Entanglement in electronic noise

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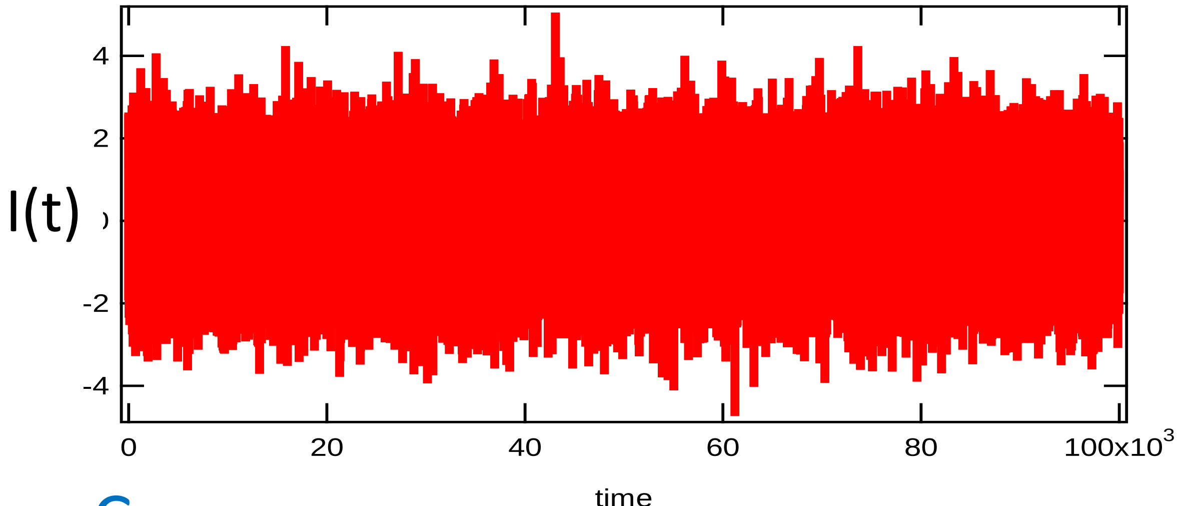
Fluctuations (noise)

Current vs. time for voltage perfectly stable: $I \neq GV$!

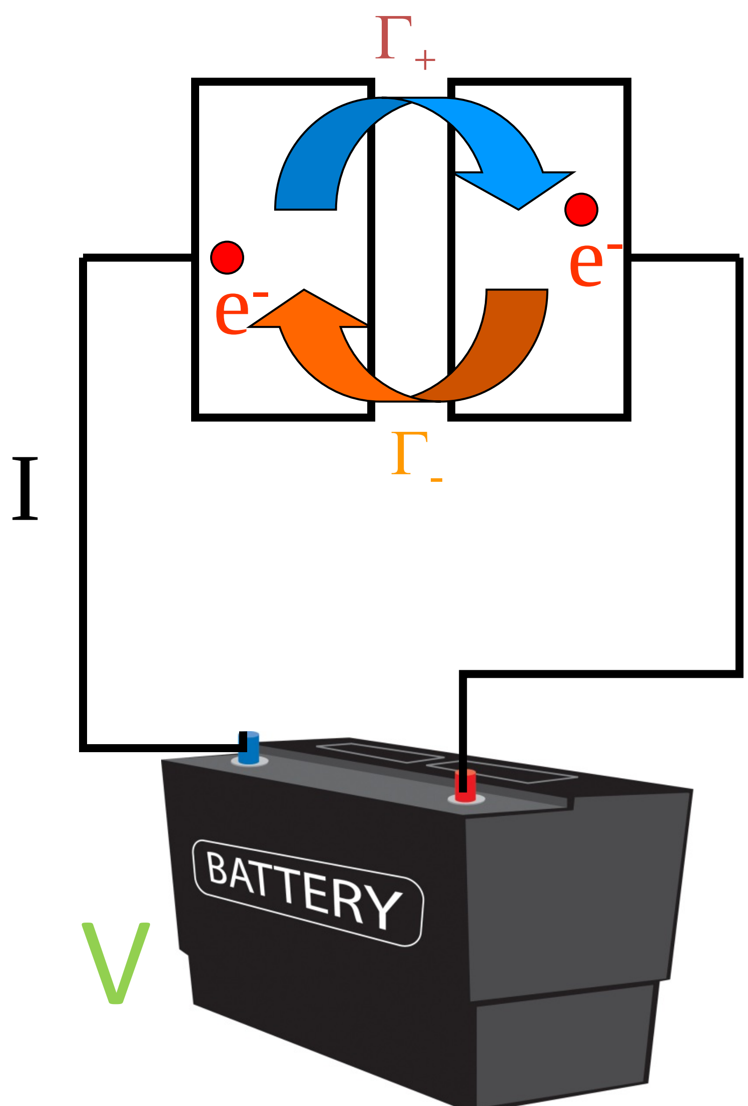


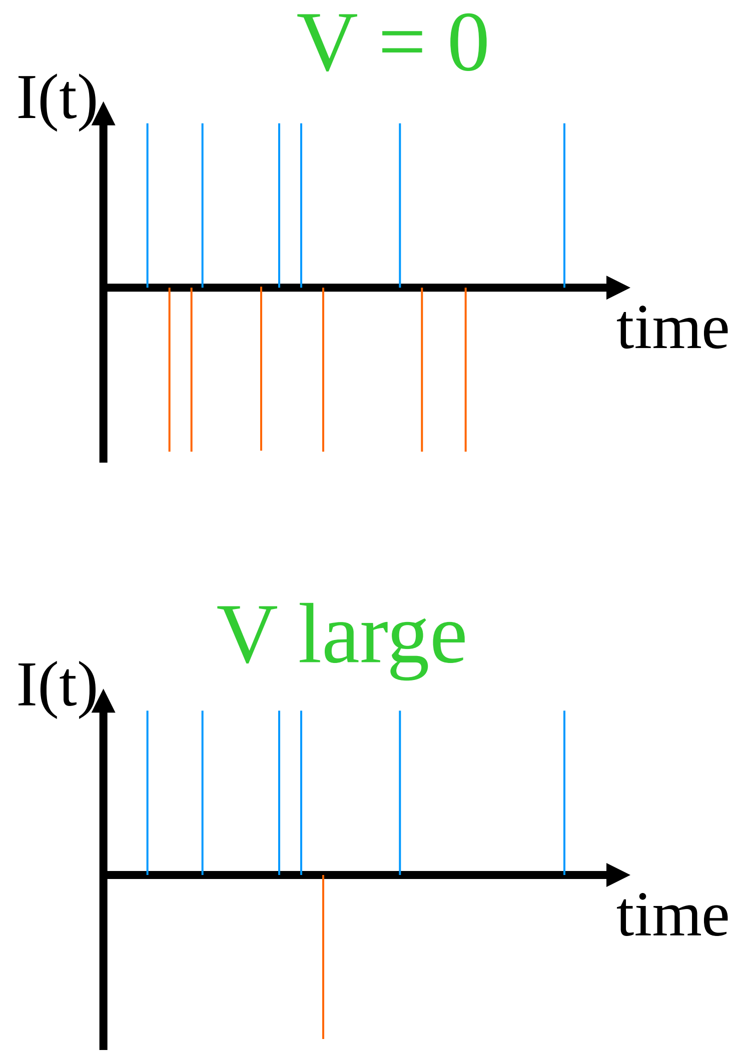
$$\langle \delta I^2(t) \rangle$$

Causes:

- Defects
- Temperature
- Discreteness of the electron charge
- Quantum mechanics itself: even vacuum fluctuates !

The system: a tunnel junction. The discreteness of charge is crucial !

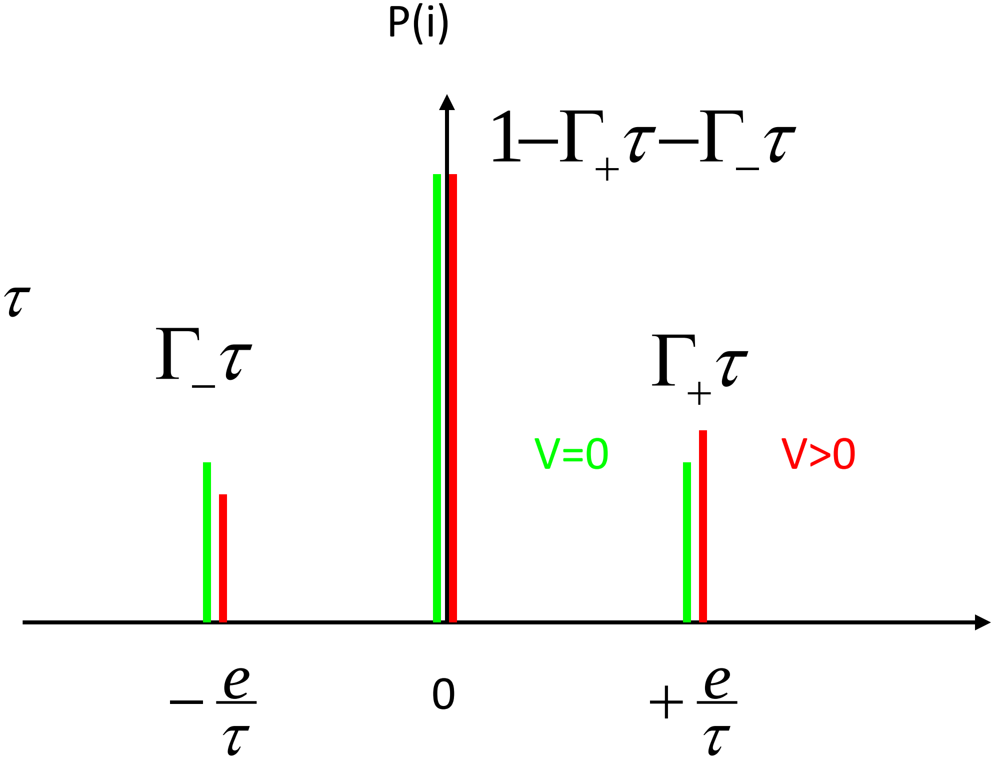




Classical statistics of the current in a tunnel junction

Each time τ :

$$P(I)=\begin{cases} P(+1e)=\Gamma_+\tau \\ P(-1e)=\Gamma_-\tau \\ P(0e)=1-\Gamma_+\tau-\Gamma_-\tau \end{cases}$$



Average current and noise in a tunnel junction (single channel)

Parameters that determine $\Gamma_{\pm}$:

- Voltage V
- Temperature T
- Conductance

$$\frac{\Gamma_+}{\Gamma_-} = \exp\left(\frac{eV}{k_B T}\right)$$

Average current : $\langle I \rangle = \left(\frac{e}{\tau}\right) \Gamma_+ \tau + \left(\frac{-e}{\tau}\right) \Gamma_- \tau = e(\Gamma_+ - \Gamma_-) = GV$

Noise: $\langle I^2 \rangle = \left(\frac{e}{\tau}\right)^2 \Gamma_+ \tau + \left(\frac{-e}{\tau}\right)^2 \Gamma_- \tau = e^2(\Gamma_+ + \Gamma_-)$

At equilibrium: V=0, $\Gamma_+ = \Gamma_-$, $\langle I \rangle = 0$ but $\langle I^2 \rangle \neq 0$

At large voltage: $\Gamma_+ \gg \Gamma_-$ so $\langle I^2 \rangle = e \langle I \rangle$

Current fluctuations in a tunnel junction at low frequency

$$\langle \delta I^2 \rangle = eIB \coth\left(\frac{eV}{2k_B T}\right) = BS_2$$

B=bandwidth

$$S_2 = \begin{cases} 2k_B TG & \text{if } eV \ll k_B T \\ eI & \text{if } eV \gg k_B T \end{cases}$$

