# Characterizing Spin Liquids and Topological Order in Wavefunctions

## ... and Model Hamiltonians


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- Topological RVB spin liquids on the kagome lattice
- Magnetic field-induced topological spin liquids
- Chiral topological spin liquids (on the square lattice)

# References

## Topological RVB spin liquids

Norbert Schuch, DP, J. Ignacio Cirac, and David Pérez-García,
Phys. Rev. B **86**, 115108 (2012)

DP, Norbert Schuch, David Pérez-García, and J. Ignacio Cirac,
Phys. Rev. B **86**, 014404 (2012)

DP and Norbert Schuch, Phys. Rev. B 87, 140407 (2013)

Norbert Schuch, DP, J. Ignacio Cirac, and David Perez-Garcia
Phys. Rev. Lett. 111, 090501 (2013)

## Field-induced topological SL

DP, Norbert Schuch, J. Ignacio Cirac, Phys. Rev. B 88, 144414 (2013)

## Chiral topological SL

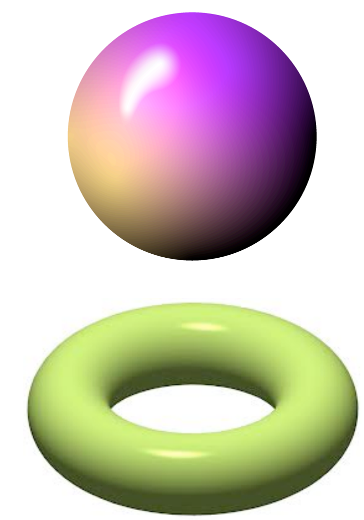
DP, J. Ignacio Cirac, and Norbert Schuch, arXiv1504.05236

# Exotic «spin liquids» beyond the «order parameter» paradigm

* no spontaneous broken symmetry
* no local  order but...

* Topological order

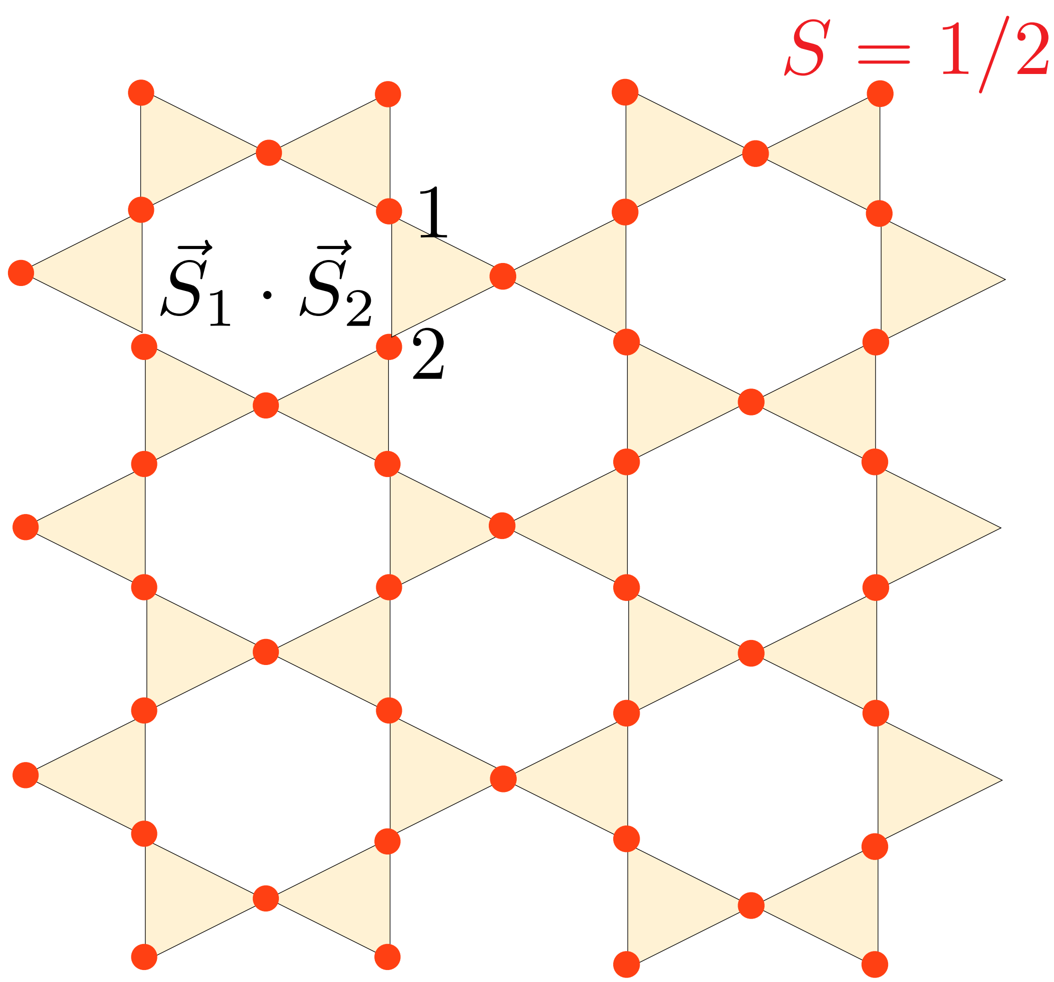
X. G. Wen



GS degeneracy (depends on **topology** of space)

- How to detect them ?

# Best candidate : spin-1/2 Heisenberg QAF on the Kagome lattice !



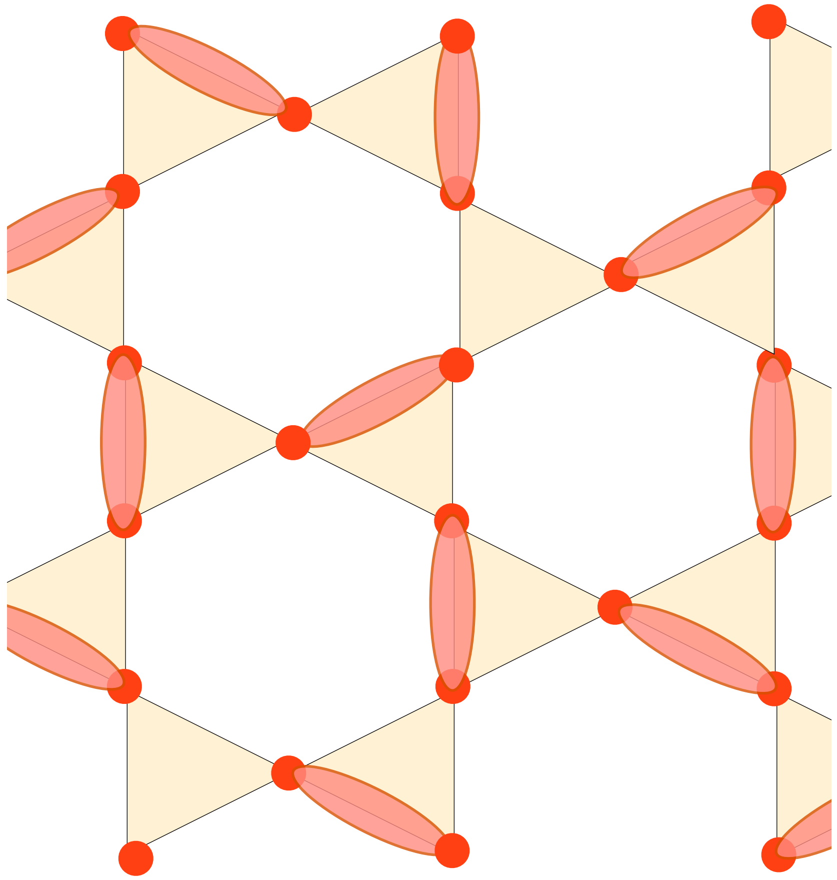

Herbertsmithite: P. Mendels (Orsay)
& Z. Hiroi (ISSP)

Numerical «evidence» (DMRG)
for a (gapped) spin liquid:

S. Yan, D.A. Huse & S. White, Science 2011
S. Depenbrock, I.P. McCulloch &
U. Schollwock, PRL 2012

# RVB spin liquid:

$S = 0$



Equal-weight superposition
of NN singlet coverings

## spin-1/2 RVB

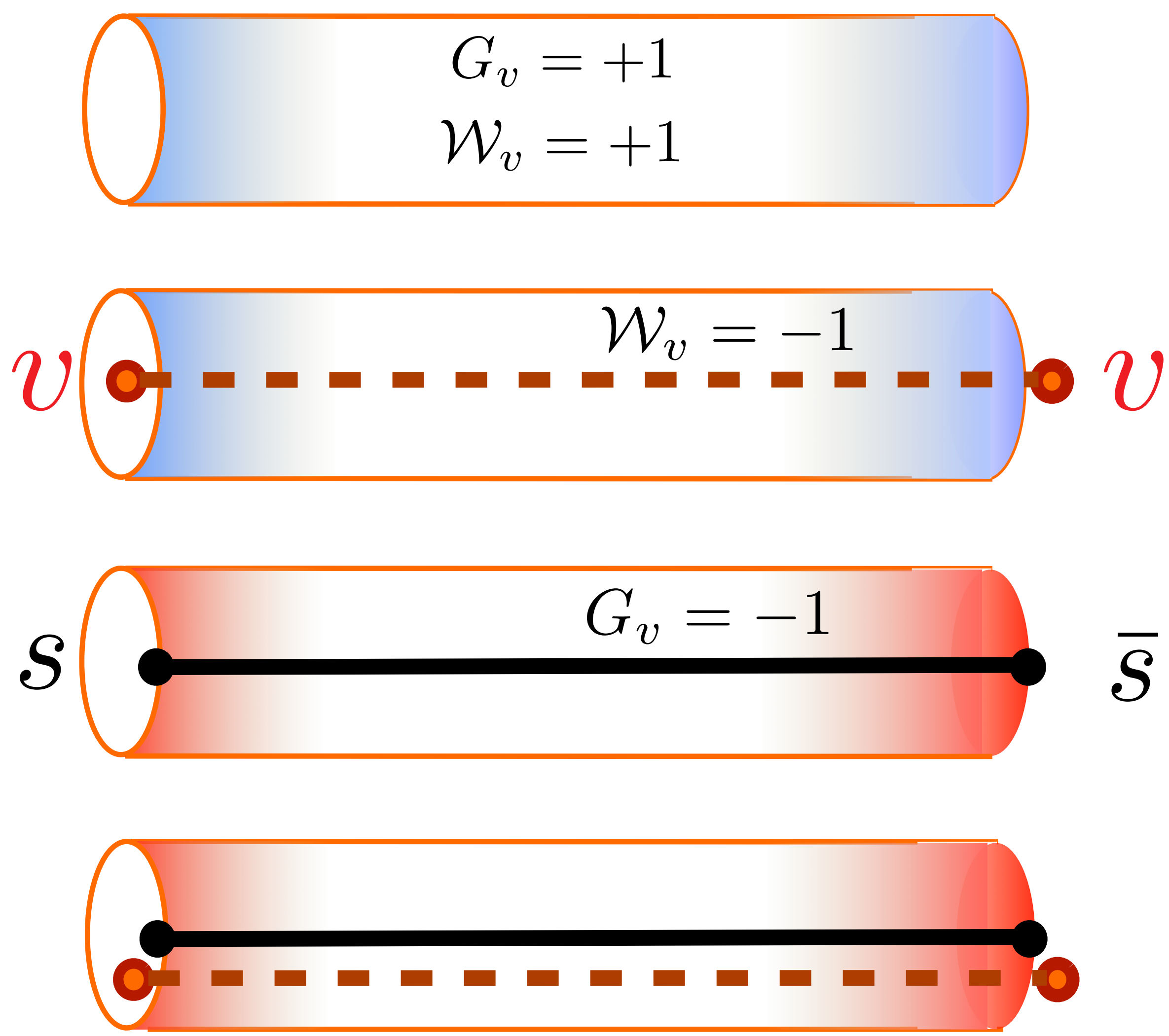
P. Fazekas and P.W. Anderson
Philosophical Magazine **30**, 423-440 (1974)

similar to RK wavefunction
-- see Masaki's talk

## Topological order

Hasting-Oshikawa-LSM theorem

# $\mathbb{Z}_2$ spin liquid : topological GS inserting «spinons» and «visons»




Kitaev's Toric Code
(fixed point $\xi = 0$ )

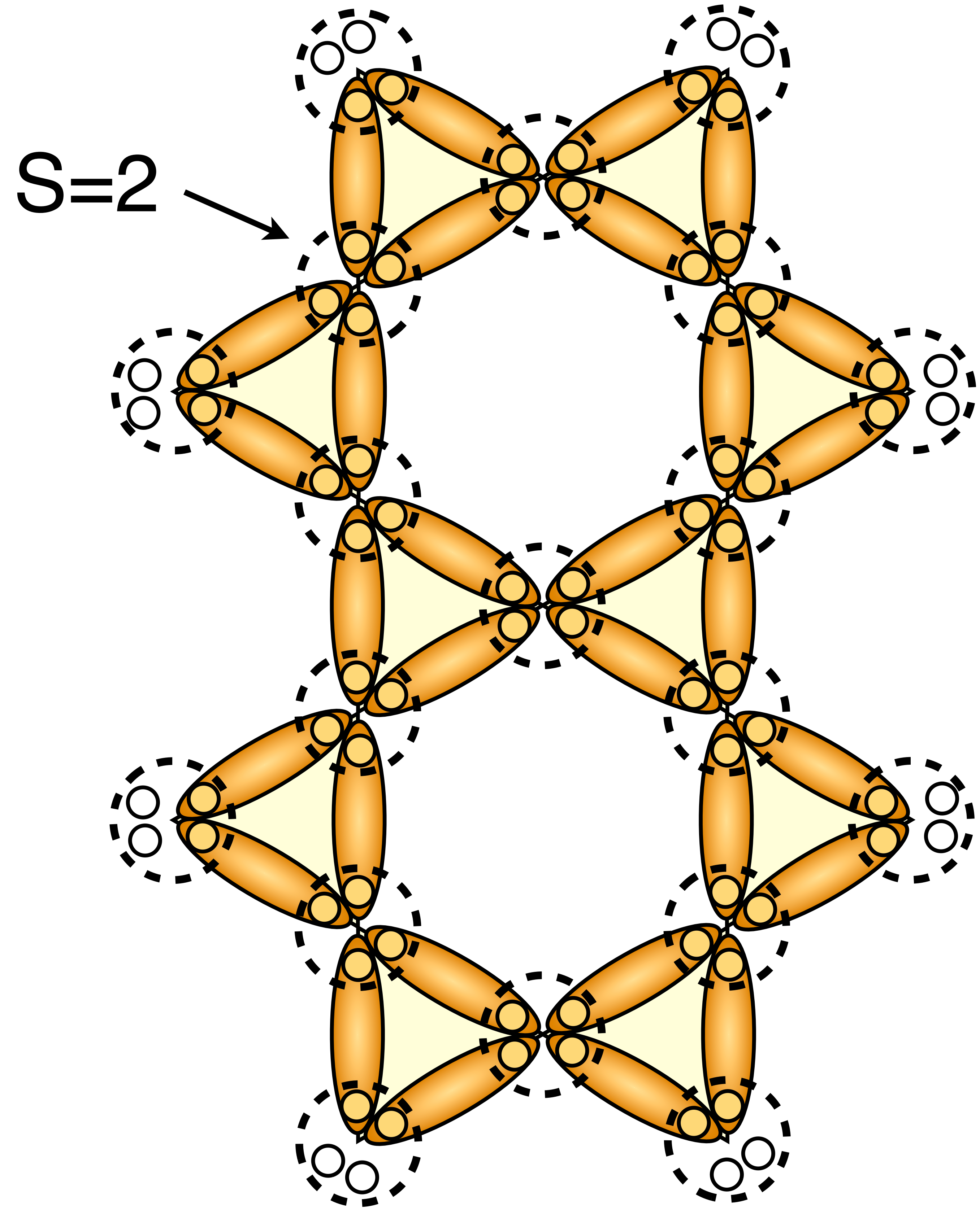
RVB SU(2) spin liquid
(finite $\xi$ ? )

# ... vs «Trivial» insulator

$$H_{\mathrm{AKLT}} = \sum_{\langle i,j \rangle} P_{ij}^{\mathbf{S}_i + \mathbf{S}_j = 4}$$

The Affleck-Kennedy-Lieb-Tasaki spin liquid

The GS is unique !

