Emergent photon + new transitions in hedgehog-suppressed O(3) σ-model

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Theme: Role of topological defects in O(3) spin model

Plan:

1. Disordering O(3) model without hedgehogs
2. Description of the phase and its ordering transitions
3. More consequences
   - Effects of Zeeman field
   - Finite temperature

3′. Easy-plane deformation

NO HEDGEHOGS, (2+1)D

4. Towards physical applications
   - Effect of finite hedgehog fugacity
   - (3+1)D Coulomb phase by hedgehog suppression
   - Origin of artificial photon in bosonic models

O(2) spins: well understood

spin waves + vortices

If no vortices → always ordered in $D \geq 3$

O(3) spins: no simple decomposition

Sharp question: what remains when all hedgehogs are suppressed

Earlier attempts: Monte Carlo studies of O(3) spin model with eliminated hedgehogs

Lau & Dasgupta (89) complete suppression;
- found only ordered phase

Kamal & Murthy (93) less restrictive (allowing hedgehog-anti hedgehog pairs);
- found also disordered phase;
- new O(3) ordering transition in $D=3$ (some complicating aspects)

This work:

- Explicit stat mech O(3) spin system with no hedgehogs and disordered phase
- Identification of topological order in this phase; gapless photon excitation
- Effective description

$$S_{NCCP^1} = -\frac{J}{2}\sum_{r\mu}\left(z_r^\dagger z_{r+\hat{\mu}} e^{i a_{r\mu}} + c.c.\right) + \frac{K}{2}\sum(\vec{\nabla}\times\vec{a})^2$$

(on a lattice)

or

$$\int d\vec{x}\left\{\left|(\vec{\nabla}-i\vec{A})\Psi\right|^2 + r|\Psi|^2 + u|\Psi|^4 + (\vec{\nabla}\times\vec{A})^2\right\}$$

$$\Psi = \begin{pmatrix}\psi_\uparrow\\ \psi_\downarrow\end{pmatrix}$$

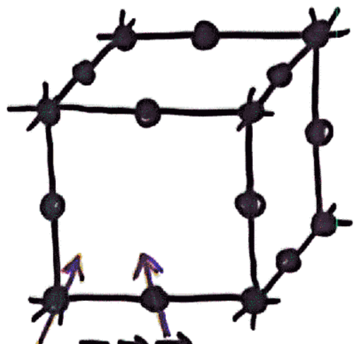
(in continuum)

gapped spin-$\frac{1}{2}$ spinons
+ noncompact gauge field

- Ordering transition described by this field theory

Our model

- Heisenberg spins on a lattice, ferro interactions



$-J\vec{n}_i\cdot\vec{n}_j$

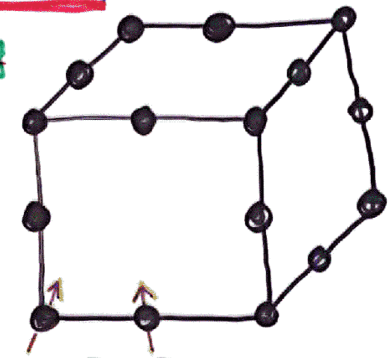
- Spin configuration $\{\vec{n}_i\}$ → → hedgehog # in each cube (formalizing

= 1 hedgehog)

- Disallow all configurations with free hedgehogs

Our model

Heisenberg spins on a sparse lattice



$-J\vec{n}_i\cdot\vec{n}_j$

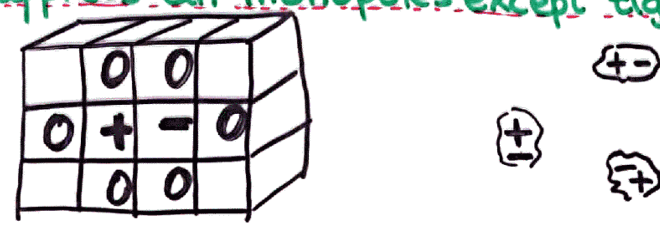
- "sparseness" needed to realize hedgehog suppression on a coarse scale; otherwise ordering effect too strong
- this model is disordered even with complete monopole suppression for $J \le 0$. (weakly AF interaction)

Monopole # in each cube defined by:

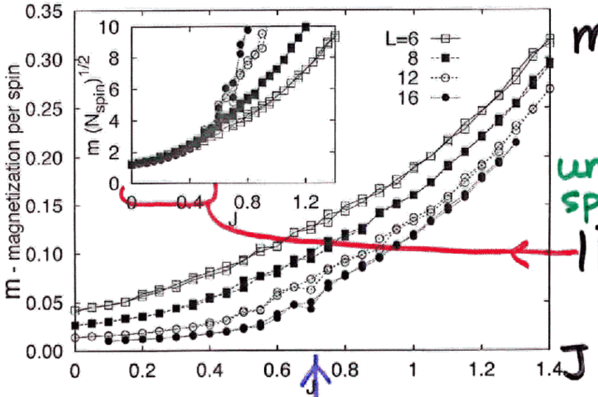
$$e^{iA_{12}} = e^{i\frac{\Omega[\vec{N}_0,\vec{n}_1,\vec{n}_2]}{2}} = \frac{1+\vec{N}_0\cdot\vec{n}_1+\vec{N}_0\cdot\vec{n}_2+\vec{n}_1\cdot\vec{n}_2+i\vec{N}_0\cdot(\vec{n}_1\times\vec{n}_2)}{\sqrt{2(1+\vec{N}_0\cdot\vec{n}_1)(1+\vec{N}_0\cdot\vec{n}_2)(1+\vec{n}_1\cdot\vec{n}_2)}}$$

$$e^{iF_\square} = e^{i(A_{12}+A_{23}+\dots+A_{n1})}; \quad k_{\text{cube}} = \frac{\sum_\square F_\square}{2\pi}$$

Suppress all monopoles except tightly bound pairs



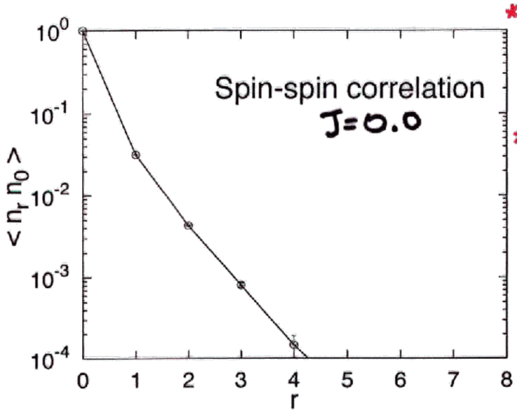
Evidence for disordered phase



$$m = \frac{\langle |\vec{M}_{tot}| \rangle}{N_{spin}}$$

uncorrelated spins $|\vec{M}_{tot}| \sim N^{1/2}$

$J_c = .725$



J=0.0

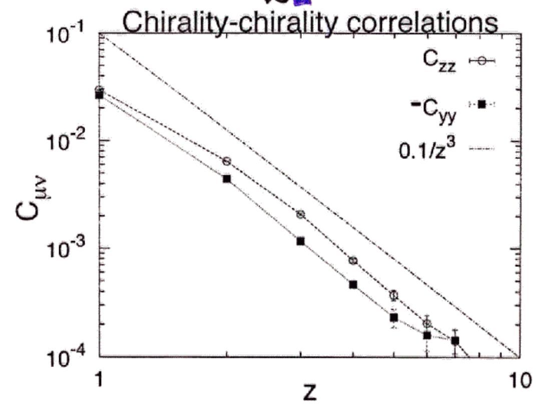
* all spin-spin correlations ferromagnetic
* decay exponent. with distance

Topological order in the P^* phase

short-ranged spin-spin correlations but...

! chirality $\chi_{123} \sim \vec{n}_1 \cdot (\vec{n}_2 \times \vec{n}_3)$ has power law !

$$C_{\mu\nu}(r) = \langle \underbrace{\sin F_\mu(r)}_{\chi} \sin F_\nu(0) \rangle$$



$C_{zz} \approx \frac{2B}{z^3}$

$C_{yy} \approx -\frac{B}{z^3}$

$$\boxed{C_{\mu\nu}(\vec{r}) \sim \frac{3 r_\mu r_\nu - \delta_{\mu\nu} r^2}{r^5}}$$

"dipole"

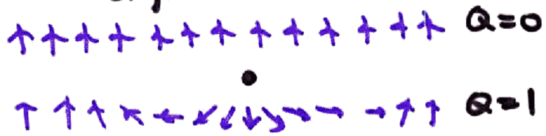
$$C_{\mu\nu}(r) \sim \frac{3 r_\mu r_\nu - \delta_{\mu\nu} r^2}{r^5}$$

Striking !

