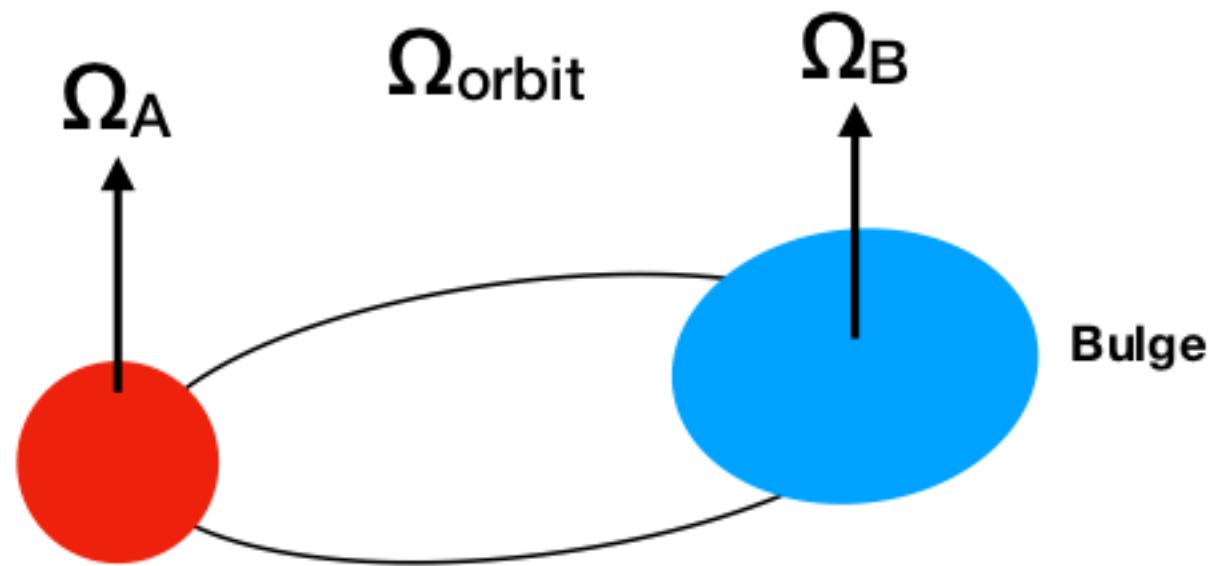


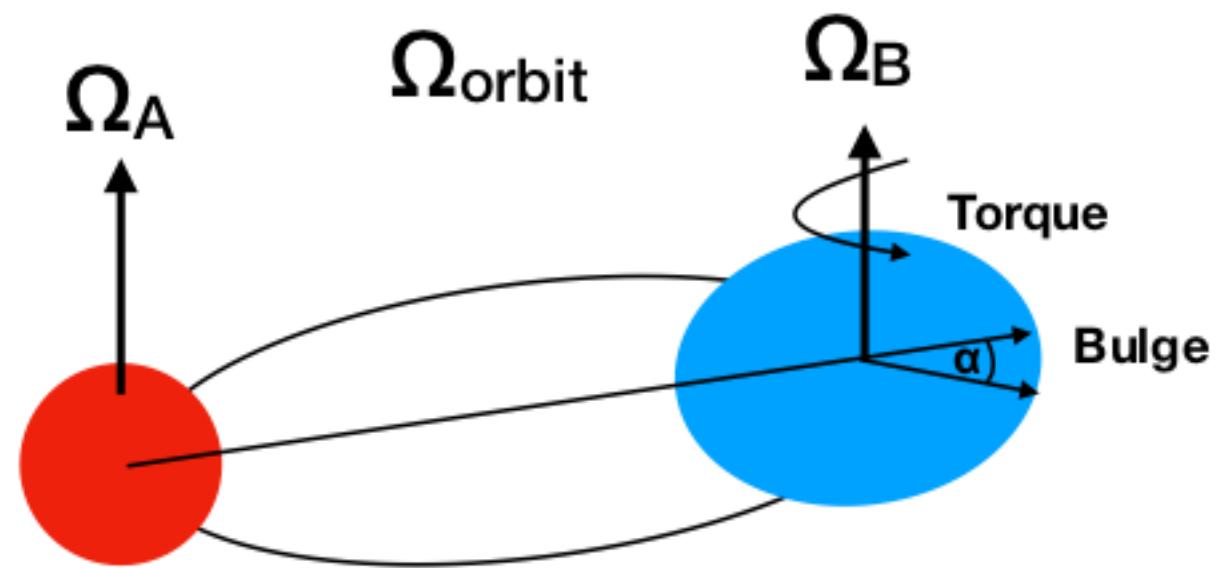
Tides, Differential Rotation, and Eclipsing Binaries

