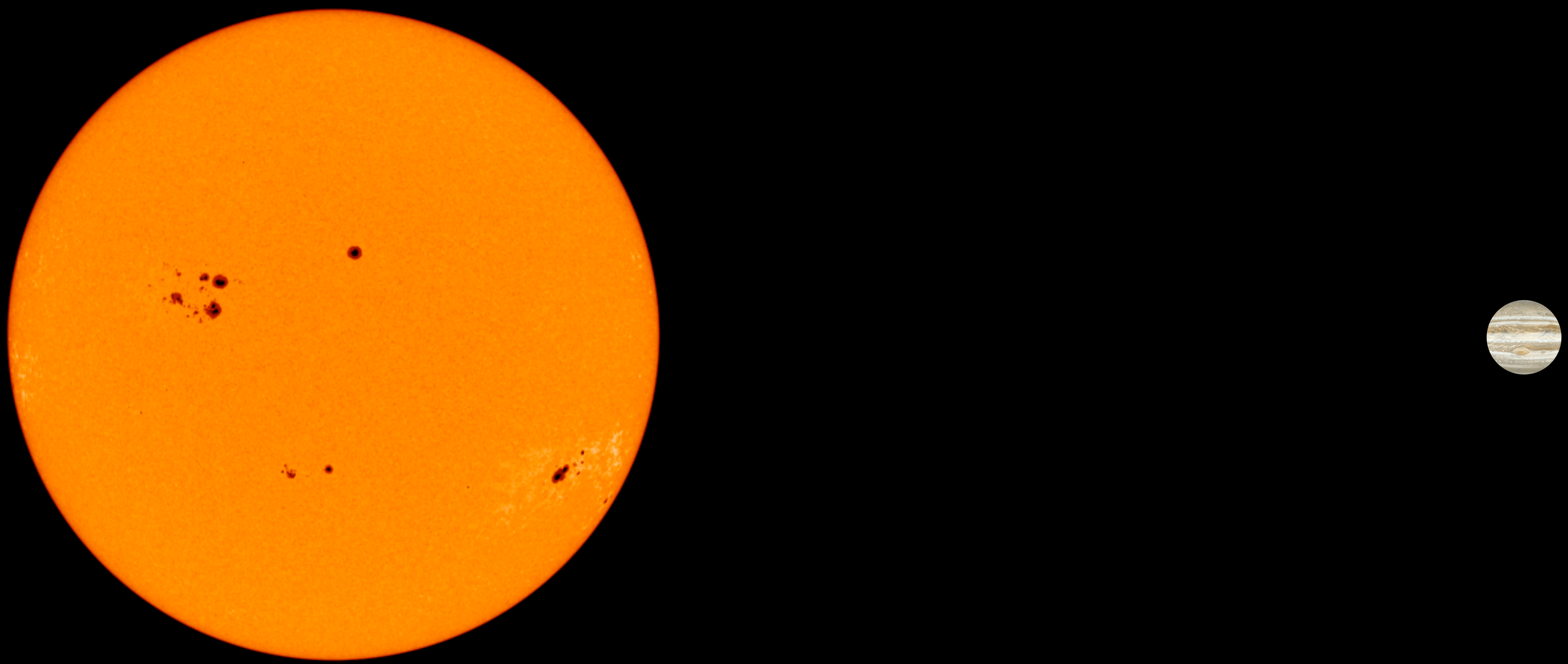
Dynamical tidal interactions of giant planets and stars

Gordon Ogilvie DAMTP, University of Cambridge



Exoplanets Rising

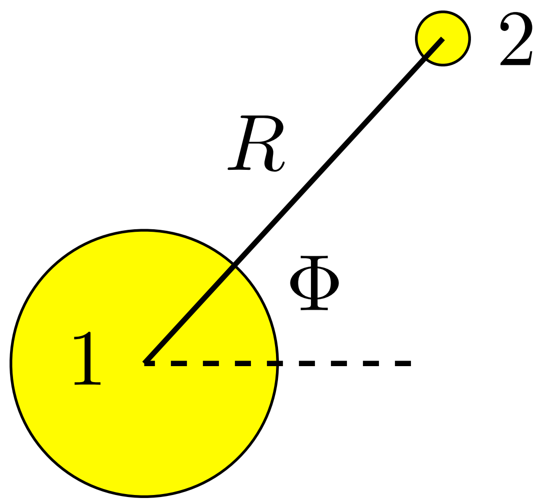
KITP 02.04.10

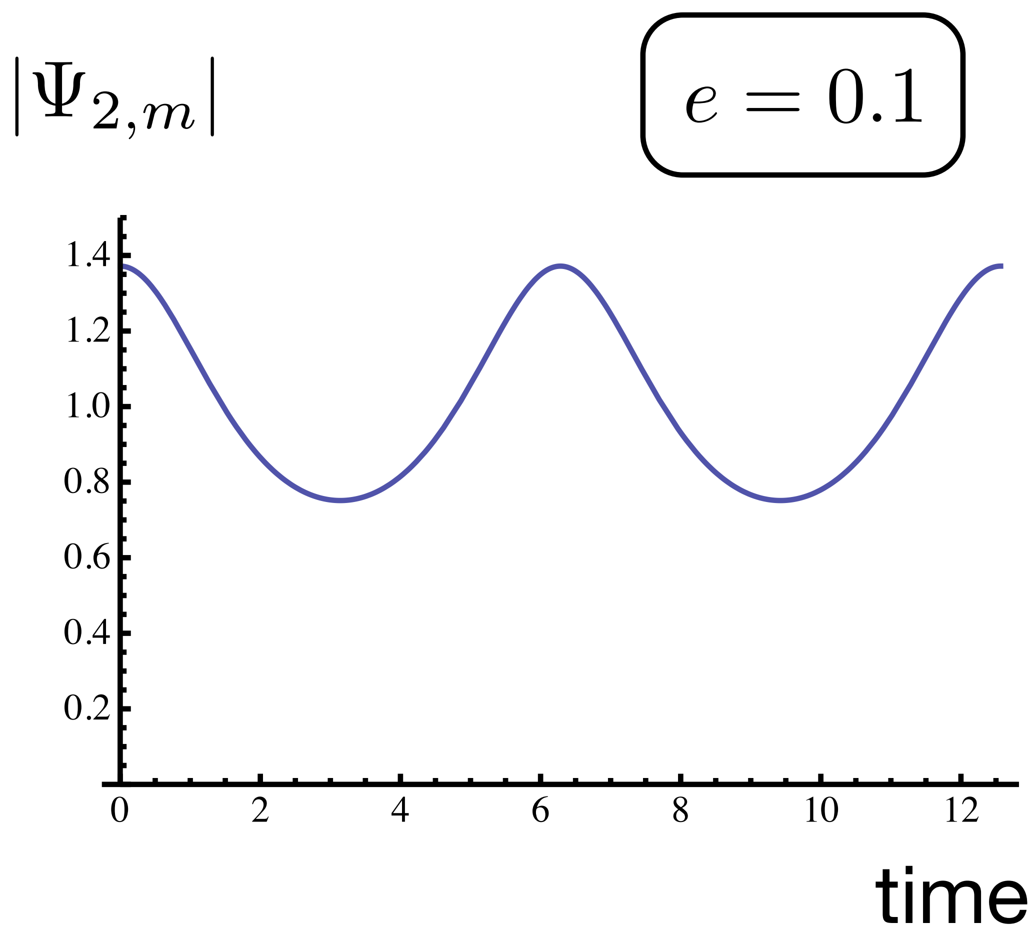
Tidal forcing

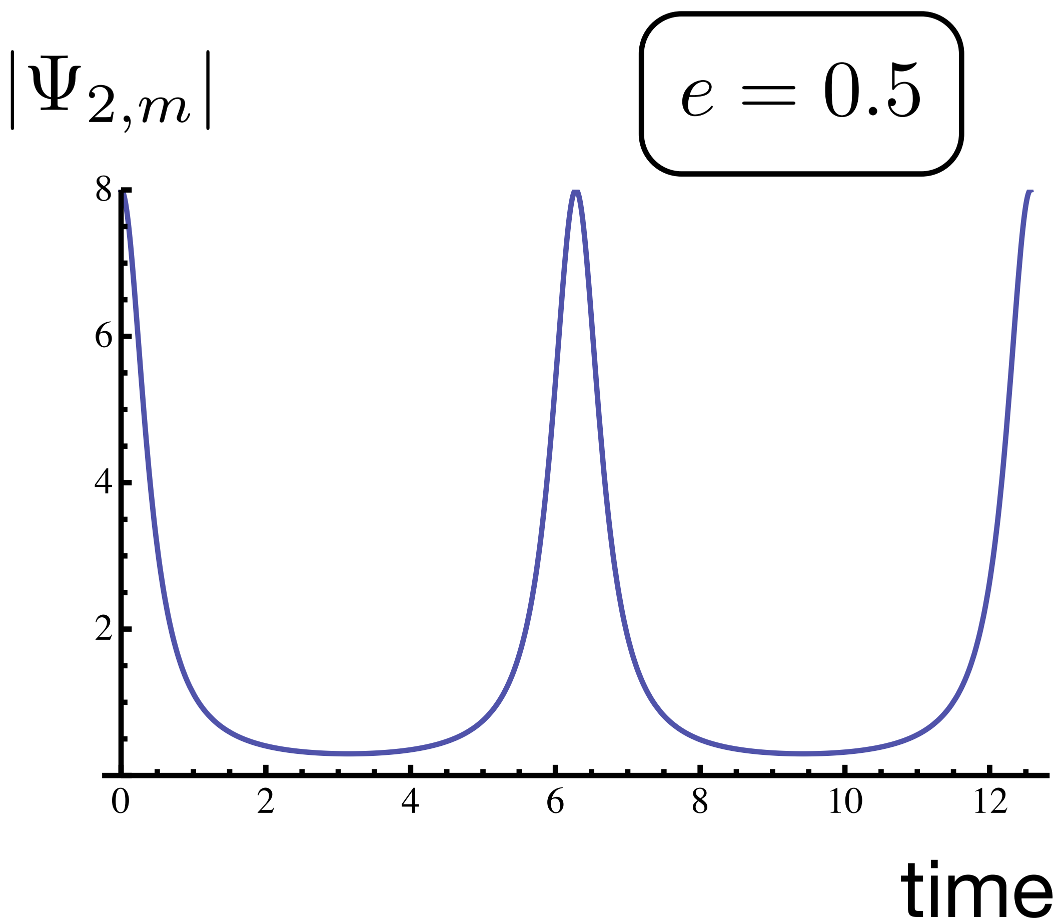
- Tidal potential experienced by body 1

$$\Psi = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \Psi_{l,m}(t) \left(\frac{r}{R_1} \right)^l Y_{l,m}(\theta, \phi)$$

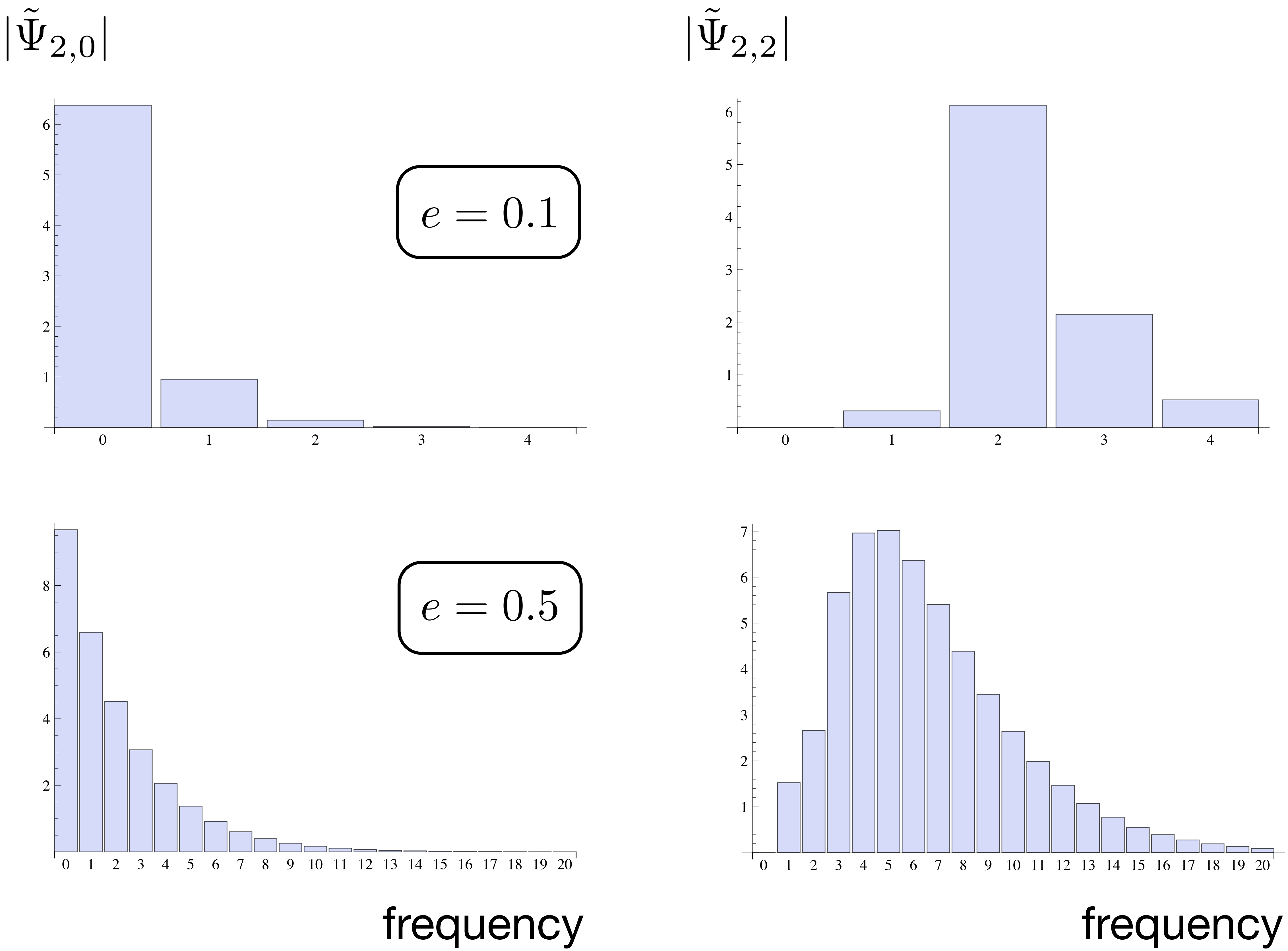
$$\Psi_{l,m} = -C_{l,m} \frac{GM_2}{R_1} \left(\frac{R}{R_1} \right)^{-(l+1)} \mathrm{e}^{-\mathrm{i}m\Phi}$$







Fourier analysis



Love number and “tidal Q”

- Consider each potential component experienced by body 1

$$\Psi = \tilde{\Psi}_{l,m} \left(\frac{r}{R_1}\right)^{l} Y_{l,m}(\theta,\phi)\,\mathrm{e}^{-\mathrm{i}\omega t}$$

- Body 1 is deformed and generates an external potential

$$\Phi' = \underline{k_{l,m}(\omega)}\,\tilde{\Psi}_{l,m} \left(\frac{r}{R_1}\right)^{-(l+1)} Y_{l,m}(\theta,\phi)\,\mathrm{e}^{-\mathrm{i}\omega t}$$

(+ orthogonal terms)

- Love number (linear response function)
- Energy transfer to orbit $\propto \omega\,\mathrm{Im}(k_{l,m})\,|\tilde{\Psi}_{l,m}|^2$
- Angular momentum transfer $\propto m\,\mathrm{Im}(k_{l,m})\,|\tilde{\Psi}_{l,m}|^2$
- $\mathrm{Im}(k) \approx \dfrac{k}{Q} \approx \dfrac{1}{Q'} \ll 1$ depends on ω, l, m (usually $l = m = 2$)

