Entanglement in Strongly-Correlated Quantum Matter @ KITP, 2015

Aspects of Holographic Quenches
Part2: Locally Excited States

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This talk is a summary of our following works:

arXiv:1302.5703 [JHEP05(2013)080])
arXiv:1401.0539 [PRL 112(2014)111602]
arXiv:1403.0702 [PRD 90(2014)041701]
arXiv:1405.5946 [PTEP 2014 (2014) 9, 093B06]
arXiv:1410.2287 [JHEP01(2015)102]
(arXiv: 1503.08161)

Collaborators

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① Summary of Main Results (Just 3 pages)

(1-1) Our setup

Take a locally excited state in a given (d+1) dim. CFT:

$$\left|O(x)\right\rangle \equiv \underline{e^{-\varepsilon H}} \cdot \underline{O(x)}\left|0\right\rangle.$$

UV regularization of local operator (Note: ε≠lattice spacing)

A primary state with dim. Δ_O

$$\Rightarrow \text{Total energy}: \int T_{tt}(x)dx^d \approx \frac{\Delta_O}{\varepsilon}$$

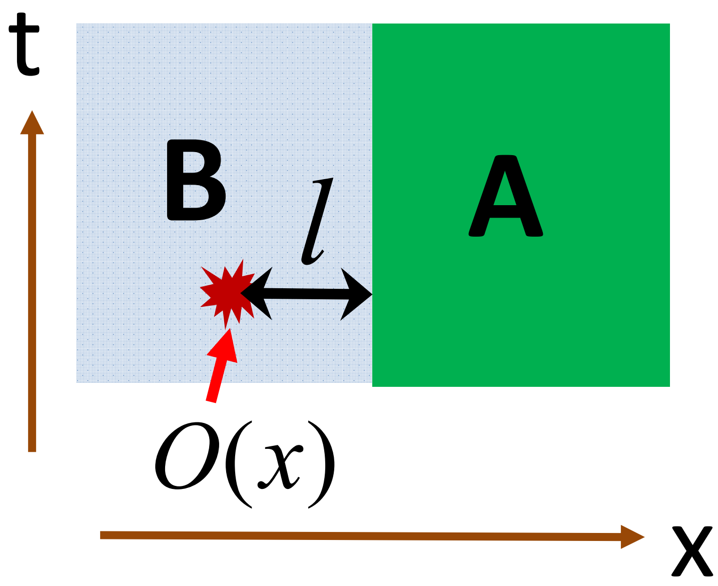
Then we consider its time evolution:

$$\left|O(x,t)\right\rangle = e^{-iHt}\left|O(x)\right\rangle.$$

(1-2) What to Compute

The growth of (n-th Renyi) entanglement entropy

$$\Delta S_A^{(n)} \equiv S_A^{(n)}\left[\left|O(x)\right\rangle\right] - S_A^{(n)}\left[\left|0\right\rangle\right].$$



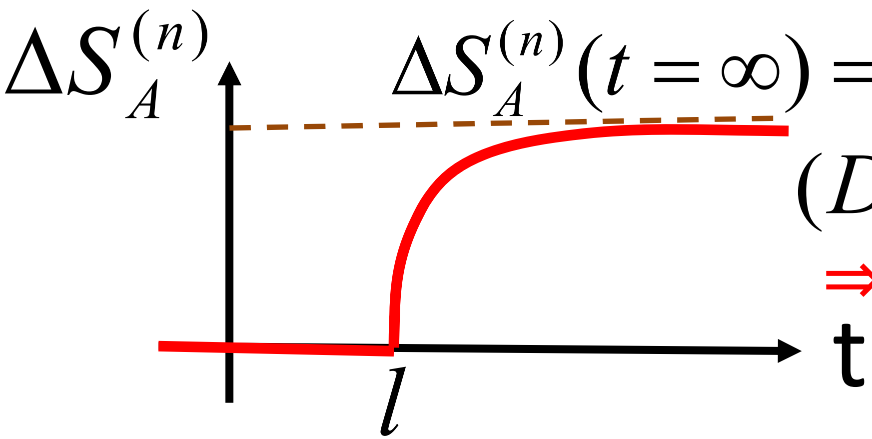
For simplicity, we choose

A = a half space .

This calculation will show propagations and generations of quantum entanglement.

(1-3) Summary of Main Results

(i) Integrable CFTs [Massless Free Fields, Minimal Models etc.]

