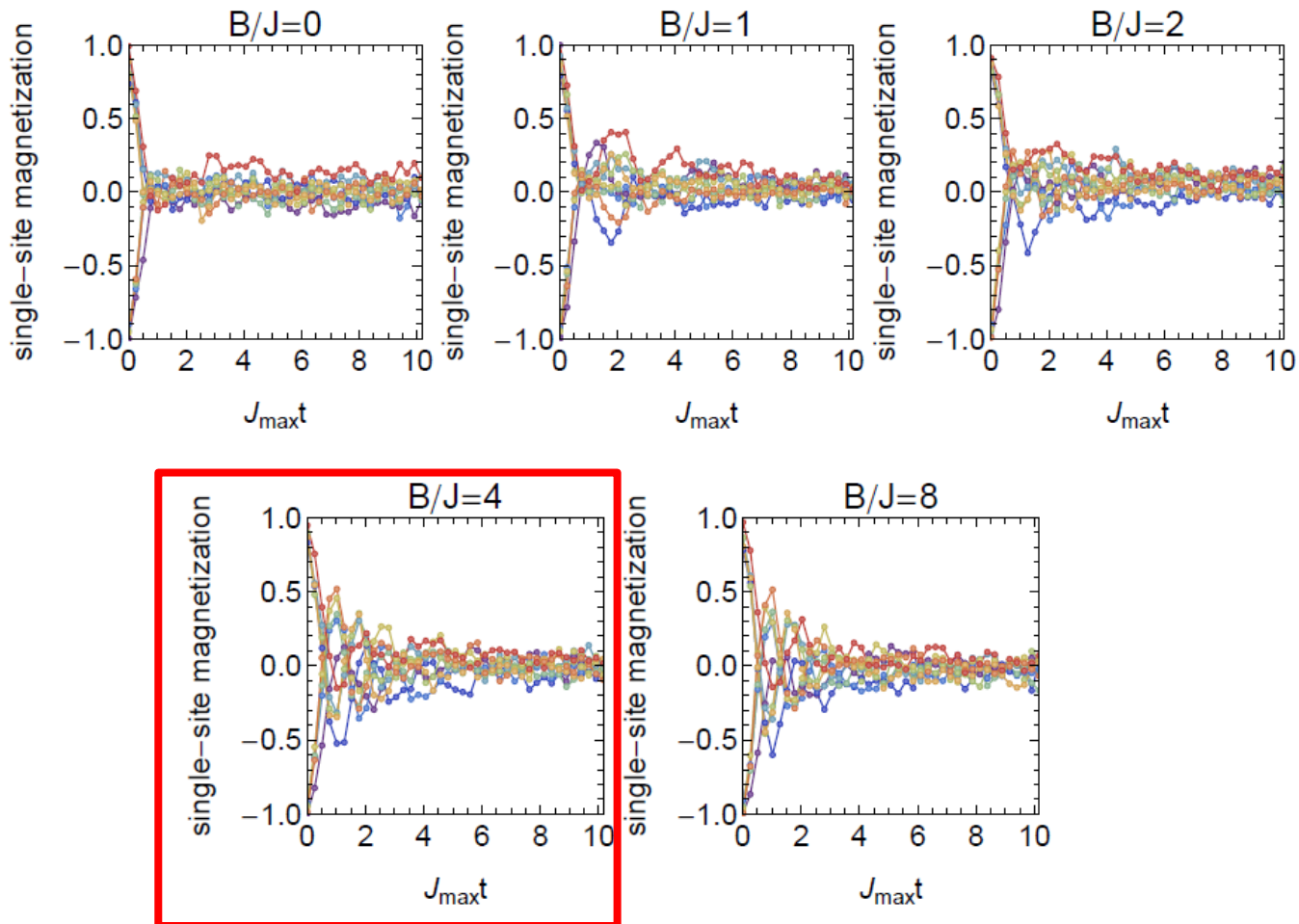
**Step 3:** Measure each spin along z after time  $t$



