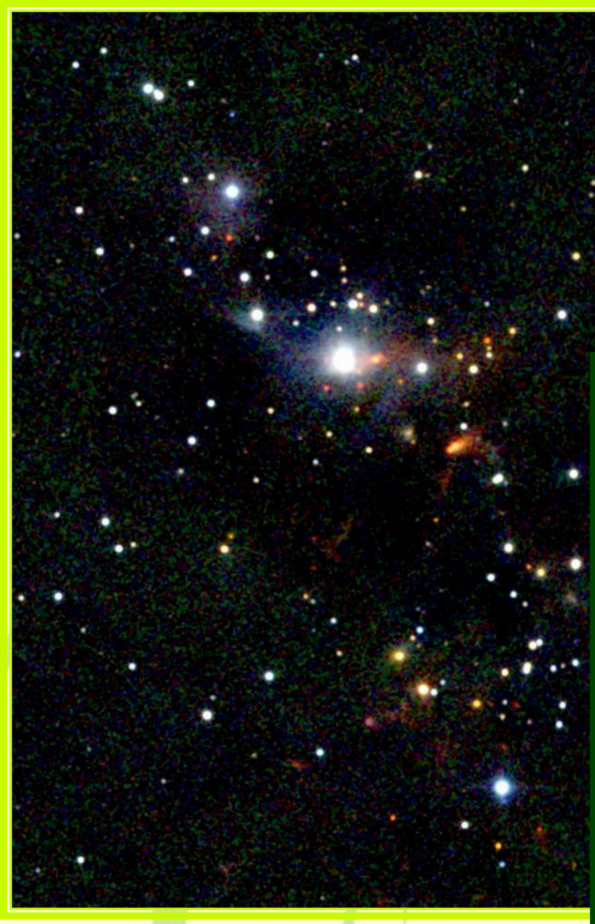
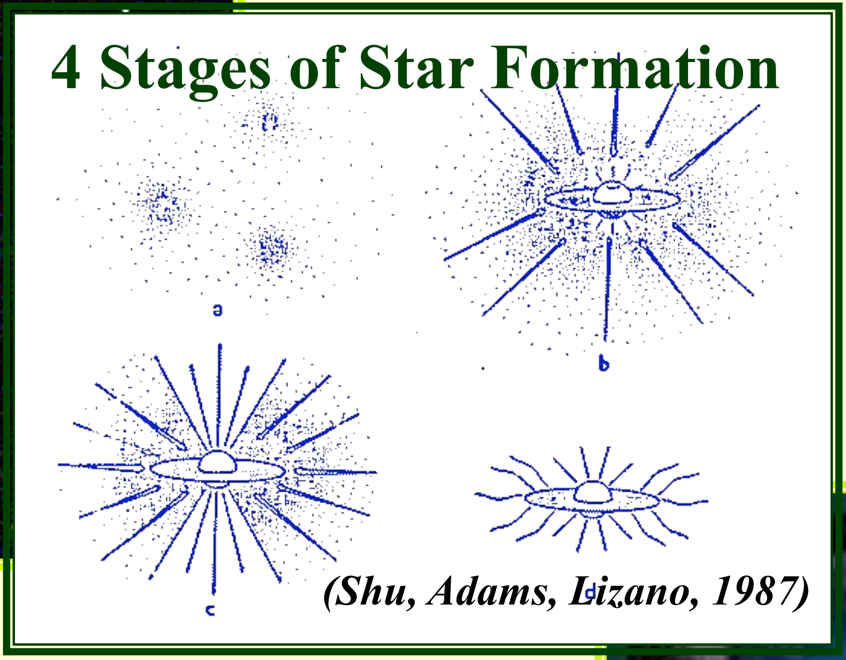
The Birth Environment of the Solar System

The Theory and Observation of Exoplanets
KITP/UC Santa Barbara, 11 May 2010
Fred C. Adams

[A. Bloch, M. Fatuzzo, J. Ketchum, D. Hollenbach, G. Laughlin, P. Myers, and E. Proszkow]

A Brief History





4 Stages of Star Formation

(Shu, Adams, Lizano, 1987)

Most Stars Form in Clusters:

[1] How does the initial cluster environment affect the formation of stars and planets?

[2] What were the basic properties of the birth cluster of our own Sun and its Solar System?

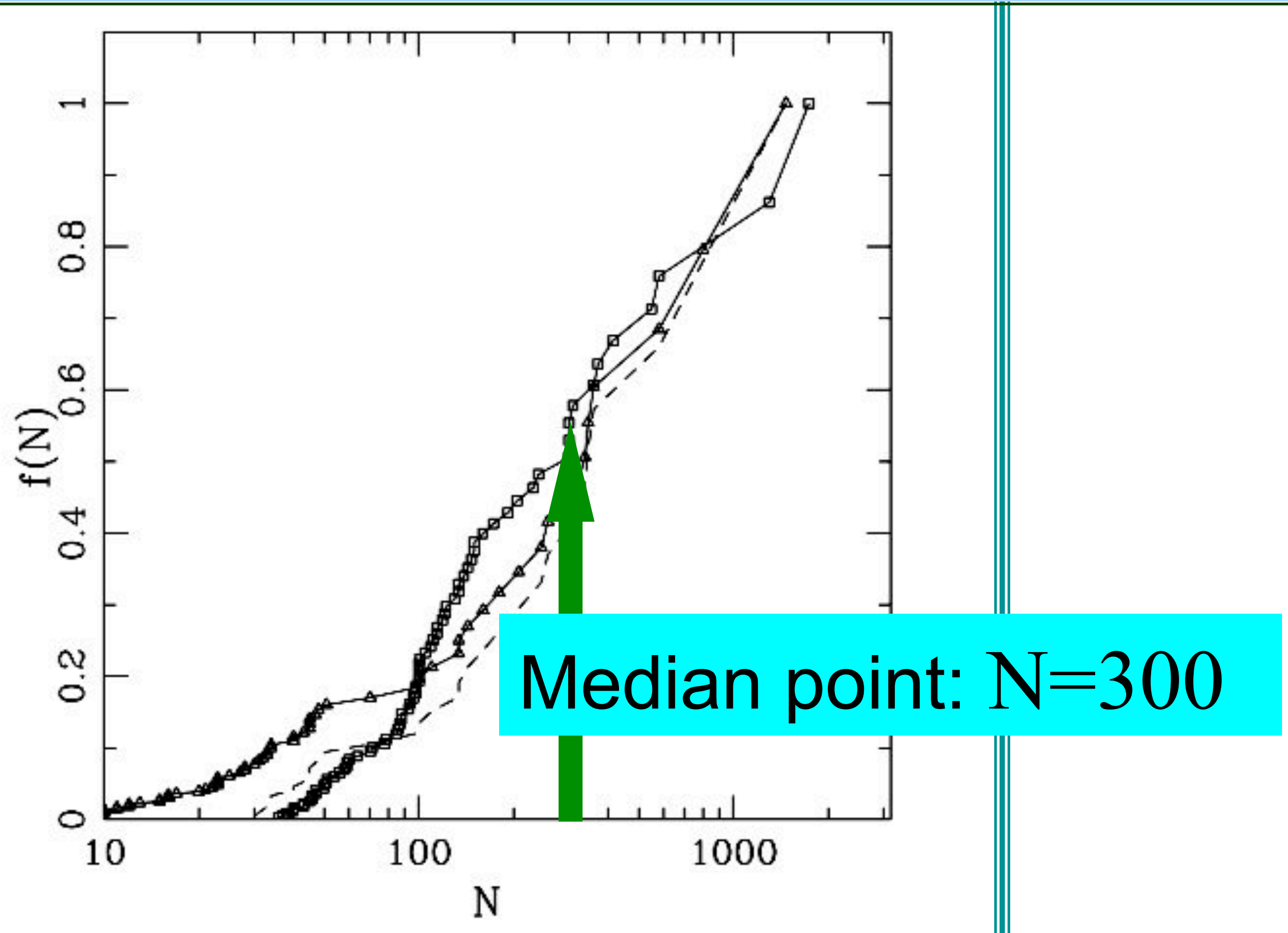
TIME SCALES

Infall-Collapse Timescale = 0.1 Myr

Embedded Cluster Phase = 3 - 10 Myr
Circumstellar Disk Lifetime = 3 - 10 Myr
Giant Planet Formation Time = 3 - 10 Myr

Terrestrial Planet Formation = 100 Myr
Late Heavy Bombardment = 600 Myr
Open Cluster Lifetime = 100 - 1000 Myr

Cumulative Distribution: Fraction of stars that form in stellar aggregates with N < N as function of N



Median point: N=300

CONJECTURE:

The cluster environment affects planet formation much more than the process of star formation

Why: Clusters have radial scale of 1 pc,
with distance between protostars of 0.24pc.
Cores are observed to move at 0.1 km/s.
During their formation time of 0.1 Myr,
protostars move only 0.01 pc << 0.24 pc…

Dynamical Studies

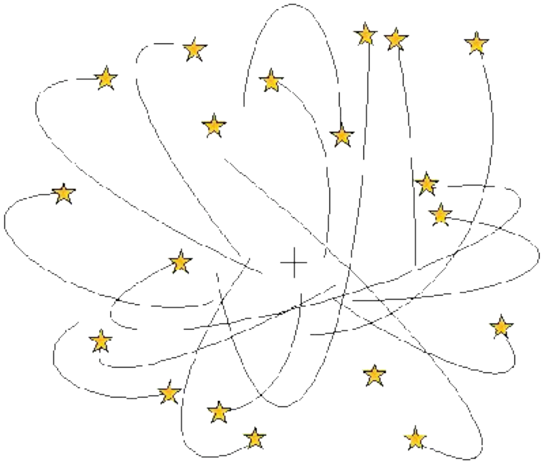
I. Evolution of clusters as astrophysical objects

II.Effects of clusters on forming solar systems (with a focus on our own system)

- **Distribution of closest approaches**
- **Radial position probability distribution**

Simulations of Embedded Clusters

- **Modified NBODY2(and 6) Codes *(S. Aarseth)***
- **Simulate evolution from embedded stage to age 10 Myr**
- **Cluster evolution depends on the following:**
  - cluster size
  - initial stellar and gas profiles
  - gas disruption history
  - star formation history
  - primordial mass segregation
  - initial dynamical assumptions
- **100 realizations are needed to provide robust statistics for output measures**



(E. Proszkow thesis 2009)

Simulation Parameters

Cluster Membership

N = 100, 300, 1000

Radius $R(N) = 1pc\left({}^{N}\!/_{300}\right)^{1/2}$

Initial Stellar Density

Gas Distribution $\rho_* \propto r^{-1}$

$$\rho_{gas} = \frac{\rho_0}{\xi(1+\xi)^3}, \quad \rho_0 = \frac{2M_*}{\pi R^3} \quad \xi = \frac{r}{R}$$

SF Efficiency = 0.33

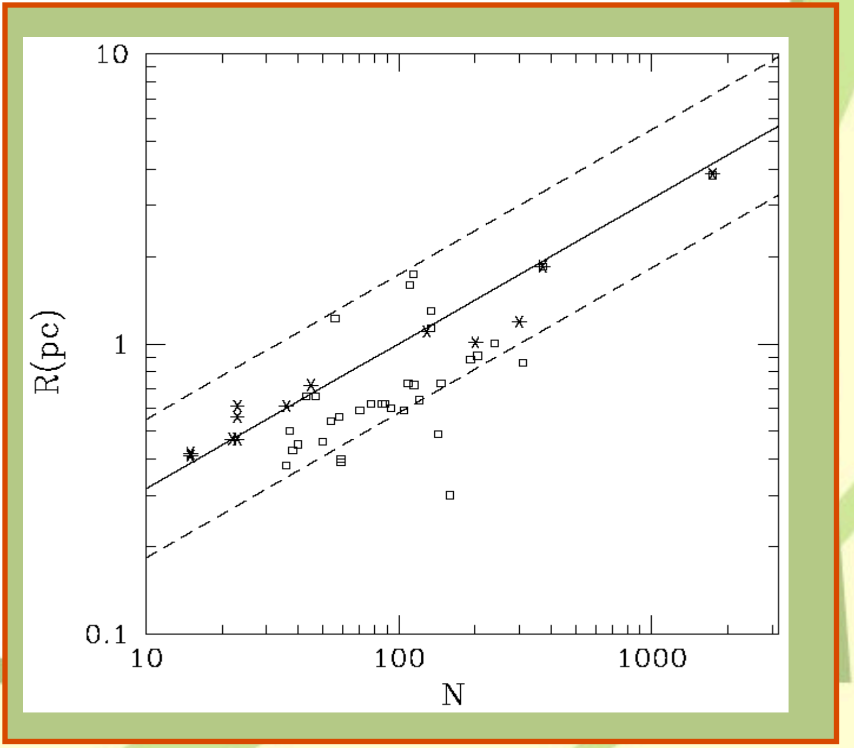
Embedded Epoch t = 0–5 Myr

SF time span t = 0-1 Myr

Virial Ratio $Q = |K/W|$

virial Q = 0.5; cold Q = 0.04

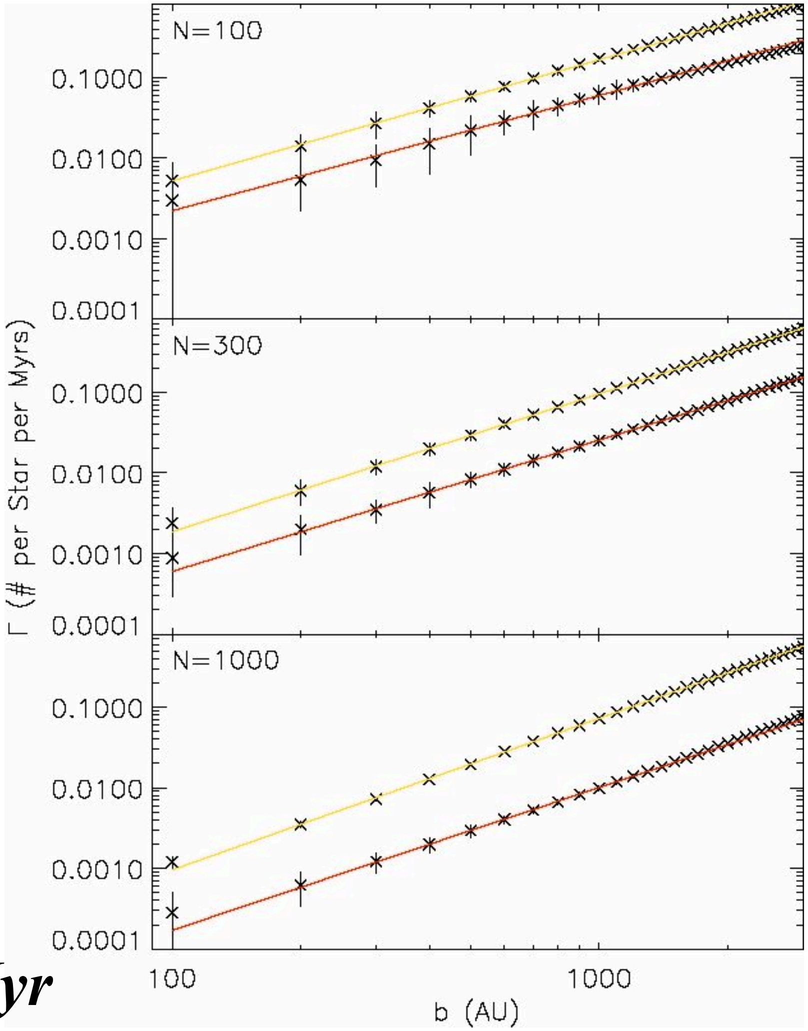
Mass Segregation: largest star at center of cluster