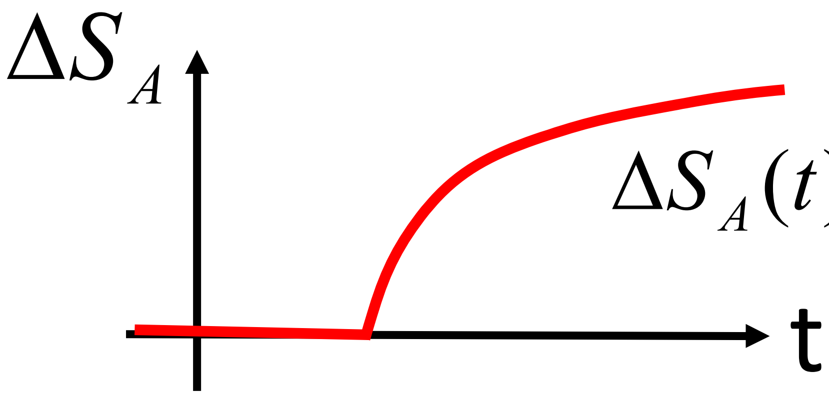


$$\Delta S_A^{(n)}(t=\infty) = \text{finite} = \log[D(O)].$$

$$(D(O) = \text{quantum dim.})$$

⇒Propagation of entangled pairs

(ii) Holographic CFTs [AdS3/CFT2]



$$\Delta S_A(t) \approx \frac{c}{6}\log\frac{t}{\varepsilon}$$

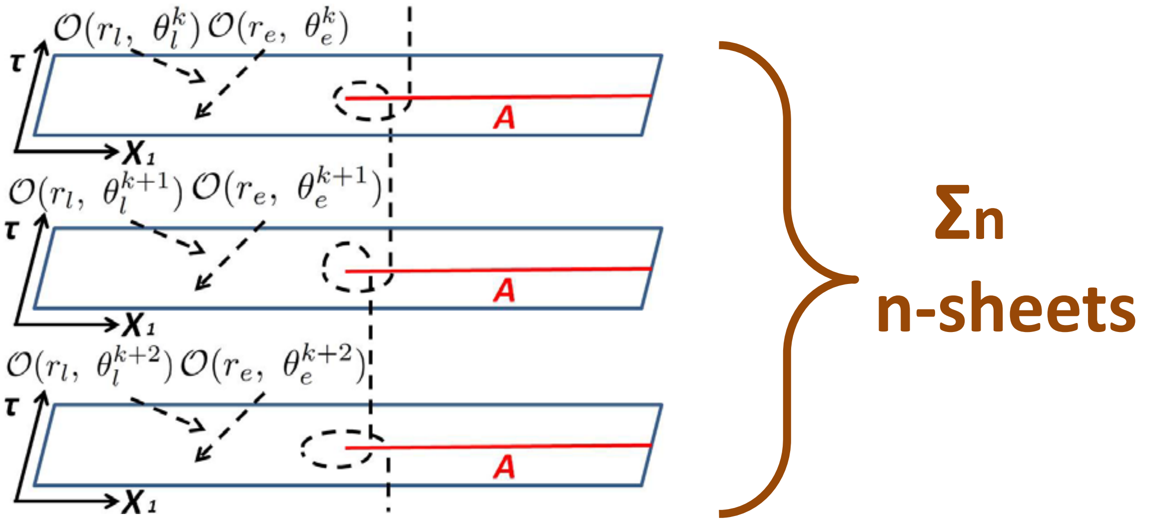
⇒Productions of entangled pairs

② Free Field Theory Calculations [Nozaki-Numasawa-TT 14]

