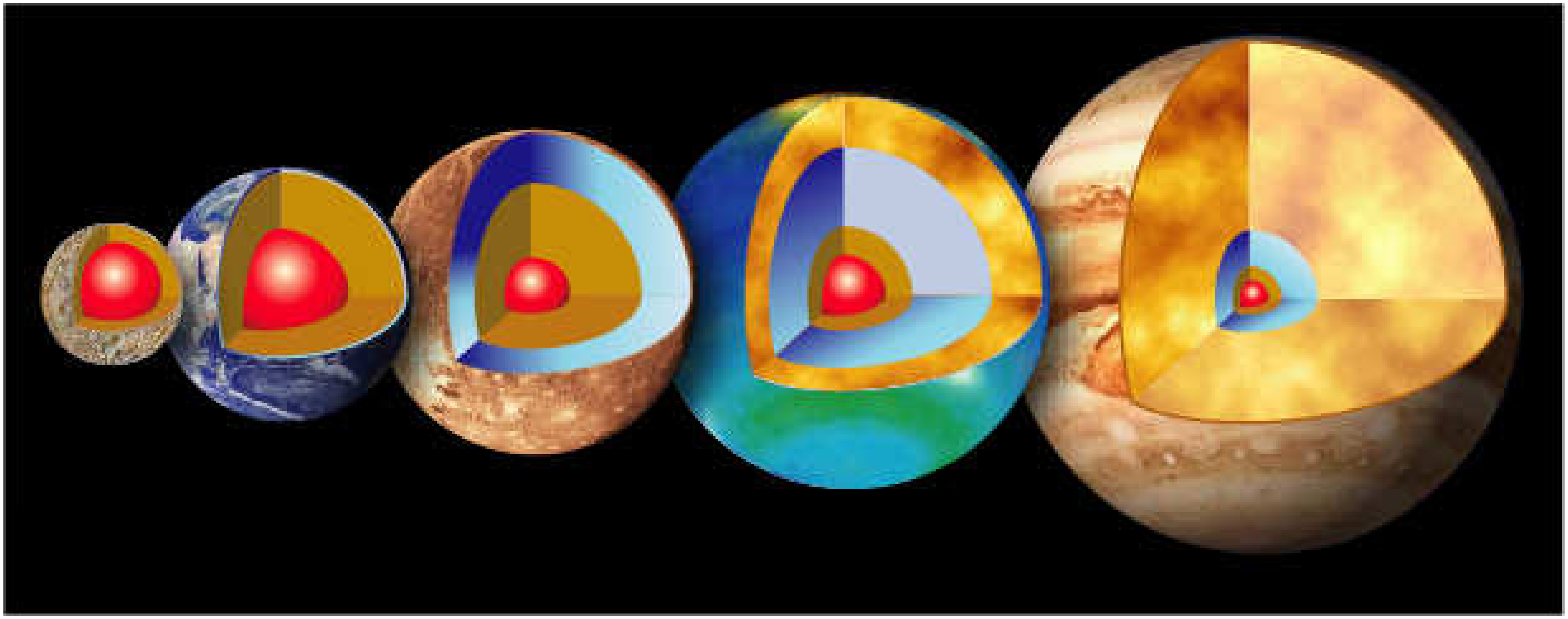
Super-Earths versus mini-Neptunes: can we tell the difference? Corot 7b versus GJ 1214b

C. Sotin, JPL/Caltech and KITP

References: Leger et al. (2004); Valencia et al. (2006); Sotin et al. (2007); Grasset et al. (2009); Fu et al. (sub.); …

- Iron planets (Mercury)
- **Terrestrial planets**
- Ocean / Icy planets
    - Icy Moons
    - **Uranus and Neptune**
- Giant planets



List of open questions in exoplanet science (wiki)

Planetary structure and evolution

- Super-Earths versus mini-Neptunes, can we tell the difference
- Equations of state for planets
- Radius versus mass
- Rotational effects (especially on gas giants)
- Solid-body and gaseous convection
- How to explain the large radii of some EGPs?
- The dynamics-structure connection