**Step 4:** Repeat for many different disorder instances and strengths

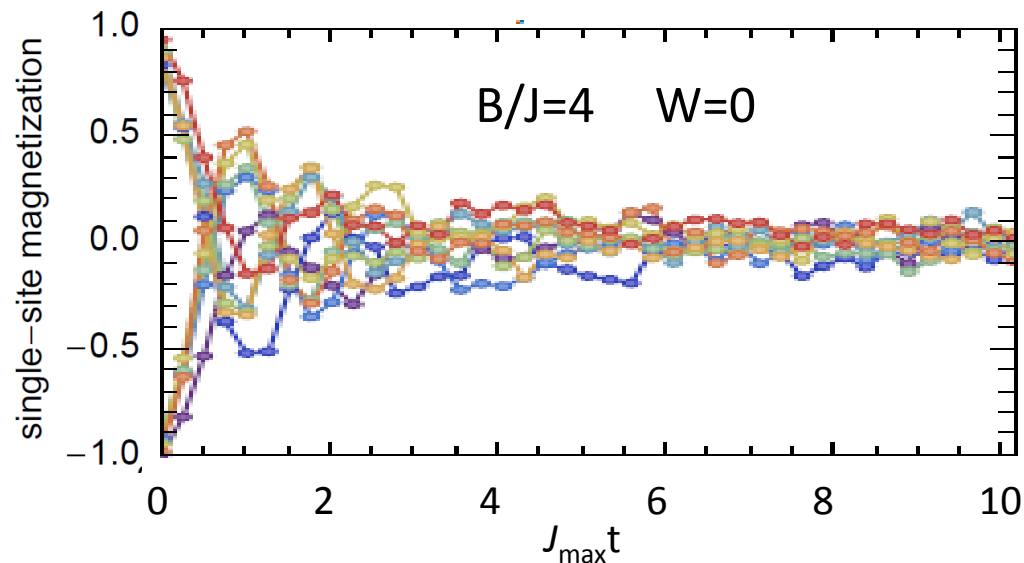
# No disorder (thermalization)

$$H = \sum_{i < j} J_{ij} \sigma_x^i \sigma_x^j + B \sum_i \sigma_z^i$$

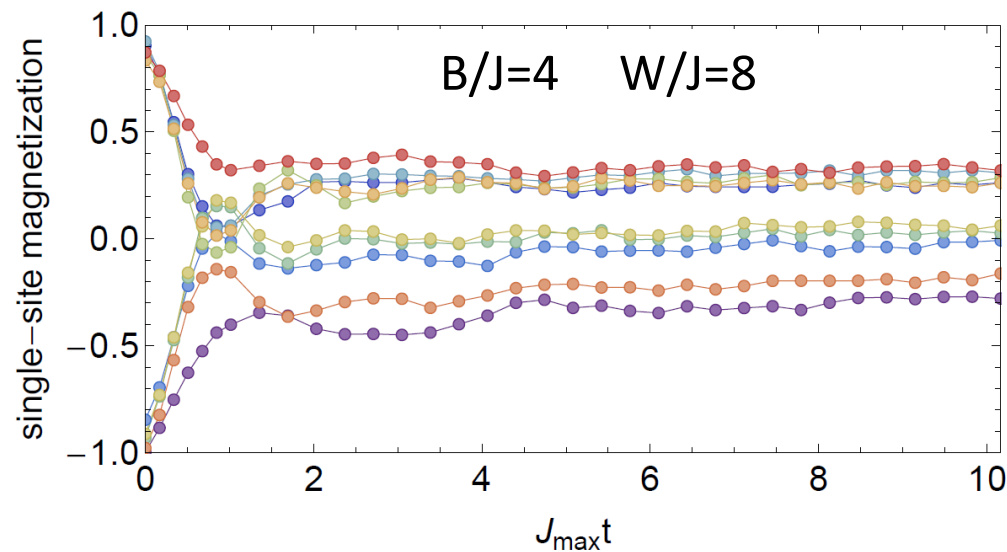


$$H_{MBL} = \sum_{i < j} J_{ij} \sigma_x^i \sigma_x^j + B_0 \sum_i \sigma_z^i + \sum_i \tilde{B}_z^i \sigma_z^i \quad \tilde{B}_z^i \in [-W, W]$$

**No disorder**



**Some disorder**



**T**rapped Ion Spin Hamiltonian Engineering

**G**round states and Adiabatic Protocols

**D**ynamics

Direct Many-Body Spectroscopy

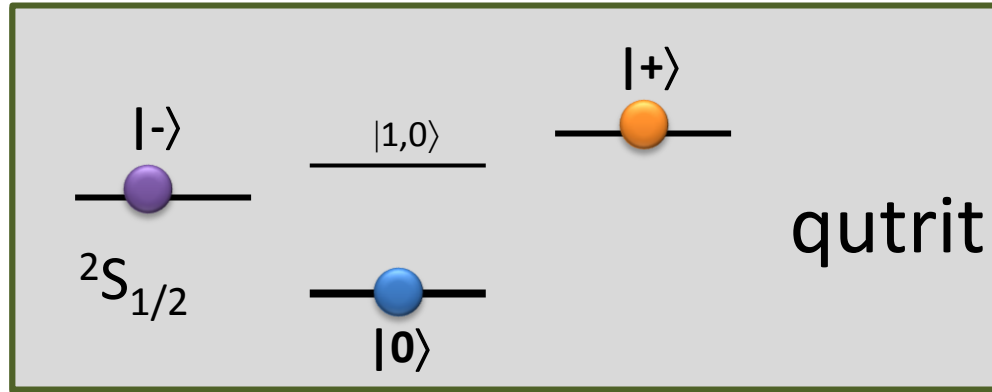
Lieb-Robinson Bounds

**M**any-Body Thermalization/Localization?

**S**pin-1

**F**uture

# Quantum simulations with Spin-1



rsb+bsb on  
both transitions

$$H = \sum_{i,j} J_{ij} S_x^i S_x^j$$

rsb  $|0\rangle \leftrightarrow |+ \rangle$   
bsb  $|0\rangle \leftrightarrow |-\rangle$