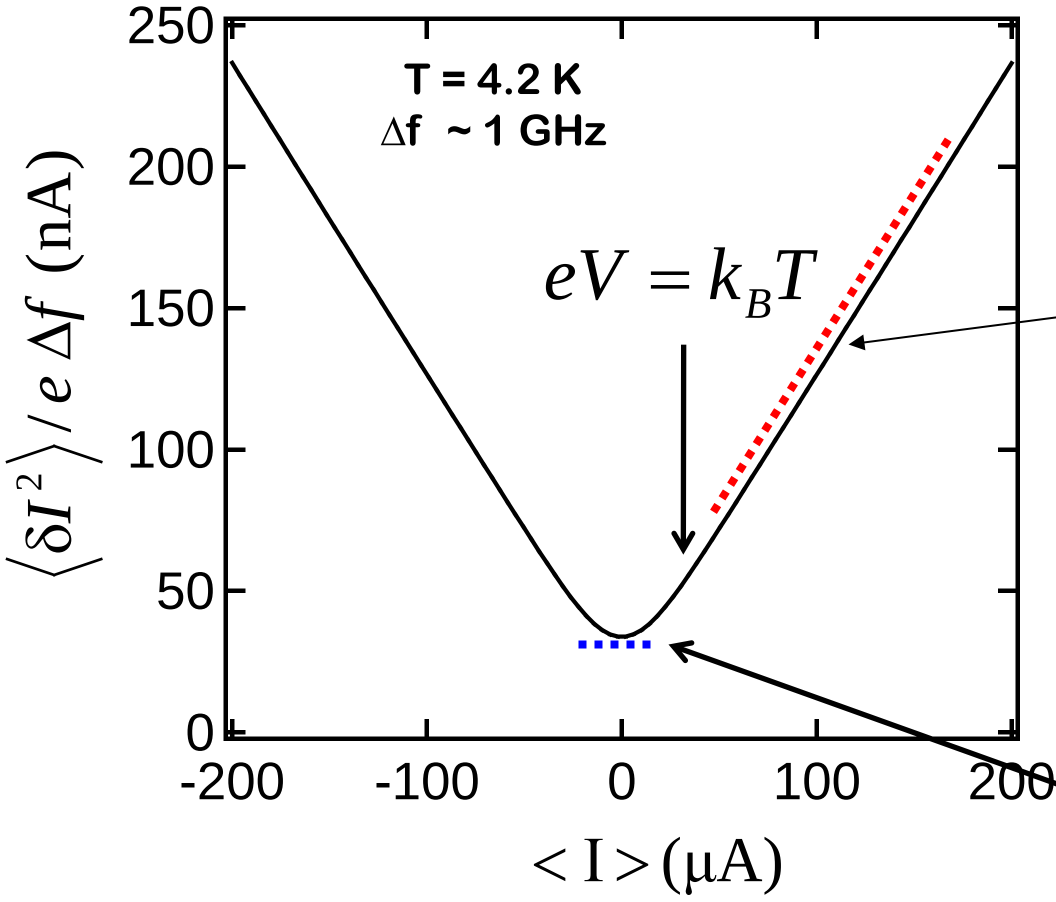
Noise spectral density in A^2/Hz

Equilibrium (Johnson) noise: macroscopic, fluctuation-dissipation theorem

Shot noise: discreteness of charge

Experiment $S_2(\omega=0,T=4.2K)$

Tunnel junction made by L. Spietz at Yale



Shot noise

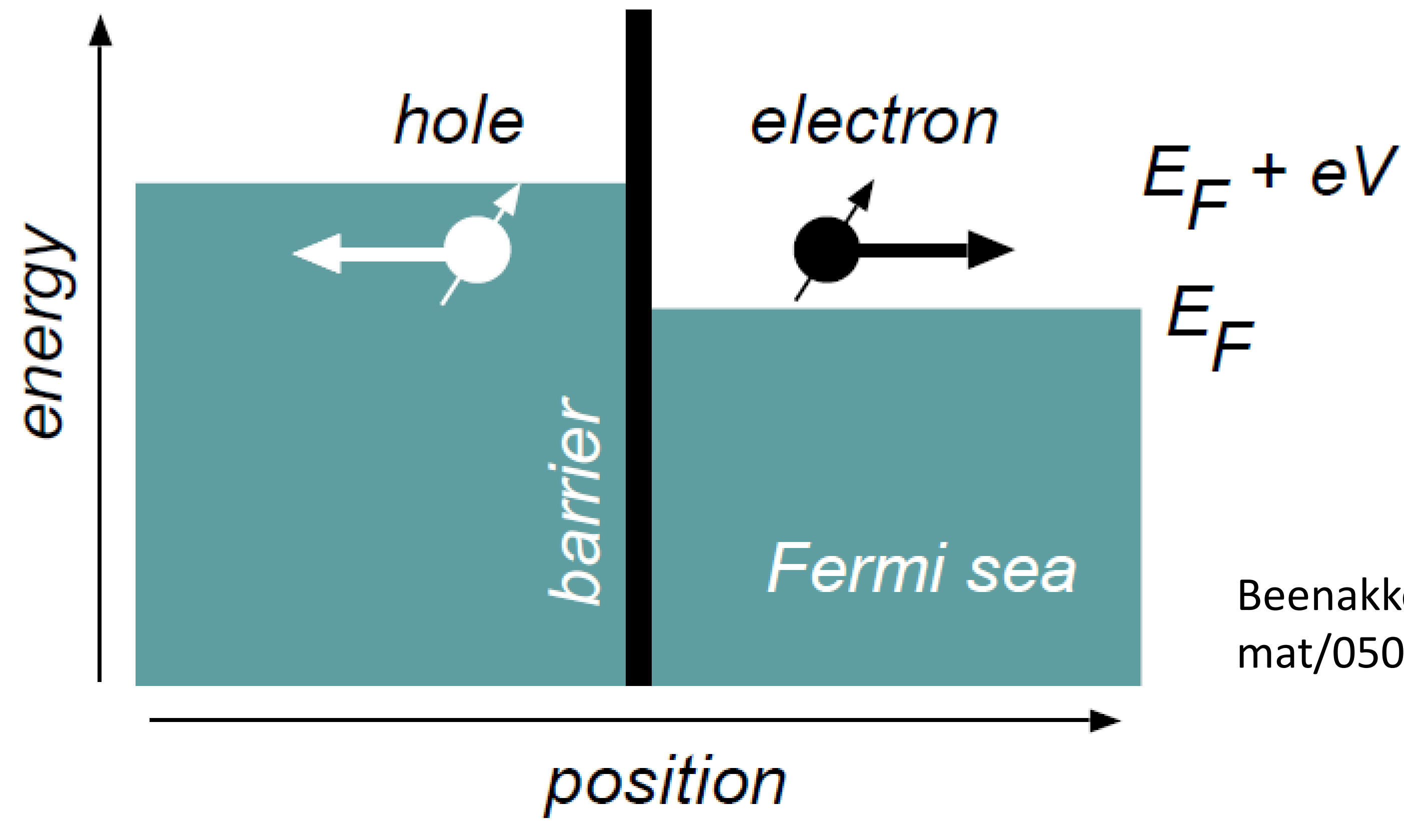
$$S_2(eV >> k_B T) = eI$$

Equilibrium noise

$$S_2(V=0) = 2k_B TG$$

What about finite frequencies ?

Electron-hole entanglement



Beenakker, cond-mat/0508488

$$|0\rangle \mapsto (1-\tau)|0\rangle - e^{2i\phi}\tau|\uparrow\downarrow\rangle_h|\uparrow\downarrow\rangle_e - e^{i\phi}\sqrt{2\tau(1-\tau)}\,\underbrace{2^{-1/2}\left(|\uparrow\rangle_h|\uparrow\rangle_e + |\downarrow\rangle_h|\downarrow\rangle_e\right)}_{\text{Bell pair}}$$

Readout: noise !

$$P_{\text{noise}} = 2eV\frac{2e^2}{h}\tau(1-\tau) = 2e^2\mathcal{E}_{\text{part}}/t_{\text{det}}$$

Variance of current fluctuations

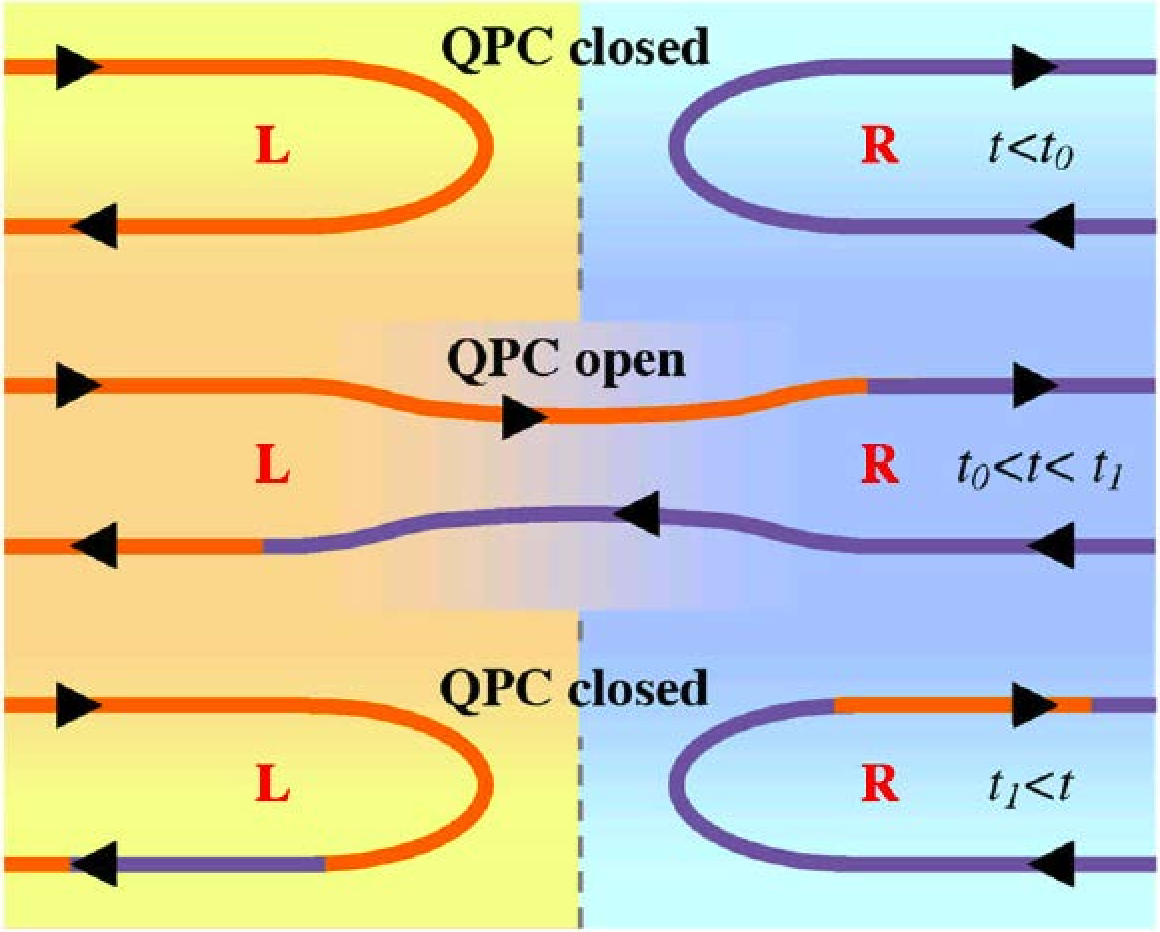
Entanglement entropy that accounts for particle conservation