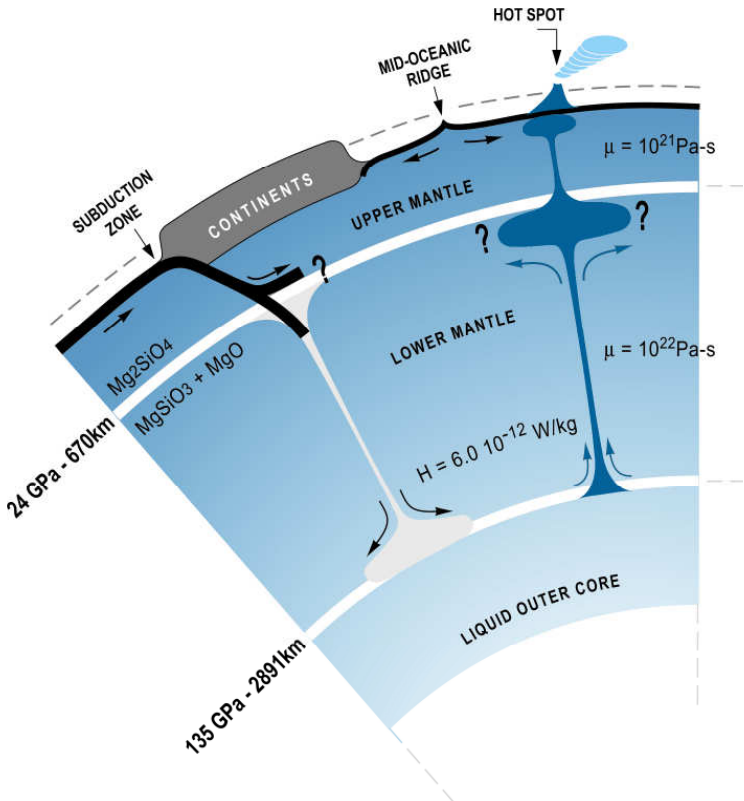
Atmospheres

- Evaporation

Habitability

- The effect of terrestrial planet mass on habitability

Internal structure of the Earth - composition

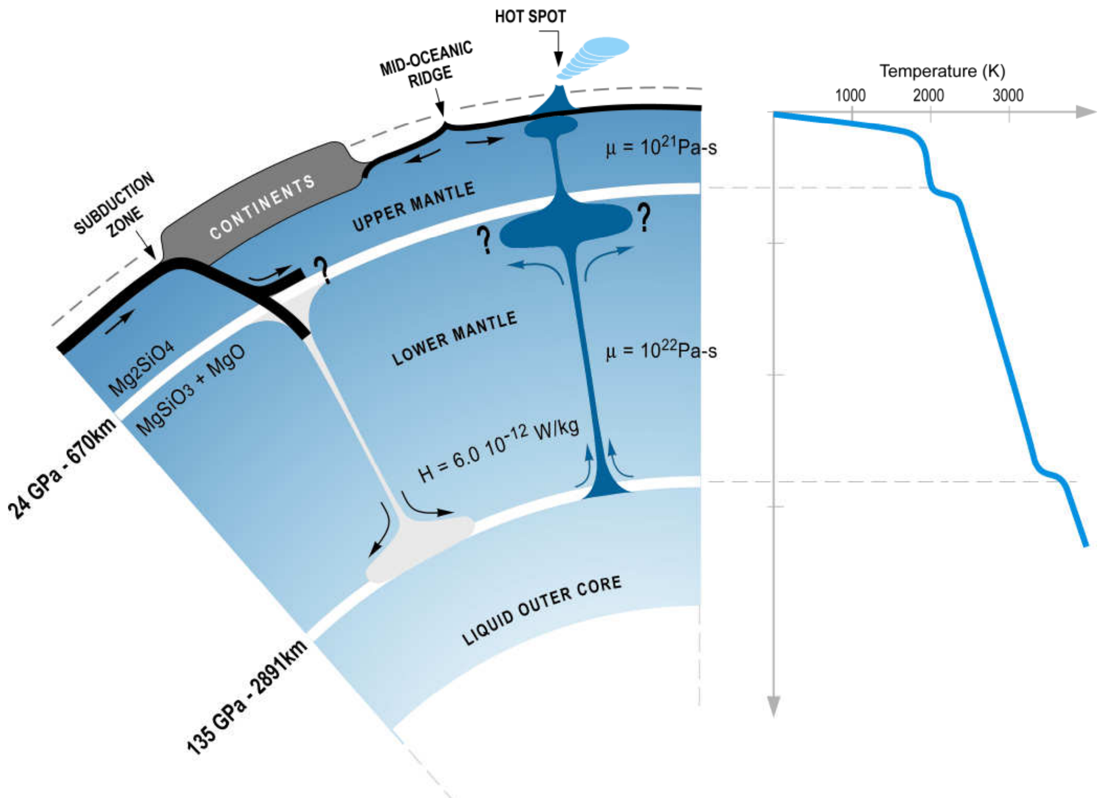


	EEH Earth model	PUM	LM	Core
O	30,28	44,76	43,8	1,61
Fe	33,39	5,89	12,69	80,25
Si	19,23	21,35	24,28	10,34
Mg	12,21	23,21	16,18	0
Total	**95,11**	**95,21**	**96,95**	**92,2**
Ni	2,02	0,25	0,71	4,99
Ca	1,01	2,32	1,2	0
Al	0,93	2,13	1,1	0
S	0,85	0,01	0,01	2,57
Total	**99,92**	**99,92**	**99,97**	**99,76**

O	30,28	44,76	43,8	1,61
Fe	35,41	6,14	13,4	85,24
Si	19,69	22,41	24,83	10,34
Mg	13,68	26,59	17,93	0

Core : Iron + light element (S, O, other).
Mantle : $(Mg,Fe)_2Si_2O_6$, $Ca(Mg,Fe)Si_2O_6$, $(Mg,Fe)_2SiO_4$ and Al phase / $(Mg,Fe)SiO_3$, $(Mg,Fe)O$ and Al phase

Modeling the mass - radius relationship. Temperature



$$\frac{dT}{dP} = \frac{\alpha T}{\rho C_P} = \frac{\gamma T}{\rho \Phi}$$

$$\Phi = \frac{K_S}{\rho} = \frac{dP}{d\rho}$$

$$\gamma = \gamma_0 \left(\frac{\rho_0}{\rho} \right)^q$$

T : Temperature

P : Pressure

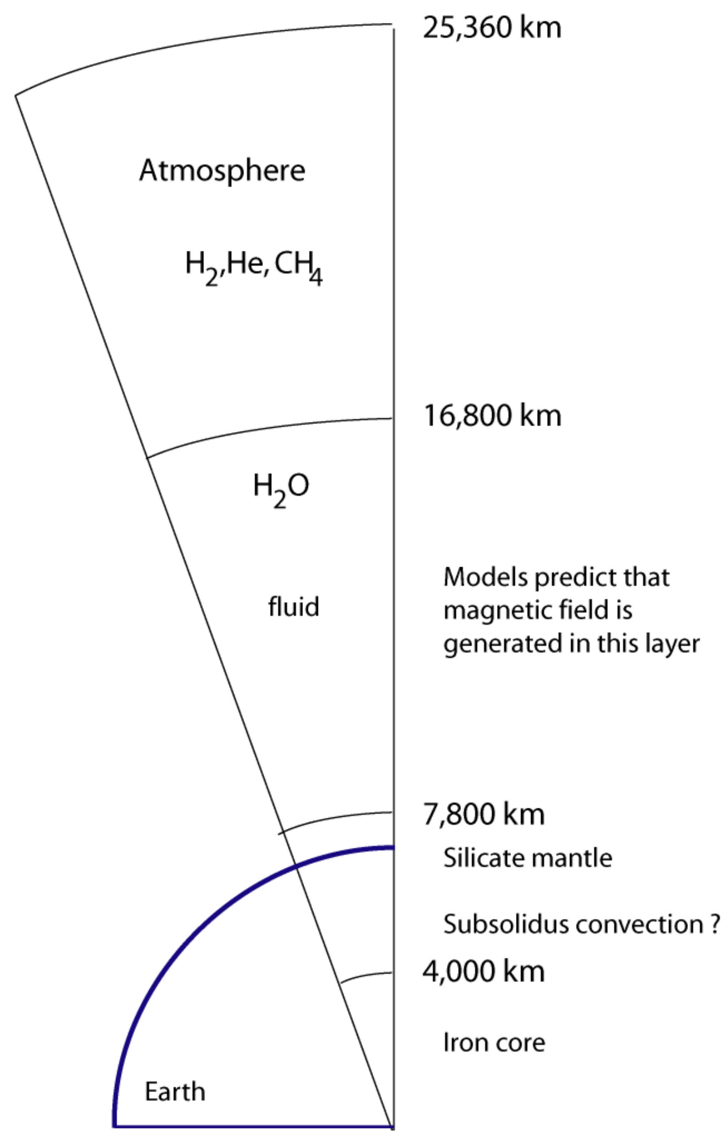
ρ: density (kg/m^3)

α: thermal expansion coefficient (K^{-1})

C_p : specific heat (J/kg/K)

