Quantum criticality in diluted 2D antiferromagnets

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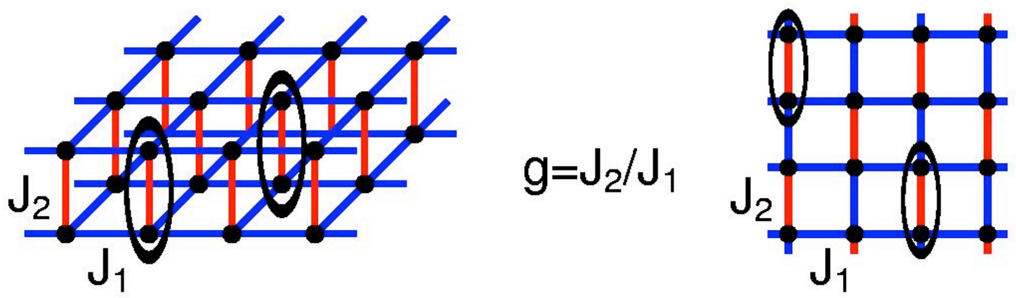
Simulations of dimer-diluted S=1/2 Heisenberg models



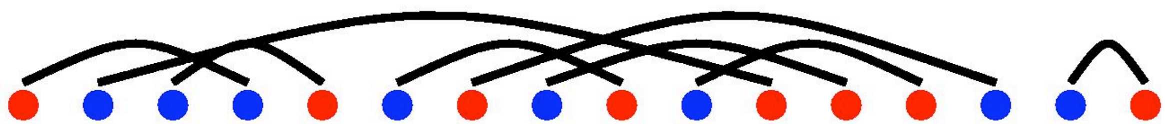
- ➪ Quantum criticality in the presence of disorder
- ➪ Geometric/quantum criticality at percolation point

Background: Random antiferromagnets

1D random J>0 Heisenberg chain: $H = \sum_i J_i \vec{S}_i \cdot \vec{S}_{i+1}$

Infinite-randomness fixed point; random-singlet phase:
Spins form singlets on all length-scales at T=0

Ma, Dasgupta, Hu, 1979; D. Fisher, 1994



Renormalization procedure:
successive decimation of strongest-coupled pairs

$$\chi \sim \frac{1}{T \ln^2(T)}$$

Dynamic exponent: $Z = \infty$

$$\omega = k^Z, \Delta_L = L^{-Z}$$

Dimerized chain: $H = \sum_i [J_i + \delta(-1)^i] \vec{S}_i \cdot \vec{S}_{i+1}$

Griffiths fixed point; $Z \propto 1/\delta$

Igloi, Juhasz, Rieger (2000)

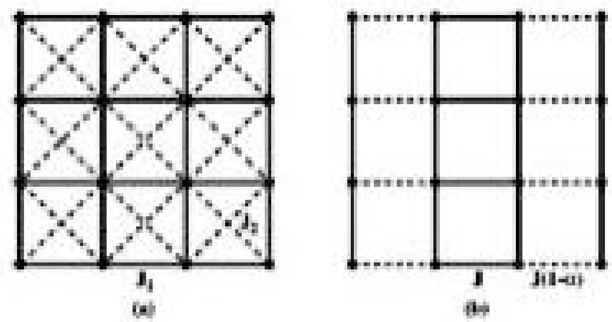
2D random transverse Ising model: $Z = \infty$

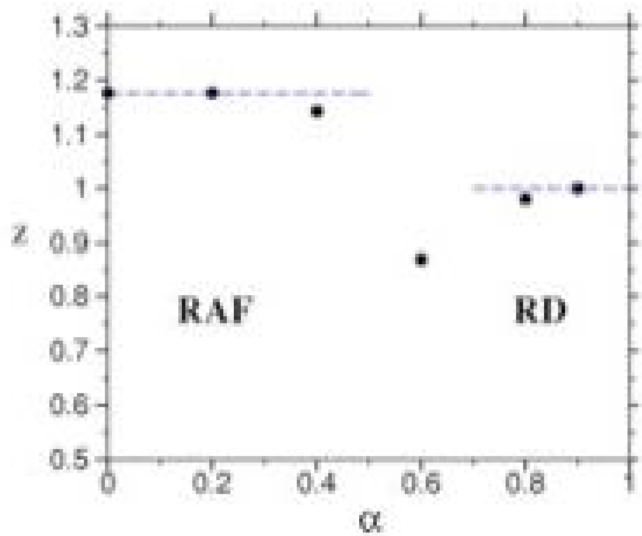
Motrunich, Mau, Huse, D. Fisher (2000); Pich, Young, Rieger, Kawashima (1998)