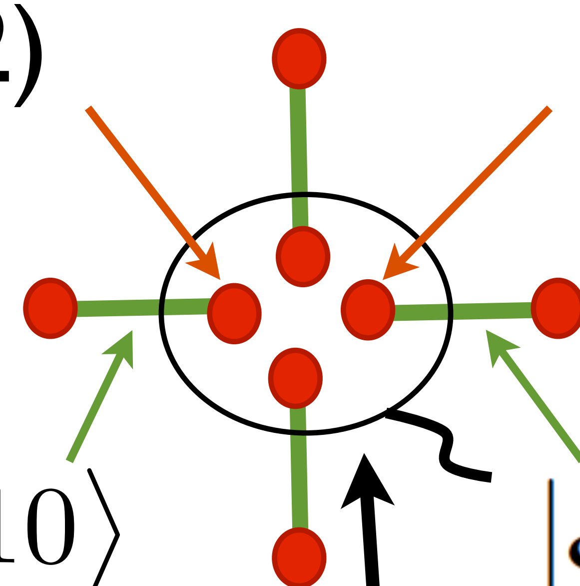


# PEPS construction

I. Cirac
F. Verstraete
G. Vidal

S=2 AKLT | spin-1/2 RVB

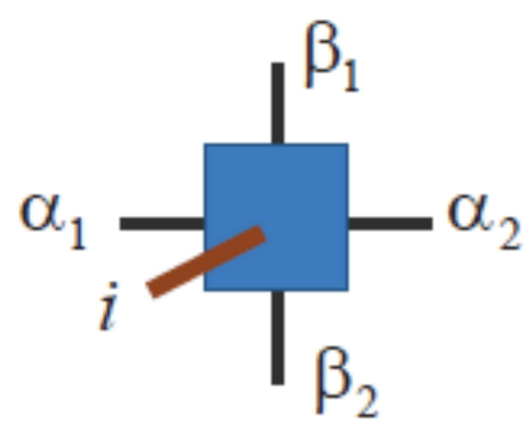
virtual states: S=1/2 (D=2) | $1/2 \oplus 0$ (D=3)



$|\mathcal{S}\rangle = |01\rangle - |10\rangle$

$|\mathcal{S}\rangle = |01\rangle - |10\rangle + |22\rangle$

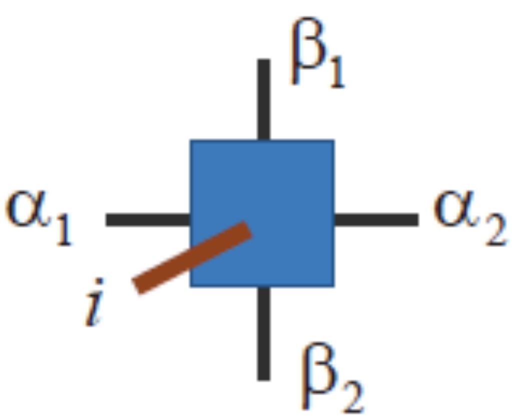
Project onto physical subspace $d = 2S_{\mathrm{phys}} + 1$:




PEPS tensor

# Tensor Network ansatz :
# Projected Entangled Paired States (PEPS)

$A^{i}_{\alpha_1,\alpha_2;\beta_1,\beta_2}$




$$i = \{1, \cdots, d_{\text{phys}}\}$$

$$\alpha, \beta = \{1, \ldots, D\}$$

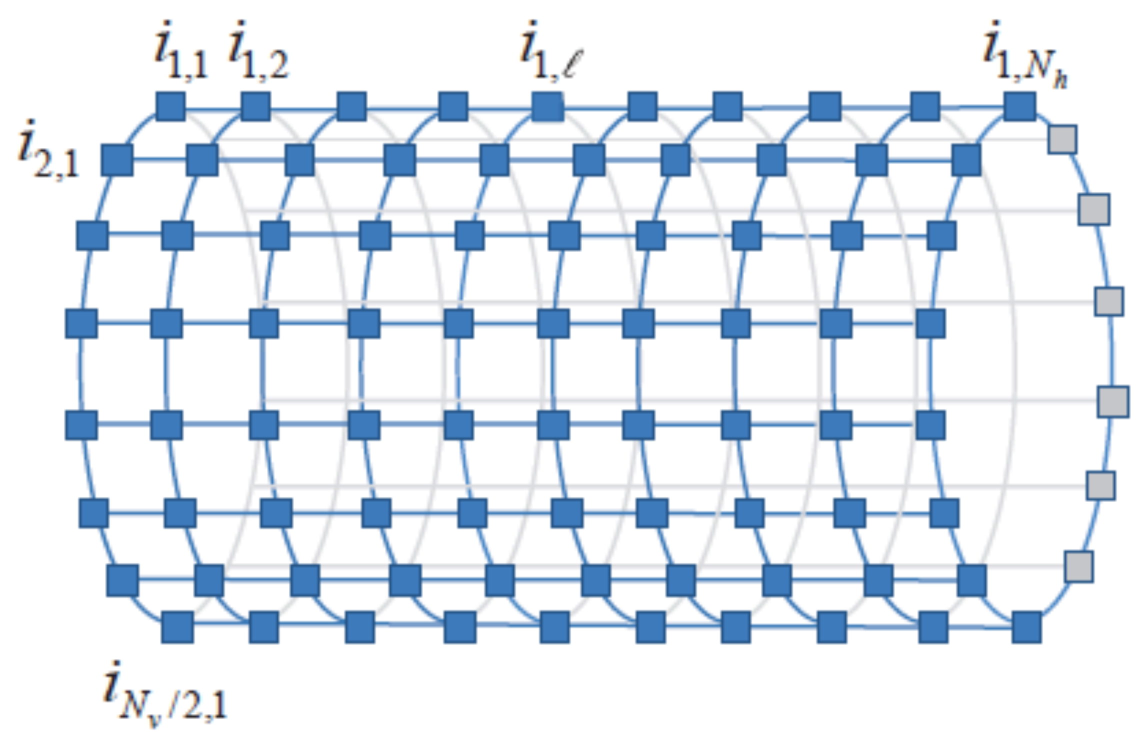
dimension of auxilary (or virtual) space

I. Cirac
F. Verstraete
G. Vidal

Coefficients $C_{\{i_{1,1},\ldots,i_{N_v,N_h}\}}$ of the wavefunction

↓

"contract" product of tensors



Two possible «routes» :

→ use simple PEPS ansatz to investigate physical (e.g. topological, etc...) properties

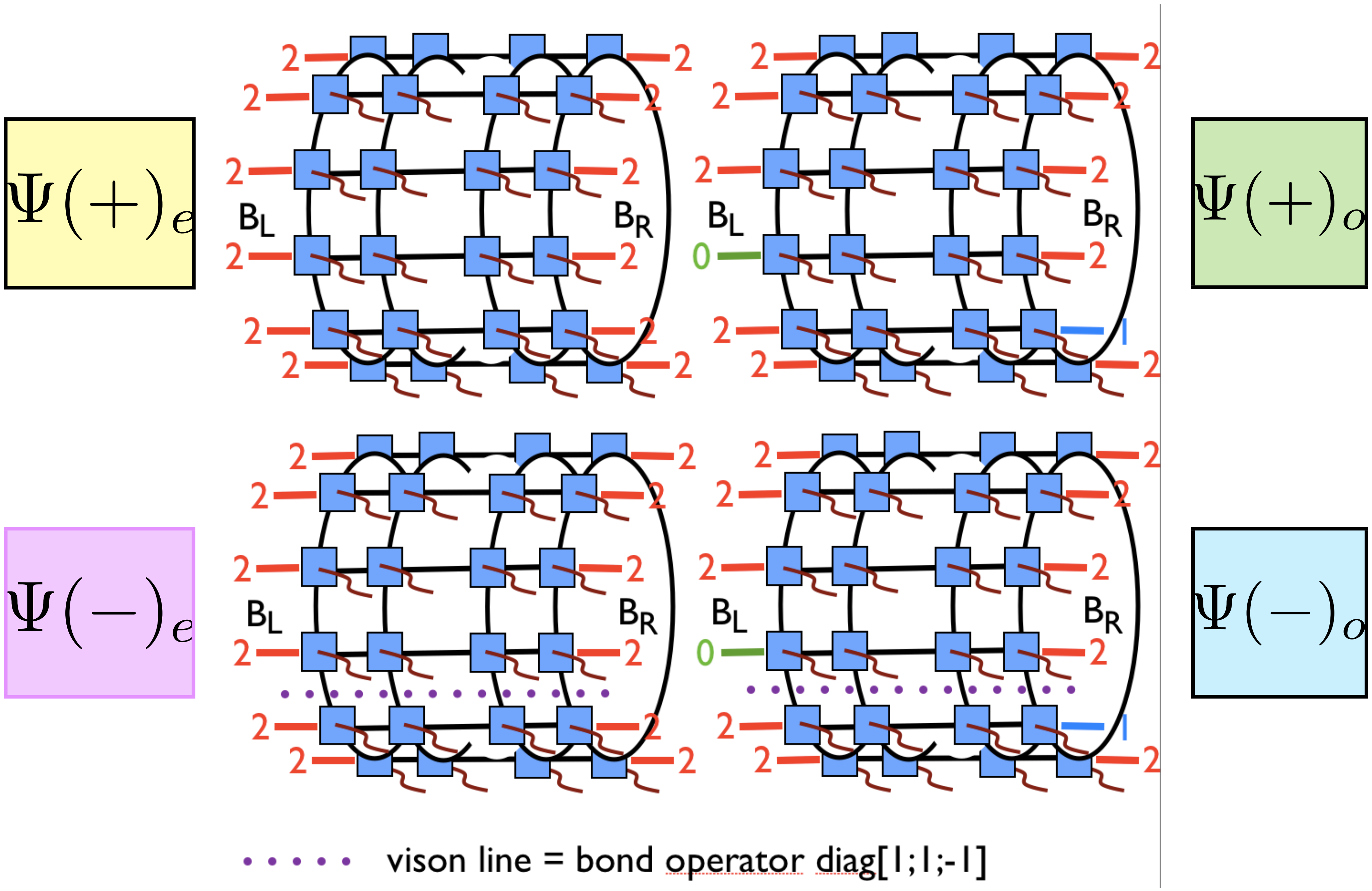
Conceptual understanding

→ optimize PEPS to construct competitive ansatz for microscopic models

G. Vidal, T. Xiang, R. Orus, P. Corboz,... & many more

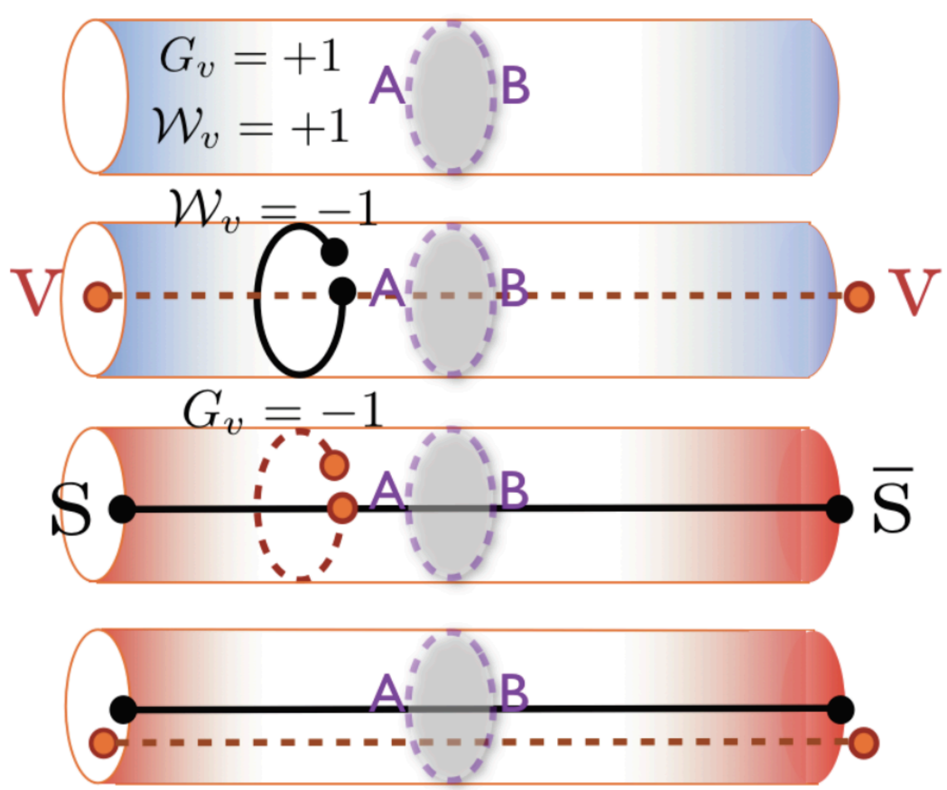
Simulations of real materials

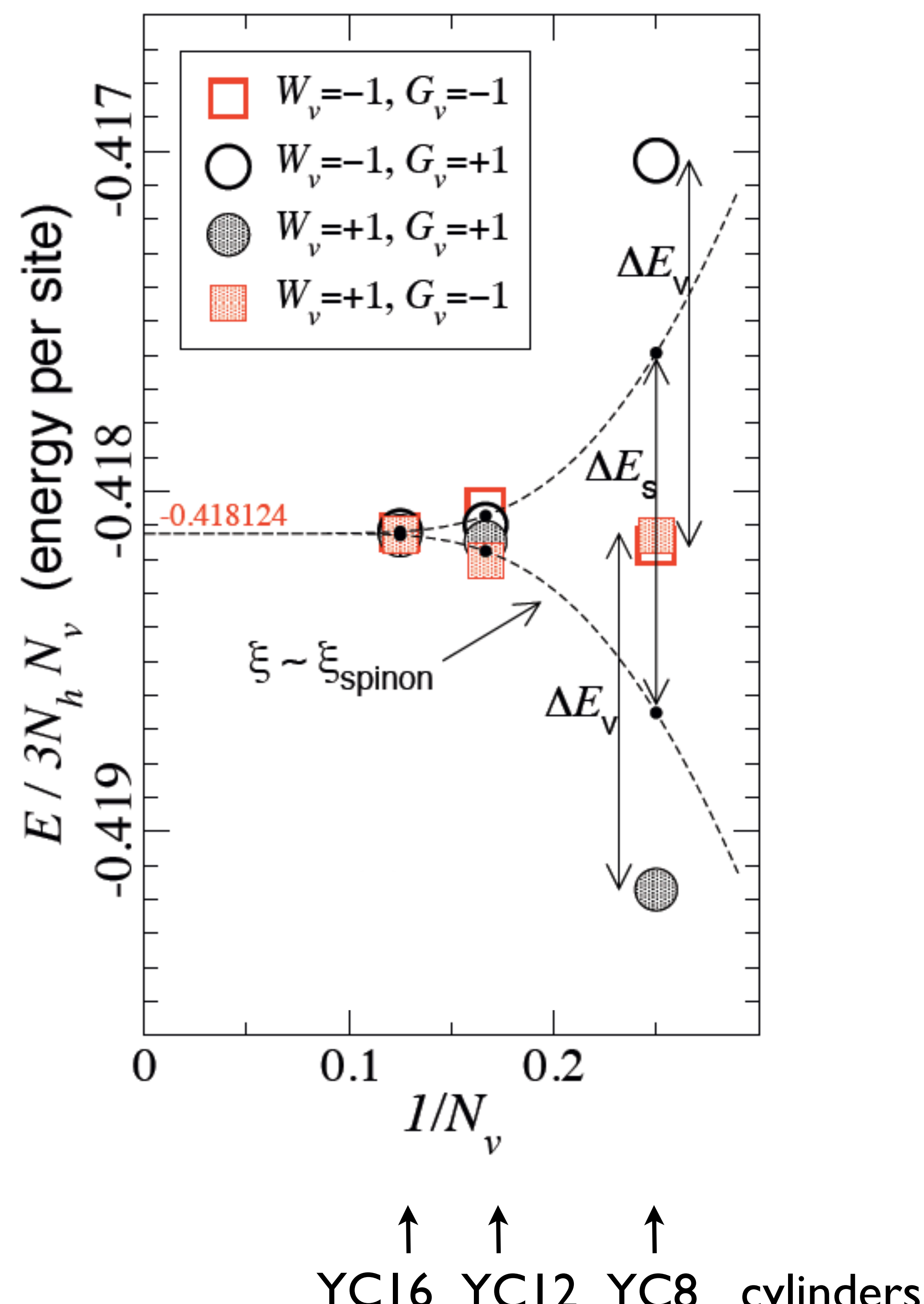
# Easy formalism to construct all topological GS !



