Critical phases:
strange elasticity of liquid-crystal rubber

University of Colorado

Outline

- Fluctuations and critical phases
- Liquid-crystals
- Rubber
- Nematic elastomers (with ***Xing, Lubensky, Mukhopadhyay***)
- Predictions

support by: ***NSF-MRSEC, Materials Theory, Packard Foundation***

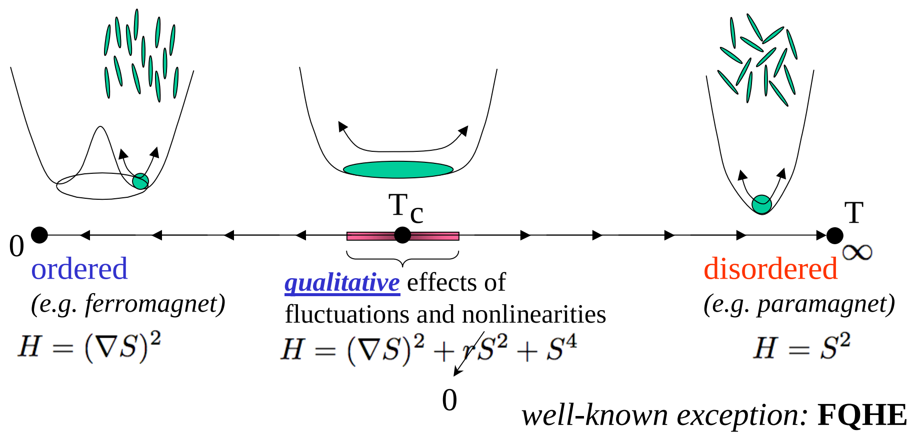
Why here?

- *classical stat. mech.* ⟷ *quantum field theory*
  (transfer matrix) (path-integral)
- *Examples:*
  - *2d crystals (xy-model)* ⟷ *1d Luttinger liquids*
  - *smectics* ⟷ *superconductor and Higgs mechanism*
  - *sliding phase of DNA-ionic complexes* ⟷ *$d > 1$ LL*
  - *Z_2 gauge theory of classical IN transition* ⟷ *modern quantum fractionalization ideas*

Fluctuations, nonlinearities and phase transitions

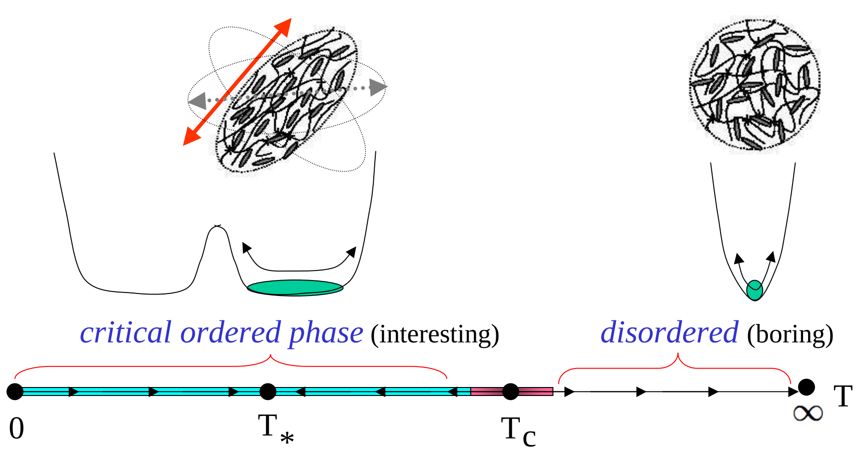
Upshot of 40 years of research on fluctuations and critical phenomena:

Fluctuations <u>and</u> nonlinearities (usually) are only important near isolated critical points (continuous phase transition)



well-known exception: **FQHE**

Critical phases



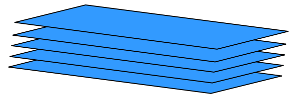
$$H = (\nabla\phi)^2 + \phi^4$$

…there are others

“Soft” elastic systems

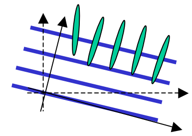
guiding principle: ***partial breaking of spatial symmetry***

