Mechanisms of tidal dissipation in planet-star interactions

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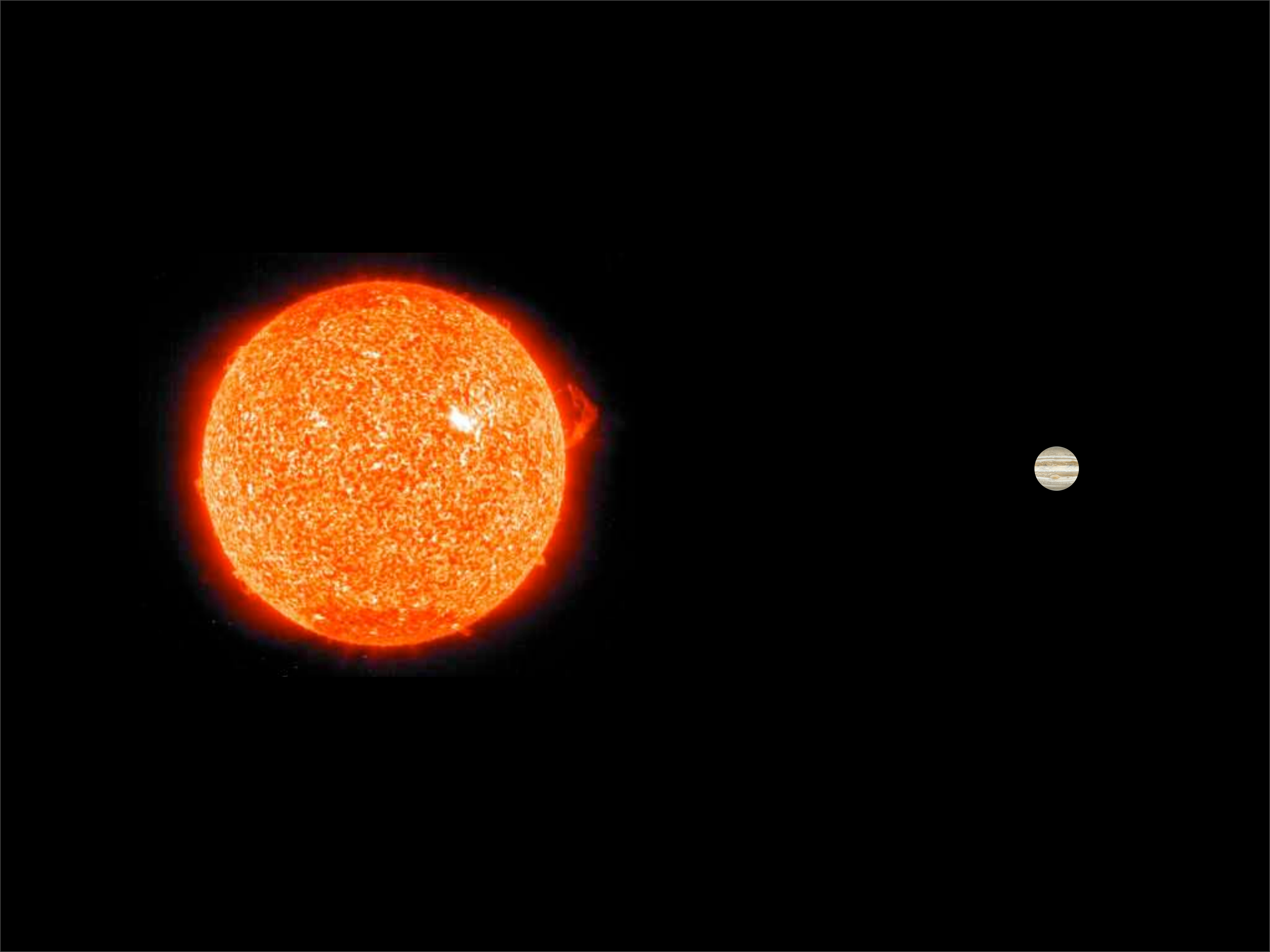
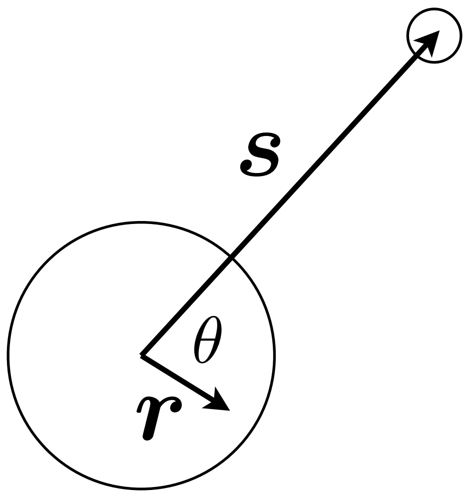
02.02.10



GENERAL TIDAL PROBLEM

Two bodies in (nearly) Keplerian orbit



$$\frac{1}{|\boldsymbol{r}-\boldsymbol{s}|}=\sum_{n=0}^{\infty}|\boldsymbol{s}|^{-n-1}r^{n}P_{n}(\cos\theta)$$

Deformation from spherical shape causes departure from Keplerian motion :

- precession (non-dissipative)
- spin-orbit evolution (dissipative)

Two important regimes :

- tidal encounter (hyperbolic / highly eccentric)
- periodic tide (small eccentricity / short period)

TIDAL COMPONENTS

Quadrupolar tide, to lowest order in e and i :

$$\Psi = A\,\frac{GM_2}{a^3} r^2 \tilde{P}_2^m(\cos\theta)\cos(m\phi - \omega t)$$

m	ω	$\hat{\omega}$	A	
0	0	0	0.316	(static)
2	$2n$	$2(n-\Omega)$	-0.775	spin / orbit
0	n	n	$0.949\,e$	circularization
2	n	$n-2\Omega$	$0.387\,e$	circularization
2	$3n$	$3n-2\Omega$	$-2.711\,e$	circularization
1	0	$-\Omega$	$0.775\,i$	alignment
1	$2n$	$2n-\Omega$	$0.775\,i$	alignment

Q-PARAMETRIZATION

Energy dissipation rate :

$$D = \frac{15}{8Q'} \frac{GM_2^2 R_1^5}{a^6} A^2 |\hat{\omega}|$$

Tidal torque :

$$|T| = \frac{15}{8Q'} \frac{GM_2^2 R_1^5}{a^6} A^2 m$$

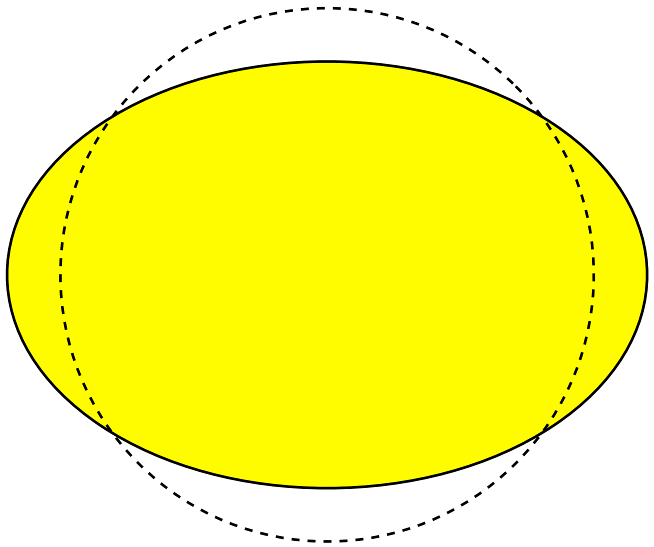
$Q' = Q$ **(quality factor) for a homogeneous fluid body**

$$Q' = Q'(\hat{\omega}, m)$$

TIDAL RESPONSE

Basic response (equilibrium tidal bulge) :

- spheroidal displacement, $\xi_r = -(\Psi + \Phi')/g, \ \boldsymbol{\nabla} \cdot \boldsymbol{\xi} = 0$
- modified in solid regions (rigidity)
- modified in convective regions
- not exact solution because $\hat{\omega} \neq 0$



Dynamical tide :

- additional (wavelike) response
- low-frequency internal waves :
  - inertial waves (convective regions)
  - inertia-gravity waves (radiative regions)

DISSIPATION MECHANISMS

Equilibrium tide :

- solid regions (viscoelastic, etc.)
  Dermott 1979
- convective regions (turbulent "viscosity")
  Zahn 1966, Goldreich & Nicholson 1977, Goodman & Oh 1997, Penev et al. 2009
- other physics (e.g. helium)
  Stevenson 1980, 1983
- nonlinear breakdown, e.g. elliptical instability
  e.g. le Bars et al. 2010

Dynamical tide :

- inertia-gravity waves in radiative regions (resonances, radiative damping, wave breaking)
  Zahn, Goldreich, Savonije, Papaloizou, Goodman, Terquem, Lubow, Witte, Barker, Ogilvie
- inertial waves in convective regions (attractors, critical latitudes, resonances)
  Ogilvie & Lin, Wu, Ivanov & Papaloizou, Goodman & Lackner, Rieutord & Valdettaro

FREQUENCY DEPENDENCE

Equilibrium tide :

- probably smooth dependence

Dynamical tide :

- frequency ranges for different wave types
- resonant peaks (coherent linear modes)
- smooth dependence (damped waves)
- complicated (attractors, etc.)