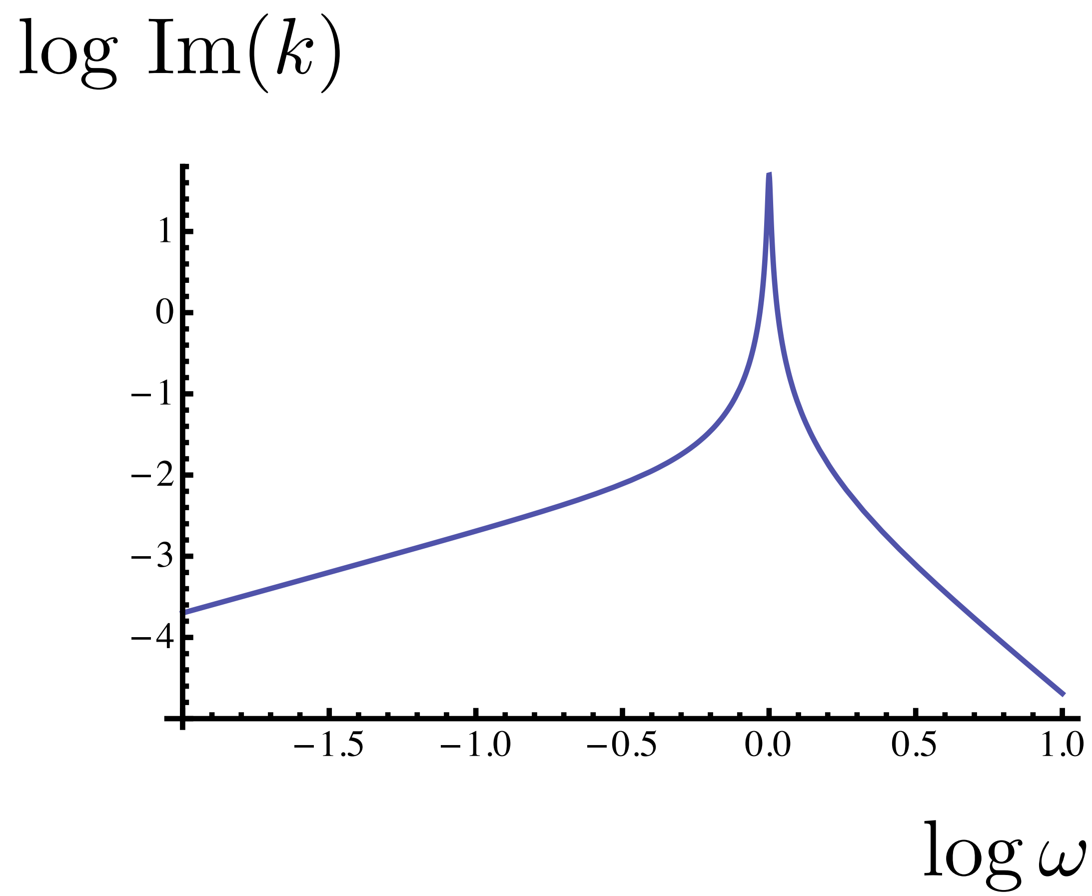
Analogy : forced harmonic oscillator

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = f\,\mathrm{e}^{-\mathrm{i}\omega t}$$

$$x = k\,\frac{f}{\omega_0^2}\,\mathrm{e}^{-\mathrm{i}\omega t}$$

$$\begin{aligned} k &= \left(1 - \frac{2\mathrm{i}\omega\gamma}{\omega_0^2} - \frac{\omega^2}{\omega_0^2}\right)^{-1} \\ &\approx \left(1 + \mathrm{i}\,Q^{-1}\right) \end{aligned}$$

$$[\,\omega, \gamma \ll \omega_0\,]$$



$$Q = \frac{\omega_0^2}{2\omega\gamma} = \frac{2\pi \times \text{maximum potential energy}}{\text{energy dissipated per cycle}} \gg 1$$

Tidal forcing problem

Viscous uniformly rotating fluid

Tidal potential Ψ and linear response proportional to $\mathrm{e}^{-\mathrm{i}\omega t}$

$$-\mathrm{i}\omega u_i + 2\epsilon_{ijk}\Omega_j u_k = -\partial_i \Phi' - \partial_i \Psi - \frac{1}{\rho}\partial_i p' + \frac{\rho'}{\rho^2}\partial_i p + \frac{1}{\rho}\partial_j T_{ij}$$

$$-\mathrm{i}\omega \rho' + u_i \partial_i \rho = -\rho \partial_i u_i$$

$$-\mathrm{i}\omega p' + u_i \partial_i p = -\Gamma_1 p \partial_i u_i$$

$$T_{ij} = 2\mu S_{ij} + \mu_\mathrm{b}(\partial_k u_k)\delta_{ij}$$

$$S_{ij} = \partial_i u_j + \partial_j u_i - \tfrac{2}{3}(\partial_k u_k)\delta_{ij}$$

$$\partial_{ii}\Phi' = 4\pi G \rho'$$

Given $\Psi = \left(\frac{r}{R}\right)^l Y_{l,m}(\theta,\phi)$

find $\Phi' = k_{l,m}\left(\frac{r}{R}\right)^{-(l+1)} Y_{l,m}(\theta,\phi) + \cdots \qquad (r > R)$

Tidal forcing problem

Energy dissipation rate

$$D = \frac{1}{2} \int \left(2\mu S_{ij}^* S_{ij} + \mu_\mathrm{b} |\partial_i u_i|^2\right) \mathrm{d}V$$

Energy input rate

$$-\frac{1}{2}\omega \,\mathrm{Im} \int \rho' \Psi^* \, \mathrm{d}V = \mathrm{Im}(k_{l,m}) \frac{(2l+1)R}{8\pi G} = D$$

Tidal torque

$$T = \frac{m}{\omega} D$$

Complications:

differential rotation, thermal diffusion, convection,

magnetic fields, nonlinearity, ...

From Goldreich (1963)

rigidity and Q the specific dissipation function

$$Q = \frac{2\pi E^*}{\oint (dE/dt)dt}, \qquad (2)$$

where E^* is the peak energy stored in the system during a cycle and $\oint(dE/dt)dt$ is the energy dissipated over a complete cycle. **Q will in general vary with the frequency and amplitude of the tide and the size of the sphere in addition to its composition.**

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cycle per second to one cycle per year (8). In this case ϵ_1 must be the dominant term and the sign of $\overline{(de/dt)}_p$ is the same as the sign of $2\omega - 3n$. **While this constant behaviour of Q with frequency may not be true for all planets (especially not the major ones)** it is still likely that the ϵ_1 term is dominant because of its relatively large coefficient. If this ϵ_1 term is dominant, we have $\overline{(de/dt)}_p > 0$ for all satellites

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small amplitude will have a phase lag which increases when its peak is reinforcing the peak of the tide of major amplitude. This non-linear behaviour cannot be treated in detail since very little is known about the response of the planets to tidal forces, except for the Earth. **In our discussions we shall use the language of linear tidal theory, but we must keep in mind that our numbers are really only parametric fits to a non-linear problem.**

Nonlinearity of tides in fluid bodies

- Equilibrium tidal amplitude

$$\frac{\xi}{R_1} \sim \frac{M_2}{M_1}\left(\frac{R_1}{a}\right)^3 \qquad\qquad \sim 1 \quad \text{for} \quad R_1 \sim \left(\frac{M_1}{M_2}\right)^{1/3} a$$

- Nonlinear breakdown through secondary instabilities when

$$\frac{\xi}{R_1} \sim \left(\frac{\nu}{R_1^2\omega}\right)^{1/2} \ll 1 \quad ?$$

- Internal wave nonlinearity

$$\sim \frac{\xi_{\text{wave}}}{\lambda}$$

Tidal Q of solar-type stars and giant planets

No simple answer!

- Q (or $k_{l,m}(\omega)$) is a response function, not a simple number
- Fluid dynamical calculations are still exploratory
- Planetary interior models are uncertain

Zahn’s categorization :

- “Equilibrium tide”

Dissipation associated with large-scale tidal bulge

- “Dynamical tide”

Dissipation associated with low-frequency waves

Tidal Q of solar-type stars and giant planets

"Equilibrium tide"

- solid regions (viscoelastic, etc.)
- convective regions (turbulent "viscosity")
- other physics (phase transitions, helium separation)
- nonlinear breakdown (elliptical instability, etc.)

"Dynamical tide"

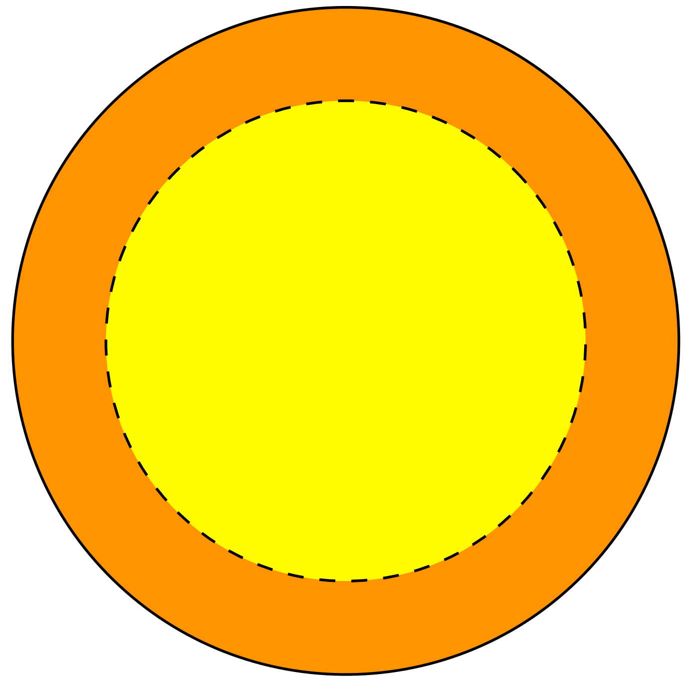
- inertial waves in convective regions
- inertia-gravity waves in radiative regions

"inertial wave" : Coriolis force

"gravity wave" : buoyancy force

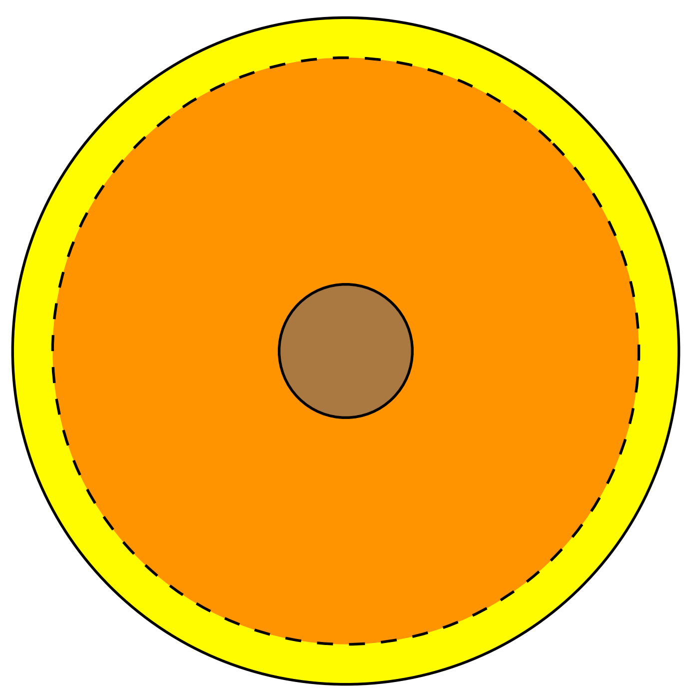
Inertial waves in convective regions

Solar-type star



[Savonije & Witte 2002]
Ogilvie & Lin 2007

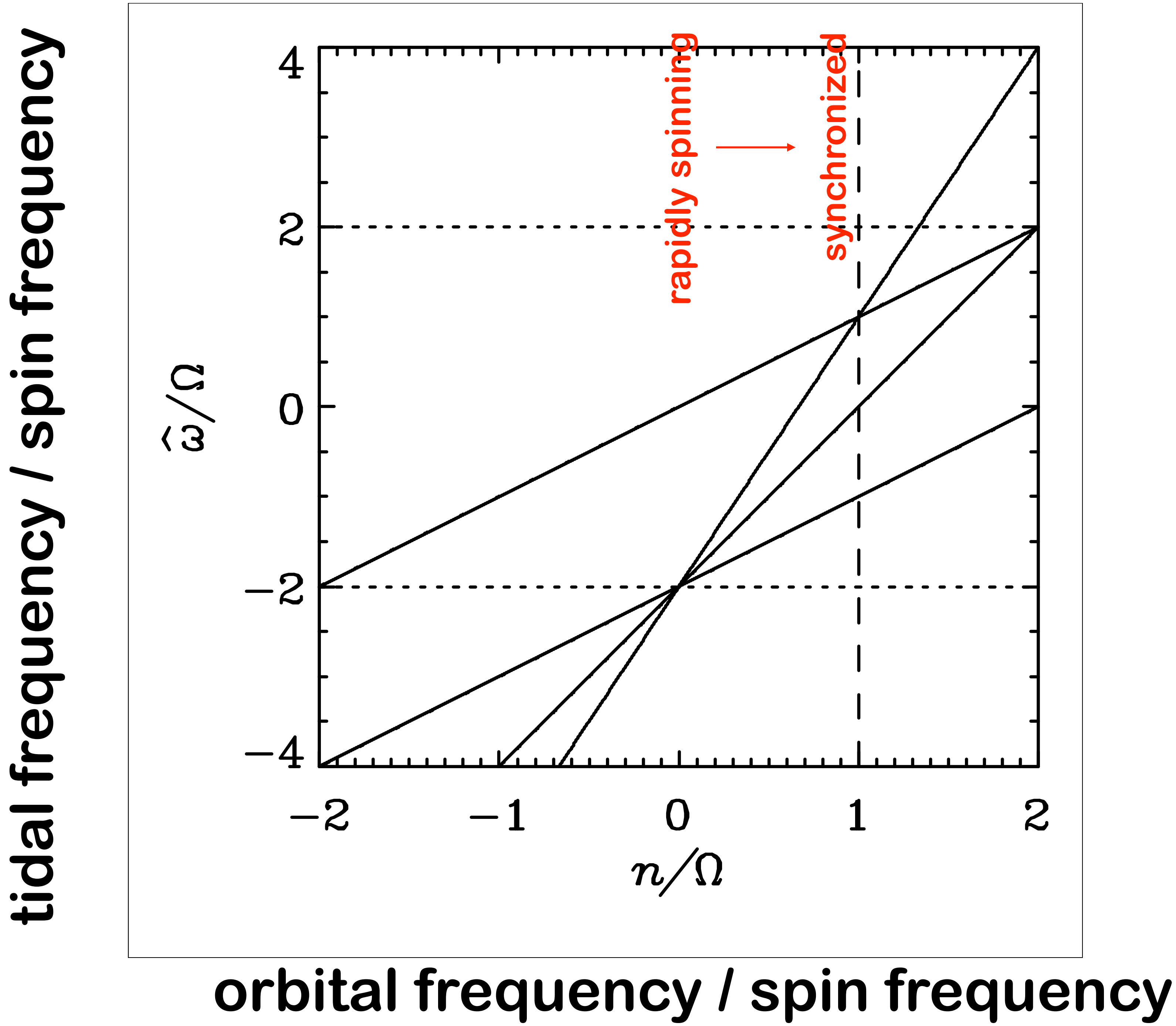
[Irradiated] giant planet



Ogilvie & Lin 2004
Wu 2005
Papaloizou & Ivanov 2005
Ivanov & Papaloizou 2007
Goodman & Lackner 2009
Ogilvie 2009
Rieutord & Valdettaro 2010

Inertial wave frequency range

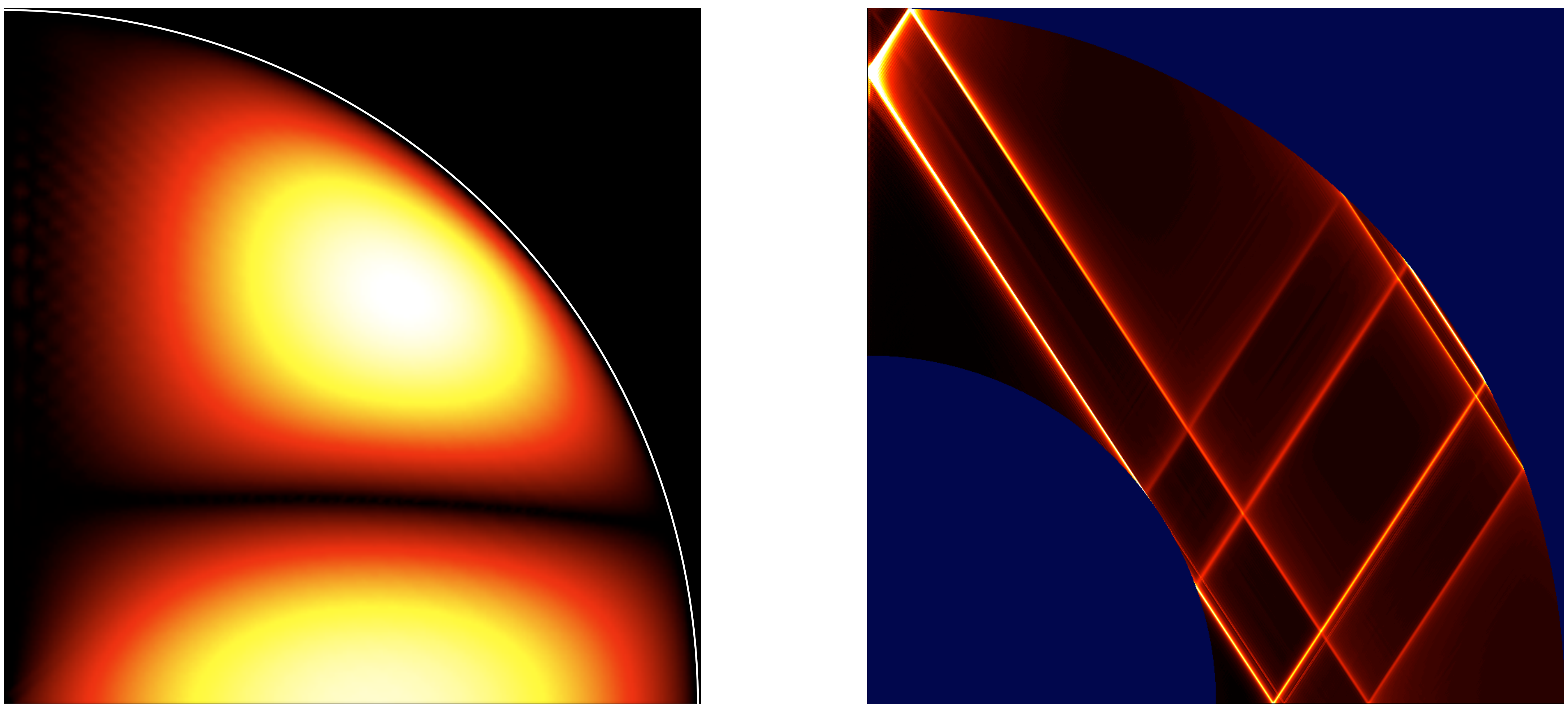
For a uniformly rotating body, $-2\Omega < \hat{\omega} < 2\Omega$



Inertial waves : modes or beams?