- **Smectic phase** (*Grinstein + Pelcovits*)



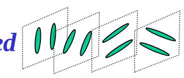
$$H = K(\nabla^2 u)^2 + B(\partial_z u + \frac{1}{2}(\nabla u)^2)^2$$

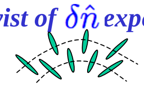
harmonic | rotational invariance | nonlinear



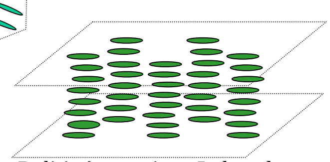
$$H = B_\perp(\nabla_\perp u - \delta n)^2 + B_z(\partial_z u)^2 + K_s(\nabla \cdot n)^2 + K_{tb}(\nabla \times n)^2$$

Higgs mechanism $\longrightarrow$ ***twist of*** $\delta\hat{n}$ ***expelled***
but not splay

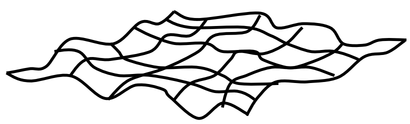




- **Columnar phase** (*L.R. + Toner*)
  (*spontaneous vortex lattice in FM superconductor*)

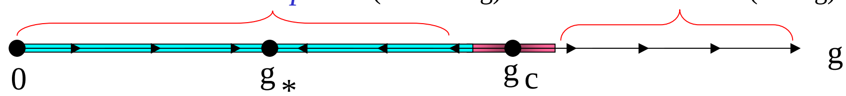


- **Tensionless polymerized membrane** (*Nelson+Peliti, Aronovitz +Lubensky, Le Doussal + L.R.*)