Each electron that crosses the barrier generates 1 entangled pair: I/e ebits/s

Problem: entanglement is sensitive to dechoherence, noise is not

Connect/disconnect 1D conductors

No bias
V=0
Gaussian for
Transmission=1

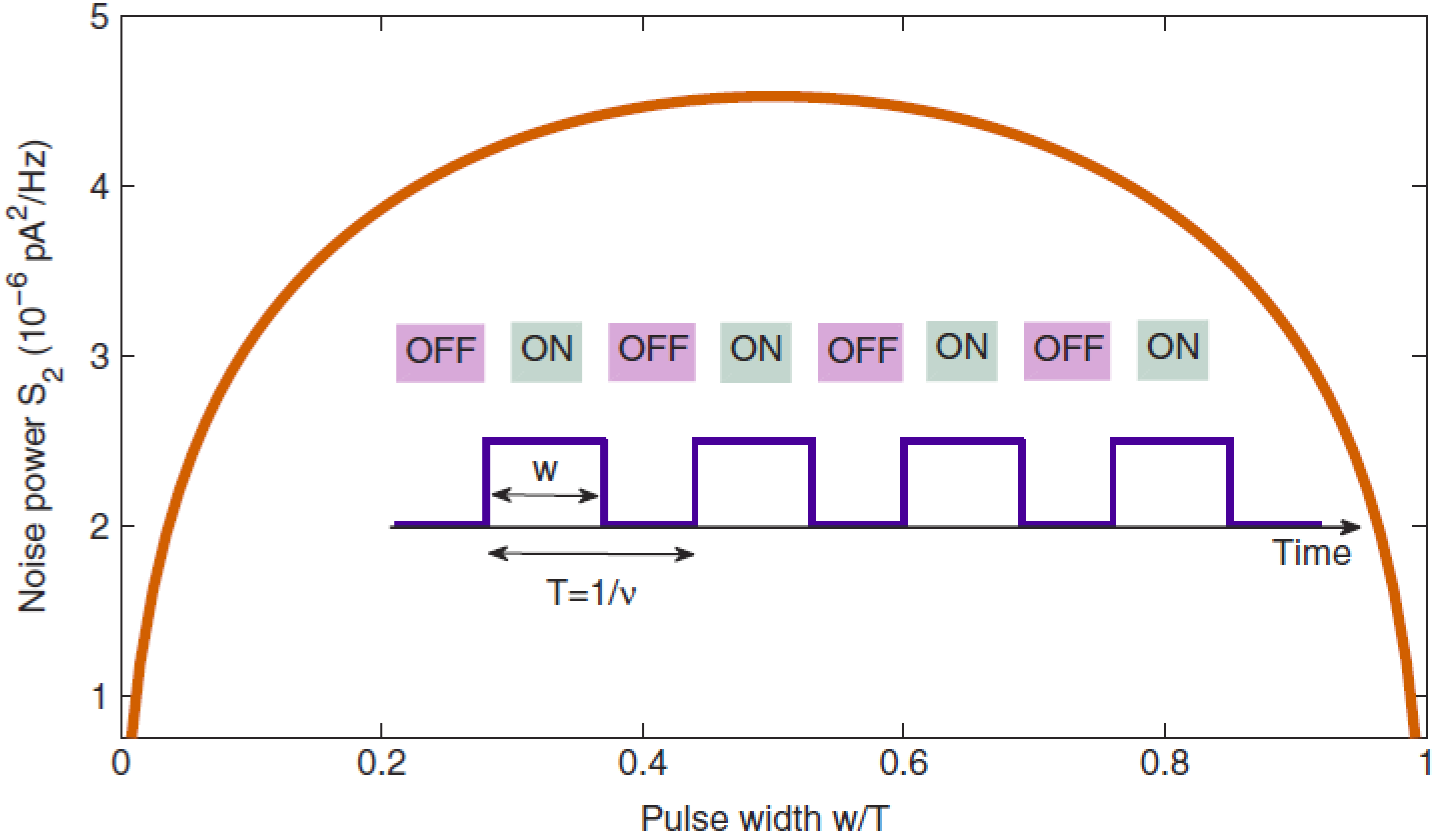


Klich, Levitov
PRL102 (09)

$$S = \frac{\pi^2}{3}C_2 + \frac{\pi^4}{15}C_4 + \frac{2\pi^6}{945}C_6 + \ldots.$$

C_n = cumulant of transfered charge

Zero frequency noise, $V_{bias}=0$



$$S = \frac{1}{3} N \log \frac{w}{\tau} \qquad \text{vs. } S = \frac{1}{3} \log L$$

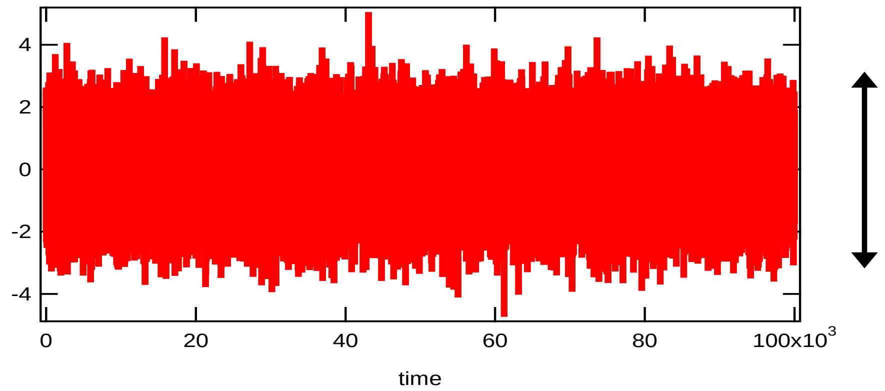
pulses

Some questions…

- Time plays the role of length. What is encoded in the frequency-dependence of the noise in terms of entropy ? Is there a spectral density for entanglement entropy ?
- Entanglement by absorption of photons: photo-assisted noise vs « energy-time » entanglement ?
- Entanglement of the electrons vs. that of the radiated field ?

Noise at finite frequency: existence of correlations

I(t)



$$C(0) = \left\langle \delta I^2(t) \right\rangle$$

Correlation function: $$C(V,\tau) = \left\langle \delta I(t+\tau)\delta I(t) \right\rangle$$

Noise spectral density: $$S(V,\omega) = \left\langle \delta I(\omega)\delta I(-\omega) \right\rangle$$

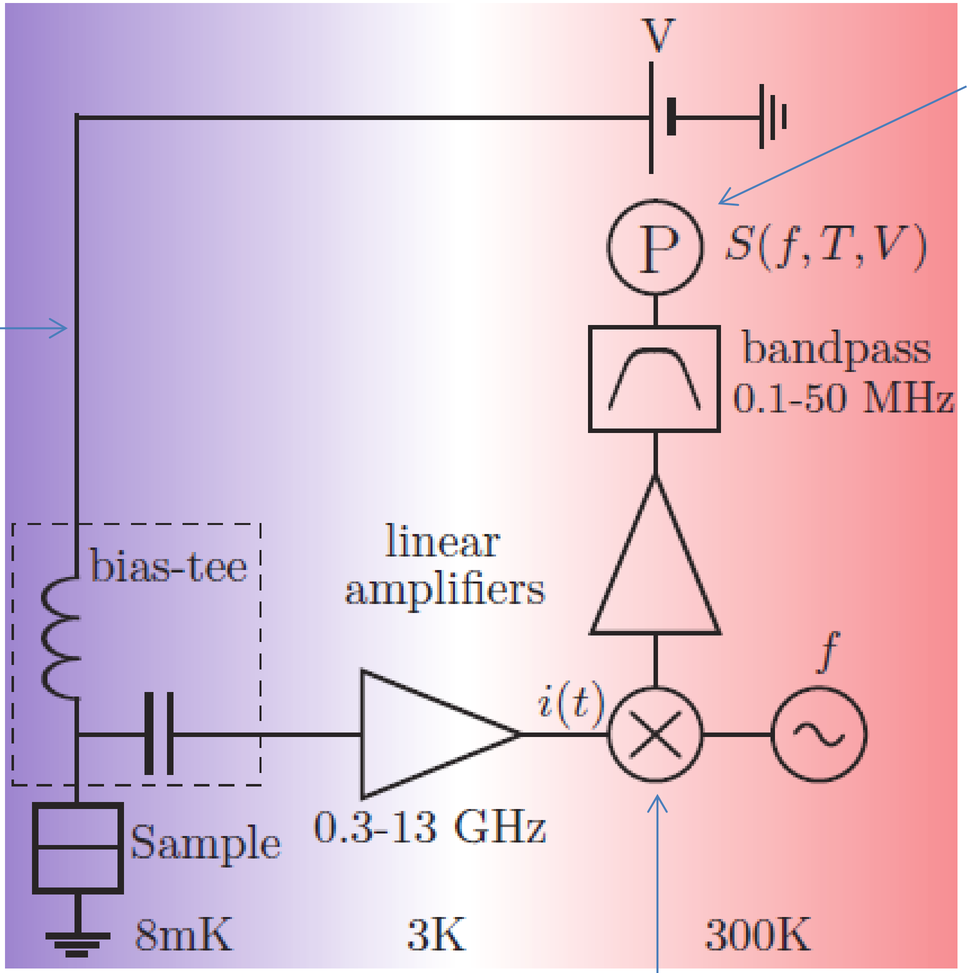
Current-current correlator in time domain: how successive electrons are correlated ?

- Average time between successive electrons per channel: $(I/e)/M$
- Conductance : $G=Mpe^2/h$, with p=transmission =probability to cross the barrier at each attempt
- Average time between attempts: h/eV
- How regular is that time ?
- Classically: no correlation (Poisson)

Method: noise spectroscopy

- Measure the power of the emitted radiation vs. frequency on a very wide bandwidth: P(f)
- Calculate (Fourier transform) the current-current correlator: $\langle I(t)I(t')\rangle$
- Relevant energy scales: millikelvin, microvolt, gigahertz !

Experimental setup



Power detector

DC bias line

Variable temperature

Frequency mixer

Calibration

What is measured:

$$P(f) = G(f)\,[\,\alpha S(f) + S_a(f)\,]$$

Gain of the amplifier, attenuation of the cables

Noise of the amplifier

Attenuation between the sample and the attenuator

Contribution of the amplifier: 5-100 K

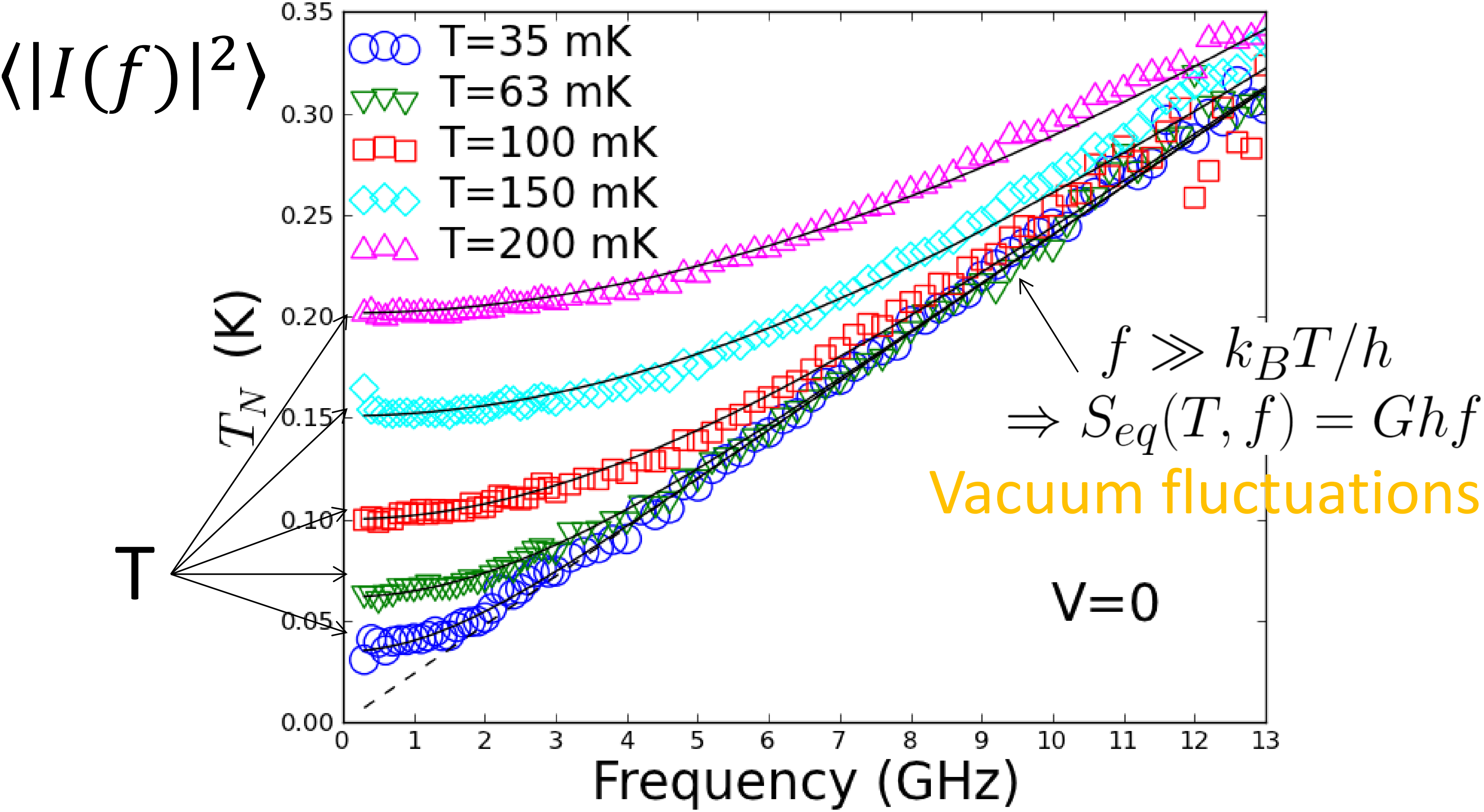
Contribution of the sample: tens of mK !