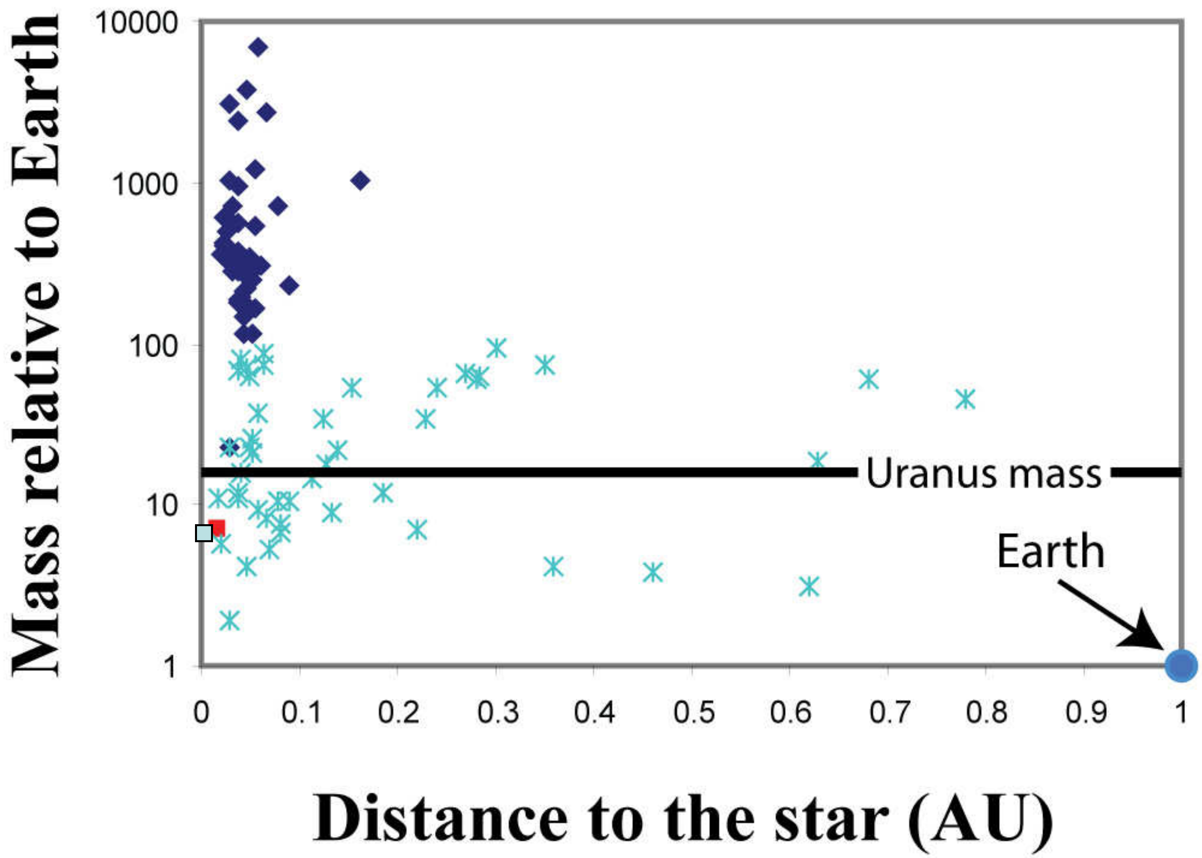
Uranus and Neptune / Earth

	Uranus	Neptune
Mass (10^{24} kg)	86.832 (**14.5**)	102.435 (**17.1**)
Volumetric radius (km)	25,364 (**3.98**)	24,625 (**3.87**)
Mean density (kg/m^3)	1,270 (2)	1,638(4)
Albedo	0.300(49)	0.290(67)
Absorbed power (x 10^{15} W)	5.26(37)	2.04(19)
Emitted power (x 10^{15} W)	5.60(11)	5.34(29)
Intrinsic power (x 10^{15} W)	0.34(38)	3.30(35)
Intrinsic flux (W/m^2)	0.042(47)	0.433(36)
Black-body temperature (K)	59.1	59.3
1-bar temperature[b] (K)	76 (2)	72 (2)
$J_{2,0}$ (x 10^{-6})	3,516(3)	3,539 (10)
$J_{4,0}$ (x10^{-6})	-35.4 (4.1)	-28(22)
$Q=\omega^2R^3/GM$	0.02951 (5)	0.02609(26)
Moment of inertia (I/MR2)	0.230	0.241



What do we know about extra-solar planets?

Star, mass (M.sin(i)), distance to the star, radius, age, atmosphere, ….



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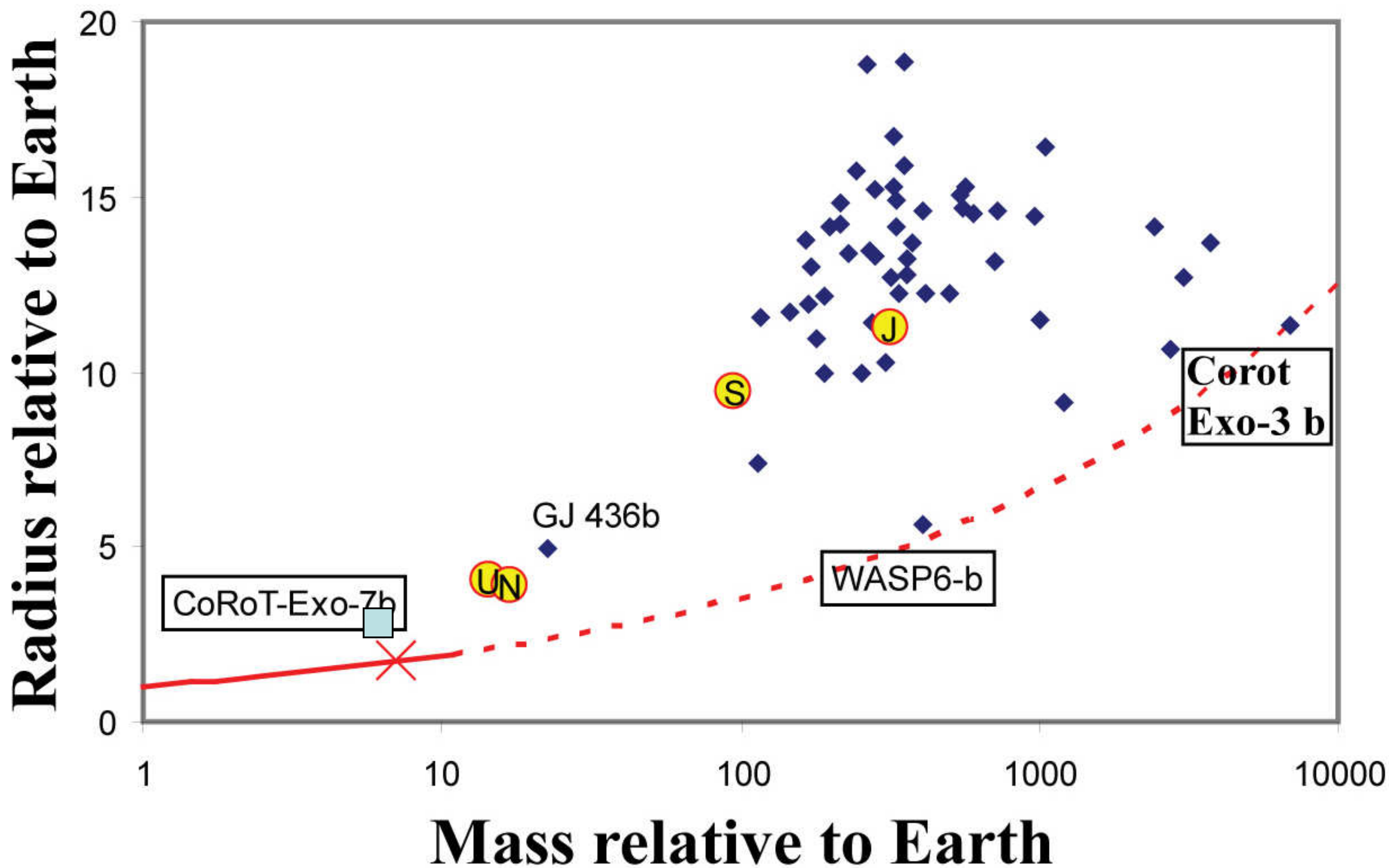
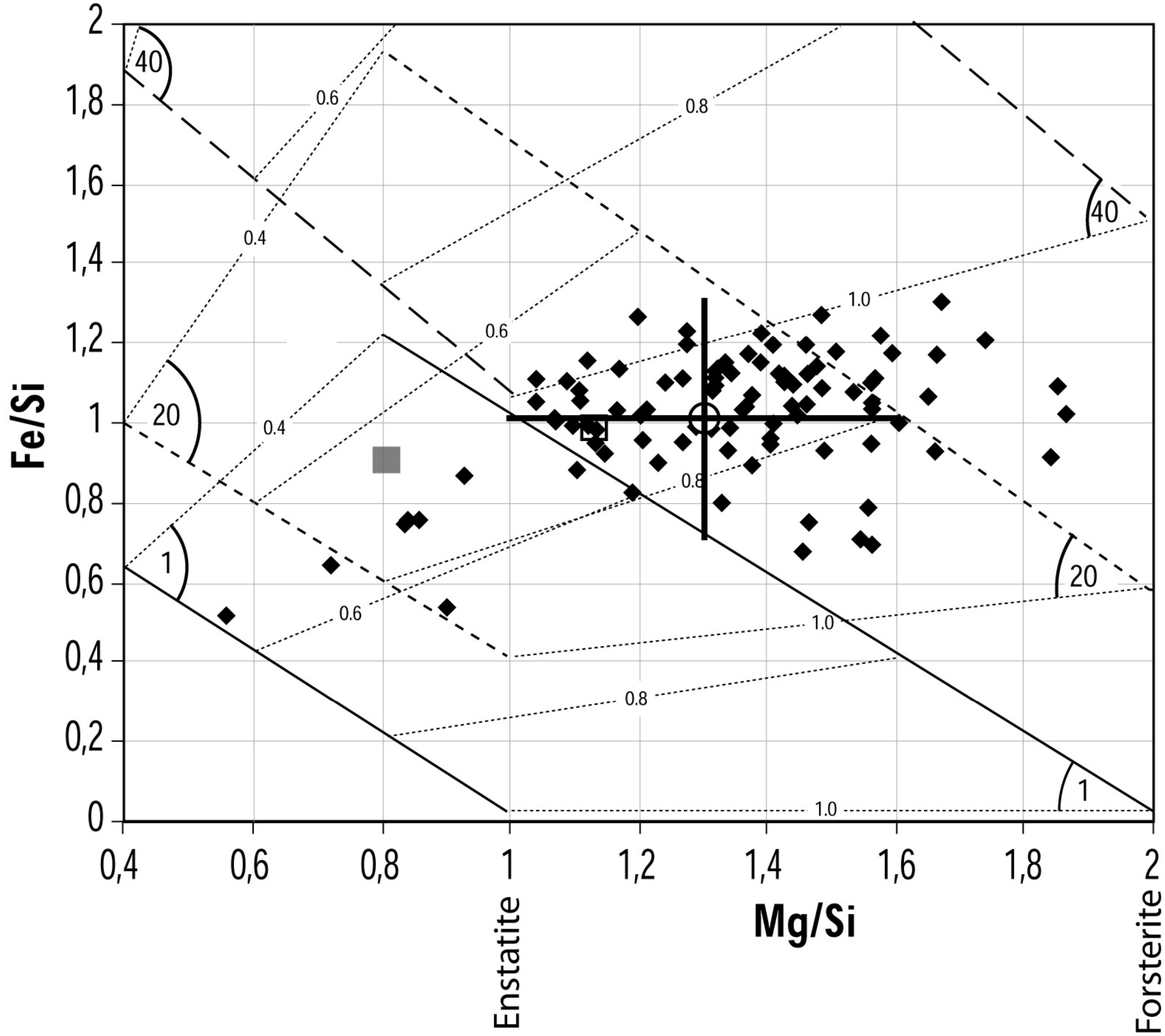