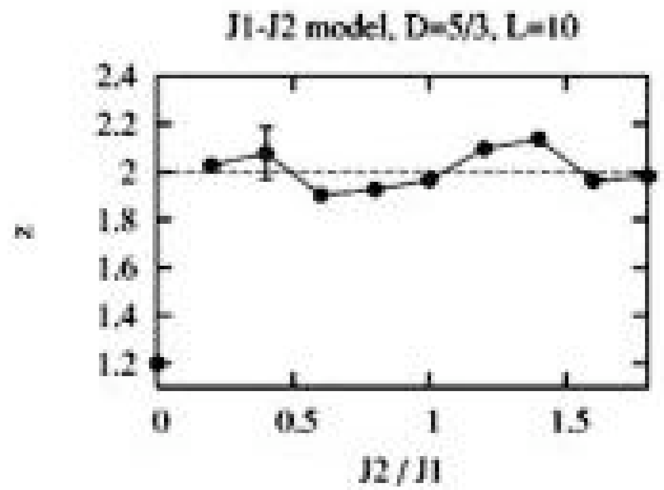
2D random J Heisenberg models:

RG procedure ⇒ conventional scaling

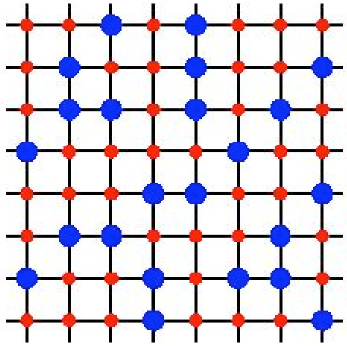
Lin, Melin, Rieger, and Igloi (2003)







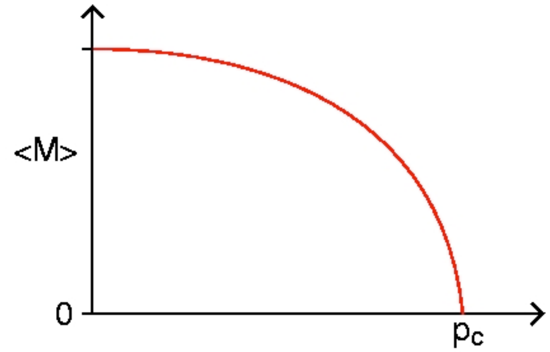
Site-diluted 2D Heisenberg model



Cuprates: Cu → Zn substitution

Fraction p of non-magnetic sites

Sublattice magnetization vs p



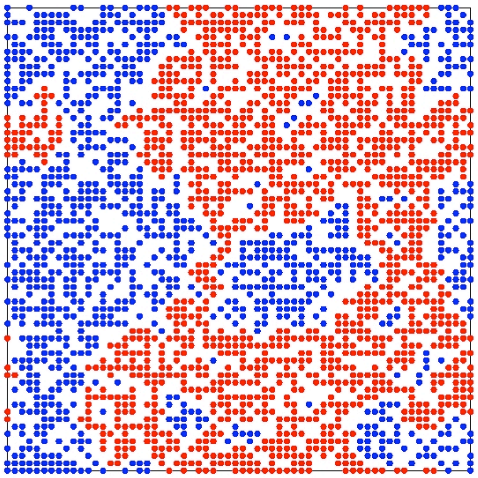
Is p_c less than percolation point $p^* \approx 0.41$?

