Characterizing spin liquids and topological orders in model wavefunctions and Hamiltonians

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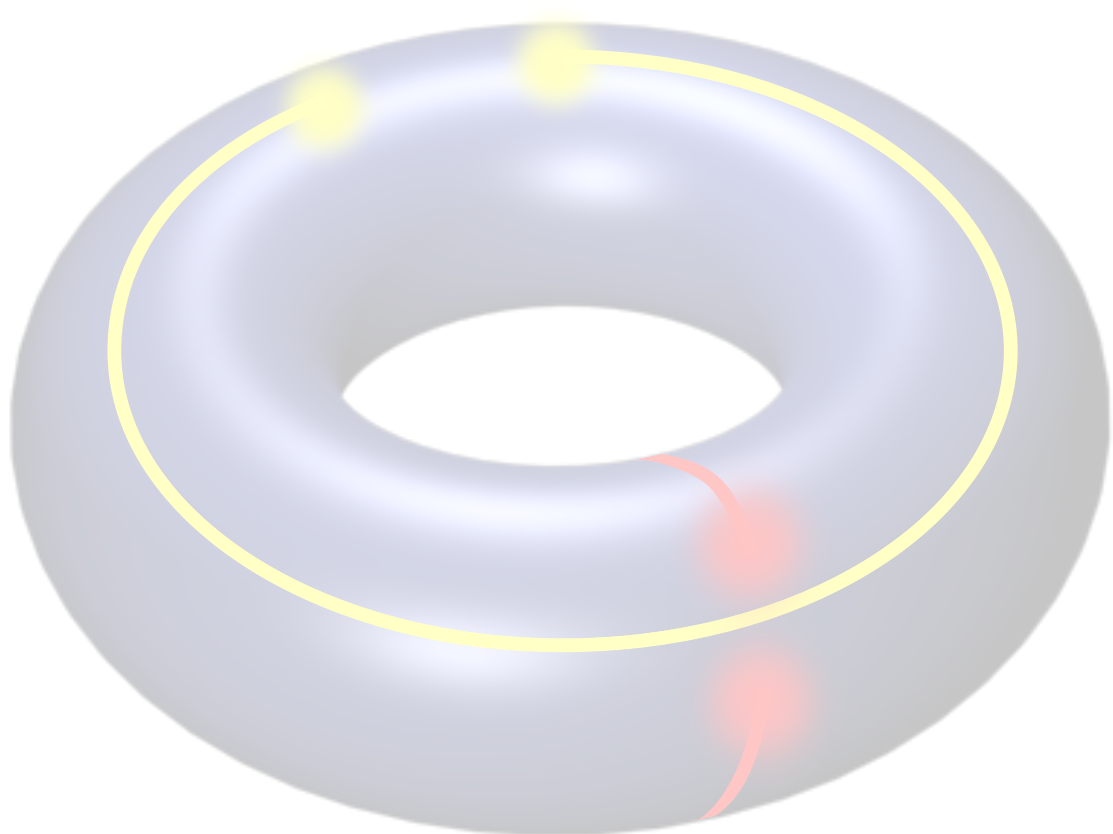
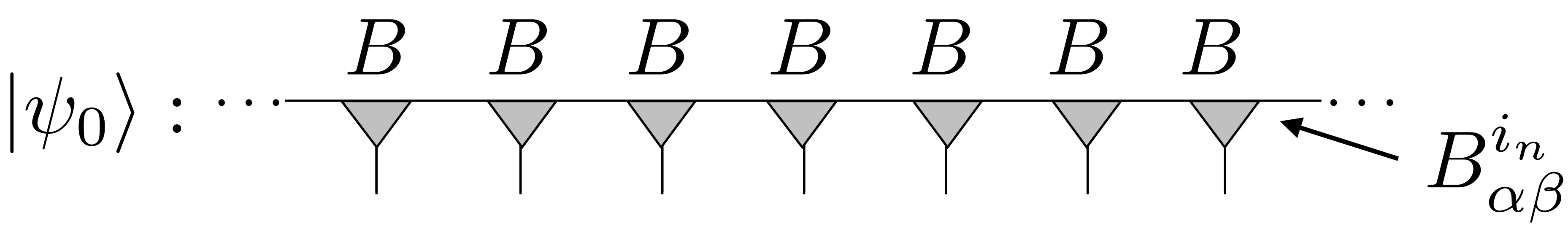


Santa Barbara, May 21st

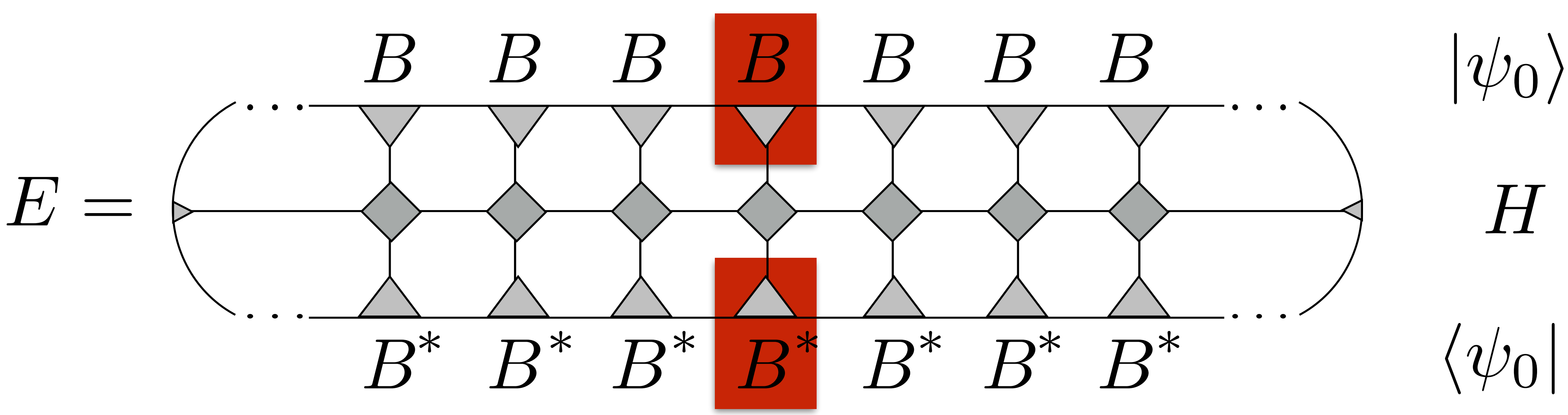
Infinite Density Matrix Renormalization Group