What do we know about extra-solar planets?

Composition:
Data from Beirao et al. (2006) and Gilli et al. (2006)

Empty square is solar composition.
Filled square is the enstatite end-member composition for the Earth's mantle.
Empty circle is barycenter of all the stellar compositions.
The large cross is typical uncertainties

Lines are values of Mg#
Areas give mass fraction of the core



Radius and mass

$$M = 4\pi \int_0^R r'^2 \rho(r') dr'$$

$$\frac{dP}{dr} = -\rho(r) g(r)$$

$$g(r) = \frac{4\pi G}{r^2} \int_0^r r'^2 \rho(r') dr'$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \frac{\alpha T}{\rho C p}$$

$$P_{th} = \int_{T0}^{T} \alpha K_T dT$$

Mass and radius are two of the few parameters.
They are related to each other through a simple equation in a 1D model.
Density depends on composition (elementary and molecular), pressure, and temperature
In the calculations, the main parameters are:

- Amount of volatiles (H_2O)
- The amount of Fe
- Distribution of Fe between iron core and mantle

We need an Equation of State (EoS) which relates density to pressure and temperature.
Example of the Birch-Murnagham EoS :

$$P = \frac{3K_{0T}}{2}\left[\left(\frac{\rho}{\rho_0}\right)^{\frac{7}{3}} - \left(\frac{\rho}{\rho_0}\right)^{\frac{5}{3}}\right]\left\{1 + \frac{3}{4}(K'_{0T} - 4)\left[\left(\frac{\rho}{\rho_0}\right)^{\frac{2}{3}} - 1\right]\right\}$$

The 3rd order Birch-Murnagham EoS

$$\begin{cases} P(\rho,T) = \frac{3}{2}K_{T,0}^{0}\left[\left(\frac{\rho}{\rho_{T,0}}\right)^{7/3} - \left(\frac{\rho}{\rho_{T,0}}\right)^{5/3}\right]\left\{1 - \frac{3}{4}\left(4 - K_{T,0}^{'}\right)\left[\left(\frac{\rho}{\rho_{T,0}}\right)^{2/3} - 1\right]\right\} \\ K_{T,0}^{0} = K_0 + a_P\left(T - T_0\right) \\ K_{T,0}^{'} = K_0^{'} \\ \rho_{T,0} = \rho_0 \exp\left(\int_{300}^{T} \alpha_{T,0}\, dT\right) \\ \alpha_{T,0} = a_T + b_T.T - c_T.T^{-2} \end{cases}$$

Used for the upper mantle

8 parameters known at ambient pressure:

- T_0: the reference temperature
- ρ_0: density
- K_0: bulk modulus
- $K'_{T,0}$ α_P: pressure and temperature derivatives of bulk modulus
- a_T, b_T, c_T: thermal expansion coefficients

The Mie-Grüneisen-Debye formulation

$$
\begin{cases}
P(\rho,T) = P(\rho,T_0) + \Delta P_{th} \\
P(\rho,T_0) = \frac{3}{2}K_0\left[\left(\frac{\rho}{\rho_0}\right)^{7/3} - \left(\frac{\rho}{\rho_0}\right)^{5/3}\right]\left\{1 - \frac{3}{4}(4 - K_0^{'})\left[\left(\frac{\rho}{\rho_0}\right)^{2/3} - 1\right]\right\} \\
\Delta P_{th} = \left(\frac{\gamma}{V}\right)\left[E(T,\theta_D) - E(T_0,\theta_D)\right] \\
E = 9nRT\left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} t^3 dt/(e^t - 1) \\
\theta_D = \theta_{D0}\left(\frac{\rho}{\rho_0}\right)^{\gamma} \\
\gamma = \gamma_0\left(\frac{\rho}{\rho_0}\right)^{-q}
\end{cases}
$$

Used for the lower mantle and core

Thermal and static pressure are dissociated. 8 parameters :

- T_0: the reference temperature
- ρ_0 : density
- K_0 and $K'_{T,0}$: bulk modulus and its pressure derivative
- θ_{D0}: reference Debye temperature
- n : number of atoms per chemical formula
- q and γ_0: scaling exponents

Other formulations & comparisons

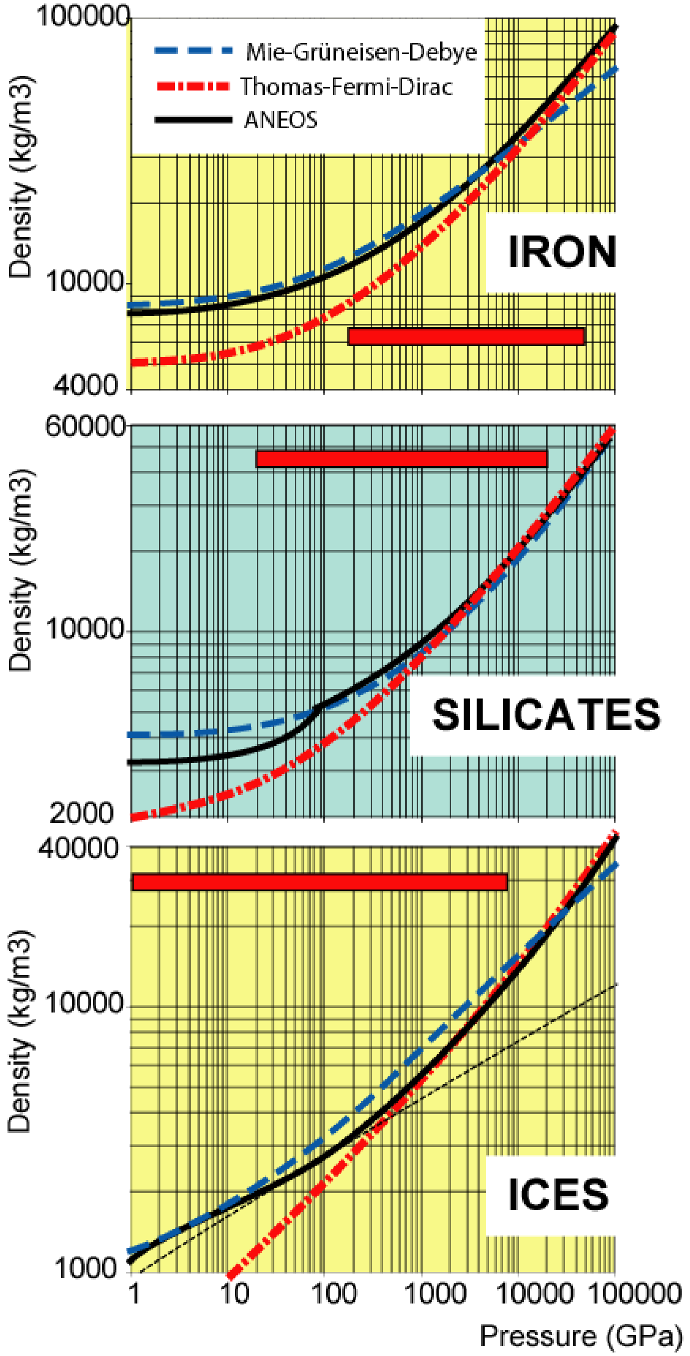
Birch-Mürnhagan EOS
- Liquid layer
- Upper silicate mantle