INFORMATION NEEDED

Planetary structure :

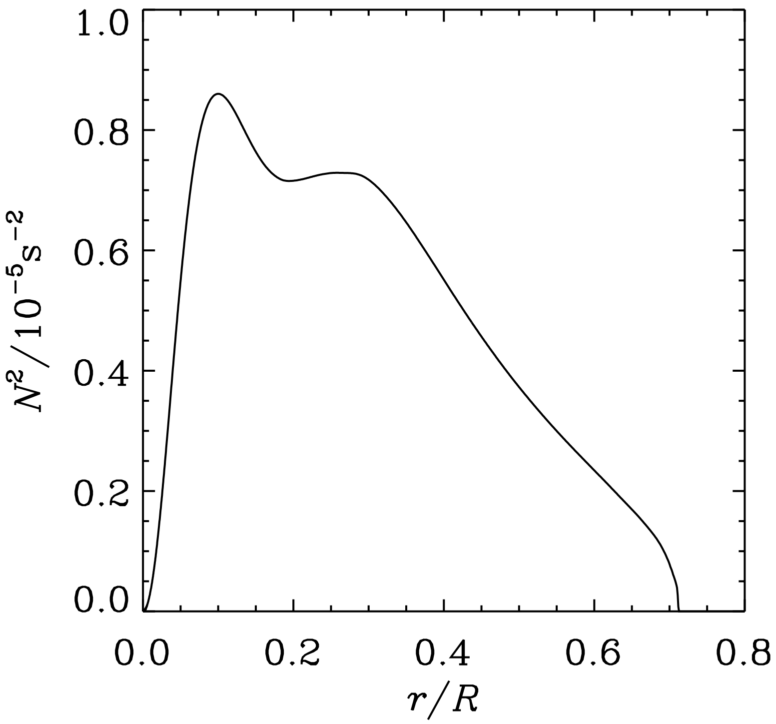
- extent and properties of dense cores
- properties of convective regions
- interface to radiative layers
- density jumps, phase transitions, thermodynamics

Stellar structure :

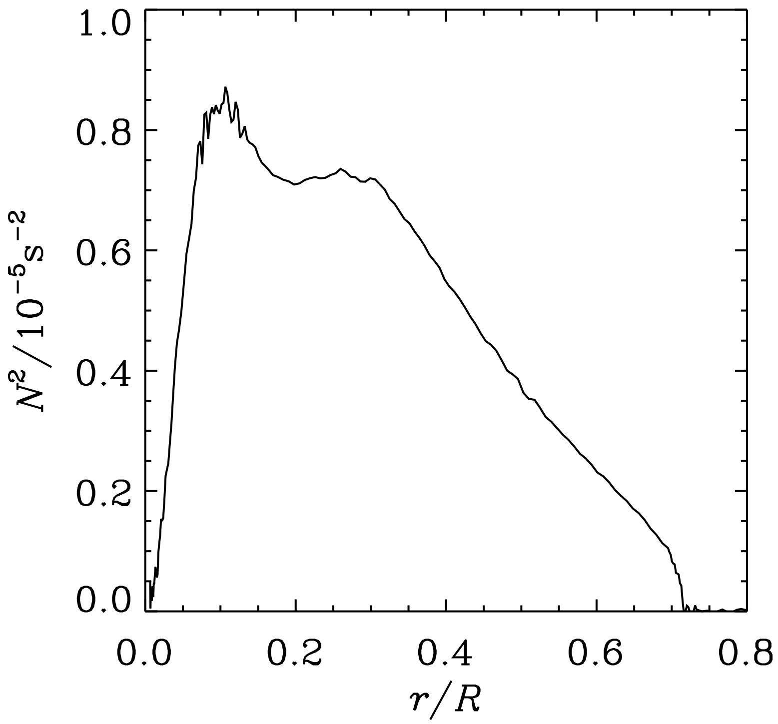
- properties of convective regions
- stratification near centre (solar-type)

STELLAR MODELS

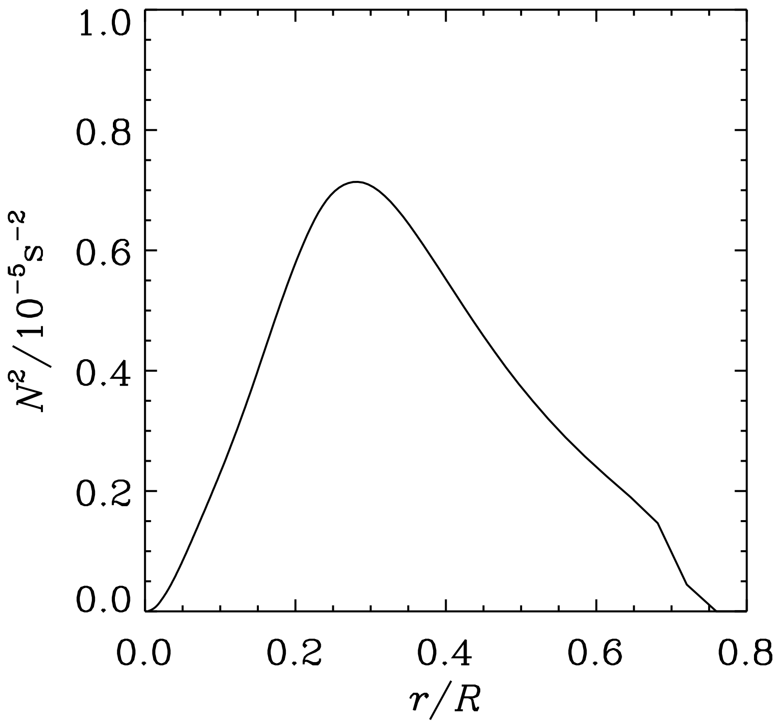
e.g. Sun, current age :



Christensen-
Dalsgaard



Bahcall

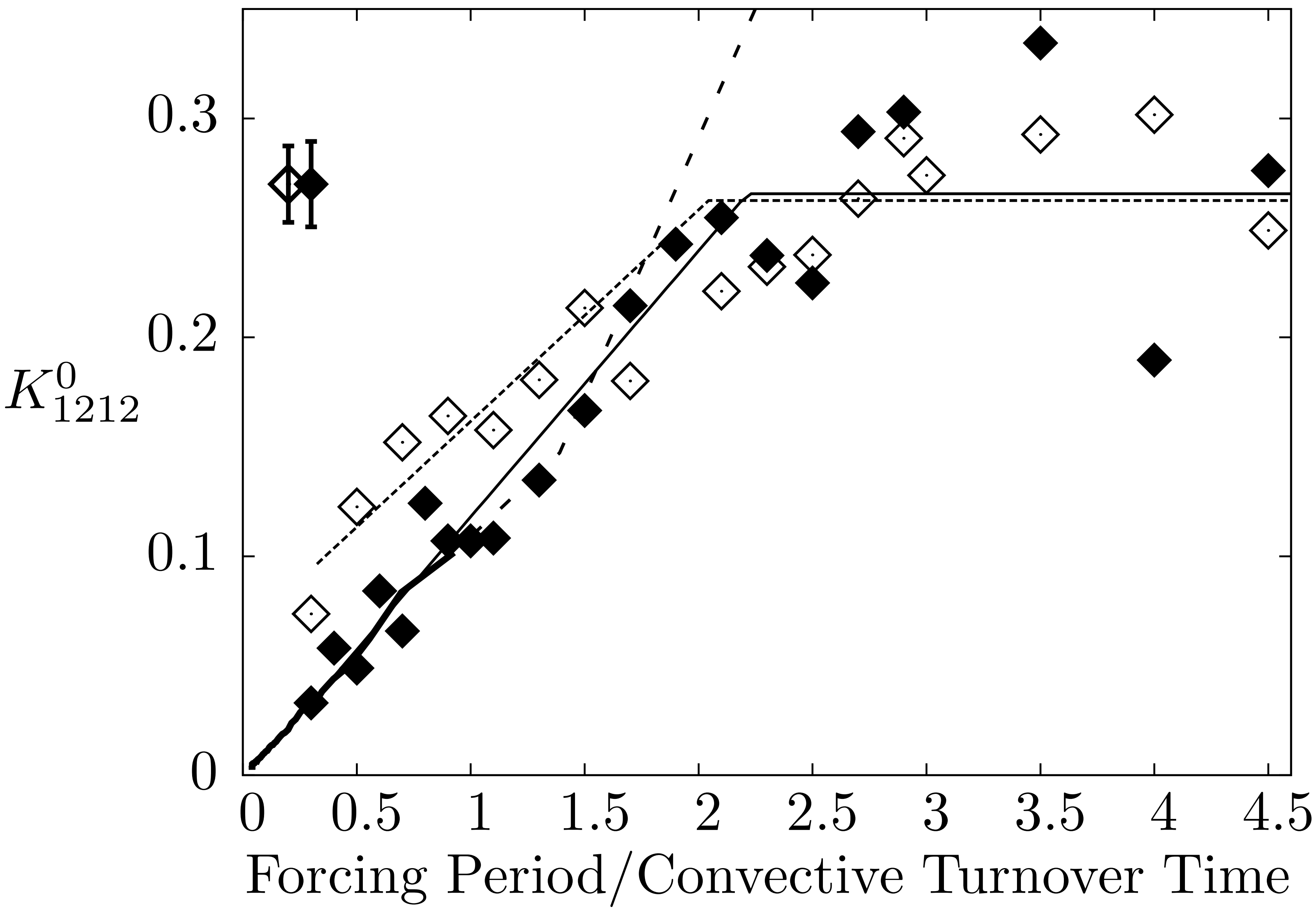


(Eggleton)