- **Matrix-Product State** representation of the ground state [Fannes et al '92]



- Area law in 1D [Hastings '07] : $d^{L\to\infty} \to d\chi^2$

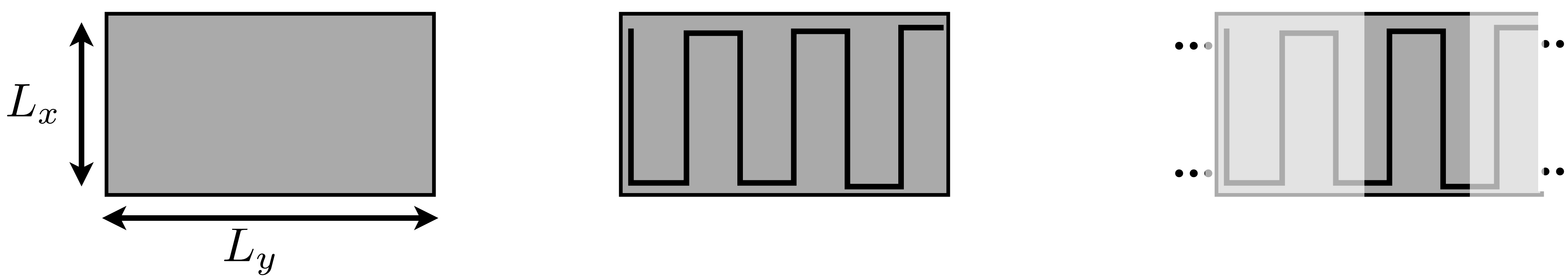
- Find the ground state iteratively [White '92, McCulloch '07]



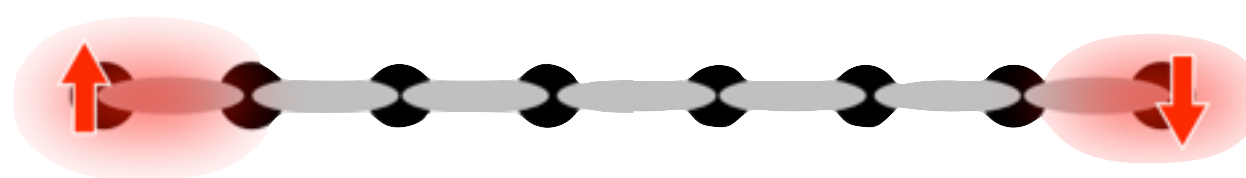
iDMRG

- Efficient variational calculation of the ground state for 2D

ED: $\mathcal{O}(e^{L_x L_y})$ DMRG: $\mathcal{O}(L_y e^{L_x})$ iDMRG: $\mathcal{O}(e^{L_x})$



1D symmetry protected topological phases

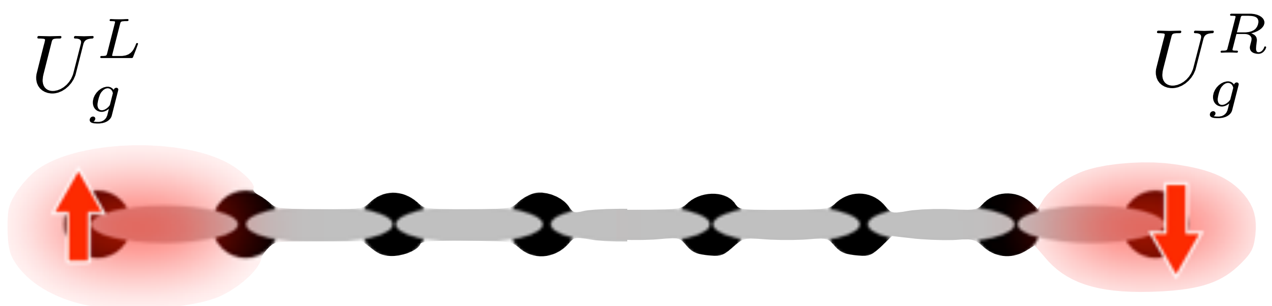


FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).
FP, E. Berg, A.M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).
F.P and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).
C.-Y. Huang, X. Chen, and FP, Phys. Rev. B **90**, 045142 (2014).

1D symmetry protected topological phases

- Hamiltonian and (gapped) ground state $|\psi_0\rangle$ **symmetric under** $g, h \in G$

$|\psi_0\rangle$:



➡ Linear on-site representation $u_g u_h = u_{gh}$

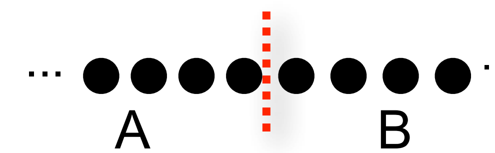
➡ **Projective representations** $U_g U_h = e^{i\phi(g,h)} U_{gh}$

- Classified by the **second cohomology** group $H^2[G, U(1)]$ ("complete" classification [Chen et al. '11; Schuch et al '11])

FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B 81, 064439 (2010).