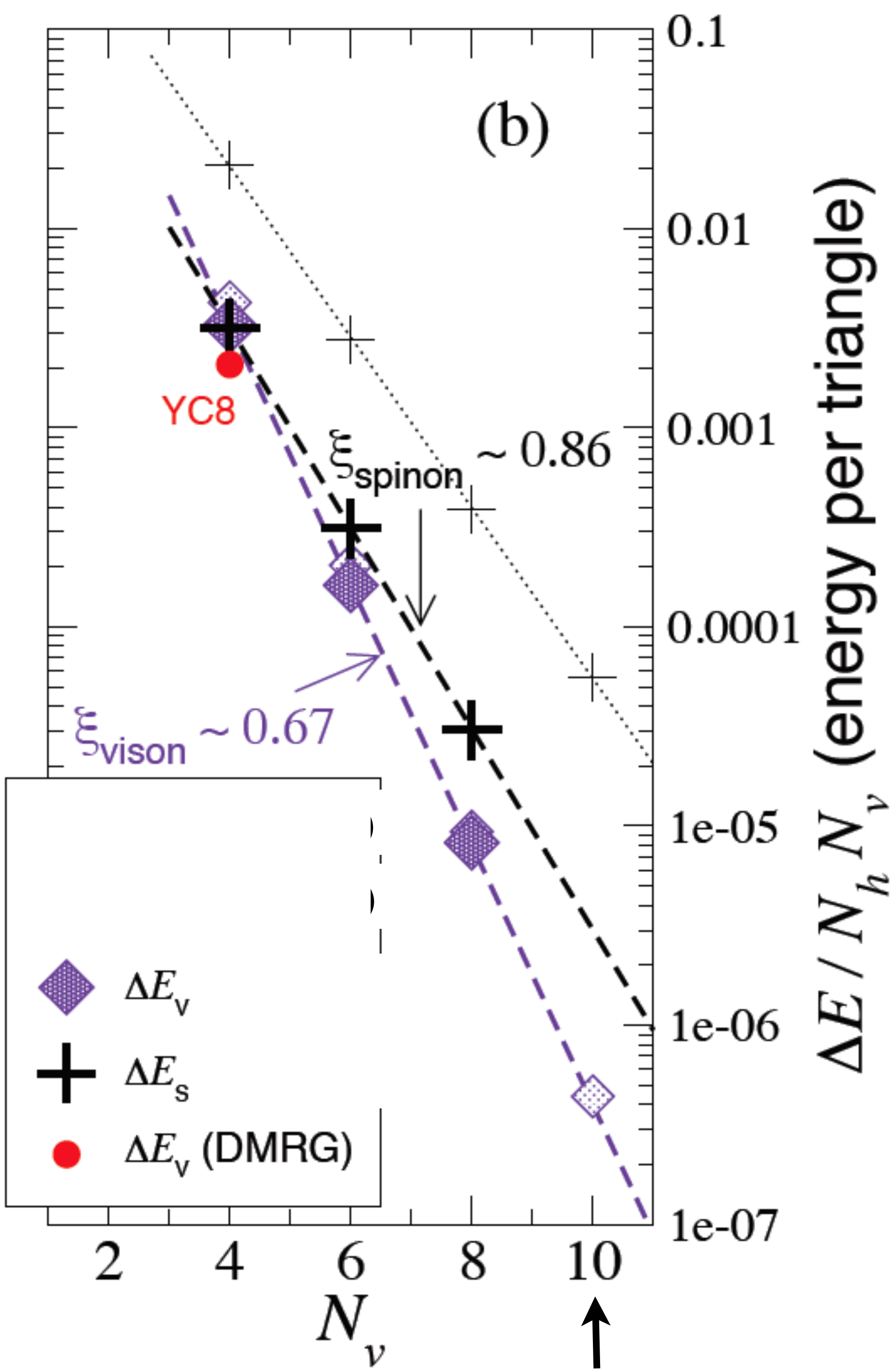

→ orthogonal in the limit of infinite cylinders $(N_h = \infty)$

# Finite size scaling of RVB energy



# Numerical data $(N_h \to \infty)$



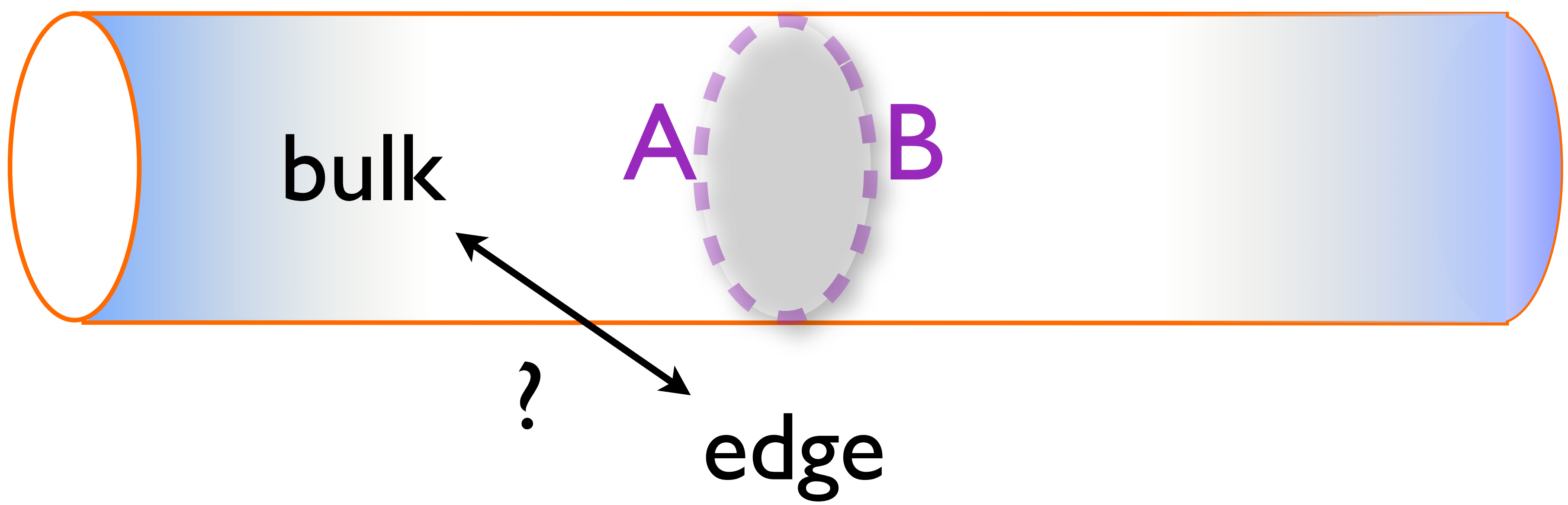

YC20
INFINITE CYLINDER

$$\Delta E_s = a N_h N_v \exp\left(-N_v/\xi_{\rm spinon}\right),$$
$$\Delta E_v = b N_h N_v \exp\left(-N_v/\xi_{\rm vison}\right),$$

Very short coherence lengths

$$\xi < 1 \text{ unit cell}$$

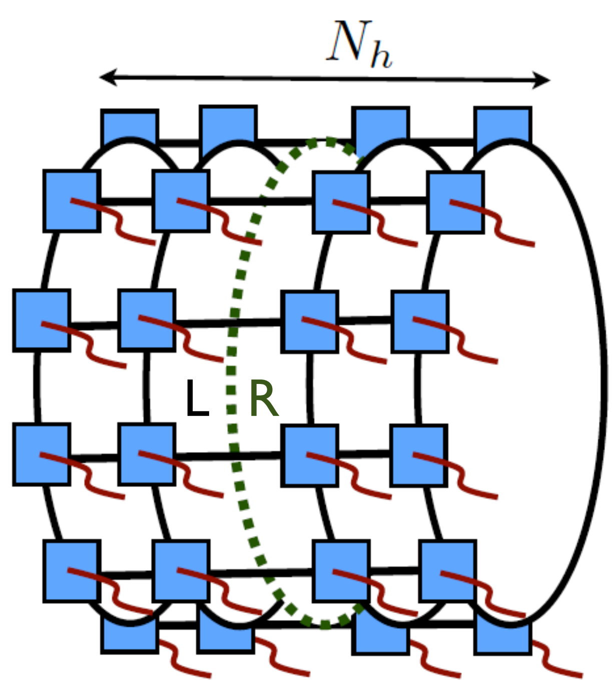
# «Holographic» framework




$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

Reduced density matrix

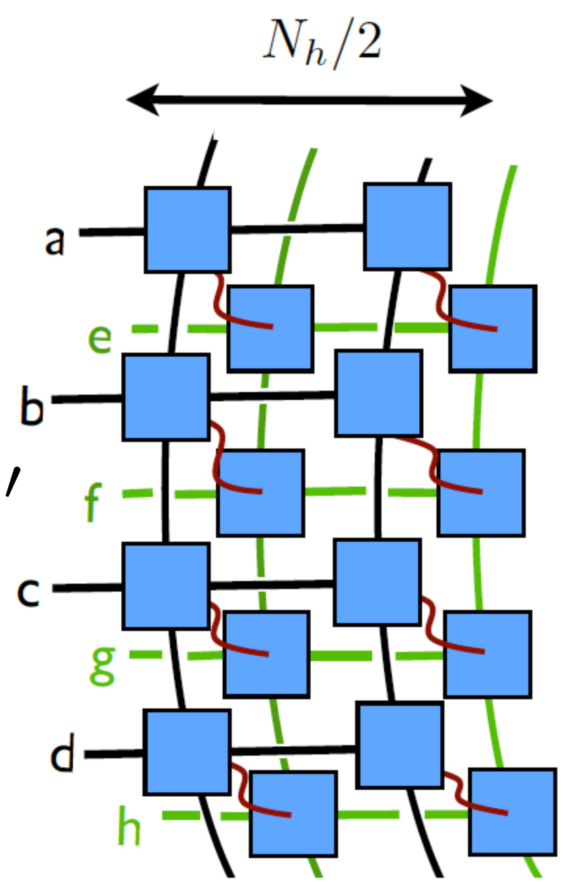
$\rho_A$




$\iota$ A B

$L$

$L'$




$\sigma_b^2$

**lives" on the boundary**

Basic formula: $\rho_A = U\sigma_b^2 U^\dagger$

isometry: maps 2D onto 1D

J. Ignacio Cirac, DP, Norbert Schuch, Frank Verstraete, Phys. Rev. B 83, 245134 (2011)

$$\sigma_b^2 = \exp\left(-H_b\right)$$

Li & Haldane

## Entanglement Hamiltonian highly non-local

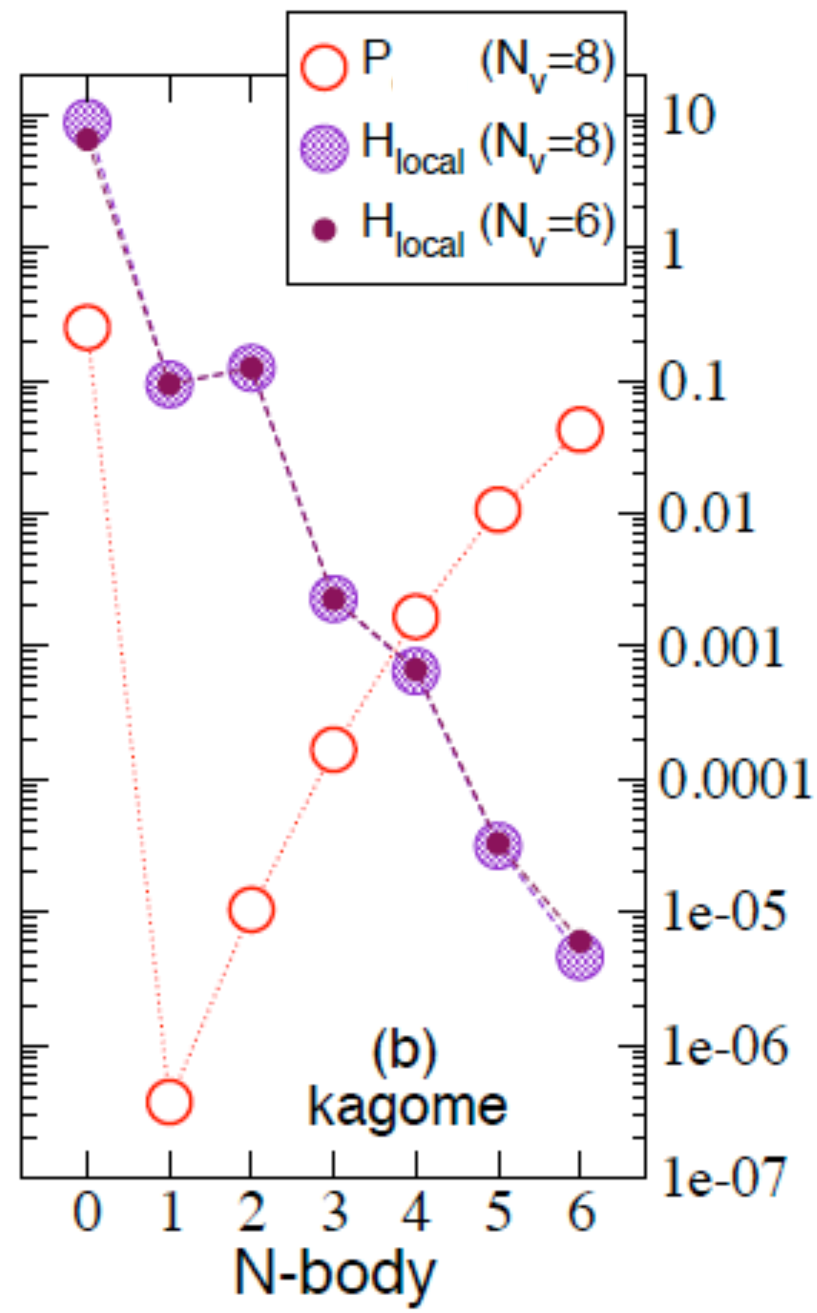
$$\tilde{H}_b = H_1 + \beta_\infty(\mathbf{1}^{\otimes N_v} - \mathcal{P})$$

$$\beta_\infty \rightarrow \infty$$

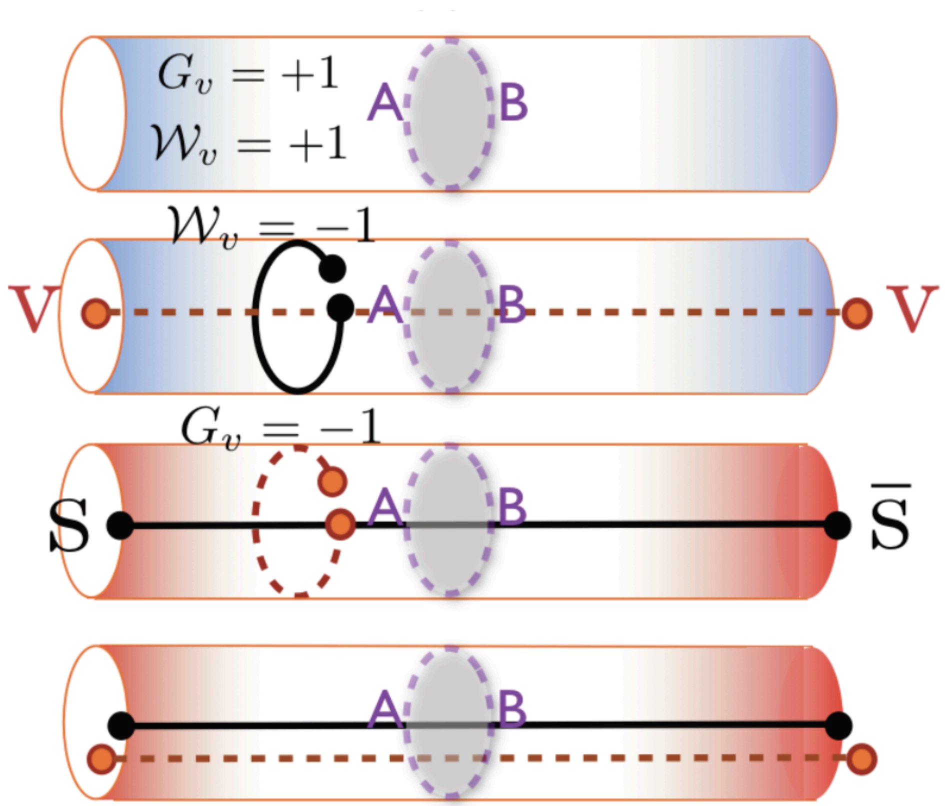
supported by the non-zero eigenvalue sector of the RDM