SOME RECENT ADVANCES

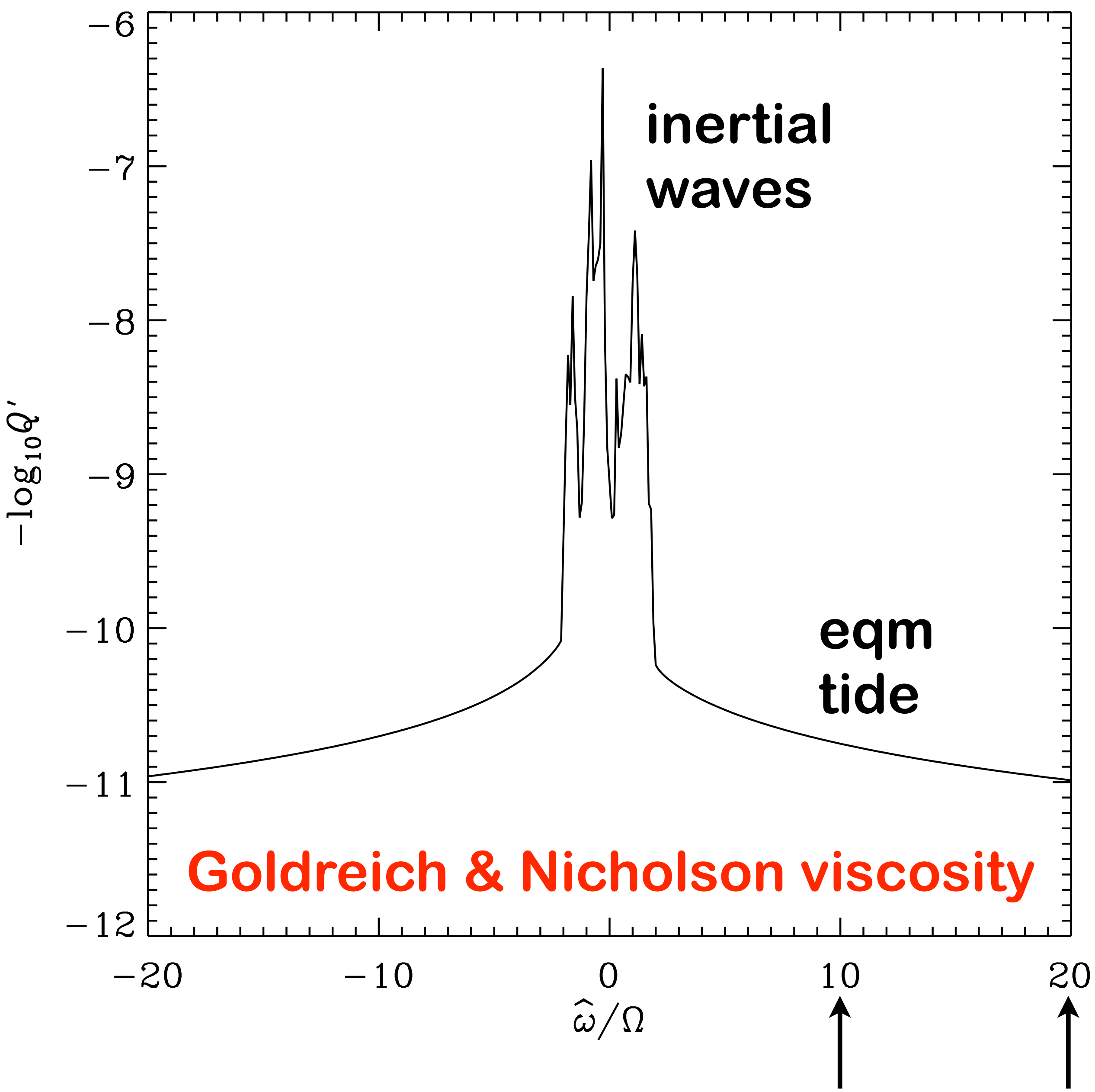
CONVECTIVE VISCOSITY

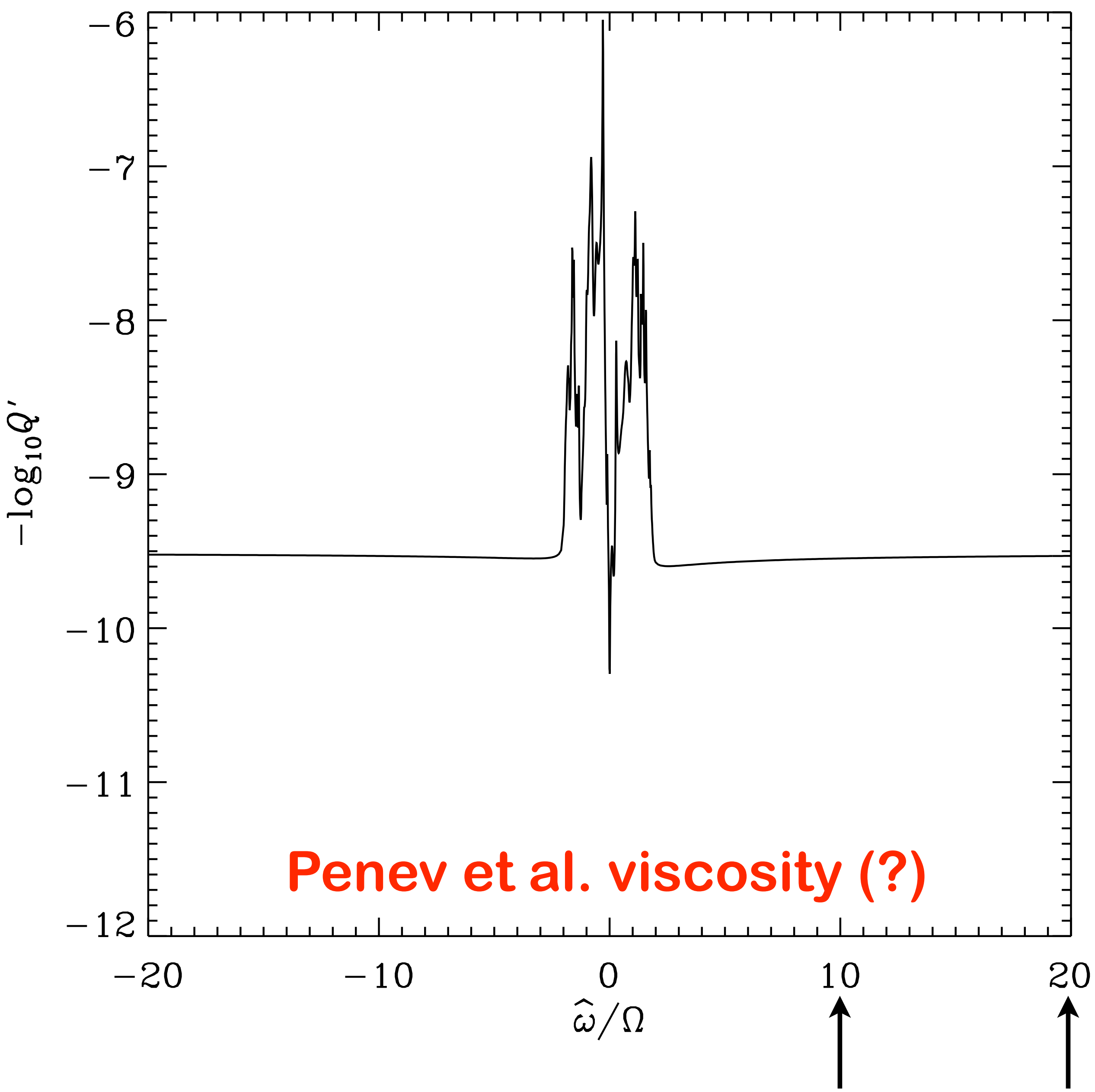
Penev et al. 2009



CONVECTIVE VISCOSITY

Application to solar CZ (but with 10-day spin period)





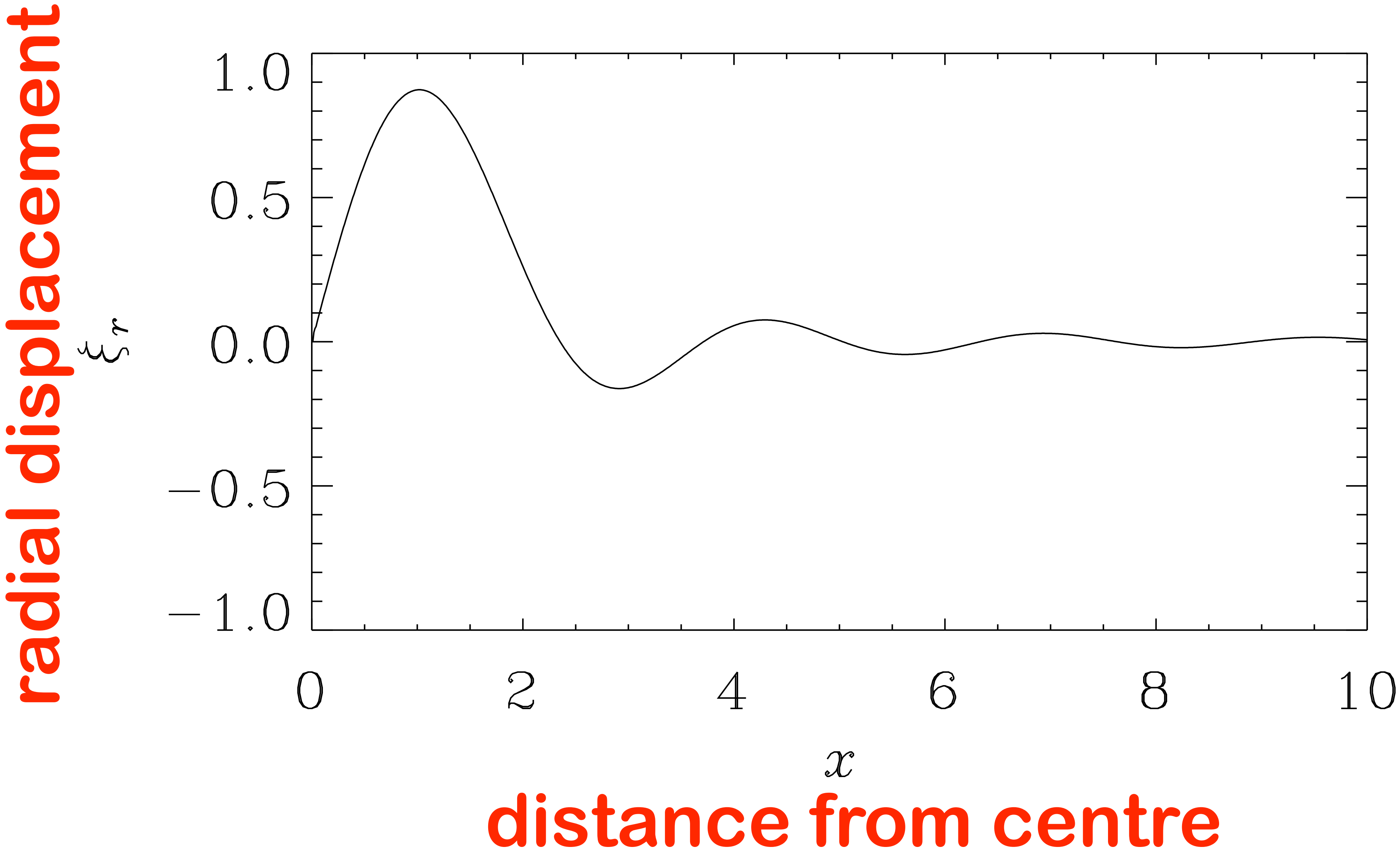
tidal period 1 day ½ day

tidal period 1 day ½ day

BREAKING GRAVITY WAVES

Barker & Ogilvie (2010)

cf. Goodman & Dickson (1998)



typical wavelength 0.001-0.01 R_{Sun}

BREAKING GRAVITY WAVES

Waves overturn and break if

$$\frac{M_\mathrm{p}}{M_\mathrm{J}} > 3.3 \left(\frac{P}{\mathrm{day}}\right)^{-1/6}$$