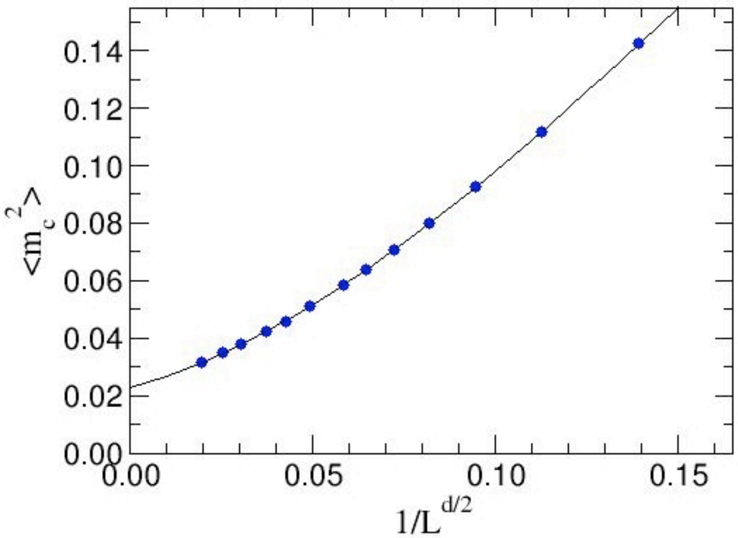
➪ Kato et al. (2000); $p_c = p^*$

➪ Sandvik (2001); percolating cluster ordered

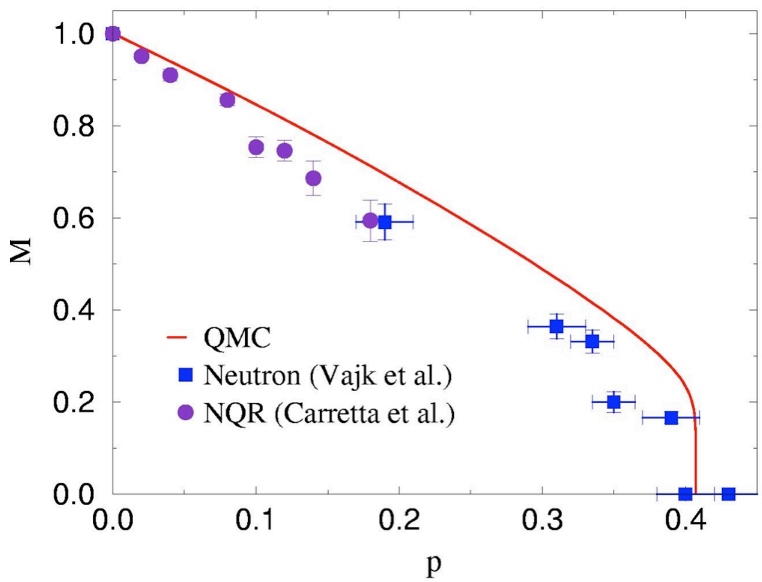
Classical percolation transition

QMC studies of percolating cluster (d=91/46)





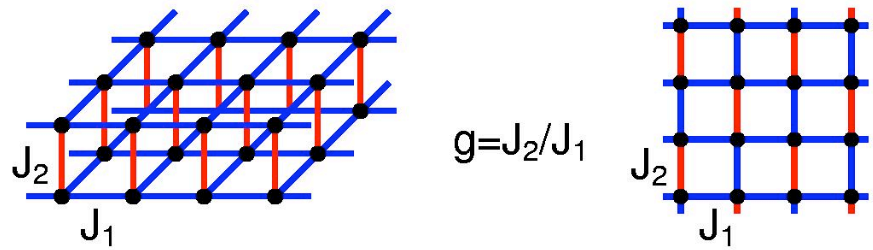
Comparison with neutron scattering; $La_2(Zn,Mg)_xCu_{1-x}O_4$



