Dynamical Tides in Exotic Stellar Binaries

Dong Lai

Cornell & KITP

2/4/2010 KITP

Three Problems:

- Neutron Star Binaries (NS/NS or NS/BH)
- White Dwarf Binaries (WD/WD, WD/NS, etc)
- An eccentric pulsar/massive MS Binary

Coalescing NS Binaries: The last three minutes

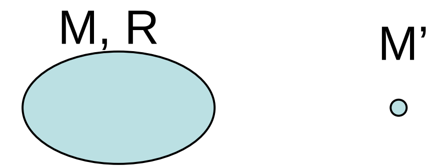
$$a \sim (70 - 2)R$$

$$f_{\rm orb} \sim (10 - 10^3)\ \mathrm{Hz}$$

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

- Most promising sources of GWs for LIGO/VIRGO
- To detect GWs, need accurate wave forms (orbital phase evolution).

Equilibrum Tide



$$\text{Distortion} \quad \epsilon \sim \frac{M'}{M}\left(\frac{R}{a}\right)^3$$

$$\text{Quadrupole} \quad k_2 M R^2 \epsilon$$

k_2=Apsidal motion const (Love number)

$$\text{Orbital Phase Shift} \quad d\Phi = d\Phi_0\left[1 - \mathcal{O}\left(\frac{1}{a^5}\right)\right]$$

==> Equilibrium tide important only at small separation (just prior to merger)

(understood in 1990s: Bildsten & Cutler, Kochenek, DL, Rasio & Shaipro , etc)

- Numerical GR Quasi-equilibrium NS binary sequence

(Baumgarte, Shapiro, Teukolsky, Shibata, Meudon group, etc. 1990s--200x)

- Recent (semi-analytic) GR calculation of tidal effect

(Hinderer, Flanagan, Poisson, Damour etc 2008-…)

Dynamical Tides: Resonant Excitation of NS Modes

occurs when $\omega_\alpha = m\Omega_{\rm orb}$

$$\xi(\mathbf{r},t) = \sum_{\alpha} c_\alpha(t)\,\xi_\alpha(\mathbf{r})$$

$$\ddot{c}_\alpha + \omega_\alpha^2 c_\alpha = \langle \xi_\alpha, \nabla U_{\rm tide}\rangle \propto \frac{M' Q_\alpha}{a^{l+1}}\, e^{-im\Omega_{\rm orb} t}$$

Overlap integral $\quad Q_\alpha = \int d^3x\, \delta\rho_\alpha^*\, (r^l Y_{lm})$

Energy transfer in a resonance $\quad \Delta E_\alpha \propto Q_\alpha^2 \left(\frac{1}{a_\alpha}\right)^{2(l+1)} t_{\rm GW}$

Orbital phase shift $\quad \Delta\Phi = -\Omega_{\rm orb}\, t_{\rm GW}\, \frac{\Delta E_\alpha}{|E_{\rm orb}|}$

Summary:
Resonant Excitations of NS Modes During Binary Inspiral

Non-rotating NS:

G-mode (Reisenegger & Goldreich 1994; DL 1994)

Rotating NS:

G-mode, F-mode, R-mode (Ho & DL 1999)
Inertial modes (DL & Wu 2006)
R-mode (excited by gravitomagnetic force; Racine & Flanagan 2006)

Results:

- For R=10 km, slow-spin NSs, the number of missing cycles < 0.1, not measurable
- Number of missing cycles $\Delta N \propto R^4$ (g mode) or $R^{3.5}$ (r mode)
  Important for larger (e.g. 15 km) NS
- For rapidly spinning NSs (> a few 100 Hz), many resonant channels lead to large GW phase error

Compact White Dwarf Binaries (WD/WD, WD/NS, WD/BH)

Ongoing work with Jim Fuller (Cornell)

$$P_{\rm orb} \sim {\rm min} - {\rm hour}$$

$$t_{\rm GW} \simeq 2000 \left(\frac{P_{\rm orb}}{\rm min}\right)^{8/3} {\rm yr}$$

- Possible SN Ia progenitors
- Sources for LISA (EM counterparts…)

Resonant tidal excitation of g-modes in WD

(assuming non-rotating WD for now…)

occurs when $\omega_\alpha = m\Omega_{\rm orb}$

$$\xi(\mathbf{r},t) = \sum_\alpha c_\alpha(t)\,\xi_\alpha(\mathbf{r})$$

$$\ddot{c}_\alpha + \omega_\alpha^2 c_\alpha = \langle \xi_\alpha, \nabla U_{\rm tide}\rangle \propto \frac{M' Q_\alpha}{a^{l+1}}\, e^{-im\Omega_{\rm orb} t}$$

Overlap integral $\quad Q_\alpha = \int d^3x\ \delta\rho_\alpha^*\ (r^l Y_{lm})$

Orbital equations that include

1. Back-reaction of excited modes on orbit
2. Gravitational radiation reaction

QuickTime™ and a
decompressor
are needed to see this picture.

$$M = 0.7 M_\odot,\ R = 8000\,\mathrm{km},\ T_{\mathrm{eff}} = 8700\ \mathrm{K}$$

$$n = 4, \quad P_{\mathrm{mode}} = 2.8\,\mathrm{min}$$

$$t_{\mathrm{res}} \sim \mathrm{hours} \ll t_{\mathrm{damp}}$$

Semi-analytic result:

QuickTime™ and a
decompressor
are needed to see this picture.

QuickTime™ and a
decompressor
are needed to see this picture.