Adam Jermyn
with Jamie Tayar and Jim Fuller

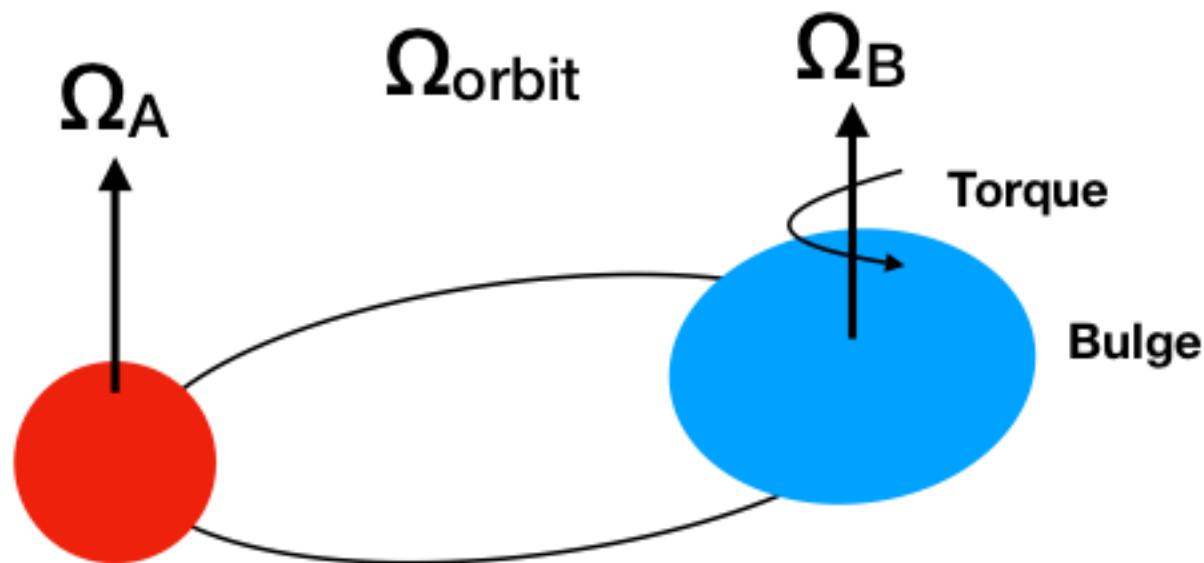
Tides



Tides

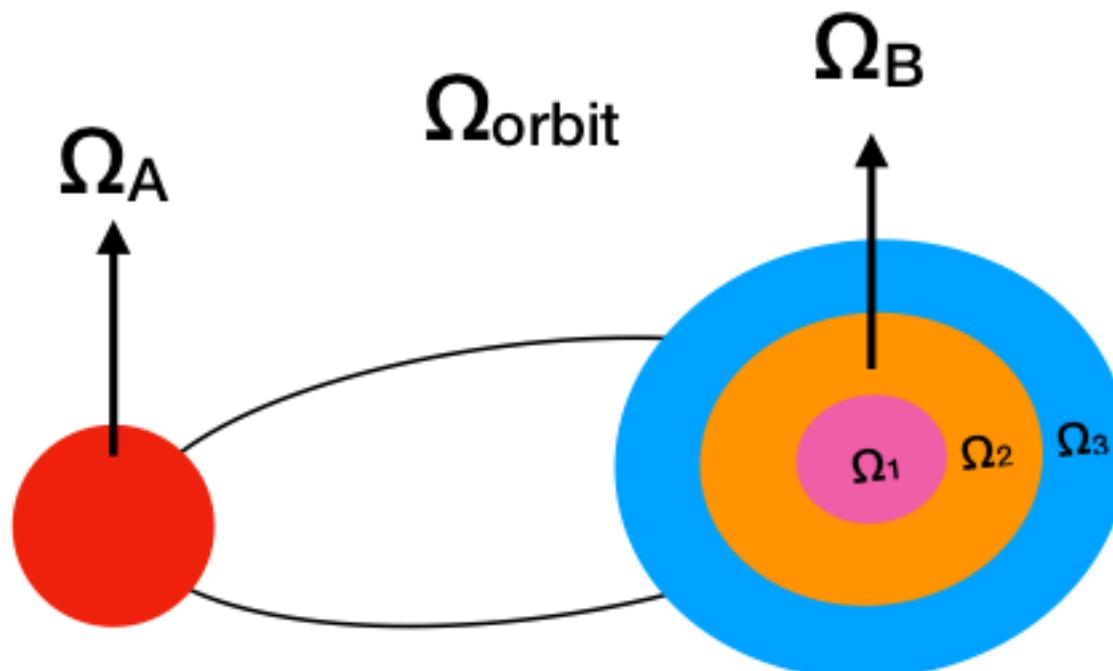


Tides



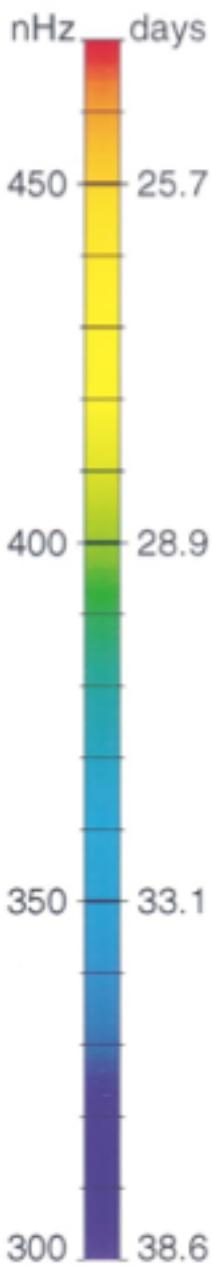
Over time, $\Omega_{A/B}$ synchronize to Ω_{orbit}

Now with differential rotation...



- 1. Does differential rotation survive a tidal torque?**
- 2. If it does, what does synchronizing mean?**

Solar Rotation Profile
(helioseismic)

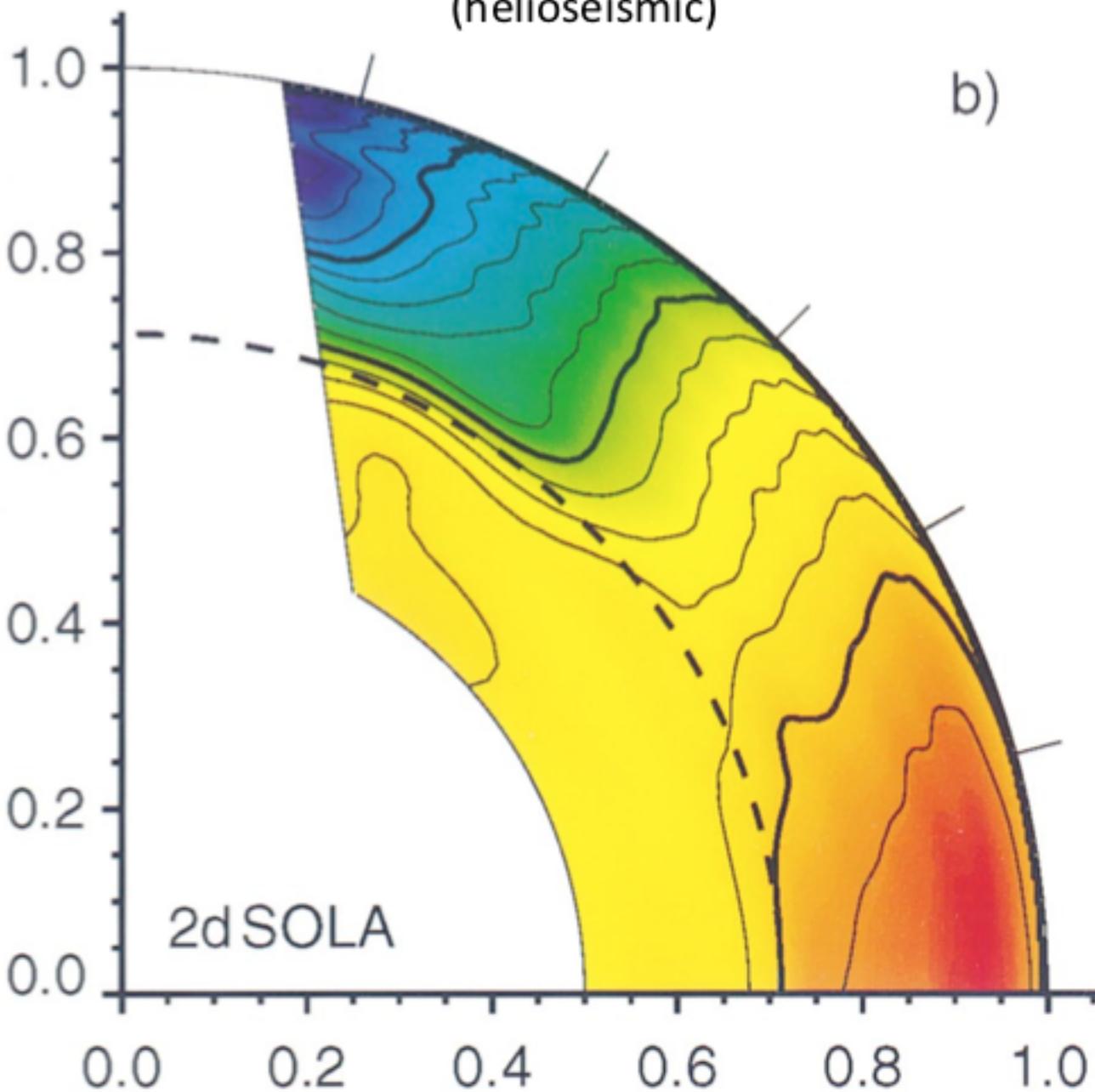


b)

Convection Zone

Tachocline

Radiative Zone



Schou et al. 1998

Does differential rotation survive?