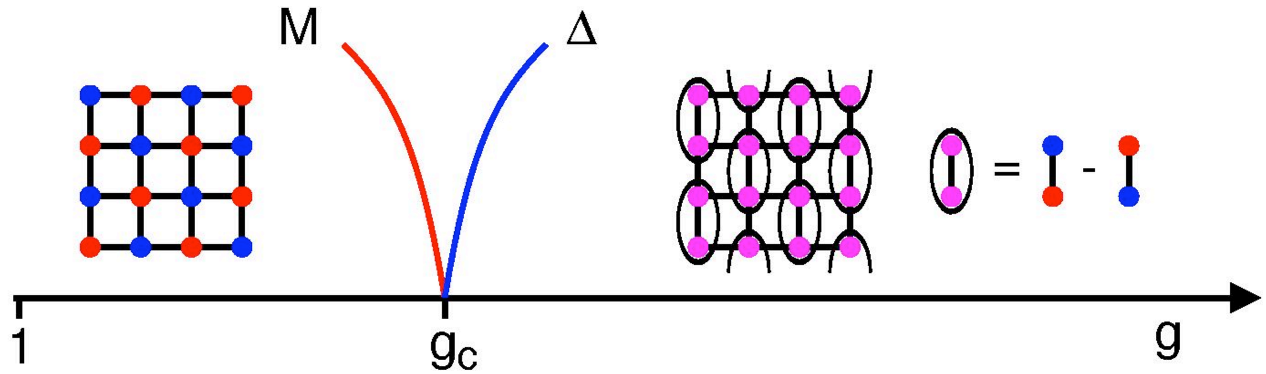
Vajk, Greven, Mang, Lynn, Gehring (2001)

Order slightly suppressed;
likely other interactions present

Quantum-criticality in dimerized 2D systems



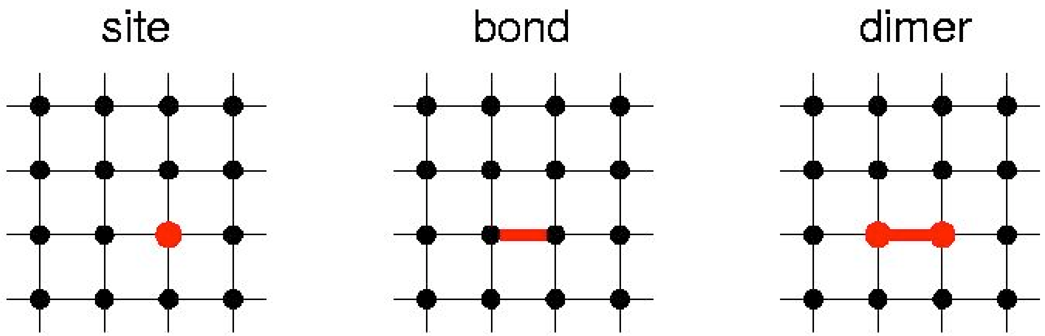
Singlet-formation at strong bonds (”dimers”) ⇒
antiferromagnetic to spin-gapped transition



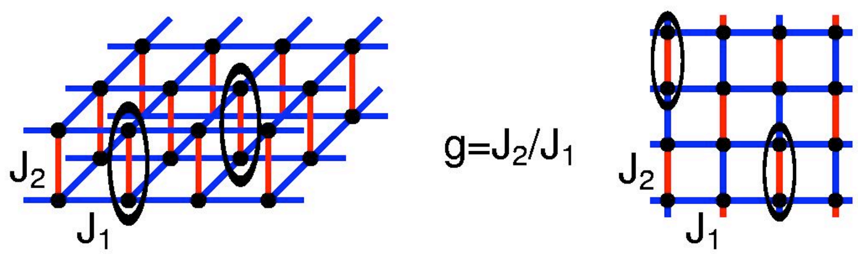
$g_c \approx 2.5$ in both bilayer and single-layer model

Quantum-critical diluted systems

Unpaired spins in site-diluted gapped system ⇒
localized moments form; order antiferromagnetically ⇒
no quantum-critical point

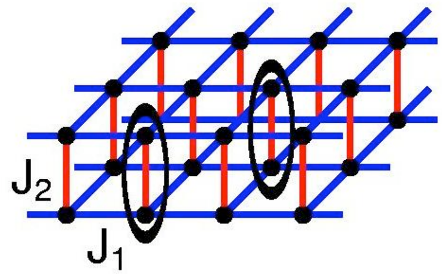


Dimer-diluted models:



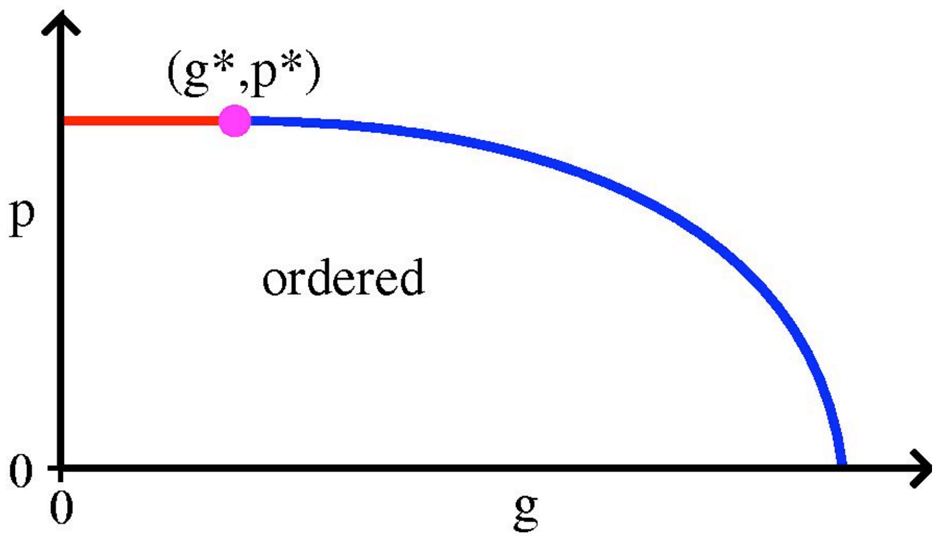
Bilayer model

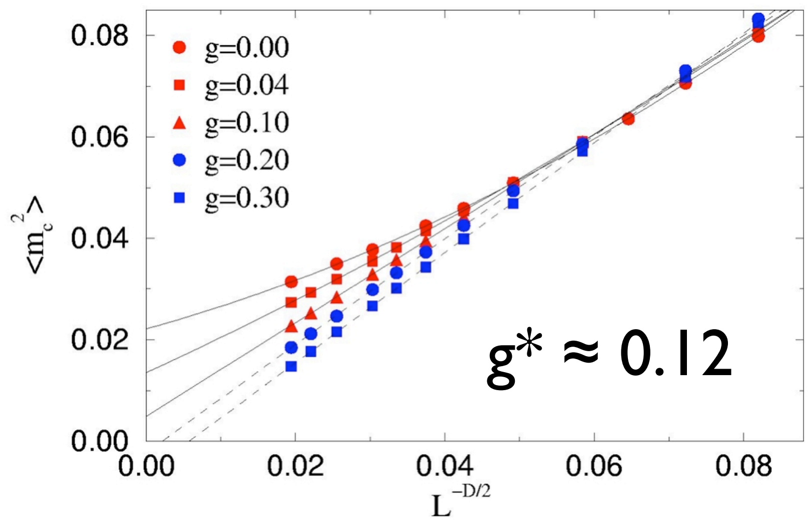
Sandvik (2002); Vajk, Greven (2002)
Sknepnek, Vojta, Vojta (2004)



$g = J_2/J_1$

Geometric percolation transition expected for $g < g^*$





Disorder relevant at transition for $g > g^*$, $p < p^*$
(Harris criterion; $\nu < 2/d$)

Confirmed in simulations of 3D classical model with columnar defects (Sknepnek, Vojta, Vojta); $Z = 1.310(6)$