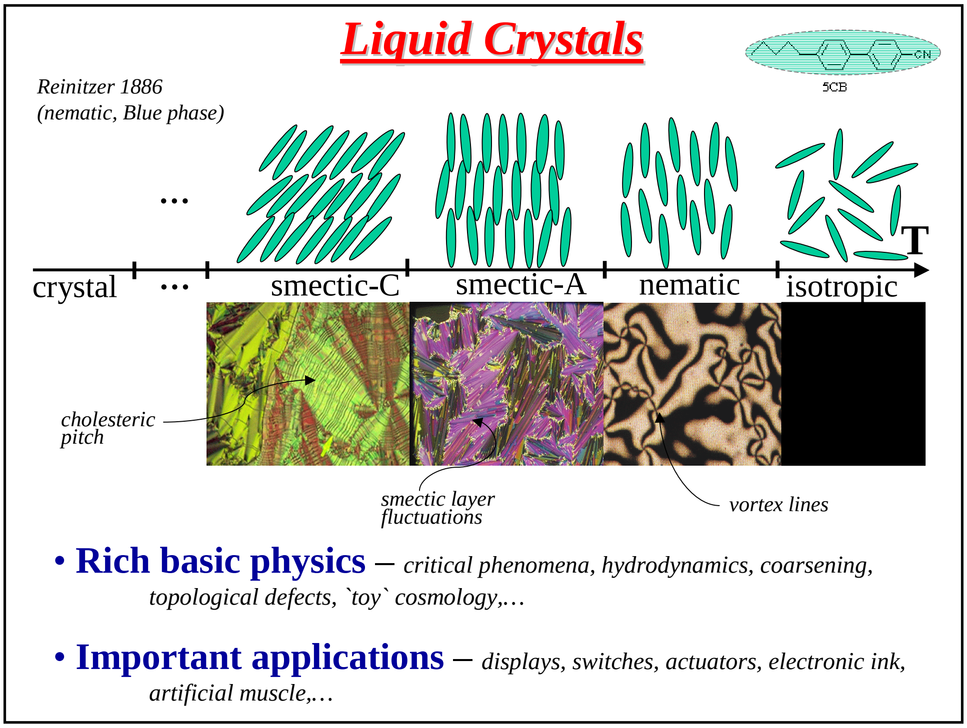
Properties of critical phases

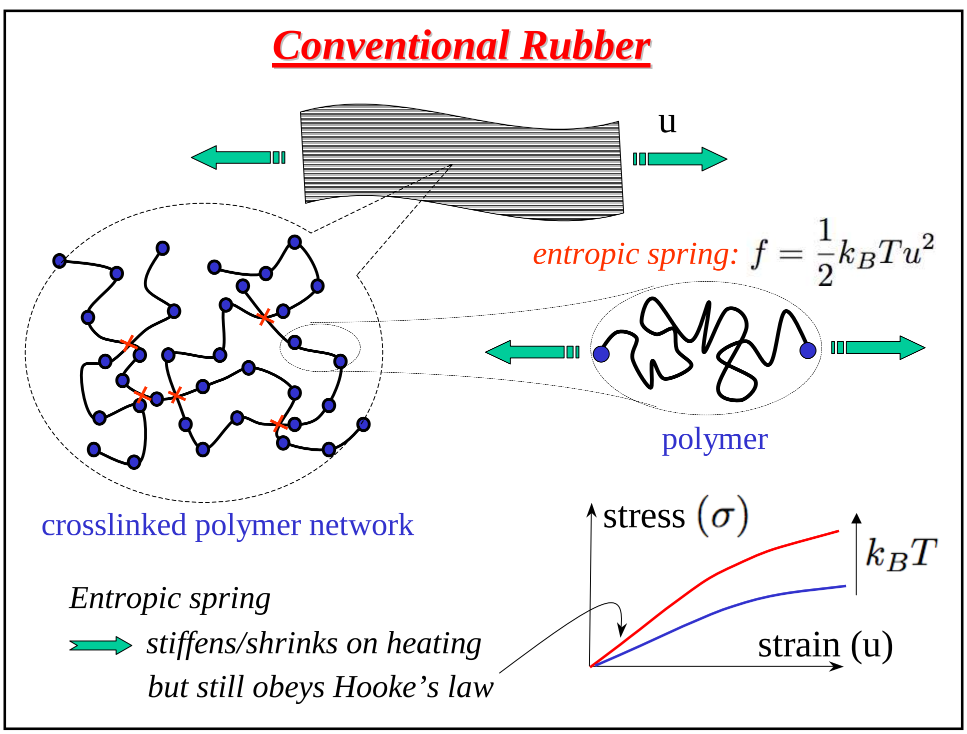
critical ordered phase (interesting) *disordered* (boring)



$$H = (\nabla^2 u)^2 + (\partial_z u + (\nabla u)^2)^2$$

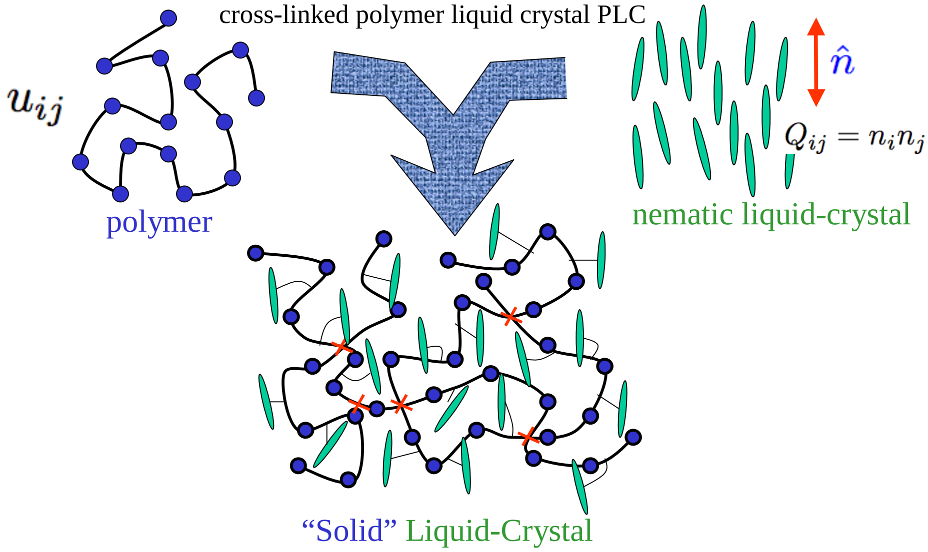
- *spontaneously broken continuous symmetry*
- *nontrivial fixed point of strongly interacting Goldstone modes (c.f. nonlinear O(N) sigma-model)*
- *universal power-law correlation functions and amplitude ratios (throughout the phase)*
- *no fine-tuning to a critical point required*
- *quantum analogs? road to 3d “Luttinger liquids”?*