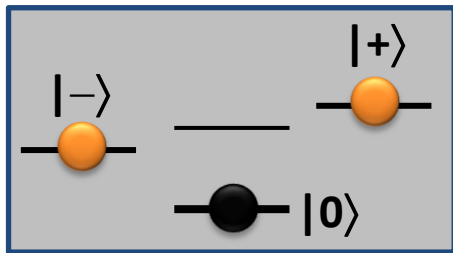
$$H = \sum_{i,j} J_{ij} (S_+^i S_-^j + S_-^i S_+^j) + D \sum_i (S_z^i)^2 + \sum_{i,m} V_{im} (2\hat{n} + 1) S_z^i$$

$$J_{ij} \approx \frac{J_0}{r^\alpha}$$

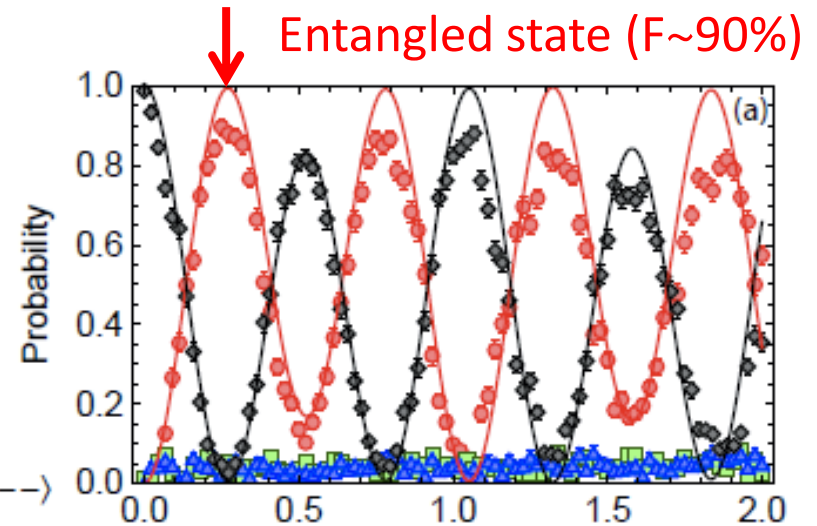
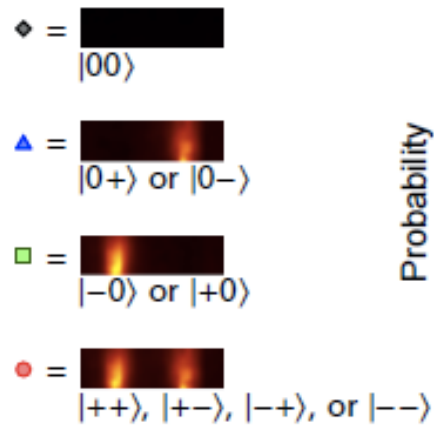
Work in  $\langle S_z^i \rangle = 0$  subspace:  $\frac{3^N}{2\sqrt{N}}$  states