Closest Approach Distributions

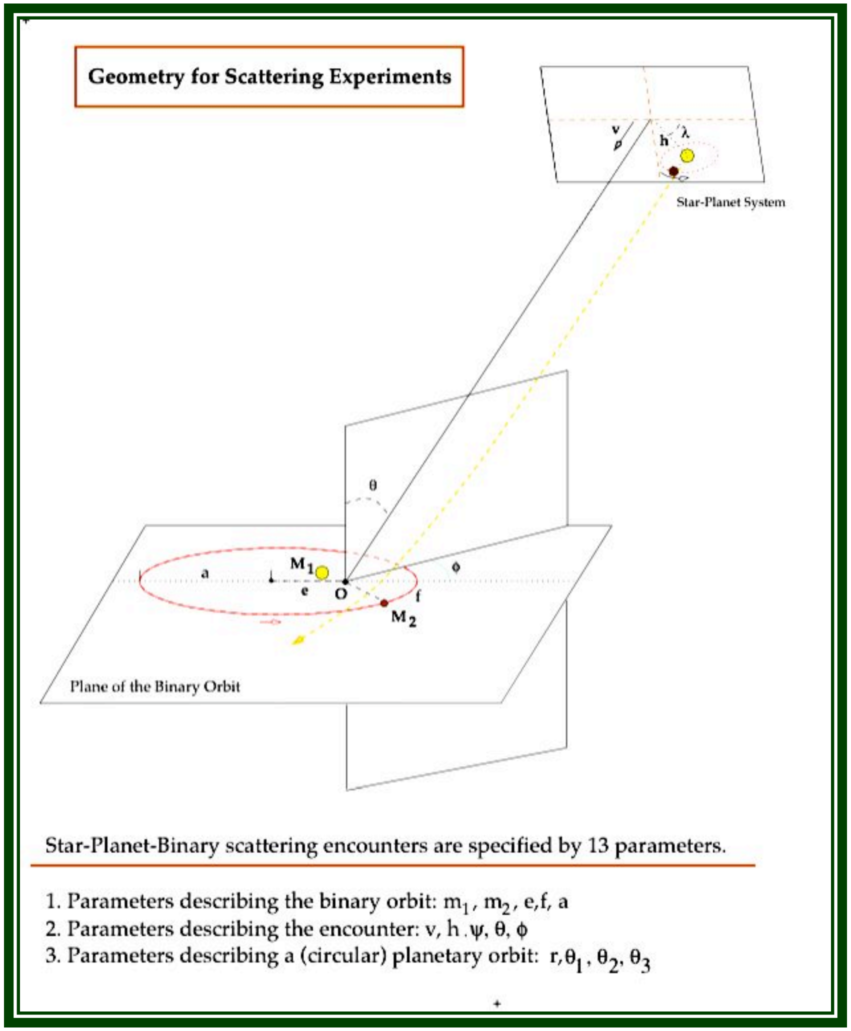
$$\Gamma = \Gamma_0\left[\frac{b}{1000\,AU}\right]^{\gamma}$$

Simulation	Γ_0	γ	b_C (AU)
100 Subvirial	0.166	1.50	713
100 Virial	0.0598	1.43	1430
300 Subvirial	0.0957	1.71	1030
300 Virial	0.0256	1.63	2310
1000 Subvirial	0.0724	1.88	1190
1000 Virial	0.0101	1.77	3650



Typical star experiences one close encounter with impact parameter b_C during time10 Myr

Solar System Scattering



Geometry for Scattering Experiments

Star-Planet-Binary scattering encounters are specified by 13 parameters.

1. Parameters describing the binary orbit: m_1, m_2, e,f, a
2. Parameters describing the encounter: v, h ,ψ, θ, φ
3. Parameters describing a (circular) planetary orbit: r,θ_1, θ_2, θ_3

Many Parameters

+

Chaotic Behavior

⬇

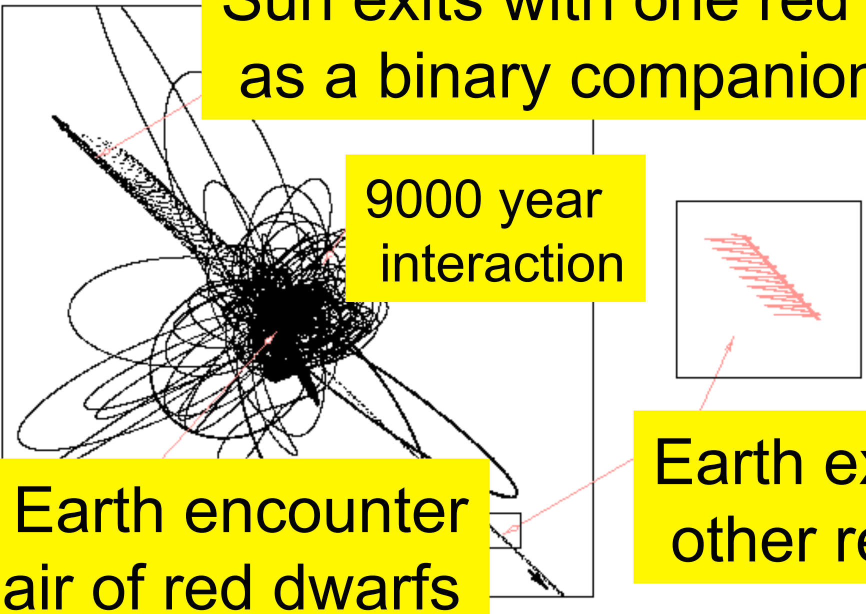
Many Simulations

Monte Carlo

Monte Carlo Experiments

- **Jupiter only, v = 1 km/s, N=40,000 realizations**
- **4 giant planets, v = 1 km/s, N=50,000 realizations**
- **KB Objects, v = 1 km/s, N=30,000 realizations**
- **Earth only, v = 40 km/s, N=100,000 realizations**
- **4 giant planets, v = 40 km/s, Solar mass, N=100,000 realizations**
- **4 giant planets, v = 1 km/s, varying stellar mass, N=100,000 realizations**

Red Dwarf Captures the Earth!



Sun exits with one red dwarf as a binary companion

9000 year interaction

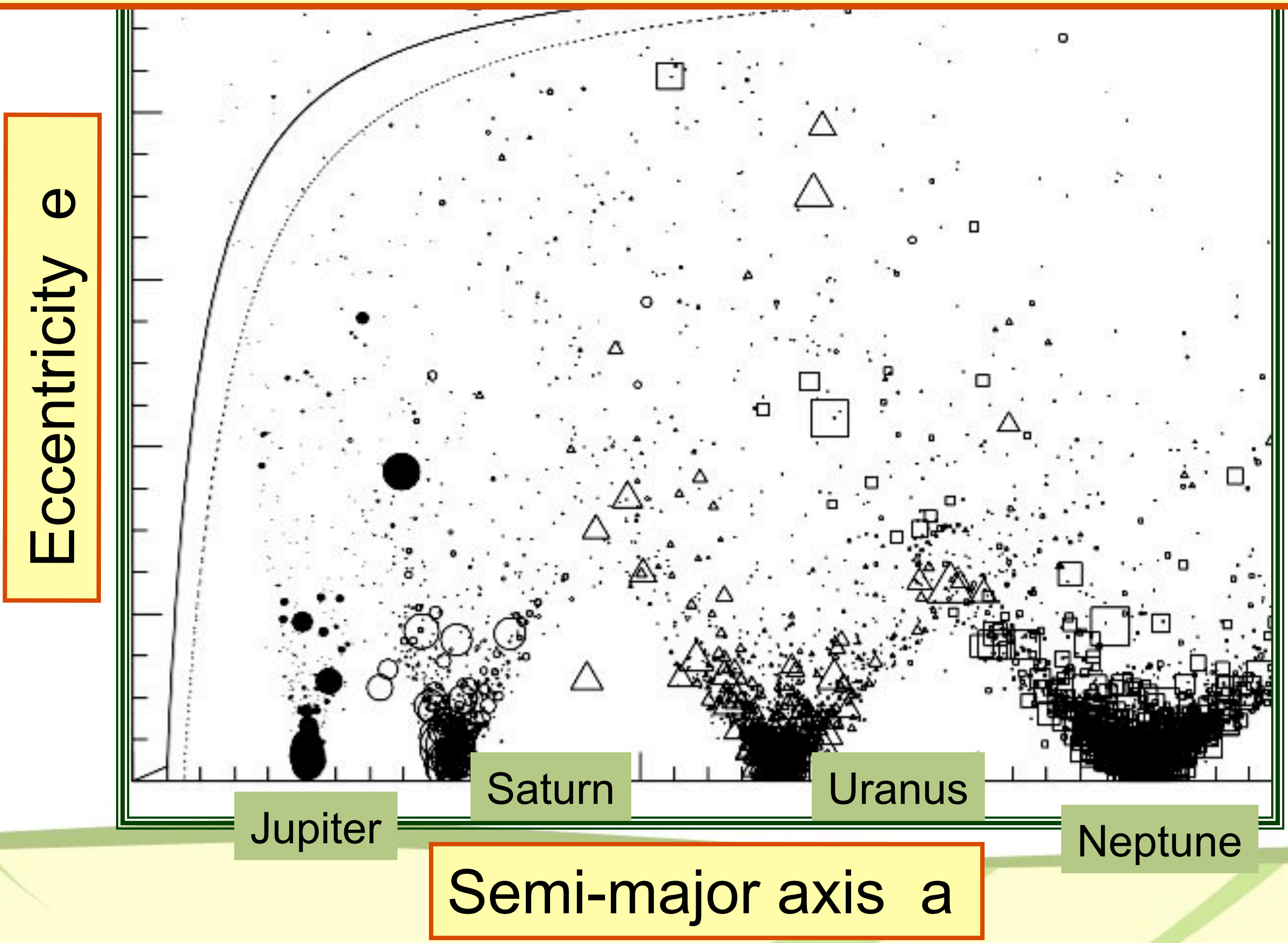
Sun and Earth encounter binary pair of red dwarfs

Earth exits with the other red dwarf

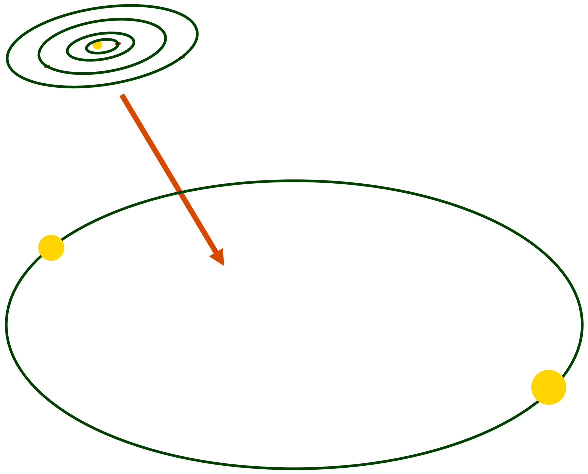
Fun Future Earth Facts

[1] Biosphere has only about 3 Gyr left
[2] Odds of Earth being scattered out of the solar system during this time = 1 in 10^5
[3] Odds of Earth being captured by passing star during this time = 1 in 3x10^6
[4] Life on Earth lasts longer if Earth leaves

Scattering Results for our Solar System



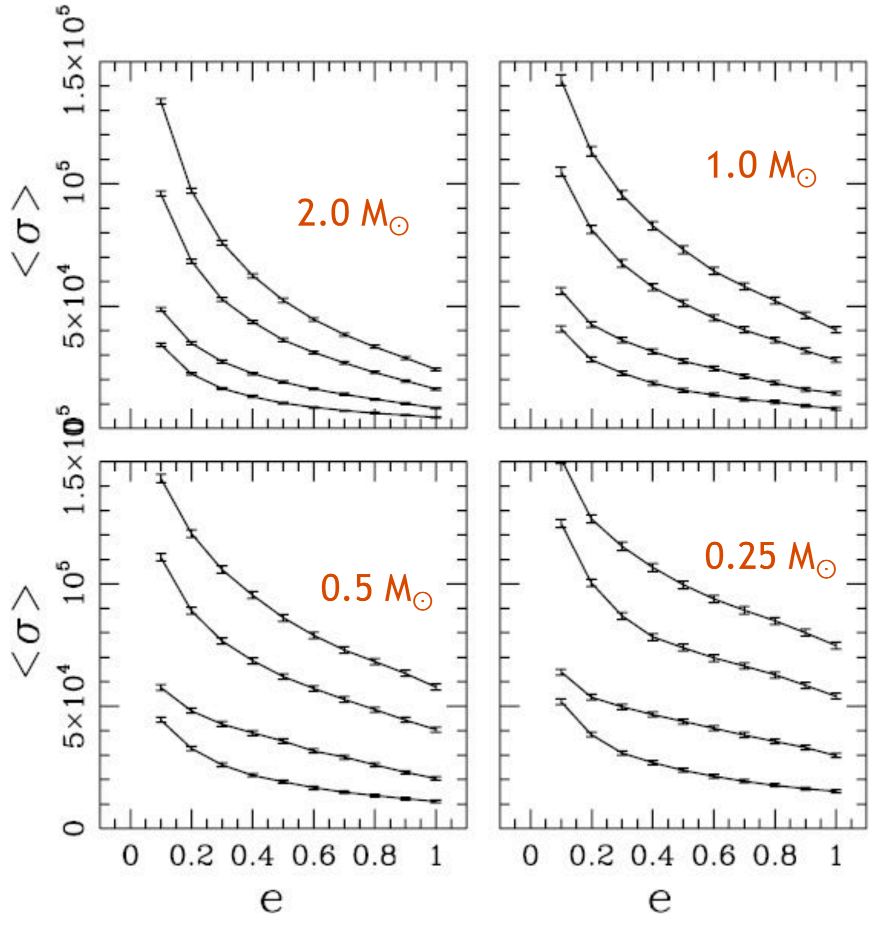
Cross Sections vs Stellar Mass



$$\langle\sigma\rangle_{ej} = C_0\left(\frac{a_P}{AU}\right)\left(\frac{M_*}{M_{sun}}\right)^{-1/2}$$

where

$$C_0 = 1350 \pm 160\ (AU)^2$$



Effects of Cluster Radiation on Forming/Young Solar Systems

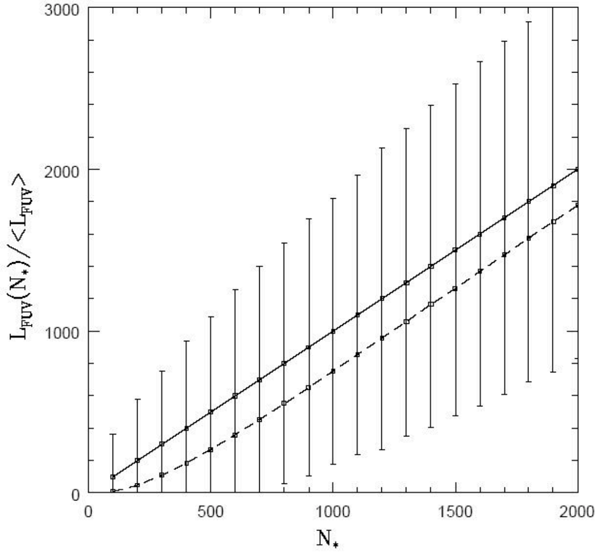
- Photoevaporation of a circumstellar disk
- Radiation from the background cluster often dominates radiation from the parent star *(Johnstone et al. 1998; Adams & Myers 2001)*
- FUV radiation (6 eV < E < 13.6 eV) is more important in this process than EUV radiation
- FUV flux of $G_0 = 3000$ will truncate a circumstellar disk to r_d over 10 Myr, where $r_d = 36AU\left[M_*/M_{sun}\right]$