...or more easily in older or more massive stars

When this occurs,

$$Q'_* \approx 1.5 \times 10^5 \left(\frac{P}{\mathrm{day}}\right)^{8/3}$$

and the planet is swallowed within

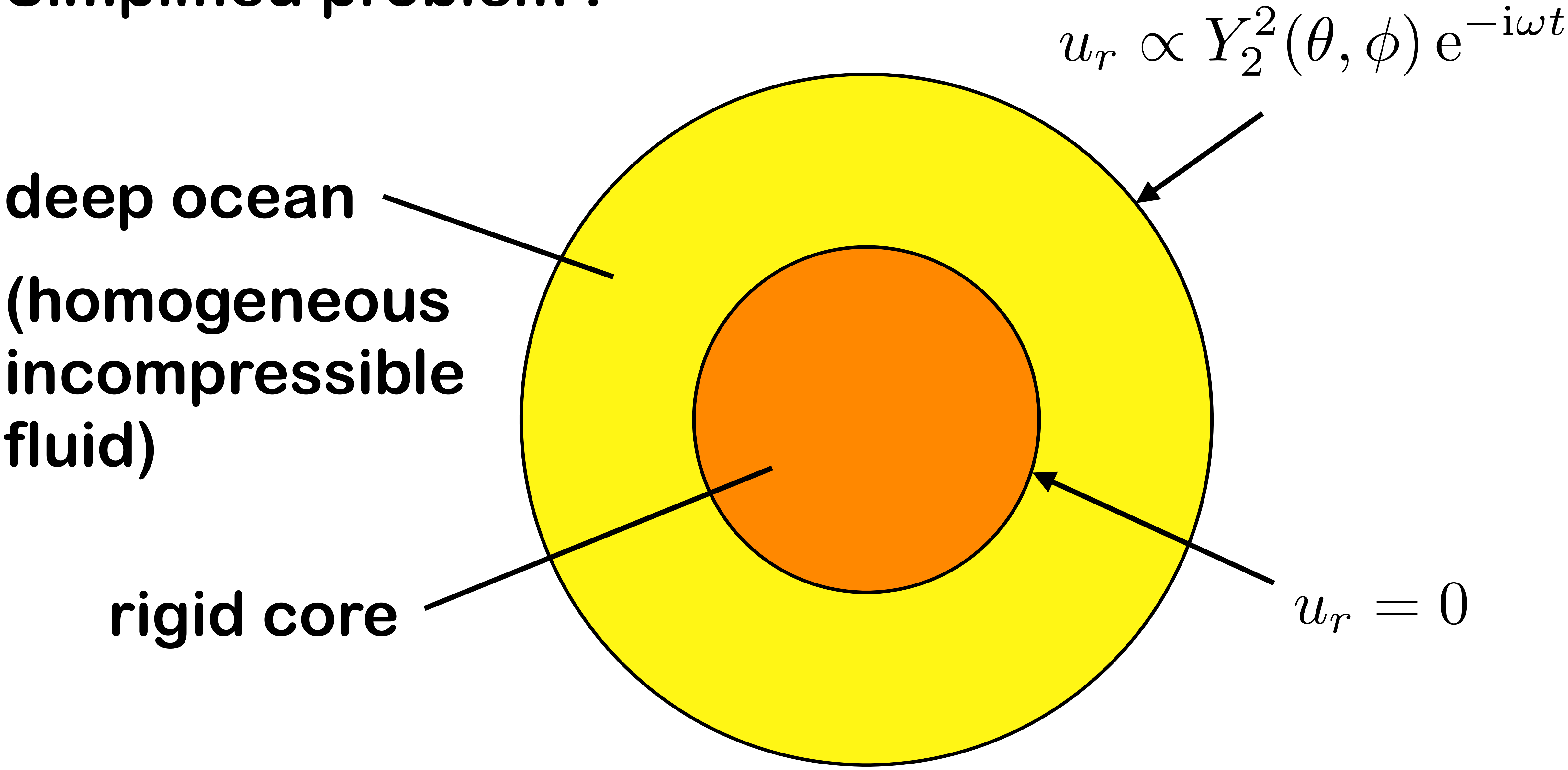
$$2.3\,\mathrm{Myr} \left(\frac{M_\mathrm{p}}{M_\mathrm{J}}\right)^{-1} \left(\frac{P}{\mathrm{day}}\right)^{7}$$

INERTIAL WAVES

Ogilvie & Lin (2004, 2007), Wu (2005), Goodman & Lackner (2009), Ogilvie (2009), Rieutord & Valdettaro (2010)

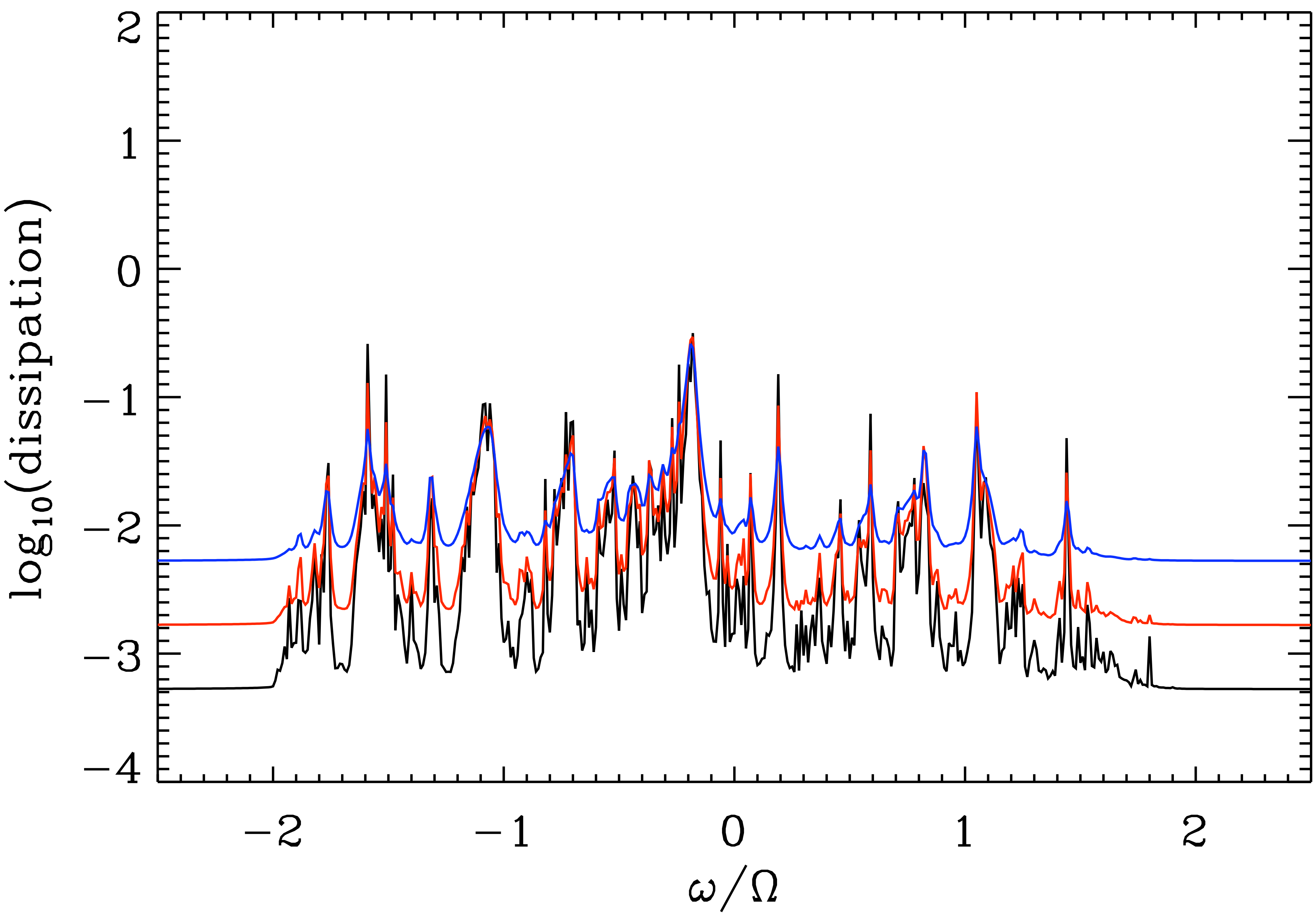
FORCED INERTIAL WAVES

Simplified problem :