$t_{\text{transport}} \gg t_{\text{tides}}$: Tides control the shear

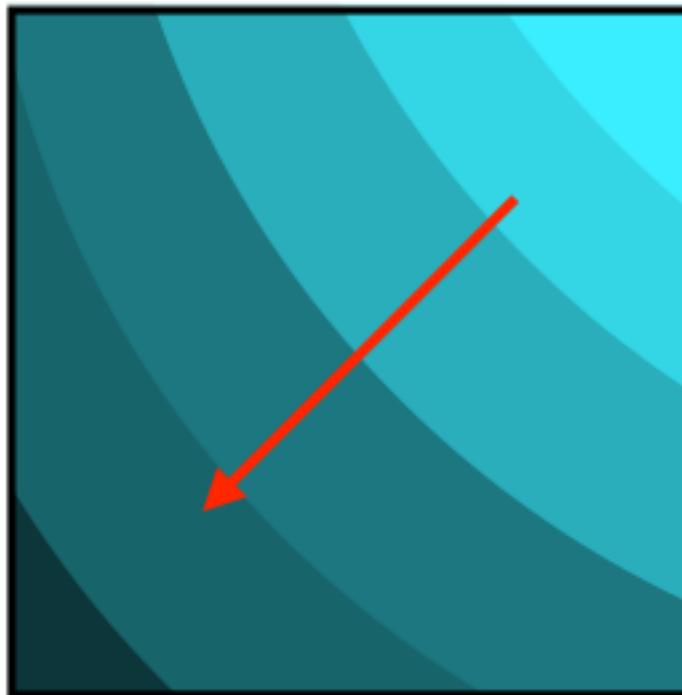
$t_{\text{transport}} \ll t_{\text{tides}}$: Intrinsic shear wins

Depends on what transports angular momentum!

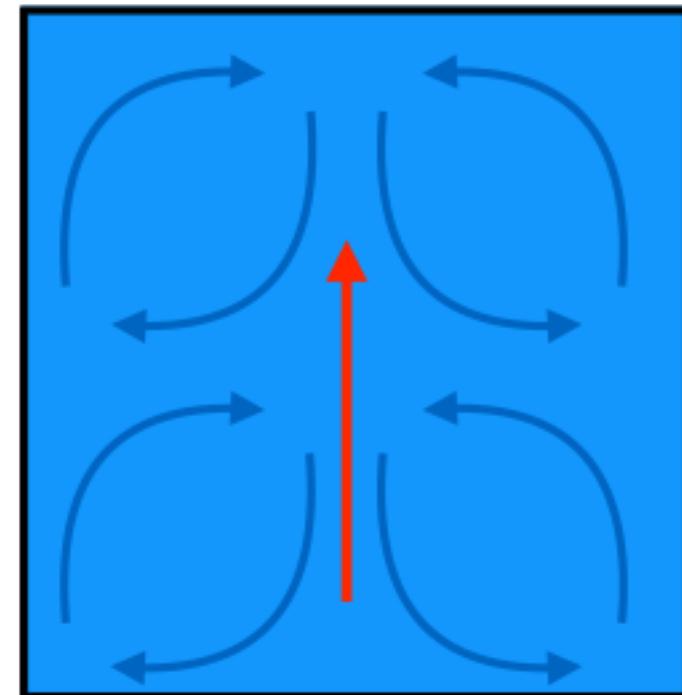
Does differential rotation survive?

Depends on what transports angular momentum!

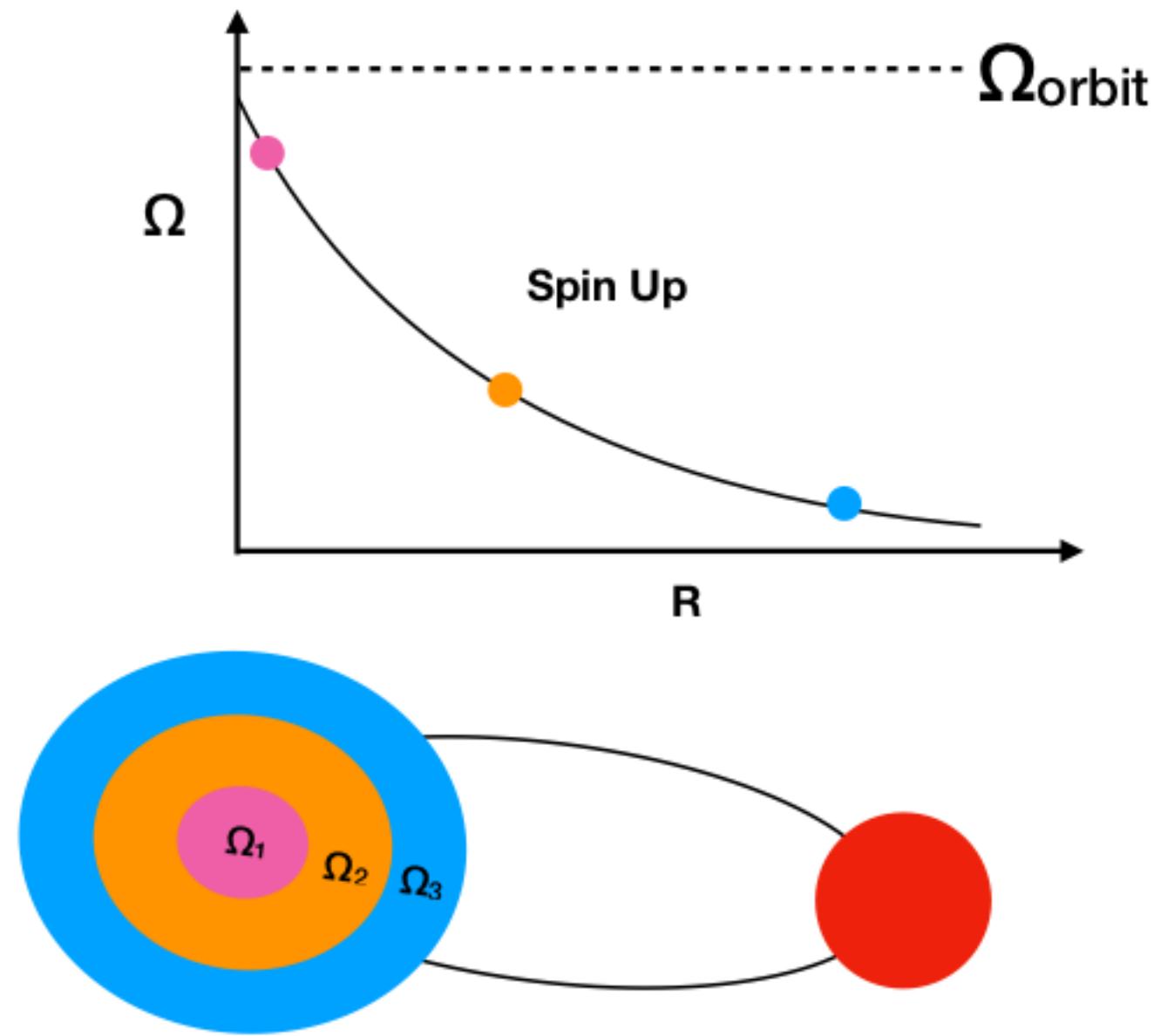
Radiative zones... unclear.