Calculation of the Radiation Field

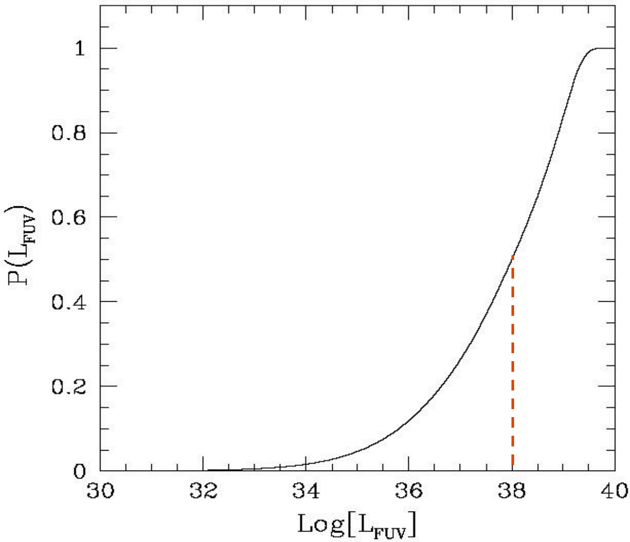
Fundamental Assumptions

- **Cluster size N = N primaries (ignore binary companions)**
- **No gas or dust attenuation of FUV radiation**
- **Stellar FUV luminosity is only a function of mass**
- **Meader's models for stellar luminosity and temperature**

Sample IMF -> $L_{FUV}(N)$



Cluster Sizes: **Expected FUV Luminosity**



Composite Distribution of FUV Flux

FUV Flux depends on:

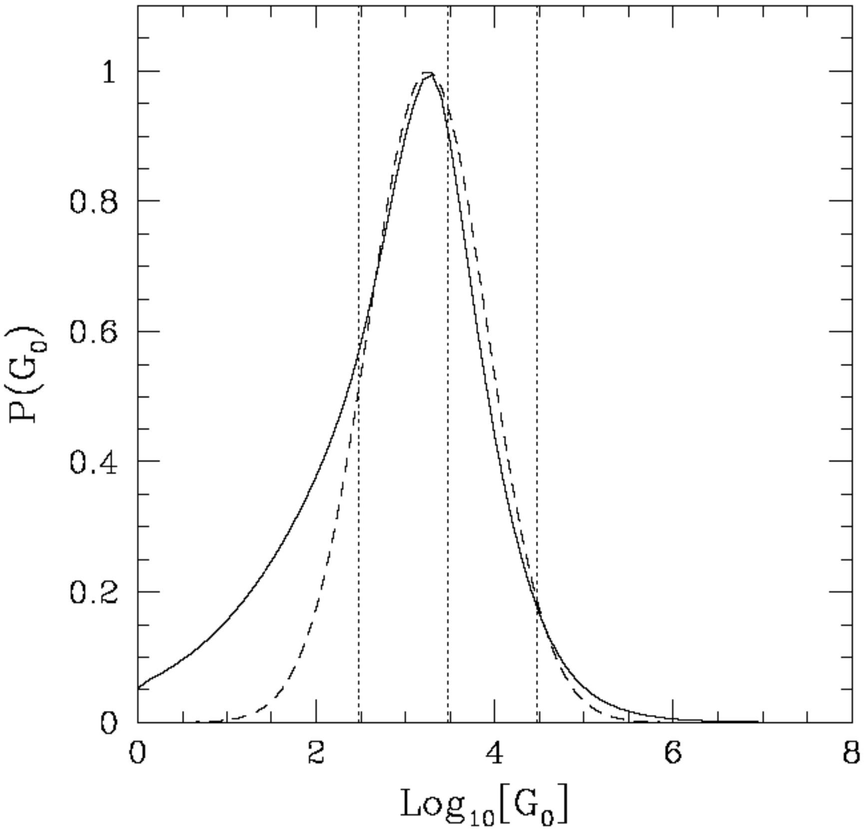
- Cluster FUV luminosity
- Location of disk within cluster

Assume:

- FUV point source at center of cluster
- Stellar density $\rho \sim 1/r$

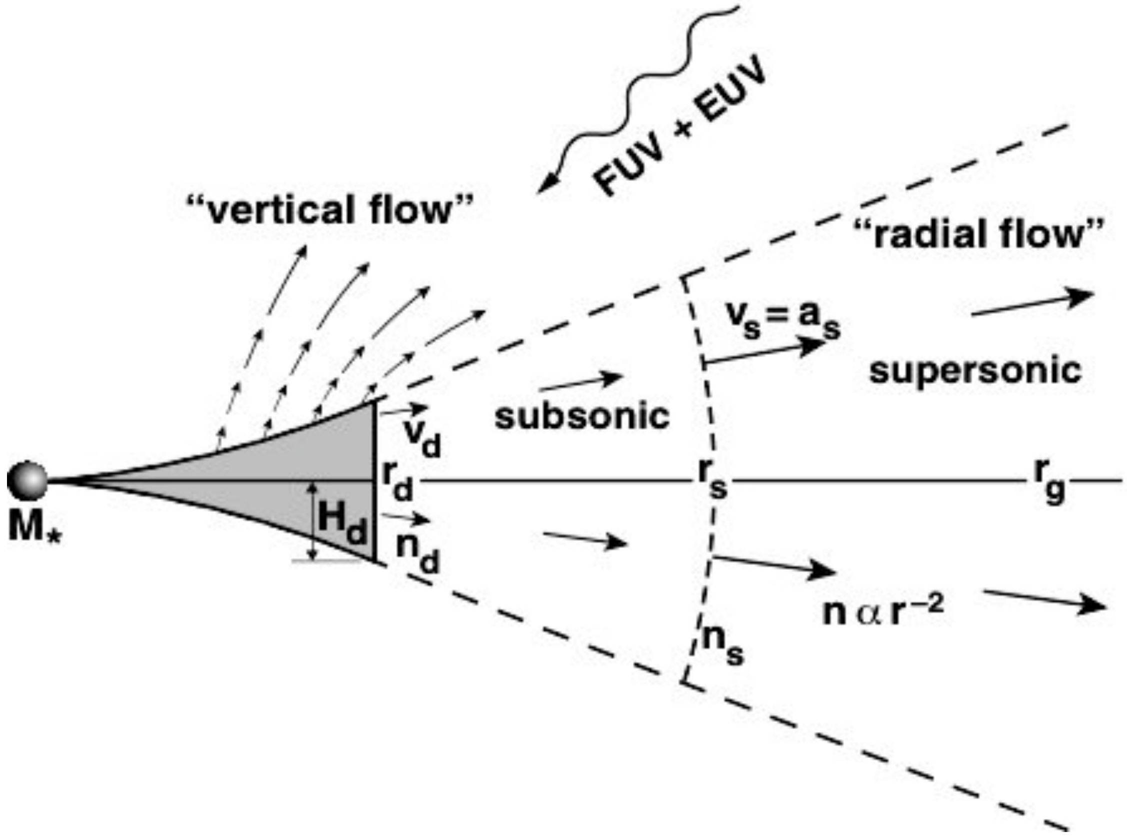
G_0 Distribution

Median	900
Peak	1800
Mean	16,500



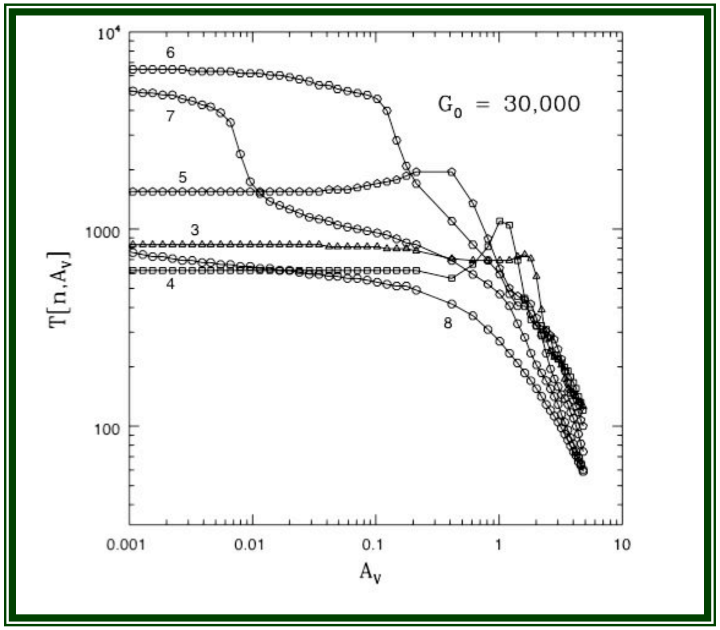
$G_0 = 1$ corresponds to FUV flux 1.6×10^{-3} erg s^{-1} cm^{-2}

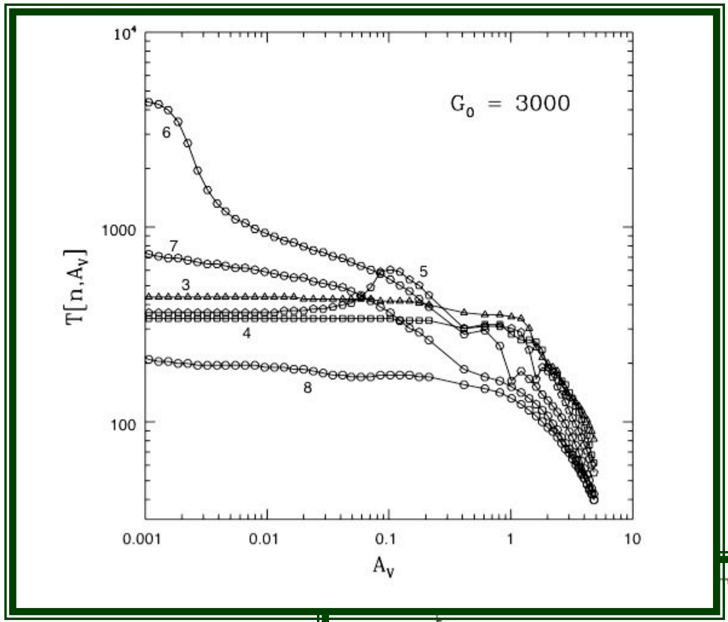
Photoevaporation Model

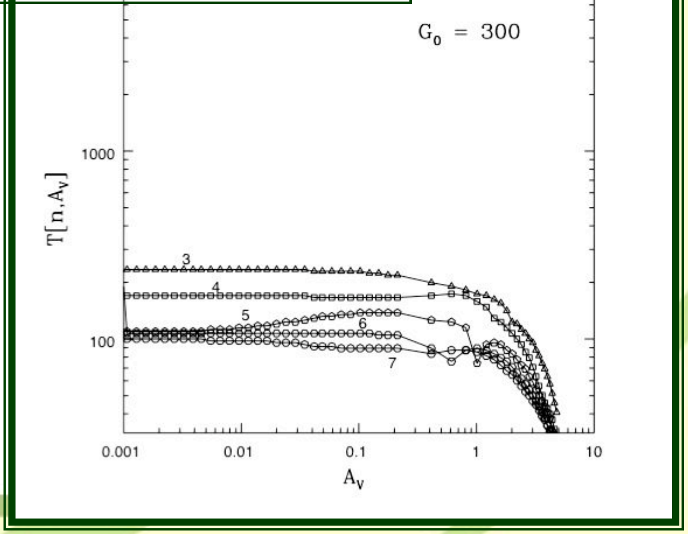


(Adams et al. 2004)

Results from PDR Code

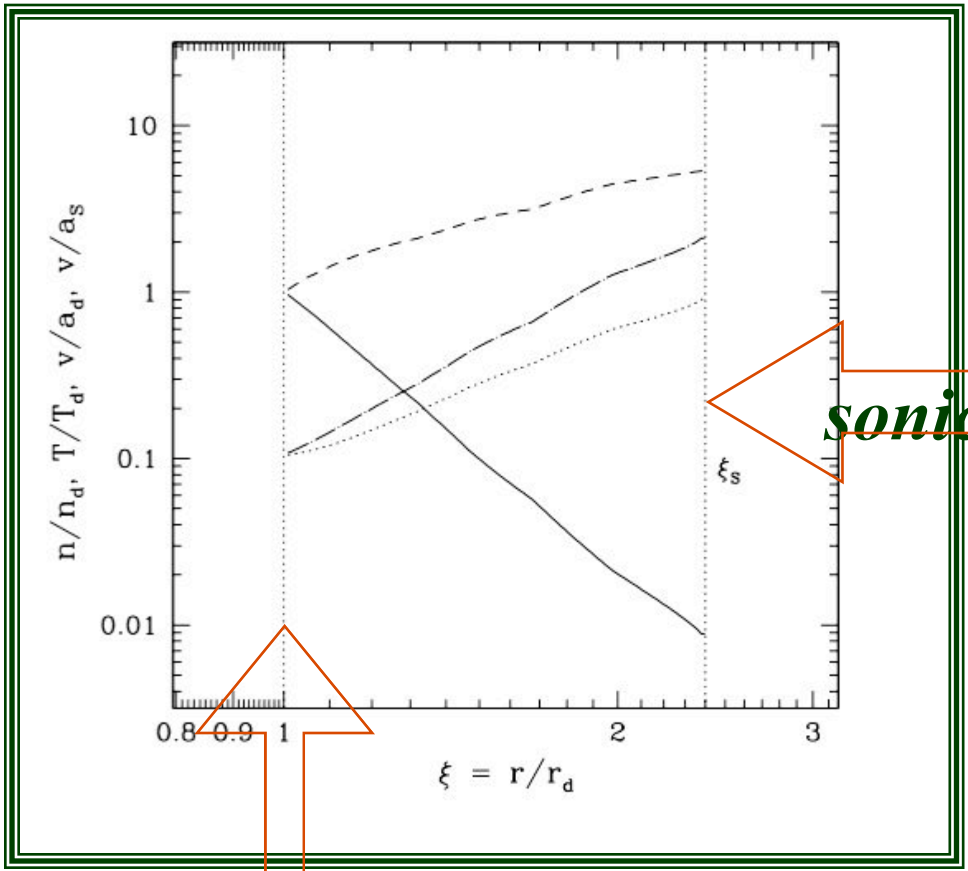






Lots of chemistry and many heating/cooling lines determine the temperature as a function of G, n, A