uniformly rotating - neglect centrifugal distortion

DISSIPATION RATE : 50% CORE



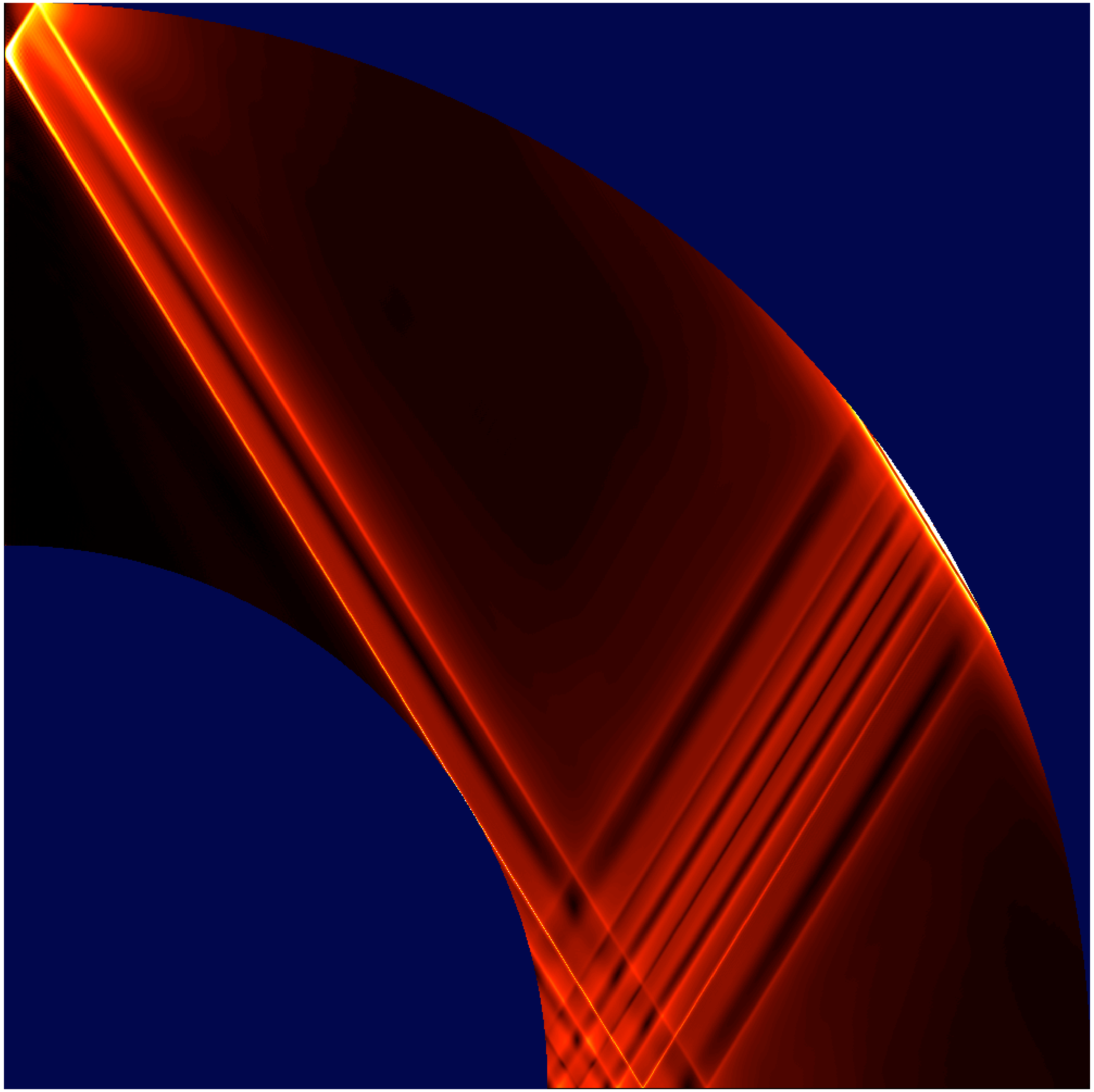
friction coefficient

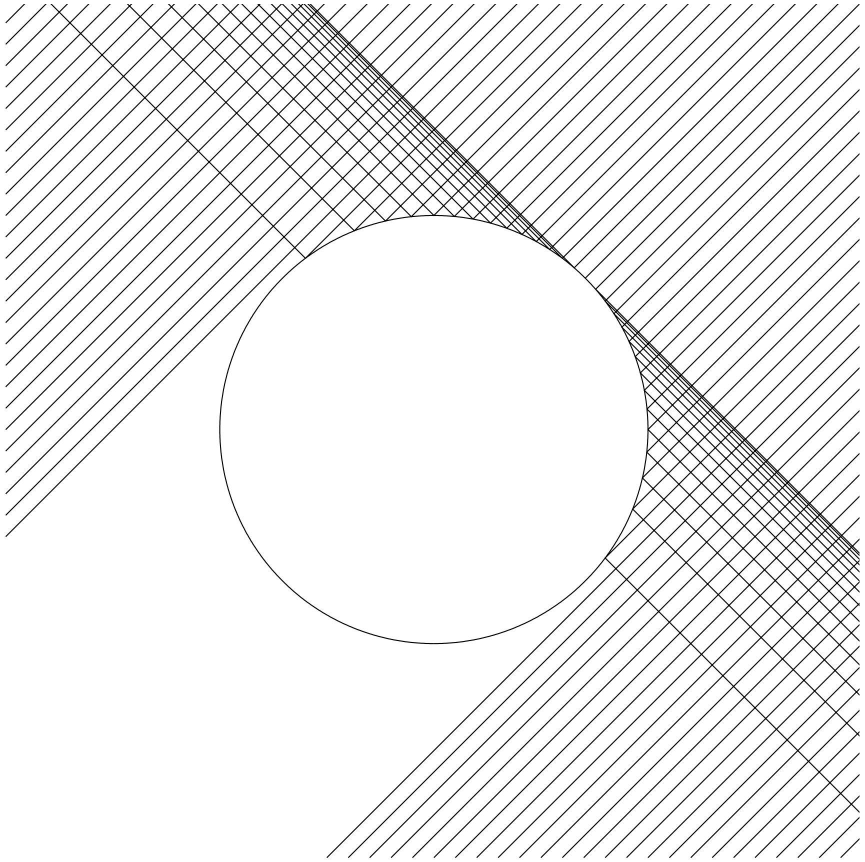
γ/Ω :