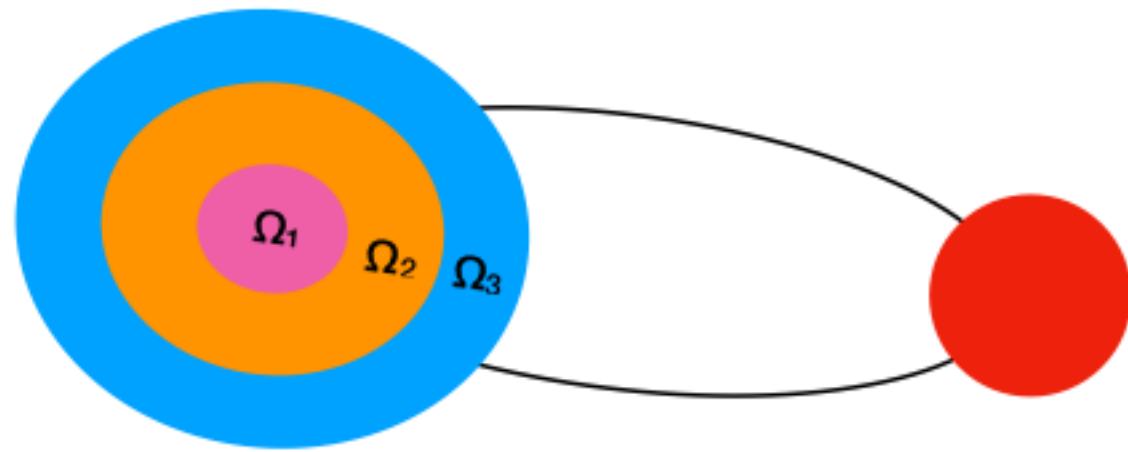
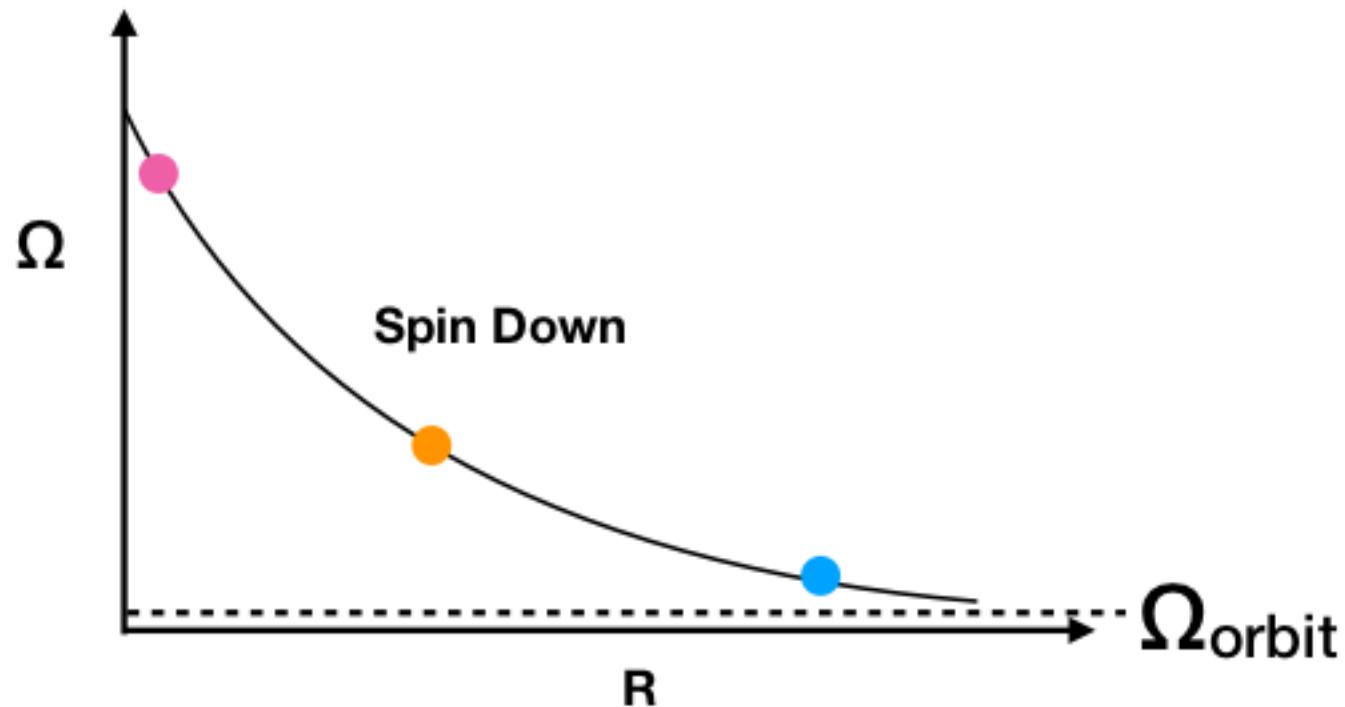
Convection zones... yes!
 $(t_{\text{conv}} \ll t_{\text{tides}})$



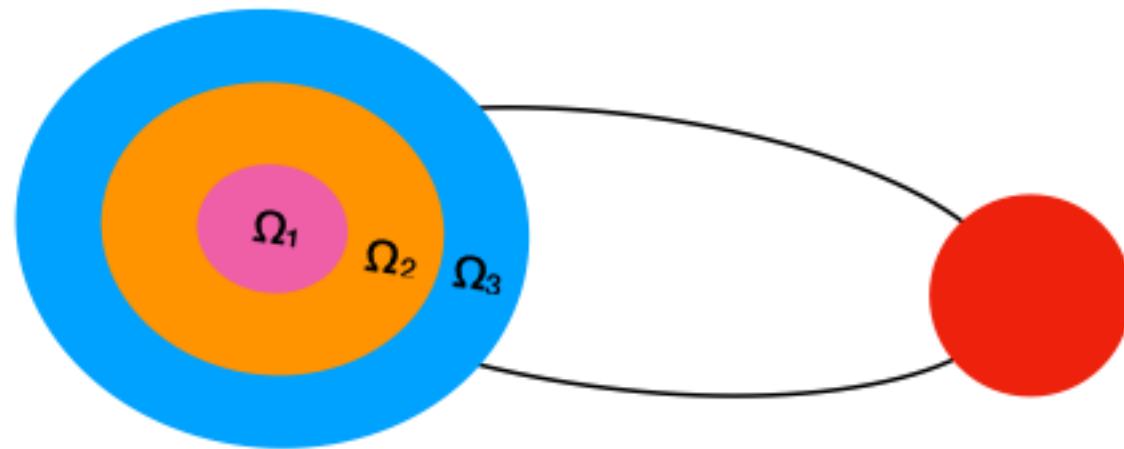
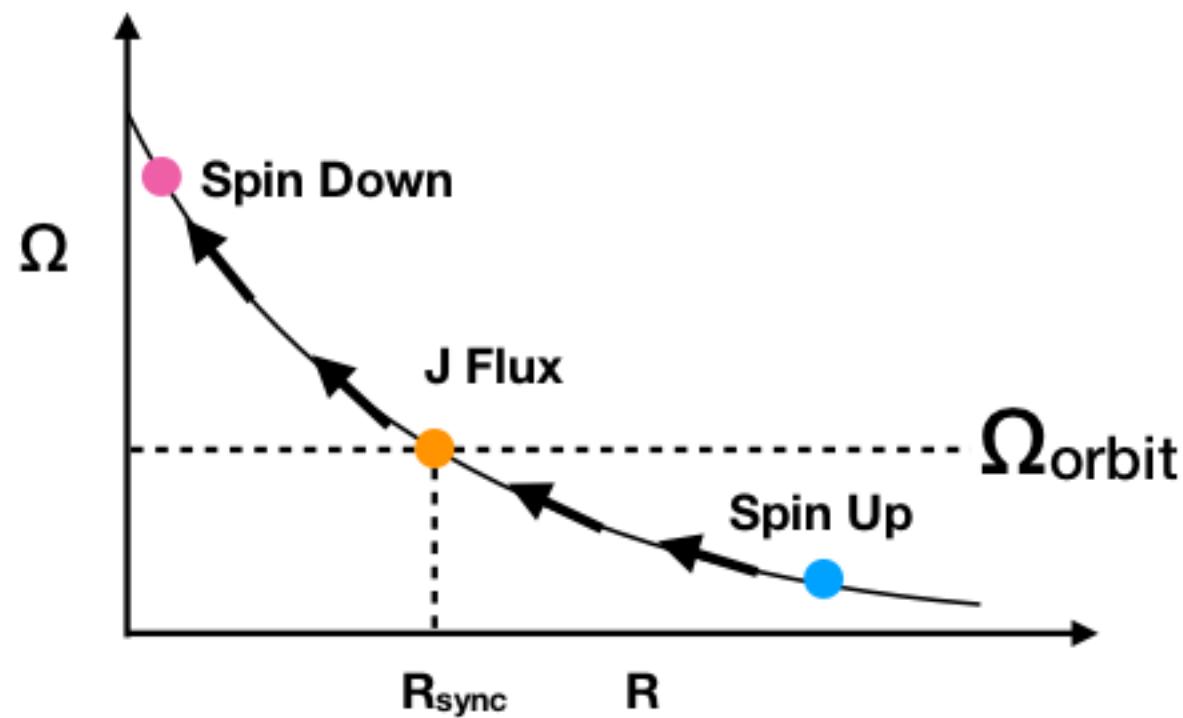
Tides in convecting stars