... but not so surprising

$$F_z = \int d^2x \, (\partial_x A_y - \partial_y A_x) = 2\pi \, Q_{\text{skyrmion}}$$

monopole $\leftrightarrow$ $\Delta Q_{\text{skyrm.}} = \pm 1$ event



Inhibited monopoles $\rightarrow$ conserved skyrmion currents j_μ

$$\partial_\mu j_\mu = 0 \;\rightarrow\; \vec{j} = \vec{\nabla} \times \vec{a}$$

Short-ranged current-current interactions

$$S \sim \int d^3r \, \vec{j}^{\,2} \sim \int (\nabla \times a)^2 \, d^3r$$

$$\Rightarrow \boxed{\langle j_\mu(r) \, j_\nu(r) \rangle \sim \frac{3 r_\mu r_\nu - \delta_{\mu\nu} r^2}{r^5}}$$

Effective NCCP^1 description

O(3) spins (on a lattice) $\leftrightarrow$ lattice CP^1

$$S_{\text{Heis}} = -\sum_{\langle ij \rangle} J \vec{n}_i \cdot \vec{n}_j \qquad S_{CP^1} = -\sum \left(J z_i^\dagger z_j e^{i a_{ij}} + h.c. \right) - K \sum \cos(\nabla \times a)$$

compact

$$\vec{n} = z^\dagger \vec{\sigma} z ; \quad z = \begin{pmatrix} z_\uparrow \\ z_\downarrow \end{pmatrix}, \; |z|^2 = 1$$

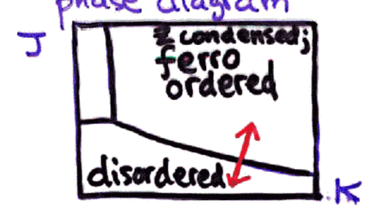
hedgehogs $\leftrightarrow$ monopoles of the gauge field

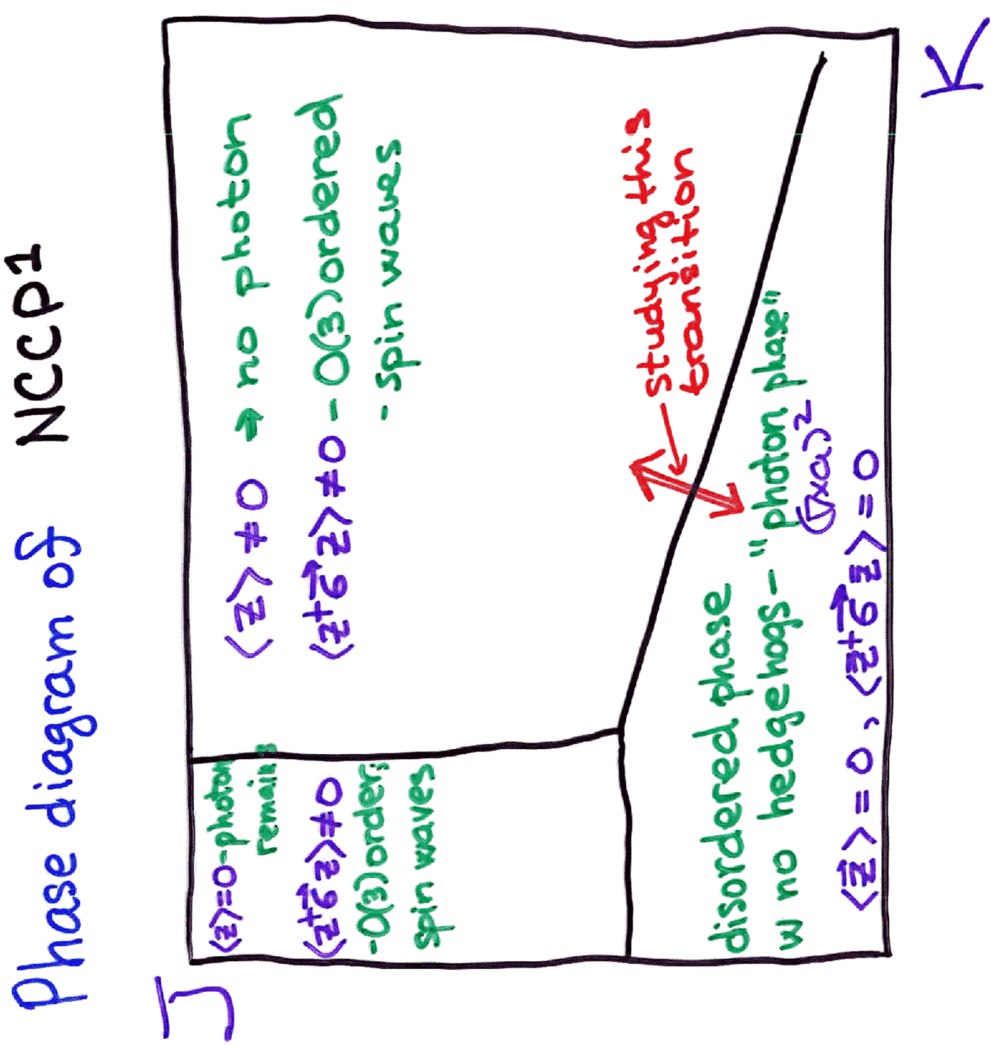
Inhibit hedgehogs $\rightarrow$ inhibit monopoles