Dense or continuous spectrum, $-2\Omega < \hat{\omega} < 2\Omega$

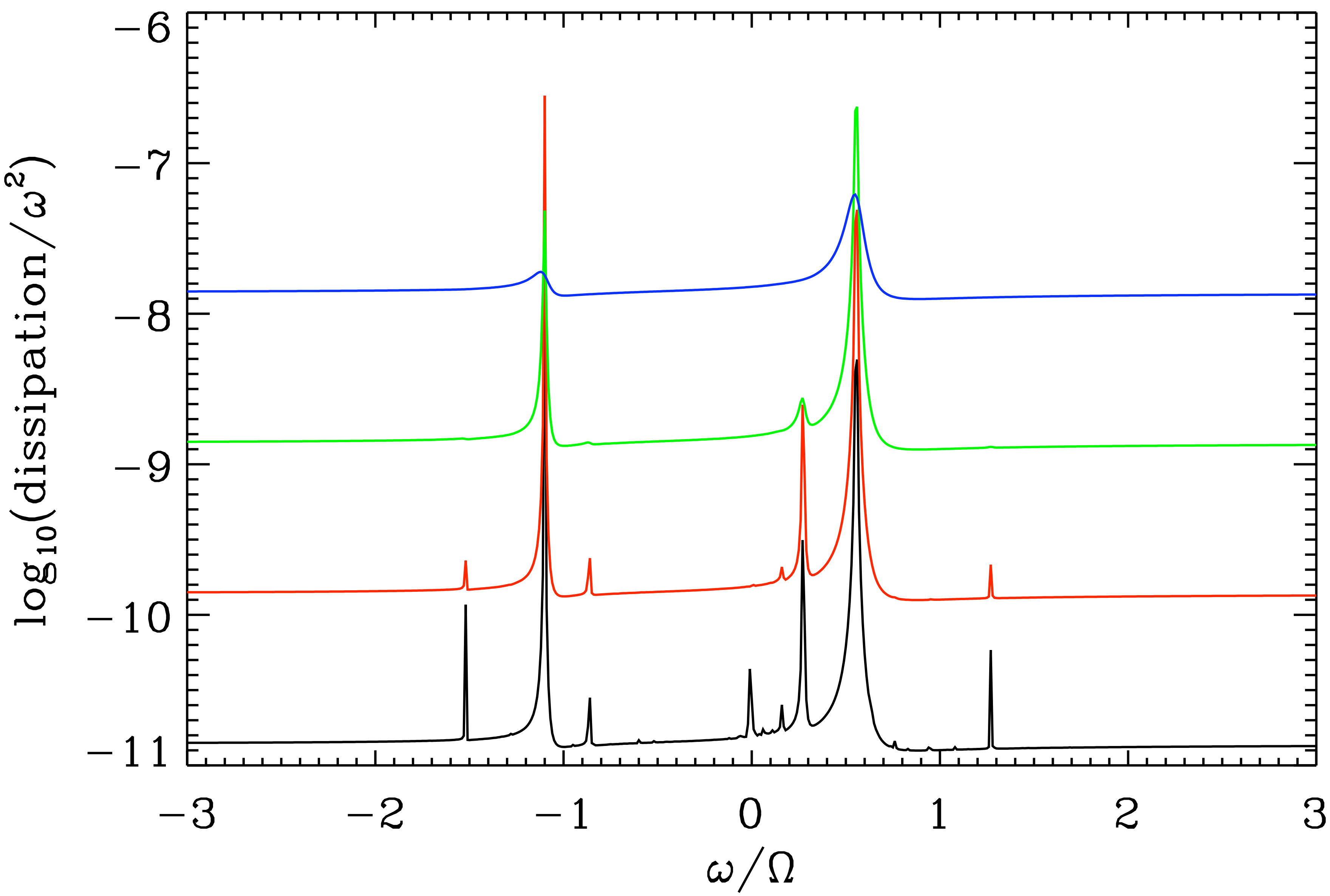
- Tidal forcing excites normal modes (Wu; Papaloizou & Ivanov)
- Tidal forcing excites narrow beams (Ogilvie & Lin; Goodman & Lackner; Rieutord & Valdettaro)

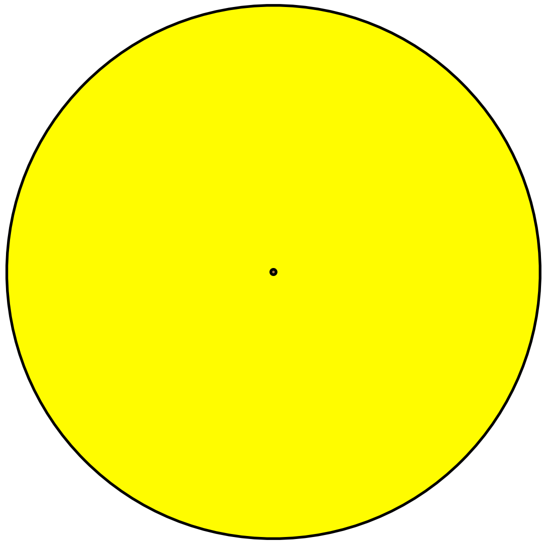


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.01





$\mathrm{Ek} = 10^{-3}$

$\mathrm{Ek} = 10^{-4}$

$\mathrm{Ek} = 10^{-5}$

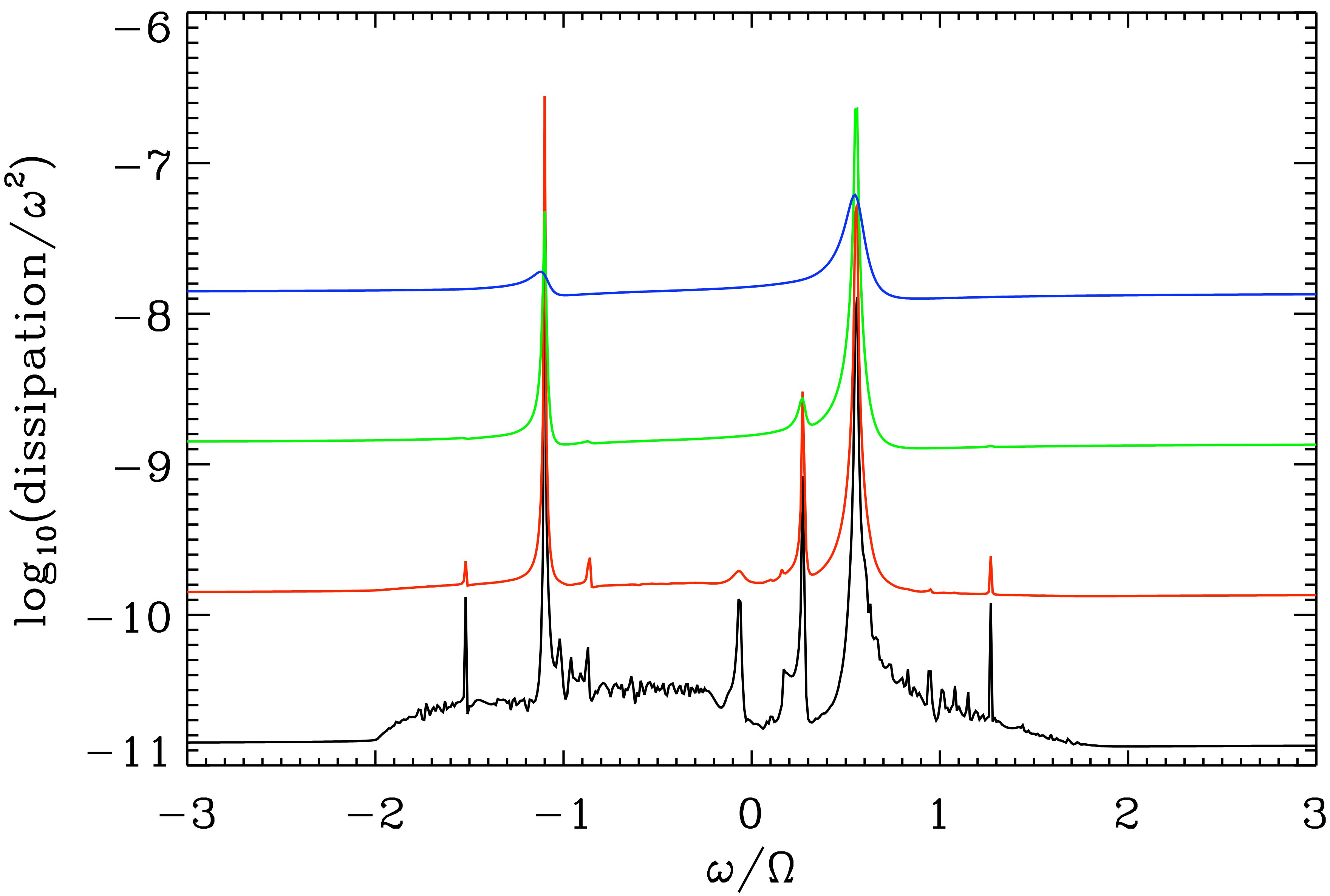
$\mathrm{Ek} = 10^{-6}$

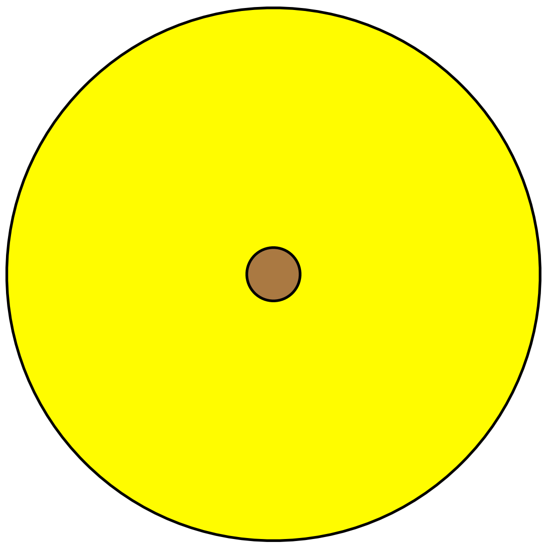
$$\left[\ \mathrm{Ek} = \frac{\nu}{2\Omega R^2}\ \right]$$

Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.1

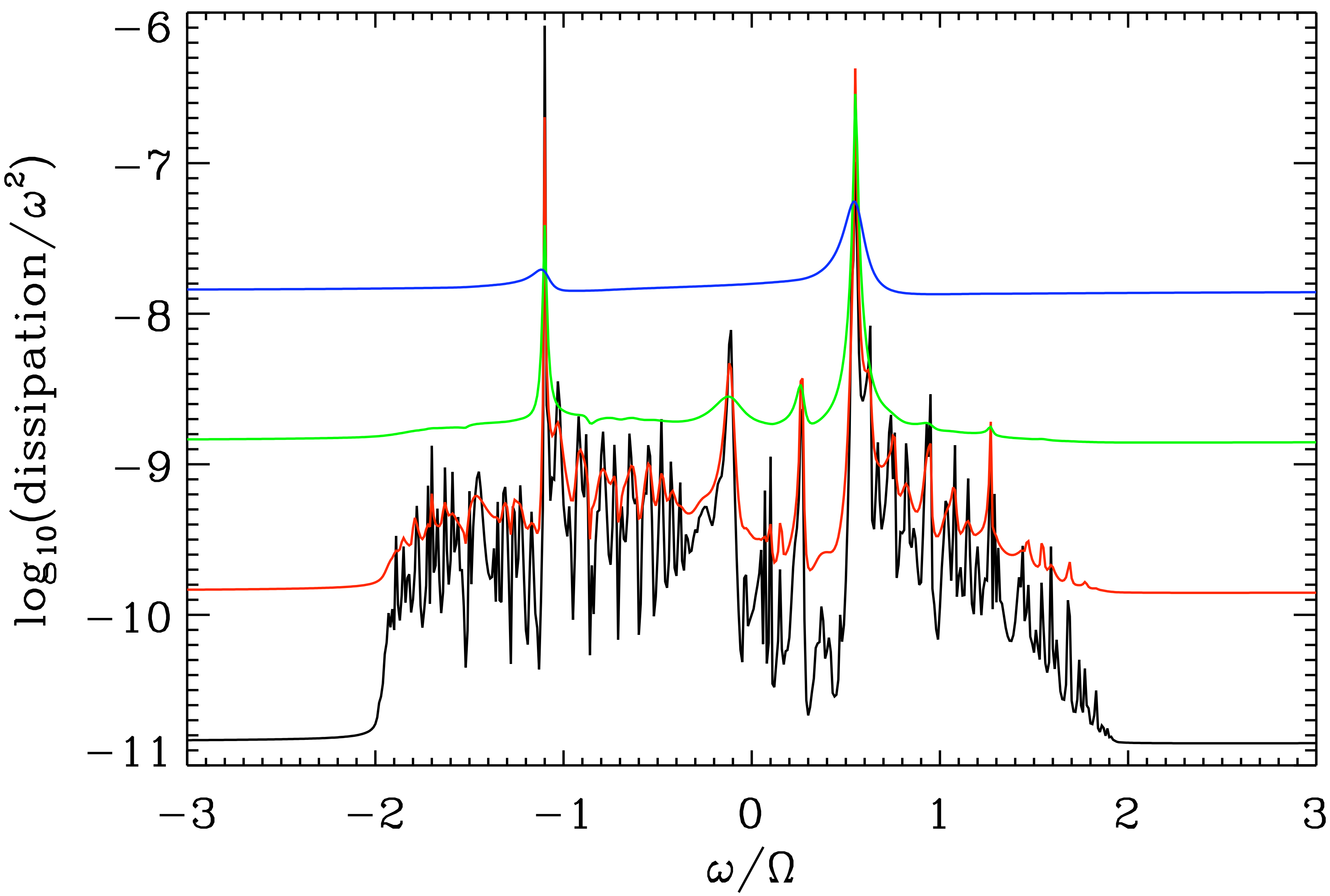


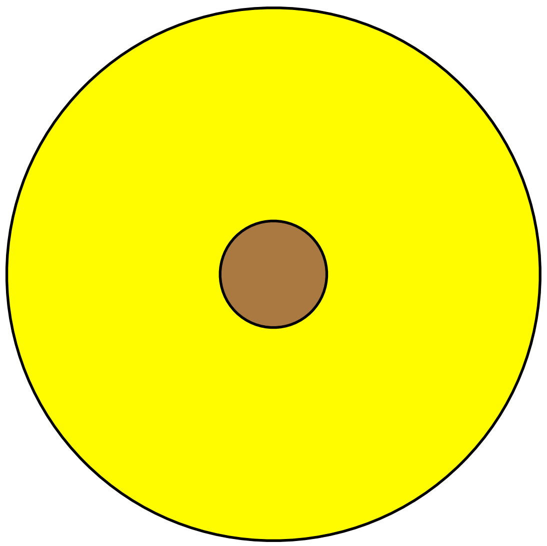


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.2

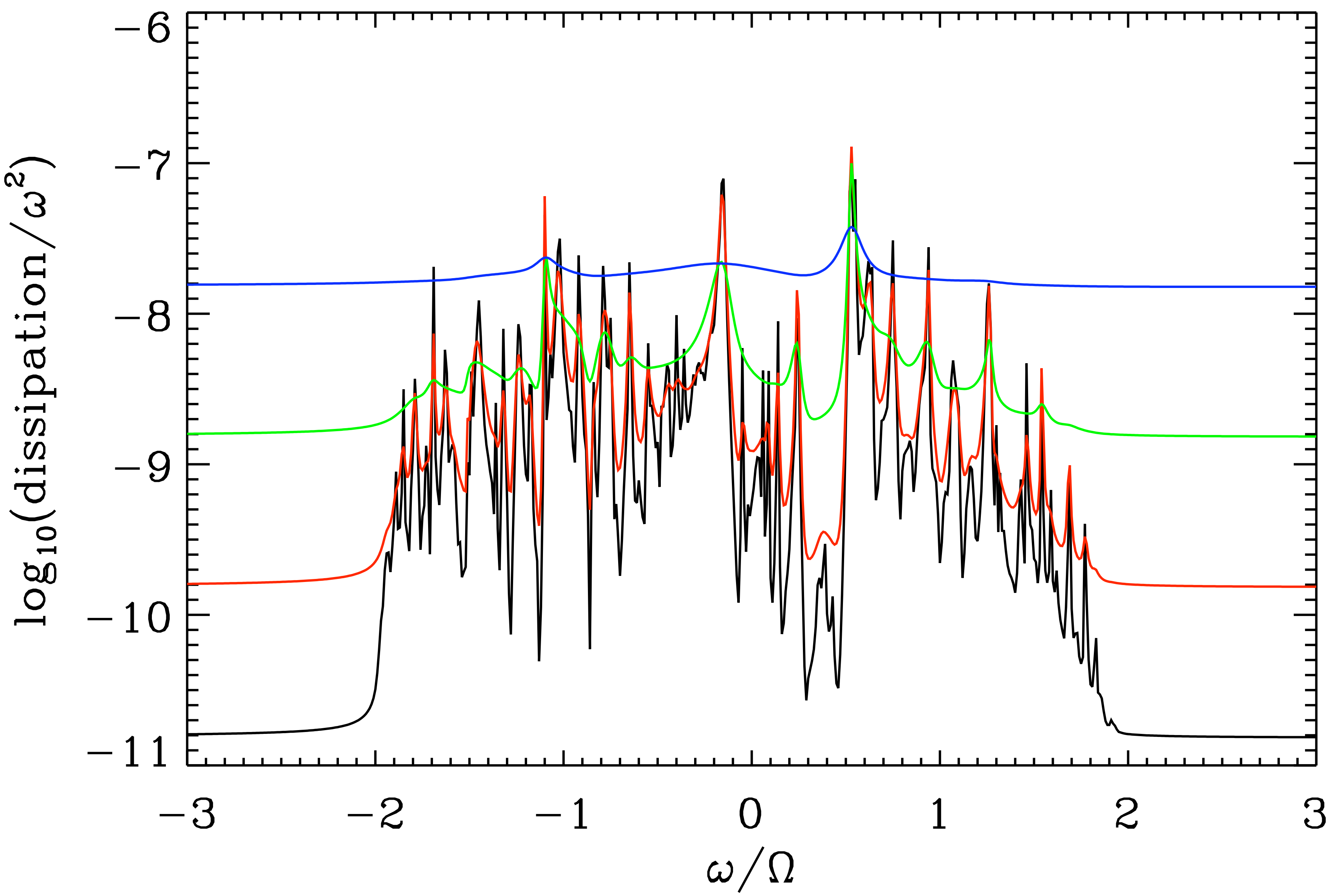


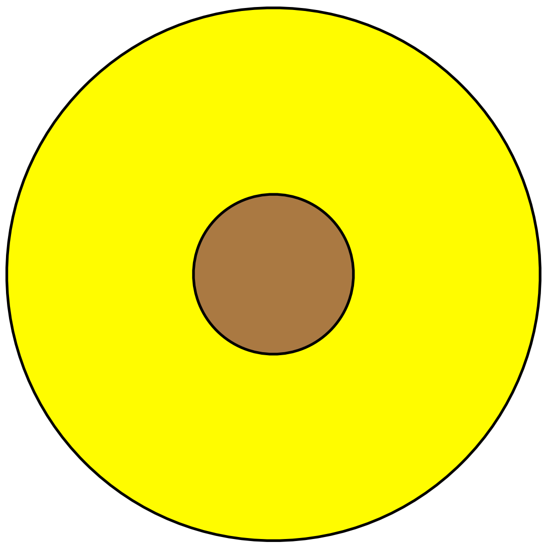


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.3

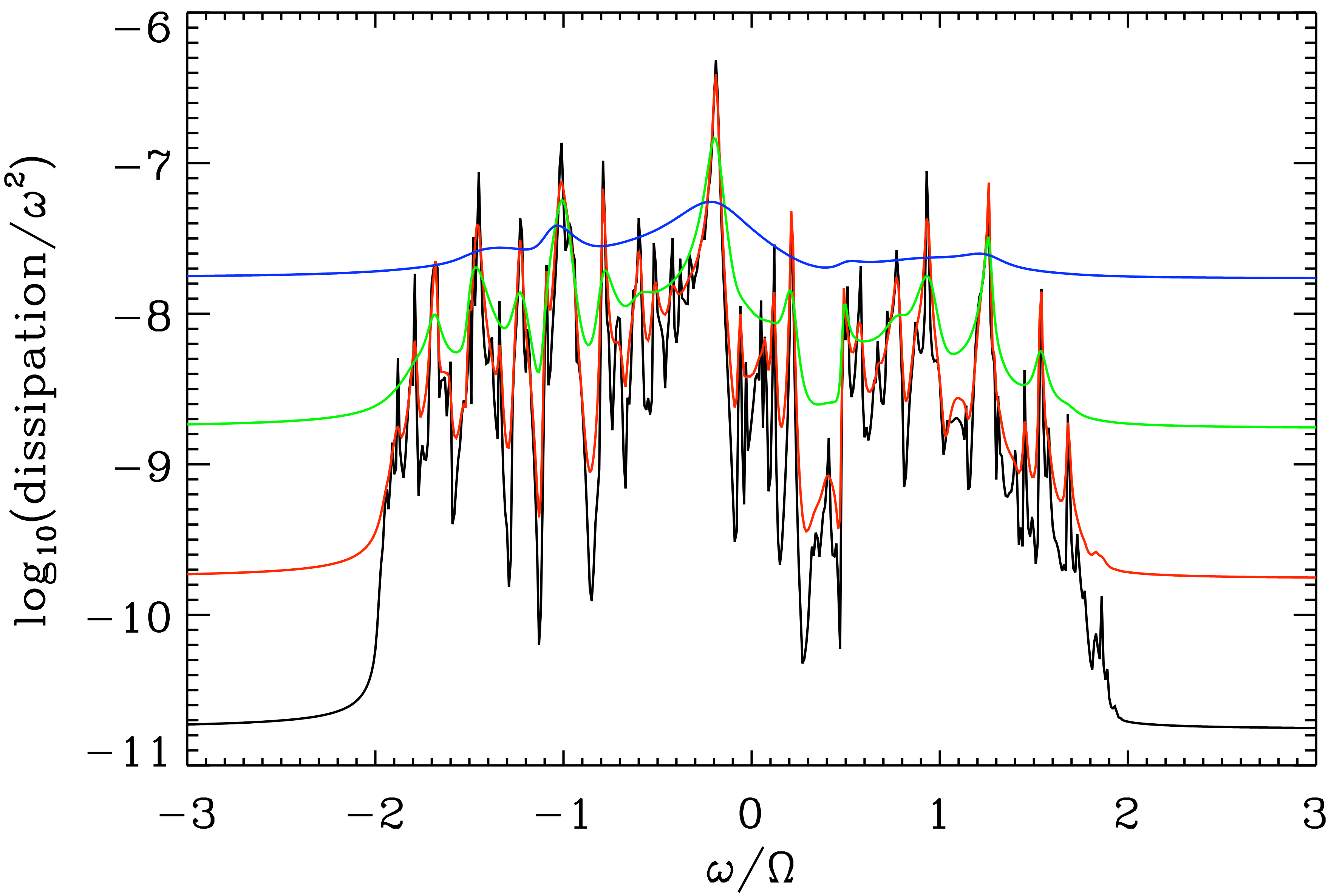


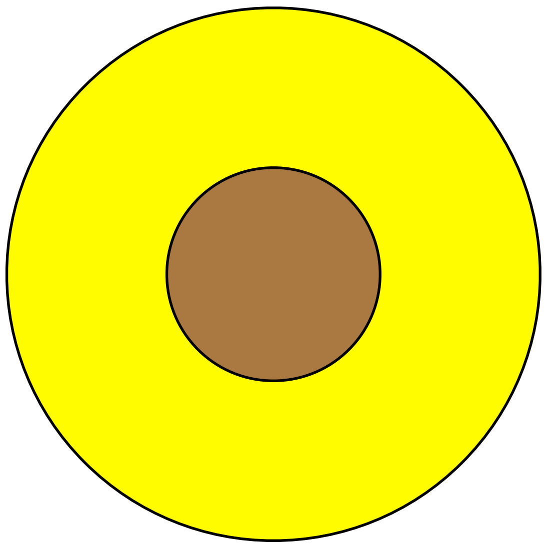


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.4

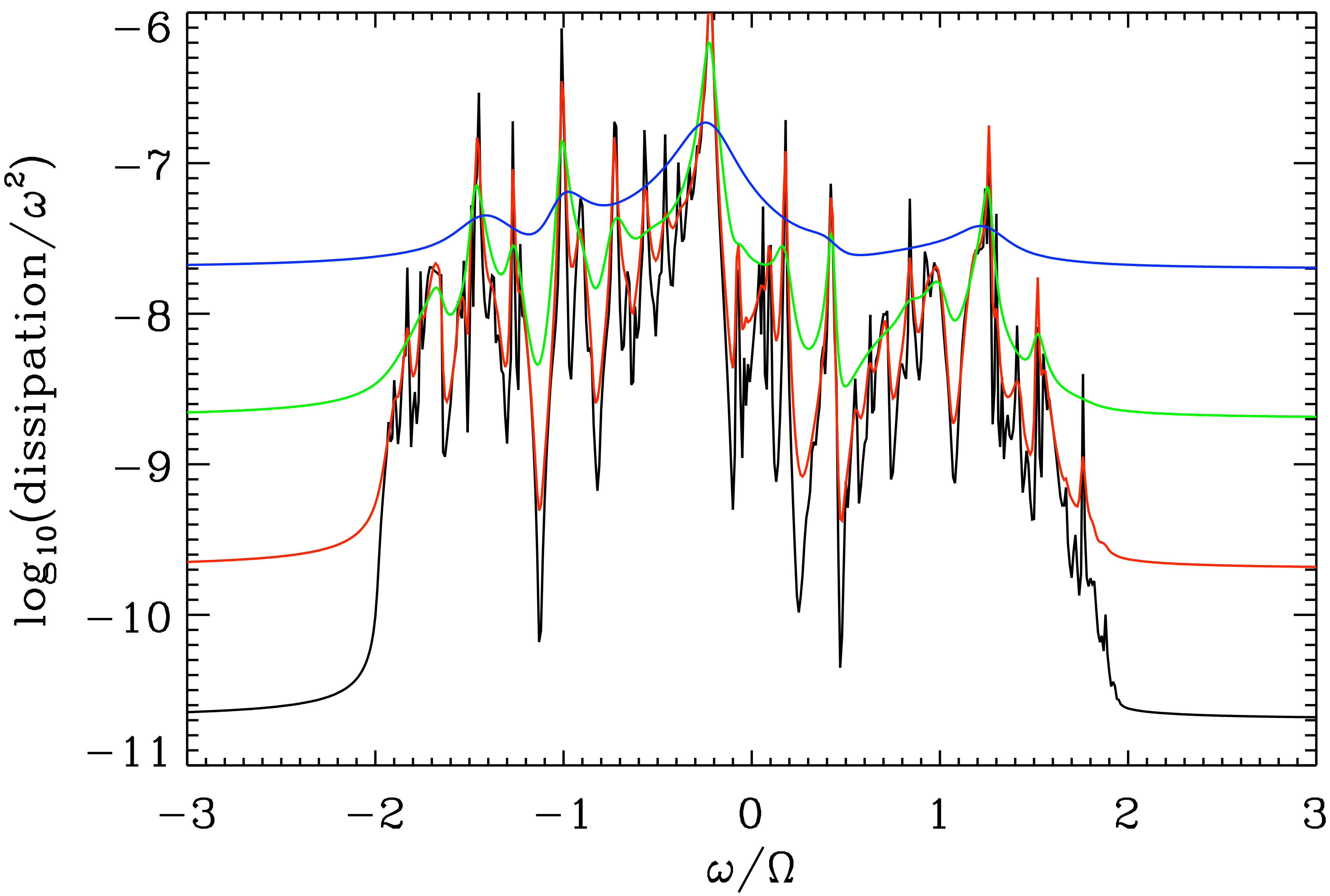


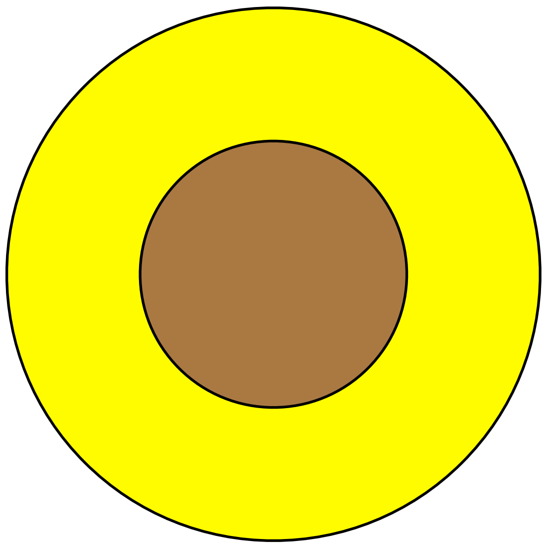


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.5