1D symmetry protected topological phases

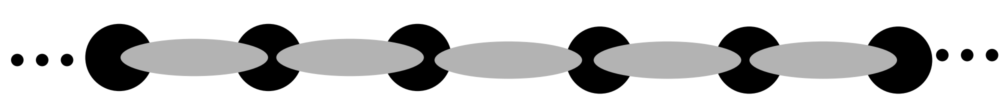
Schmidt decomposition (SVD)



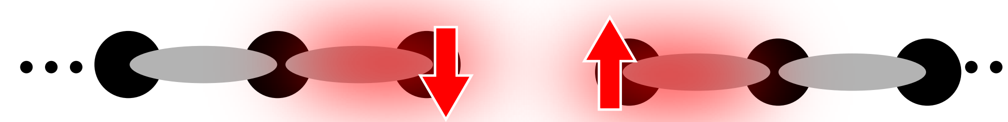
- Decompose a state with respect to a bipartition:

$$|\psi\rangle \;=\; \sum_{i=1}^{N_A}\sum_{j=1}^{N_B} C_{ij}|i\rangle_A|j\rangle_B = \sum_{\gamma}\lambda_\gamma|\phi_\gamma\rangle_A|\phi_\gamma\rangle_B$$

- "Artificial" edges give access to the edge modes



$|\psi\rangle$



$|\phi_\gamma\rangle_A$ $\qquad$ $|\phi_\gamma\rangle_B$

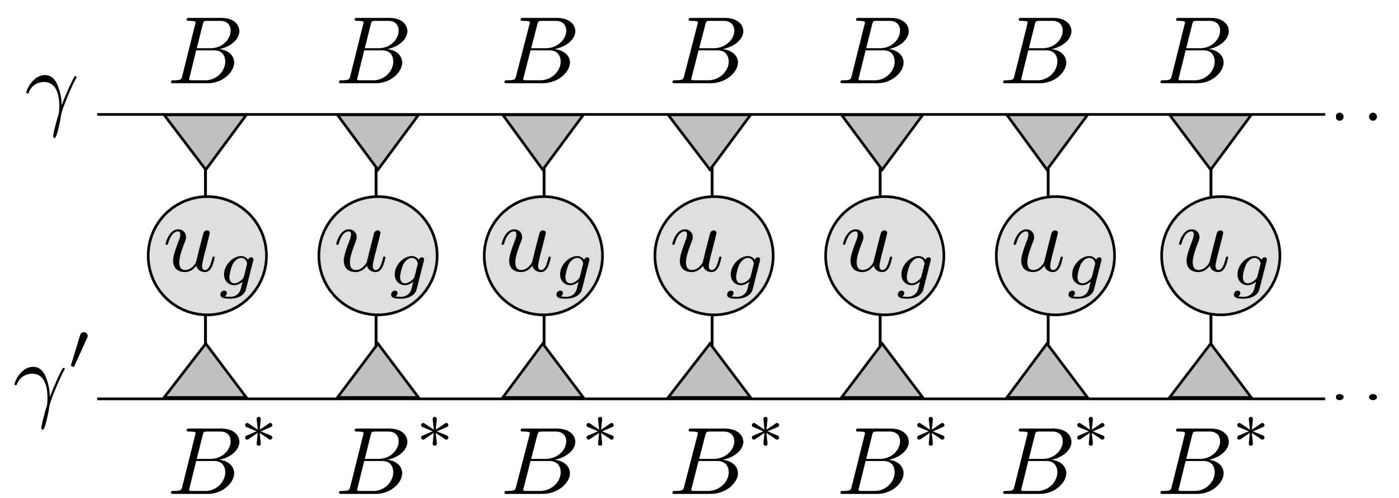
- Projective representation in terms of Schmidt states

$$[U_g]_{\gamma\gamma'} = \langle\phi_\gamma|\bigotimes_j u_g|\phi_{\gamma'}\rangle$$

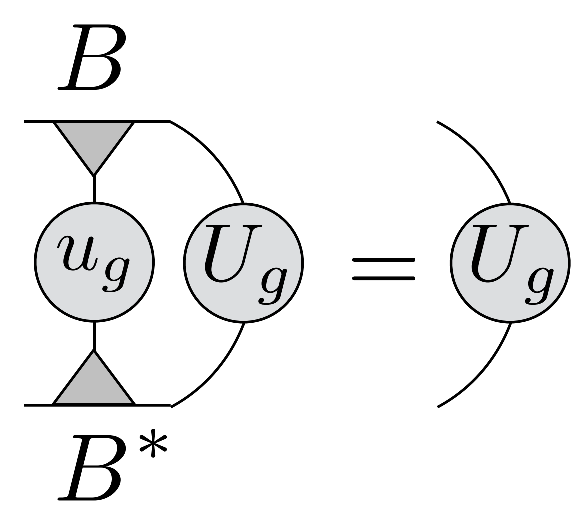
1D symmetry protected topological phases

- Matrix-product state gives direct access to the Schmidt decomposition (canonical form)