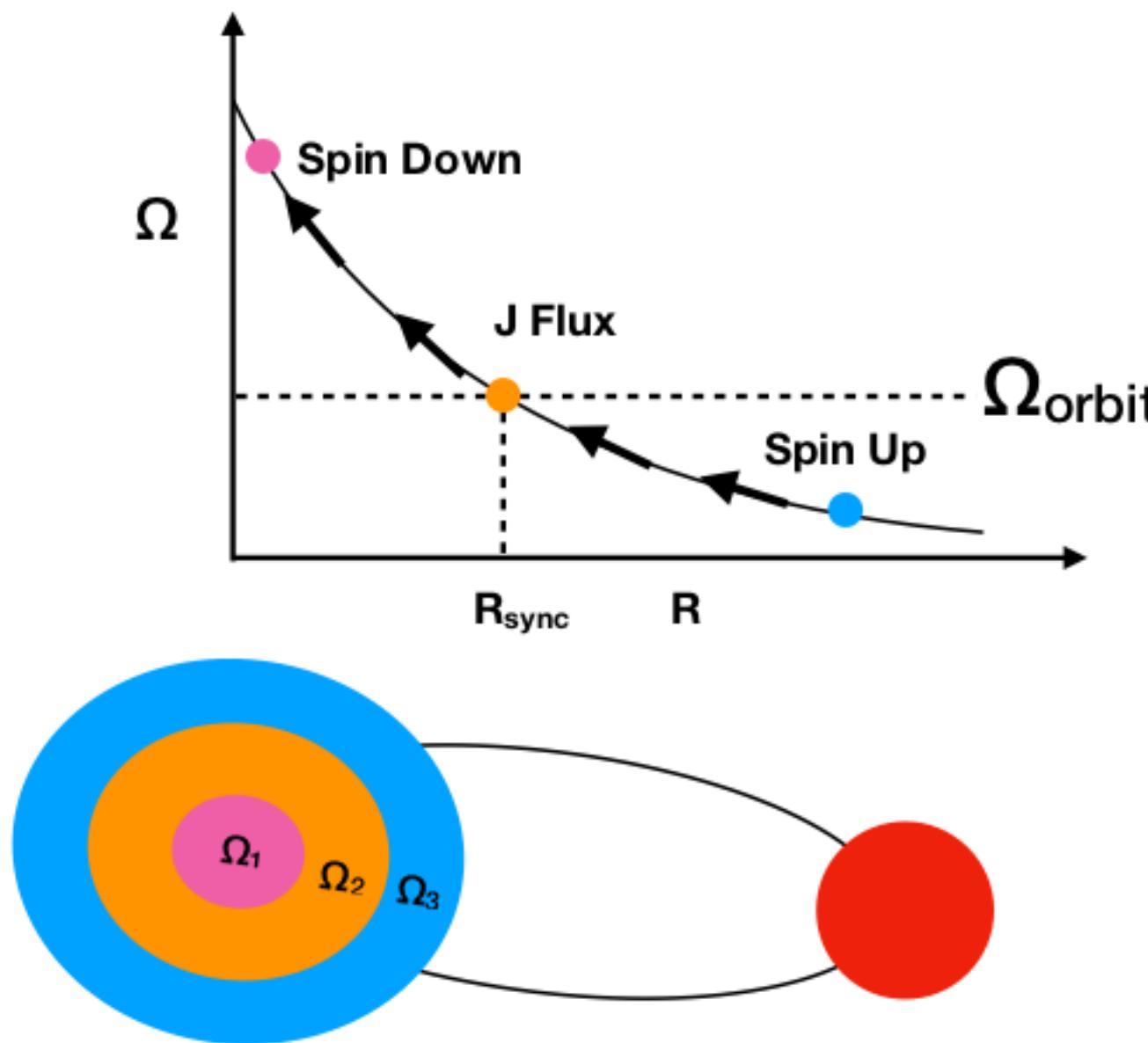
Tides in convecting stars



Tides in convecting stars



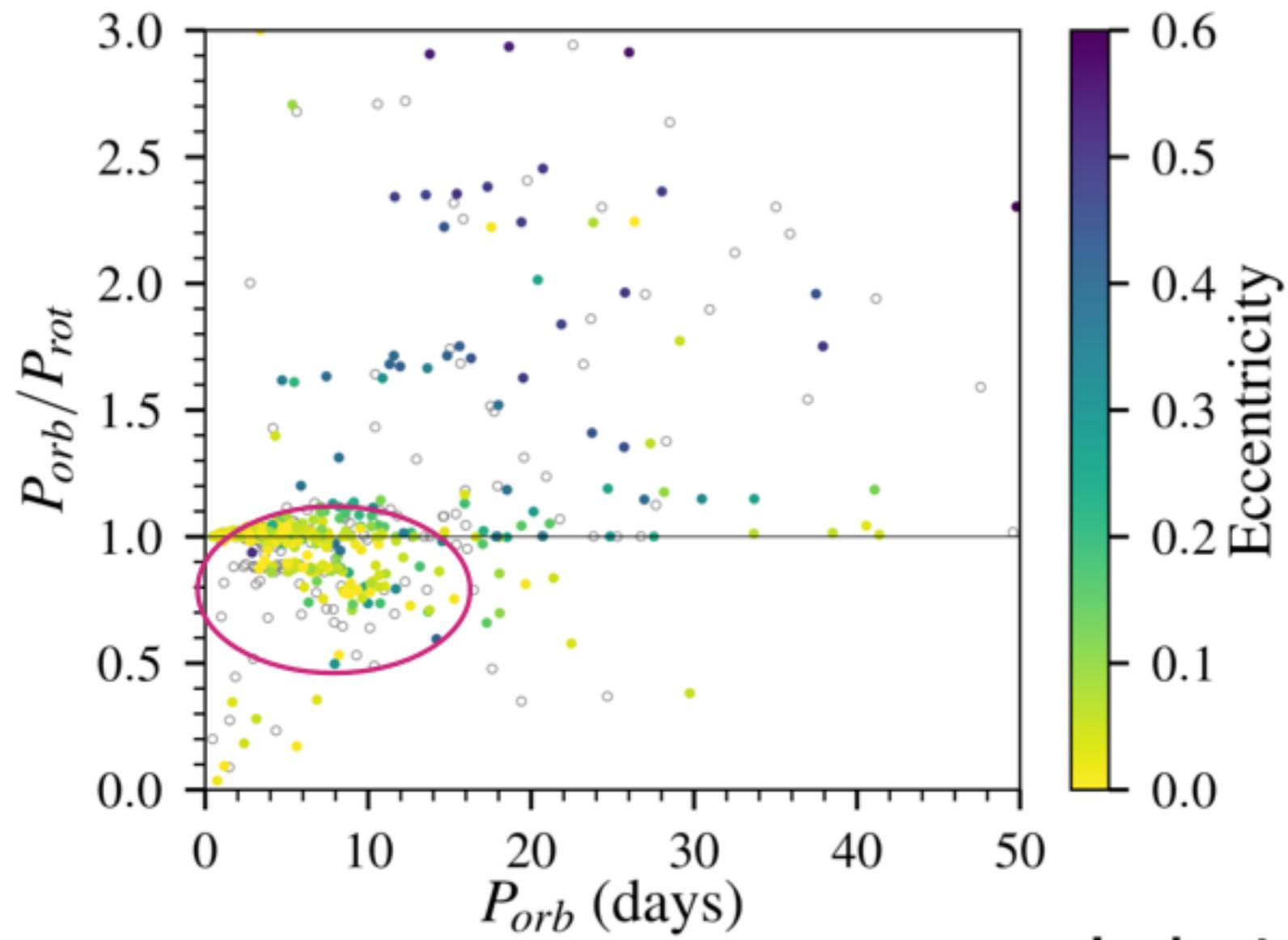
Tides in convecting stars



Synchronized:

- Net torque on each star is zero.
- Surface period may not equal orbital period!

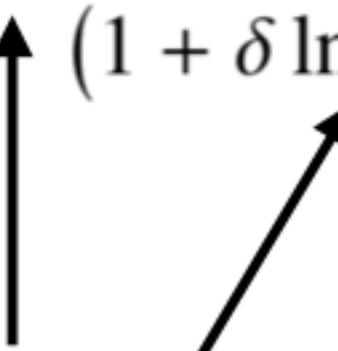
Eclipsing Binaries



Lurie et al. 2017

Inference Problem

Toy Model: $\Omega(r, \theta) = \Omega_0 \left(\frac{r}{R_\star} \right)^\beta (1 + \delta \ln \Omega P_2(\cos \theta))$