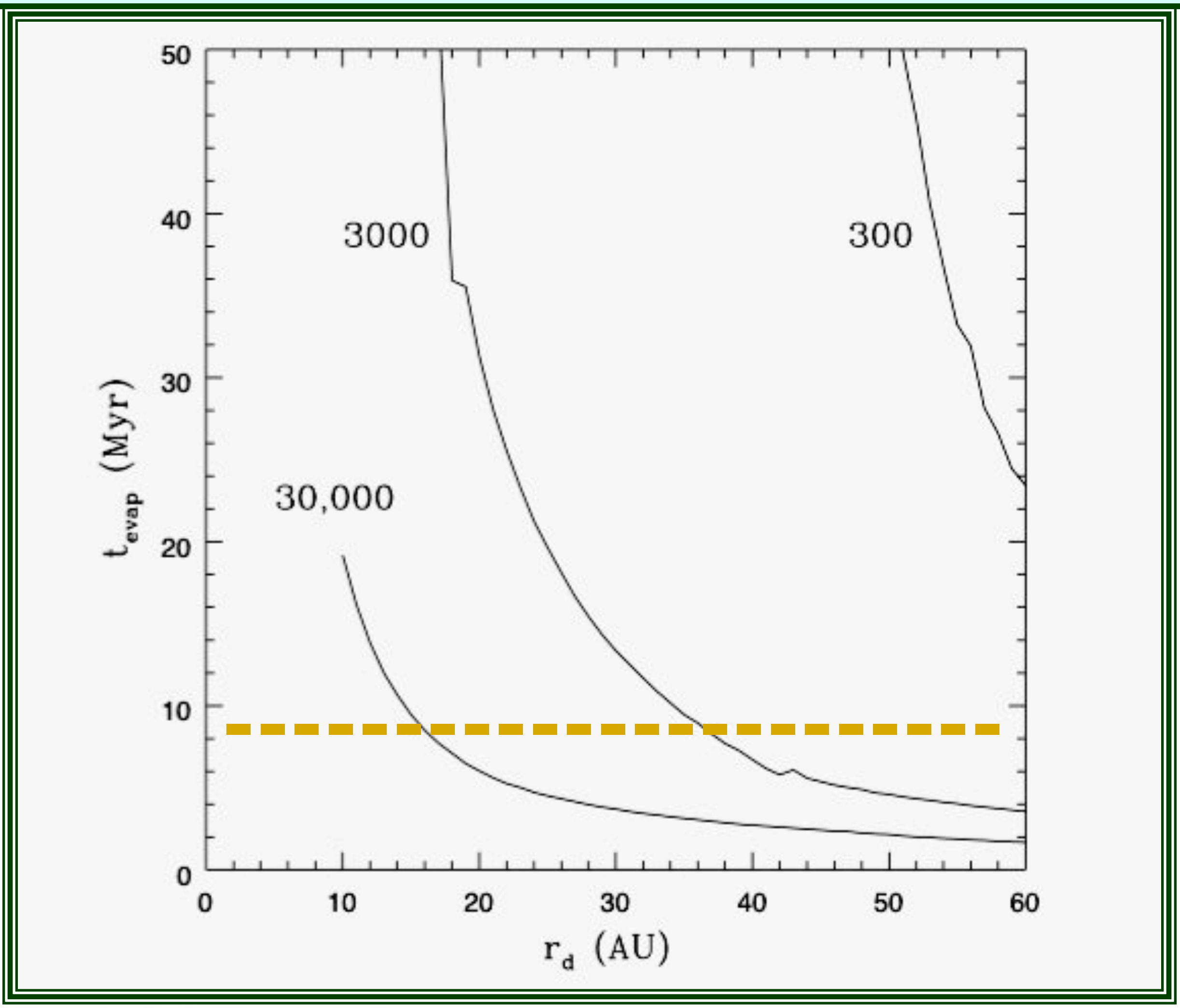
Solution for the Fluid Fields



sonic surface

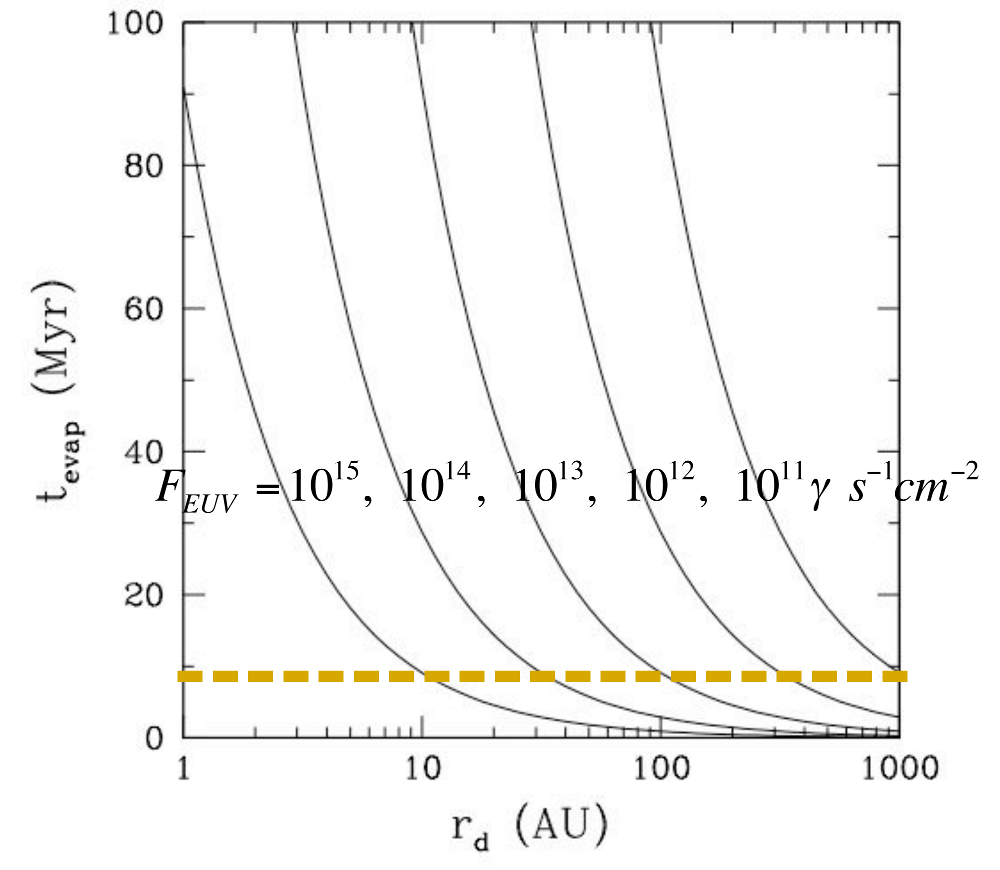
outer disk edge

Evaporation Time vs FUV Field



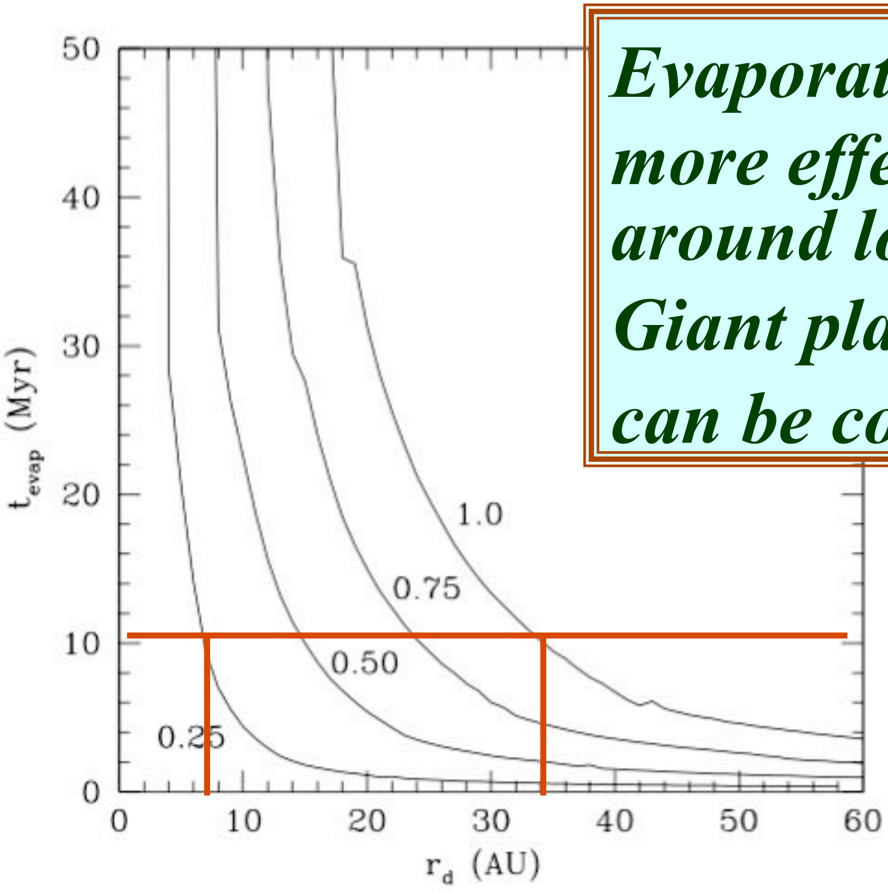
(for disks around solar mass stars)

Evaporation Time vs EUV Field



(FUV radiation has larger effect on solar nebula than EUV)

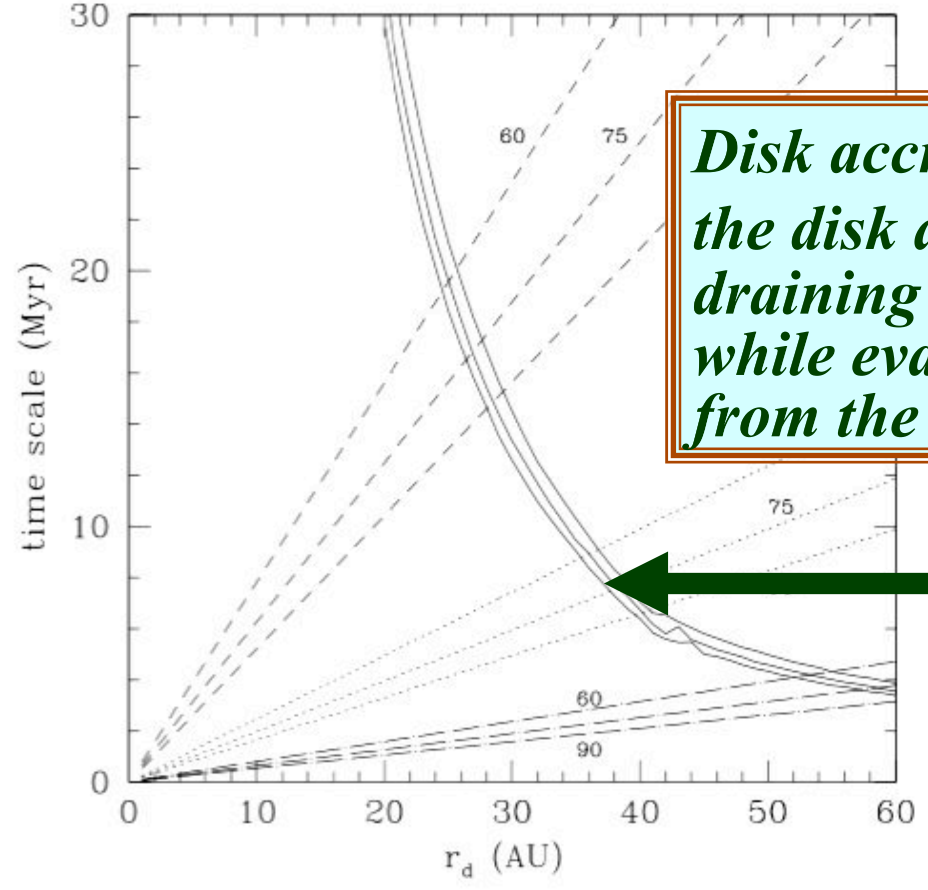
Evaporation Time vs Stellar Mass



Evaporation is much more effective for disks around low-mass stars: Giant planet formation can be compromised

G=3000

Evaporation vs Accretion



Disk accretion aids and abets the disk destruction process by draining gas from the inside, while evaporation removes gas from the outside . . .

Total time scale of 8 Myr, consistent with observations…

Conclusion [1]

Clusters have a moderate effect on the solar systems forming within them -- environmental effects are neither dominant nor negligible:

Closest approaches of order 1000 AU
Disks truncated dynamically to 300 AU
Disks truncated via radiation to 40 AU
Lifetimes have environmental upper limit
Planetary orbits are moderately altered
Only a few planetary ejections per cluster

(these effects must be described via probabilities)

Where did we come from?

Solar System Properties

Enrichment of short-lived radioactive nuclear species

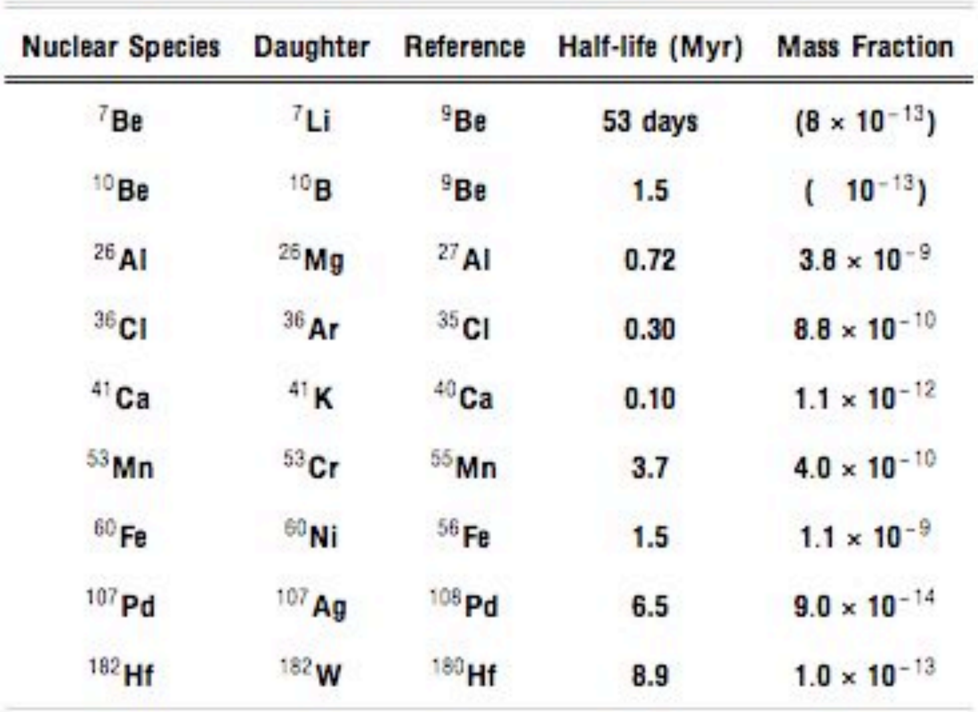
Planetary orbits are well-ordered (ecc. & inclination)