Liquid Crystals
Reinitzer 1886
(nematic, Blue phase)
5CB
CN
T
crystal
…
smectic-C
smectic-A
nematic
isotropic
cholesteric pitch
smectic layer fluctuations
vortex lines
• Rich basic physics – critical phenomena, hydrodynamics, coarsening, topological defects, `toy` cosmology,…
• Important applications – displays, switches, actuators, electronic ink, artificial muscle,…


Conventional Rubber
u
entropic spring: $f = \frac{1}{2} k_B T u^2$
polymer
crosslinked polymer network
stress (σ)
$k_B T$
strain (u)
Entropic spring
stiffens/shrinks on heating
but still obeys Hooke's law

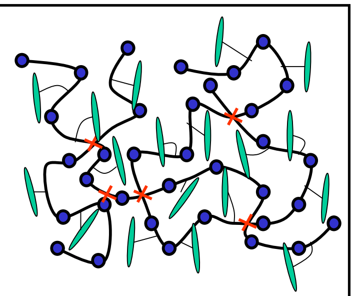
Nematic Elastomer

Terentjev
Finkelmann
Ratna



exhibits most conventional liquid-crystal phases (I, N, Sm-A, Sm-C,…)

Questions of Interest



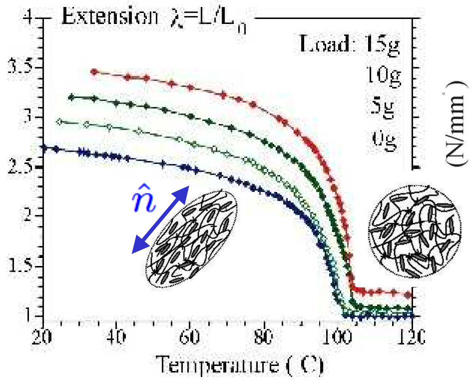
- **Effects of polymer matrix on liquid crystal order**
  cf. liquid crystals in random matrix (e.g., aerogel)
  (L.R.+Toner; Feldman; Gingras; Clark, Garland, Birgeneau)

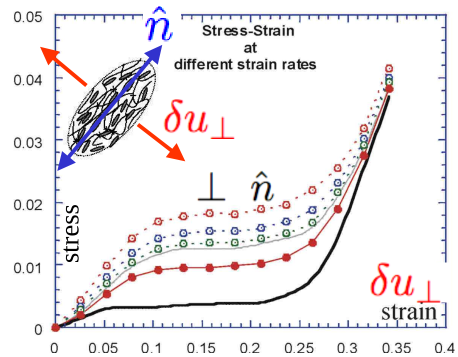
- **Effect of liquid crystal order on rubber (gel) elasticity**
  (de Gennes, Golubovic+Lubensky, Warner, Terentjev)

Must understand both questions self-consistently to understand liquid crystal elastomers and gels

Experiments

Terentjev





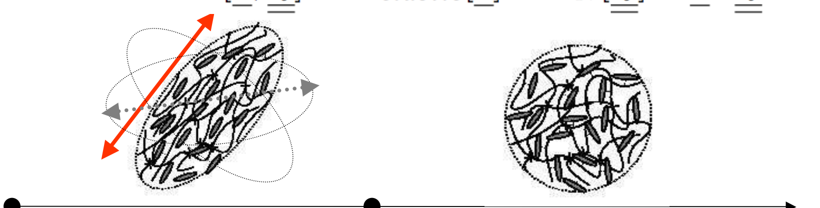
Properties:
- *spontaneous distortion (~ 400%) at T_{IN}, thermoelastic*
- *"soft" elasticity*
- *giant electrostriction*

Applications:
- *plastic displays*
- *switches*
- *actuators*
- *artificial muscle*

also smectic elastomers
W. Lehmann, et al, Nature '01

Nematic Elastomer Elasticity (no fluctuations)