$\rightarrow$!! NONcompact gauge field

$$\boxed{S_{NCCP^1} = -\sum J z_i^\dagger z_j e^{i a_{ij}} + h.c. + \frac{K}{2} \sum (\vec{\nabla} \times \vec{a})^2}$$

phase diagram



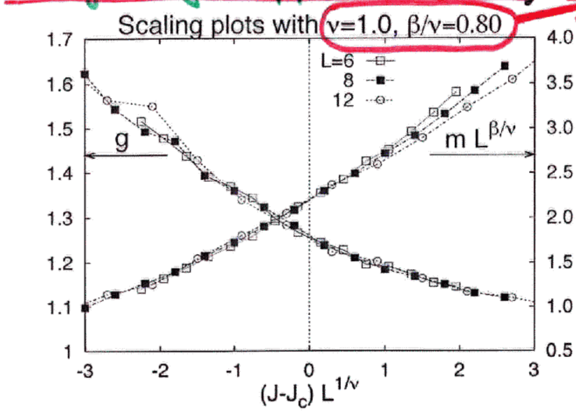


Critical properties

correlation length $\xi \sim \delta^{-\nu}$
magnetization $m \sim \delta^{\beta}$

Hedgehog-suppressed O(3)

$\eta \approx .6$

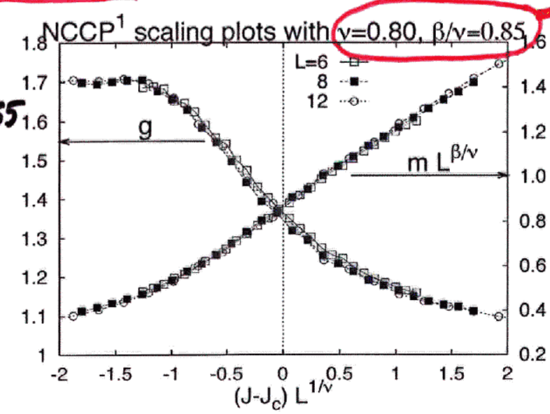


continuous transition

NCCP1

$K = .6$
$J_c = 1.255$

$\eta \approx 0.7$



O(3) Heisenberg

3

$\nu \approx .705$; $\beta/\nu \approx .517$

$\eta \approx .033$

Effect of Zeeman field

O(3) spin model : apply $-h \sum_i n_i^z$

NCCP1 $-h \sum (|z_\uparrow|^2 - |z_\downarrow|^2)$

→ will induce $\langle n_i^z \rangle > 0$ for $h > 0$

h=0 spinons are gapped $m(|z_\uparrow|^2 + |z_\downarrow|^2)$, $m>0$

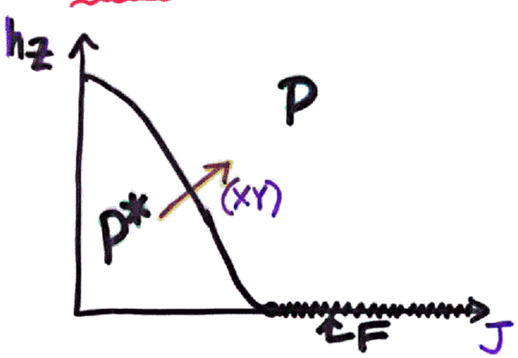
small h>0 the photon phase is stable

photon → $\frac{1}{r^3}$ chirality correlations

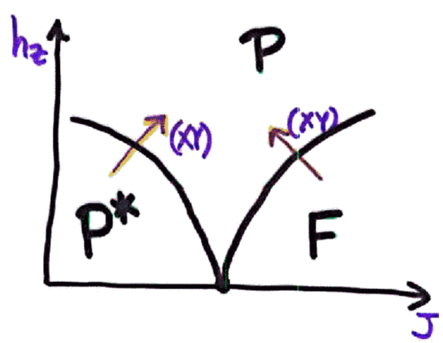
increasing h $z_\uparrow$ condenses; usual paramagnet (inverted XY transition)