$|\phi_\gamma\rangle_B$:
$[U_g]_{\gamma\gamma'} =$



- Dominant eigenvector of the “mixed” transfer matrix

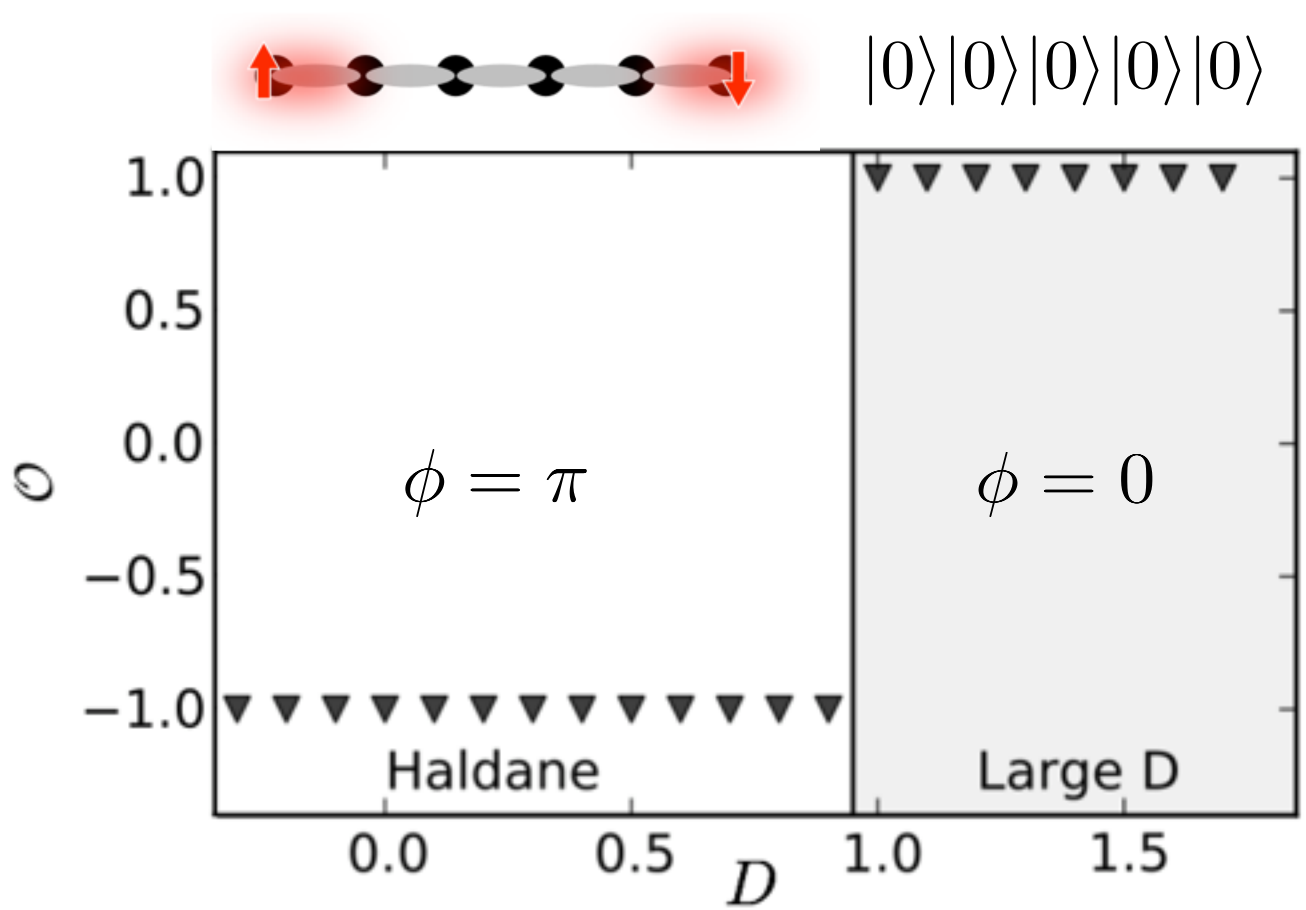


FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B 81, 064439 (2010).
FP and A.M. Turner, Phys. Rev. B 86, 125441 (2012).

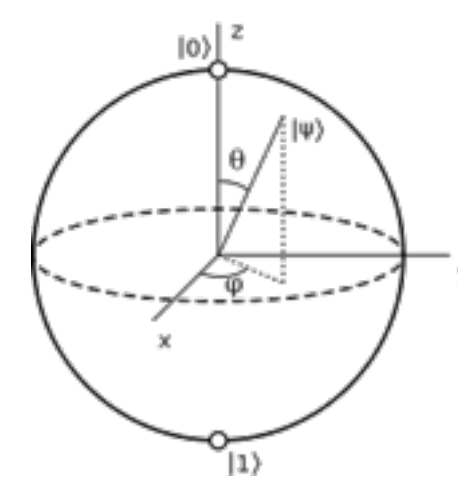
1D symmetry protected topological phases

- $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry protects the Haldane phase