$$H_1 = H_{\mathrm{local}}\mathcal{P}$$

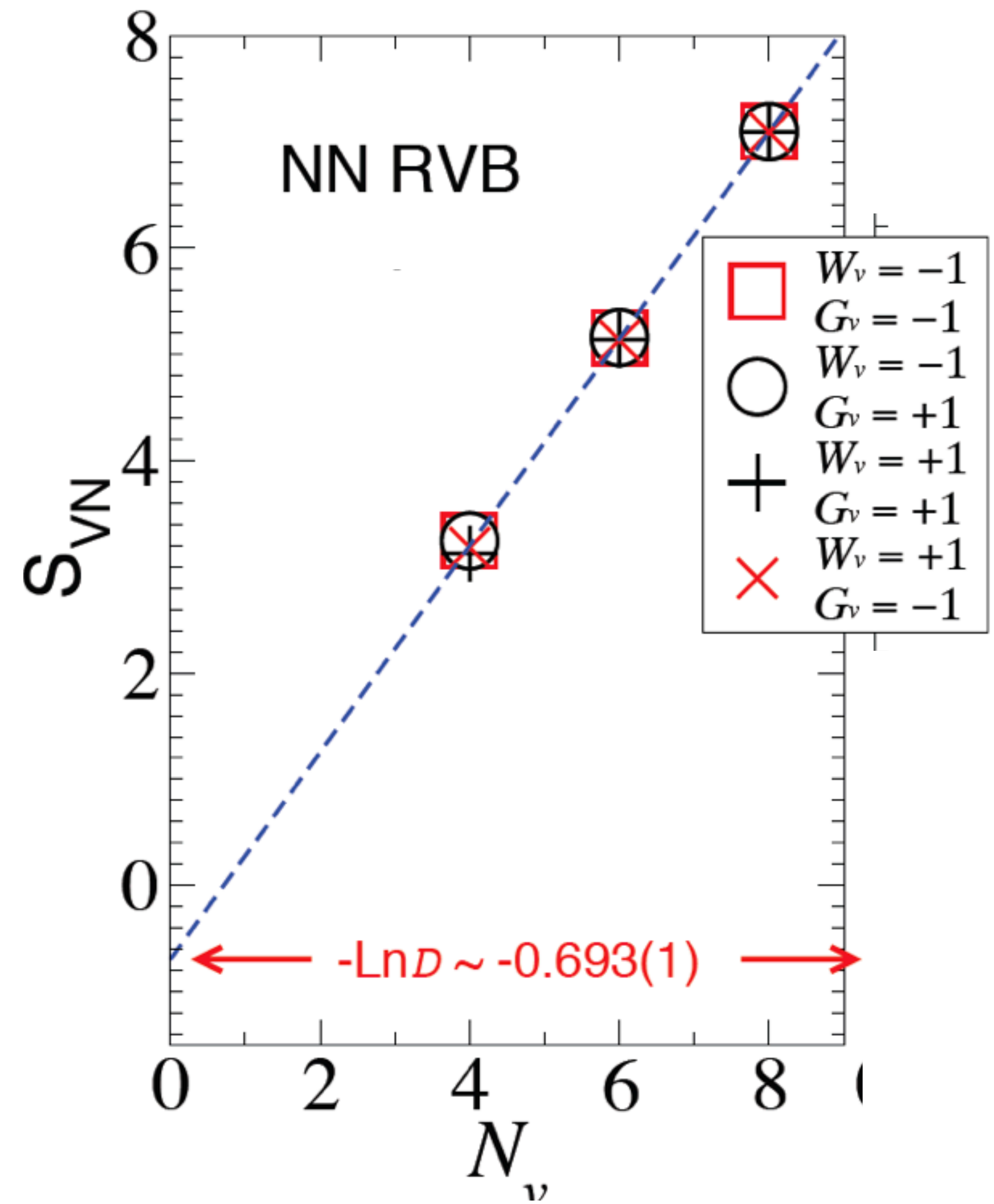
projector characterizing topological sectors



# Numerical results




$$S_{VN} = -\mathrm{Tr}(\sigma_b^2 \ln \sigma_b^2)$$



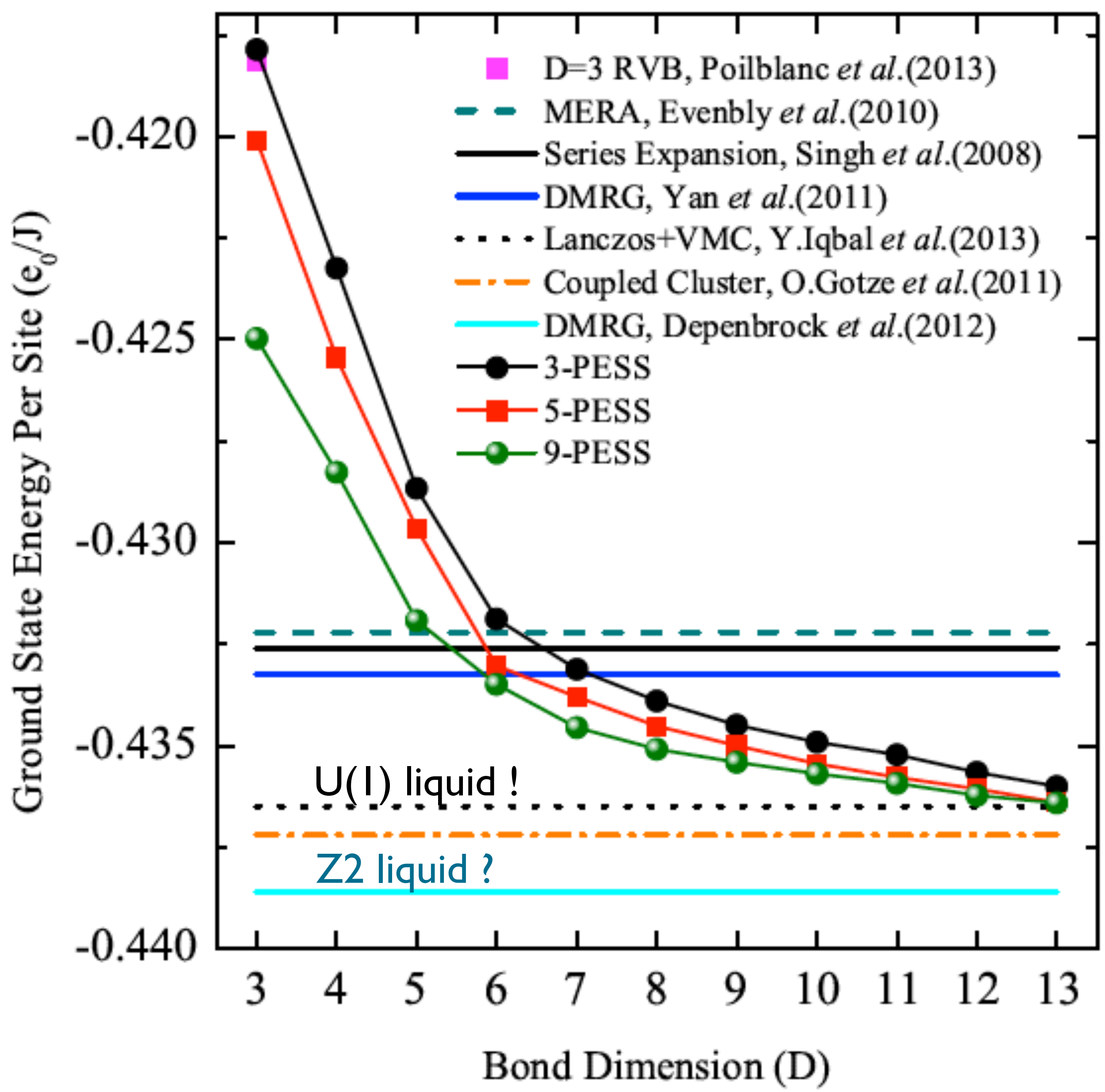

Kitaev & Preskill, 2006

Levin & Wen, 2006

$$S_{\mathrm{VN}} \sim C N_v - \ln D$$

# Improving the RVB / PEPS ...

Z. Y. Xie, J. Chen, J. F. Yu, X. Kong, B. Normand, and T. Xiang, arXiv:1307.5696




Simple update method
based on imaginary-time evolution

# Adding a magnetic field...

$$\mathcal{H} = -\gamma H S_Z = -h S_Z$$

Continuous U(1) symmetry

If U(1) symmetry spontaneously broken:

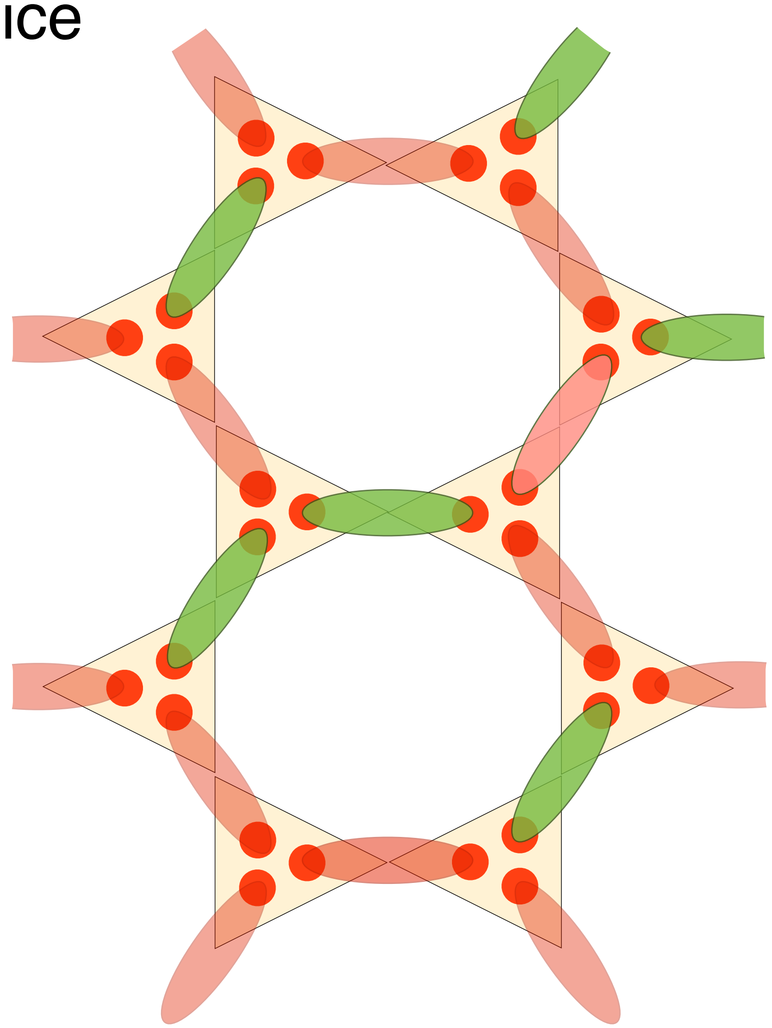
⟹ LR magnetic (tranverse) order similar to superfluid

Could a U(1)-invariant topological liquid be stabilized ?

# Simple exemple:
# spin-3/2 AKLT in a magnetic field

Let's construct a simple ansatz !