Calibration: $S(V, T, f) = eI$ at high voltage

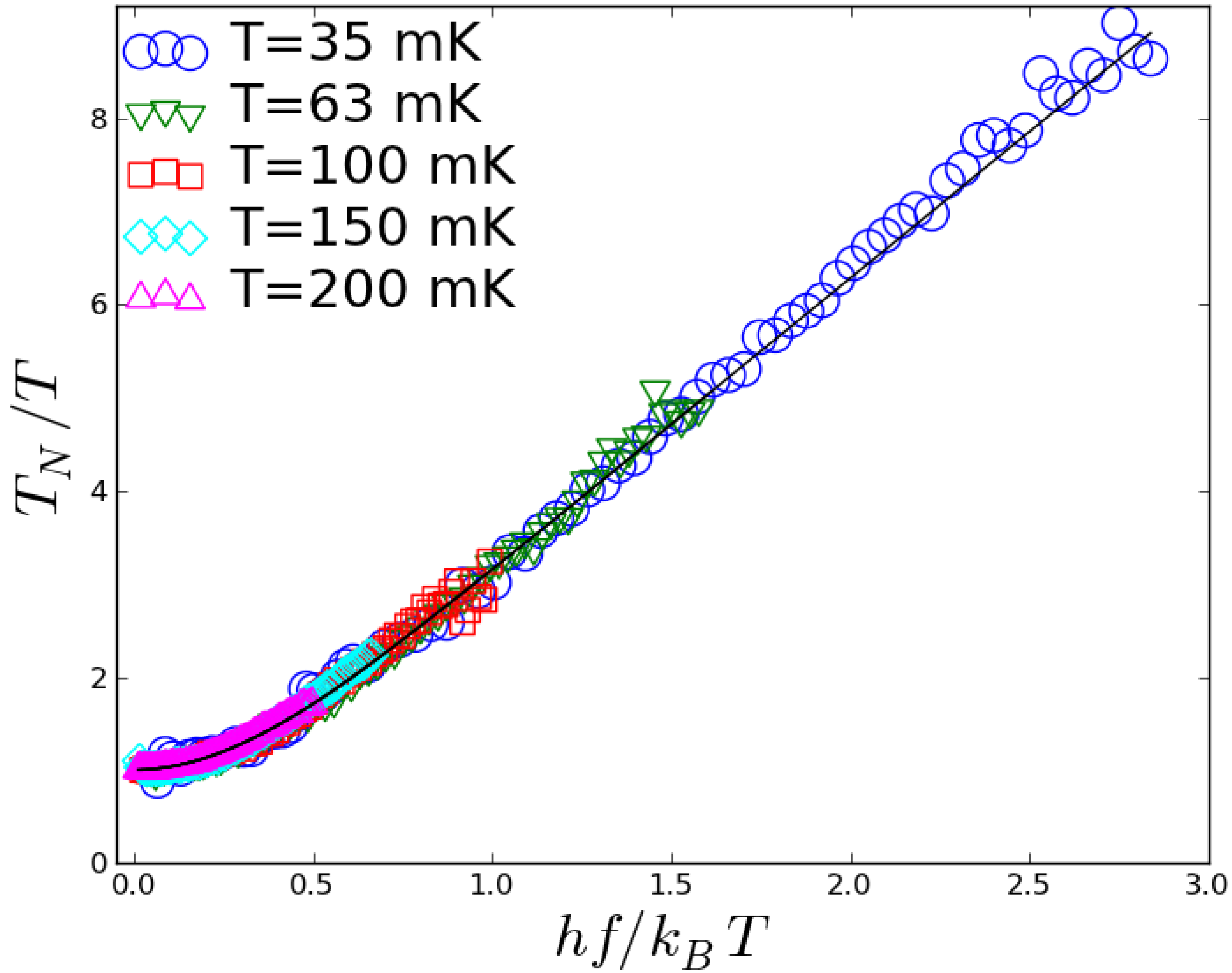
Noise temperature: noise as an equivalent temperature

- Noise at equilibrium: $S=2k_BTG$
- In any situation, one defines the noise temperature: $T_N=S/(2k_BG)$
- T_N is the temperature at which a macroscopic resistor produces as much noise as the sample
- At equilibrium, $T_N=T$

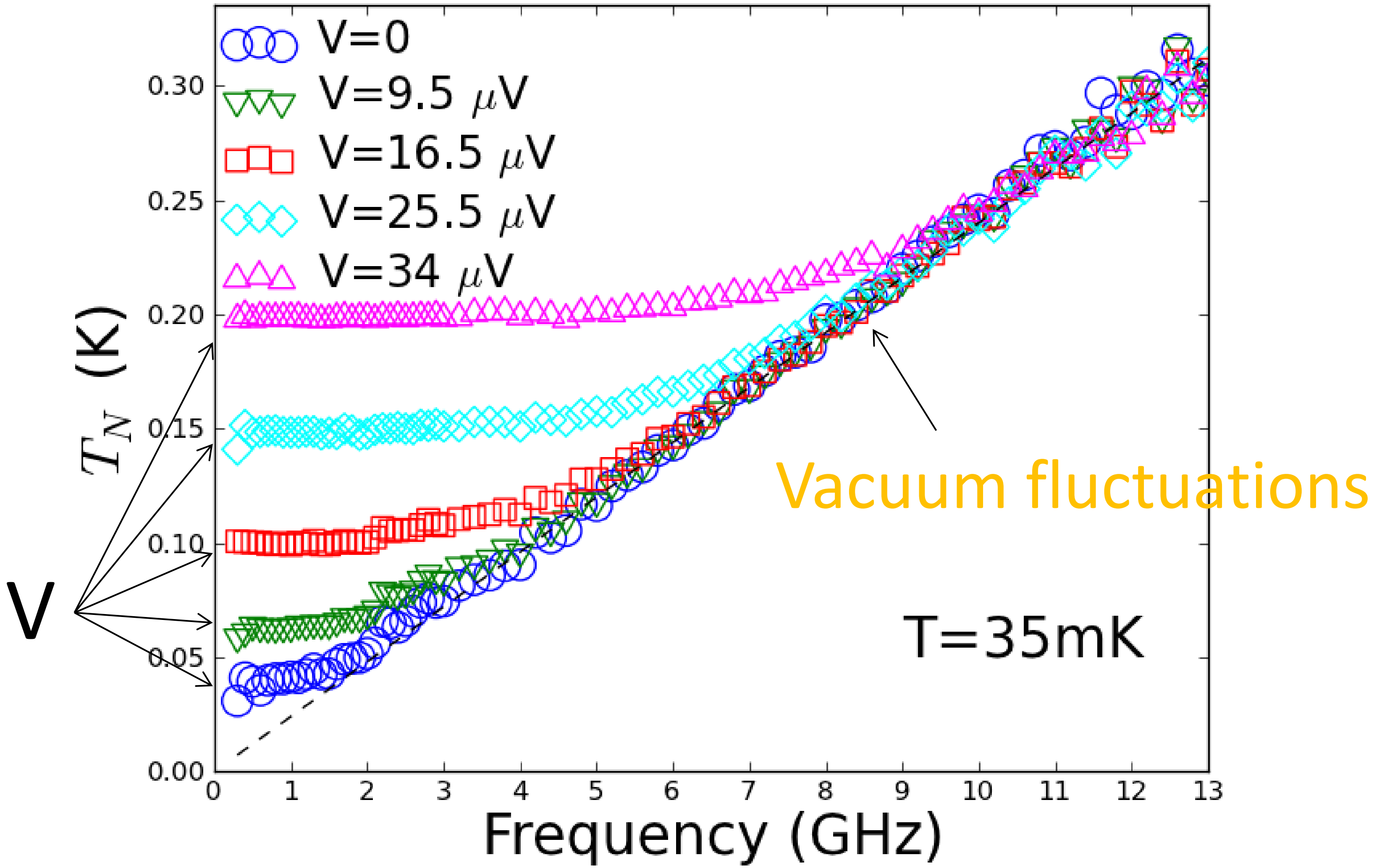
Result: the tunnel junction at equilibrium (1D Planck's law)



Rescaling: one timescale, h/k_BT



The tunnel junction out of equilibrium: $V \neq 0$



Noise at equilibrium in time-domain: $C_{eq}(t) = \langle I(t')I(t'+t)\rangle_{eq}$

Problem: S diverges at high frequency because of vacuum fluctuations !

Our solution: we subtract the T=0 contribution: Thermal excess noise:

$$\Delta C_{eq}(t,T) = C_{eq}(t,T) \text{-} C_{eq}(t,T=0)$$

Out-of-equilibrium noise in time-domain: $C(t) = \langle I(t')I(t'+t)\rangle$

Theory: $C(t,V,T) = C_{eq}(t,T)\cos\frac{eVt}{h}$

Thermal excess noise:

$\Delta C(t,T,V) = C(t,T,V)$-$C(t,T=0,V)$

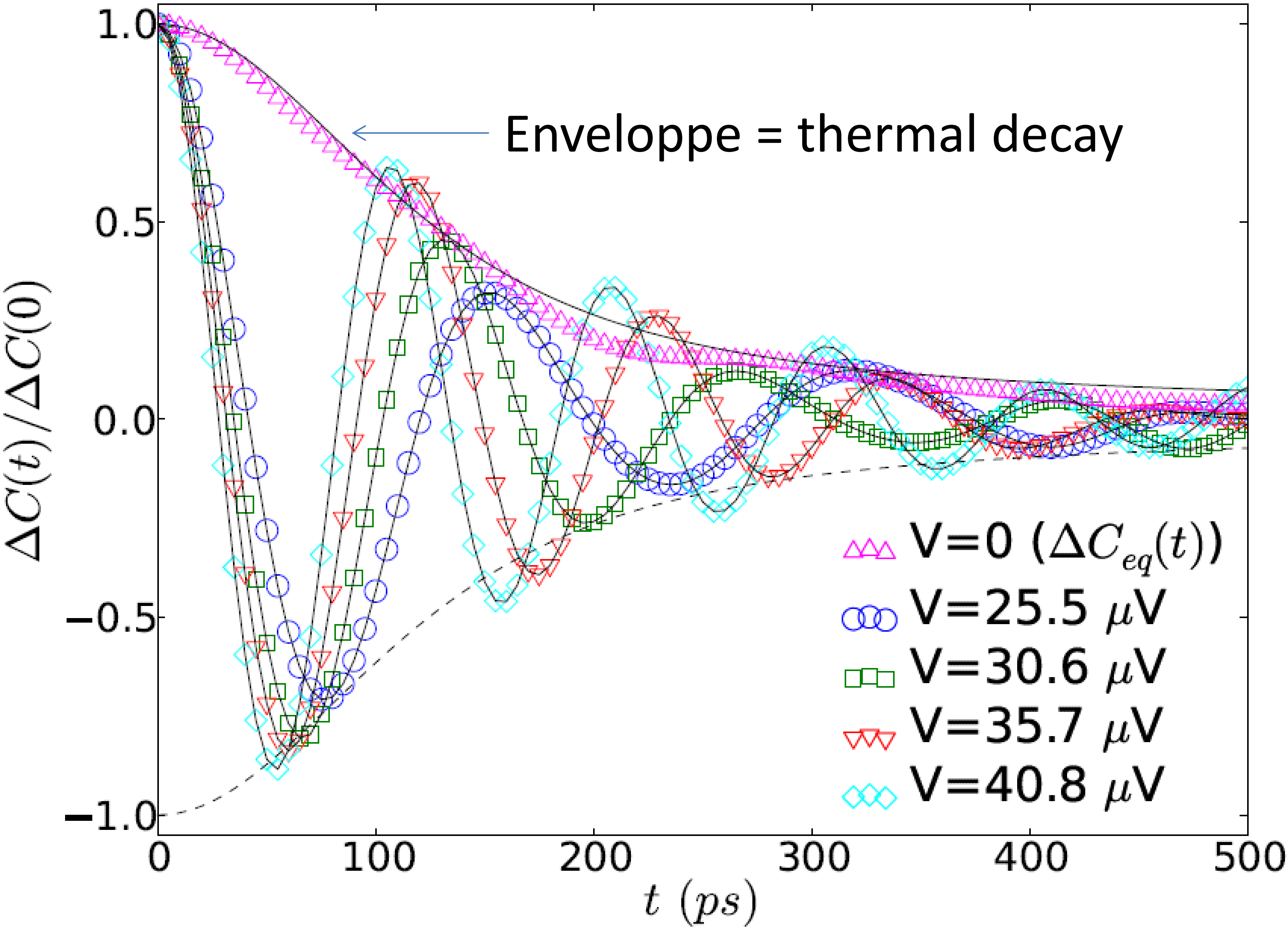
Expected:

$$\Delta C(t,V,T) = \Delta C_{eq}(t,T)\cos\frac{eVt}{h}$$

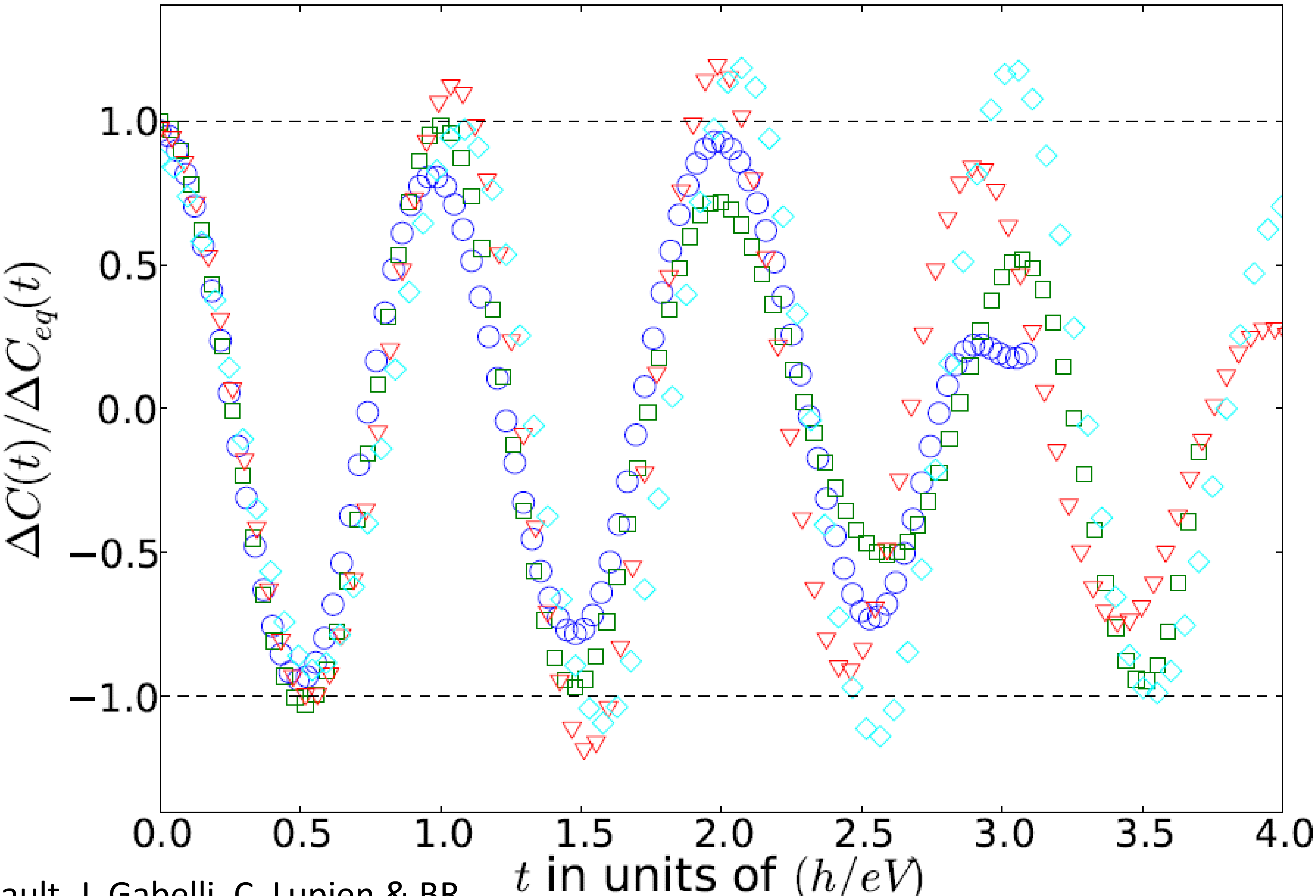
Enveloppe: thermal noise

Oscillation !

Time-domain:ΔC(t)



Oscillations with period τ=h/eV !



K. Thibault, J. Gabelli, C. Lupien & BR, arXiv1410.0931

Interpretation

- Electrons try to cross the barrier REGULARLY with a period h/eV. The temperature adds a jitter, typically given by $h/k_B T$.
- Interpretation: Pauli + Heisenberg principles: $eV \geq \Delta E \geq h/\Delta t$
- At equilibrium, only the thermal jitter remains.

Current noise / electromagnetic radiation

Current / voltage fluctuations = fluctuating electromagnetic field
= white light !

Average power in a bandwidth Δf ~ intensity of light:

$$\langle P \rangle = R\langle \delta I^2 \rangle = R S_2(f)\Delta f$$
$$= [n(f) + \tfrac{1}{2}]hf$$

Noise = average photon number

At equilibrium: Thermal (Johnson) noise = blackbody radiation !

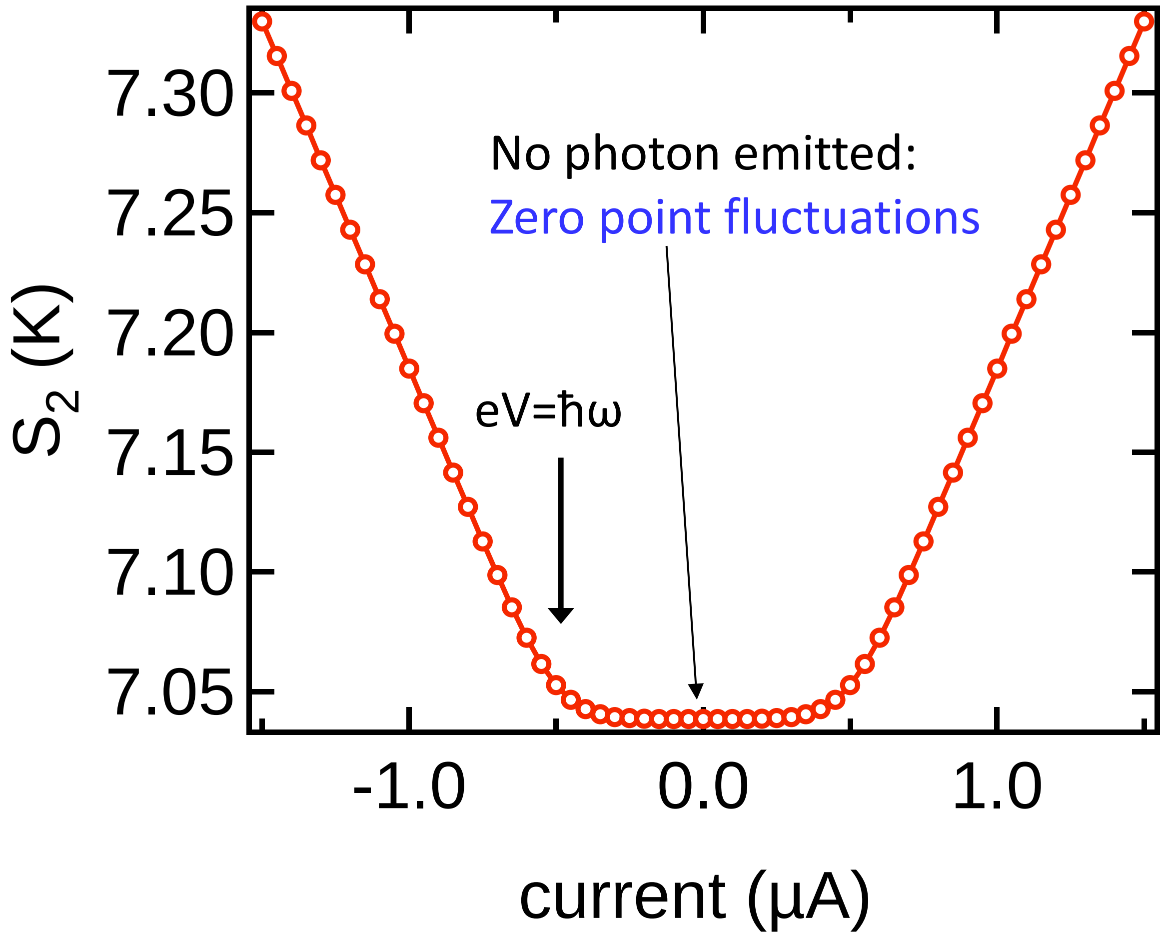
S_2 in the quantum regime $\hbar\omega > k_B T, eV$

$T_{phonons}$= 22 mK
$T_{electrons}$= 27 mK

f = 5.5 - 6.5 GHz
hf/k_B = 290 mK
Ghf/e = 0.50μA

There is a huge contribution of the amplifier that adds to the ZPF !

Tunnel junction R=50Ω



Squeezing ?

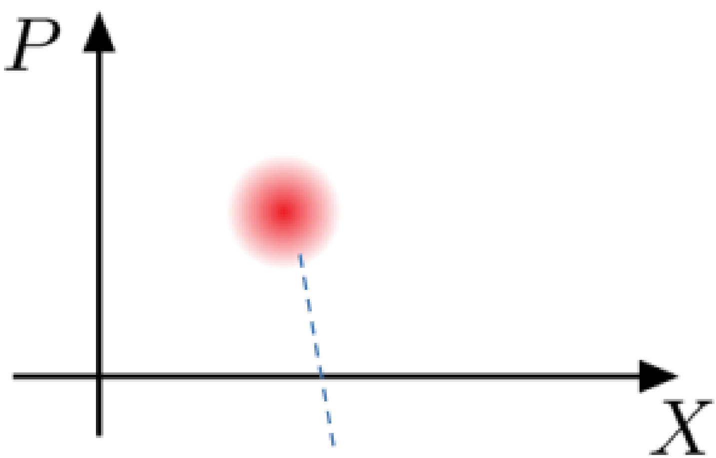
$$[\hat{X}, \hat{P}] \neq 0$$

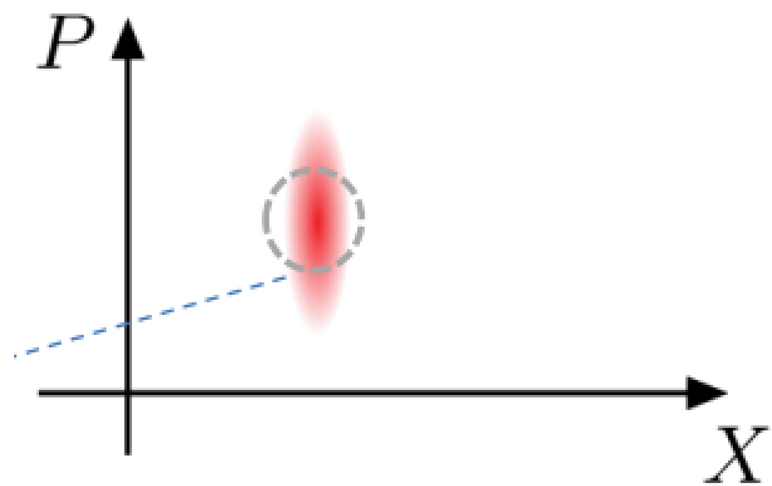
Heisenberg principle :

$$\Delta X^2 \Delta P^2 \geqslant \frac{1}{4} \left\langle \left[\hat{X}, \hat{P}\right] \right\rangle^2$$

$$\Delta X^2 \Delta P^2 = \frac{\hbar^2}{4}$$

$$\Delta X \neq \Delta P$$





Current squeezing?

Quadratures?