* directly tested

O(3) phase diagram



Easy plane



Effect of finite temperature

(T>0 ↔ 2D classical)

T=0 P* phase : gapped spinons, carry gauge charges → interact logarithmically

small T>0 finite density of thermally excited particles;

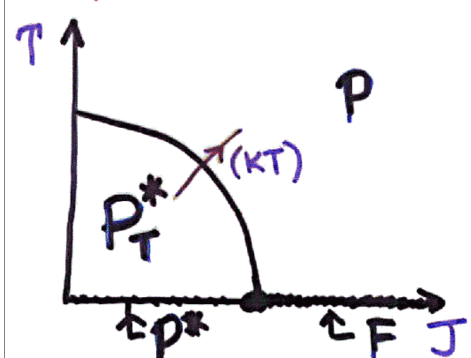
charges are bound into gauge-neutral pairs

→ the photon phase is stable – P^*_T ("thermal" P^* at $T \neq 0$) (dielectric)

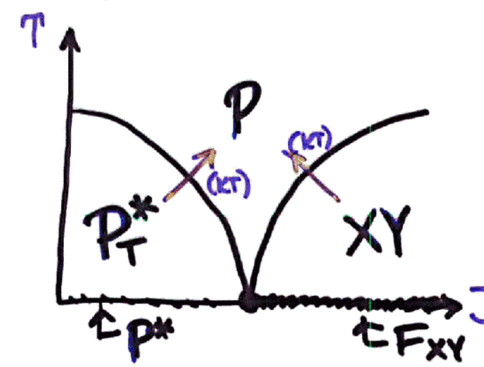
increasing T unbinding (KT) transition

plasma of gauge charges

O(3)



Easy-plane



Easy-plane deformation

Spin model:

$$J\,\vec{n}_i\cdot\vec{n}_j \longrightarrow J_{xy}(n_i^x n_j^x + n_i^y n_j^y) + J_z n_i^z n_j^z$$
$$J_{xy} > J_z$$

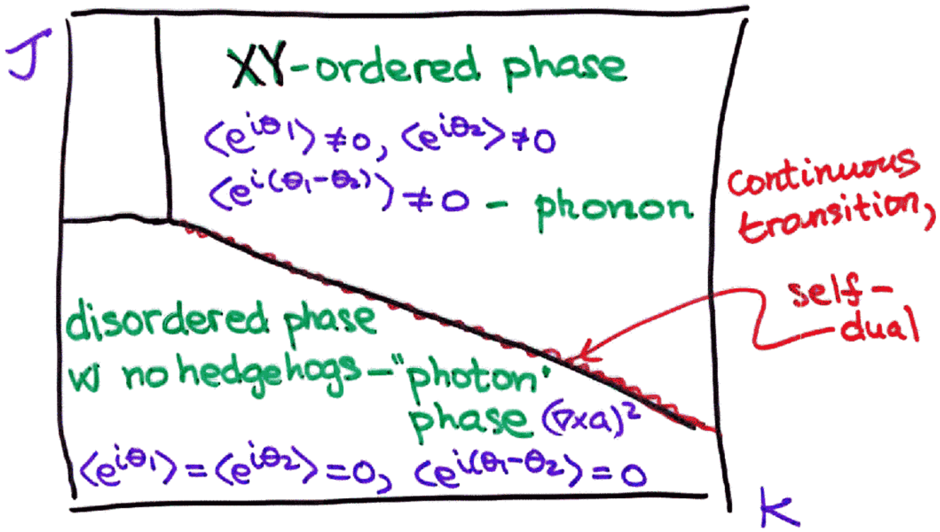
NCCP^1 model: add $-v(|z_\uparrow|^2 - |z_\downarrow|^2)^2$

$v > 0$ prefers $|z_\uparrow| \approx |z_\downarrow|$

Extreme easy-plane: $|z_\uparrow| = |z_\downarrow|$

$z_\uparrow \sim e^{i\theta_1}$, $z_\downarrow \sim e^{i\theta_2}$

$$S = -J\sum[\cos(\nabla\theta_1 - a) + \cos(\nabla\theta_2 - a)] + K\sum(\nabla\times a)^2$$