Hexagonal lattice



$|22\rangle$

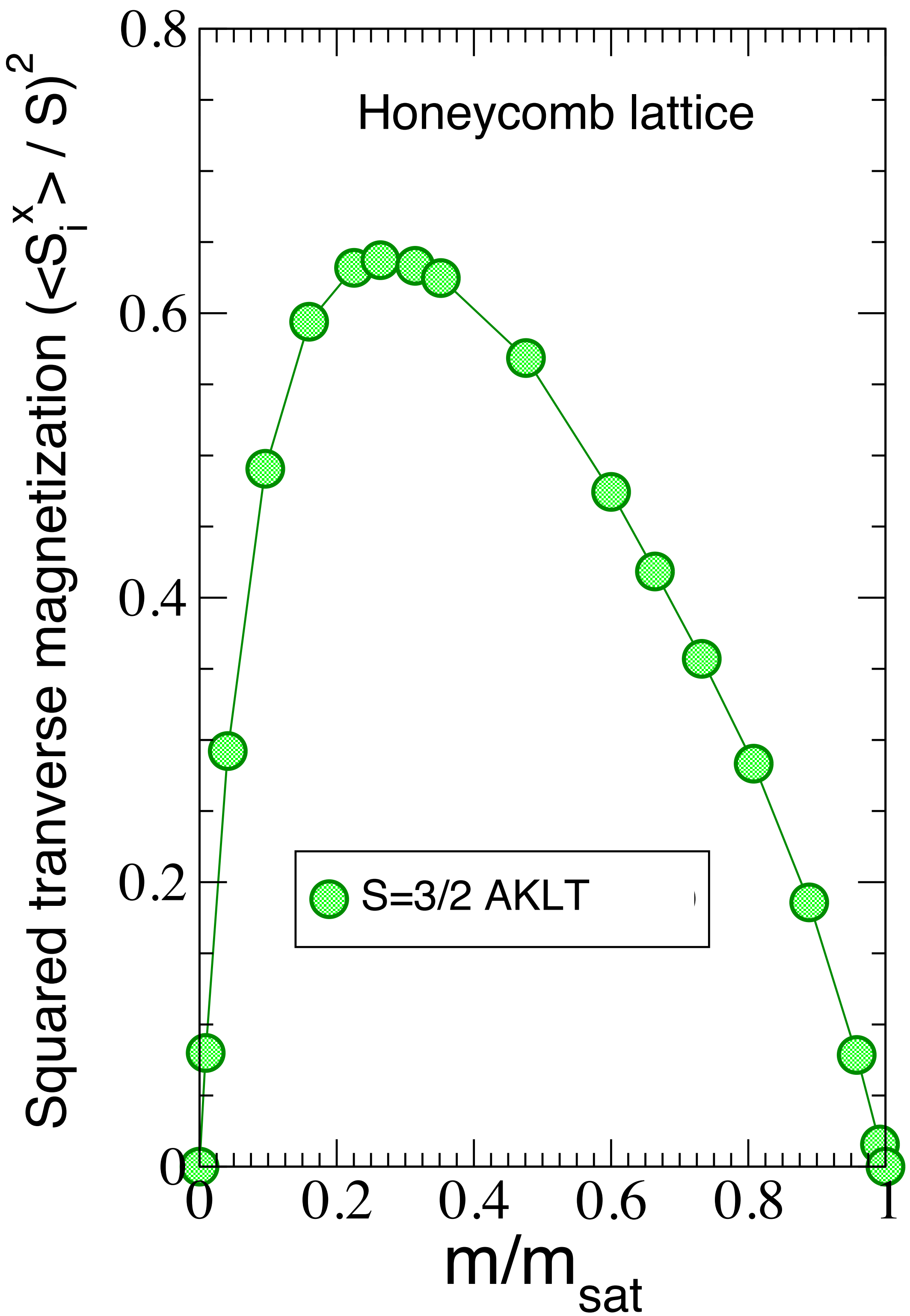
$S_z = +1$

triplons

$|01\rangle - |10\rangle + \beta|11\rangle$

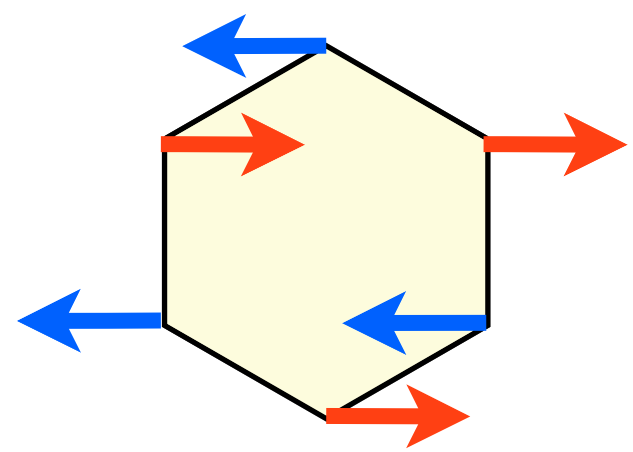
2-parameter family of D=3 PEPS

# Spontaneous U(1) symmetry breaking does occur !




Transverse Néel order

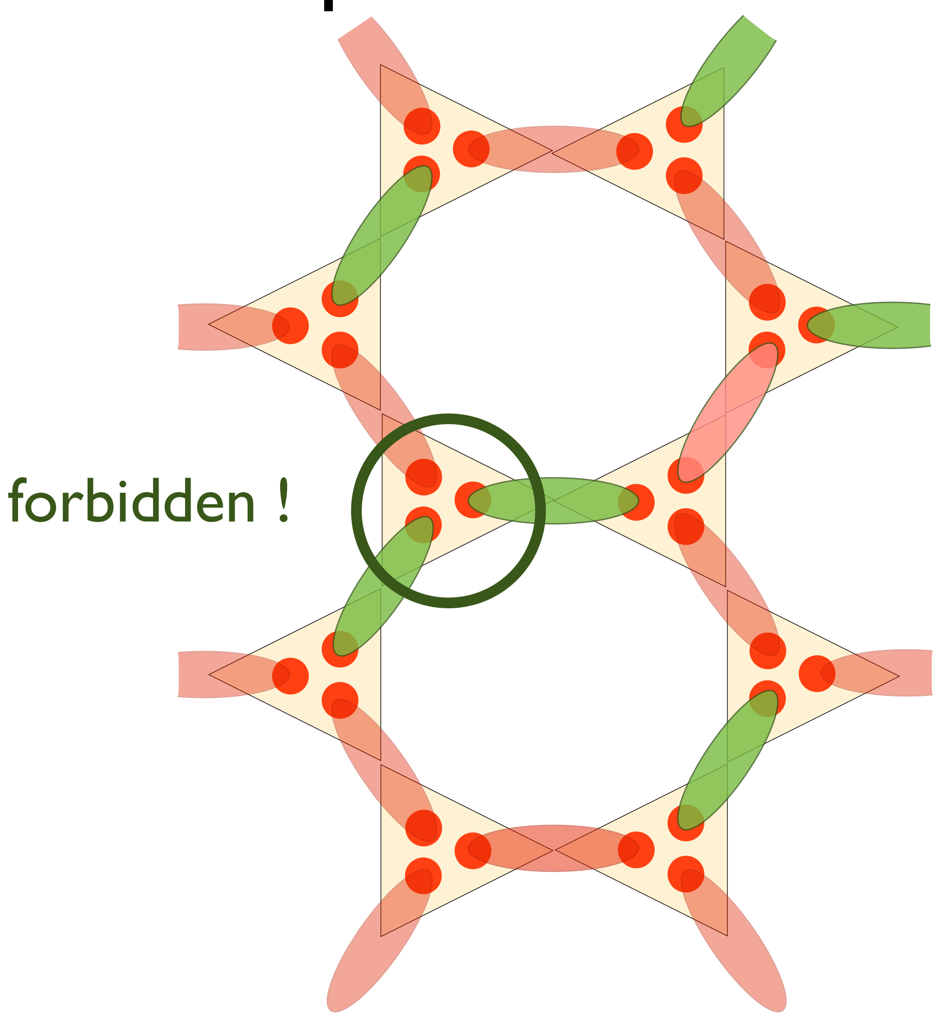
~ superfluid density

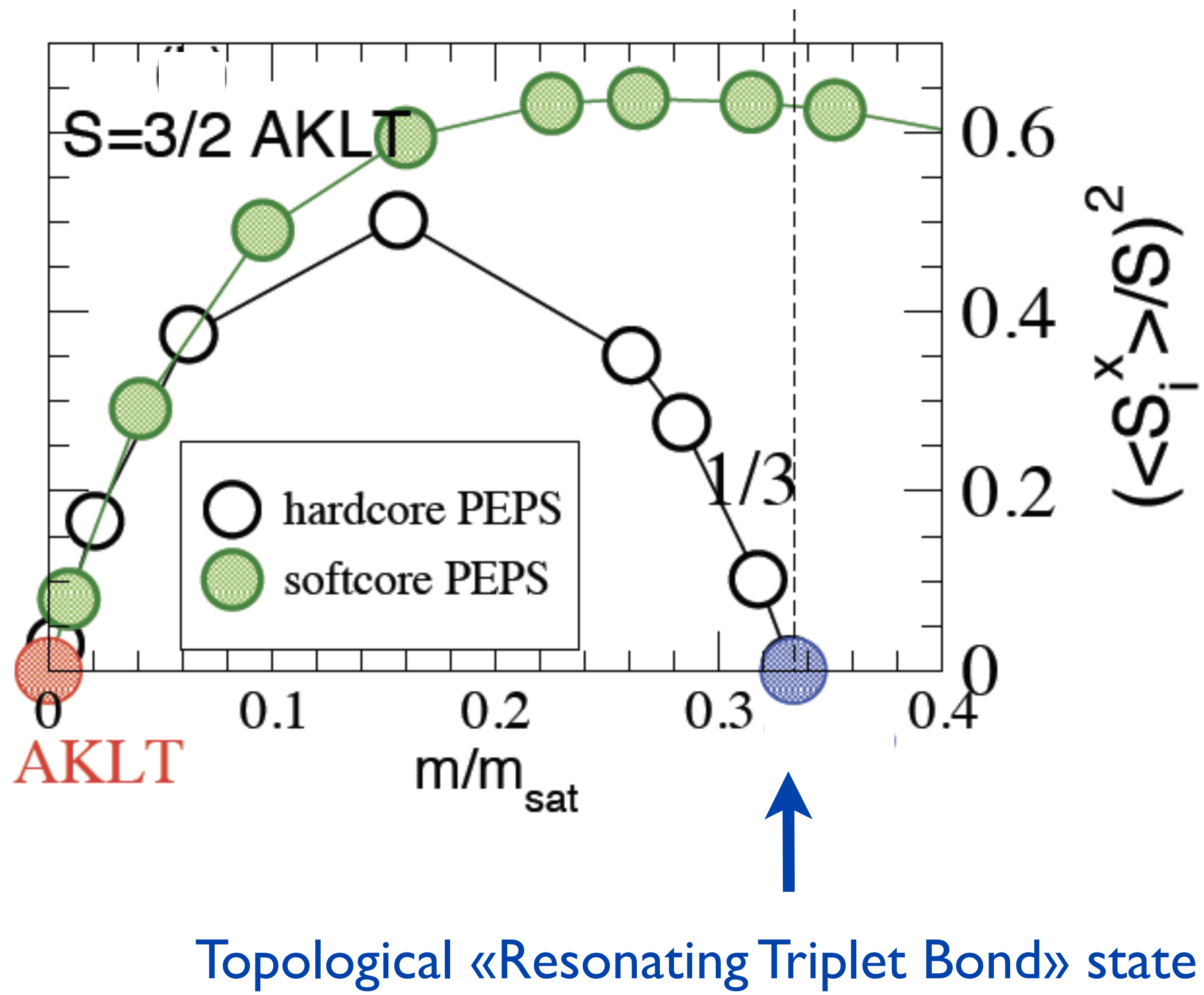


# Topological field-induced spin liquid

IDEA : make the triplets HARDCORE

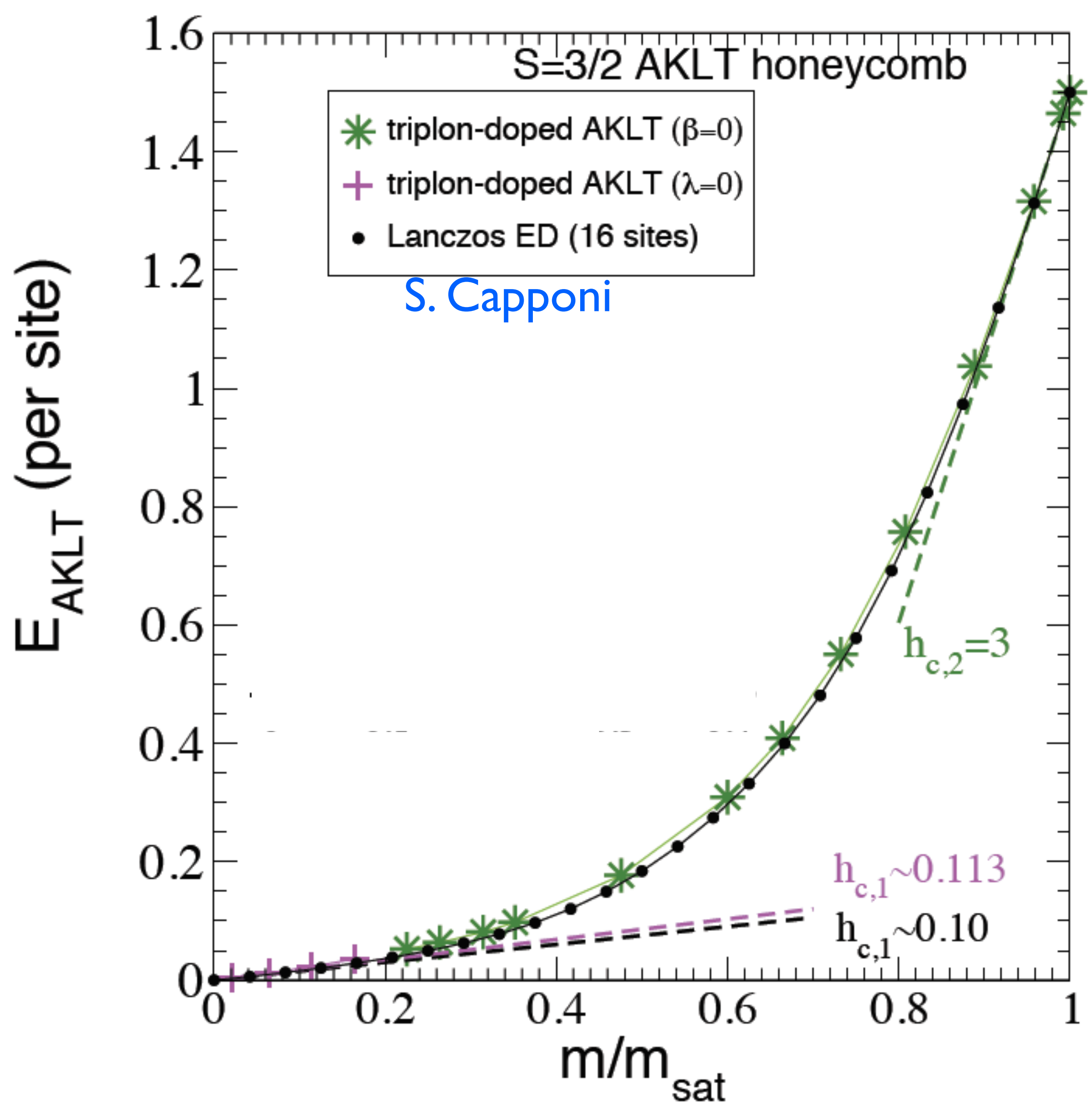
spin-3/2 AKLT




samedi 23 mai 15

Non-topological ansatz is excellent for AKLT hamiltonian + Zeeman term



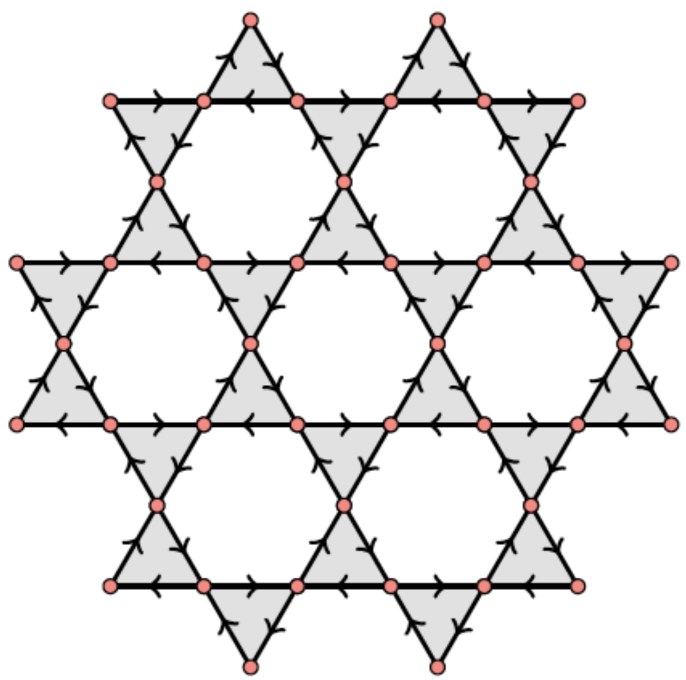

$$h = \partial E / \partial m$$

# Topological chiral spin liquid

bosonic analog of the FQHS

Kalmeyer & Laughlin, PRL 1987

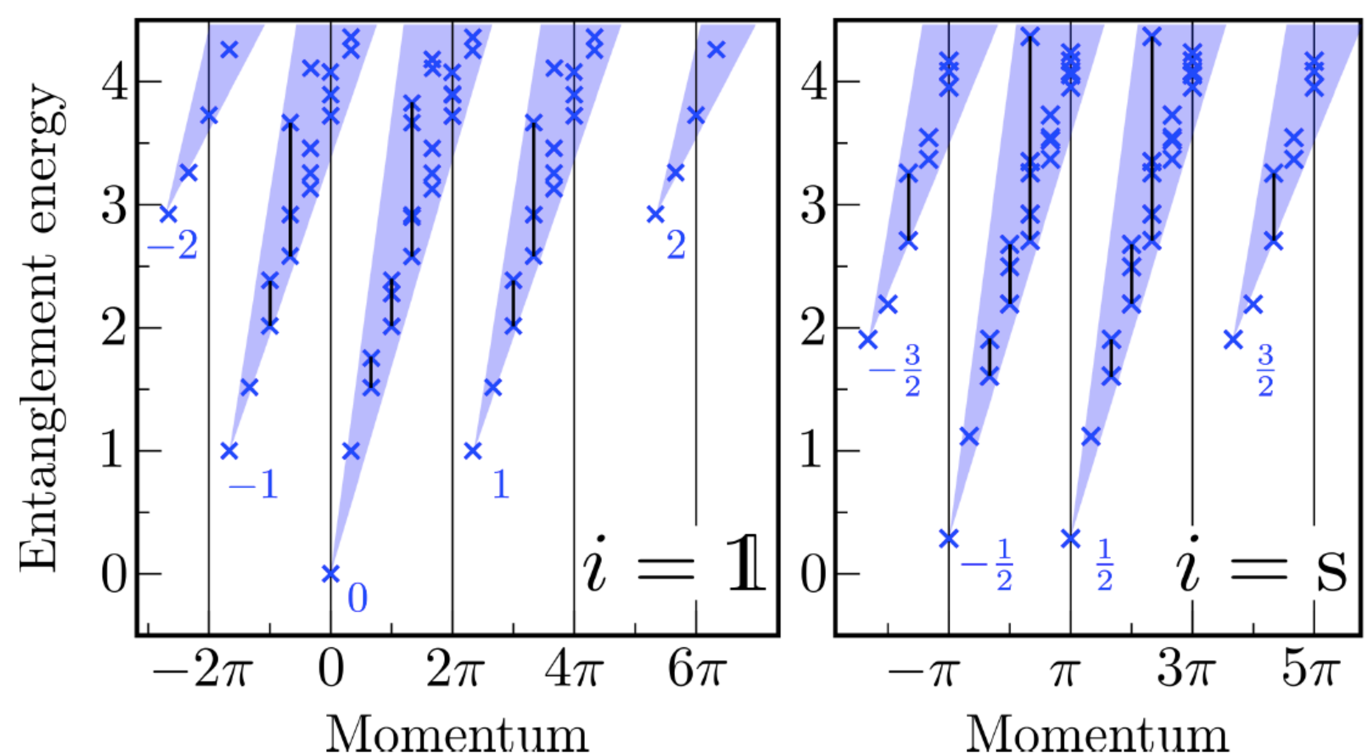
B. Bauer et al., Nature Communications 5, 5137 (2014)



$$\cdots + J_\chi \sum_{i,j,k\in\triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