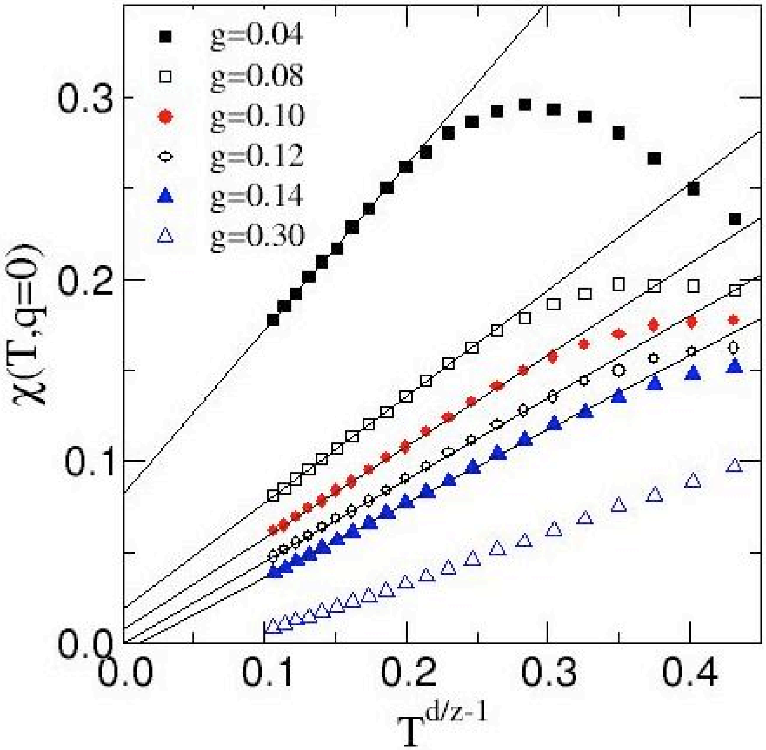
Uniform magnetic susceptibility

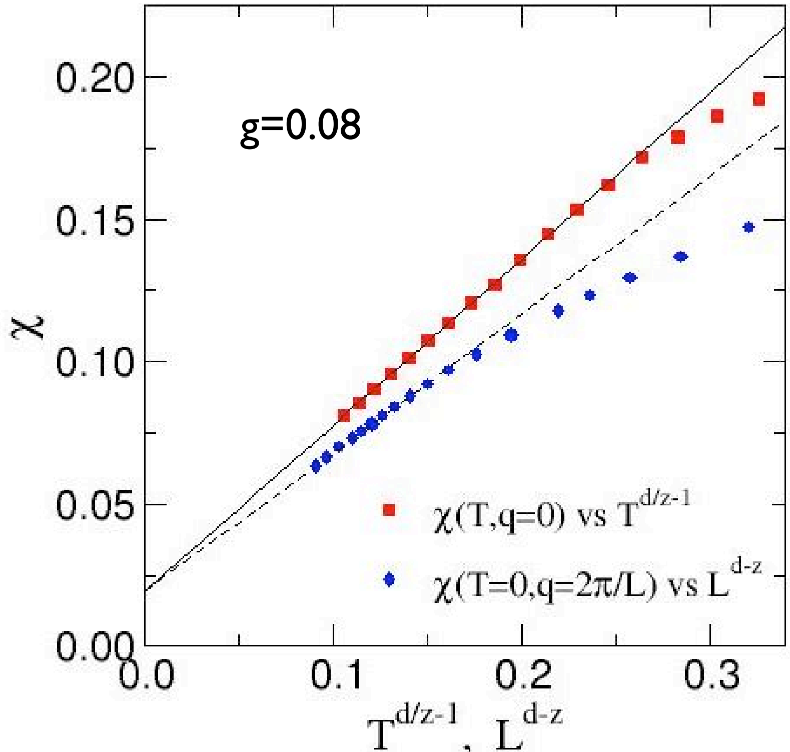
T > 0 quantum-critical form: $\chi = a + bT^{d/z-1}$