===>

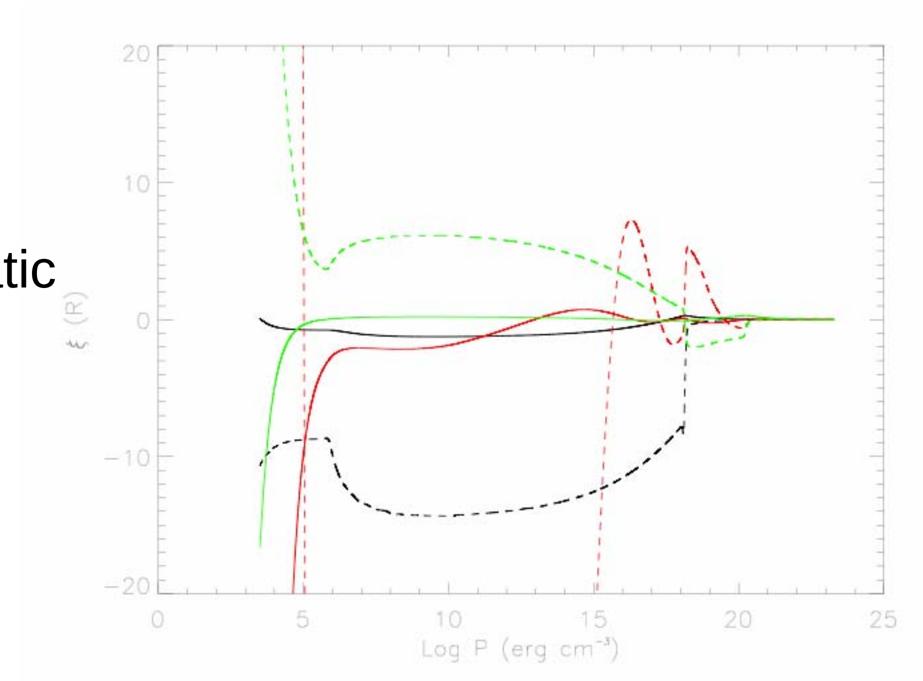
Resonant mode amplitude up to ~8%

Resonant mode energy up to $6 \times 10^{-4} \frac{GM^2}{R} \sim 10^{47}$ erg

Bad News:

Displacement ~ R in envelope
==> Linear calculation is problematic

Nonlinear damping important



QuickTime™ and a decompressor are needed to see this picture.

QuickTime™ and a decompressor are needed to see this picture.

Tentative Summary:
Dynamical Tides in Compact WD Binaries

- Long before merger, may transfer a lot of energy to internal modes/waves (nonlinear), making WD bright.
  More works to be done…

- Main venue for angular momentum transfer (synchronization)
- Relevant to GW detection by LISA, SN Ia progenitors

Dynamical Tides in an Eccentric Pulsar/MS Binary

DL96,97; cf. Kumar & Quataert 97,98

Relevant facts:

PSR J0045-7319

B star companion: $M = 9M_\odot$ (spinning, S & L misaligned)

Orbit: $P_{\rm orb} = 51$ days, $e = 0.8$, $a = 20R$, $a_p = 4R$

Key observation:

orbital decay $P_{\rm orb}/\dot{P}_{\rm orb} = -0.5$ Myr

Mechanism of Orbital Decay

(Equilibrium tide does not work, regardless of tidal Q…)

Dynamic tide:

At each periastron passage, energy (and J) is transferred to internal modes (g-modes), which is (partially) damped ==> orbital decay

Dominant modes are those with $\omega_\alpha \sim 2\Omega_{\rm peri}$

Energy in modes

Orbital energy

QuickTime™ and a
decompressor
are needed to see this picture.

Steady state:

$$\langle E_{\rm mode} \rangle = \sum_\alpha \langle E_\alpha \rangle$$

$$\langle E_\alpha \rangle \sim \Delta E_\alpha$$ typically

Secular orbital evolution:

$$\frac{dE_{\rm orb}}{dt} = - \sum_\alpha \frac{\langle E_\alpha \rangle}{t_{{\rm damp},\alpha}}$$

$$\frac{dJ_{\rm orb}}{dt} = \cdots$$

Energy in modes

Steady state:

$$\langle E_{\rm mode}\rangle = \sum_\alpha \langle E_\alpha\rangle$$

$$\langle E_\alpha\rangle \sim \Delta E_\alpha$$ typically

Orbital energy

QuickTime™ and a
decompressor
are needed to see this picture.

Secular orbital evolution:

$$\frac{dE_{\rm orb}}{dt} = -\sum_\alpha \frac{\langle E_\alpha\rangle}{t_{{\rm damp},\alpha}}$$

$$\frac{dJ_{\rm orb}}{dt} = \cdots$$

Mean motion resonance:

$$\langle \Delta E_\alpha\rangle = \frac{\Delta E_\alpha}{4\sin^2(\pi\omega_\alpha/\Omega_{\rm orb}) + (P_{\rm orb}/2t_{{\rm damp},\alpha})^2}$$

$$\gg \Delta E_\alpha$$

when $\omega_\alpha \simeq N\Omega_{\rm orb}$

Spin Effects

J transfer

QuickTime™ and a
decompressor
are needed to see this picture.

Energy transfer

- Retrograde rotation increases energy transfer to modes
  ==> faster orbital decay

- Pseudo-synchronization

$$\frac{dJ_s}{dt} = 0 \quad \text{at} \quad \Omega_s = 1.7\Omega_{\rm peri}$$

(for e=0.8)

By contrast, equilibrium tidal eqn gives

$$\frac{dJ_s}{dt} = 1 - f(e)\frac{\Omega_s}{\Omega_{\rm peri}} = 0 \quad \text{for} \quad \Omega_s = 1.02\Omega_{\rm peri}$$

- All these will be complicated by differential rotation