Entanglement spectrum:
chiral edge modes

$SU(2)_1$ CFT

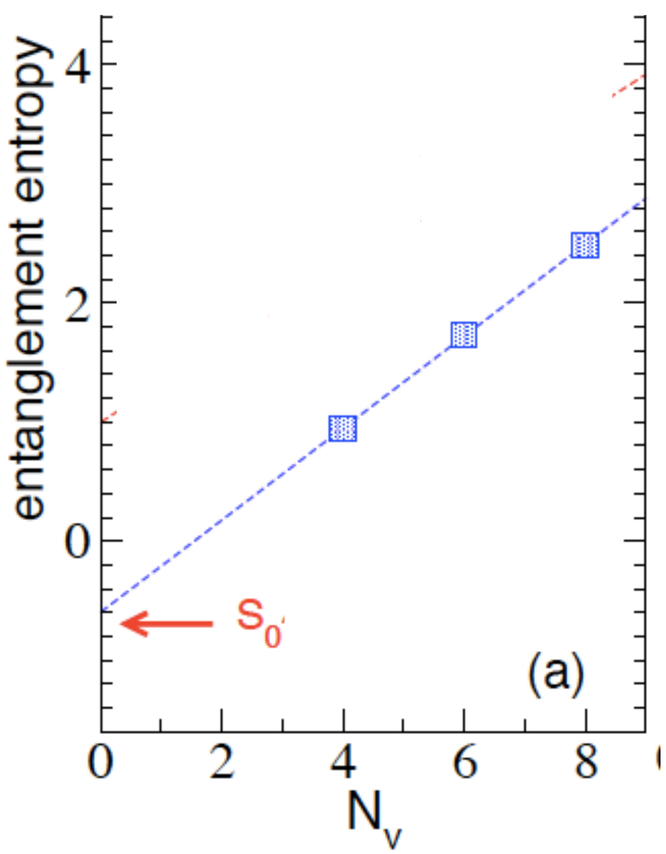


# Idea : «deform» the D=3 RVB tensor... (square lattice)

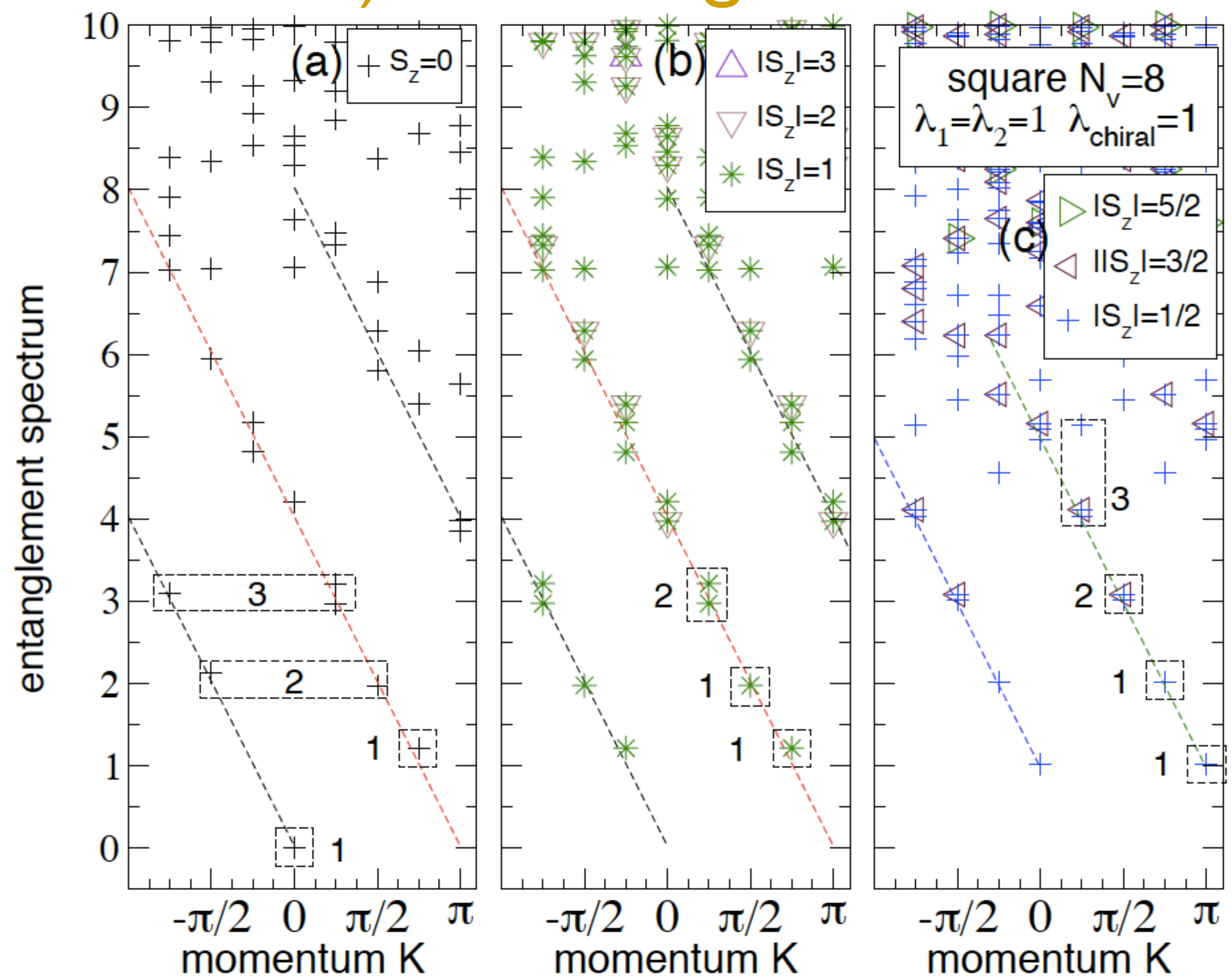
$d + i\, d$ **symmetry:** $A = A_{d_{x^2-y^2}}(\lambda_1) + i\,\lambda_2\, I_{d_{xy}}$

$$\Psi = \Psi_s + i\Psi_g \rightarrow_{\text{reflec.}} \Psi_s - i\Psi_g = \Psi^*$$

1) four topological sectors




2) chiral edge modes




CFT ?

3) infinite correlation length !

# Summary

* Qualitative understanding of (simple) correlated phases (topo SL, SF, SC,...)

* Systematic improvement can be made for physical Hamiltonians : iPEPS

* Can chiral PEPS describe GS of physical Hamiltonian ?

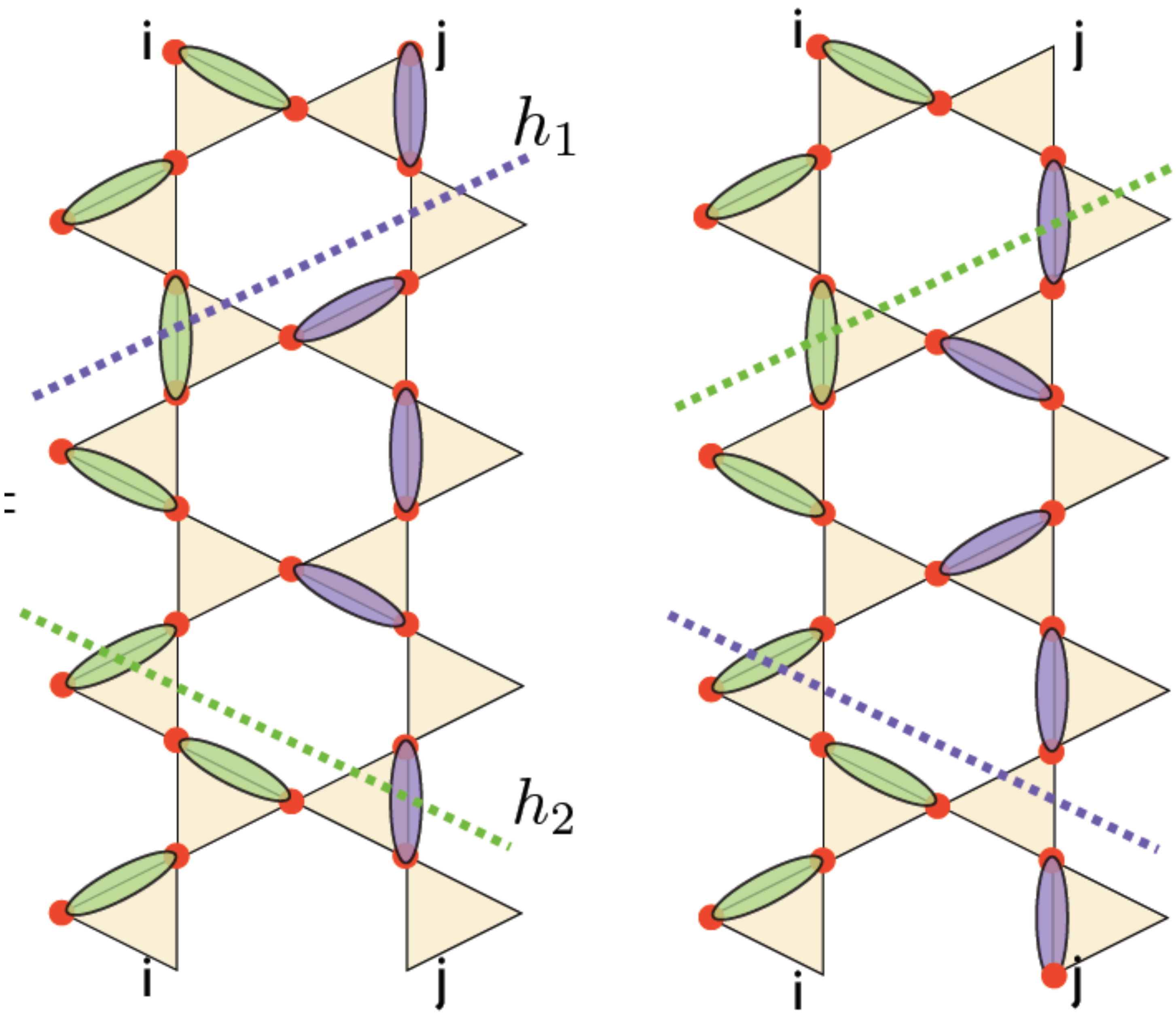
* Can chiral PEPS describe **gapped** chiral SL?

# Supplementary slides

# Disconnected topological sectors in the space of dimer lattice coverings

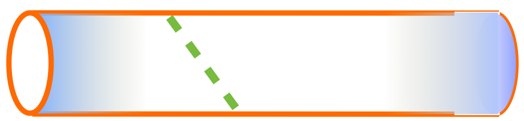
On a **cylinder**:

$4 \times 2$ cylinders

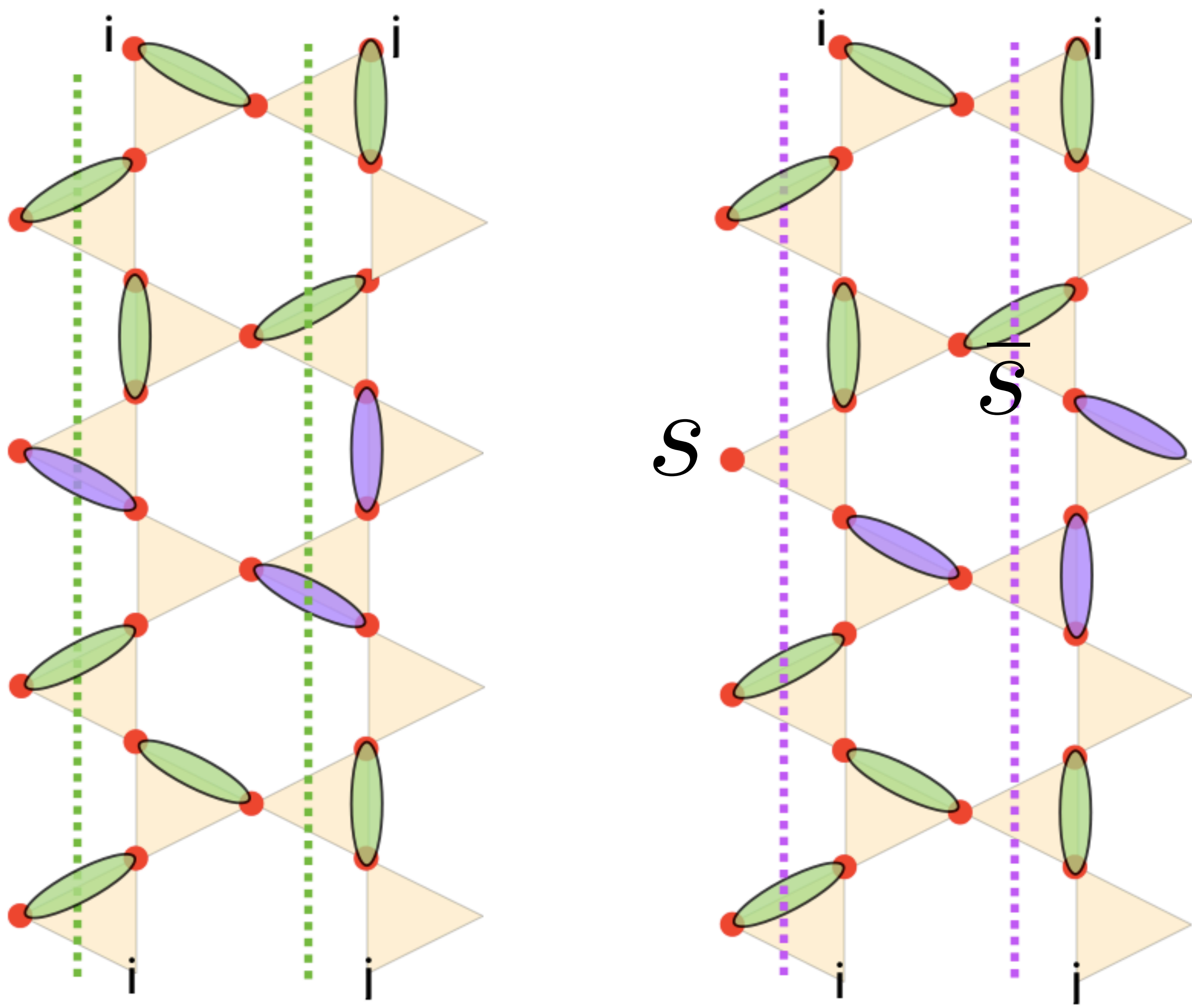



conserved parity: $G_h = +1$ $G_h = -1$

# Fix the cylinder boundaries



cylinder geometry



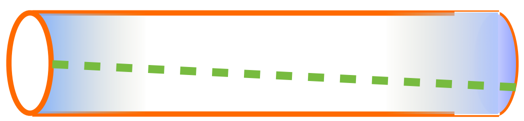

«even»

$G_v = +1$

«odd»