The equation is annotated with two arrows. One arrow points vertically upwards from the text "Two free parameters" to the term $\left(\frac{r}{R_\star} \right)^\beta$. Another arrow points diagonally upwards and to the right from the same text to the term $(1 + \delta \ln \Omega P_2(\cos \theta))$.

Two free parameters

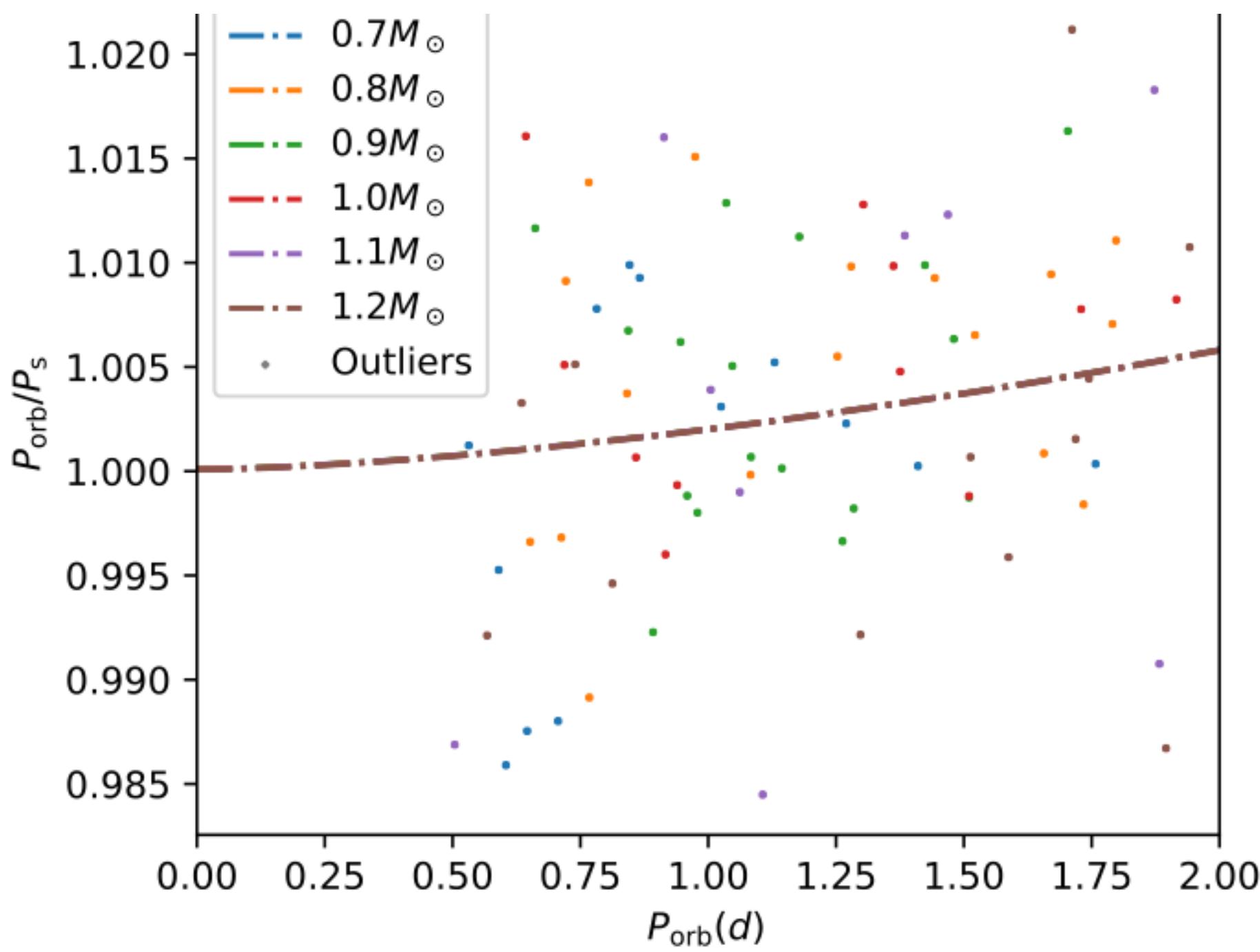
Inference Problem

Toy Model: $\Omega(r, \theta) = \Omega_0 \left(\frac{r}{R_\star} \right)^\beta (1 + \delta \ln \Omega P_2(\cos \theta))$