Mie-Grüneisen-Debye EOS
- Lower silicate mantle

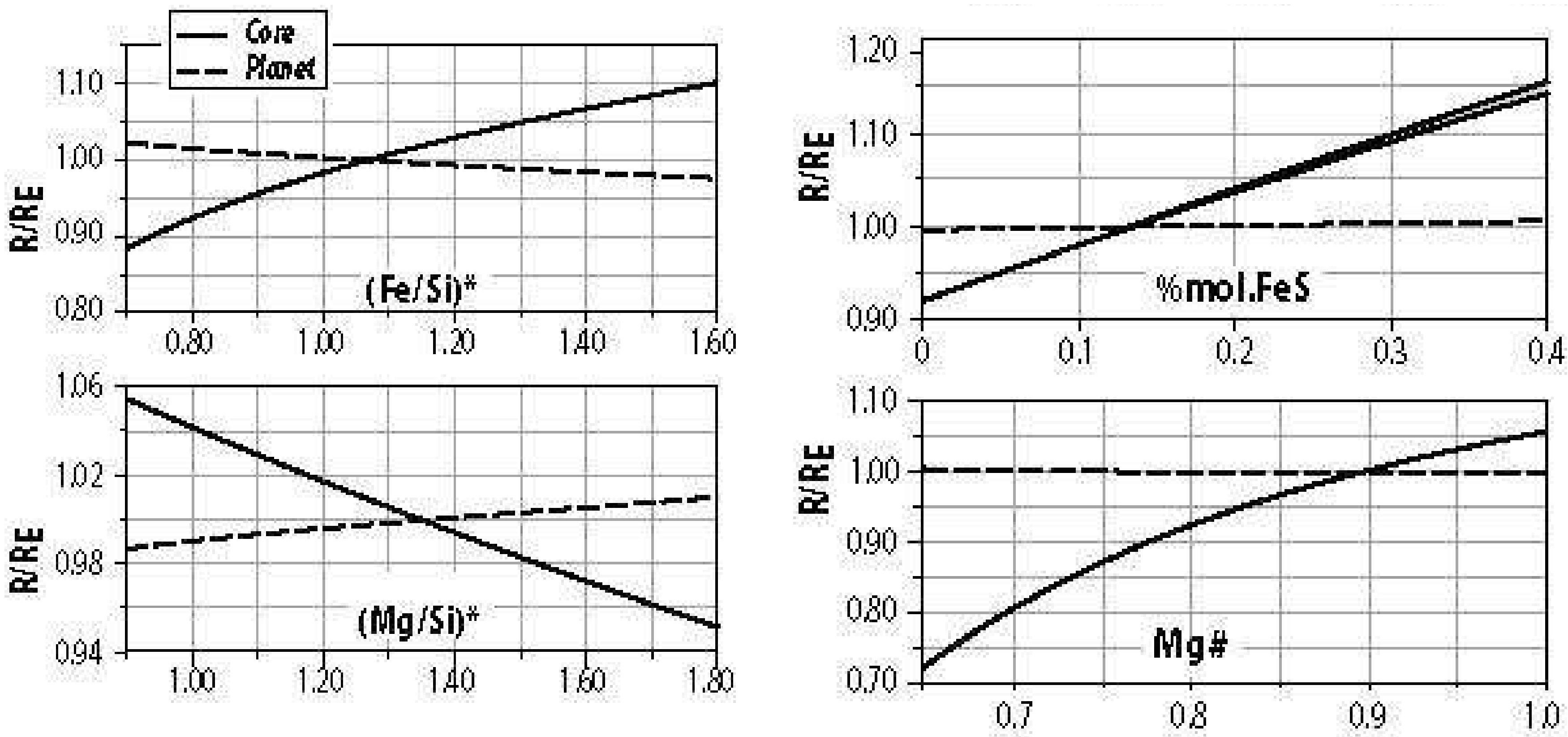
Thomas-Fermi-Dirac
- Icy mantle
- Metallic core (P> 10 TPa)

Vinet EoS

ANEOS (Thompson, 1990)



Radius versus composition (1 M_E < M < 10 M_E)



Total radius does not vary significantly with on the composition
The amount of Fe plays a significant role for the radius of the core

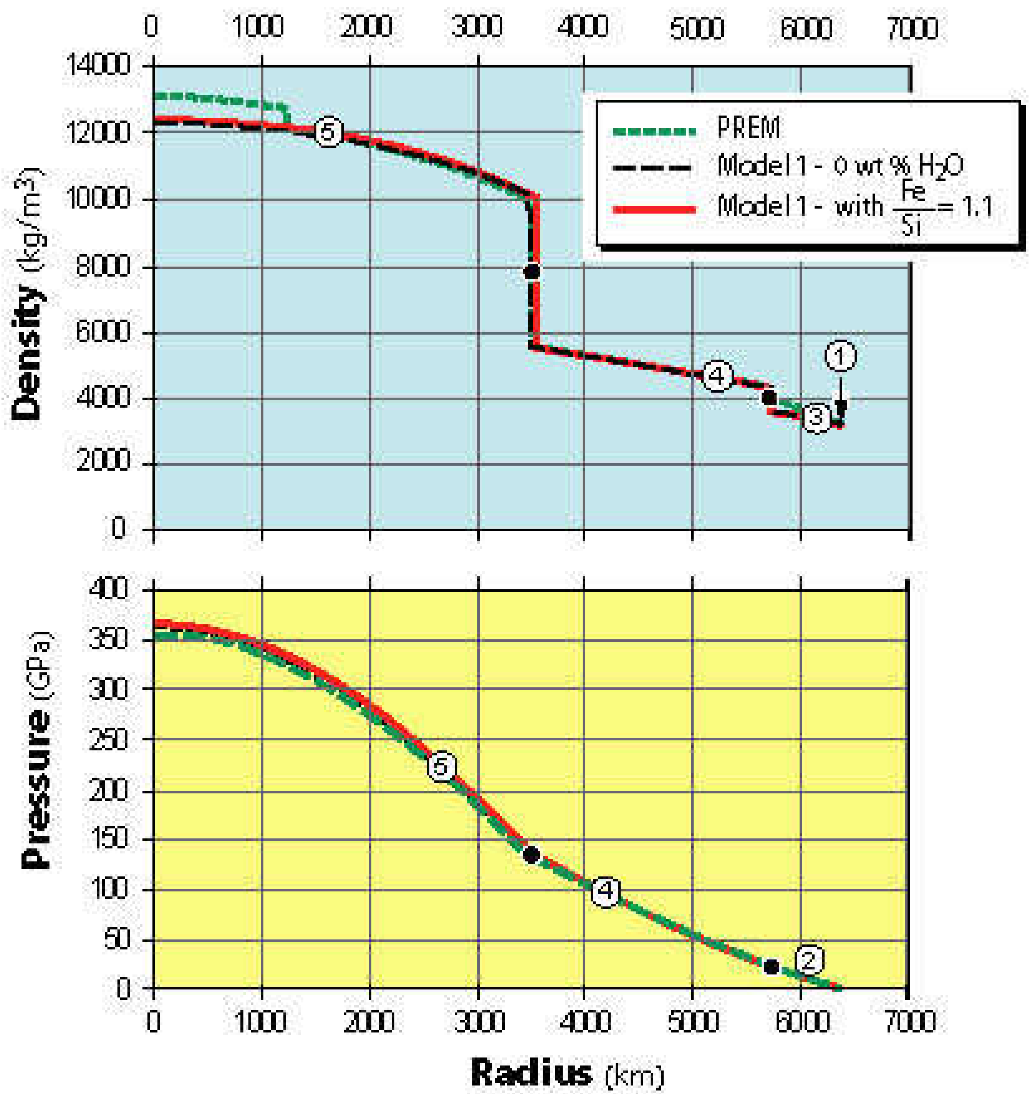
Results : Validation of the model - Earth

Model :

- Fe/Si = 0.987
- Mg/Si = 1.136
- Mg# = 0.9
- H_2O: 0.01 wt %

$M=M_{Earth}$

R=6414 km (0.6%)



How much water to add?