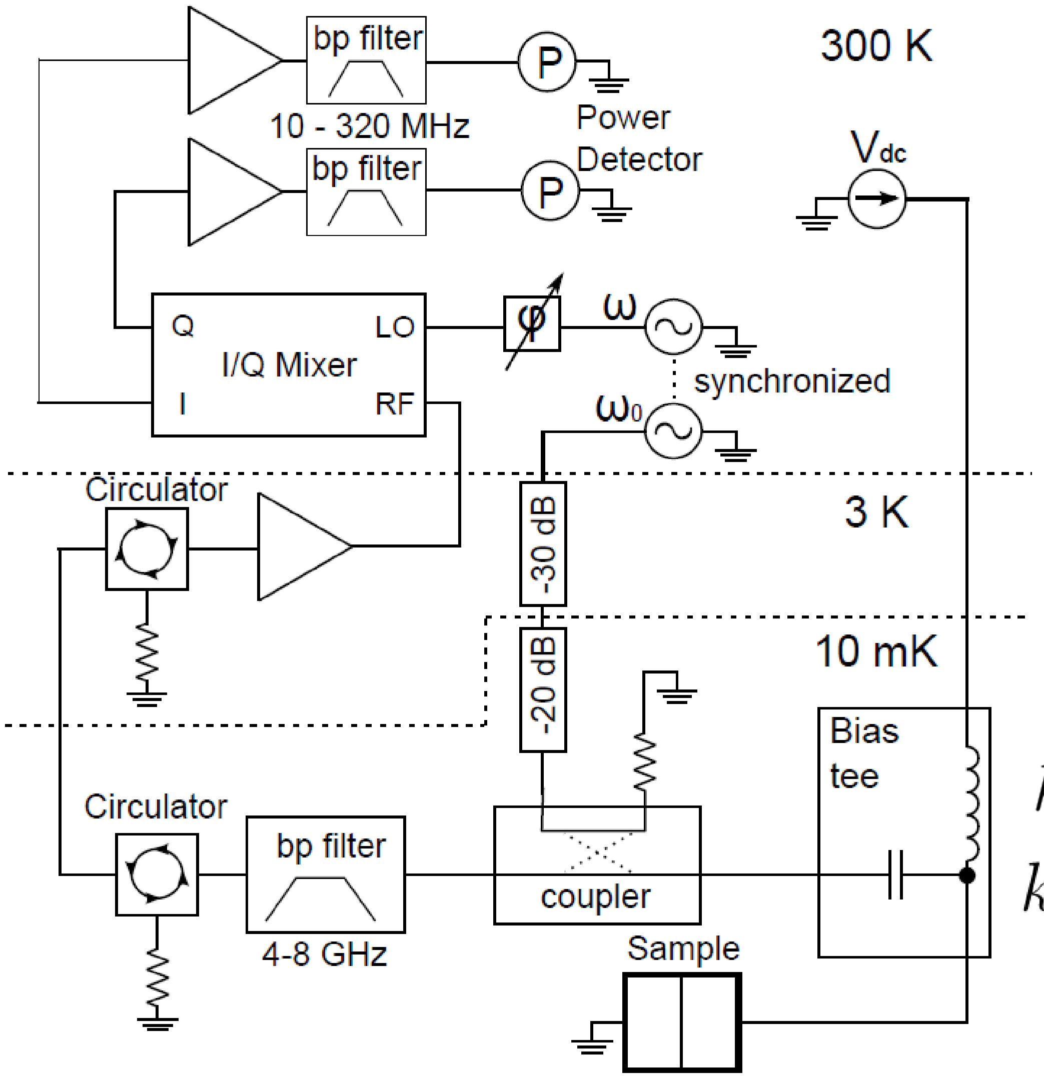
Optics | Electronics

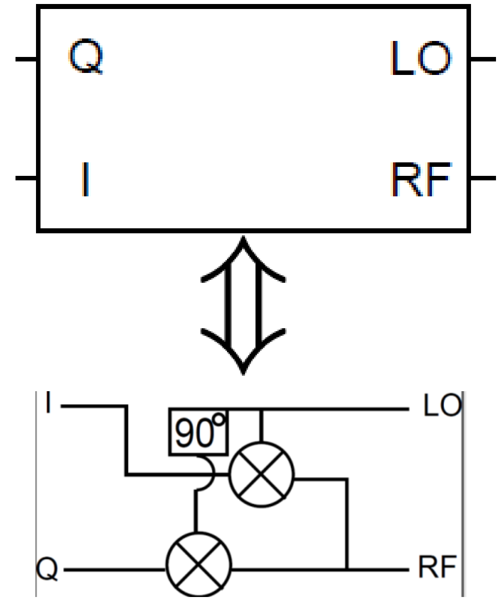
$$\frac{1}{\sqrt{2}}(a + a^\dagger) \rightleftharpoons \frac{1}{\sqrt{2}}(I(\omega) + I(\omega)^\dagger) = A$$

$$\frac{i}{\sqrt{2}}(a - a^\dagger) \rightleftharpoons \frac{i}{\sqrt{2}}(I(\omega) - I(\omega)^\dagger) = B$$

$$S(\omega) = \frac{\Delta A^2 + \Delta B^2}{2}$$

Experimental set-up



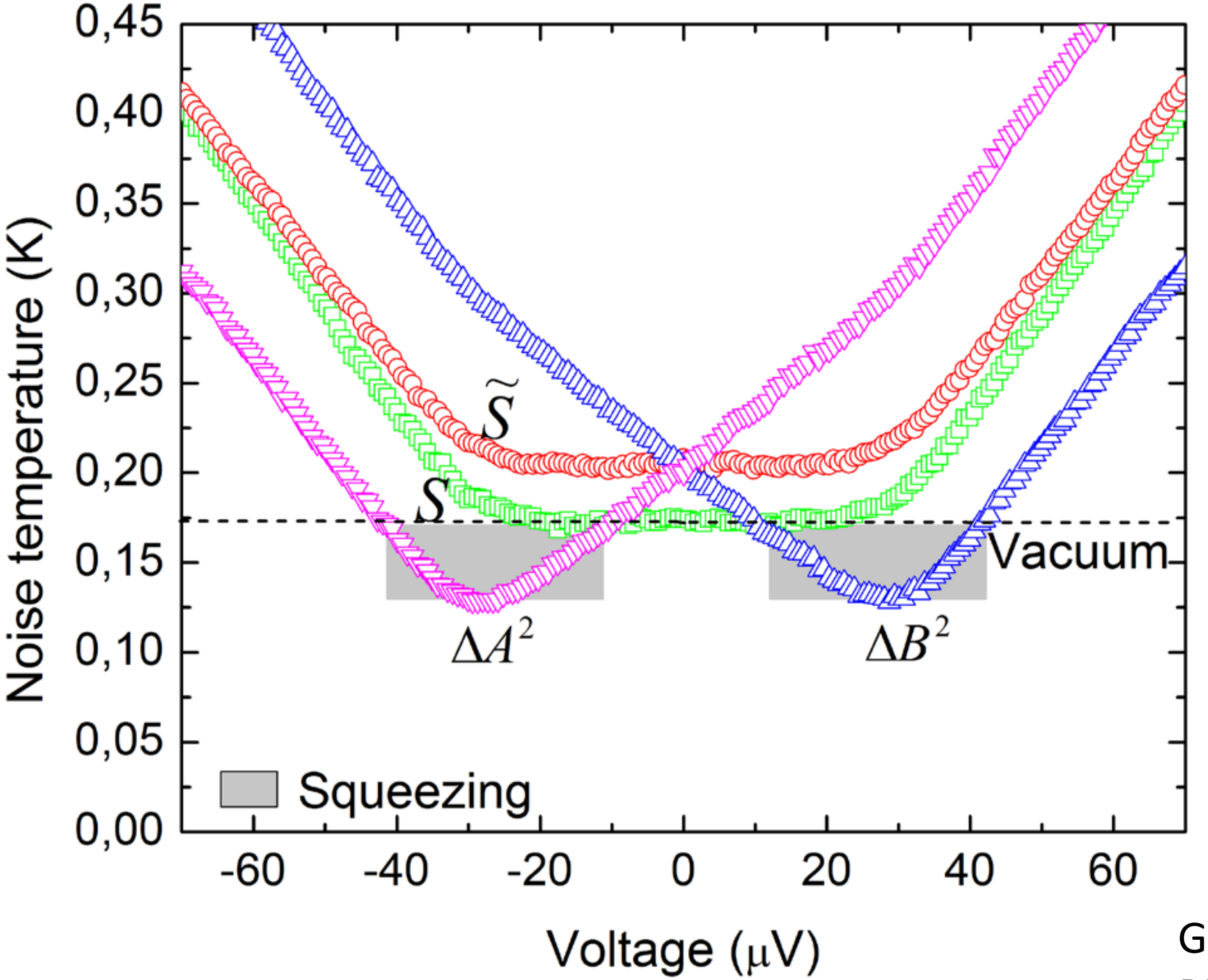


Quantum regime :

$$k_B T_e \ll eV \qquad 86\ \mu\mathrm{V} = 1\ K$$

$$k_B T_e \ll \hbar\omega_0 \qquad 1\ \mathrm{GHz} = 1\ \mathrm{K}$$

Result $\omega_0 = 2\omega$



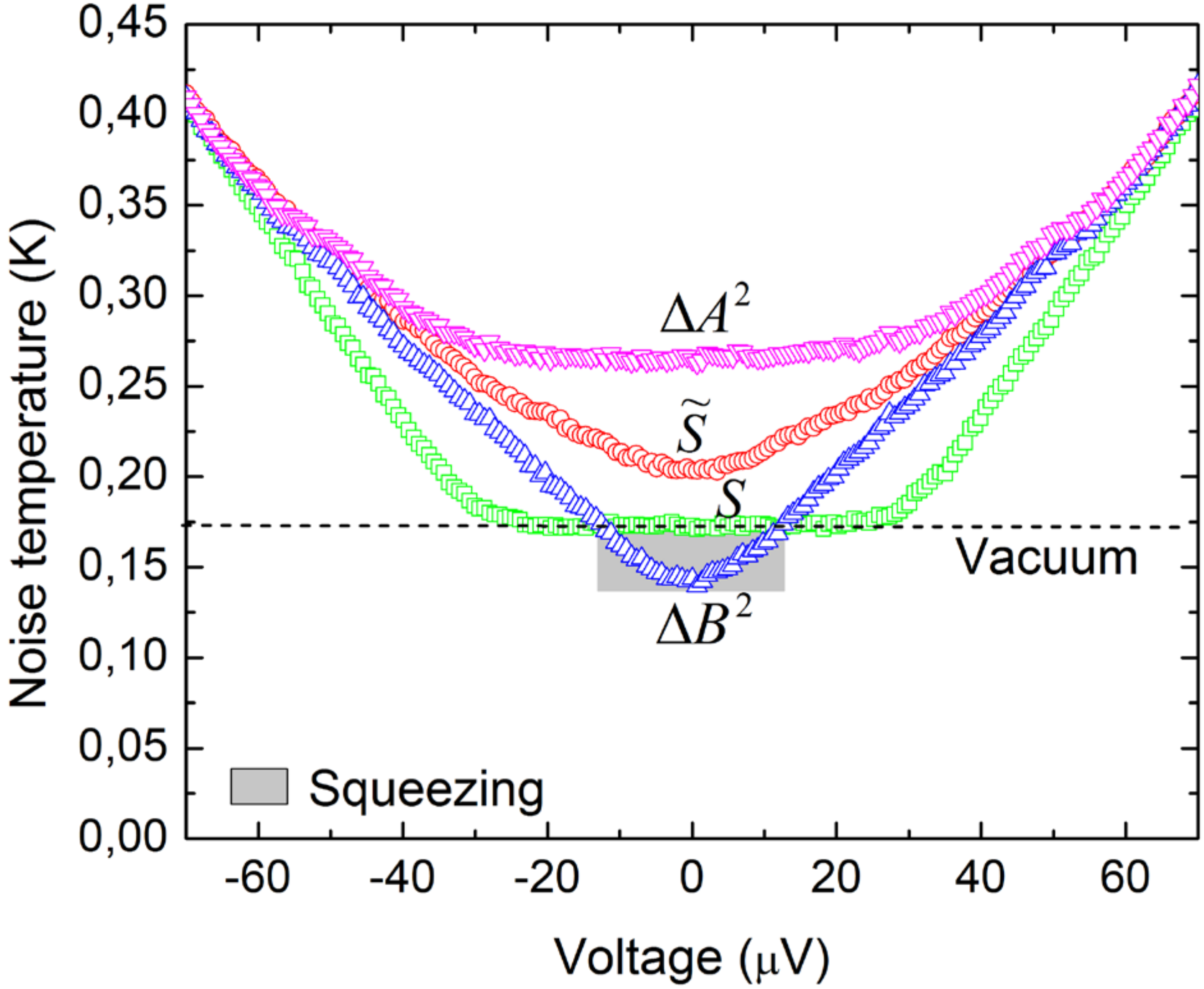
$\omega = 7.2$ GHz

$\omega_0 = 14.4$ GHz

$\alpha = 0.74$

G. Gasse, C. Lupien & BR,
PRL 111, 136601 (2013)

Result $\omega_0 = \omega$



$\omega = 7.2\ \text{GHz}$

$\omega_0 = 7.2\ \text{GHz}$

$\alpha = 0.82$

Two-mode squeezing ?