Edge of early solar nebula -- gas disk -- at 30 AU

Observed edge of Kuiper belt at around 40 - 50 AU

Orbit of dwarf planet Sedna: e = 0.82 and p = 70 AU

Short-Lived Radio Isotopes



Nuclear Species	Daughter	Reference	Half-life (Myr)	Mass Fraction
^{7}Be	^{7}Li	^{9}Be	53 days	(8×10^{-13})
^{10}Be	^{10}B	^{9}Be	1.5	(10^{-13})
^{26}Al	^{26}Mg	^{27}Al	0.72	3.8×10^{-9}
^{36}Cl	^{36}Ar	^{35}Cl	0.30	8.8×10^{-10}
^{41}Ca	^{41}K	^{40}Ca	0.10	1.1×10^{-12}
^{53}Mn	^{53}Cr	^{55}Mn	3.7	4.0×10^{-10}
^{60}Fe	^{60}Ni	^{56}Fe	1.5	1.1×10^{-9}
^{107}Pd	^{107}Ag	^{108}Pd	6.5	9.0×10^{-14}
^{182}Hf	^{182}W	^{180}Hf	8.9	1.0×10^{-13}

Solar Birth Requirements (1.0)

Supernova enrichment requires large N

Well ordered solar system requires small N

$$M_* > 25 M_\odot$$

$$\varepsilon(Neptune) < 0.1$$

$$F_{SN} = 0.000485$$

$$\Delta\Theta_j < 3.5^\circ$$

Probability of Supernovae

$$\boxed{P_{SN}(N) = 1 - f_{not}^{N}}$$

Probability of Supernovae

$$P_{SN}(N) = 1 - f_{not}^{N} = 1 - (1 - F_{SN})^{N}$$

Cross Section for Solar System Disruption

$$\boxed{\langle \sigma \rangle \approx (400\,AU)^2}$$

$$\varepsilon(Neptune) \geq 0.1 \quad and/or \quad \Delta\theta \geq 3.5^\circ$$

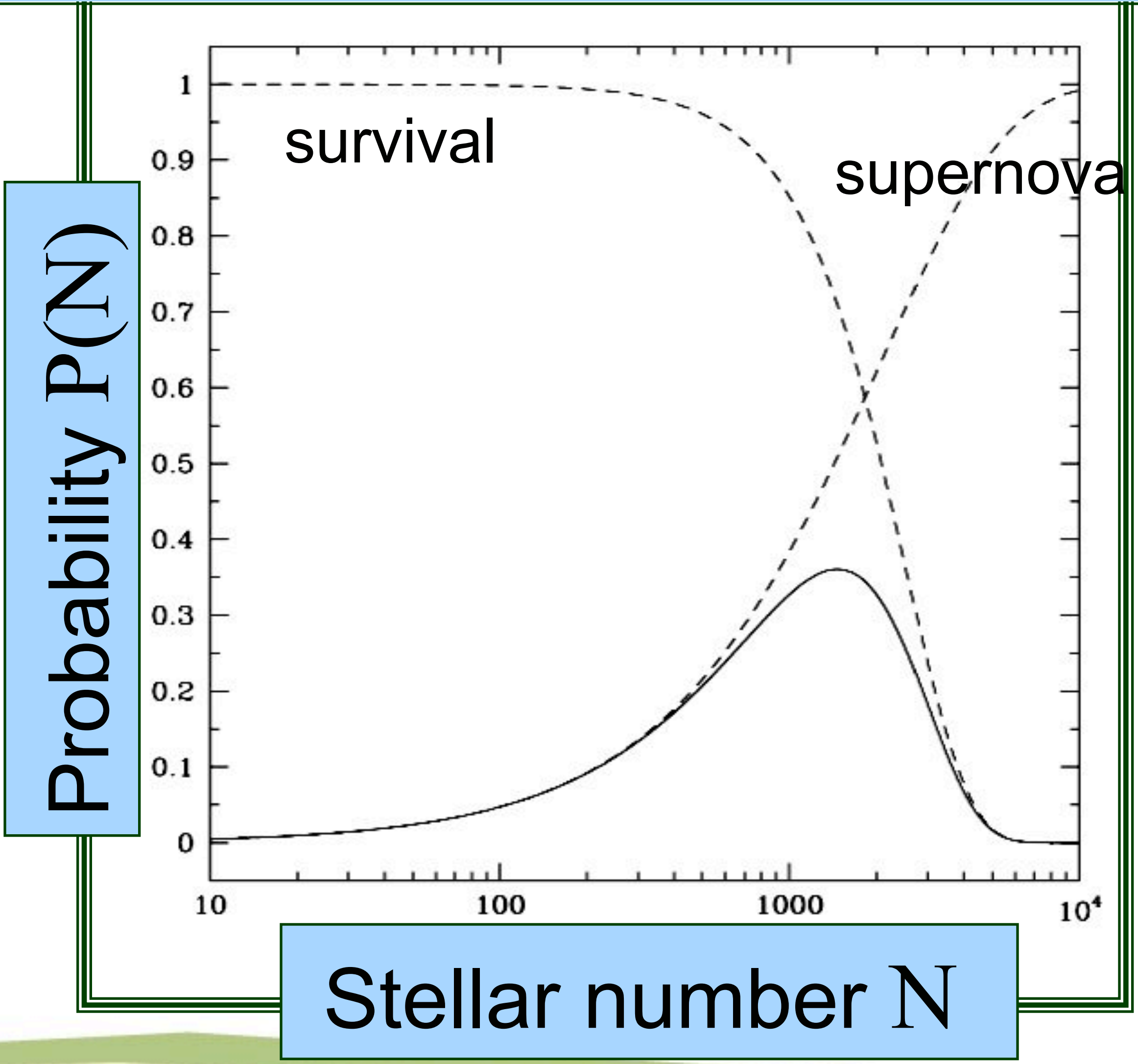
Probability of Scattering

Scattering rate: $\Gamma = n\langle \sigma v \rangle$

Survival probability: $P_{survive} \propto \exp\left[-\int \Gamma dt\right]$

Use the calculated rate of close encounters and interaction cross sections $\langle \sigma \rangle$

Expected Size of the Stellar Birth Aggregate



Adams & Laughlin, 2001, Icarus, 150, 151

Constraints on the Solar Birth Aggregate

$$\boxed{\langle N \rangle \approx 2000 \pm 1100}$$

$$\boxed{P \approx 0.017}$$

(1 out of 60)

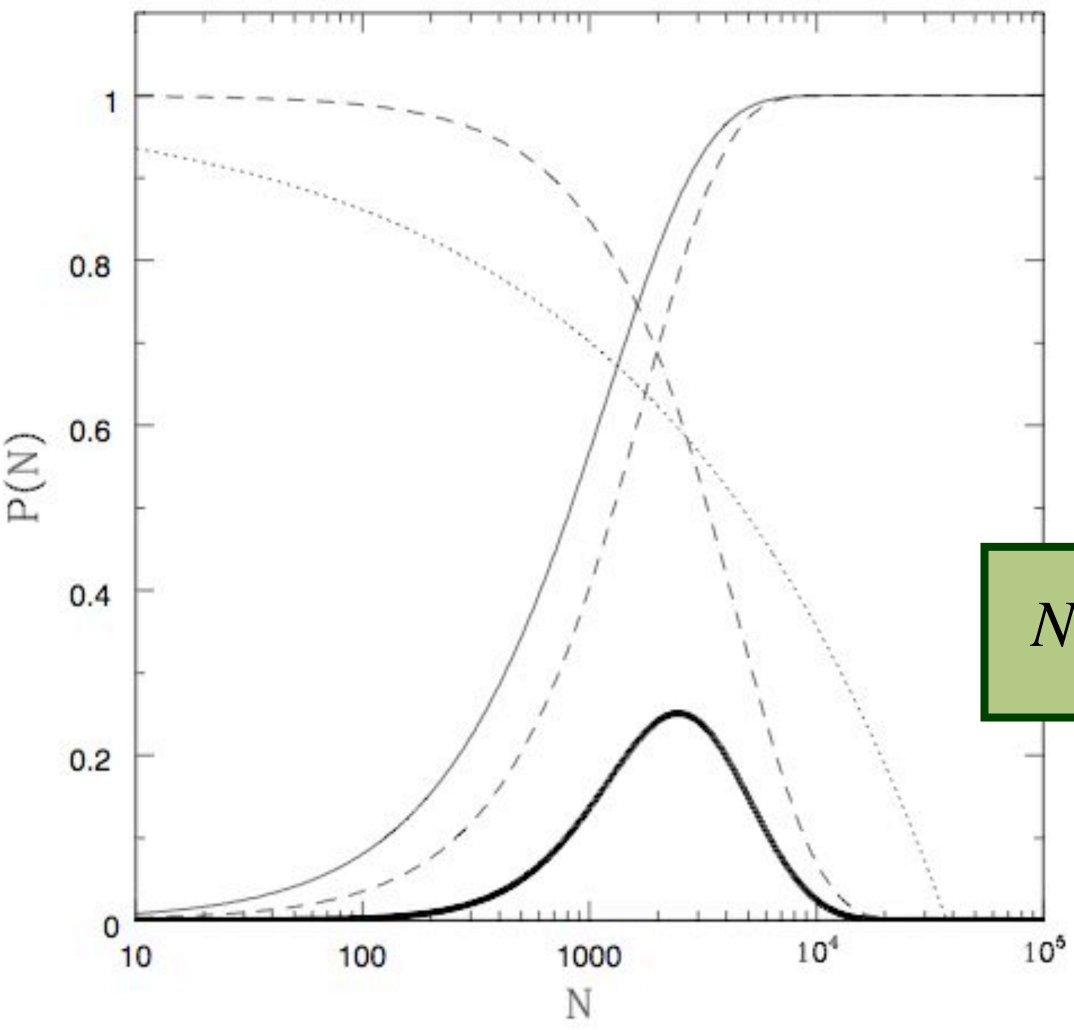
(Adams & Laughlin 2001 - updated)

Extended Constraints

SEDNA: Orbit can be produced via scattering encounter with b = 400 - 800 AU. Need value near lower end to explain edge of Kuiper Belt (Kenyon, Bromley, Levison, Morbidelli, Brasser)

RADIATION: FUV radiation field G < 3000. Implies constraint on available real estate in Birth Cluster (will be function of size N)

Extended Constraints



Supernova
Neptune
Sedna
Radiation

$N = 4300 \pm 2800$

[2] CONSISTENT SCENARIO for Solar Birth Aggregate

Cluster size: N = 1000 - 7000

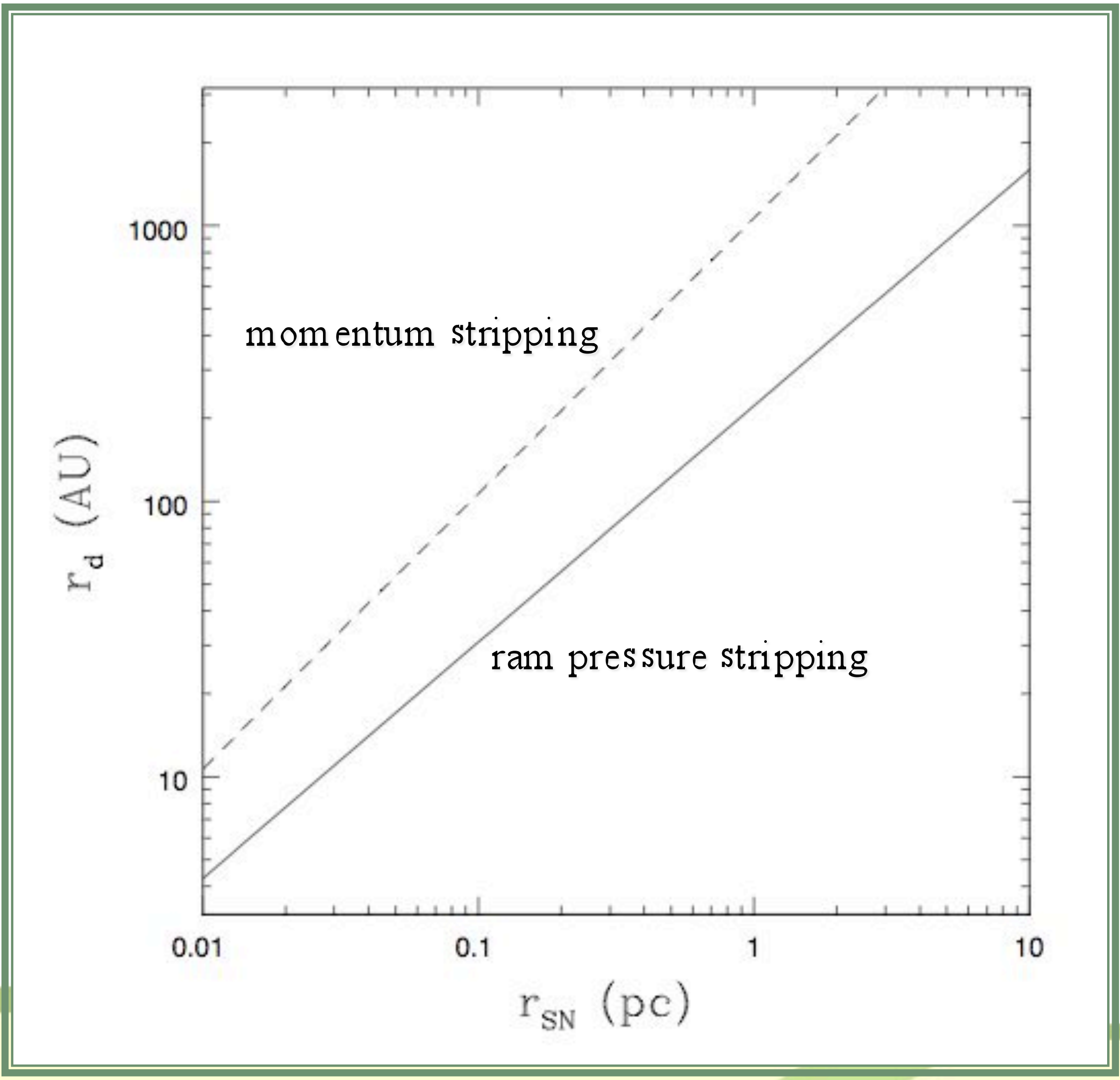
Reasonable *a priori* probability (few percent)

Allows meteoritic enrichment and scattering survival

UV radiation field evaporates disk down to 30 AU

Scattering interactions truncate Kuiper belt at 50 AU
leave Sedna and remaining KBOs with large (a,e,i)

Disk Truncation Radii due to SN Blast



Timing and Tuning Issues

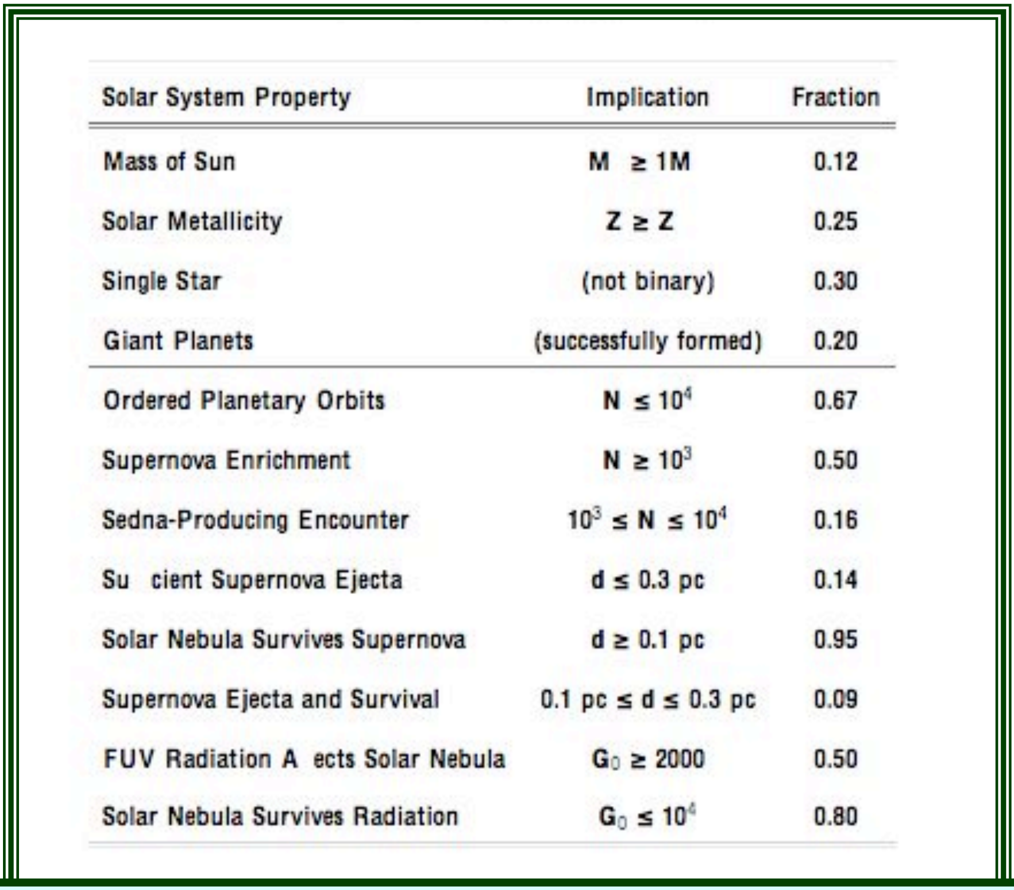
[1] The 25 Msun SN progenitor lives for 7.5 Myr, solar nebula must live a bit longer than average.