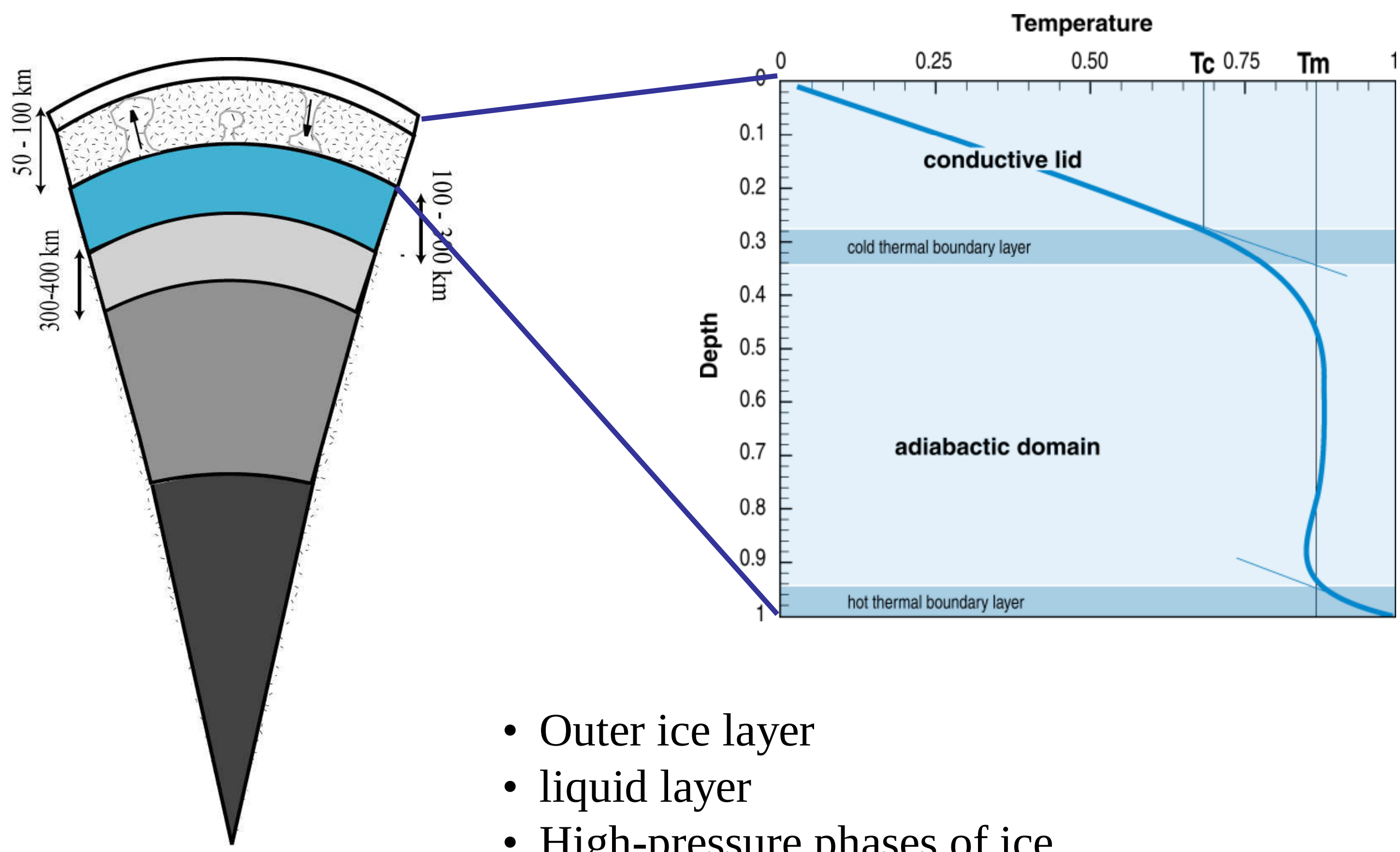
	Solar[a]	Solar[b]		EH[a]	EH[b]	
		Model 1	Model 2		Model 3	Model 4
M_{H_2O}	–	5×10^{-2}–50	5×10^{-2}–50	–	5×10^{-2}–50	5×10^{-2}–50
(Fe/Si)	0.977	0.986	0.986	0.878	0.909	0.909
(Mg/Si)	1.072	1.131	1.131	0.734	0.803	0.803
Mg# (silicates)	–	0.9	0.7	0.9–0.7	0.9	0.7

Four models: Solar and Enstatite and two different Mg#.

Name	Mass/M_{Earth}	Planetary radius					Best fit			
		Measured	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Water-rich							M_{H_2O} (%)			
Europa	*0.008*	*1565*	1854	1865	1852	1860	15	13	16	14
Callisto	0.0181	2410	2396	2407	2397	2403	51	49	50	50
Ganymede	0.0248	2631	2641	2655	2638	2650	50	47	49	48
Titan	0.0225	2575	2563	2577	2559	2575	51	49	51	50
Earth-like							Fe/Si			
Mercury	*0.055*	*2437*	2705	2723	2706	2715	8	8	7.5	7.5
Mars	0.107	3389	3349	3366	3342	3357	0.78	0.84	0.71	0.79
Venus	0.81	6051	6056	6071	6008	6032	0.96	1.03	0.80	0.85
Earth	1	6371	6414	6447	6379	6405	1.10	1.19	0.92	0.99
Moon	*0.0123*	*1738*	1600	1642	1591	1621	0.22	0.48	0.30	0.30

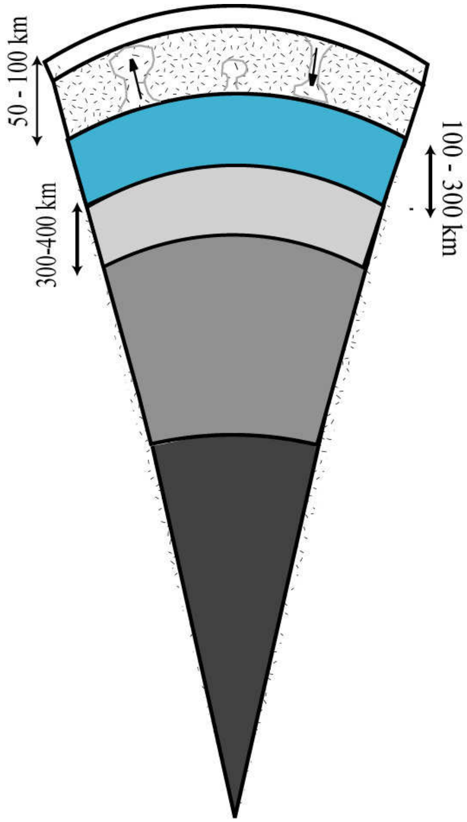
The right part of the table indicates the required value of M_{H_2O} (ocean-planet) or Fe/Si (Earth-like planet) in order to get the value of the measured radius for each body and for each of the four models described in Table 2.