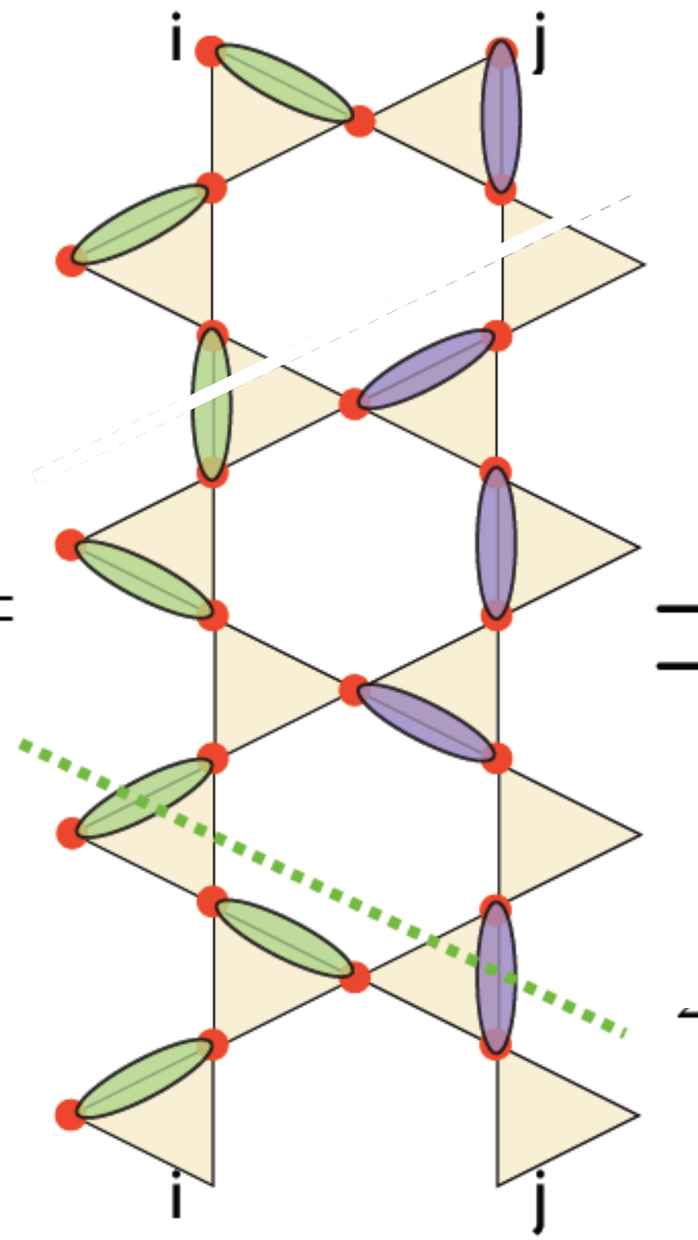
$G_v = -1$

# Eigenstates of a «Wilson loop» operator

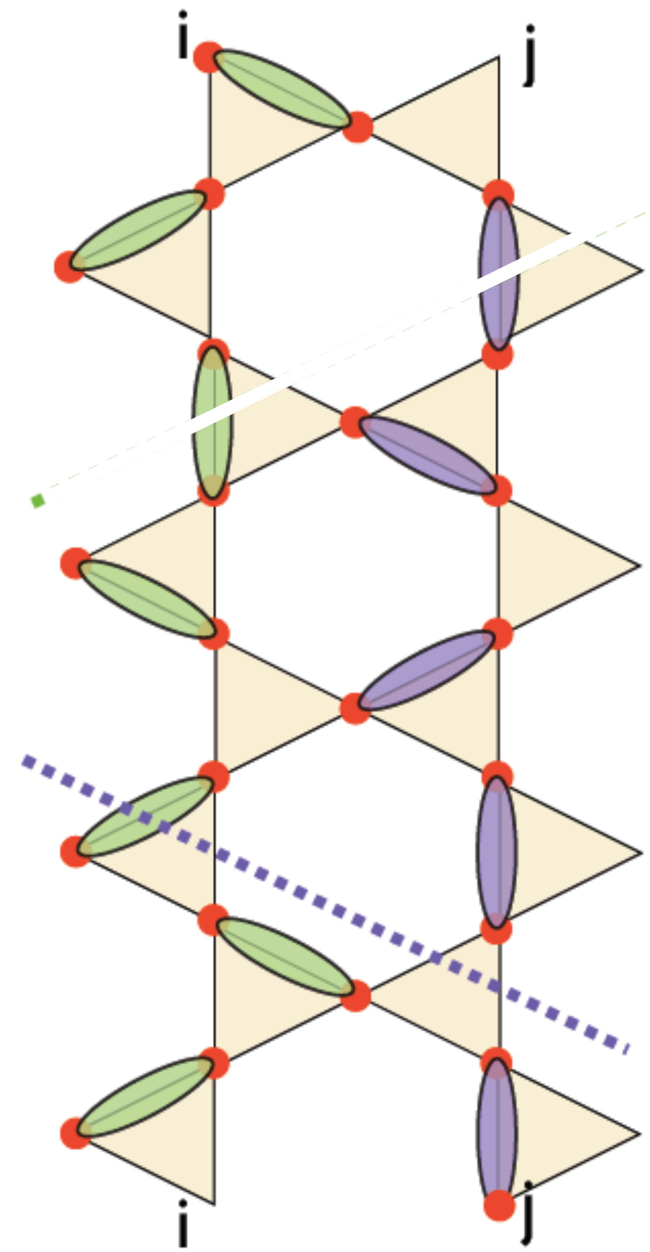


cylinder geometry

$\Psi^{\pm}_{\mathrm{RVB}} =$




$\pm$

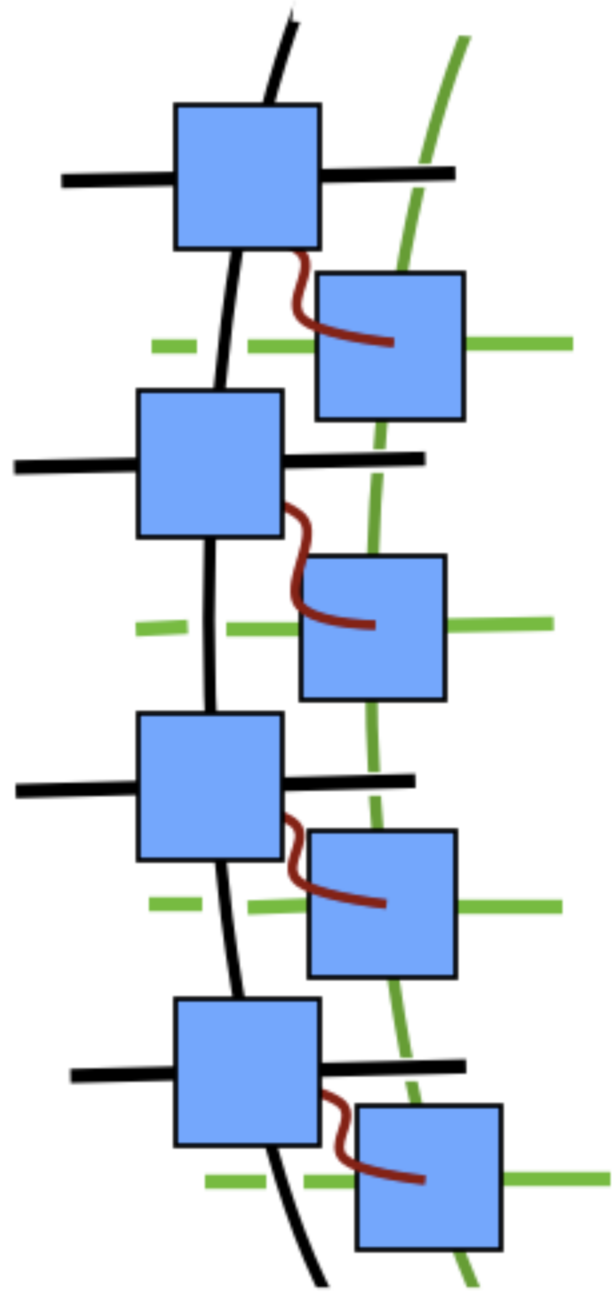



$+$ no vison flux: $w_v = +1$

$-$ $\mathbb{Z}_2$ vison flux: $w_v = -1$

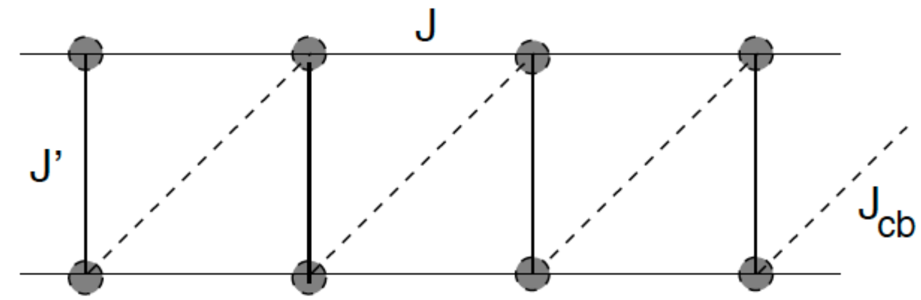
# Build «double layer» tensor network by contracting physical variables

$\langle \Psi | \Psi \rangle$ ⇒

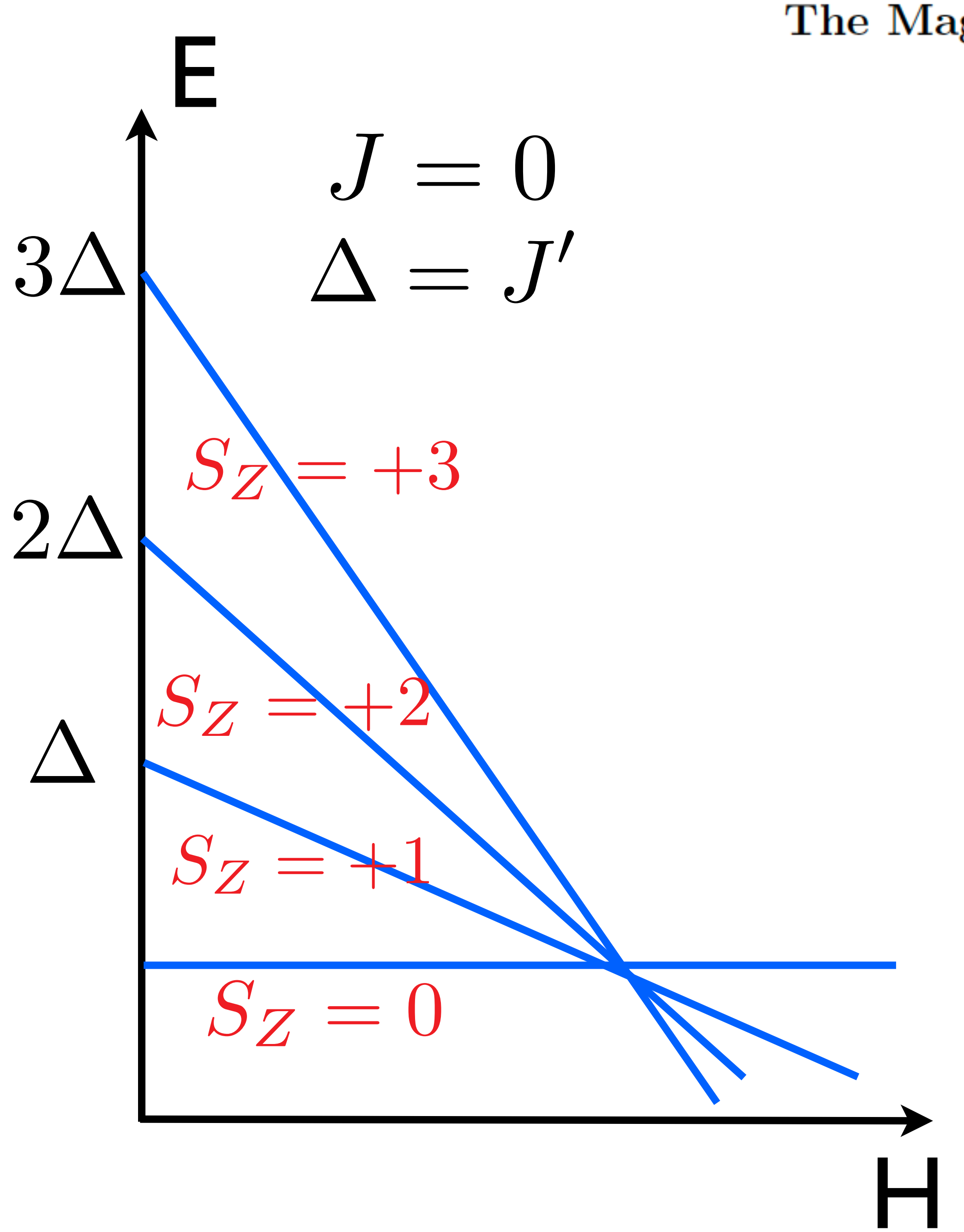


«transfer matrix»

if D small enough **exact contractions** possible...

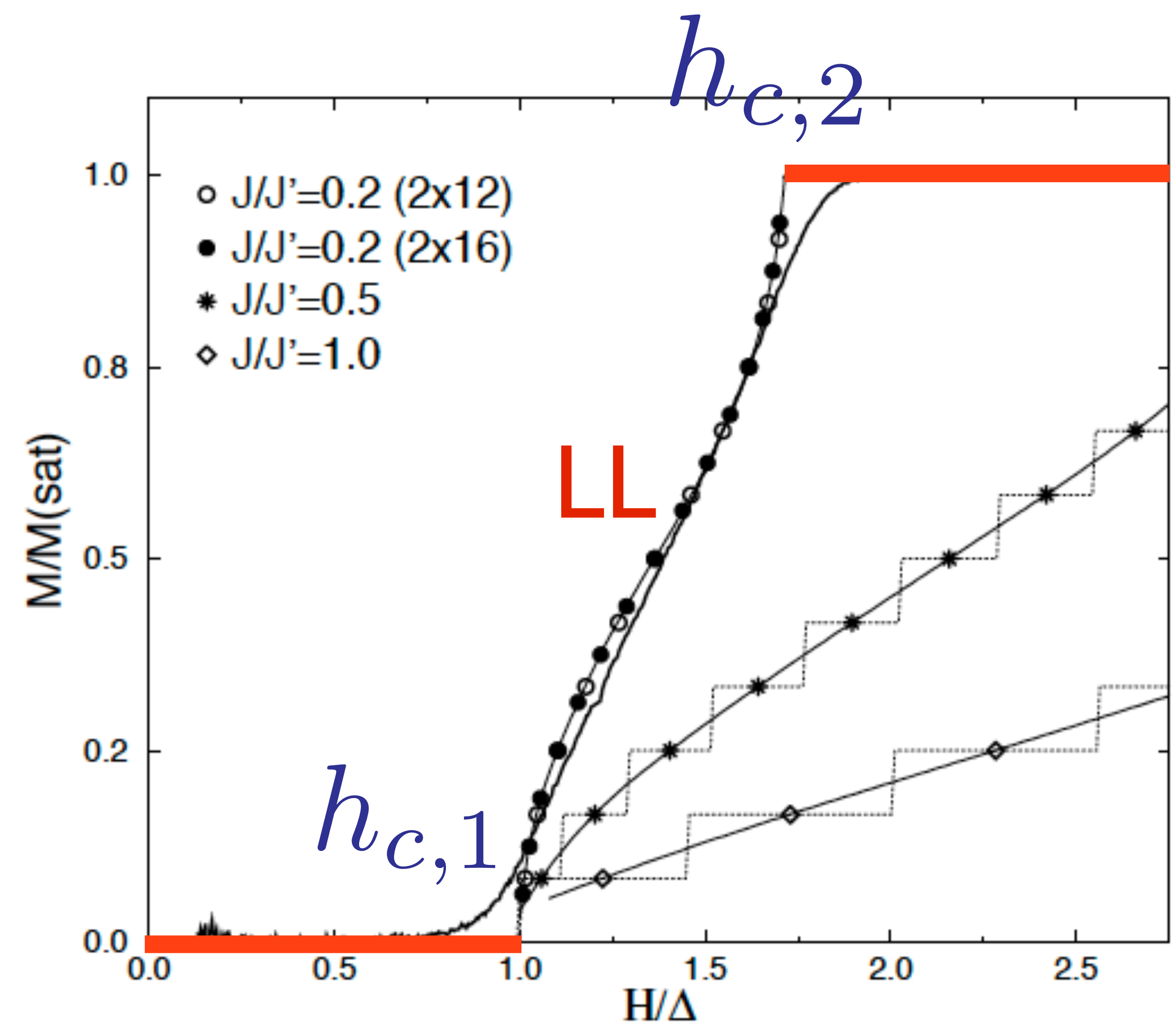



# Decoupled dimers




The Magnetization of $Cu_2(C_5H_{12}N_2)_2Cl_4$ : A Heisenberg Spin Ladder System.