(2-1) Replica formulation

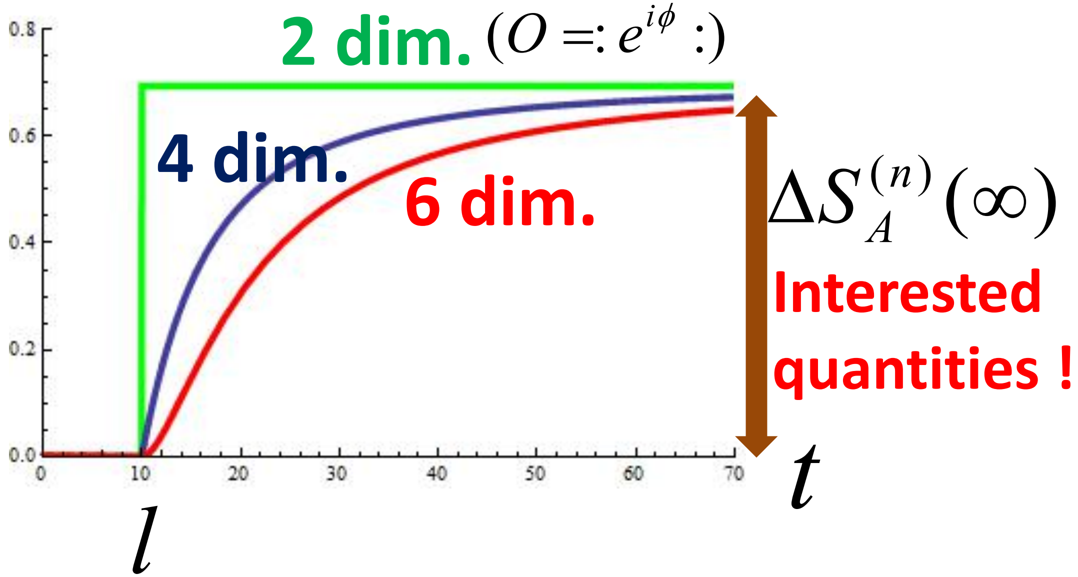
The n-th Renyi EE can be expressed in terms of 2n-point correlation functions on Σn :

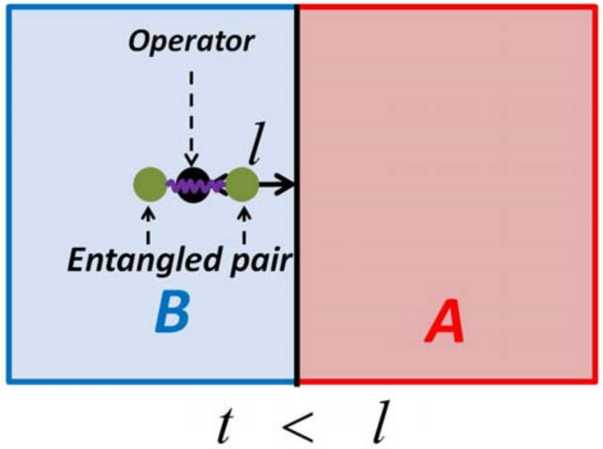
$$\Delta S_A^{(n)} = \frac{1}{1-n}\cdot\Big[\log\Big\langle O(r_l,\theta_l^n)O(r_e,\theta_e^n)\cdots O(r_l,\theta_l^1)O(r_e,\theta_e^1)\Big\rangle_{\Sigma_n} - n\cdot\log\Big\langle O(r_l,\theta_l)O(r_e,\theta_e)\Big\rangle_{\Sigma_1}\Big].$$

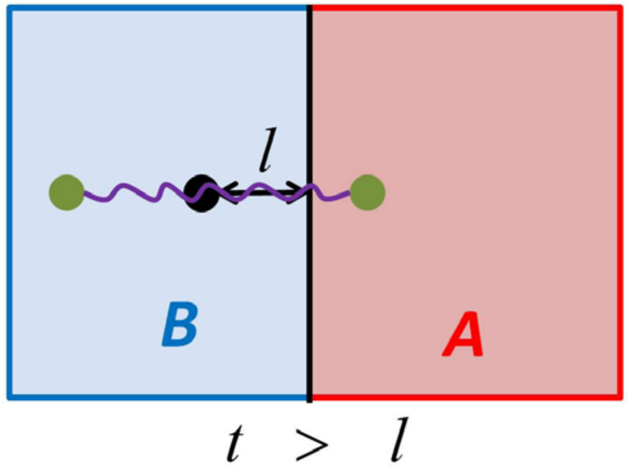


(2-2) Results in free massless scalar theory

$\Delta S_A^{(2)}$ for $O =: \phi :$ (i.e. $k = 1$)







$$E.g.\quad \Delta S_{A(4\,\mathrm{dim})}^{(2)} = \log\left(\frac{2t^2}{t^2 + l^2}\right).$$

Note :

$\Delta S_A^{(n)f}$ is `topologically invariant'

under deformations of A.

$\Delta S_A^{(n)f}$ for $O=\phi^k$ in $d+1>2$ dim.