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + D \sum_j (S_j^z)^2$$



$$\mathcal{O} \propto \mathrm{tr}(U_x U_z U_x^\dagger U_z^\dagger)$$

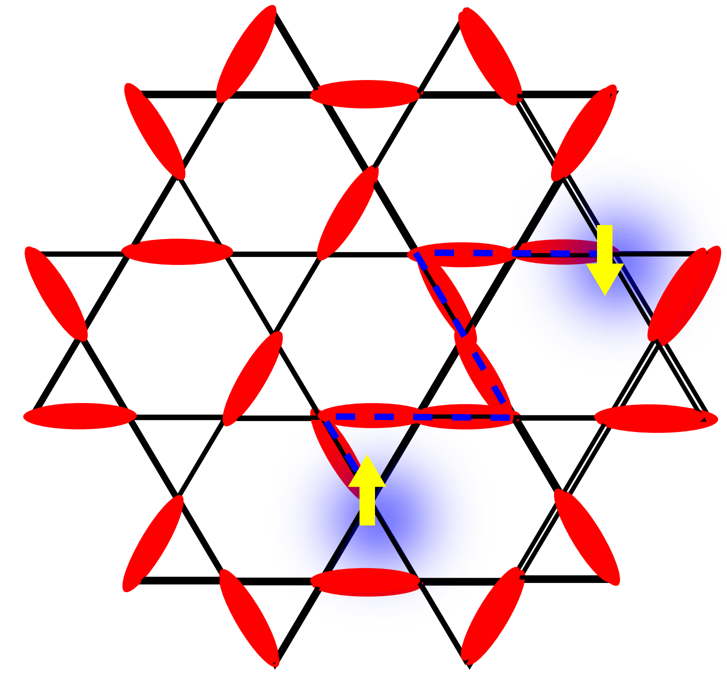


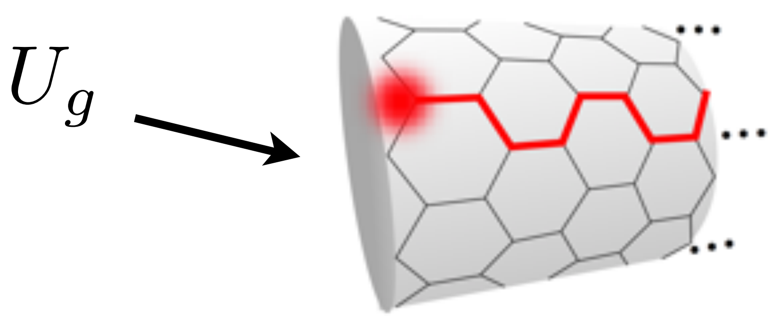
- Time reversal and lattice inversion symmetries

FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B 81, 064439 (2010).
FP and A.M. Turner, Phys. Rev. B 86, 125441 (2012).

2D Symmetry enriched topological order

- RVB state on the kagome lattice: $\mathbb{Z}_2$ spin liquid states (spin 1/2 singlets)
- e and f-particles in projective ($S = 1/2$) representation





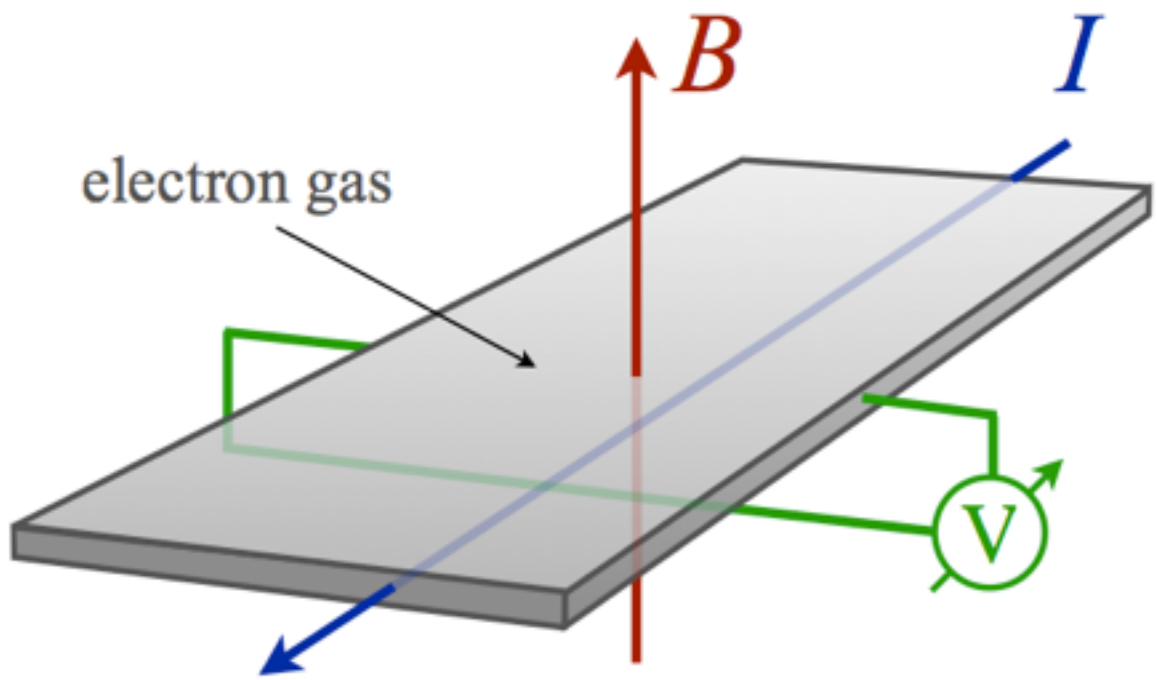
QP	1	e	m	f
$U_x U_z U_x^{-1} U_z^{-1}$	1	-1	1	-1

$\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

Space group symmetries: M. Zaletel et al., arXiv:1501.01395

C.-Y. Huang, X. Chen, and FP, Phys. Rev. B 90, 045142 (2014).