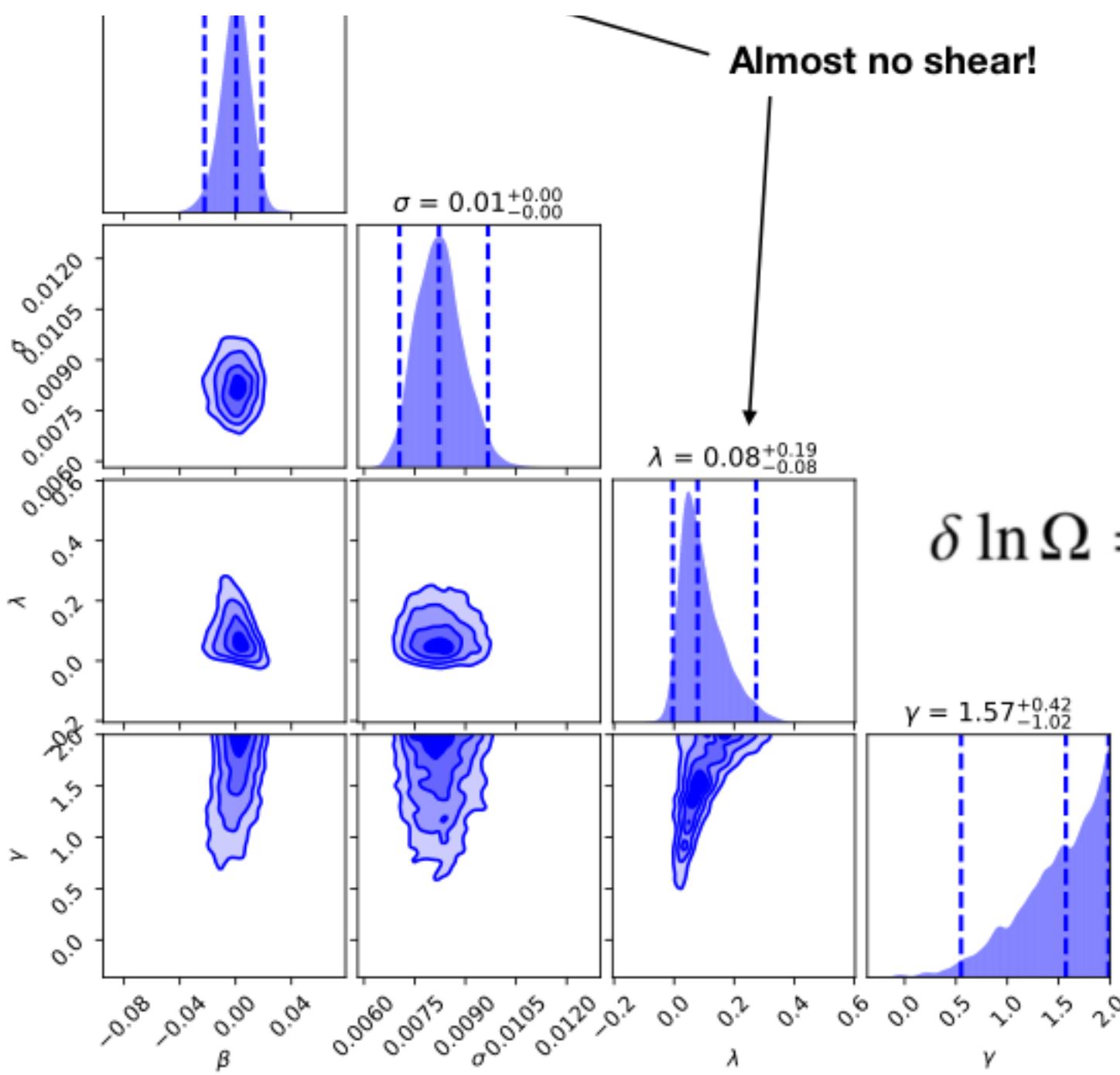
Prediction: $\frac{P_{\text{orb}}}{P_s} \approx k_\star^{-1}(\beta) \frac{1 + \frac{5}{8}\delta \ln \Omega}{1 - \frac{2}{7}\delta \ln \Omega}$ **(from tidal integrals)**



- Depends on stellar structure and .
- Assume MS, pick mass to match Kepler temperature