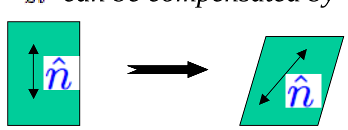
- **Isotropic phase:** $\mathcal{H}_{elastic}[\underline{\underline{u}}] = \mu\, \underline{\underline{u}}^2$ *(conventional)*
- **IN transition:** $\mathcal{H}[\underline{\underline{u}}, \underline{\underline{Q}}] = \mathcal{H}_{elastic}[\underline{\underline{u}}] + \mathcal{H}_N[\underline{\underline{Q}}] - \underline{\underline{u}} \cdot \underline{\underline{Q}}$

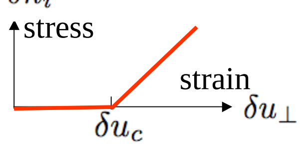


$\langle \underline{\underline{Q}} \rangle \neq 0 \neq \langle \underline{\underline{u}} \rangle$ T_{IN} $\langle \underline{\underline{Q}} \rangle = 0 = \langle \underline{\underline{u}} \rangle$ T

- **Nematic phase:** $\mathcal{H} = \mu(\underline{\underline{u}} - \underline{\underline{Q}})^2 + \mathcal{H}_{lc}[\underline{\underline{Q}}]$ *nemato-elastic Goldstone mode*

$\delta\underline{\underline{u}}^{\perp} = u_{zi}$ *can be compensated by* $\delta\underline{\underline{Q}}^{\perp} = \delta\hat{n}_i$





Goal

Xing + L.R., PRL, EPL
Lubensky + Stenull, EPL

- Construct rotationally invariant elastic theory of deformations about $\underline{\underline{u}}_0$
- Study fluctuations and heterogeneities about $\underline{\underline{u}}_0$

Must incorporate underlying rotational invariance of the nematic state

⟹ *some distortions cost no energy:* ***"soft" uniaxial solid***

$$f[\vec{R}(\mathbf{x})] = f[O_T\vec{R}(O_R\mathbf{x})]$$

- Vanishing energy cost for: $\delta\underline{\underline{u}} = \underline{\underline{O}}\cdot\underline{\underline{u}}_0\cdot\underline{\underline{O}}^T - \underline{\underline{u}}_0$
- Harmonic elasticity about nematic state: $\underline{\underline{\varepsilon}} = \underline{\underline{u}} - \underline{\underline{u}}_0$

$$\mathcal{H}^0_{NE} = \mu_{zi}\varepsilon^2_{zi} + B_z\varepsilon^2_{zz} + \mu_\perp\varepsilon^2_{ij} + \lambda\varepsilon^2_{ii} + \lambda_{zi}\varepsilon_{zz}\varepsilon_{ii}$$

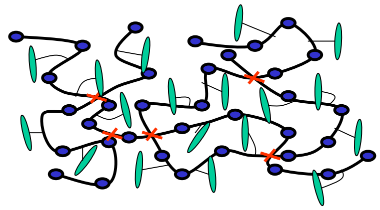
$\mu_{zi} \to 0$, *required by rotational invariance*

- Nonlinear elasticity about nematic state:

$$\mathcal{H}_{NE} = B_z w^2_{zz} + \mu_\perp w^2_{ij} + \lambda w^2_{ii} + \lambda_{zi} w_{zz} w_{ii}$$

$$w_{zz} = \partial_z u_z + \frac{1}{2}(\nabla u_z)^2 \qquad w_{ij} = \frac{1}{2}(\partial_{(i}u_{j)} - \partial_i u_z \partial_j u_z)$$

Fluctuations and Heterogeneity



- Thermal fluctuations: $\mathcal{Z} = \mathrm{Trace}_u[e^{-\beta\mathcal{H}[u]}]$
- Heterogeneity ⟹ random torques and stresses:

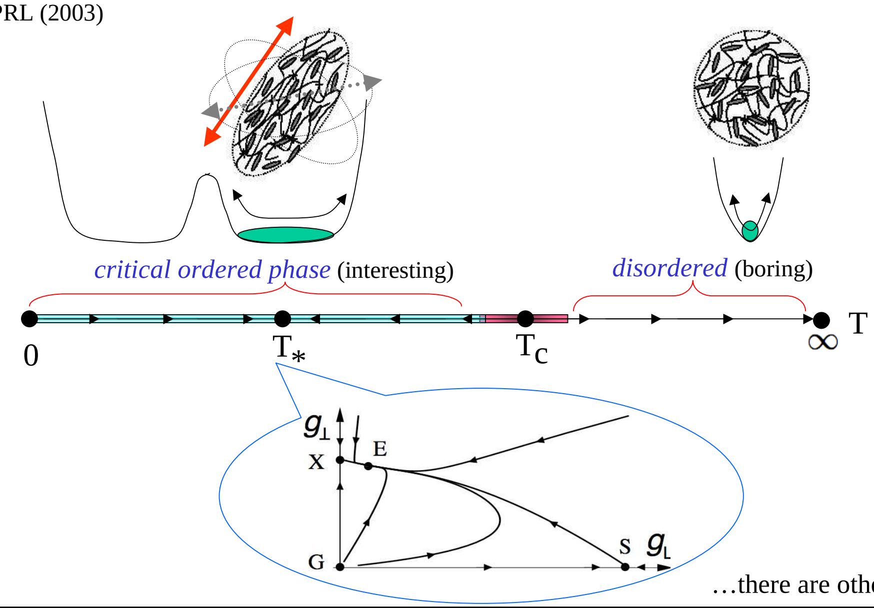
nematic elastomers are only statistically homogeneous and isotropic

$$\mathcal{H}^{real}_{NE} = \mathcal{H}_{NE}[\underline{\underline{u}}] - \underbrace{\underline{\underline{u}}\cdot\underline{\underline{\sigma}}(\mathbf{r}) - (\hat{n}\cdot\vec{g}(\mathbf{r}))^2}_{\text{encodes heterogeneity}}$$

Elastic "softness" leads to strong qualitative effects of thermal fluctuations and network heterogeneity

Critical phase in nematic elastomer

Xing + L.R., PRL (2003)

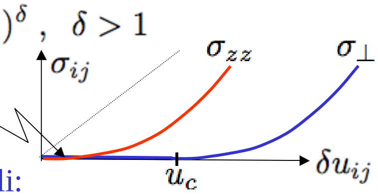


…there are others

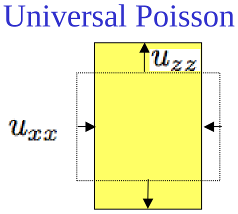
Predictions

Xing + L.R., PRL (2003)

- *Universal* elasticity: $\overline{\langle|\delta u(q)|^2\rangle} \sim q_\perp^{-4+\eta}$, for $r_\perp > \xi_\perp \sim K^2/\Delta$
- Non-Hookean elasticity: $\sigma_{zz} \sim (u_{zz})^\delta$, $\delta > 1$
  (cf. non-Fermi liquid)
  vanishing slope
  no linear response



- Length-scale dependent elastic moduli:
  $K_{\text{eff}}(L) \sim L^\eta, \quad \mu_{\text{eff}}(L) \sim L^{-\eta_\mu}, \quad B_{\text{eff}}(L) \sim B_0$
- Macroscopically incompressible: $\kappa_{\text{eff}} \sim \mu_{\text{eff}}(L)/B_{\text{eff}}(L) \to 0$
- Universal Poisson ratios:



$$u_{xx} > 0 \Rightarrow \begin{cases} u_{yy} = \frac{5}{7}u_{xx} \\ u_{zz} = -\frac{12}{7}u_{xx} \end{cases}$$

$$u_{zz} > 0 \Rightarrow u_{xx} = u_{yy} = -\frac{1}{2}u_{zz}$$

Summary and Conclusions

- Critical phases in "soft" condensed matter
- Liquid-crystal rubber (nematic elastomer) --- *a "liquid" solid*
- Dramatic influence of thermal fluctuations and heterogeneities on macroscopic elasticity and orientational correlations

- *Dynamics?*
- *Other liquid crystal phases (e.g., smectic rubber)?*
- *Quantum realizations of critical phases?*
- *Defects?*