10^{-2}

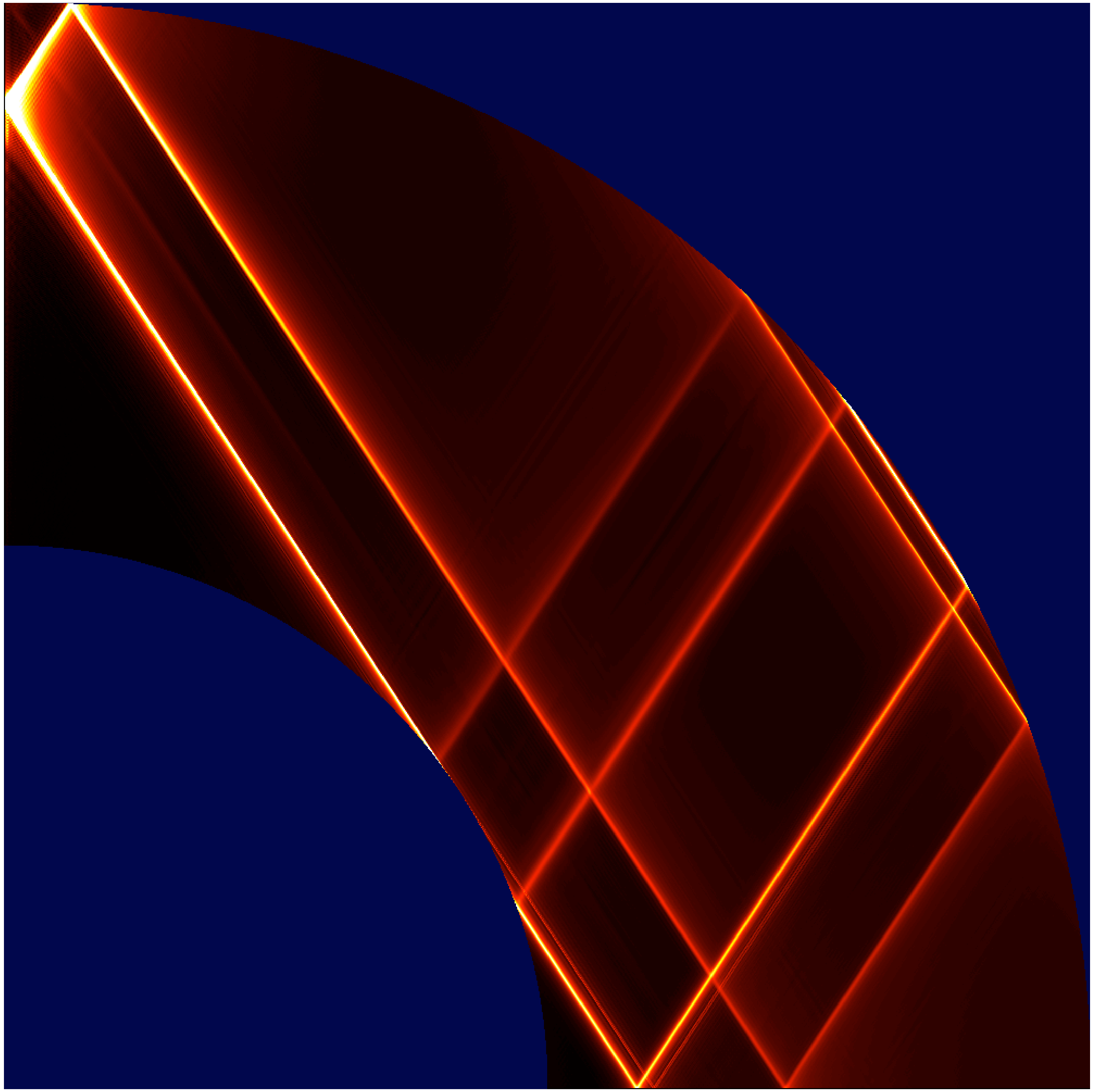
$10^{-2.5}$

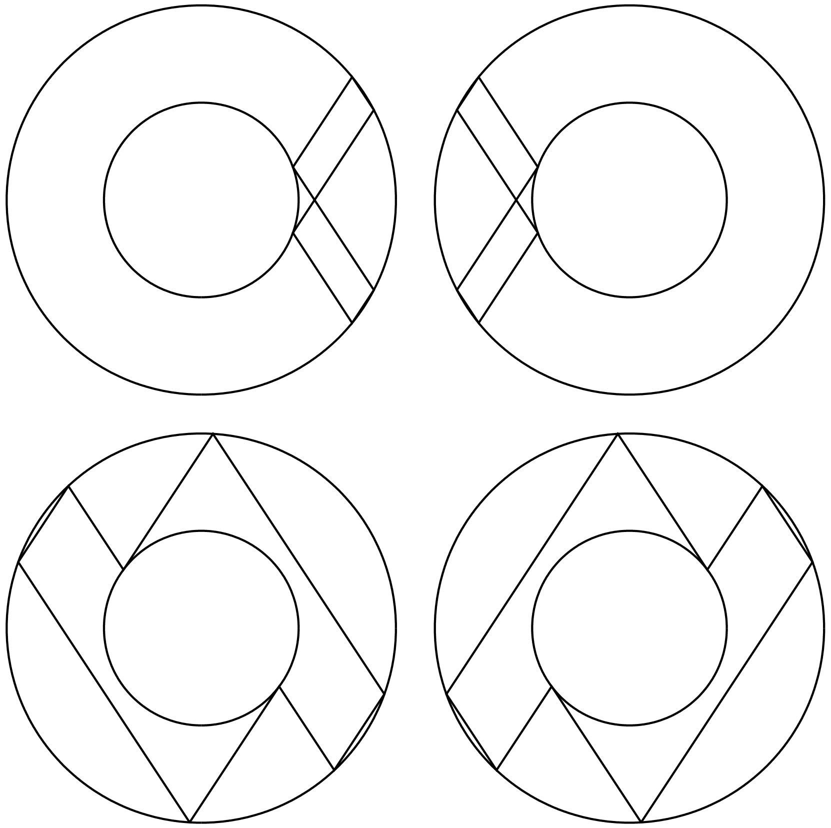
10^{-3}





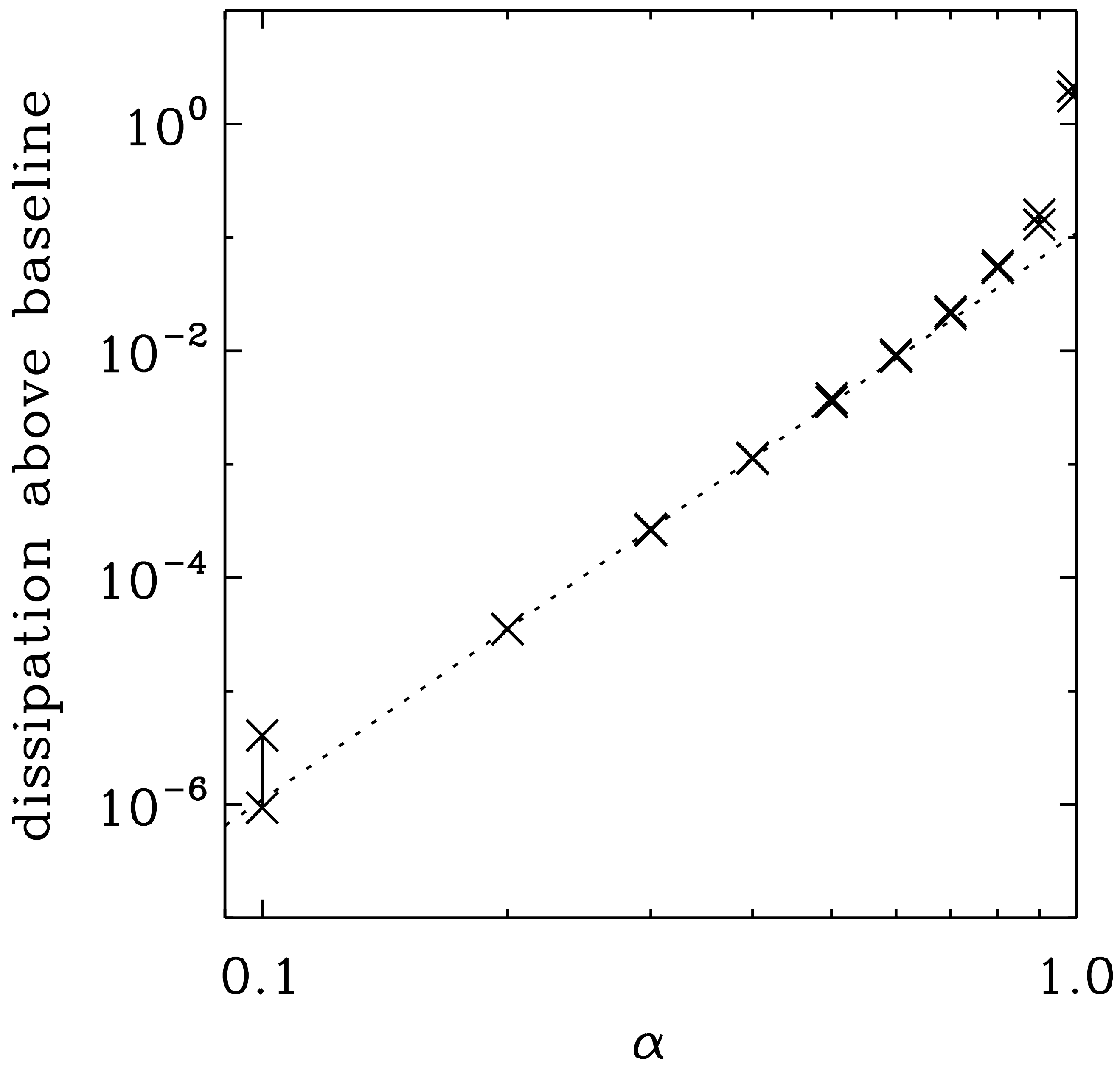
critical latitude singularity



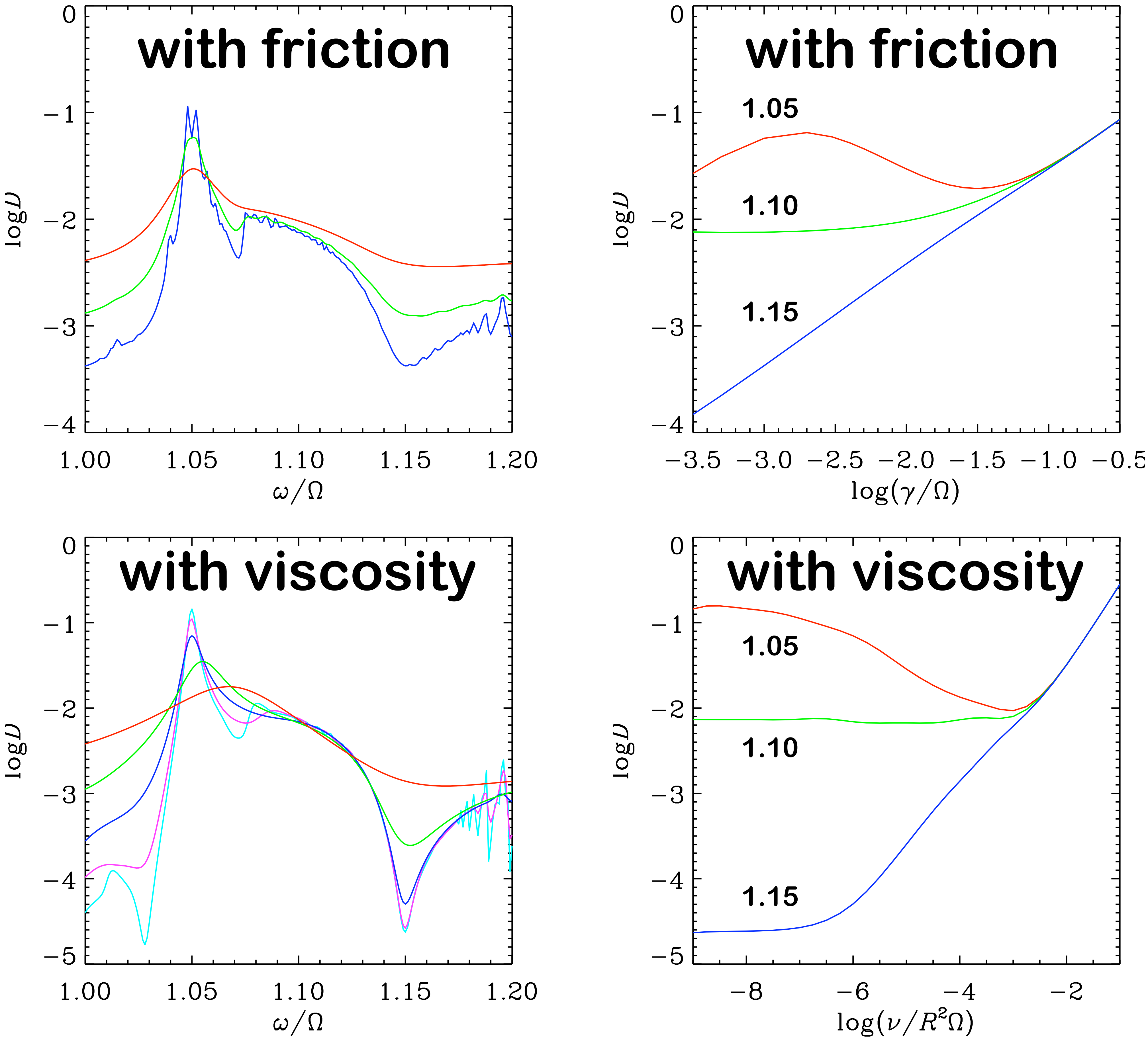


wave attractors

DEPENDENCE ON CORE SIZE



- **frequency-averaged dissipation** $\propto \alpha^5$ **(Goodman & Lackner 2009, Ogilvie 2009)**