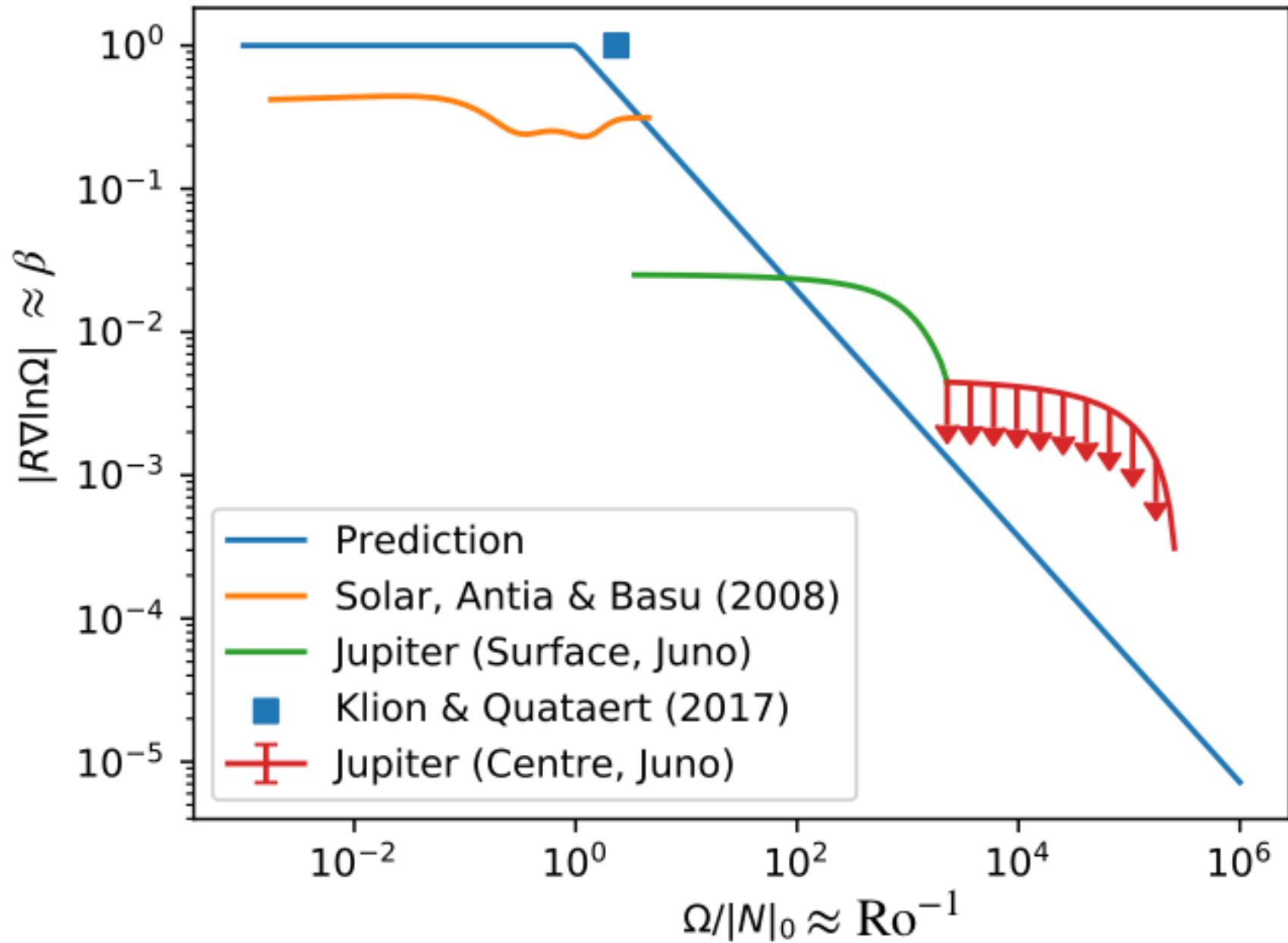


$$\beta = 0.00^{+0.02}_{-0.02}$$

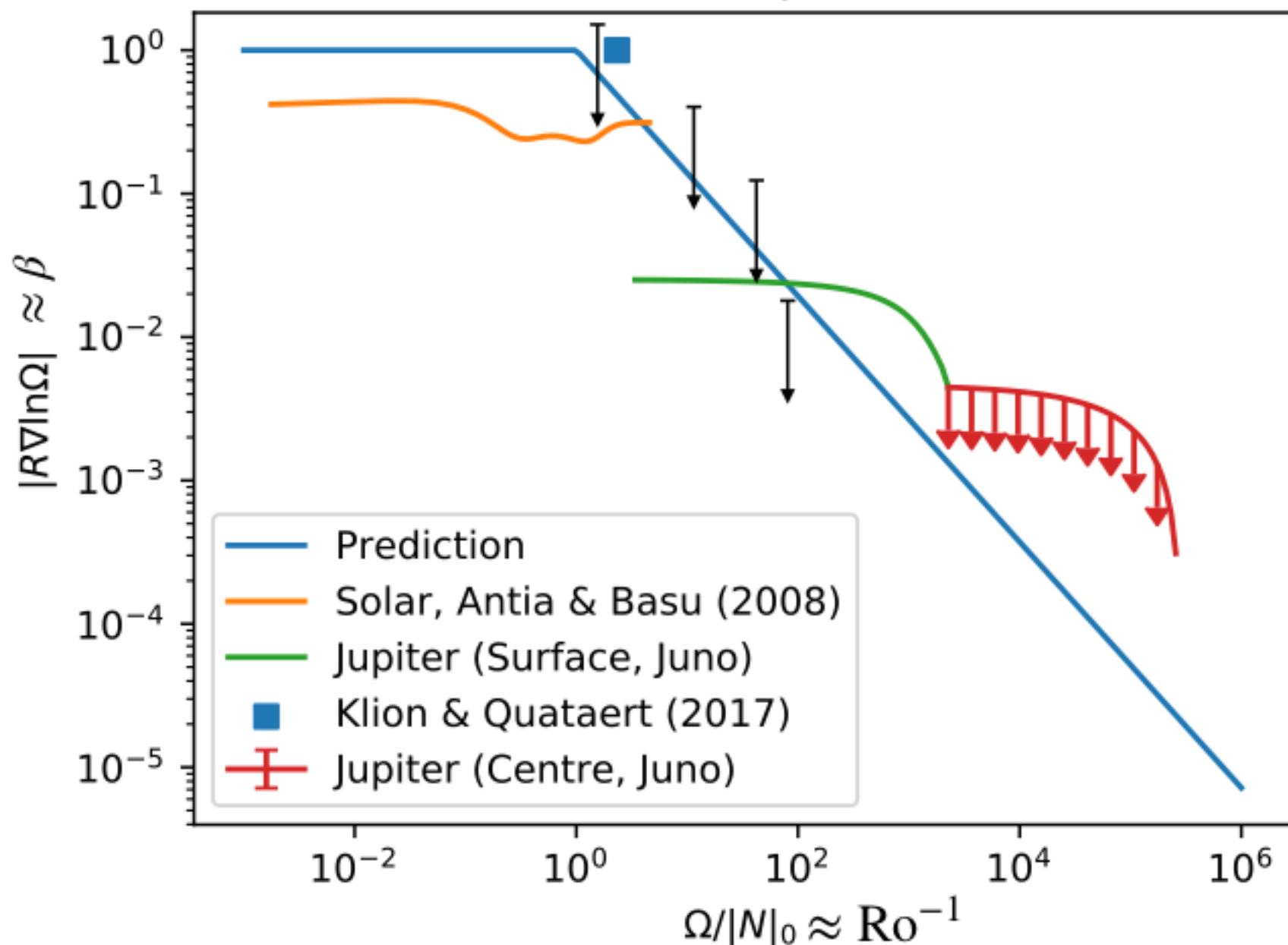


$$\delta \ln \Omega = \lambda \left(\frac{P_s}{10d} \right)^\gamma$$

$P_{\text{orb,min}}$	$P_{\text{orb,max}}$	β	$\beta_{1-\sigma}$	$\beta_{2-\sigma}$
0	50	0.152	0.251 -0.482	0.337 -0.632
0	2	0.000	0.010 -0.010	0.019 -0.022
2	6	0.031	0.091 -0.252	0.139 -0.346
6	10	0.066	0.373 -0.330	0.677 -1.186
10	50	0.264	1.329 -1.911	2.267 -2.778



Jermyn+2020 arXiv:2008.09125



$$\Omega / |N|_0 \approx Ro^{-1}$$

Jermyn+2020 arXiv:2008.09125

Questions?

$$\begin{aligned}
 &= \int (R-r)^2 r^3 \left(Q_s \left(\frac{r}{R}\right) - Q_o \right) dr \\
 &= -Q_o \int r^3 (R-r)^2 dr + Q_s \int r^3 \left(\frac{r^3-p}{R^3} \right) dr \\
 &+ Q_s \int \frac{r^3-p}{R^3} (R-r)^2 dr \\
 &= R^5 \left[Q_s Q_o - C_o Q_o \right] \\
 &= R^5 \left[\frac{20Q_o}{120-11p} - Q_o \right] \rightarrow Q_o = \frac{1}{110-p} R^5
 \end{aligned}$$

$$\begin{aligned}
 C_o &= \int_0^1 (1-x)^2 x^3 dx = \int_0^1 x^3 x^5 2x^4 dx = \frac{1}{4} + \frac{1}{6} - \frac{1}{5} = \frac{30+20-48}{120} = \frac{2}{120} = \frac{1}{60} \\
 C_s &= \int_0^1 x^3 (-x)^2 dx = \int_0^1 x^3 x^5 2x^4 dx = \frac{1}{4} + \frac{1}{6} - \frac{1}{5} = \frac{(4-p)(5-p)+(4-p)(5-p)}{120} \quad \left| \begin{array}{l} V_c \sim (F, g)^{\frac{1}{3}} \\ Q \sim (h^2 N)^{-1} (r^2 - Q_o) \end{array} \right. \\
 &= \frac{(4-p)(5-p)+(4-p)(5-p)}{120} \quad \left| \begin{array}{l} hN \sim (F, g)^{\frac{1}{3}} \\ p \sim Kg^{\frac{5}{3}}, h \sim \frac{1}{80} \sim \frac{Kg^{\frac{5}{3}}}{9} \sim R - C \end{array} \right. \\
 &\qquad\qquad\qquad \rightarrow \int^{\frac{1}{3}} \frac{h^2}{K} \sim \frac{h^2}{r^2} - \frac{(R-r)}{r^2} \\
 &\qquad\qquad\qquad \rightarrow \int^{\frac{1}{3}} \frac{h^2}{K} \sim \frac{h^2}{r^2} - \frac{(R-r)}{r^2} \\
 &\qquad\qquad\qquad \left. \begin{array}{l} \sim \left(\frac{F}{84} \right)^{\frac{1}{3}} \sim \left(\frac{F}{985} \right)^{\frac{1}{3}} \sim \frac{T^{4/3}}{J^{10} K} \\ \sim \frac{T^{4/3}}{m^{5/3}} - \left(\frac{C_o N}{12} \right)^{\frac{1}{3}} \sim \left(\frac{1}{2} \right)^{\frac{1}{3}} \end{array} \right\} \\
 &\qquad\qquad\qquad \int \frac{dI}{dm} dm = \int \frac{1}{a} \left(\frac{a}{m} - 1 \right) \left(\frac{6M_c^2}{r} \right)^{\frac{1}{2}} \frac{1}{m} dm \\
 &\qquad\qquad\qquad \frac{20-11p+20-11p+18+20p}{120} \quad \left. \begin{array}{l} \frac{48M_c^2}{m_k} \int \frac{1}{a} \left(\frac{a}{m} - 1 \right) r^{\frac{1}{2}} dr = 0 \\ \int \frac{1}{a} \left(\frac{a}{m} - 1 \right) r^{\frac{1}{2}} dr = \int \frac{h^2}{r^2} \sim \int \frac{h^2}{r^2} (S_L - S_{L_o}) dr \end{array} \right\} \\
 &\qquad\qquad\qquad = \frac{2}{120} = \frac{1}{60} = \frac{2}{(4+11p-11)(6-p)} \\
 &\qquad\qquad\qquad \approx \frac{2}{110-74p}
 \end{aligned}$$