# Spin-1 dynamics (N=2)

$$H = \sum_{i,j} J_{i,j} (S_i^+ S_j^- + S_i^- S_j^+)$$



Dark =  $|0\rangle$ ; Bright = not  $|0\rangle$



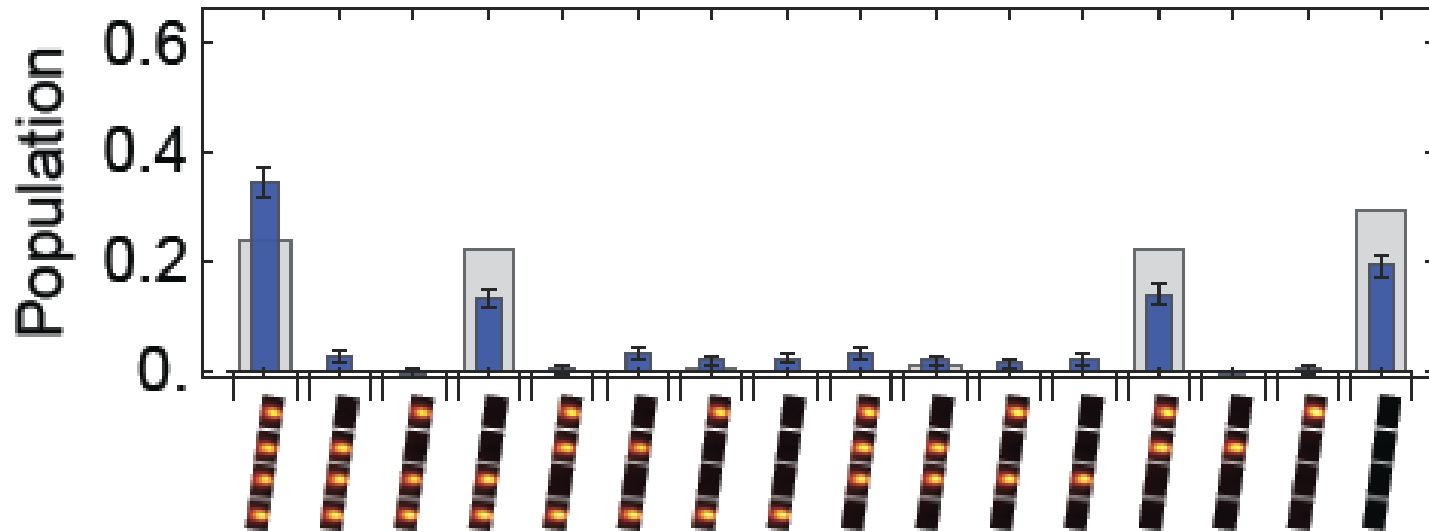
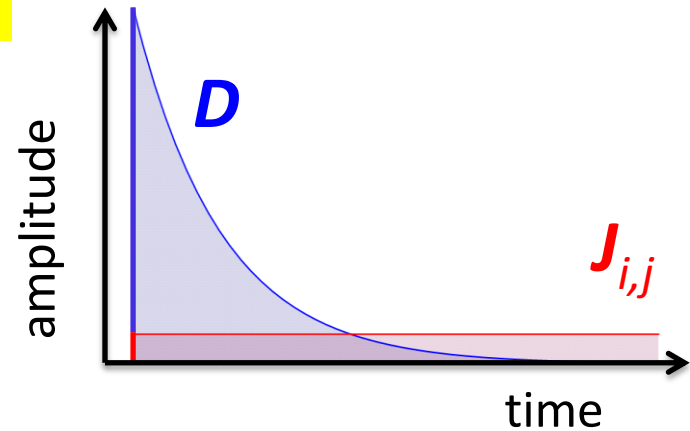


# Spin-1 adiabatic ramp (N=4)

$$H = \sum_{i,j} J_{i,j} (S_i^+ S_j^- + S_i^- S_j^+) + D(t) \sum_i (S_z^i)^2$$

Step 1: prepare  $|00\dots 0\rangle$

Step 2: ramp from  $D \gg J$  to  $D \ll J$

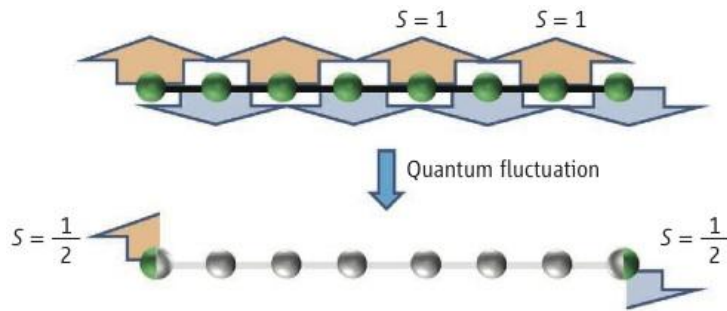


# Quantum simulations with Spin-1

$$H = \sum_{i,j} J_{ij} \left( S_+^i S_-^j + S_-^i S_+^j + \lambda S_Z^i S_Z^j \right) + D \sum_i (S_Z^i)^2$$

(long-range) Haldane  
Hamiltonian

D. Haldane, PRL 50, 1153 (1983)



Adiabatically transform  $D, \lambda \rightarrow 0$   
“Haldane phase” – topological edge states

**ground state:  
nonlocal string order**

$$\lim_{|i-j| \rightarrow \infty} \langle S_x^i | \exp \left[ i\pi \left( S_x^{i+1} + S_x^{i+2} + \dots + S_x^{j-1} \right) \right] | S_x^j \rangle$$