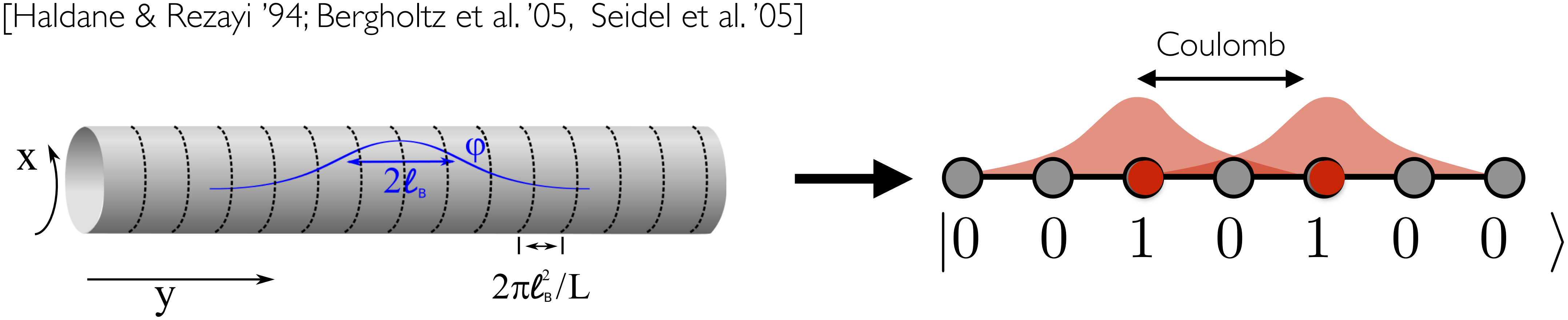
Fractional Quantum Hall



M. P. Zaletel, R. S. K. Mong, FP, Phys. Rev. Lett. **110**, 236801 (2013).
M. P. Zaletel, R. S. K. Mong, FP, and E. H. Rezayi, Phys. Rev. B **91**, 045115 (2015).
R. S. K. Mong, M. P. Zaletel, FP, Z. Papic, arXiv:1505.02843.

Fractional Quantum Hall

- Consider the FQHE on an infinitely long cylinder
  - Orbitals are localized along the cylinder: Quasi 1D model using an occupation number basis $|\ldots, j_0, j_1, \ldots\rangle$
    [Haldane & Rezayi '94; Bergholtz et al. '05, Seidel et al. '05]



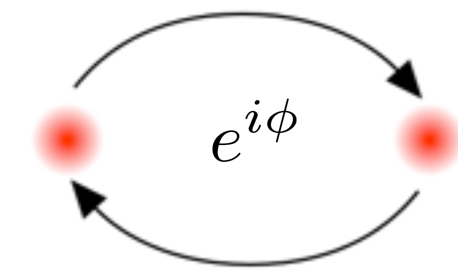
  - Coulomb interactions yield quantum many-body problem

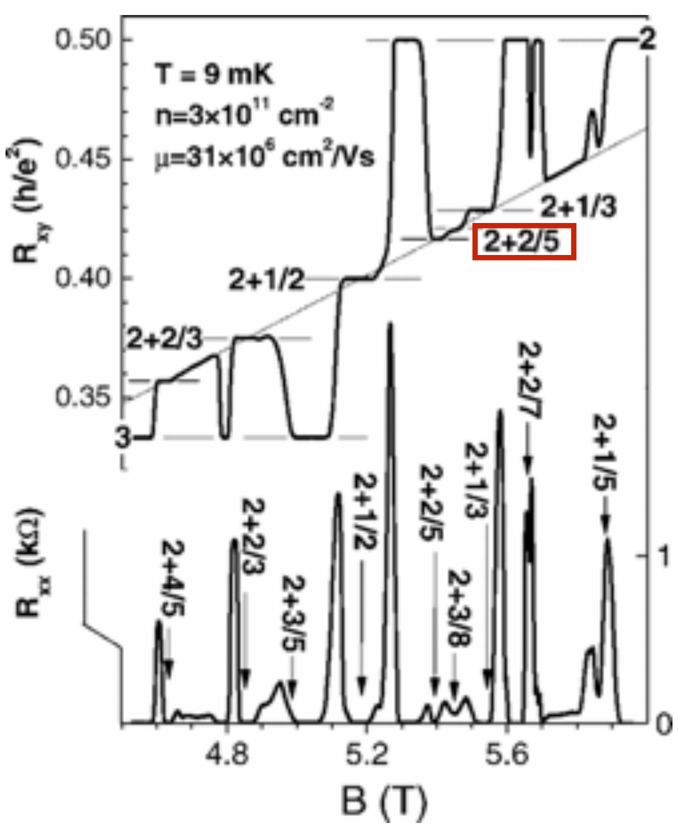
$$\hat{H} = \sum_n \sum_{k \geq |m|} V_{km} c^\dagger_{n+m} c^\dagger_{n+k} c_{n+m+k} c_n$$

M. P. Zaletel, R. S. K. Mong, FP, PRL 110, 236801 (2013).
M. P. Zaletel, Roger S. K. Mong, FP, and E. H. Rezayi, Phys. Rev. B 91, 045115 (2015).