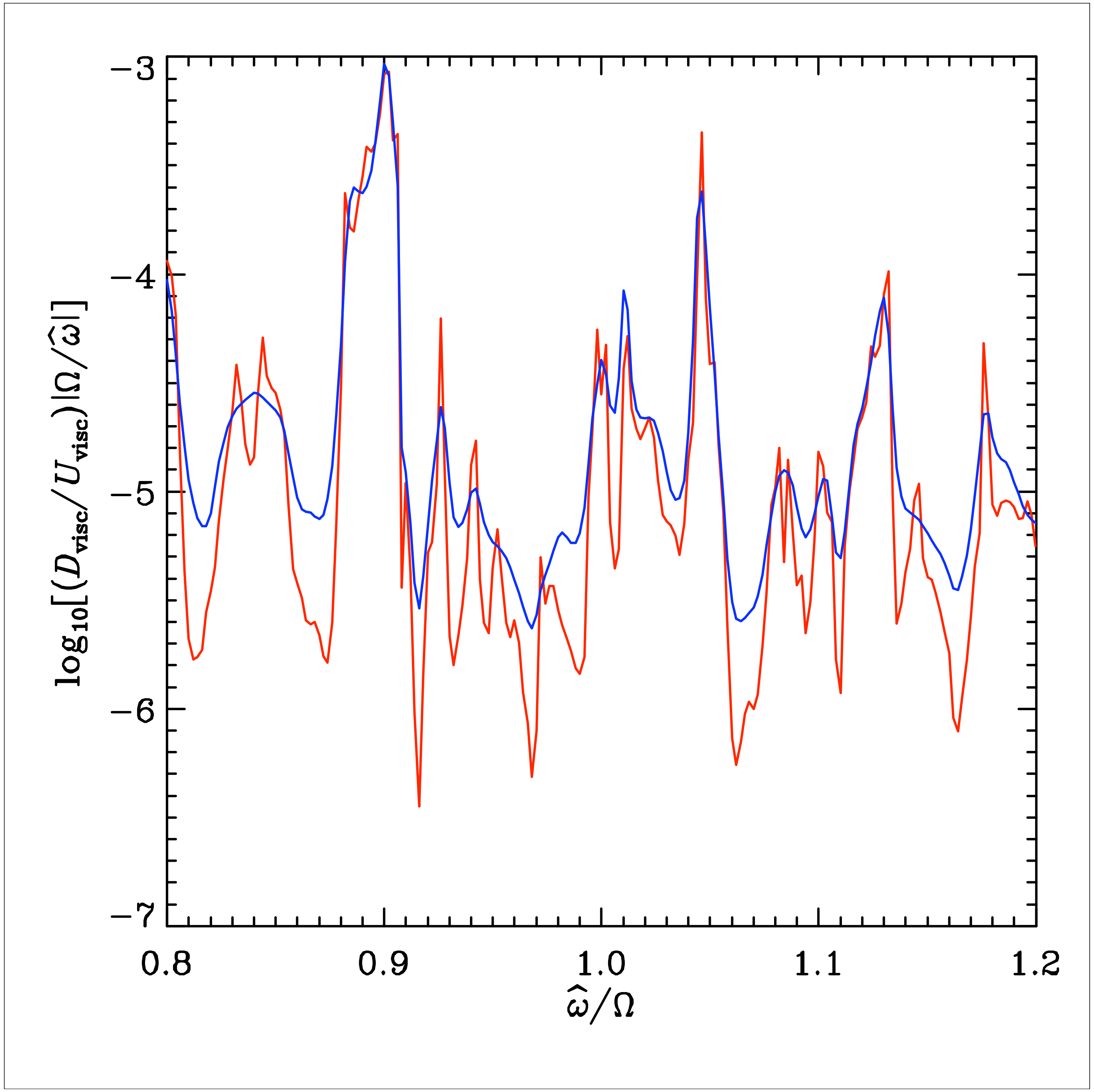


with friction
with friction
with viscosity
with viscosity
1.05
1.10
1.15
1.05
1.10
1.15
log D
log D
log D
log D
ω/Ω
log(γ/Ω)
ω/Ω
log(ν/R²Ω)
0
−1
−2
−3
−4
−5
1.00
1.05
1.10
1.15
1.20
−3.5
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
−8
−6
−4
−2

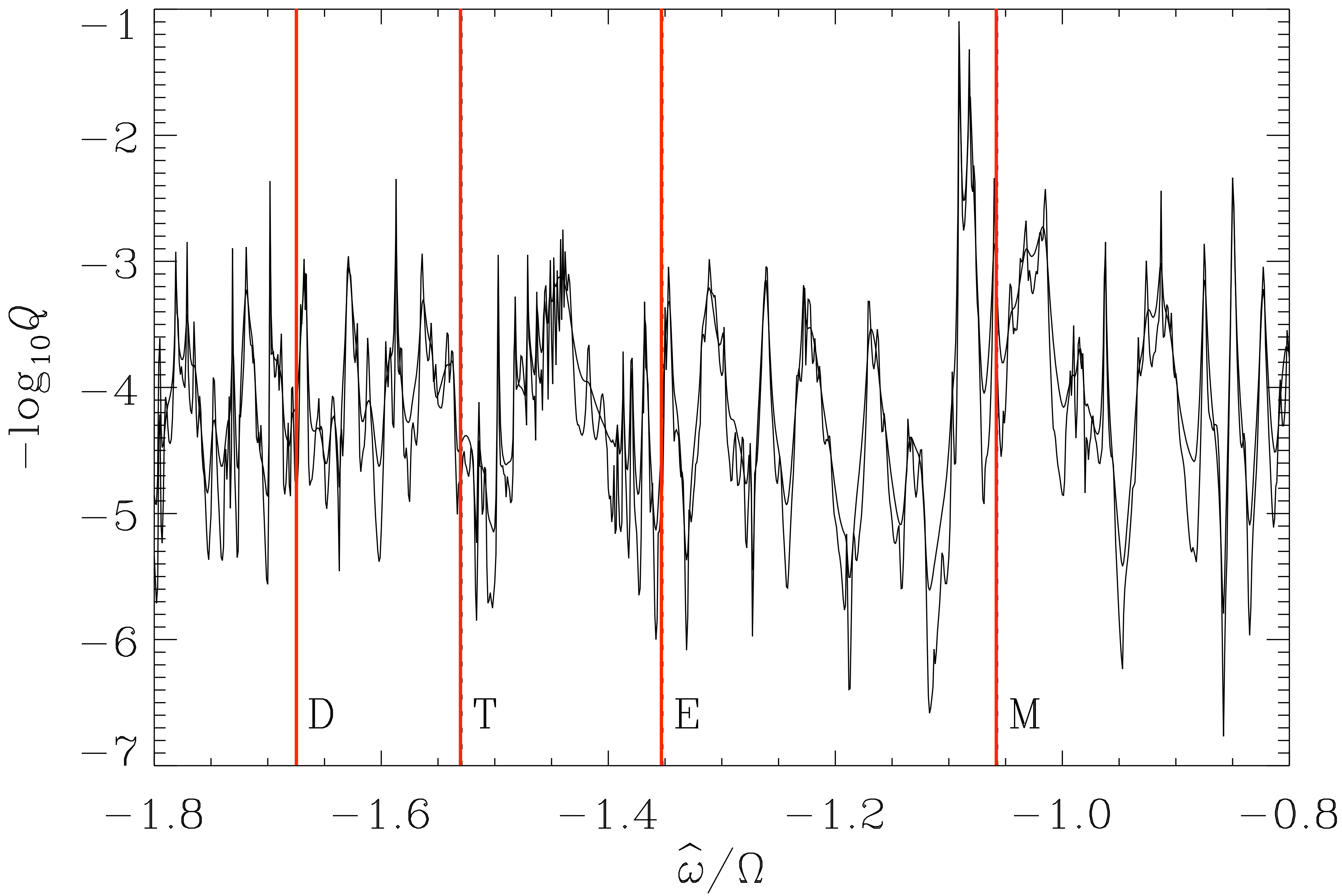
cf. GIO & LIN (2004)