I. Cohen and A. Retzker, PRL **112**, 040503 (2014)

C. Senko, et al., arXiv 1410.0937 (2014)

**T**rapped Ion Spin Hamiltonian Engineering

**G**round states and Adiabatic Protocols

**D**ynamics

Direct Many-Body Spectroscopy

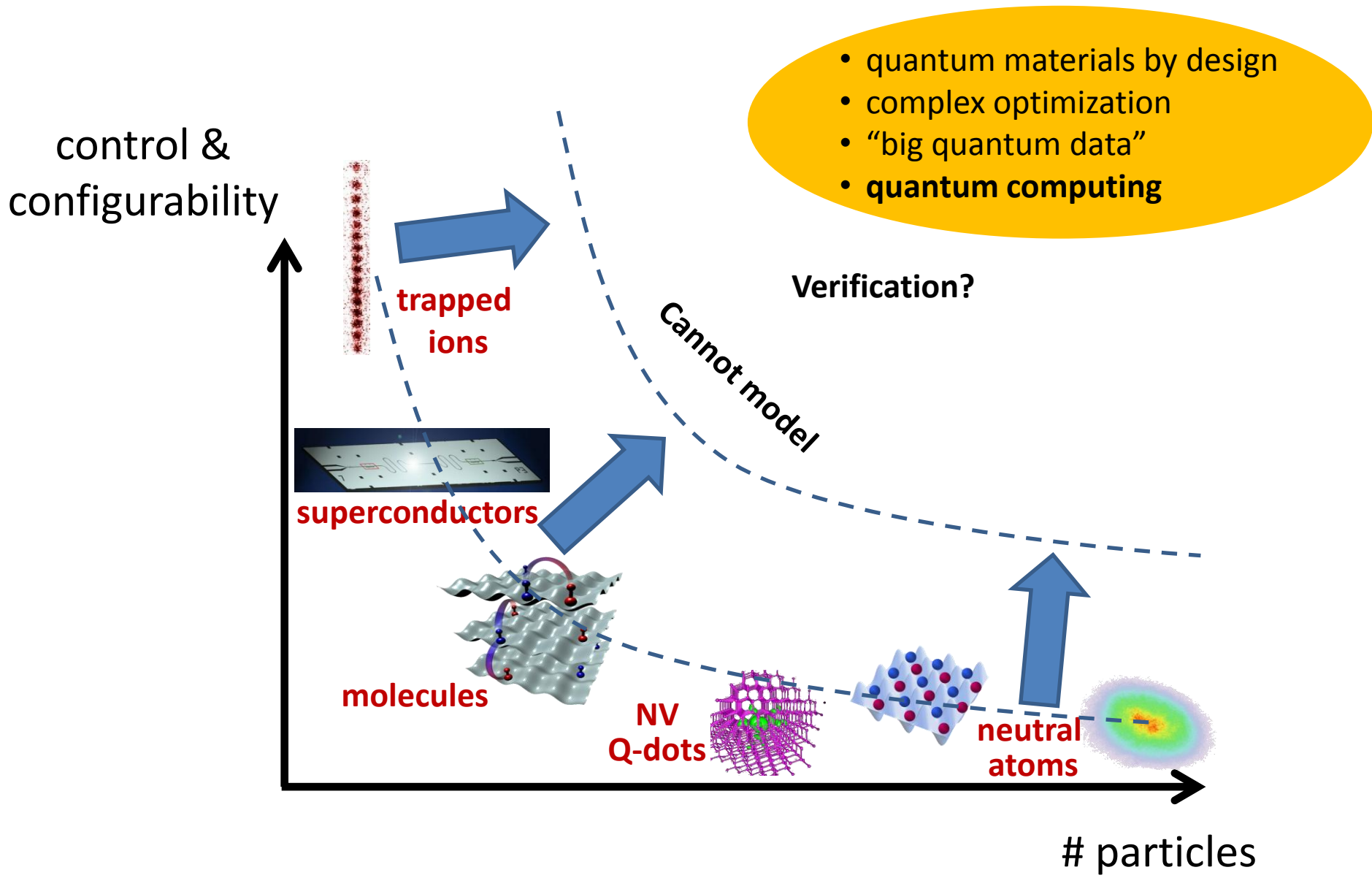
Lieb-Robinson Bounds

**M**any-Body Thermalization/Localization?

**S**pin-1

**F**uture

# Implementations of Quantum Simulators



# Arbitrary Fully-Connected Spin Models

$$H = \sum_{i \neq j} J_{i,j} \hat{\sigma}_x^{(i)} \hat{\sigma}_x^{(j)}$$

$$\frac{N(N-1)}{2} \text{ couplings } J_{i,j}$$

$$J_{i,j} = \left( \frac{\hbar \Delta k^2}{2m} \right) \Omega_i \Omega_j \sum_m \frac{b_i^m b_j^m}{\mu^2 - \omega_m^2}$$

**$N+1$  controls**



$\Omega_i$

# Arbitrary Fully-Connected Spin Models

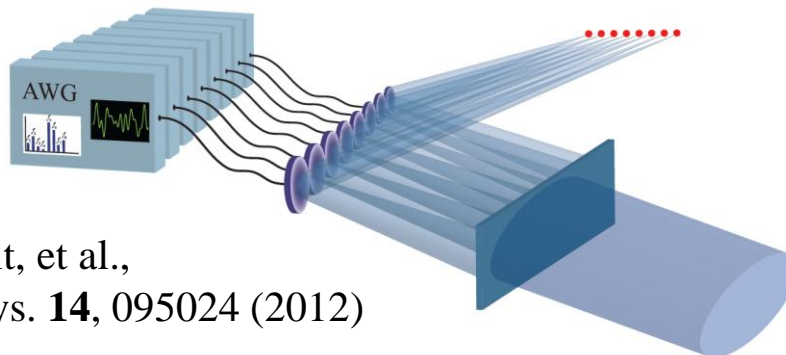
$$H = \sum_{i \neq j} J_{i,j} \hat{\sigma}_x^{(i)} \hat{\sigma}_x^{(j)}$$