Correlation between quadratures at two different frequencies:

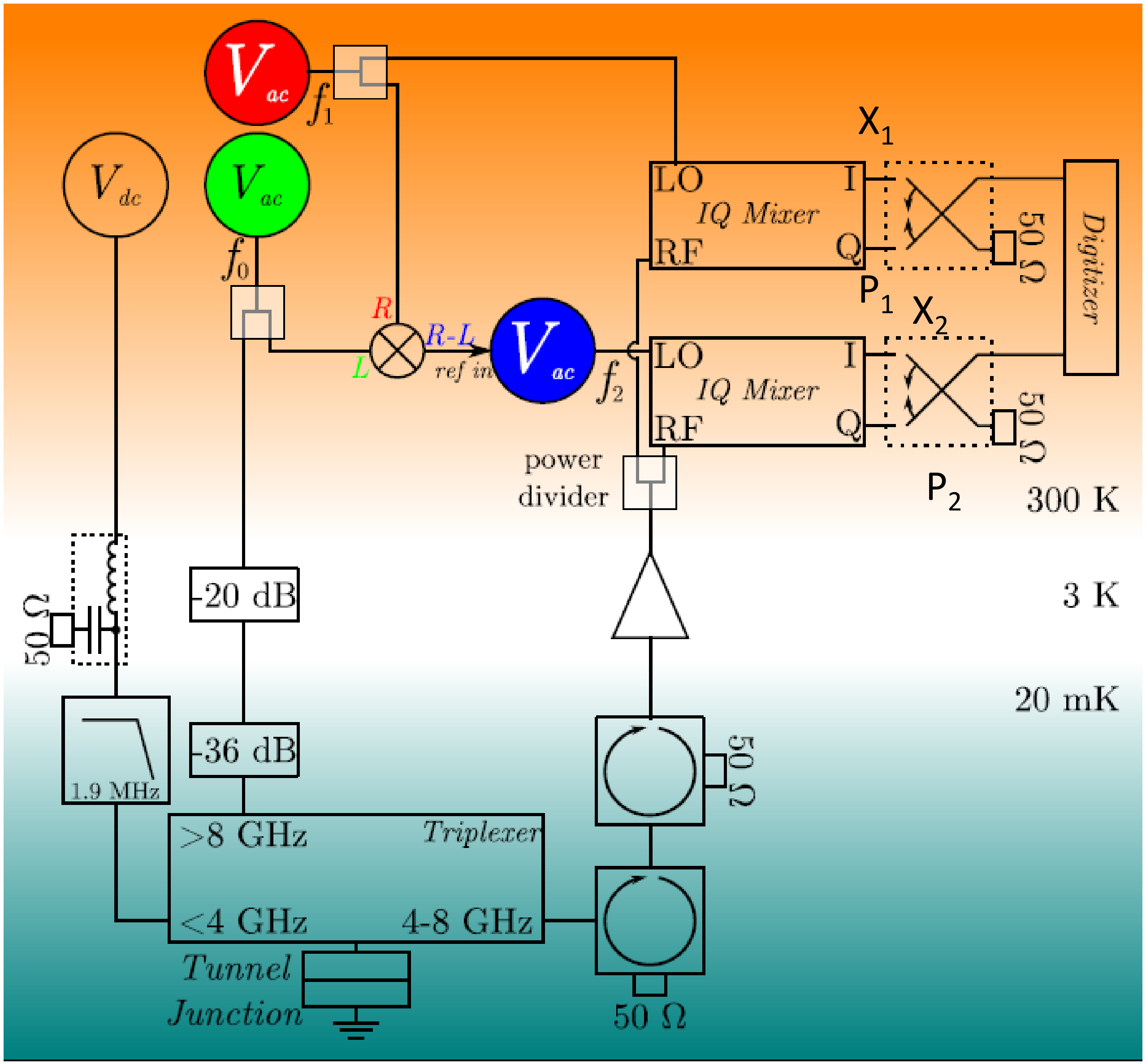
$$\langle X_1 X_2 \rangle \neq 0 \,, \langle P_1 P_2 \rangle \neq 0$$

Bell-like inequality to proove entanglement:

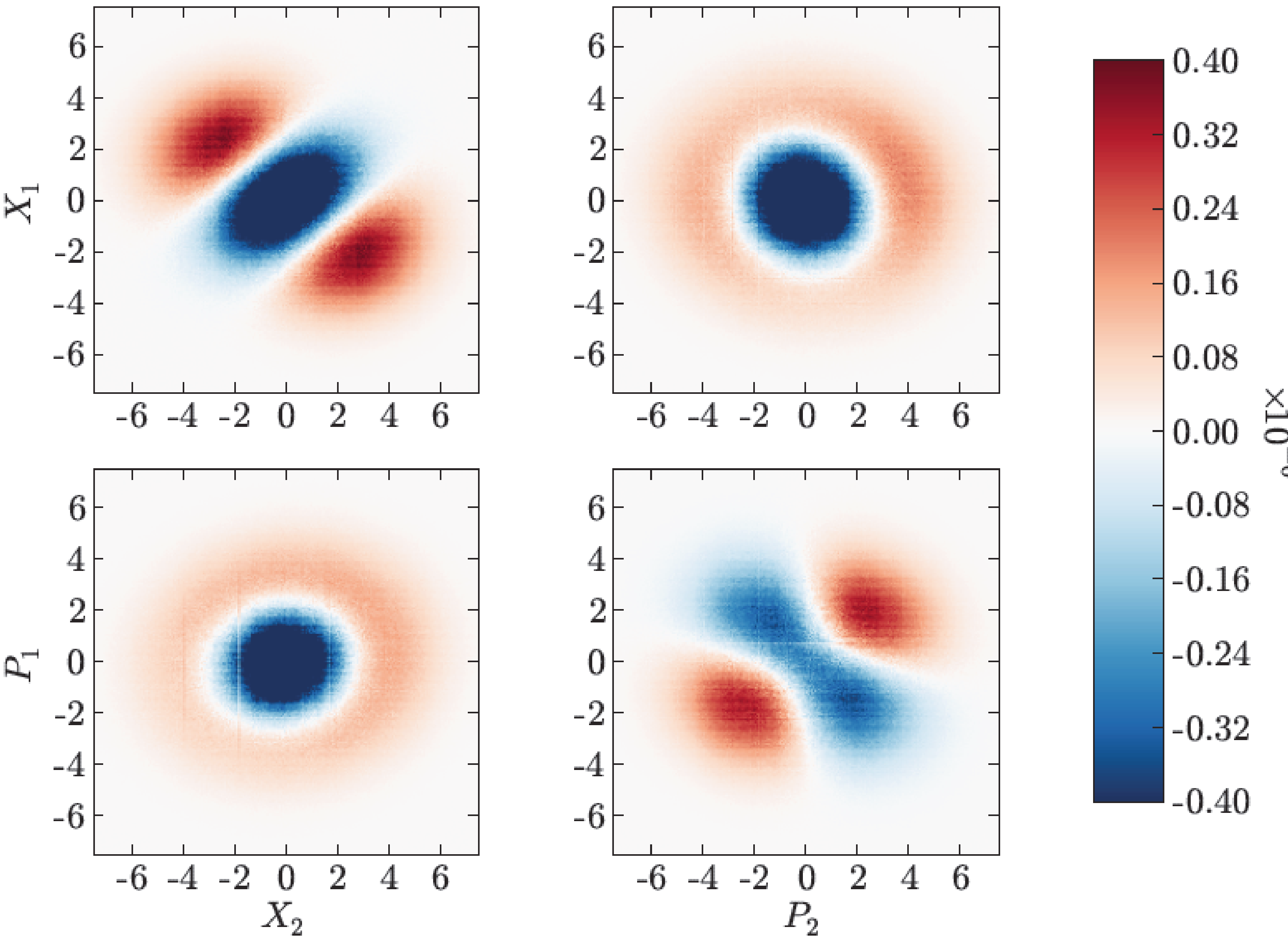
$$\langle (X_1 - X_2)^2 \rangle + \langle (P_1 + P_2)^2 \rangle \geq 4$$

$$|\Psi\rangle \approx |\cos\omega_1 t\rangle |\cos\omega_2 t\rangle + |\sin\omega_1 t\rangle |\sin\omega_2 t\rangle$$
$$\approx |\uparrow\rangle \ |\uparrow\rangle \ + \ |\downarrow\rangle \ |\downarrow\rangle$$