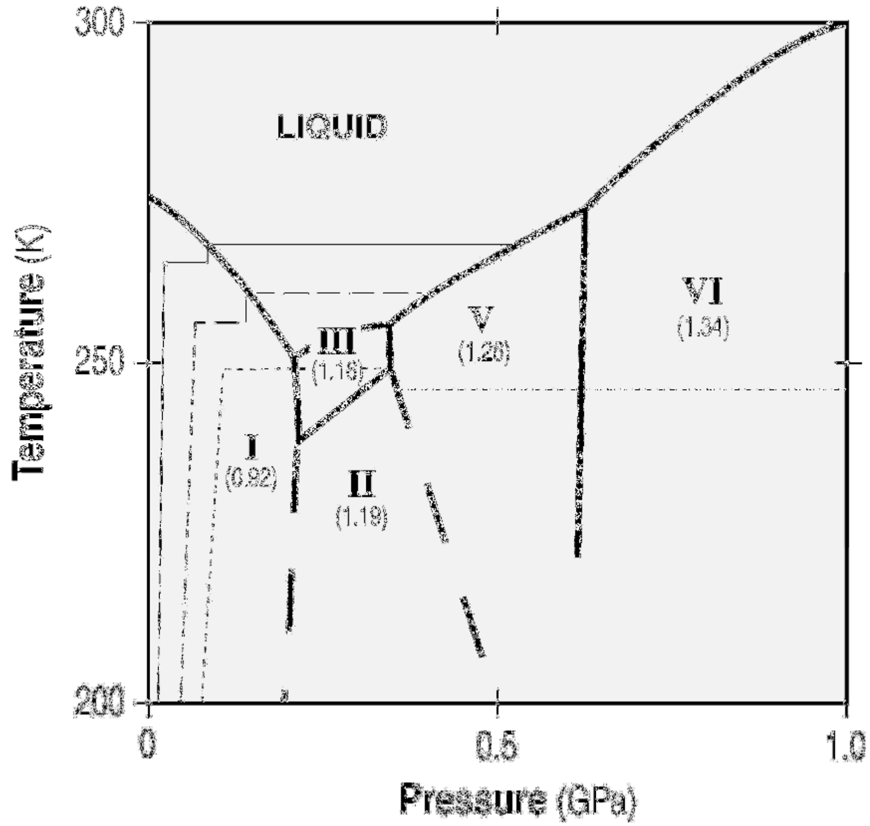
Internal structure of large icy satellites: a model for icy exoplanets (ocean exoplanets)



- Outer ice layer
- liquid layer
- High-pressure phases of ice
- Silicate layer
- Iron core

Internal structure of large icy satellites (2/2)



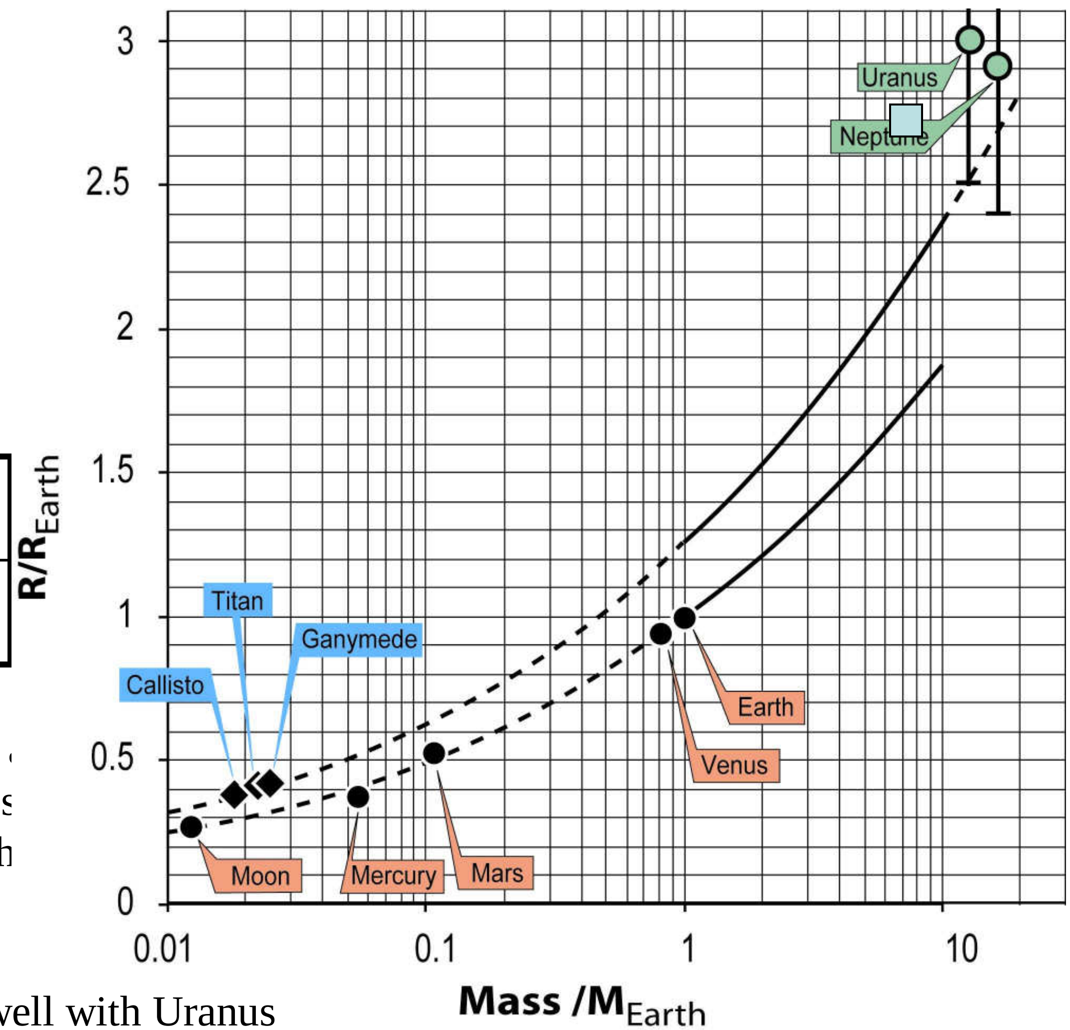


Results : Validation of the model – Solar system

$$\frac{R}{R_{Earth}} = \left(\frac{M}{M_{Earth}}\right)^{0.274}$$

	Earth-like		Ocean/Icy	
0.01-1	1.00	0.306	1.258	0.302
1-10	1.00	0.274	1.262	0.275

A planet with 50% water is 26% larger than
planet without water (for the same total mass
The points Uranus and Neptune have 1 Earth
radius of atmosphere removed.



GJ1214 has more than 50% ice in it. It fits well with Uranus and Neptune without their H2/He atmosphere

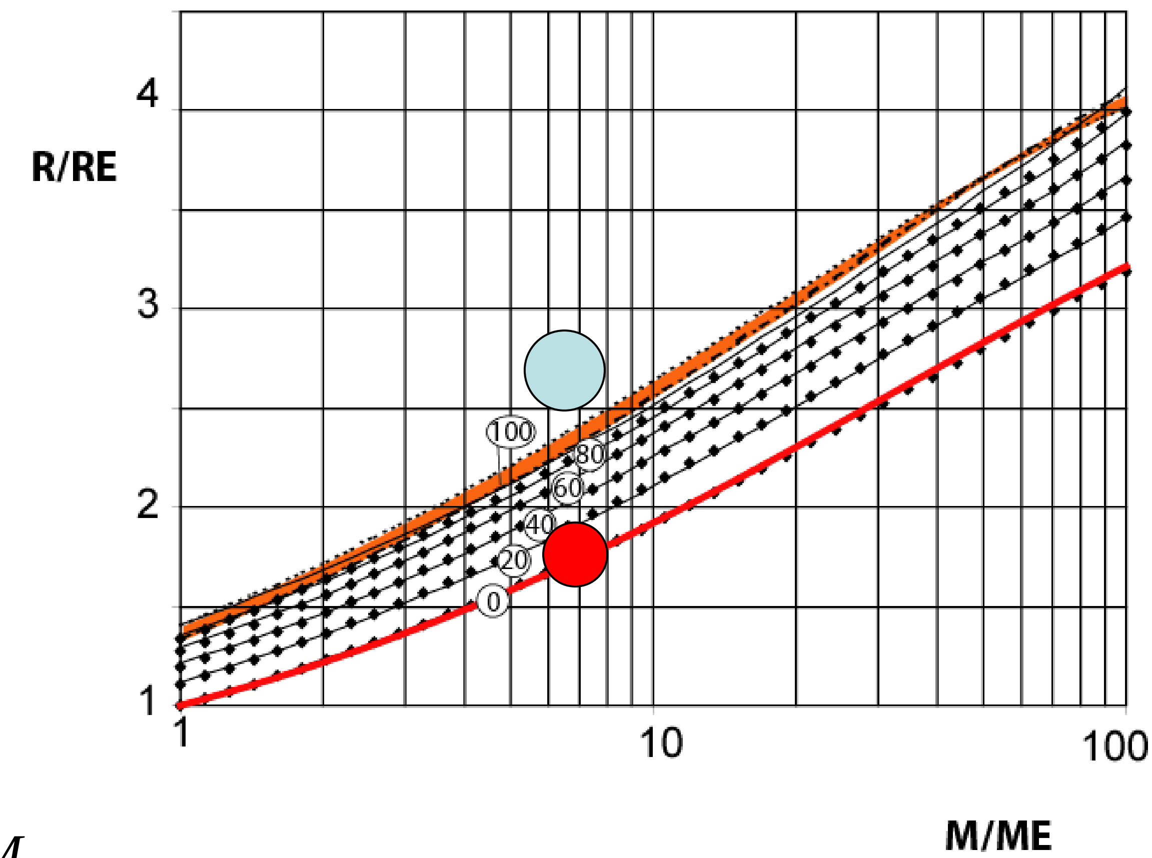
Results : Extrapolation to larger planets