$$\frac{N(N-1)}{2} \text{ couplings } J_{i,j}$$

$$J_{i,j} = \left( \frac{\hbar \Delta k^2}{2m} \right) \sum_m \Omega_{i,m} \Omega_{j,m} \frac{b_i^m b_j^m}{\mu_m^2 - \omega_m^2}$$

$$\Omega_{i,m}, \mu_m$$

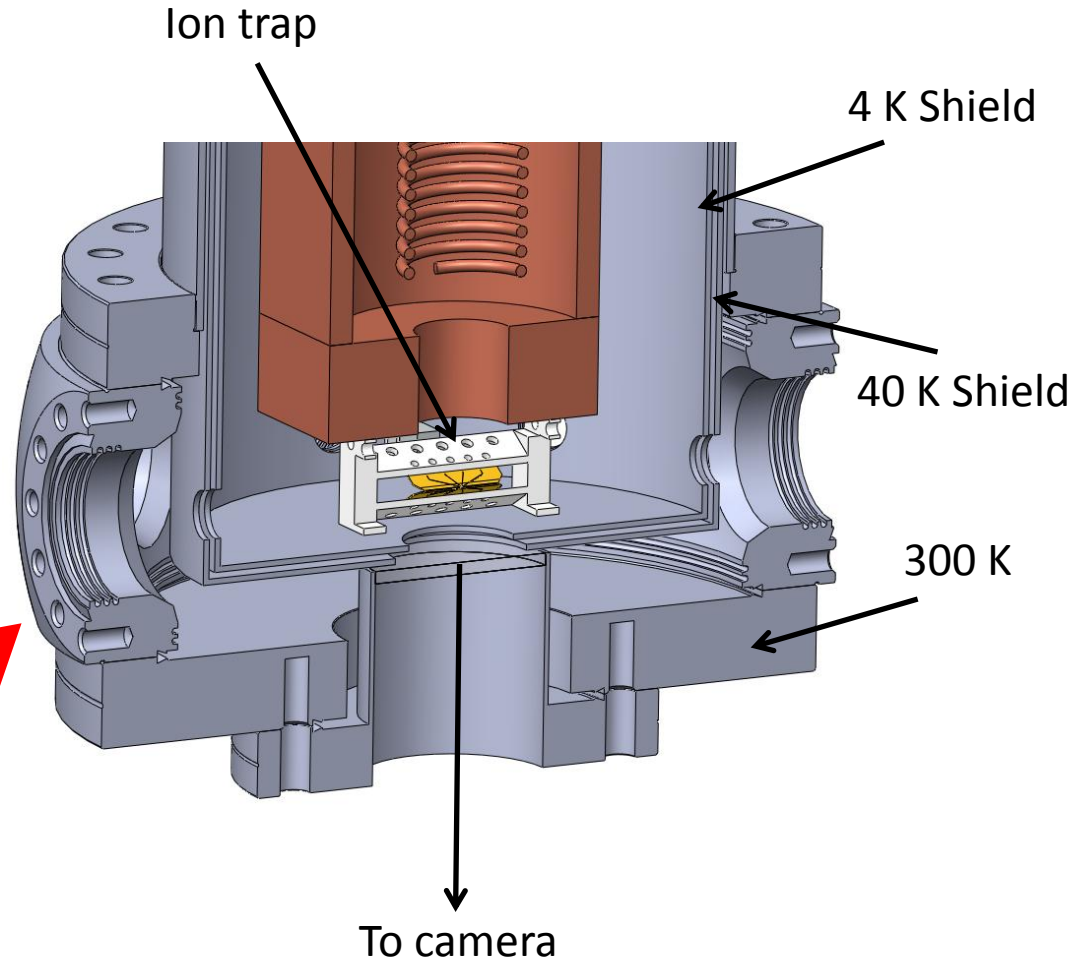
**$N^2$  controls**



S. Korenblit, et al.,  
New. J. Phys. **14**, 095024 (2012)

32-channel optical modulator

# Scaling Up: 4K to get lower pressure



P. Richerme

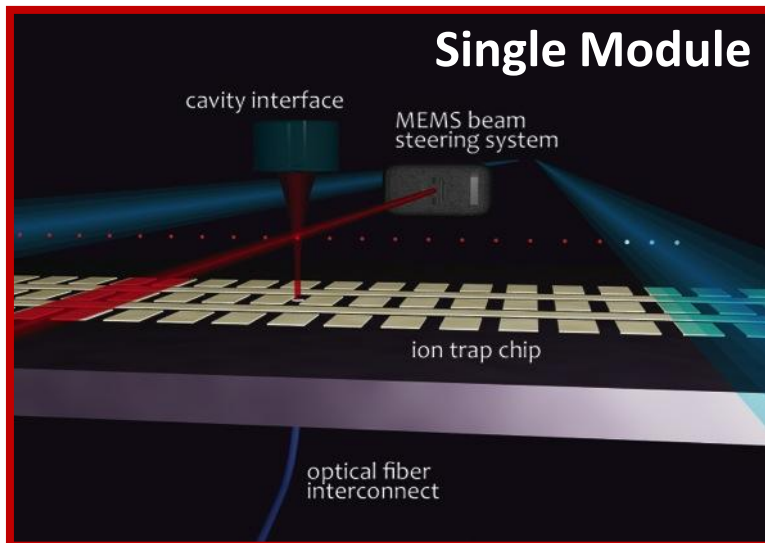
# Modular Scaling to $10^6$ qubits?