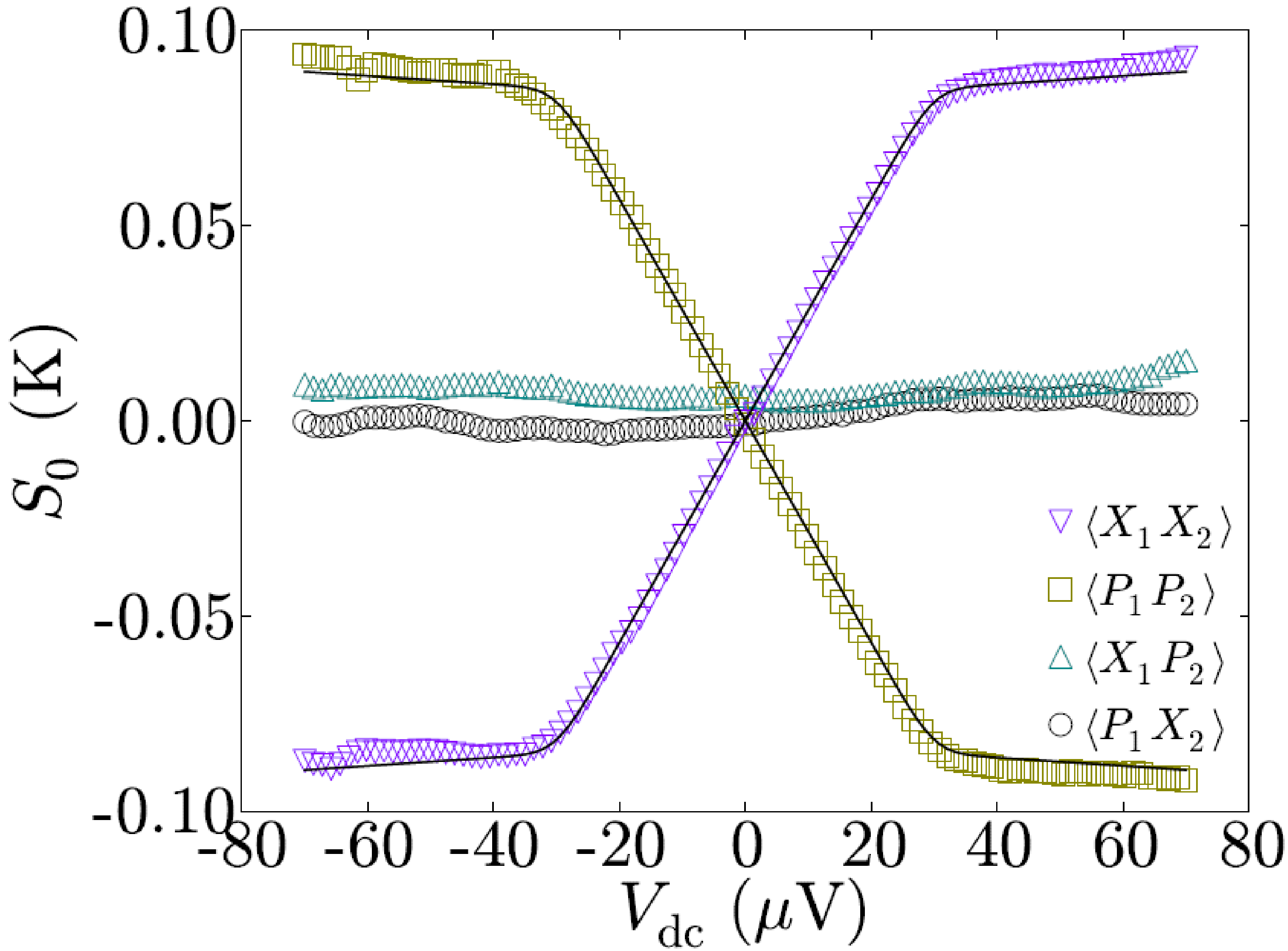
Experimental setup



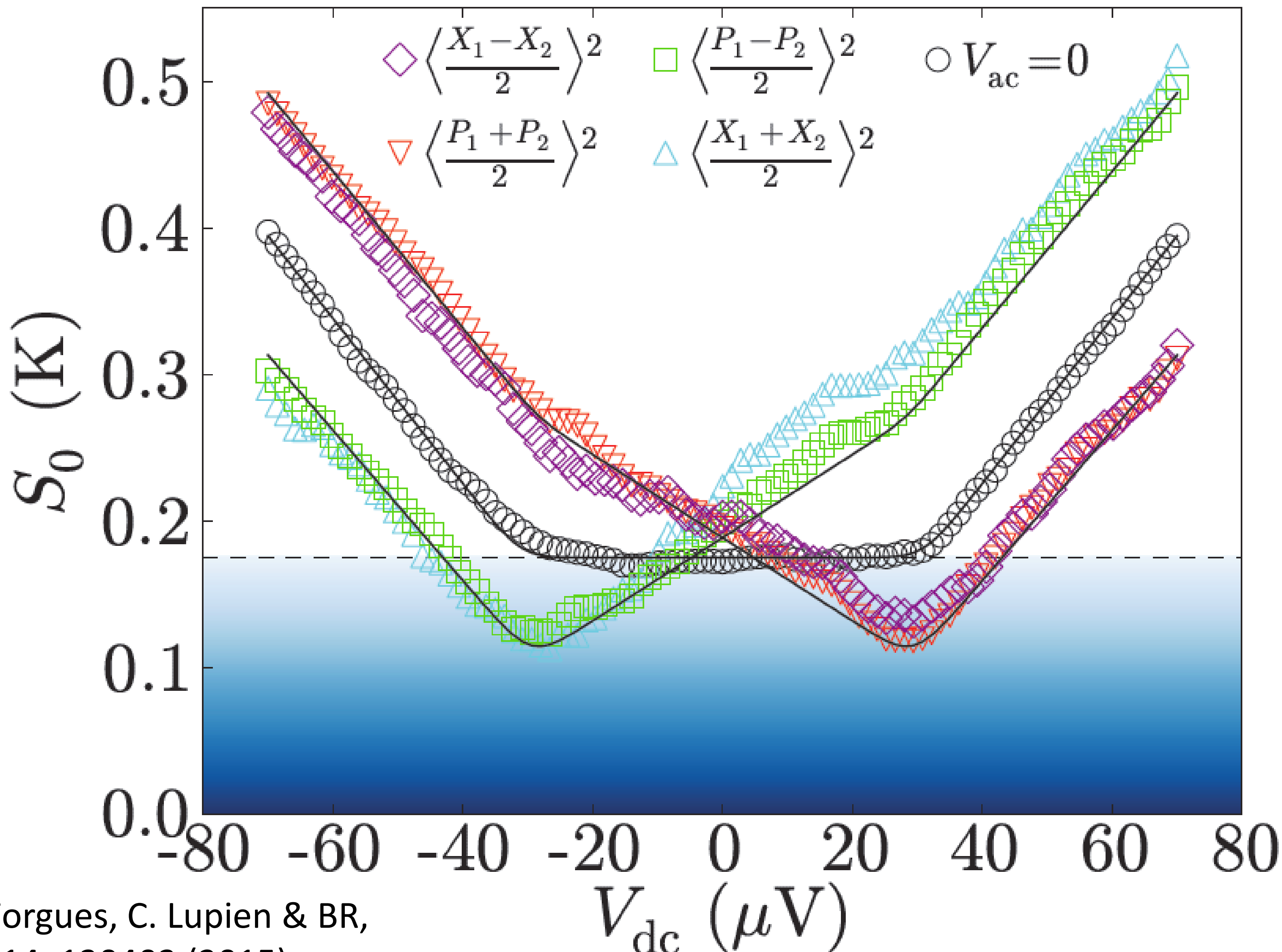
Qualitative results: $P(V_{ac})-P(V_{ac}=0)$



Quantitative results: correlations



Criterion for entanglement !



J.C. Forgues, C. Lupien & BR,
PRL114, 130403 (2015)

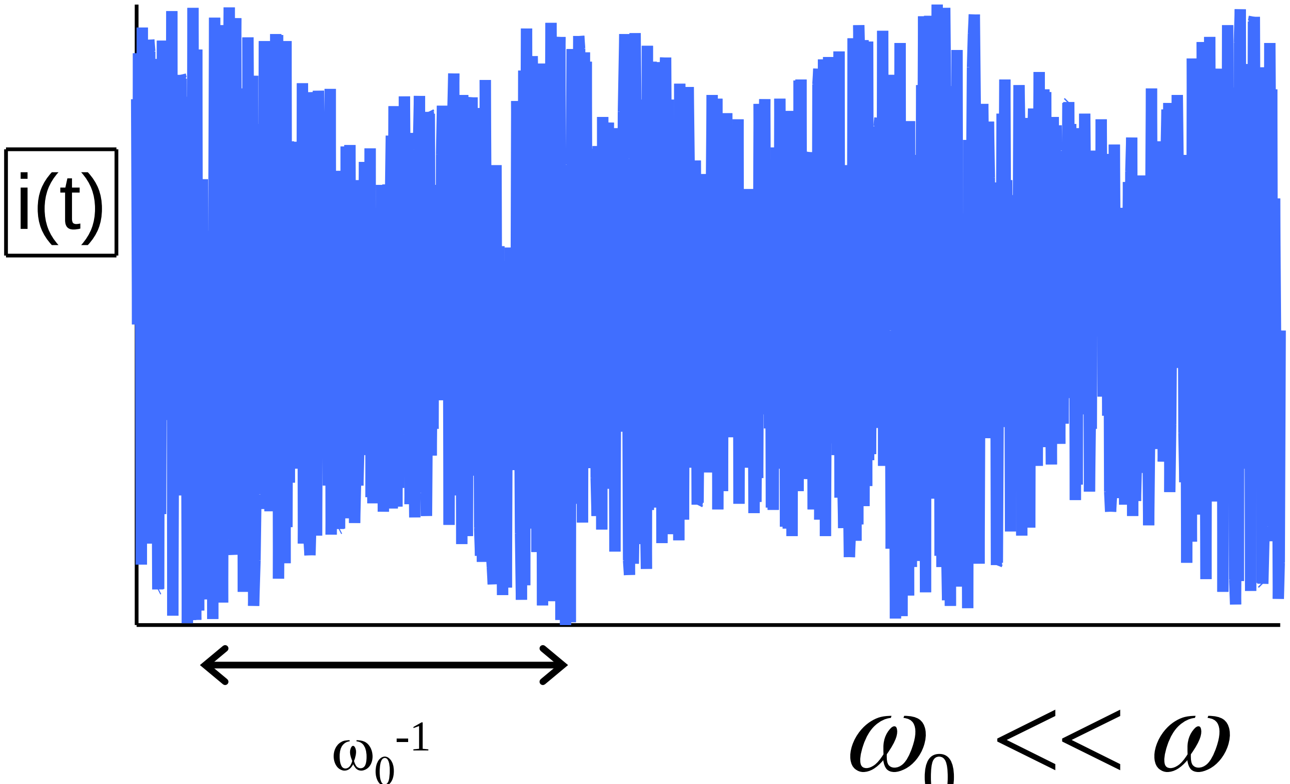
Interpretation: noise modulation

- Noise at frequency f_1 modulated at f_2+f_1 gives a sideband at $-f_2$ with a well-defined phase, and vice-versa
- This works even at the single photon level !
- One can modulate the zero point fluctuations !

Noise susceptibility – How fast can one modulate noise ?

$$V(t) = V_{dc} + \delta V \cos \omega_0 t$$

$$\langle \delta I(\omega) \delta I(\omega_0 - \omega) \rangle$$

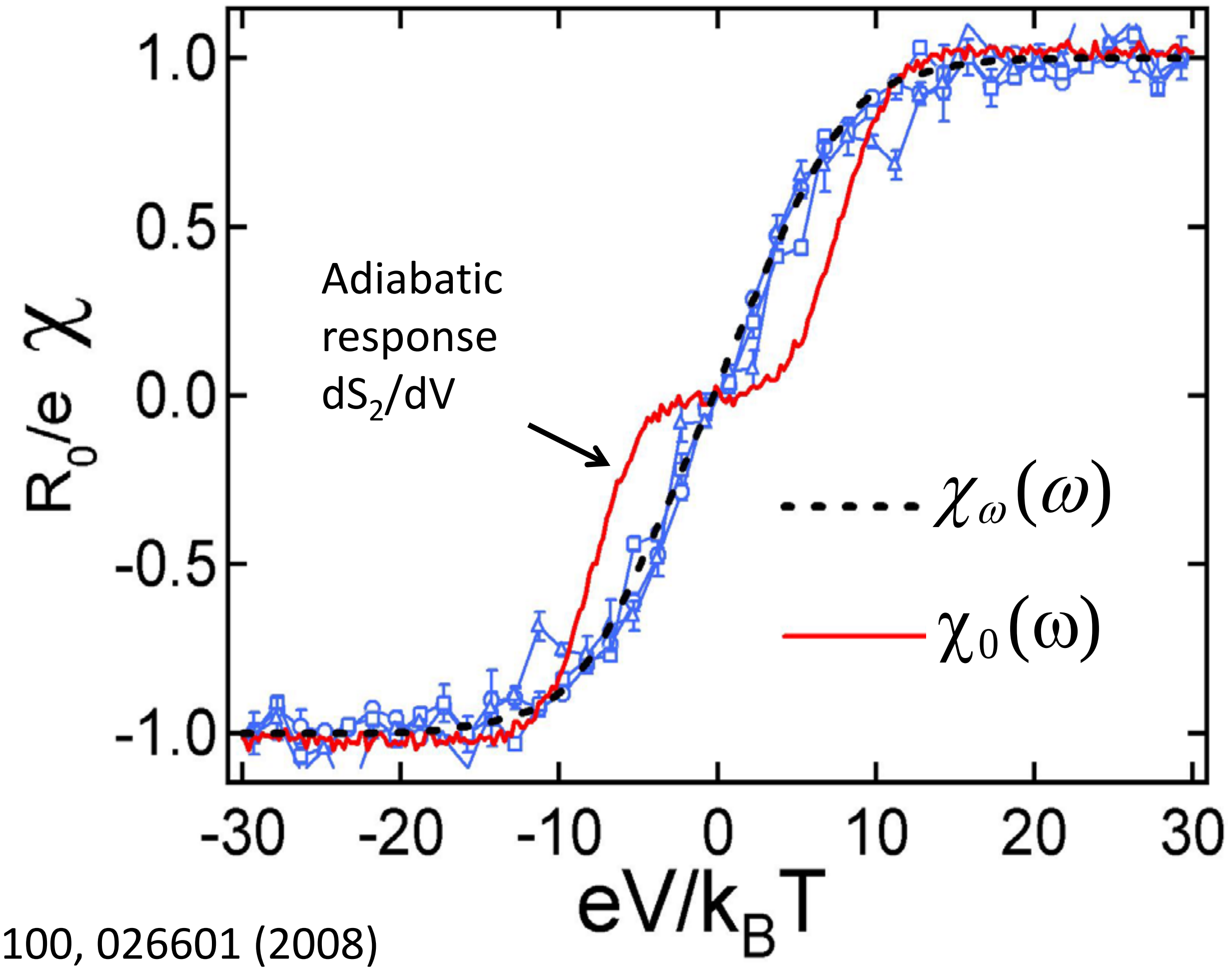


$$\frac{\partial S_2(\omega)}{\partial V_{\omega_0}}$$

$$\omega_0 << \omega$$

Noise susceptibility – the quantum regime: experiment

ω_0~ ω~ 6 GHz
T=35 mK
$\hbar\omega/k_BT$~8.5



J. Gabelli &BR, PRL 100, 026601 (2008)

Related ongoing projects

- Theory: how to link properties of the electrical current (i.e., electrons), with that of the radiated electromagnetic field (F. Qassemi, A. Blais)
- Experiment: measurement of the photon statistics of the squeezed electromagnetic field with linear detectors : photon pairs (J.-O. Simoneau, S. Virally)