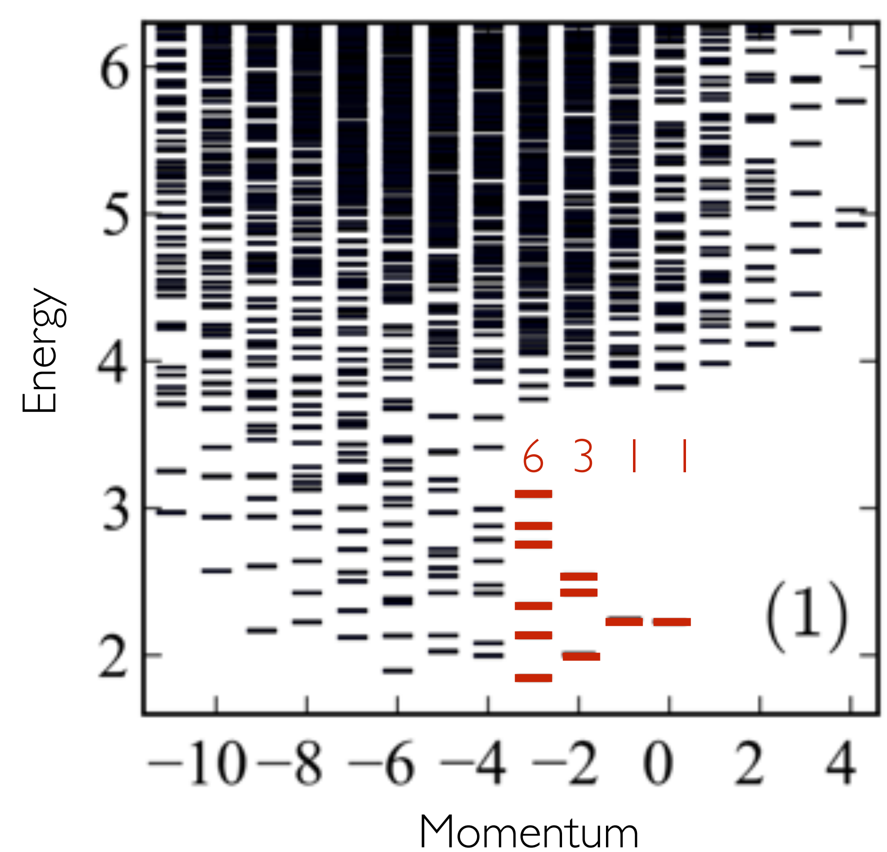
Fractional Quantum Hall

- Finding the ground state with filling factor $\nu = 12/5$ using the DMRG algorithm ($L = 28\ell_B$)
- Charge e/5 quasiparticles: Fibonacci anyons
  [Read Rezayi '98]

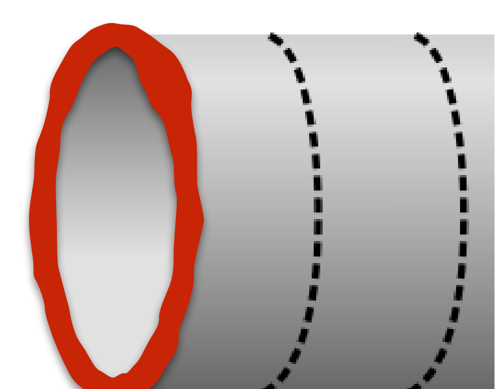




[Xia et al '04]



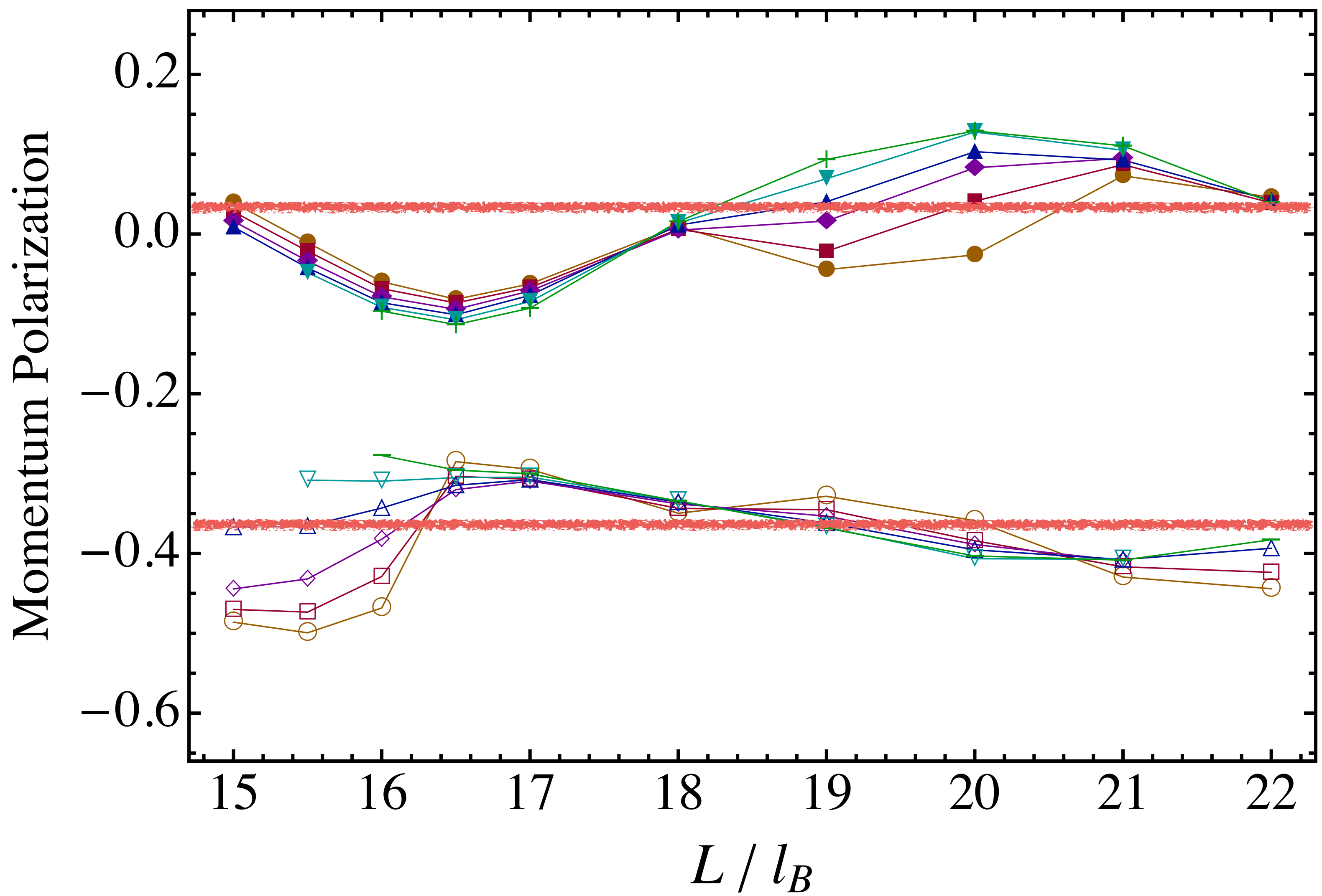
six primary fields of the Z3 parafermion CFT: $1,\ \psi,\ \psi^\dagger,\ \varepsilon,\ \sigma,\ \sigma^\dagger.$