## Quantum Computation in Small Quantum Registers

- Linear ion chain with 20-100 ions (Elementary Logic Unit, or ELU)
- **Arbitrary quantum logic operation among the qubits in the chain**

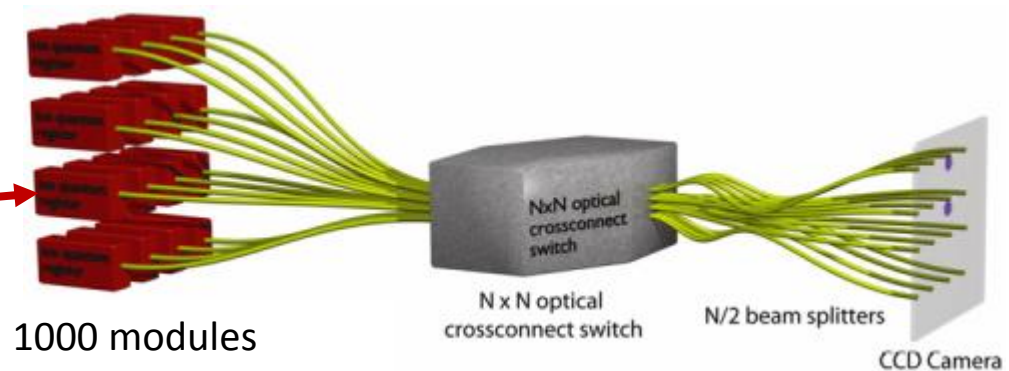
## Interconnect of Multiple Such Registers via Photonic Channel

- Reconfigurable interconnect using optical crossconnect switches
- **Efficient optical interface for remote entanglement generation**



100 qubits / module

## 100,000 qubit quantum computer





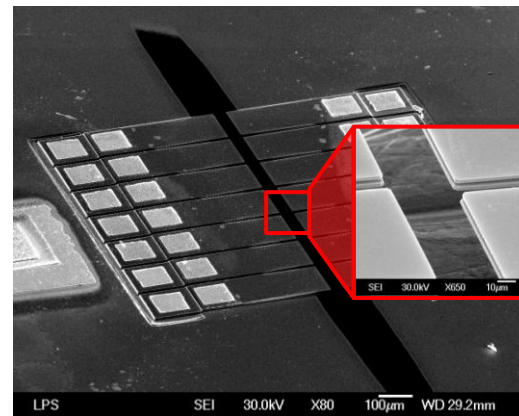
# Science

8 March 2013 | \$10

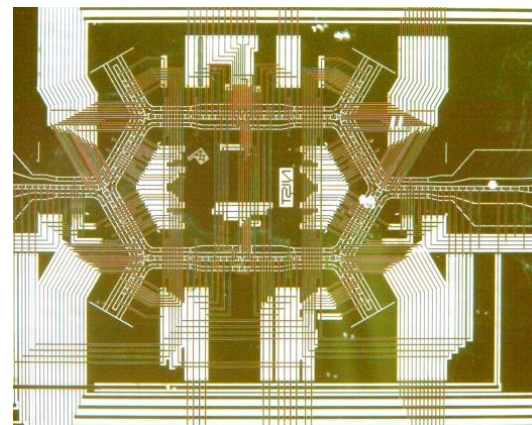
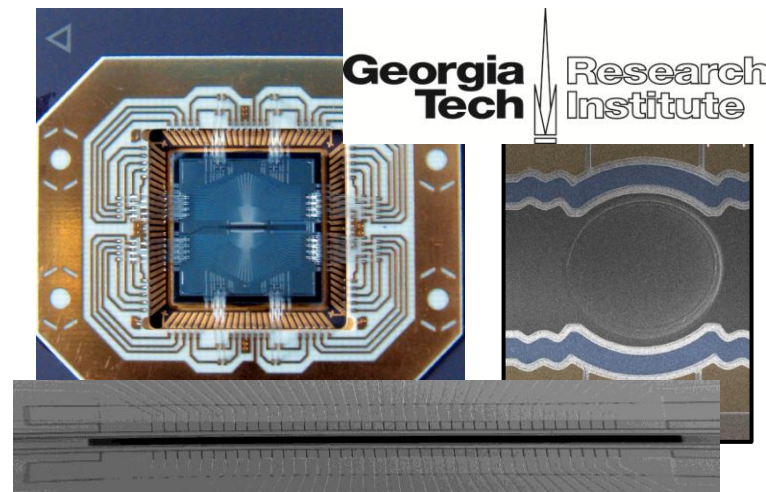
Quantum  
Information  
Processing