TABLE I. $\Delta S_A^{(n)f}$ and $\Delta S_A^{f}\left(=\Delta S_A^{(1)f}\right)$ for free massless scalar field theories in dimensions higher than two ($d>1$).

	n	$k=1$	$k=2$	$\cdots$	$k=l$
$\Delta S_A^{(n)f}$	2	$\log 2$	$\log\frac{8}{3}$	$\cdots$	$-\log\left(\frac{1}{2^{2l}}\sum_{j=0}^{l}\left({}_lC_j\right)^2\right)$
	3	$\log 2$	$\frac{1}{2}\log\frac{32}{5}$	$\cdots$	$\frac{-1}{2}\log\left(\frac{1}{2^{3l}}\sum_{j=0}^{l}\left({}_lC_j\right)^3\right)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	m	$\log 2$	$\frac{1}{m-1}\log\frac{2^{2m-1}}{2^{m-1}+1}$	$\cdots$	$\frac{1}{1-m}\log\left(\frac{1}{2^{ml}}\sum_{j=0}^{l}\left({}_lC_j\right)^m\right)$
ΔS_A^{f}	1	$\log 2$	$\frac{3}{2}\log 2$	$\cdots$	$l\log 2-\frac{1}{2^l}\sum_{j=0}^{l}{}_lC_j\log{}_lC_j$

Renyi Entropy

von-Numann EE

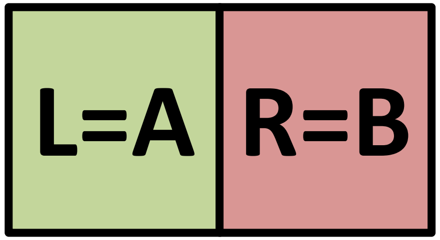
EPR state !

[For a proof: Nozaki, arXiv:1405.58754]

Heuristic Explanation

First , notice that in free CFTs, there are definite particles moving at the speed of light.

$$\Rightarrow \quad \phi \approx \underset{\text{left-moving}}{\phi_L} + \underset{\text{right-moving}}{\phi_R} \cdot$$



$$\phi^k |\mathrm{vac}\rangle \approx \sum_{j=0}^{k} {}_kC_j \cdot (\phi_L)^j \cdot (\phi_R)^{k-j} |\mathrm{vac}\rangle$$

$$= 2^{-k/2} \sum_{j=0}^{k} \sqrt{{}_kC_j}\, |j\rangle_L |k-j\rangle_R .$$

$$\Rightarrow \quad \Delta S_A^{(n)f} = \frac{1}{1-n} \log\left[2^{-nk} \sum_{j=0}^{k} ({}_kC_j)^n\right],$$

$$\Delta S_A^f = k \log 2 - 2^{-k} \sum_{j=0}^{k} {}_kC_j \cdot \log[{}_kC_j].$$

Agree with replica Calculations !

③ Rational 2d CFTs [He-Numasawa-Watanabe-TT 14]

(3-1) Free Scalar CFT in 2d

Consider following two operators in the free scalar CFT:

(i) $O_1 = e^{i\alpha\phi}, \qquad \Rightarrow \qquad \Delta S_A^{(n)f} = 0.$

$$|O_1\rangle = e^{i\alpha\phi_L}|0\rangle_L \otimes e^{i\alpha\phi_R}|0\rangle_R$$ ⇒Direct product state

(ii) $O_2 = e^{i\alpha\phi} + e^{-i\alpha\phi}, \qquad \Rightarrow \qquad \Delta S_A^{(n)f} = \log 2.$

$$|O_2\rangle = e^{i\alpha\phi_L}|0\rangle_L \otimes e^{i\alpha\phi_R}|0\rangle_R + e^{-i\alpha\phi_L}|0\rangle_L \otimes e^{-i\alpha\phi_R}|0\rangle_R$$

$$\approx |\uparrow\rangle_L|\uparrow\rangle_R + |\downarrow\rangle_L|\downarrow\rangle_R$$ ⇒ EPR state

(3-2) Rational 2d CFTs (e.g. minimal models, WZW models)

2nd Renyi EE $\Rightarrow$ **4 pt. function:** $\langle O(\infty)O(1)O(z)O(0)\rangle.$