[2] Solar system must live near edge of cluster for most of the time to avoid radiation, but must lie at distance of 0.1 - 0.2 pc at time of explosion.

[3] Solar system must experience close encounter at $b = 400$ AU to produce Sedna, but no encounters with $b < 225$ AU to avoid disruption of Neptune, etc.

[4] Solar system must live in its birth cluster for a relatively long time (100 Myr), a 10 percent effect.

Constraint Summary



Solar System Property	Implication	Fraction
Mass of Sun	$M \geq 1M$	0.12
Solar Metallicity	$Z \geq Z$	0.25
Single Star	(not binary)	0.30
Giant Planets	(successfully formed)	0.20
Ordered Planetary Orbits	$N \leq 10^4$	0.67
Supernova Enrichment	$N \geq 10^3$	0.50
Sedna-Producing Encounter	$10^3 \leq N \leq 10^4$	0.16
Sufficient Supernova Ejecta	$d \leq 0.3$ pc	0.14
Solar Nebula Survives Supernova	$d \geq 0.1$ pc	0.95
Supernova Ejecta and Survival	0.1 pc $\leq d \leq 0.3$ pc	0.09
FUV Radiation Affects Solar Nebula	$G_0 \geq 2000$	0.50
Solar Nebula Survives Radiation	$G_0 \leq 10^4$	0.80

$$P_* = \Gamma_M \Gamma_Z \Gamma_B \Gamma_P \Gamma_e \Gamma_{SN} \Gamma_{rad} \Gamma_{Sedna} \cdots$$

Alternative Scenarios for Nuclear Enrichment

[1] Internal enrichment -- X-wind models (Shu et al.)
[2] AGB stars -- low probability (Kastner, Myers)
[3] WR stars -- also low probability (need m > 60)
[4] Distributed enrichment (Gounelle)
[5] Supernova enrichment with varying progenitors

[a] Need some combination: Stellar source for 60Fe, spallation for 7Be and 10Be, both for 26Al…
[b] Sedna constraint almost same as SN constraint, so prediciton for solar birth aggregate unchanged

Conclusions:

[1] Initial cluster environment has moderate effect on disks and planets (less effect on star formation itself)

[2] Birth aggregate of Solar System was a moderately large cluster with stellar membership N = 4000 +/- 2000

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- *Modes of Multiple Star Formation, 2001, Adams & Myers, ApJ, 553, 744*
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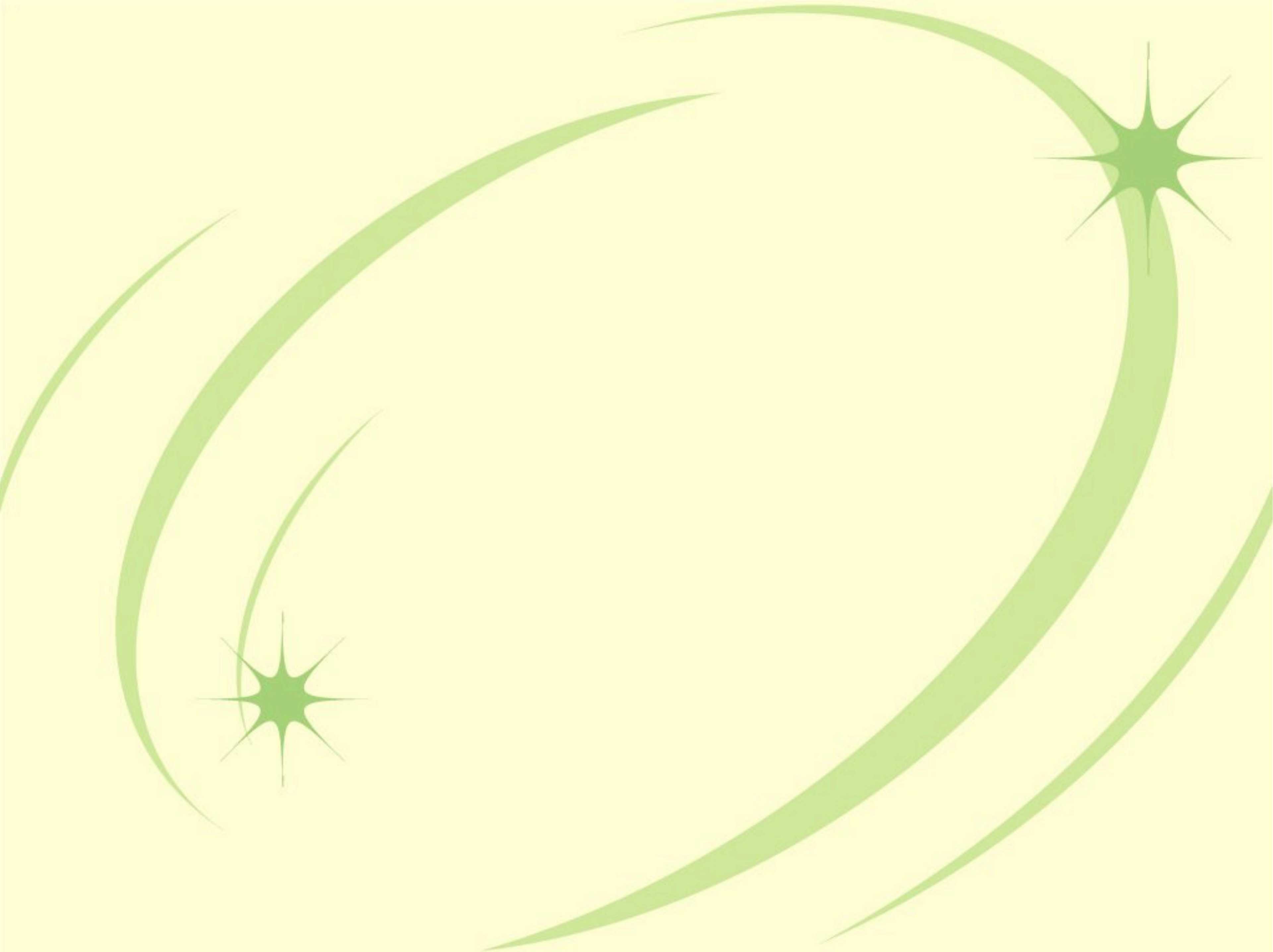
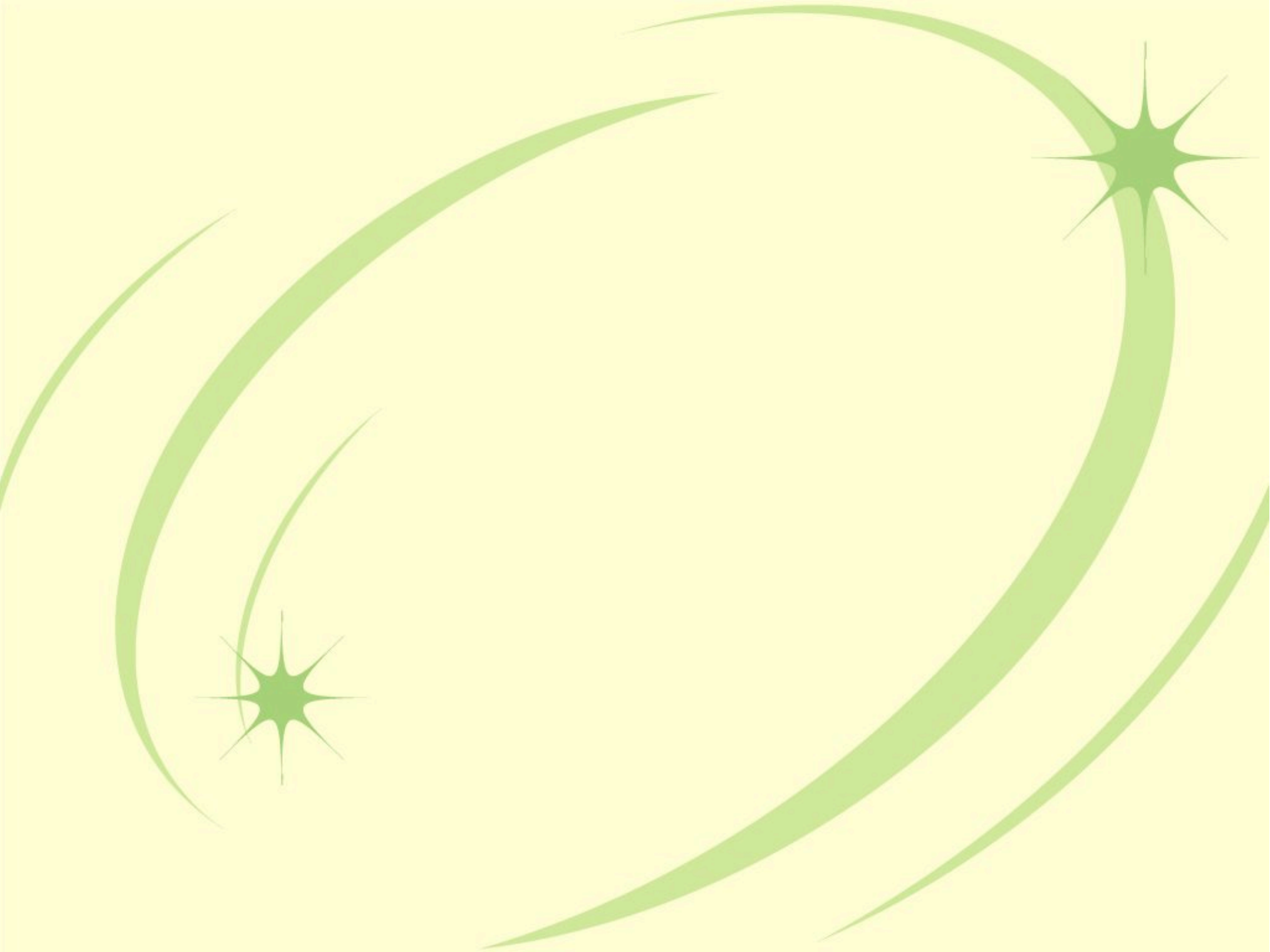
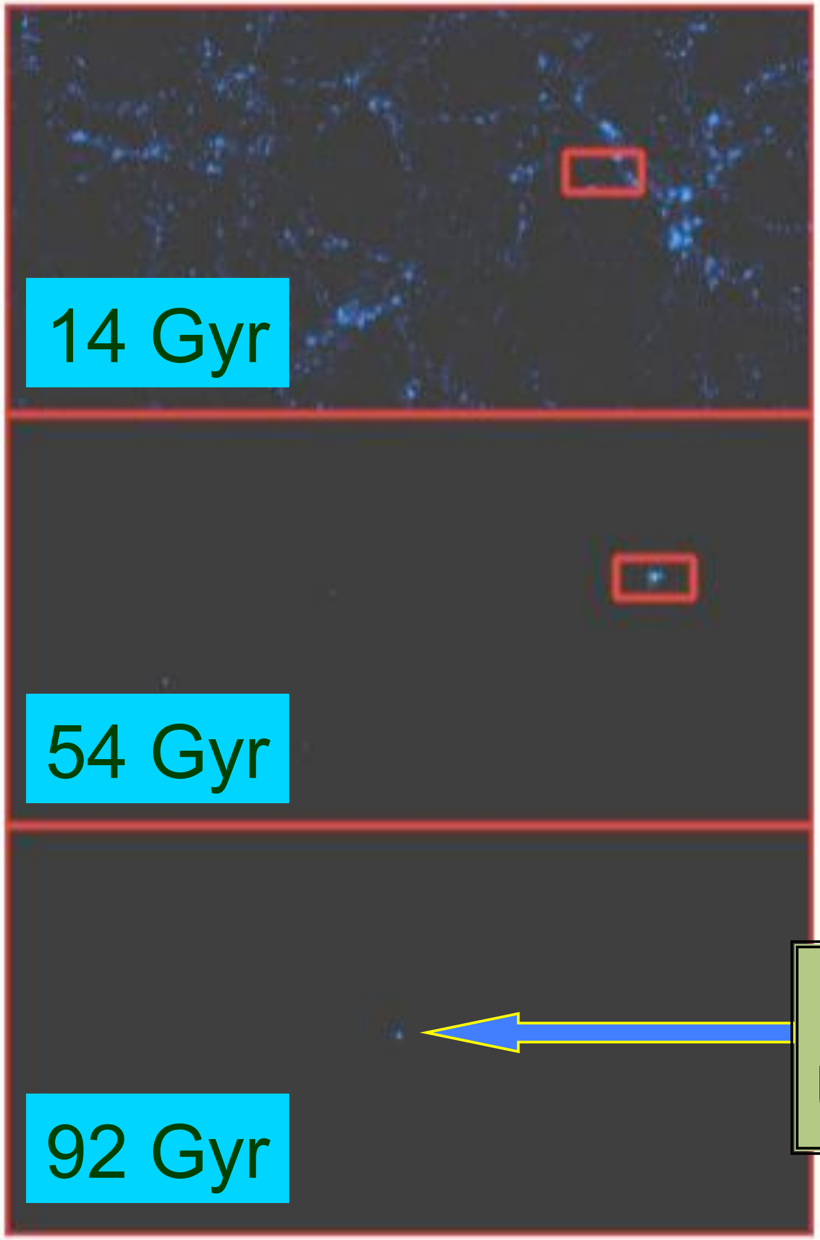




ORBITS:

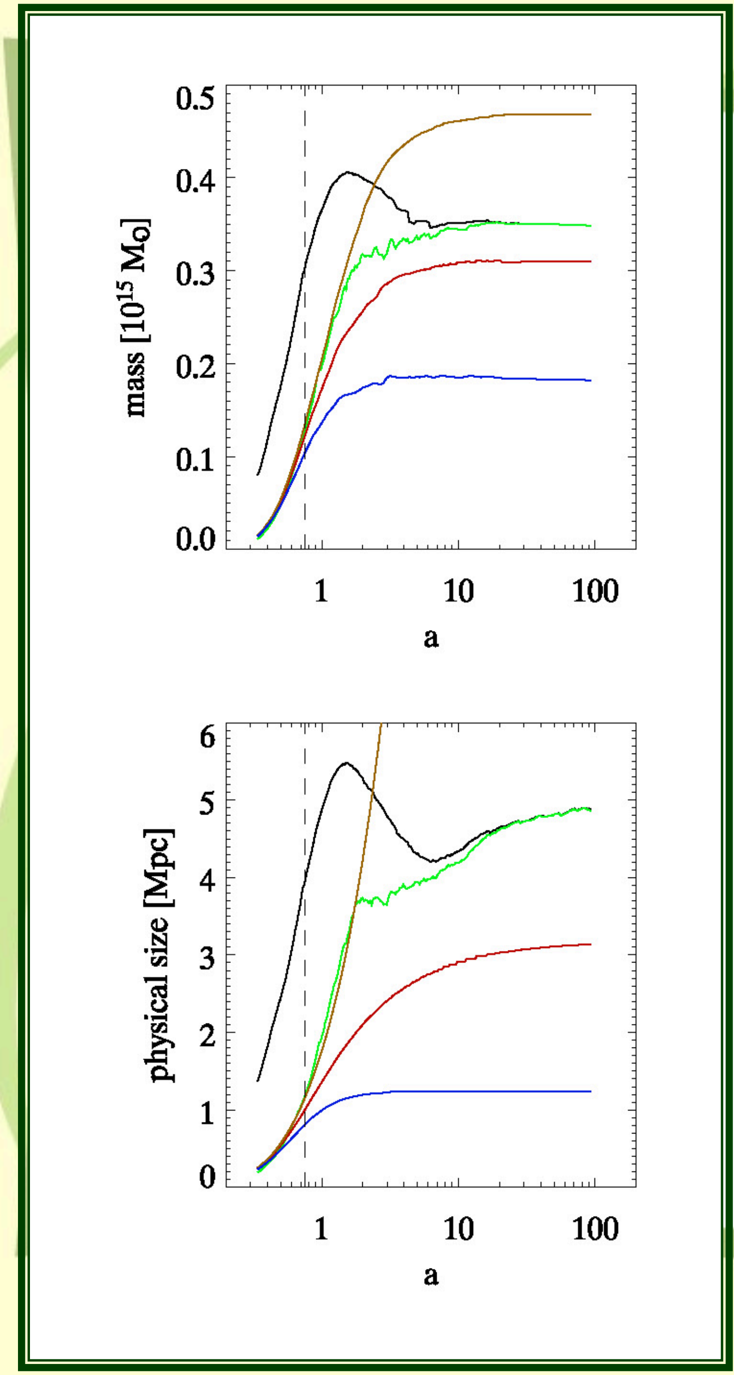
Rounding out Young Embedded Star Clusters, Future Structure of Dark Matter Halos, Unambiguous Definition of Galactic Masses, Orbital Instability in Triaxial Cusp Potentials, and Stochastic Hill's Equations

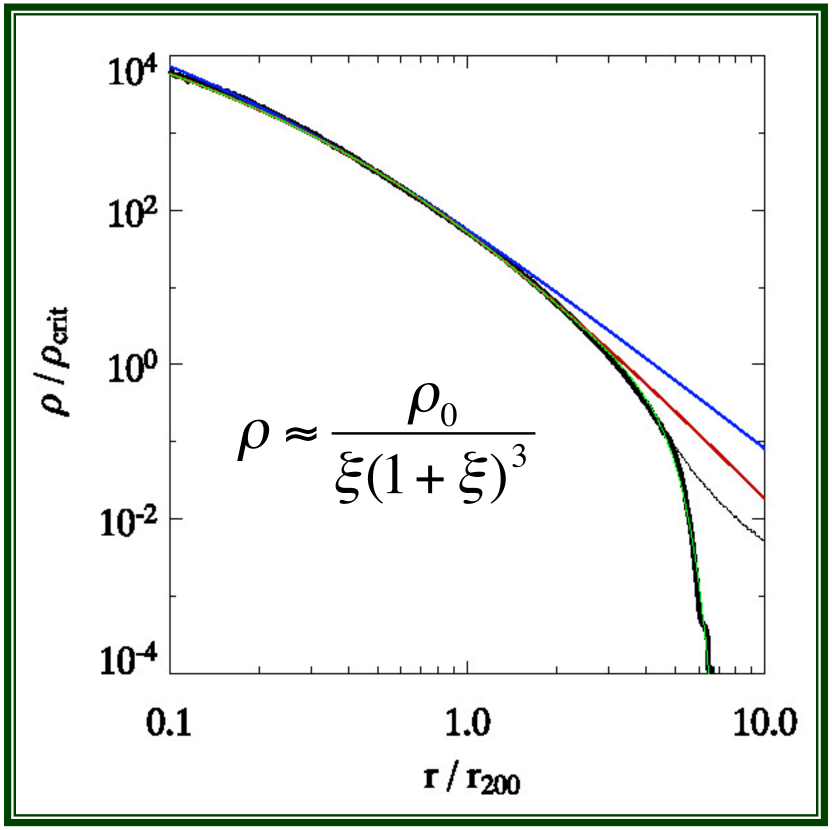


Island Universe

(M. Busha et al. 2003)

Dark matter halos approach a well-defined asymptotic form with unambiguous total mass, outer radius, & density profile





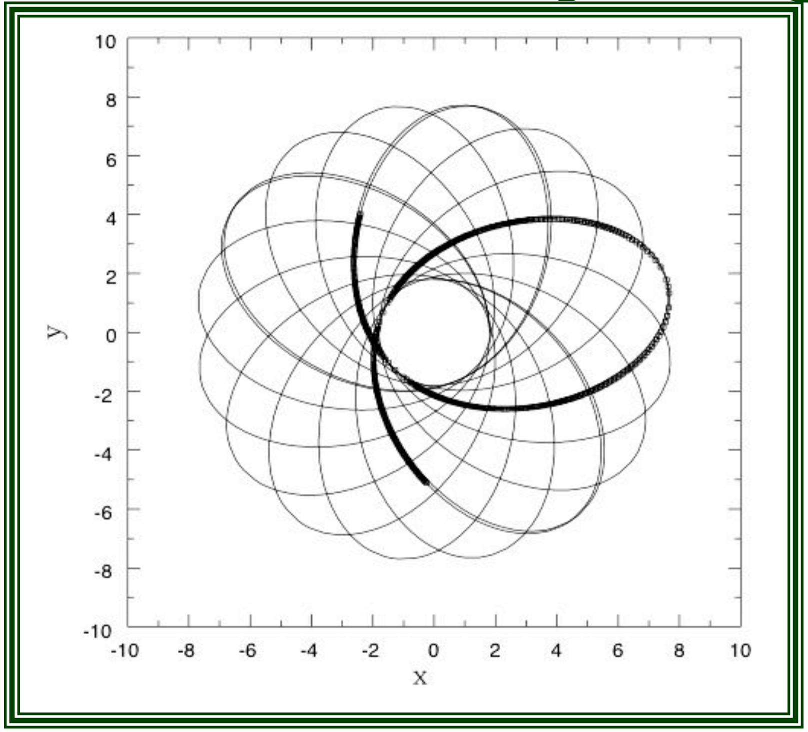
$$\rho \approx \frac{\rho_0}{\xi(1+\xi)^3}$$

(Busha et al. 2005)

WHY THESE ORBITS?

- Most of the mass is in dark matter
- Most dark matter resides in these halos
- Halos have the universal form found here (nfw/hq) for most of their lives
- Most orbital motion that will EVER occur will be THIS orbital motion $\left(factor \;\; of \;\; 10^{74} \right)$

Spherical Limit: Orbits look like Spirographs



Orbits in Spherical Potential

$$\rho = \frac{\rho_0}{\xi(1+\xi)^3} \Rightarrow \Psi = \frac{\Psi_0}{1+\xi}$$

$$\varepsilon \equiv |E|/\Psi_0 \quad and \quad q \equiv j^2/2\Psi_0 r_s^2$$

$$\varepsilon = \frac{\xi_1 + \xi_2 + \xi_1\xi_2}{(\xi_1+\xi_2)(1+\xi_1+\xi_2+\xi_1\xi_2)}$$

$$q = \frac{(\xi_1\xi_2)^2}{(\xi_1+\xi_2)(1+\xi_1+\xi_2+\xi_1\xi_2)}$$

$$q_{\max} = \frac{1}{8\varepsilon}\frac{(1+\sqrt{1+8\varepsilon}-4\varepsilon)^3}{(1+\sqrt{1+8\varepsilon})^2}$$

(angular momentum of the circular orbit)

$$\xi_* = \frac{1-4\varepsilon+\sqrt{1+8\varepsilon}}{4\varepsilon}$$

(effective semi-major axis)

$$\frac{\Delta\theta}{\pi} = \frac{1}{2} + \left[(1+8\varepsilon)^{-1/4} - \frac{1}{2}\right]\left[1 + \frac{\log(q/q_{\max})}{6\log 10}\right]^{3.6}$$

$$\lim_{q \to q_{\max}} \Delta\theta = \pi(1+8\varepsilon)^{-1/4}$$

(circular orbits do not close)

These results determine the radiation exposure of a star, averaged over its orbit, as a function of energy and angular momentum:

$$\langle F_{fuv} \rangle \approx \frac{L_{fuv}}{8 r_s^2 \sqrt{q}} \frac{A \varepsilon^{3/2}}{\cos^{-1}\sqrt{\varepsilon} + \sqrt{\varepsilon}\sqrt{1-\varepsilon}}$$

$$where \quad 1 \leq A(q) \leq \sqrt{2}$$

Triaxial Density Distributions

- Relevant density profiles include NFW and Hernquist

$$\rho_{nfw} = \frac{1}{m(1+m)^2} \qquad \rho_{Hern} = \frac{1}{m(1+m)^3}$$

- Isodensity surfaces in triaxial geometry

$$m^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \qquad a > b > c > 0$$

- In the inner limit both profiles scale as 1/r

$$m << 1 \quad \Longrightarrow \quad \rho \propto \frac{1}{m}$$

Triaxial Potential

$$\Phi = \int_0^\infty du \frac{\psi(m)}{\sqrt{(u+a^2)(u+b^2)(u+c^2)}} \qquad \psi(m) = \int_\infty^{m^2} \rho(m)\, dm^2$$

- In the inner limit the above integral can be simplified to

$$\Phi = -I_1 + I_2$$

where I_1 is the depth of the potential well and the effective potential is given by

$$\boxed{I_2 = 2\int_0^\infty du \frac{\sqrt{\xi^2 u^2 + \Lambda u + \Gamma}}{(u+a^2)(u+b^2)(u+c^2)}}$$

ξ, Λ, Γ are polynomial functions of x, y, z, a, b, c

Triaxial Forces

$$F_x = \frac{-2\,\mathrm{sgn}(x)}{\sqrt{\left(a^2-b^2\right)\left(a^2-c^2\right)}}\ln\left(\frac{2G(a)\sqrt{\Gamma}+2\Gamma-a^2\Lambda}{2a^2\xi G(a)+\Lambda a^2-2a^4\xi^2}\right)$$

$$F_y = \frac{-2\,\mathrm{sgn}(y)}{\sqrt{\left(a^2-b^2\right)\left(b^2-c^2\right)}}\left[\sin^{-1}\left(\frac{\Lambda-2b^2\xi^2}{\sqrt{\Lambda^2-4\Gamma\xi^2}}\right)-\sin^{-1}\left(\frac{2\Gamma/b^2-\Lambda}{\sqrt{\Lambda^2-4\xi^2\Gamma}}\right)\right]$$

$$F_z = \frac{-2\,\mathrm{sgn}(z)}{\sqrt{\left(a^2-c^2\right)\left(b^2-c^2\right)}}\ln\left(\frac{2G(c)\sqrt{\Gamma}+2\Gamma-c^2\Lambda}{2c^2\xi G(c)+\Lambda c^2-2c^4\xi^2}\right)$$