$$\frac{R}{R_{Earth}} = \left(\frac{M}{M_{Earth}}\right)^{0.274}$$

Reference Case :

- **Fe/Si = 1.10 ***
- **Mg/Si = 1.25 ***
- **Mg# = 0.8**
- **H_2O: 0.01 wt %**

* Averaged from Gilli et al., A&A, 2006

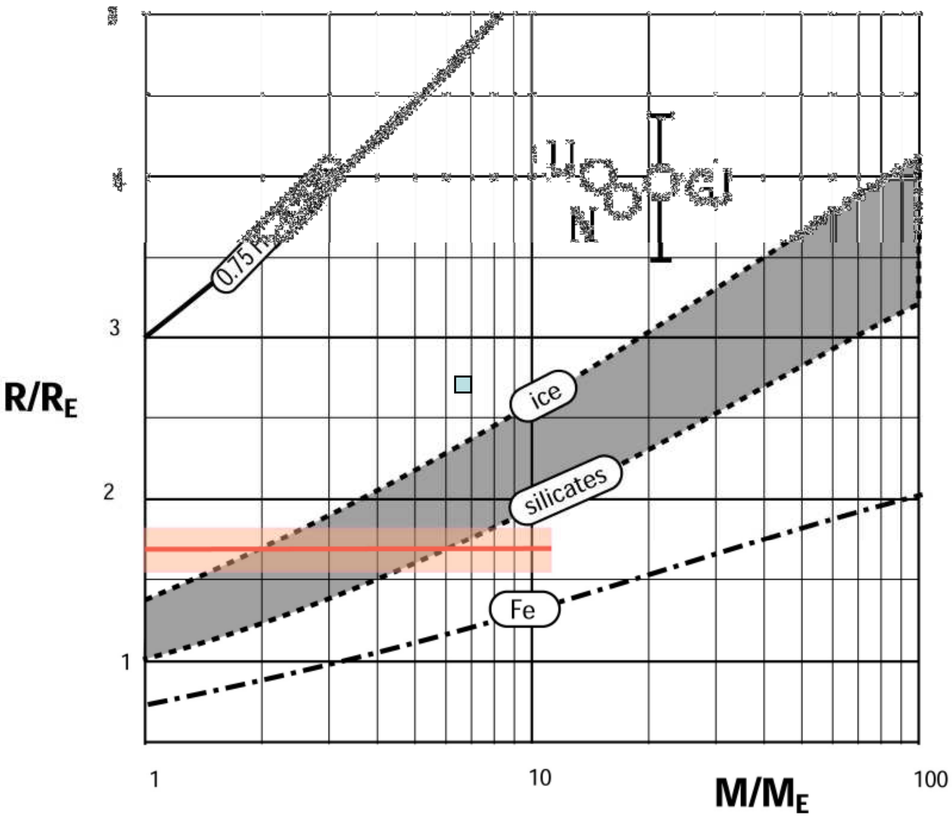


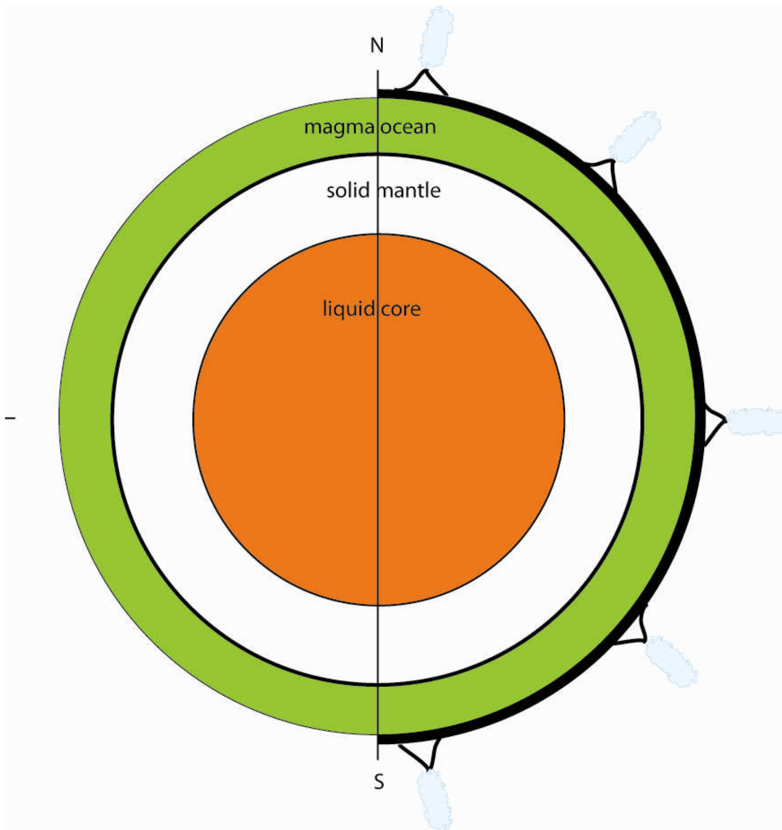
$$\log\left(\frac{R}{R_E}\right) = \log(\alpha) + \left(\beta + \gamma\frac{M}{M_E} + \varepsilon\left(\frac{M}{M_E}\right)\right)\log\left(\frac{M}{M_E}\right)$$

Each coefficient depends on the amount of water (X)

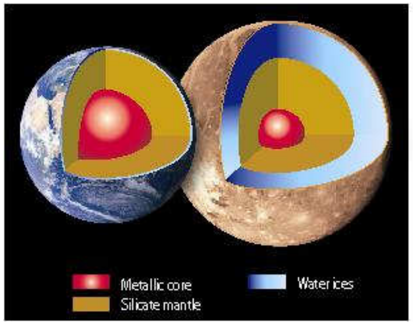
$$\xi = \sum_{i=0}^{2} \xi_i \, X_w^{\,i-1}$$

CoRoT Exo-7b & GJ1214b





Conclusions



- Models give very good prediction of radii
- Amount of water is a first order parameter
- **Radius is 26 % larger for an Ocean planet with 50 %wt of ices**
- Temperature is a second order parameters.
- Composition and Mg# control the size of the core.
- **If Mass and Radius are perfectly known, the amount of water can be known at ± 4.4 %**
- **If 10% uncertainty of mass and radius, then the amount of water can be known at ± 20 %**
- Corot7b and GJ1214b provide a good study case
- YES, super-Earths and mini-Neptunes can be distinguished
- BUT [Super-Earths with H2/He atm] give same values than mini-Neptunes

- Future study: escape rate of the H2/He atm and implications for the interior structure of mini-Neptunes