T=0; $\chi = 0$ for finite system. Use $\chi(q \to 0^+)$

$$\xi \sim T^{-z}; \chi = a + b\xi^{z-d} \quad \xi \to L$$

$$\chi(T = 0, q \to 0^+) = a + bL^{z-d}$$

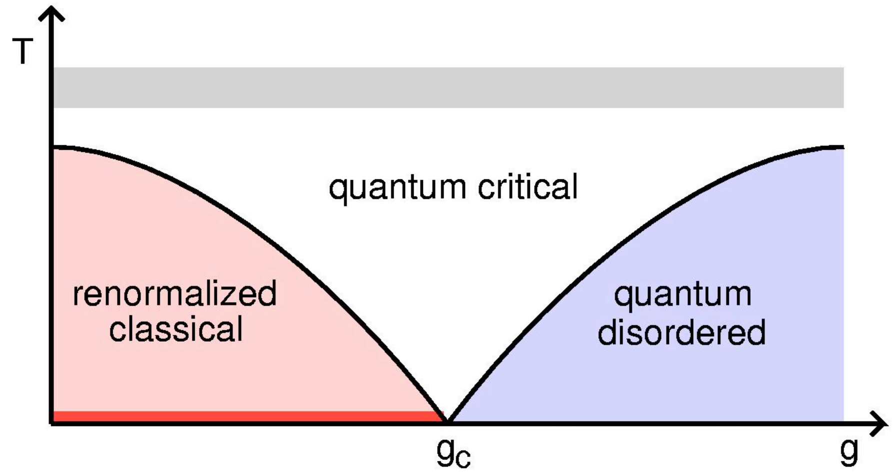




Z=1.36(2)

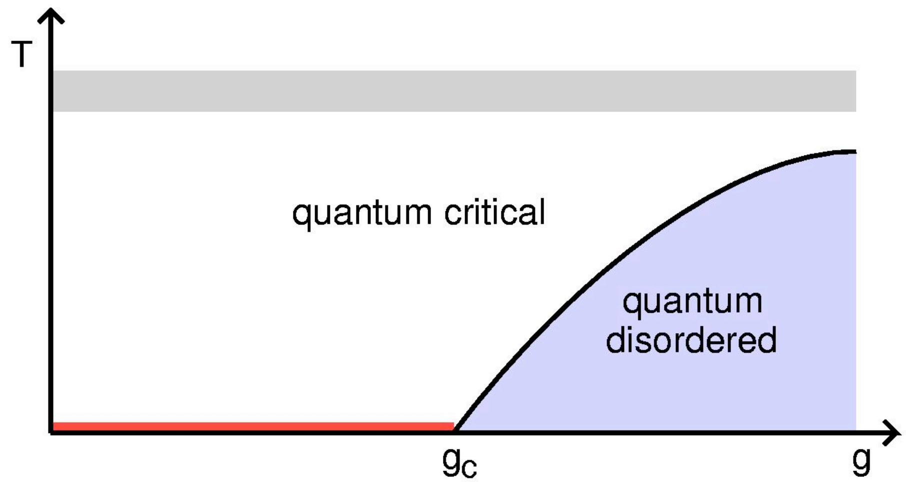
No sign of quantum-critical to renormalized classical cross-over

Pure 2D antiferromagnet (Chakravarty, Halperin, Nelson)