C.A. Hayward[1], D. Poilblanc[1] and L.P. Lévy[2] (1996)



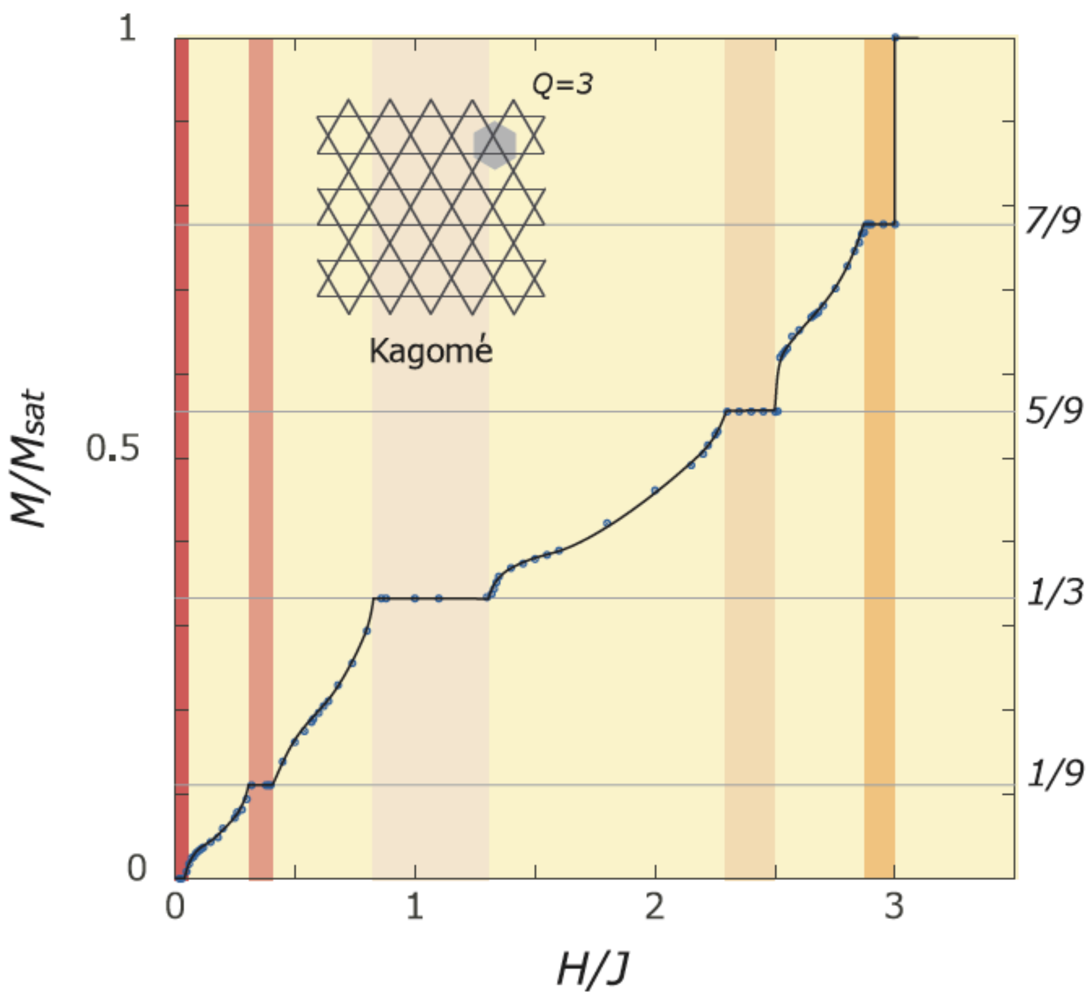

What happens in 2D ?

# Uncompressible plateaus ?

motivation : magnetization curve can exhibit fractional plateaus corresponding to uncompressible phases

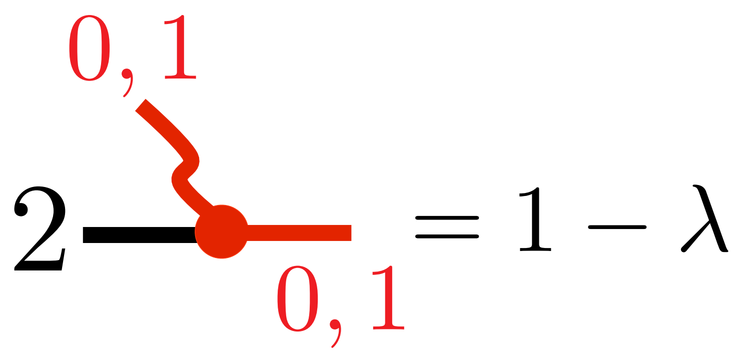
Sylvain Capponi, Oleg Derzhko, Andreas Honecker, Andreas M. Lauchli, and Johannes Richter, arXiv:1307.0975.

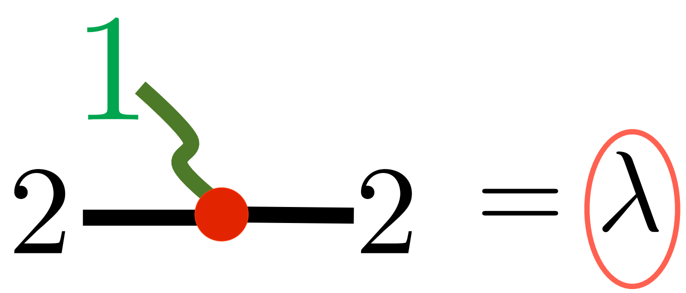
Satoshi Nishimoto, Naokazu Shibata, and Chisa Hotta, Nature Communications (in press), arXiv:1307.3710.



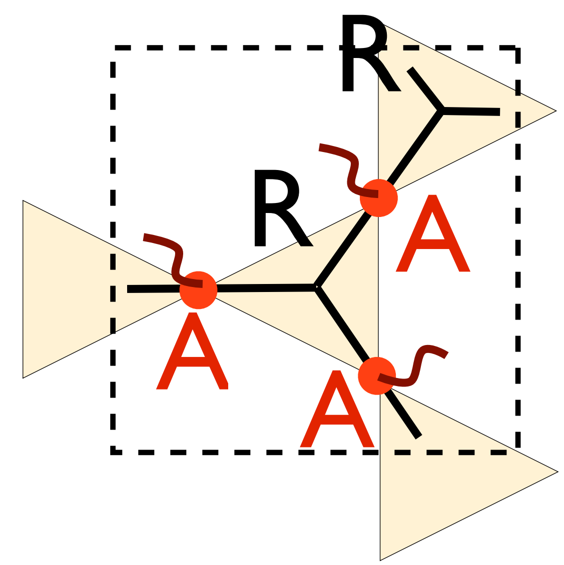

Can it be described at the (simple) PEPS variational level ?

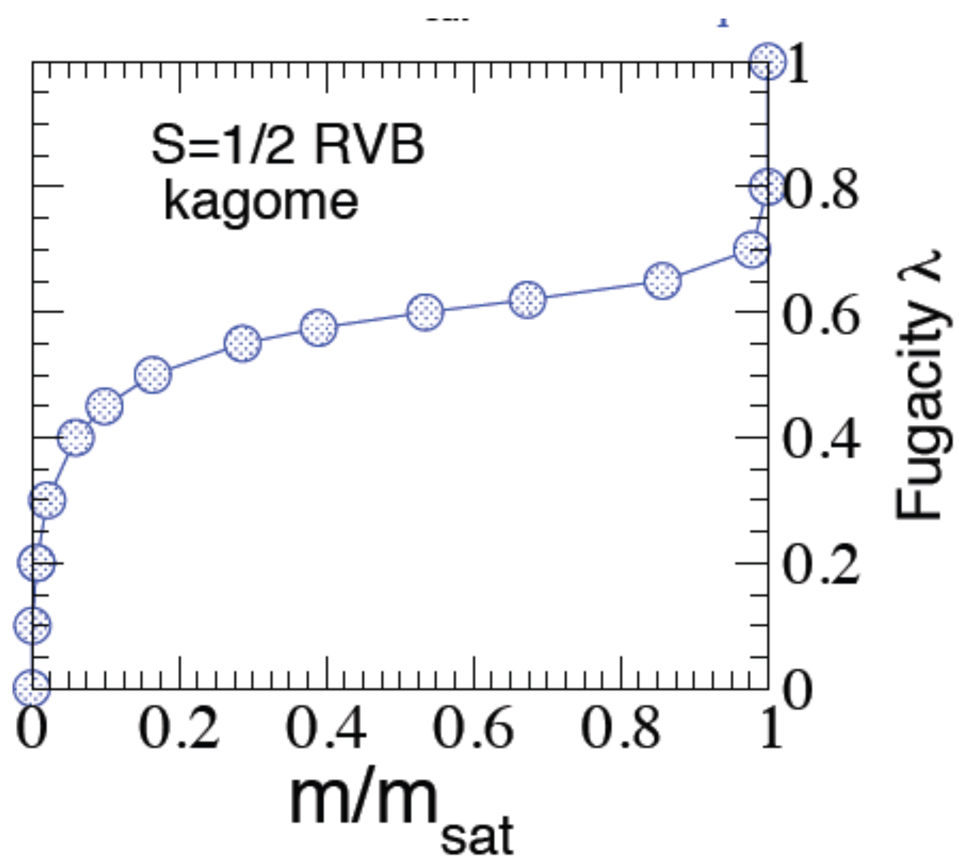
# Example : spinons in D=3




RVB kagome



# Entanglement Hamiltonian (acting on the edge)

$$\sigma_b^2 = \exp\left(-H_b\right)$$

The spectrum of Hb is in one-to-one with the true edge spectrum !

Li & Haldane

edge site operators :

$$D \times D \text{ matrix} \Rightarrow \text{basis of } D^2 \text{ operators } x_\lambda$$

$$H_b = c_0 N_v + \sum_{\lambda,i} c_\lambda \hat{x}^i_\lambda + \sum_{\lambda,\mu,r,i} d_{\lambda\mu}(r)\,\hat{x}^i_\lambda \hat{x}^{i+r}_\mu + \sum_{\lambda,\mu,\nu,r,r',i} e_{\lambda\mu\nu}(r,r')\,\hat{x}^i_\lambda \hat{x}^{i+r}_\mu \hat{x}^{i+r'}_\nu + \cdots ,$$

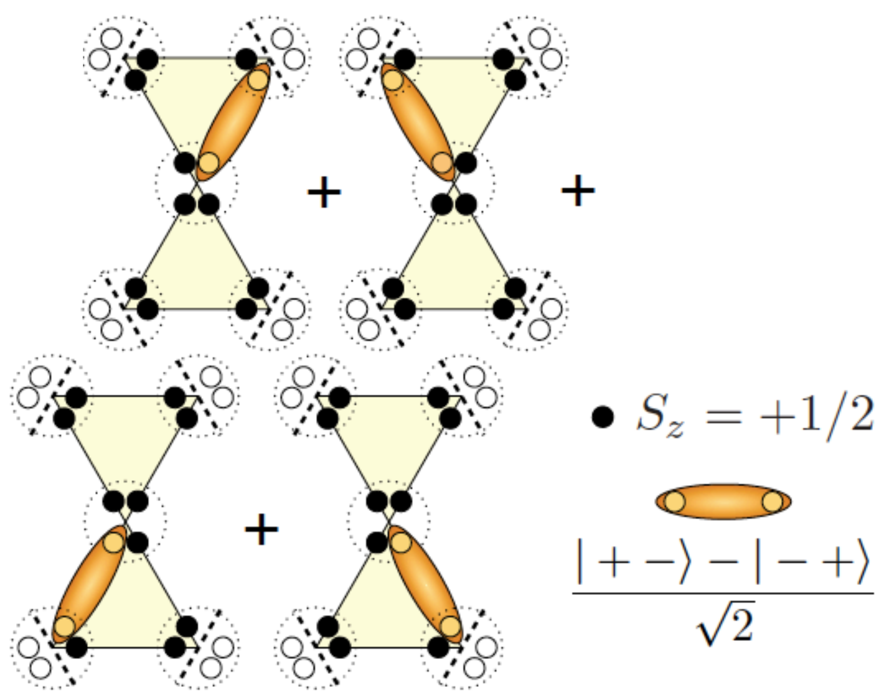
Is it local ?

More interesting physics on the kagome lattice !

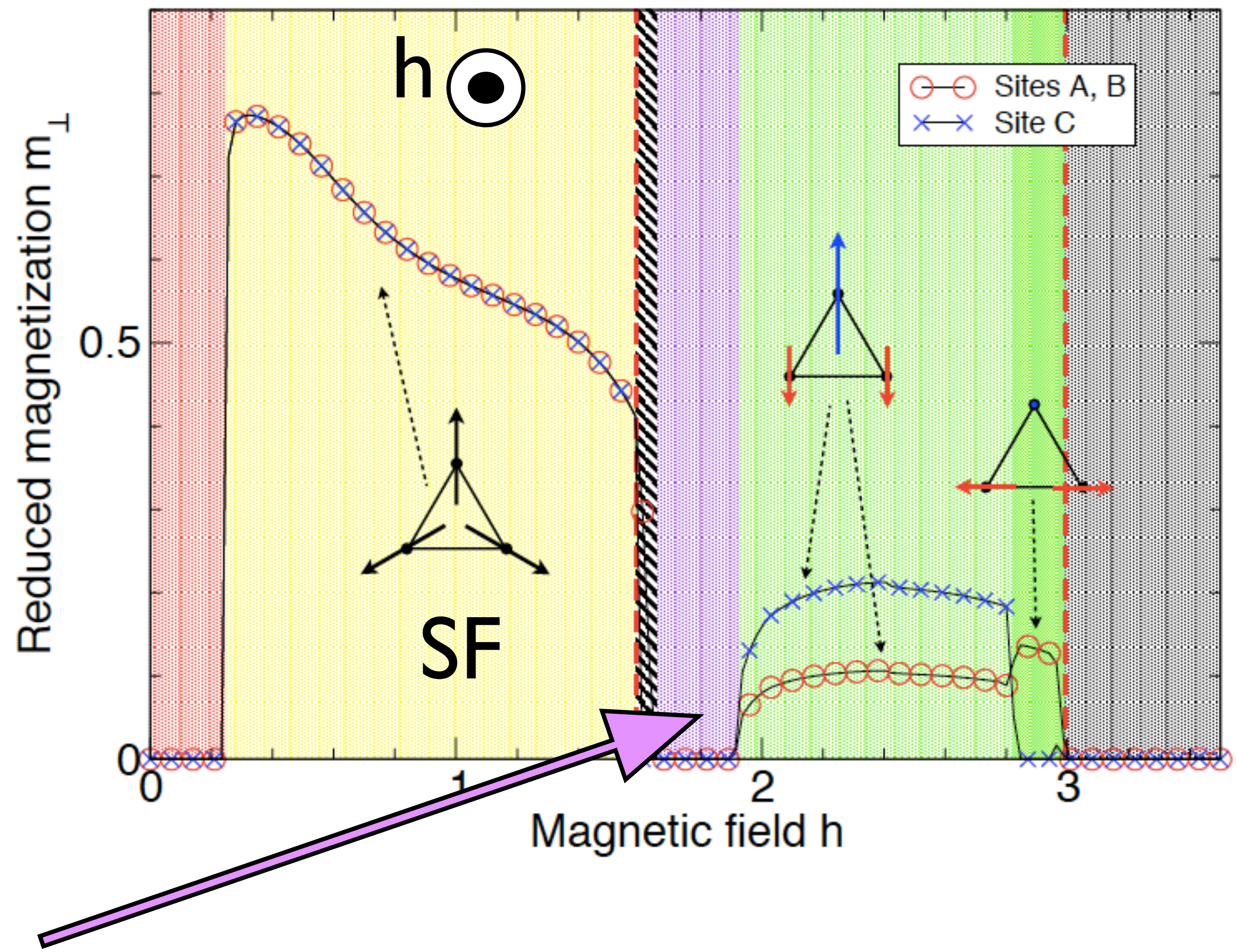
S=2 AKLT

uncompressible «nematic»

$m/m_{\text{sat}} = 5/6$




SF + N




Thibaut Picot and DP, PRB 2015

(iPEPS + TEBD)