$$G(u)=\xi^2u^4-\Lambda u^2+\Gamma$$

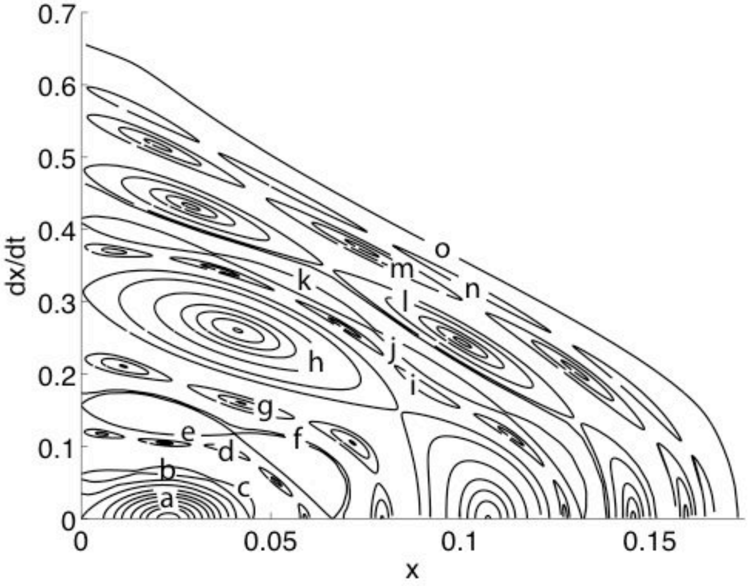
$$\xi^2=x^2+y^2+z^2$$

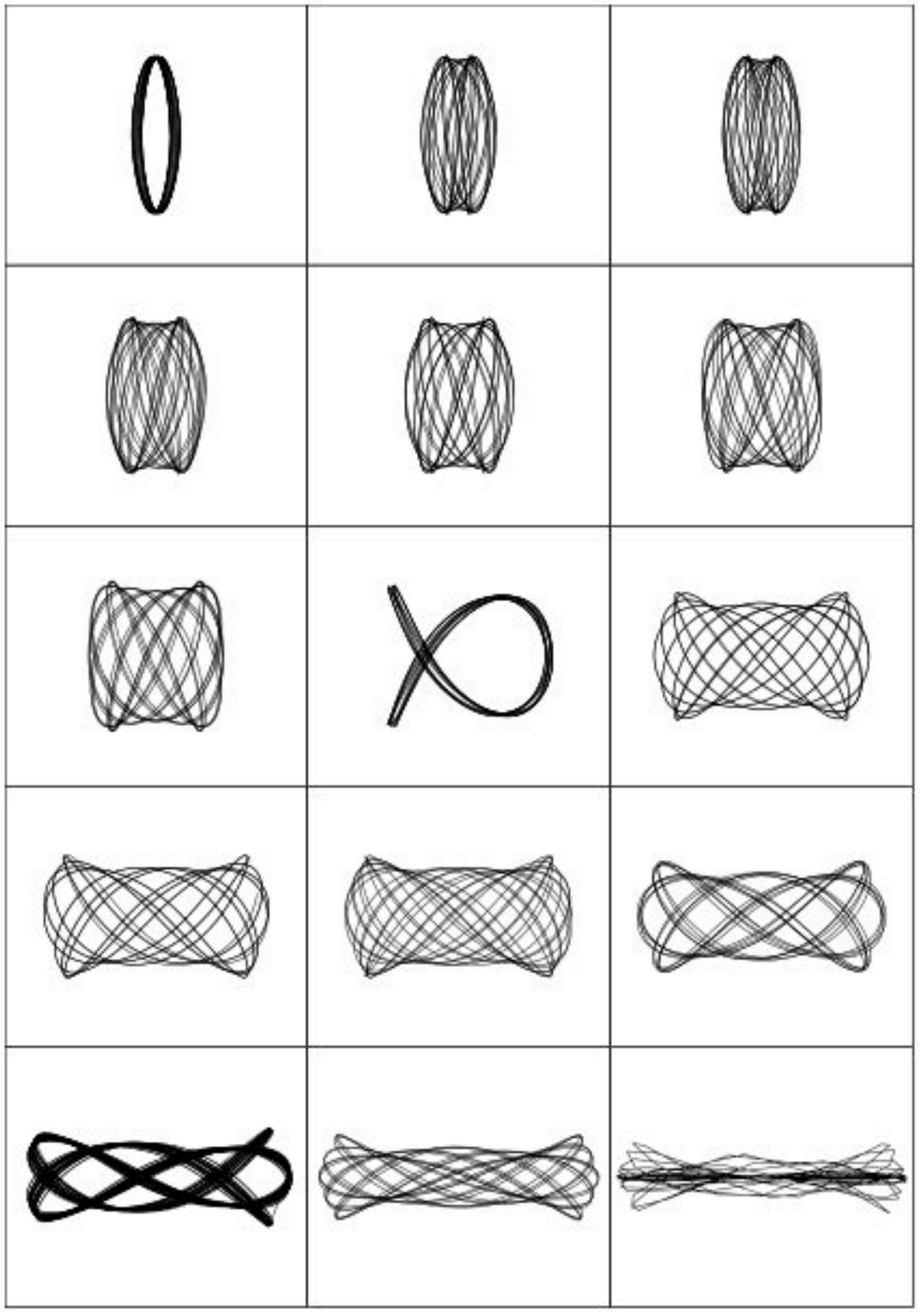
$$\Lambda=\left(b^2+c^2\right)x^2+\left(a^2+c^2\right)y^2+\left(a^2+b^2\right)z^2$$

$$\Gamma=b^2c^2x^2+a^2c^2y^2+a^2b^2z^2$$

(Adams, Bloch, Butler, Druce, Ketchum 2007)

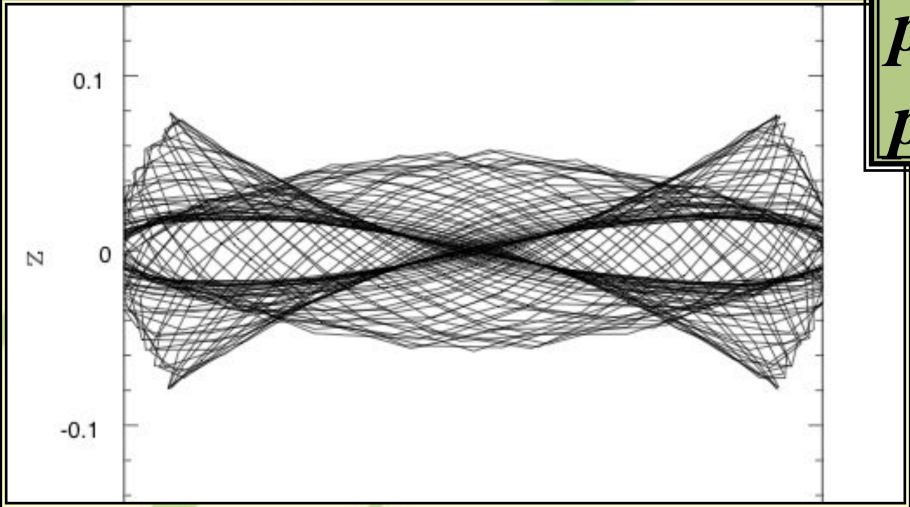
Orbit Gallery



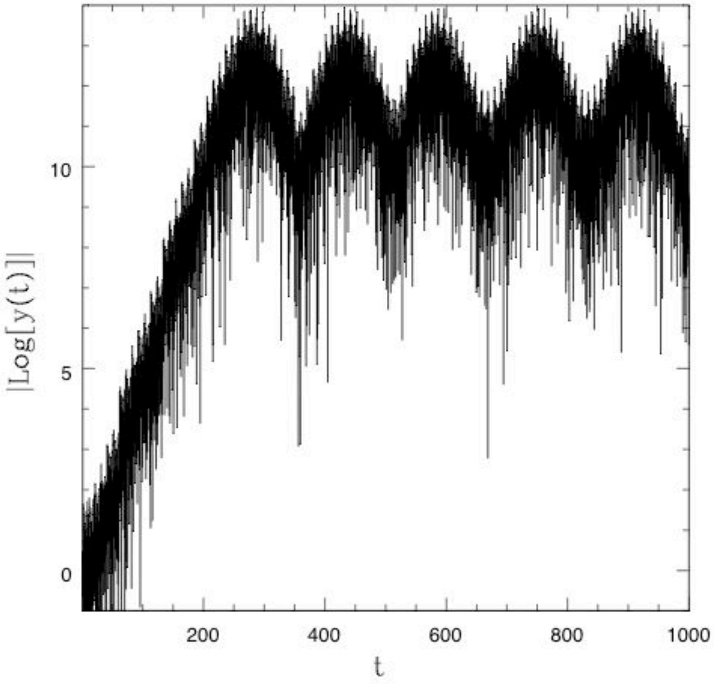


INSTABILITIES

Orbits in any of the principal planes are unstable to motion perpendicular to the plane.



Unstable motion shows:
(1) exponential growth,
(2) quasi-periodicity,
(3) chaotic variations, &
(4) eventual saturation.



Perpendicular Perturbations

- Force equations in limit of small x, y, or z become

$$F_x \approx -\left(\frac{4}{a\left(\sqrt{c^2y^2+b^2z^2}+a\sqrt{y^2+z^2}\right)}\right)x$$

$$F_y \approx -\left(\frac{4}{b\left(\sqrt{c^2x^2+a^2z^2}+b\sqrt{x^2+z^2}\right)}\right)y$$

$$F_z \approx -\left(\frac{4}{c\left(\sqrt{b^2x^2+a^2y^2}+c\sqrt{x^2+y^2}\right)}\right)z$$

⇒

$$F_x \approx -\omega_x^2 x$$

$$F_y \approx -\omega_y^2 y$$

$$F_z \approx -\omega_z^2 z$$

- Equations of motion perpendicular to plane have the form of Hill's equation
- Displacements perpendicular to the plane are unstable

Hill’s equation

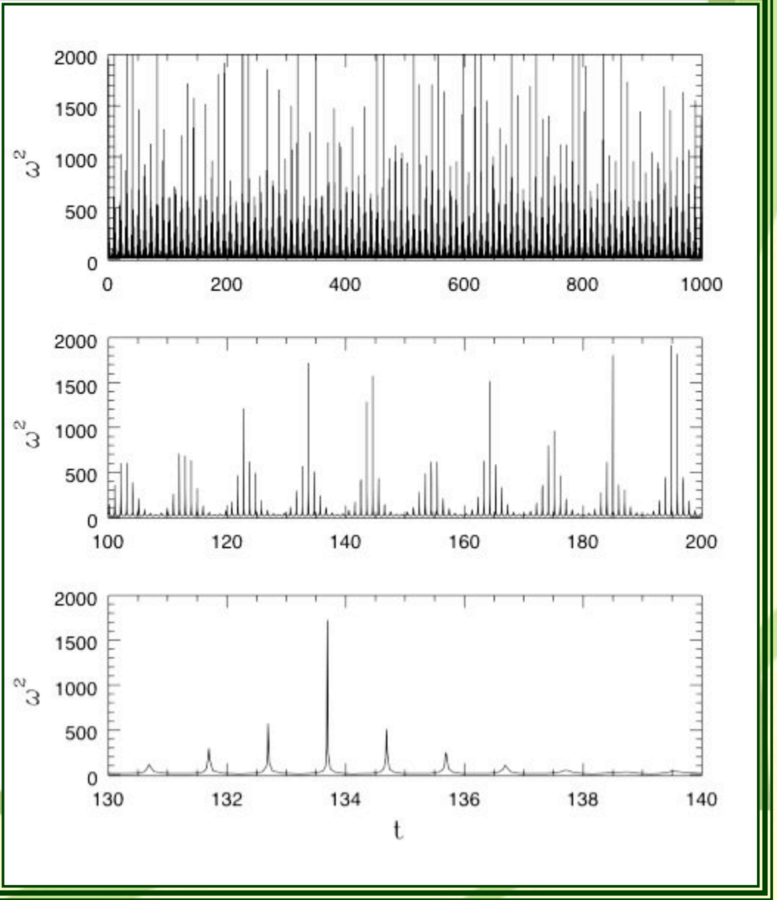
$$\frac{d^2y}{dt^2} + \frac{4/b}{\sqrt{c^2x^2 + a^2z^2} + b\sqrt{y^2 + z^2}}\,y = 0$$

$$\Downarrow$$

$$\frac{d^2y}{dt^2} + \left[\lambda_k + q_k Q(\mu_k t)\right]y = 0$$

$$\Downarrow$$

$$\frac{d^2y}{dt^2} + \omega^2(t)\,y = 0$$



Floquet's Theorem

For standard Hill's equations (including Mathieu equation) the condition for instability is given by Floquet's Theorem (e.g., Arfken & Weber 2005; Abramowitz & Stegun 1970):

$$|\Delta| \geq 2 \quad required \quad for \quad instability$$

$$where \quad \Delta \equiv y_1(\pi) + dy_2/dt(\pi)$$

Need analogous condition(s) for the case of stochastic Hill's equation…

CONSTRUCTION OF DISCRETE MAP

To match solutions from cycle to cycle, the coefficients are mapped via the 2x2 matrix:

$$\begin{bmatrix} \alpha_b \\ \beta_b \end{bmatrix} = \begin{bmatrix} h & (h^2-1)/g \\ g & h \end{bmatrix} \begin{bmatrix} \alpha_a \\ \beta_a \end{bmatrix}$$

where $h = y_1(\pi), \; g = dy_1/dt(\pi)$

and where $y_k(t) = \alpha_k y_{1k}(t) + \beta_k y_{2k}(t)$

The dynamics reduced to matrix products:

$$M^{(N)} = \prod_{k=1}^{N} M_k(q_k, \lambda_k)$$

GROWTH RATES

The growth rates for the matrix products can be broken down into two separate components, the asymptotic growth rate and the anomalous rate:

$$\gamma_\infty = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \gamma(q_k, \lambda_k) \to \langle \gamma \rangle$$

[where individual growth rates given by Floquet's Theorem]
Next: take the limit of large q, i.e., unstable limit: $h >> 1$

$$\Delta\gamma = \lim_{N\to\infty} \frac{1}{\pi N} \sum_{k=1}^{N} \ln(1 + x_{k1} / x_{k2}) - \frac{\ln 2}{\pi}$$

$$where \;\; x_k \equiv h_k / g_k$$

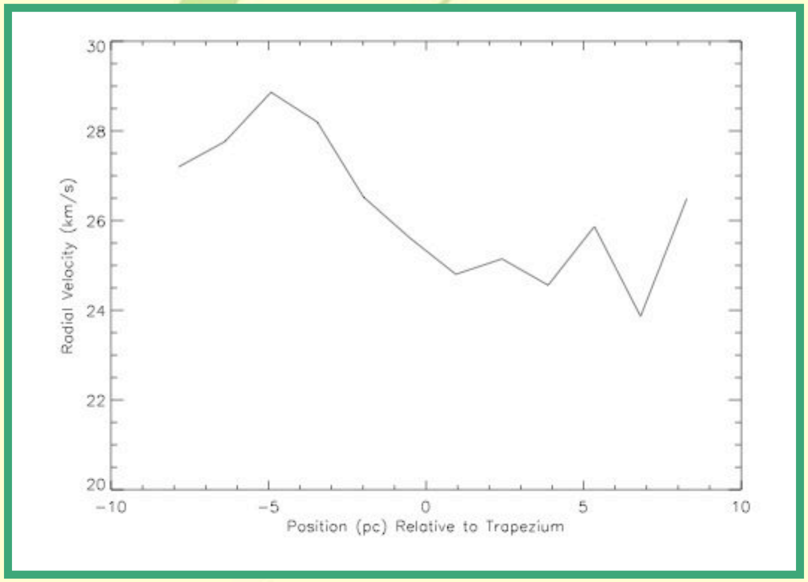
Astrophysical Applications

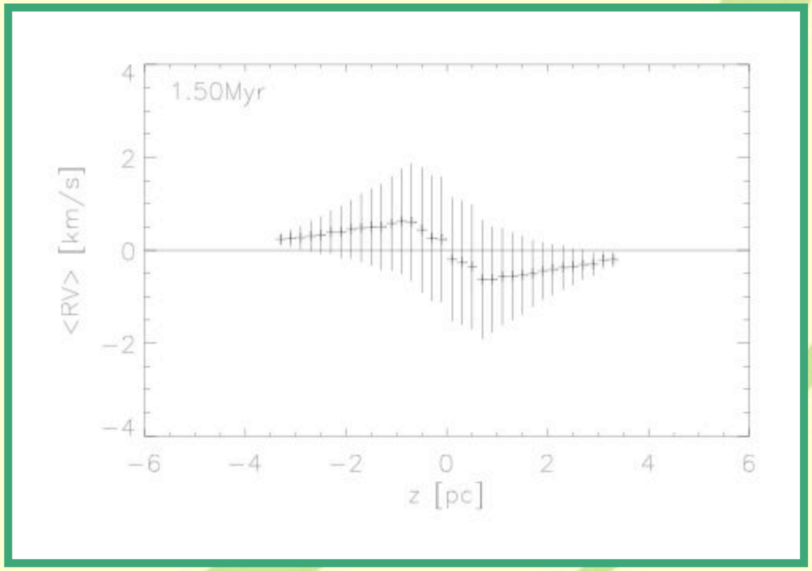
- ***Dark Matter Halos***: Radial orbits are unstable to perpendicular perturbations and will develop more isotropic velocity distributions.
- ***Tidal Streams***: Instability will act to disperse streams; alternately, long-lived tidal streams place limits on the triaxiality of the galactic mass distribution.
- ***Galactic Bulges***: Instability will affect orbits in the central regions and affect stellar interactions with the central black hole.
- ***Young Stellar Clusters***: Systems are born irregular and become rounder: Instability dominates over stellar scattering as mechanism to reshape cluster.

New Cluster Result

Kinematic observations of the Orion Nebula Cluster show that the system must have:

- Non-spherical geometry
- Non-virial initial conditions
- Viewing angle not along a principal axis





(with E. Proszkow, J. Tobin, and L. Hartmann, 2009)