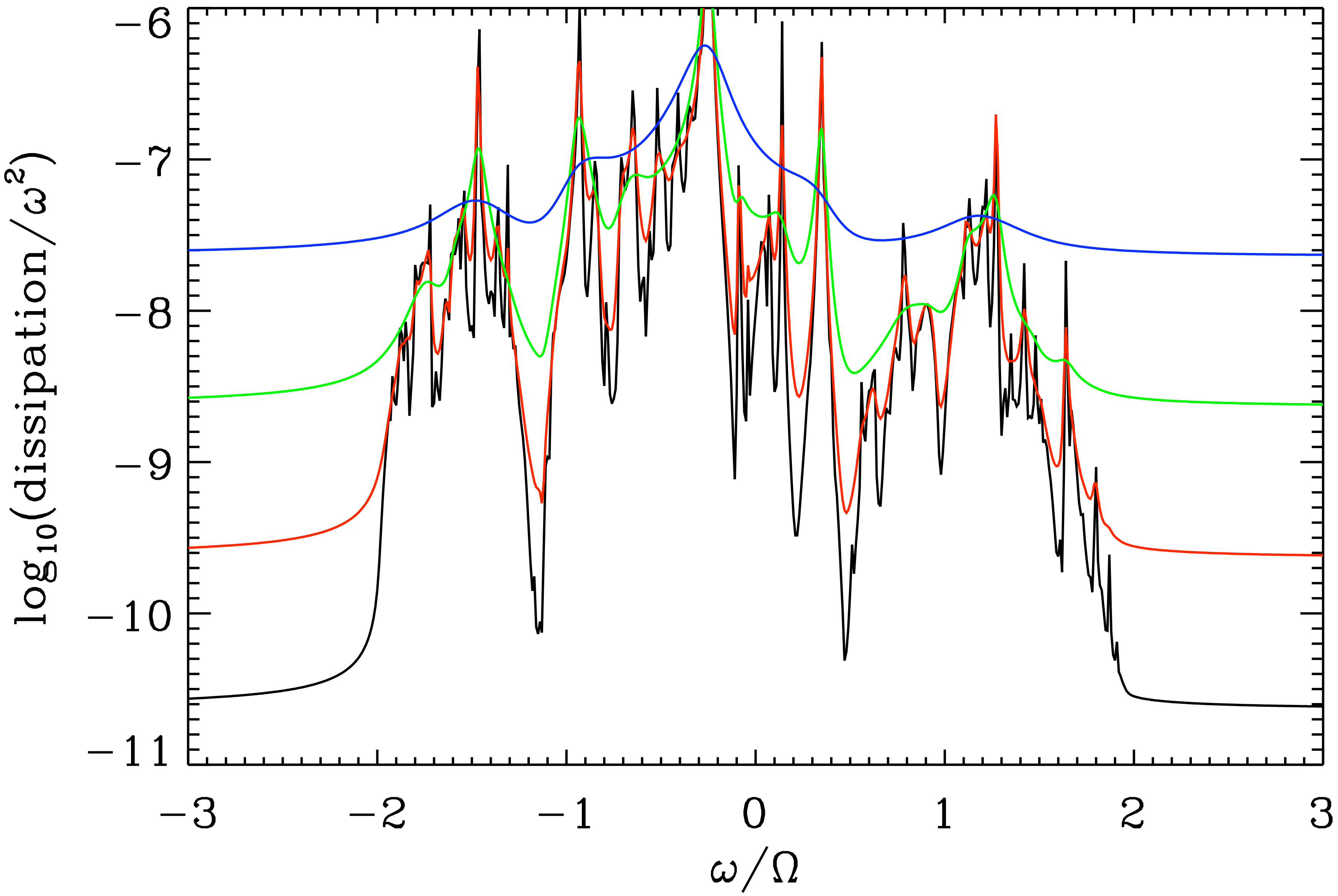


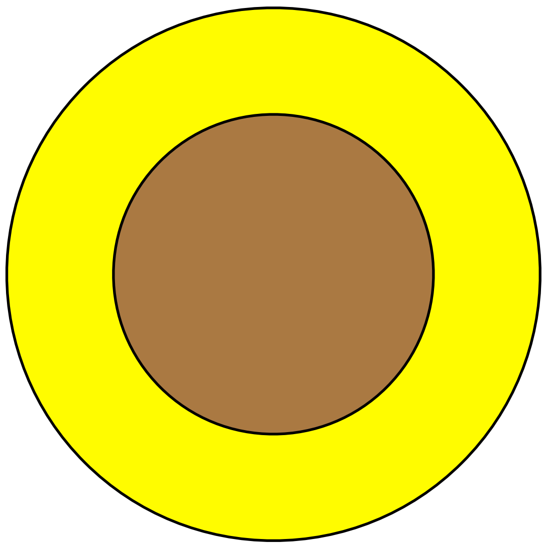


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.6

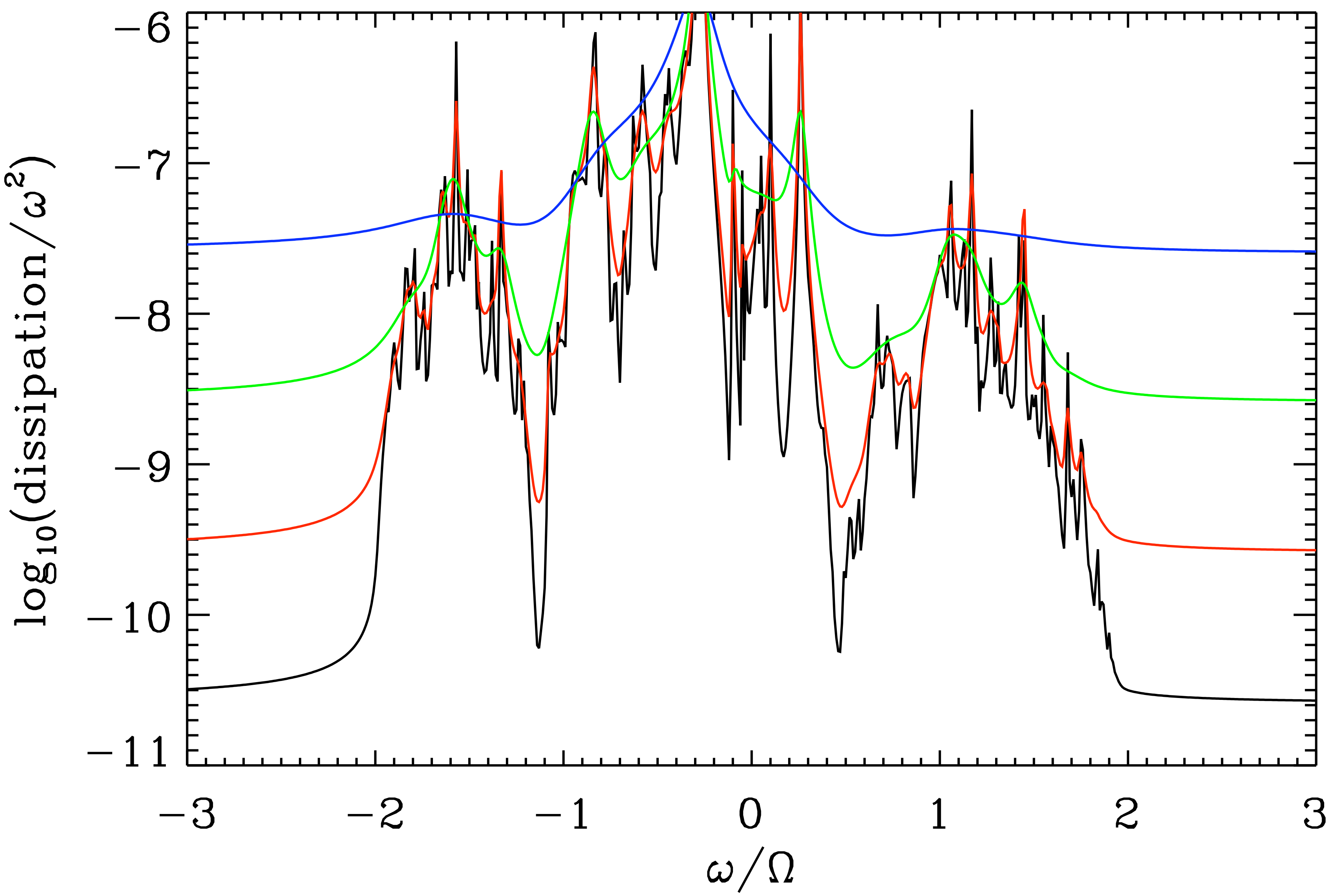


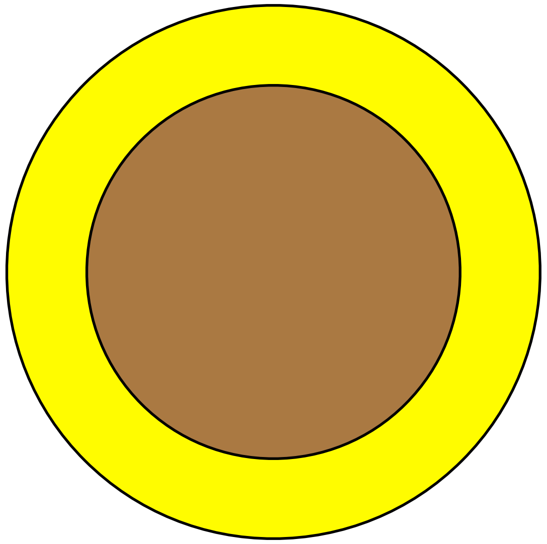


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.7

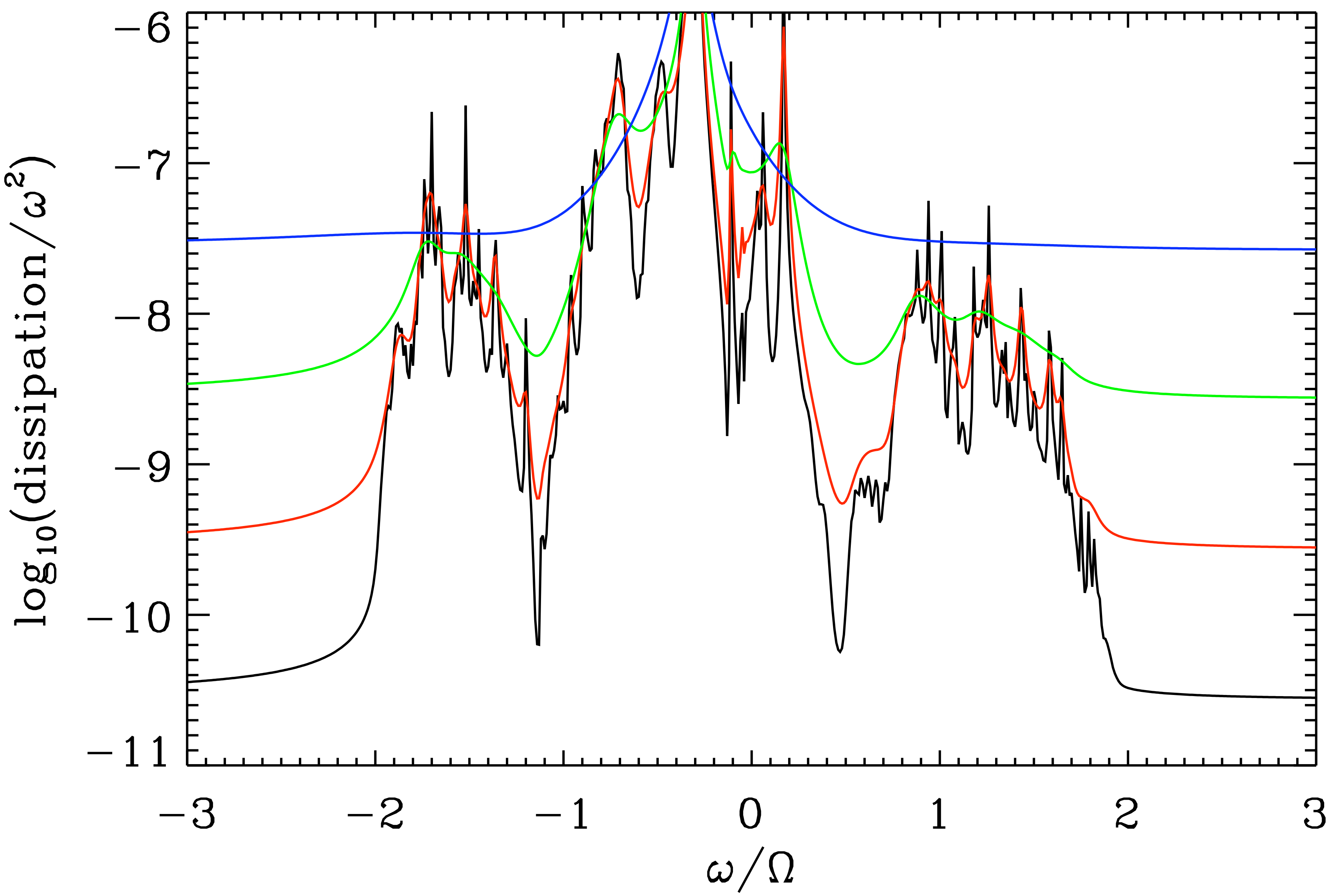


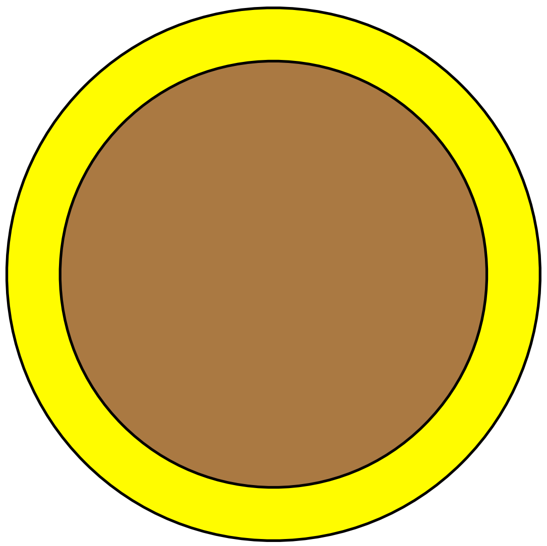


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.8

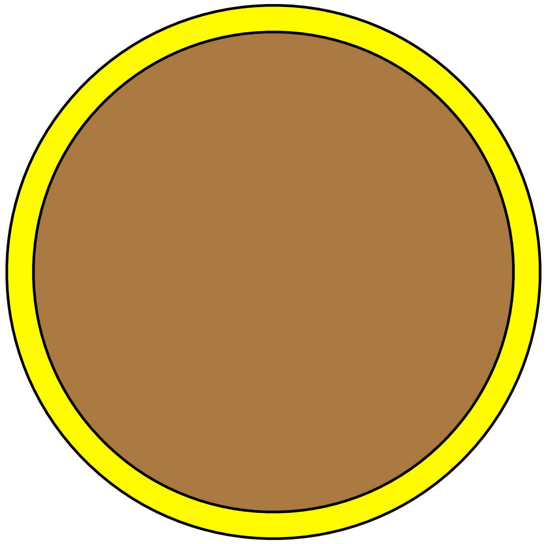


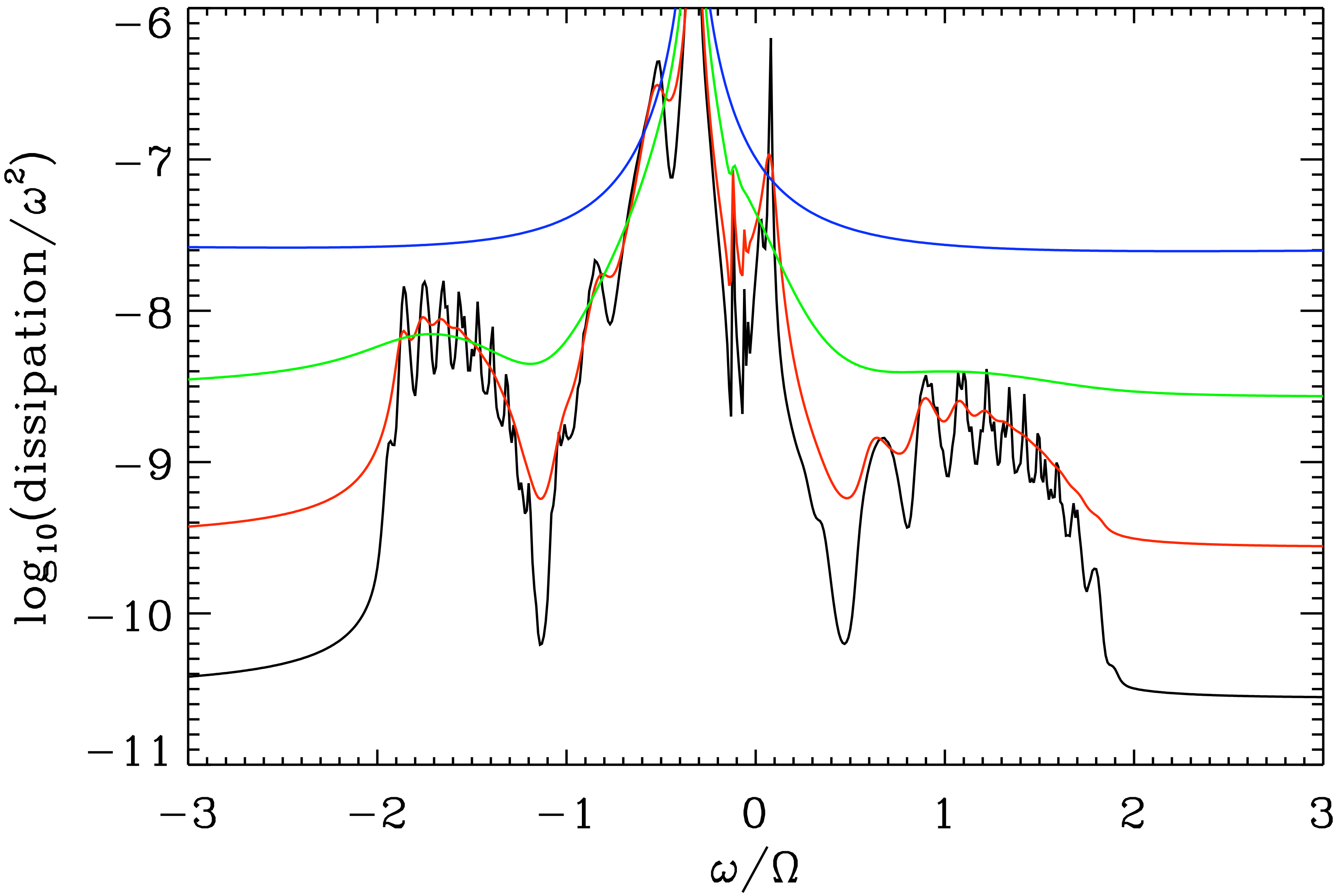


Responses of spheres and shells

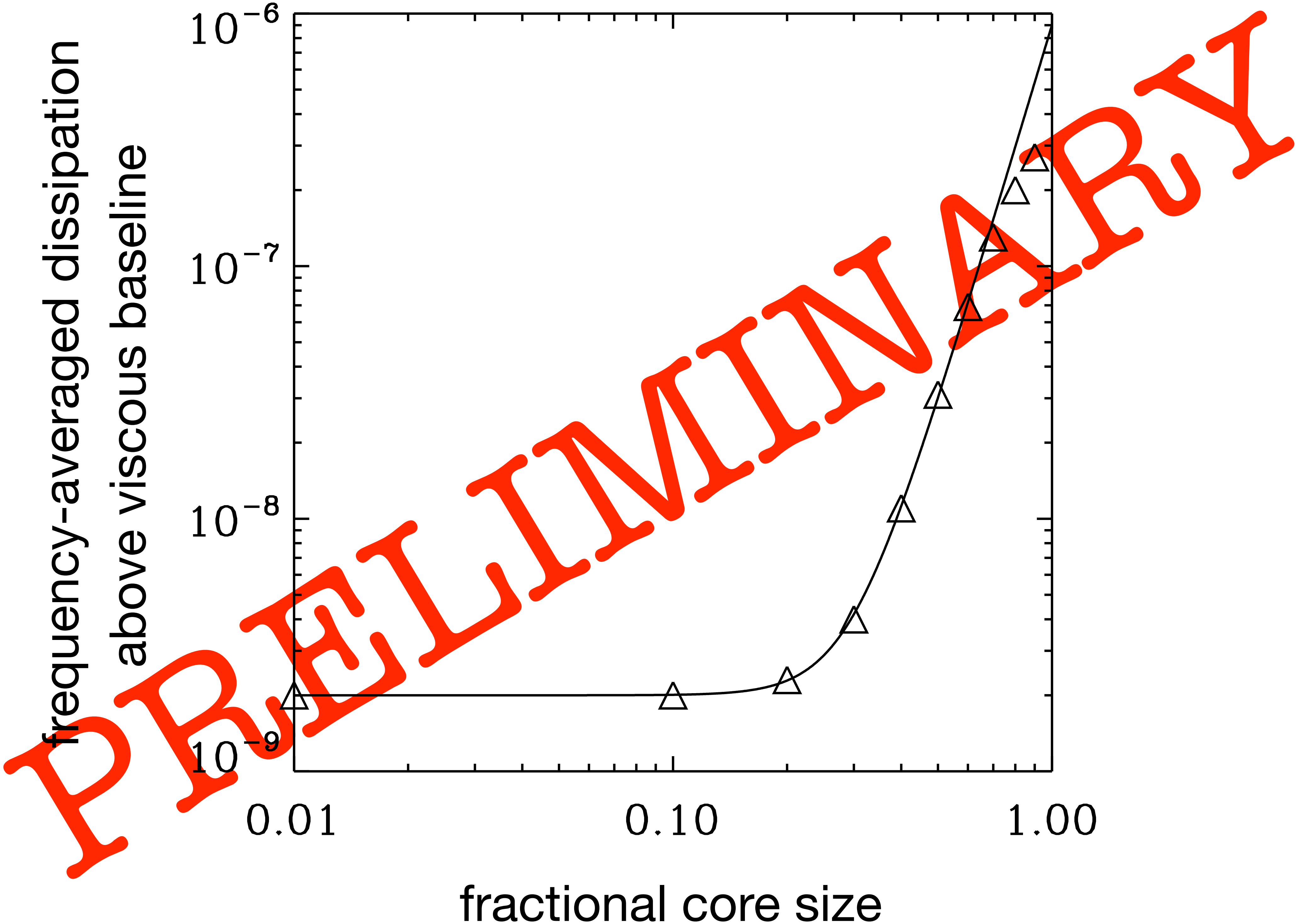
Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.9