(i) Early time $(0 < t < l): (z, \bar{z}) \to (0,0).$

Chiral Fusion Transformation
z→1-z

(ii) Late time $(t \geq l): (z, \bar{z}) \to (1,0).$

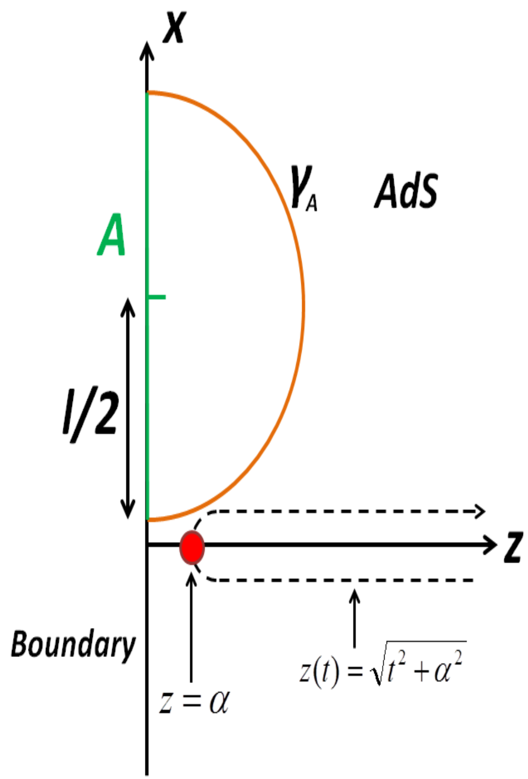
This allows us to prove $\Delta S_A^{(n)} = \log D(O)$ **for any n.**

quantum dim.

Ex. Ising model : $\Delta S_A^{(n)}[I] = \Delta S_A^{(n)}[\varepsilon] = 0,$

$$\Delta S_A^{(n)}[\sigma] = \log\sqrt{2}.$$

④ Holographic Analysis



A locally excited state

~ A falling particle in AdS.

$$\Delta_O \approx mR$$

[e.g. stress tensors agree with the CFT.]

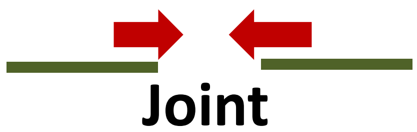
We can find an analytical metric using the Horowitz-Itzhaki map.

$$\Delta S_A \approx \frac{c}{6}\log\left(\frac{t}{\varepsilon}\right).$$

[Holographic Calculations: Numasawa-Nozaki-TT 13, Caputa-Nozaki-TT 14]

[Large c CFT computations: Asplund-Bernamonti-Galli-Hartman 14]

$cf.\quad \Delta S_A \approx \frac{c}{3}\log t$, for local quenches



in the sense of Calabrese-Cardy 2007. [Holographic Calculation: Ugajin 13]

⑤ Finite Temperature Analysis [Caputa-Simon-Stikonas-TT 14]

Consider a local operator excited state at finite temp.

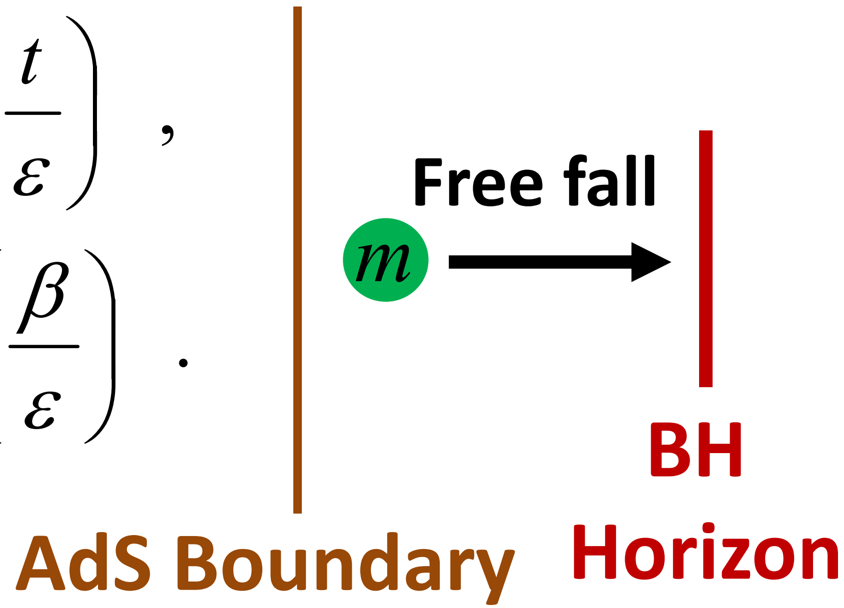
(i) Integrable CFTs ⇒ Only O(ε/β) differences.

(ii) Holographic CFTs (=AdS Blackholes)

⇒ **EE saturates at late time !**

$$0 < t << \beta : \quad \Delta S_A^{(1)} \approx \frac{c}{6}\log\left(\frac{t}{\varepsilon}\right) \quad ,$$

$$t >> \beta : \quad \Delta S_A^{(1)} \approx \frac{c}{6}\log\left(\frac{\beta}{\varepsilon}\right) \quad .$$



A Sketch of Time Evolution