giant planet, 20% core

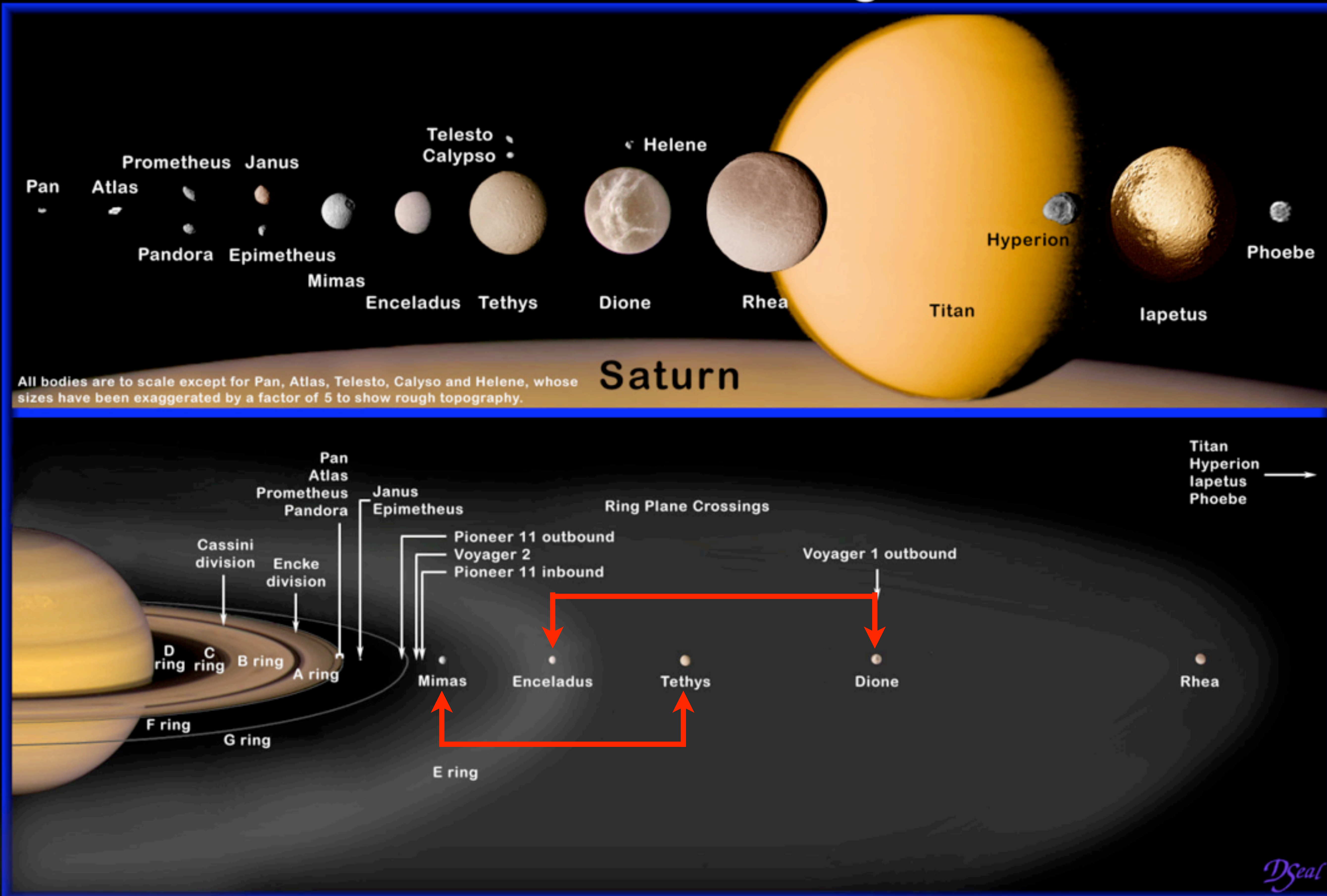
Ek = 10^{-7}, 10^{-8}



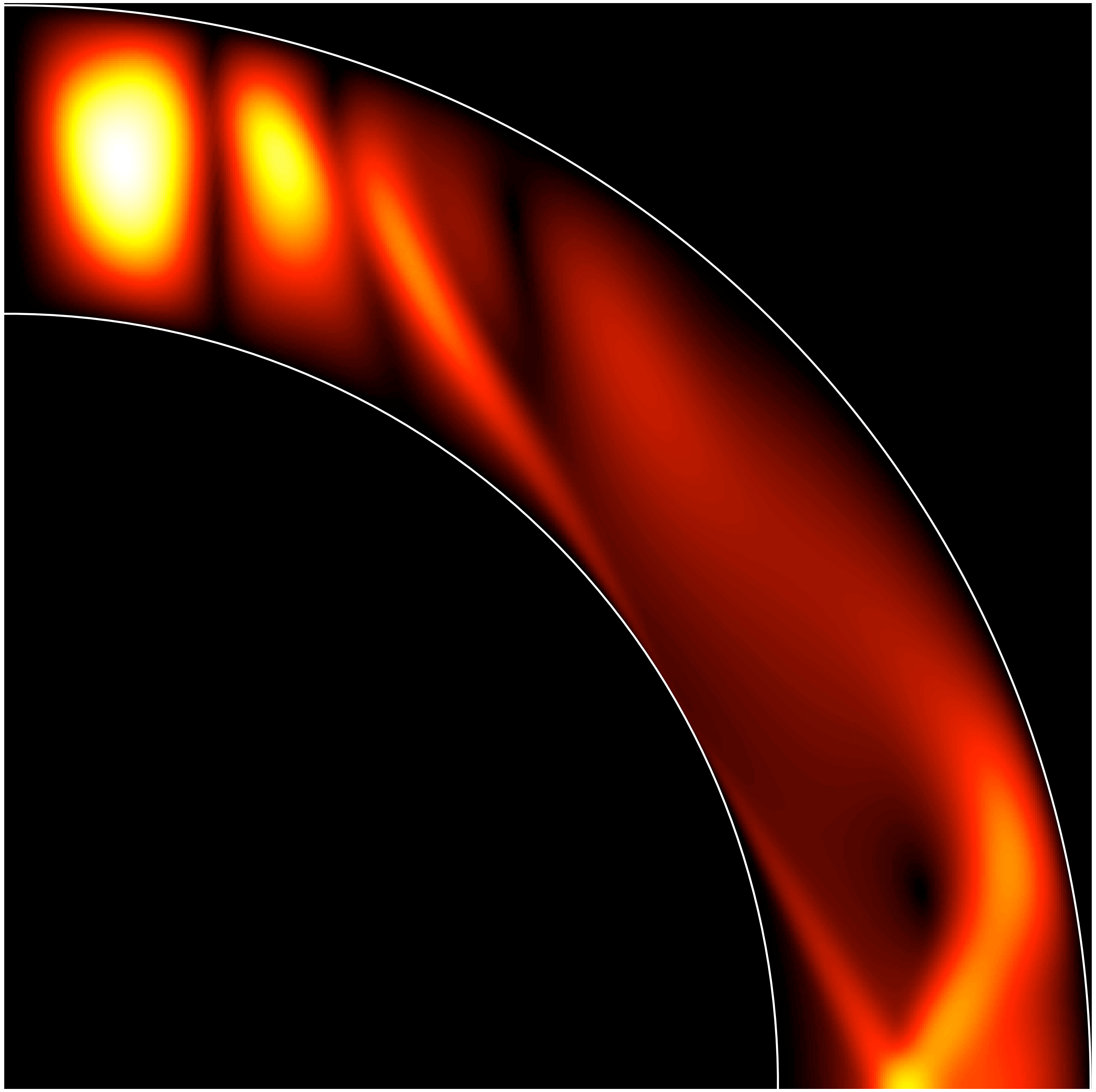
INERTIAL WAVES IN SATURN



Saturn's Satellites and Ring Structure



STELLAR APPLICATION



inertial-wave response of convective zone

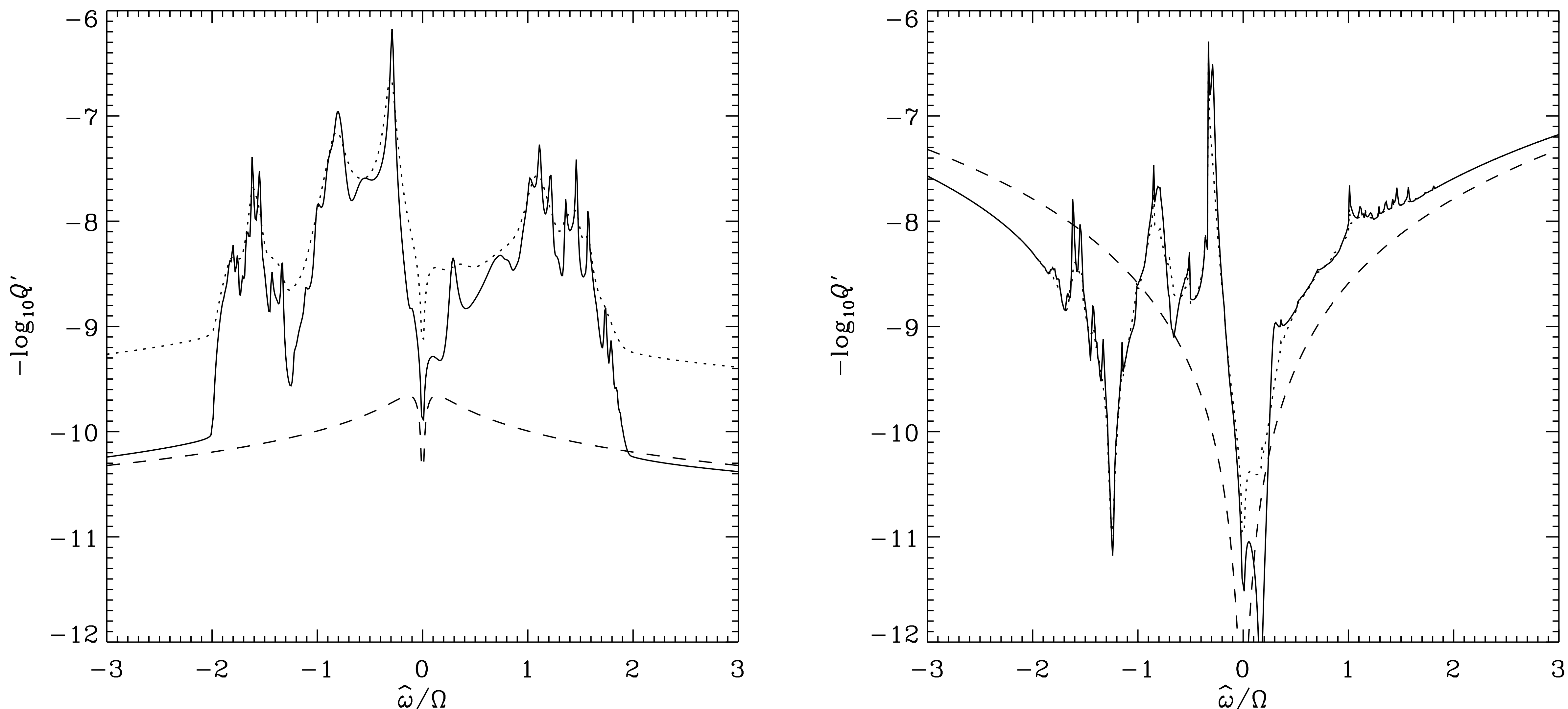
tidal frequency equal to spin frequency

relevant to binary circularization but not planet hosts

cf. GIO & LIN (2007)

solar model, spin period 10 d

inertial waves   Hough waves



-log Q

tidal frequency

CONCLUSIONS

- Q' is not a constant for stars or giant planets
- Linear and nonlinear fluid mechanisms require much further study
- Linear waves give an intricate frequency dependence of Q', still only partly understood
- Cleanly launched and fully damped waves give a robust, smooth frequency dependence
- Extrasolar systems need to be examined on an individual basis owing to structural differences
- Better interior models and physics are needed