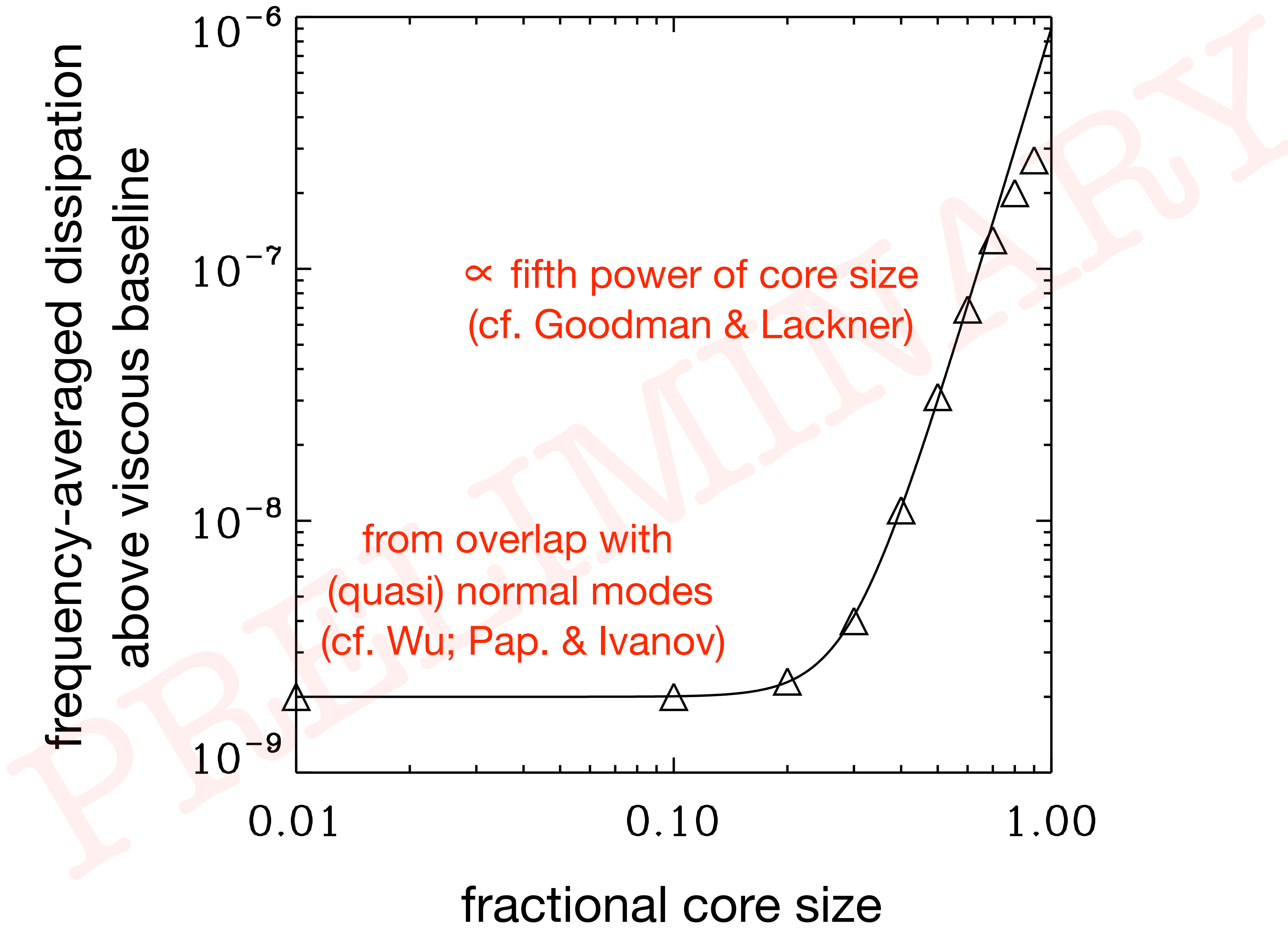




Dependence on core size



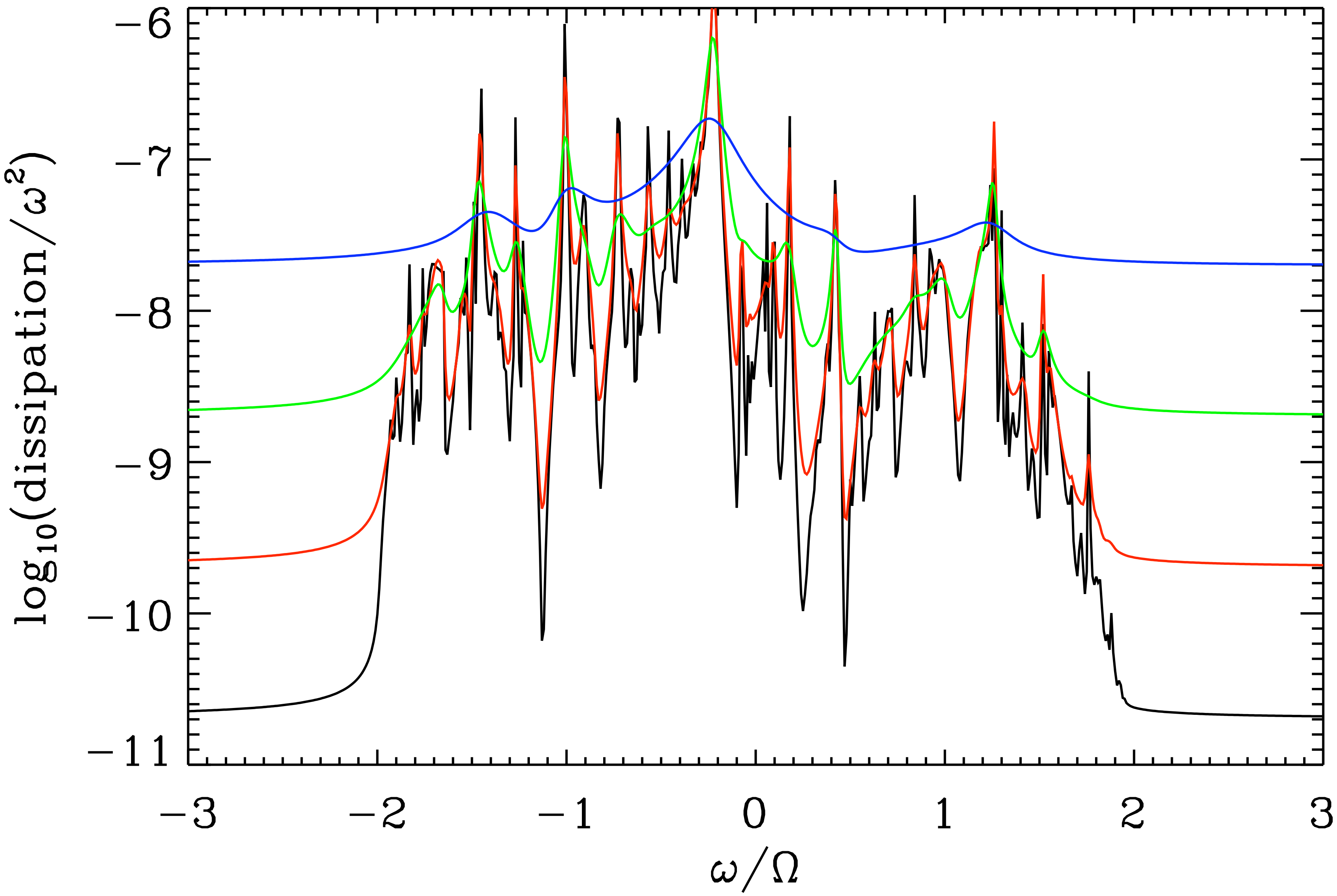
Dependence on core size

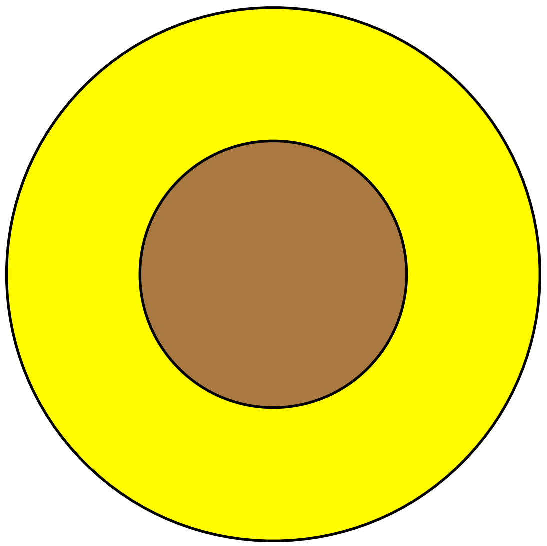


Rigid versus fluid core

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core

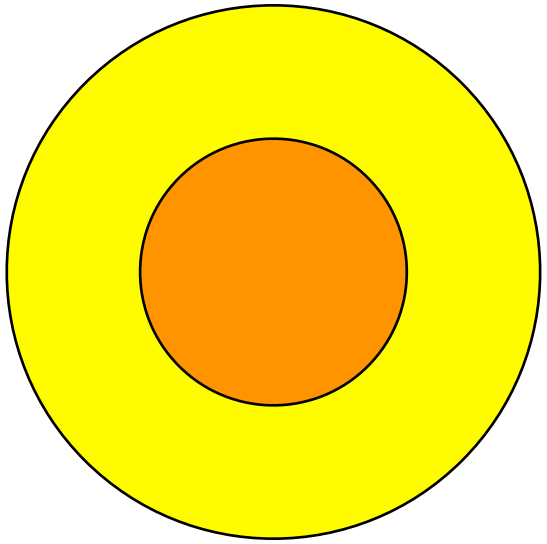


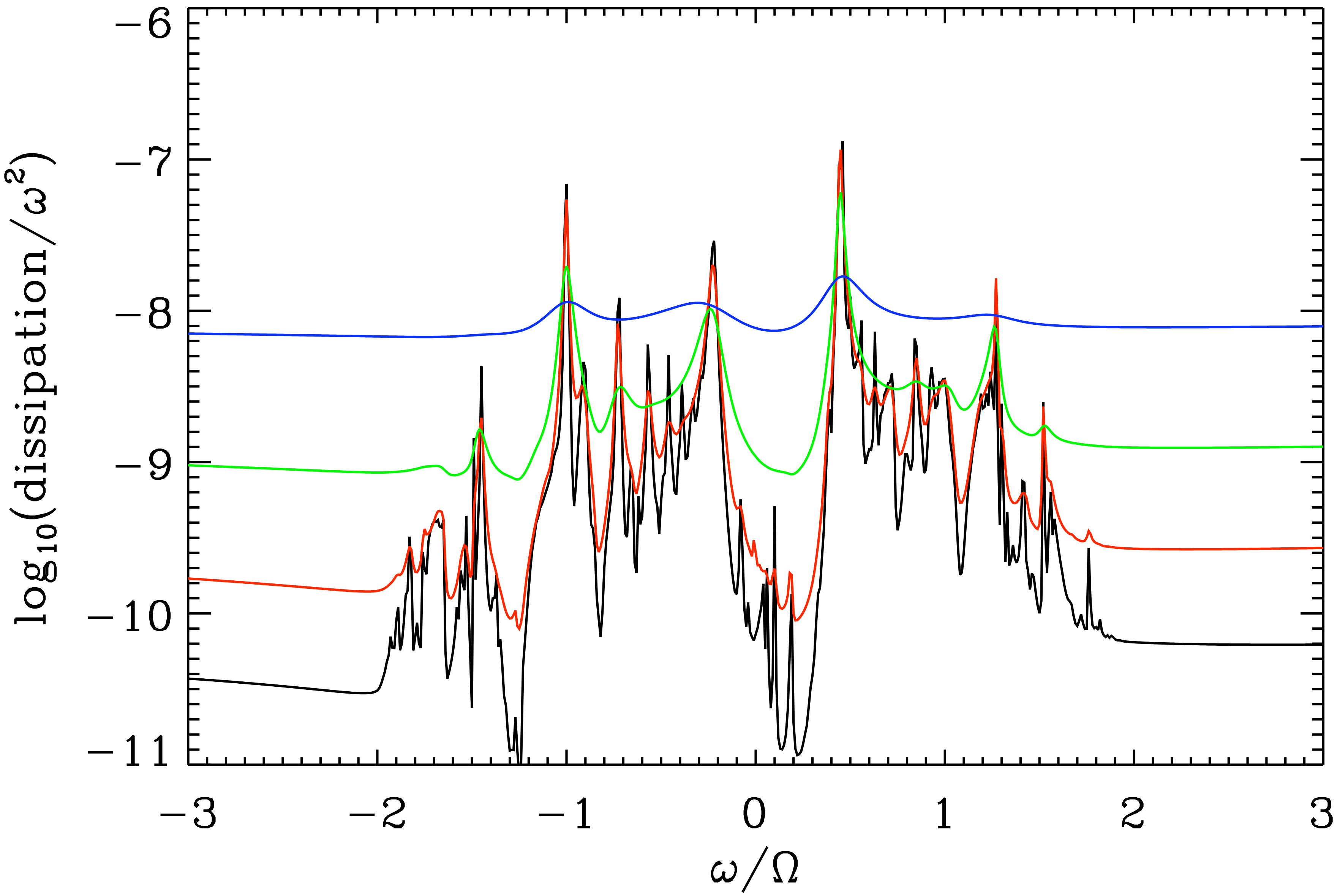


Rigid versus fluid core

Idealized problem : isentropic rotating fluid in spherical geometry

- Fluid core, density jump by factor 2





Responses of spheres and shells

- Full spheres with smooth density profiles support normal modes
- Some tidal overlap with normal modes occurs, leading to resonant peaks in the response, if the density is non-uniform
- Presence of a core and/or density jumps enhances tidal response but concepts of normal modes and resonance are less relevant
- Enhanced dissipation for tidal frequencies (in rotating frame) $-2\Omega < \omega < 2\Omega$ relevant for synchronization and circularization
- Frequency-averaged Q strongly dependent on internal stucture but not on viscosity; for intermediate core sizes,