Self-duality in the easy-plane model

Particle description

two particles: $b_1^+ \sim e^{i\theta_1}$, $b_2^+ \sim e^{i\theta_2}$

both carry gauge charge

$\boxed{n_1 + n_2 \neq 0}$ - - - log-interaction - - - $\boxed{n_1' + n_2' \neq 0}$

$\boxed{\begin{array}{c} n_1 + n_2 = 0 \\ n_1 - n_2 \text{ arbitrary} \end{array}}$ - - - gauge-neutral short-ranged interaction - - - $\boxed{\begin{array}{c} n_1' + n_2' = 0 \\ n_1' - n_2' \text{ arb.} \end{array}}$

Vortex description

two vortices: ℓ_1-vortex in θ_1; ℓ_2-vortex in θ_2

$\boxed{\ell_1 - \ell_2 \neq 0}$ cannot be screened - - - log-interaction - - - $\boxed{\ell_1' - \ell_2' \neq 0}$

$\boxed{\begin{array}{c} \ell_1 - \ell_2 = 0 \\ \ell_1 + \ell_2 \text{ arb.} \end{array}}$ screened by a - - - short-ranged interactions - - - $\boxed{\begin{array}{c} \ell_1' - \ell_2' = 0 \\ \ell_1' + \ell_2' \text{ arb.} \end{array}}$

Transition

$P^* \to$ XY-ord. – condensation of particles

XYord $\to P^*$ – condensation of vortices

critical – both descriptions are identical

Towards physical applications

This work: Suppressed hedgehogs by hand
(& for all parameter values)

If hedgehogs are allowed w finite fugacity

Polyakov → hedgehogs proliferate and
destroy the photon phase

assumes gapped matter

This may no longer be true in the presence of gapless matter fields (e.g. at the critical point) — connection of this work w the next two talks

- hedgehogs may remain suppressed in the effective sense

(3+1)D Coulomb by hedgehog suppression

In principle, can disorder (3+1)D O(3) model without hedgehogs

- haven't succeeded in the spin model (hard to fight against ordering), but certainly happens in $NCCP^1$ model

Start in such (3+1)D phase, allow finite hedgehog fugacity:
the photon phase is stable since monopoles are point particles (worldlines in spacetime)

- fractionalization in a model with full SU(2) symmetry

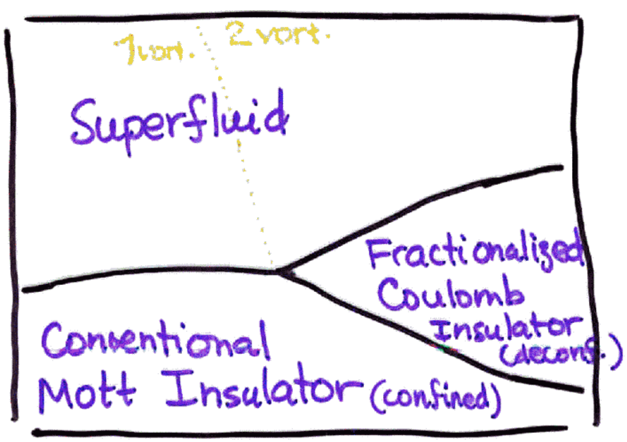
Perspective on the origin of artificial photon in three-dimensional bosonic models

(with T. Senthil)

Explicit bosonic models that realize Coulomb phase

* Foerster, Nielsen, Ninomiya '80
* X.G.Wen '01 (ferms)
* O. Motrunich and T. Senthil '02
* X.G. Wen '02
* M. Hermele, L. Balents, and M.P.A. Fisher '03
* D. Huse, W. Krauth, R. Moessner, and S.L. Sondhi '03
* R. Moessner and S. Sondhi '03

Phase diagram

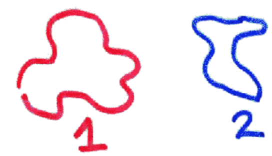


Proper description

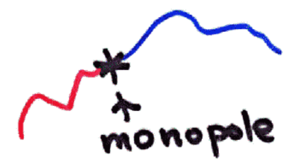
$$S = -J\sum\left[\cos(\nabla\theta_1 - a) + \cos(\nabla\theta_2 - a)\right] - K\sum\cos(\nabla\times a)$$

compact!

If no monopoles → two vortices (separately conserved)



Include monopoles



(3+1)D – stable possibility for such "domain wall" particle to remain gapped

→ TWO DISTINCT PHYSICAL VORTICES in SF

proliferate both vortices w equal amplitude

→ Coulomb phase