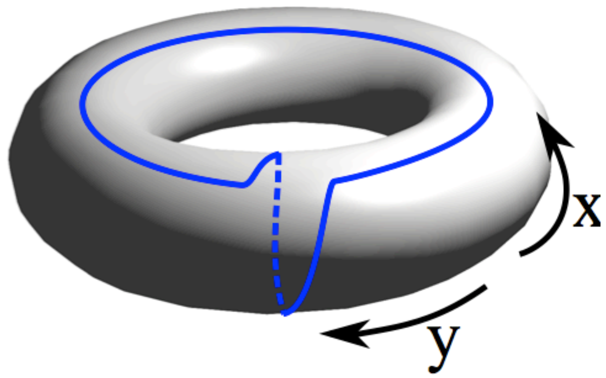


R. S. K. Mong, M. P. Zaletel, FP, Z. Papic, arXiv:1505.02843

Fractional Quantum Hall

- iDMRG on FQHE at $\nu = 12/5$: Numerical evidence for the existence of Fibonacci anyons!





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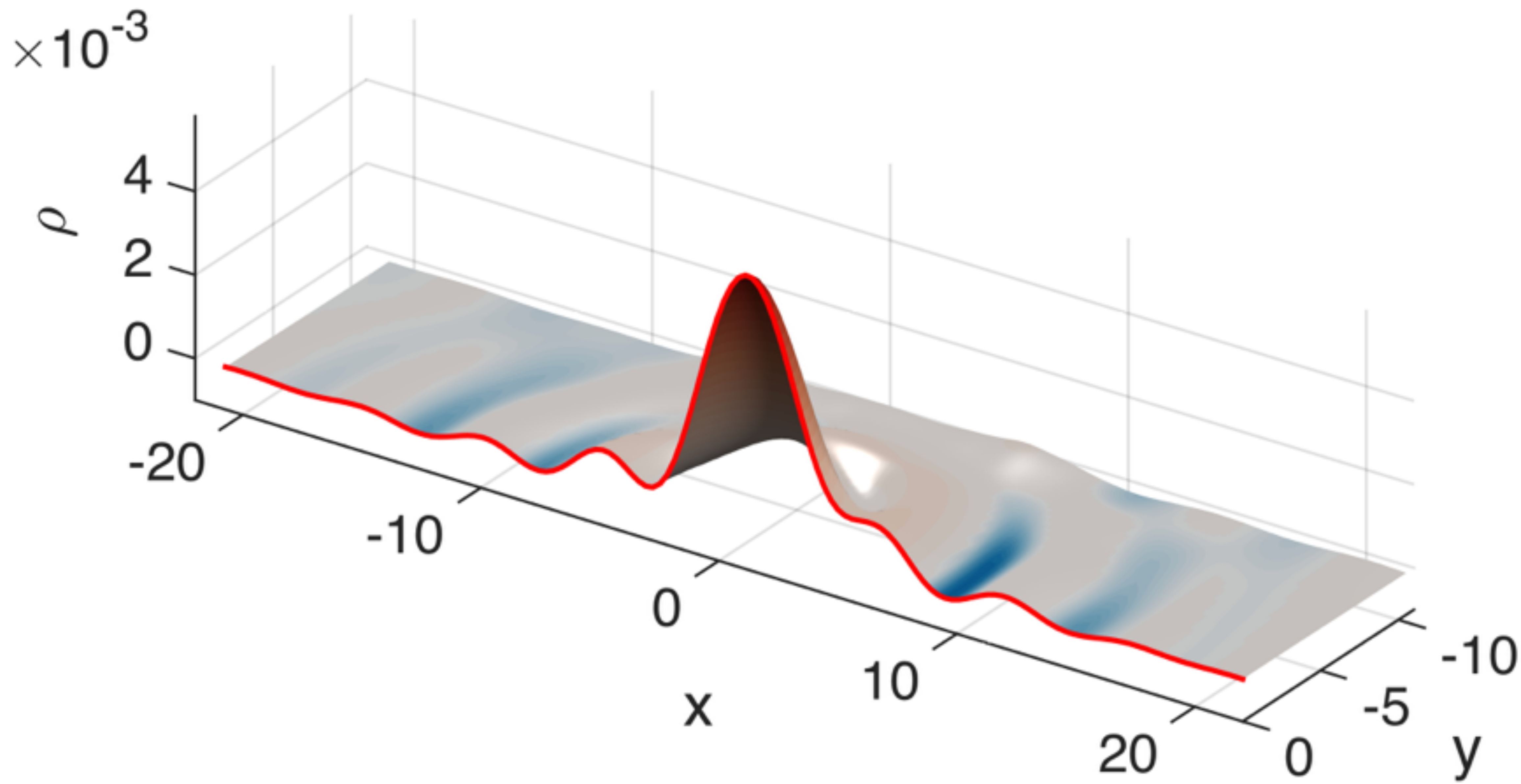
Fractional Quantum Hall



Density profile for
anyons with charge e/5

R. S. K. Mong, M. P. Zaletel, FP, Z. Papic, arXiv:1505.02843