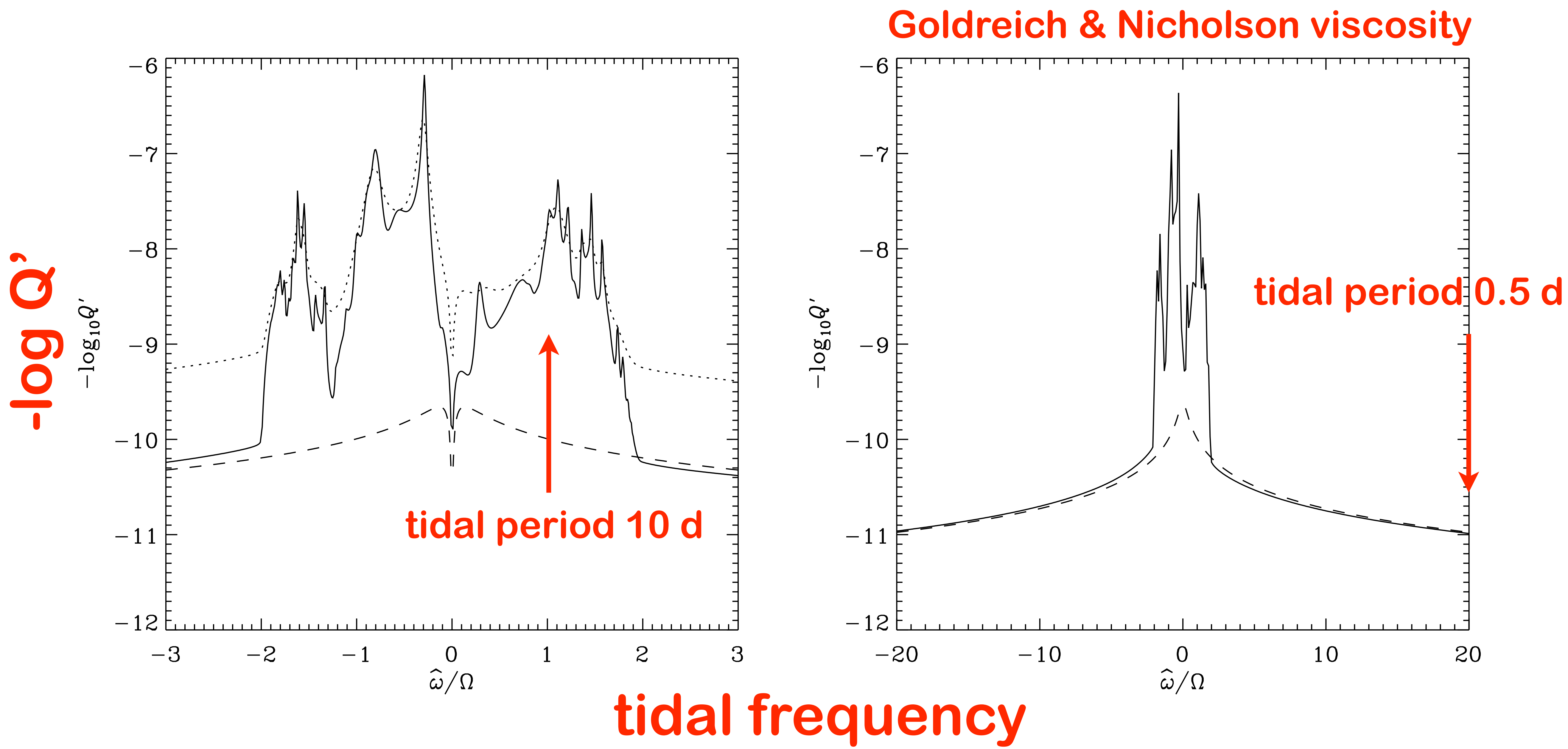
$$\left\langle \frac{1}{Q'} \right\rangle_\omega \approx \left(\frac{R_\mathrm{c}}{R_\mathrm{p}}\right)^5 \left(\frac{\Omega}{\Omega_\mathrm{dyn}}\right)^2$$

- Strong frequency dependence in cases of low viscosity

Inertial waves in a solar-type star

Ogilvie & Lin (2007)

- solar model, but spin period 10 days
- dissipation in convective zone only



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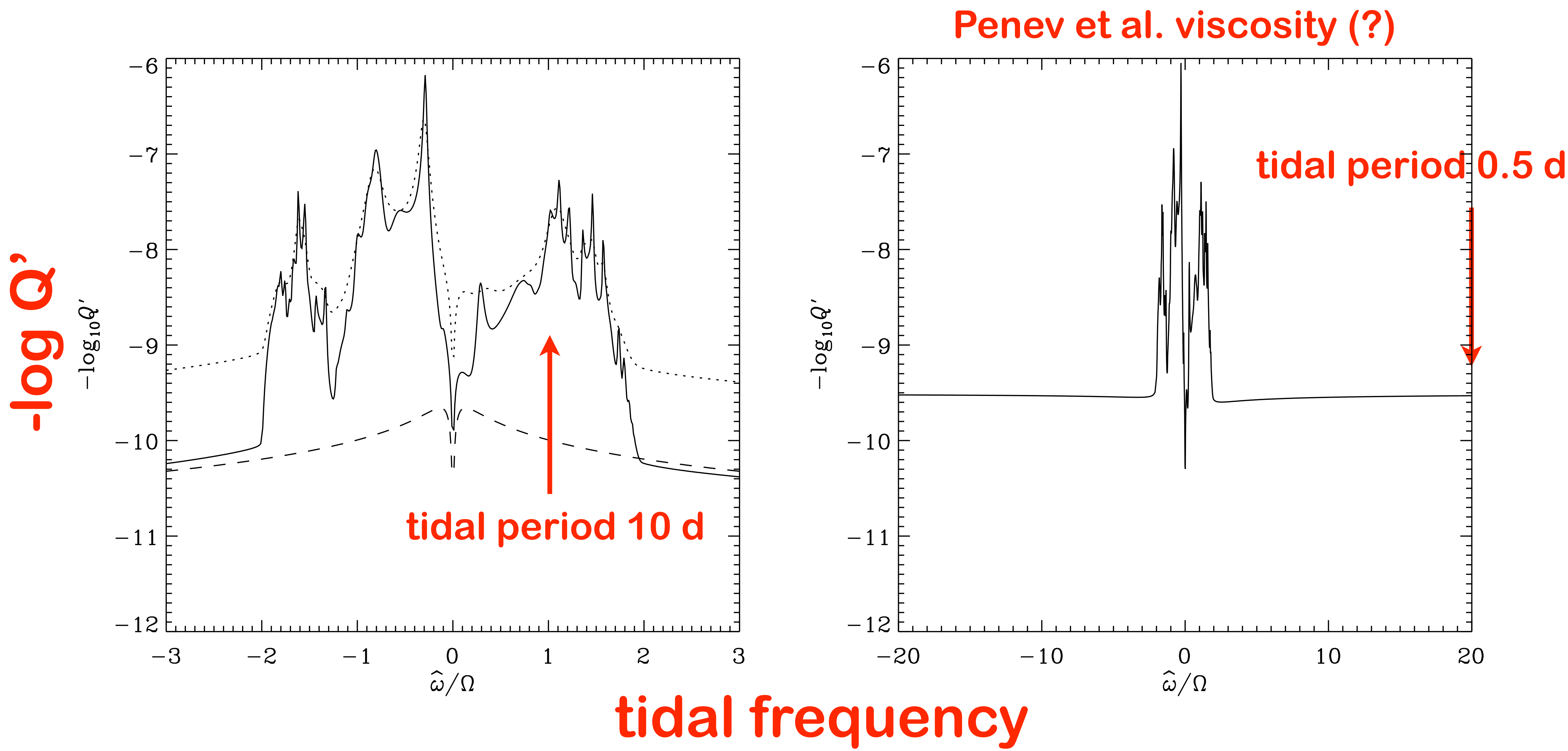


Inertial waves in convective regions

Complications:

- convection (dissipation, scattering)
- magnetic fields (regular and irregular)
- imperfect reflections
- nonlinearity

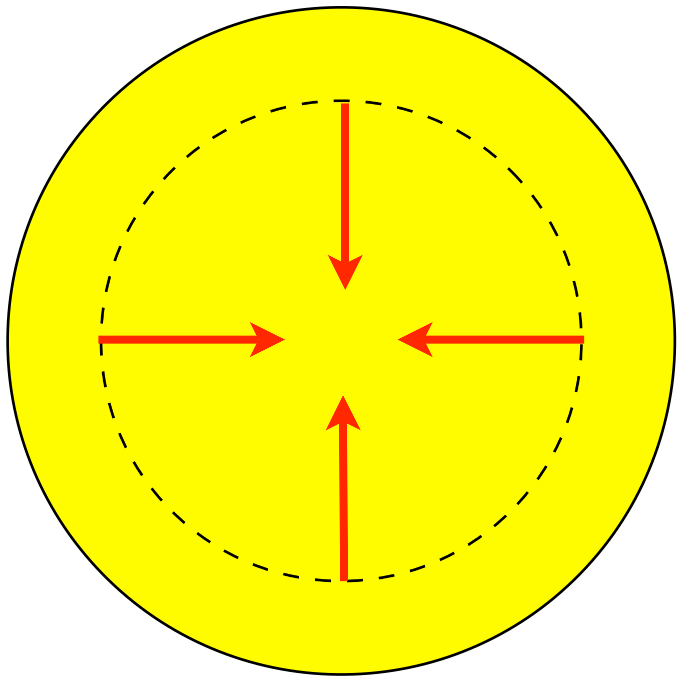
All difficult to model accurately and may wash out some of the frequency-dependence of Q

Needed from planetary structure:

- density profile
- density / entropy jumps
- size and rigidity of core

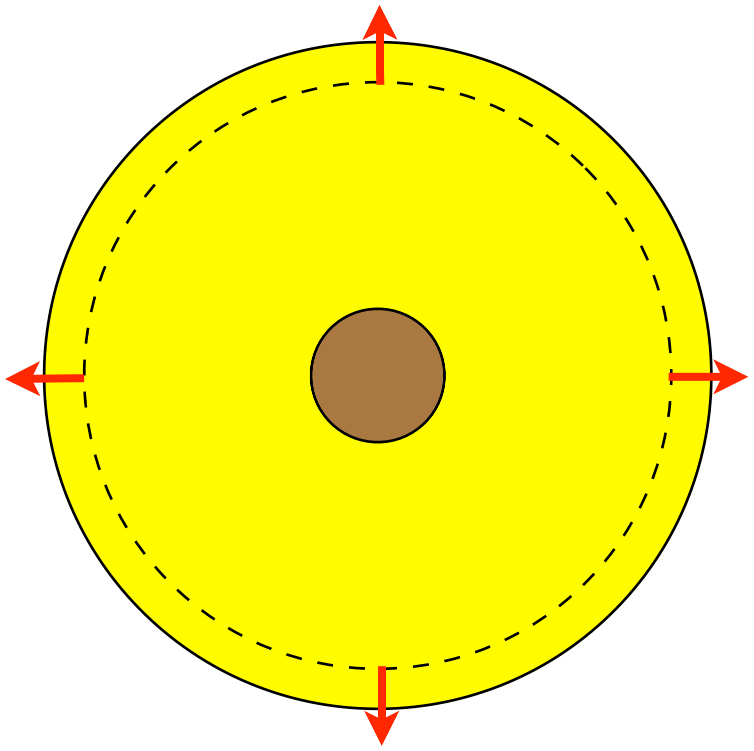
Inertia-gravity waves in radiative regions

Solar-type star



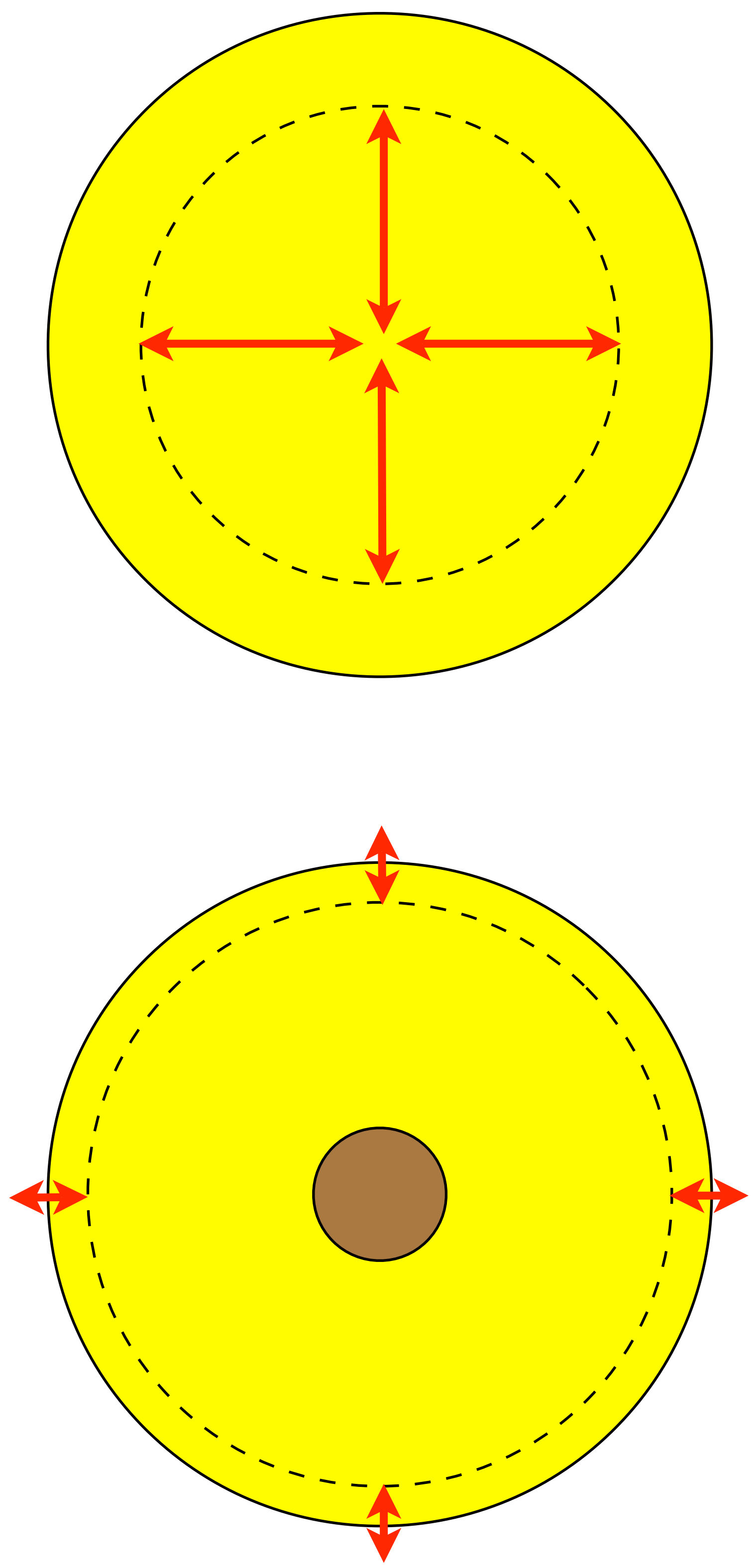
Goodman & Dickson 1998
Terquem et al. 1998
Savonije & Witte 2002
Ogilvie & Lin 2007
Barker & Ogilvie 2010

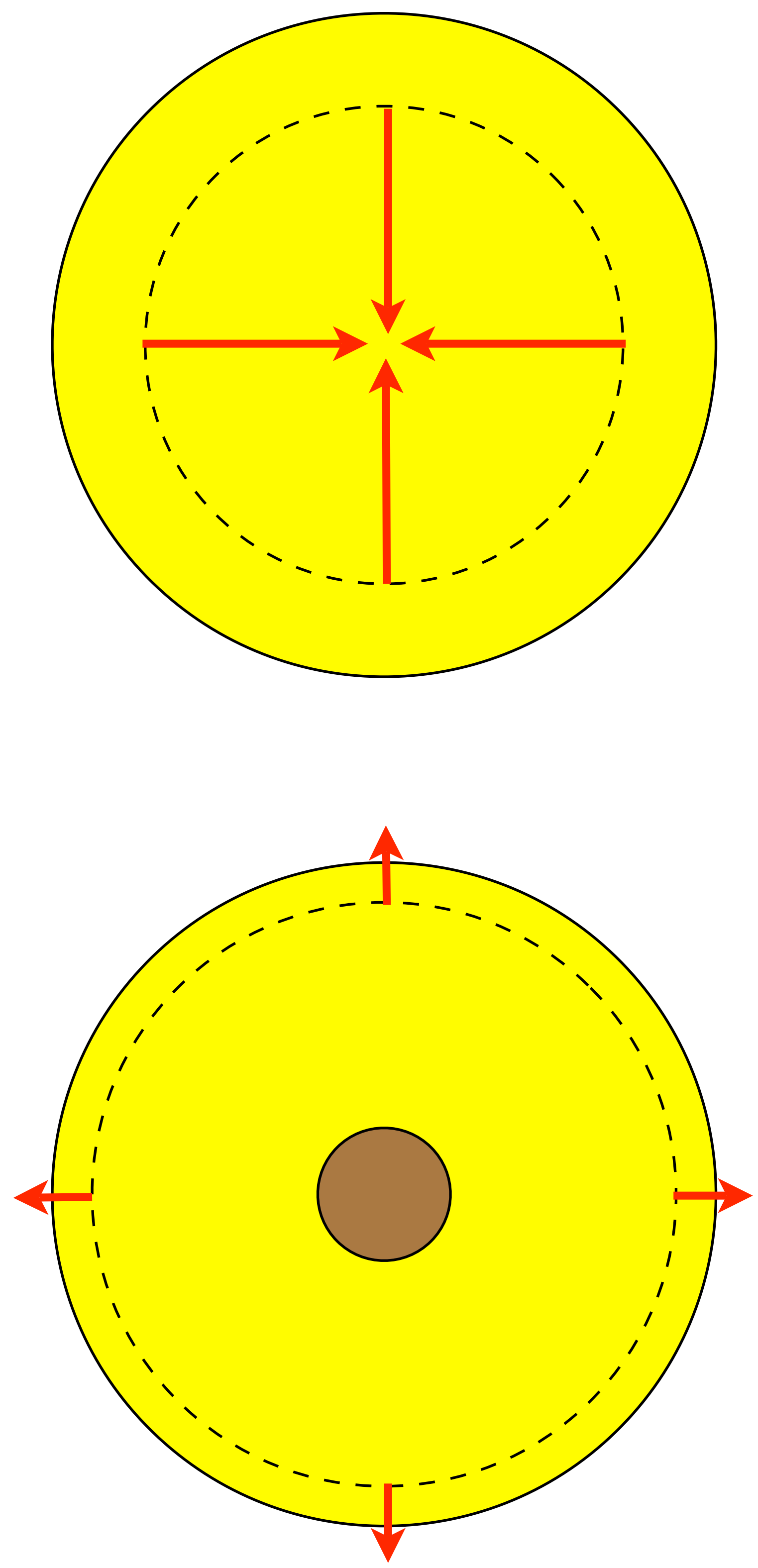
Irradiated giant planet



[Ioannou & Lindzen 1993]
Lubow et al. 1997
Ogilvie & Lin 2004
[Gu & Ogilvie 2009]
[Arras & Socrates 2010]

Inertia-gravity waves: resonant modes or breaking waves?