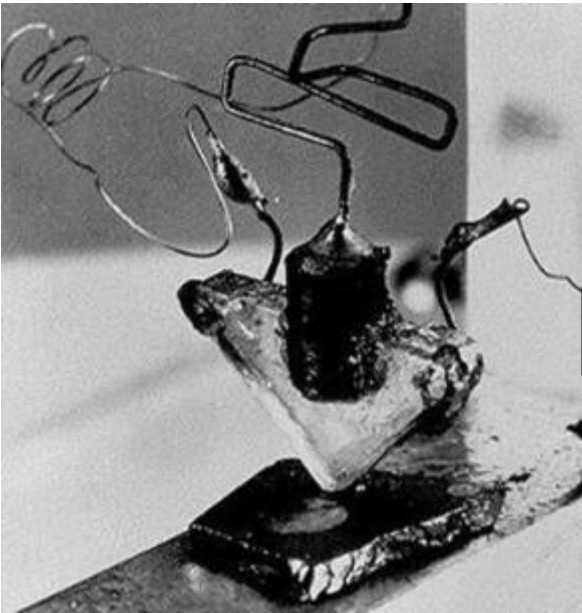
Sandia  
National  
Laboratories



The Laboratory for Physical Sciences



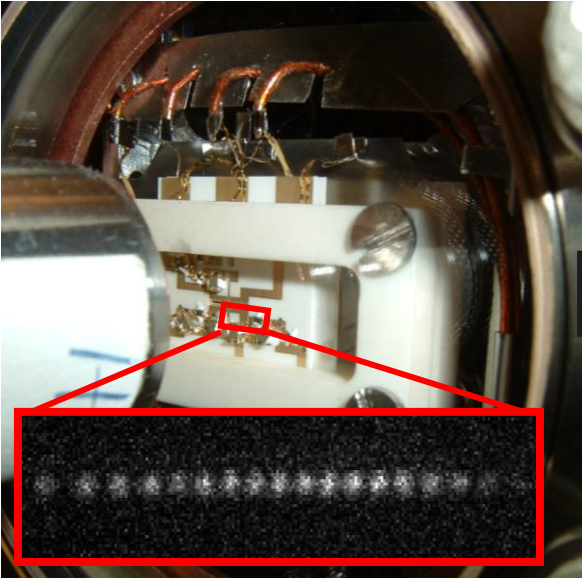
NIST



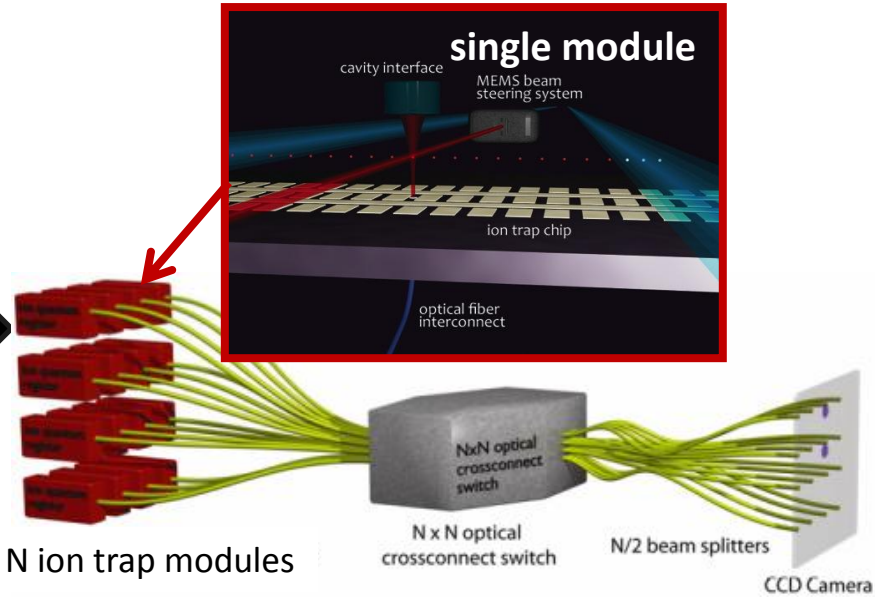
1947: first transistor



2000: integrated circuit



2015: qubit collection



Large scale quantum network?





**Grad Students**

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- Shantanu Debnath
- Caroline Figgatt
- David Hucul
- Volkan Inlek
- Aaron Lee
- Kale Johnson
- Harvey Kaplan
- Lexi Parsagian
- Chris Rickerd
- Crystal Senko
- Ksenia Sosnova
- Jake Smith
- Ken Wright

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- Kate Collins
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- Lenore Koenig



**Postdocs**

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- Norbert Linke
- Brian Neyenhuis
- Phil Richerme
- Grahame Vittorini

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- Howard Carmichael
- Jim Freericks
- Alexey Gorshkov
- Jungsang Kim (Duke)
- Jake Taylor



NSA ARO