Inertia-gravity waves in radiative regions

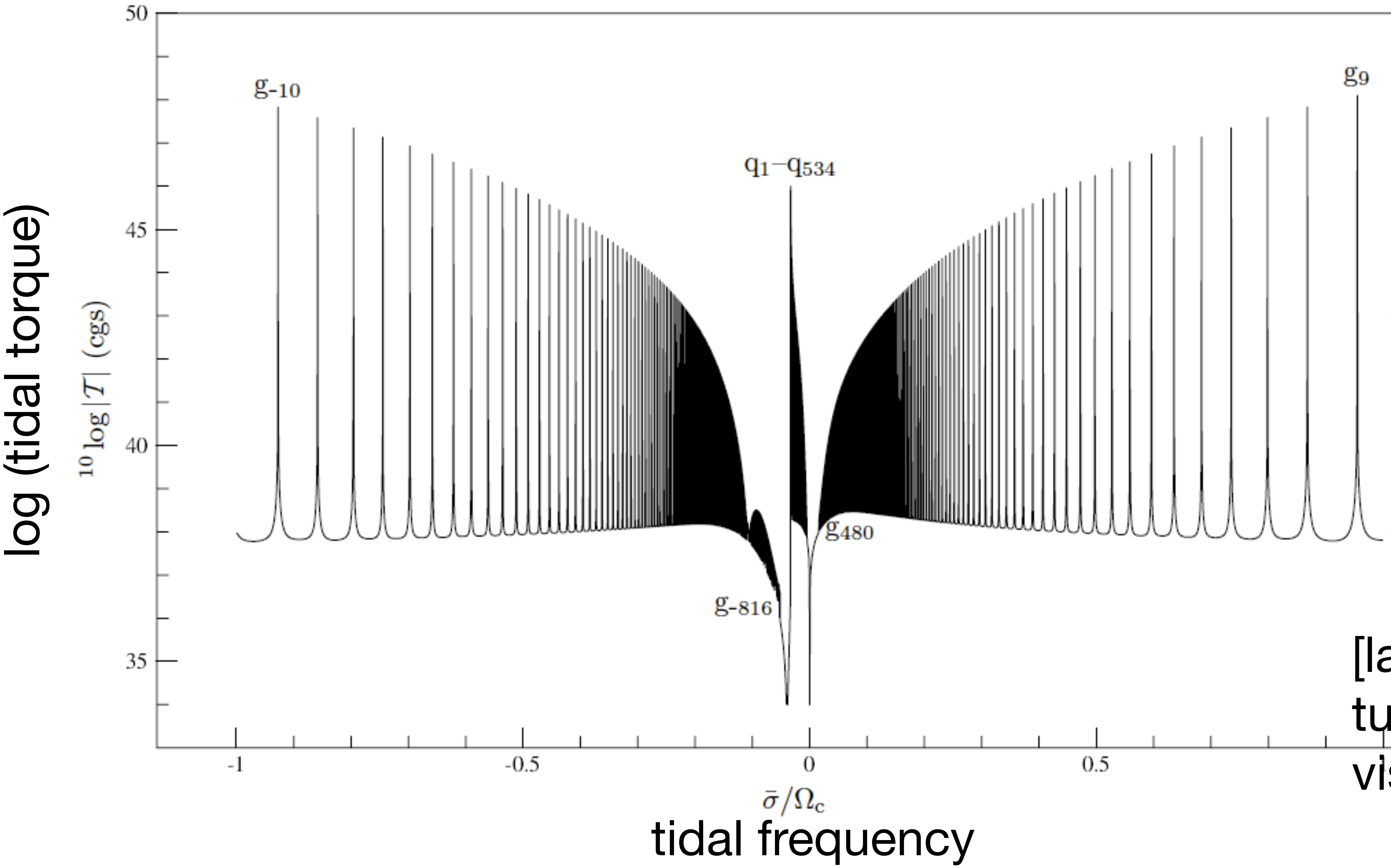
Savonije & Witte 2002

- linear tidal response of 1-solar mass star
- realistic stellar model and evolution
- Coriolis force (traditional approximation)
- radiative diffusion
- turbulent viscosity [large?]

Inertia-gravity waves in radiative regions (star)

Savonije & Witte 2002 (cf. Terquem et al. 1998)

- resonant excitation of normal modes



$Q' \approx 10^2$

$Q' \approx 10^7$

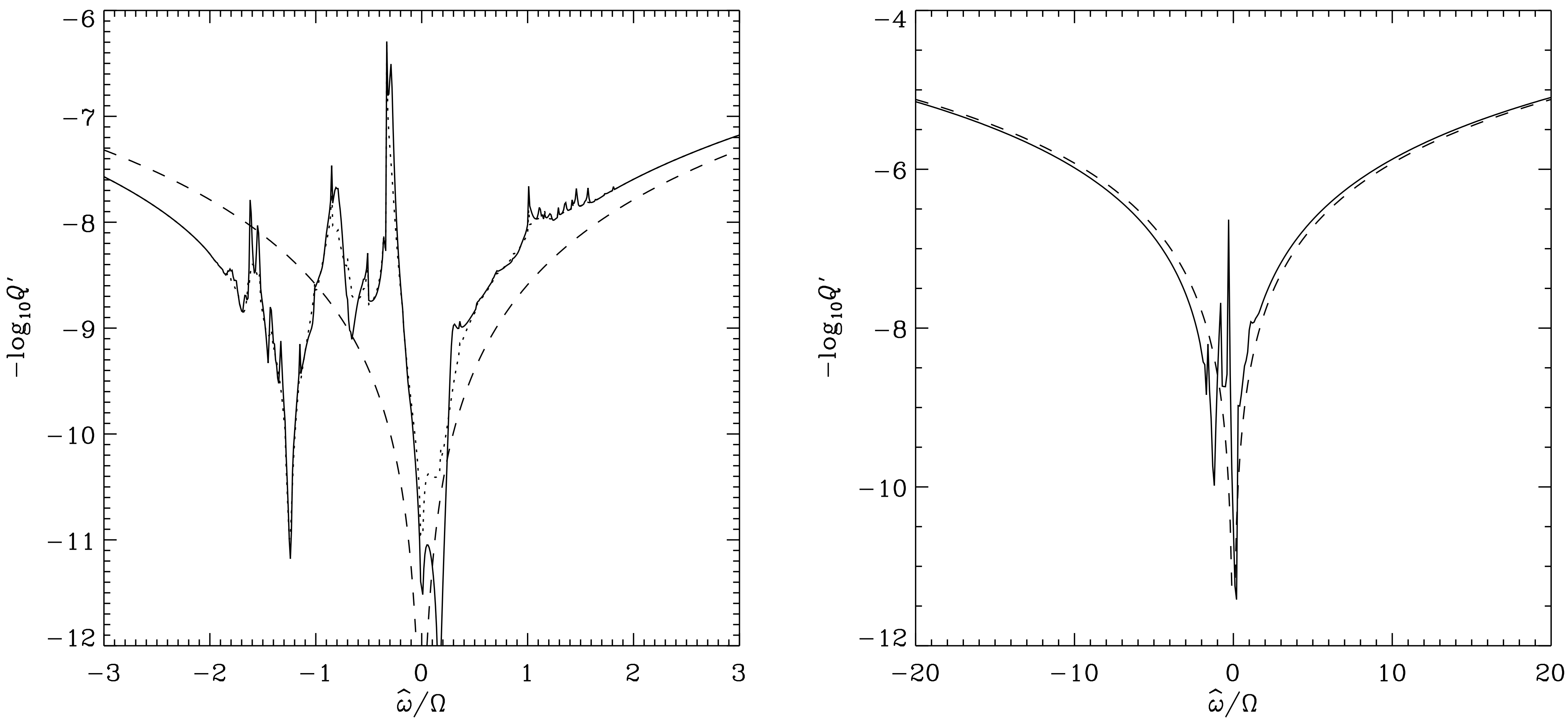
[large turbulent viscosity?]

Inertia-gravity waves in radiative regions (star)

Ogilvie & Lin 2007 (cf. Goodman & Dickson 1998)

- assumes waves do not reflect from stellar centre

-log Q'



tidal frequency

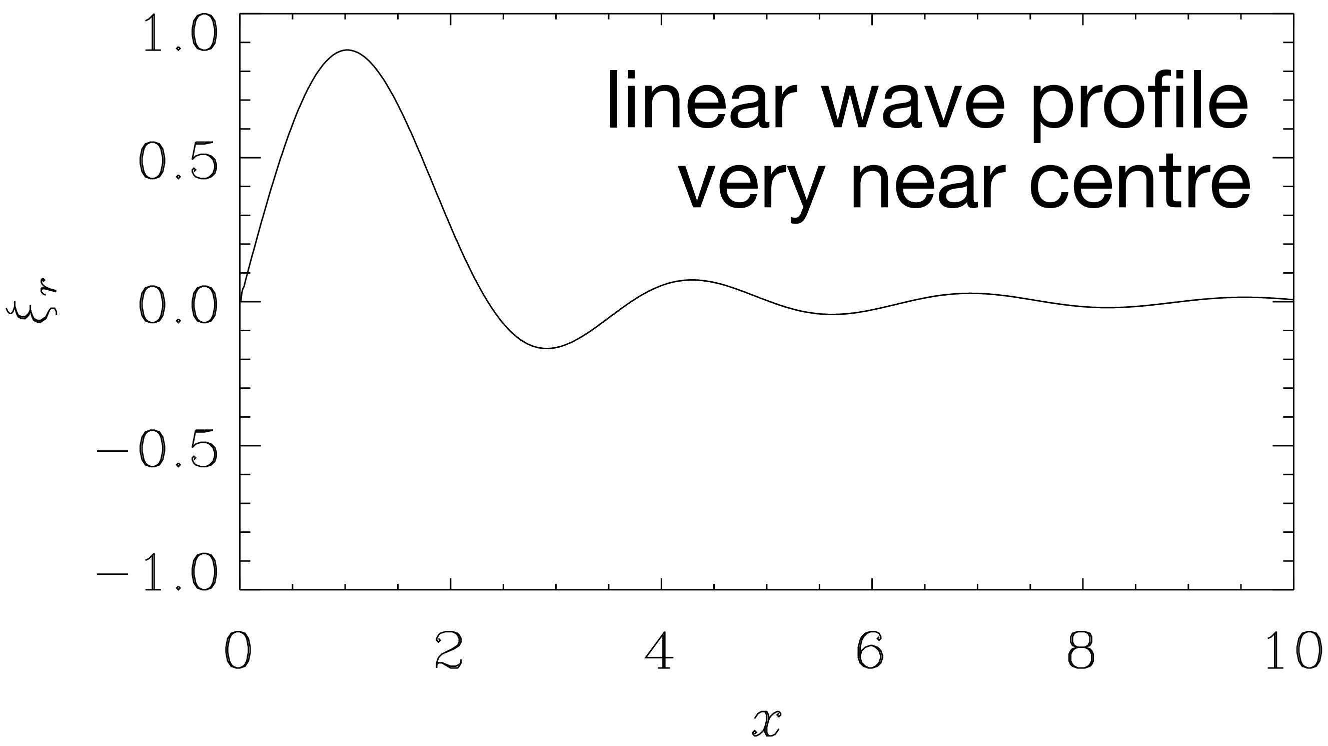
Inertia-gravity waves in radiative regions (star)

Barker & Ogilvie 2010

- waves break at centre if

$$\frac{M_\mathrm{p}}{M_\mathrm{J}} > 3.3 \left(\frac{P_\mathrm{orb}}{\mathrm{day}}\right)^{-1/6}$$

  or more easily in older or slightly more massive stars



- if this occurs, then $Q'_* \approx 1.5 \times 10^5 \left(\frac{P_\mathrm{orb}}{\mathrm{day}}\right)^{8/3}$

  and planet is devoured within $2.3\,\mathrm{Myr} \left(\frac{M_\mathrm{p}}{M_\mathrm{J}}\right)^{-1} \left(\frac{P_\mathrm{orb}}{\mathrm{day}}\right)^{7}$

For smaller forcing amplitudes, resonant locking (Savonije & Witte) may need to be reexamined allowing for wave breaking

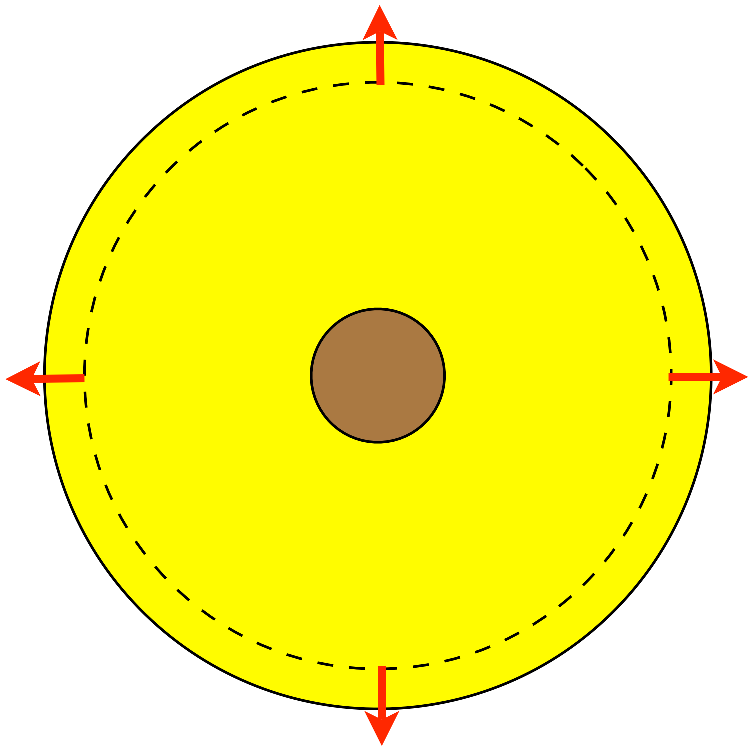
Inertia-gravity waves in radiative regions (planet)

Lubow et al. 1997

- rough application of Zahn / Goldreich-Nicholson approach to hot Jupiters

$$\frac{1}{Q'_{\rm p}} \propto \frac{\rho_{\rm b}}{\bar{\rho}} \left(\frac{H}{R_{\rm p}}\right)^{2/3} \left(\frac{\omega}{\omega_{\rm dyn}}\right)^{8/3}$$

$$Q'_{\rm p} \approx 10^{5-6} \left(\frac{P_{\rm tide}}{\rm day}\right)^{8/3}$$



Issues still needing to be addressed (atmospheric dynamics simulations?) :

- role of Coriolis force and winds
- suppression of wave generation in thin atmosphere
- wave reflection and nonlinearity
- spin evolution of atmosphere

Summary

Equilibrium tides

- possibly interesting contribution in stars with convective envelopes
- maybe relevant for giant planets with nonlinearity or exotic physics

Inertial waves in convective regions

- enhanced dissipation for $-2\Omega < \omega < 2\Omega$
- strong dependence on internal structure and (probably) frequency
- application to stars less clear because of vigorous convection

Inertia-gravity waves in radiative regions

- typical dependence $1/Q' \propto \omega^{8/3}$ but suppressed in HJ atmosphere at high frequencies

Conclusions

- Tidal evolution probably determines the fate of short-period extrasolar planets
- Linear theory of idealized models predicts an intricate frequency-dependence of Q', which may be modified in reality
- Nonlinear aspects (wave breaking, mode coupling, etc.) can be important even for "weak" tides. Extrasolar planets may be in a different regime from solar-system planets
- Better models of planetary (and stellar) interiors are needed and more understanding of the interaction of tides with convection, magnetic fields, etc.
- Thermal and magnetic tides also require further investigation
- Extrasolar systems are diverse and can reveal much when examined on an individual basis