Low-T cross-overs controlled by spin-stiffness/gap

Conjecture: Stiffness = 0 on percolating cluster; no RC cross-over

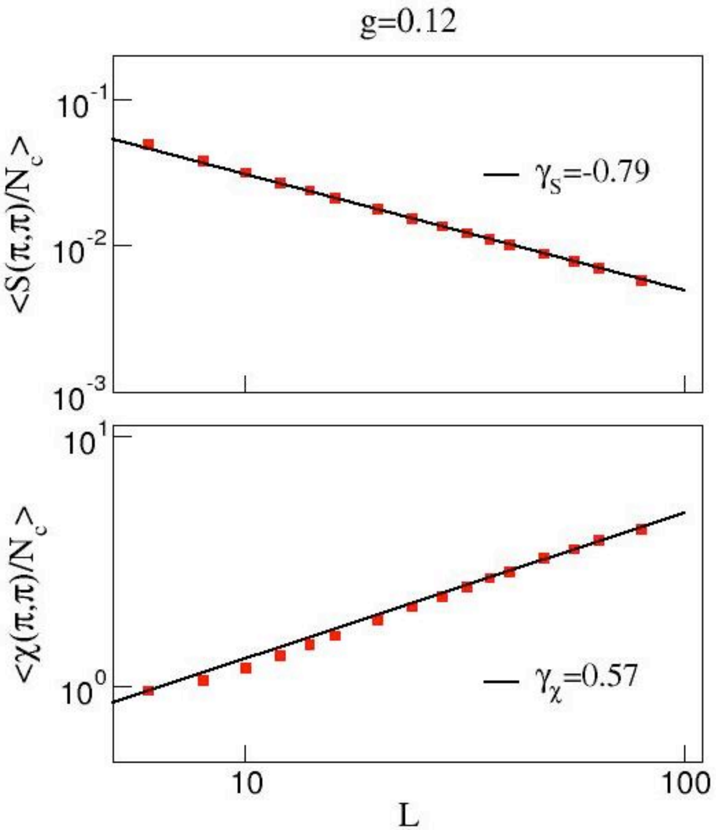


Check of scaling (Z) using antiferromagnetic quantities

$$q = (\pi, \pi)$$

$$S(q) = \langle S^z_{-q} S^z_q \rangle = \int d\omega S(q,\omega) \sim L^{1-\eta}$$

$$\chi(q) = \int_0^{1/T} d\tau \langle S^z_{-q}(\tau) S^z_q(0) \rangle = \int d\omega \frac{1}{\omega} S(q,\omega) \sim L^{1+z-\eta}$$

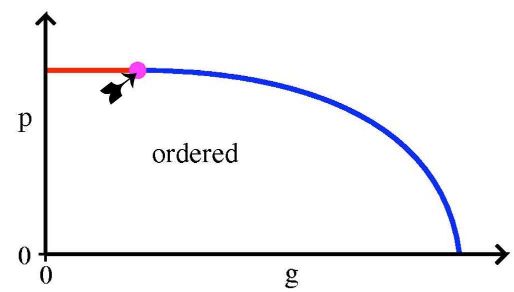


$$1 - \eta = \gamma_S$$

$$1 + z - \eta = \gamma_\chi$$

Data at g close to g* consistent with

$$z = \gamma_\chi - \gamma_S \approx 1.36$$



Correlation length measured by neutron scattering in $La_2(Zn,Mg)_xCu_{1-x}O_4$

Vajk, Greven, Mang, Lynn, Gehring (2001)

Close to percolation: $\xi \sim 1/T^z, z \approx 1.4$

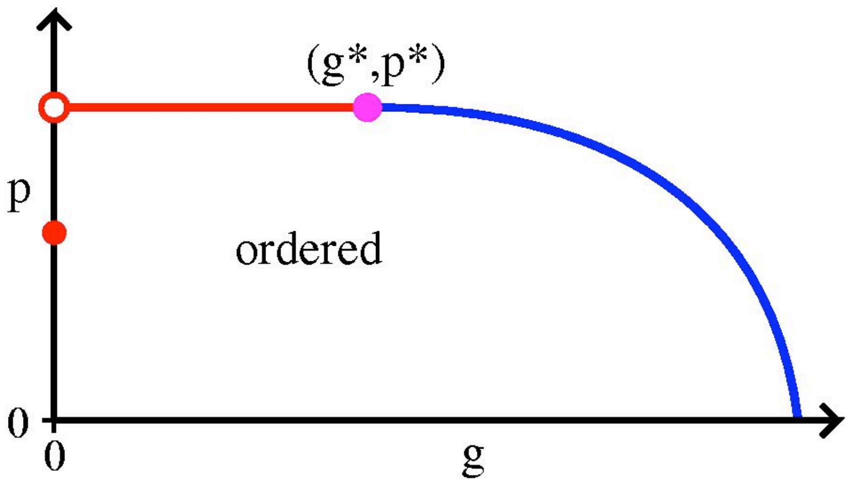
Could this be due to multi-critical point (g*, p*) in extended parameter space?

How universal is the behavior found for the diluted bilayer antiferromagnet?

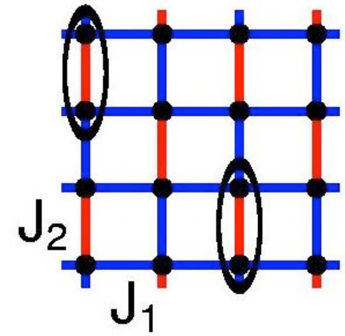
Can similar behavior be observed in a single layer?

Single layer with staggered dimers

Expected T=0 phase diagram

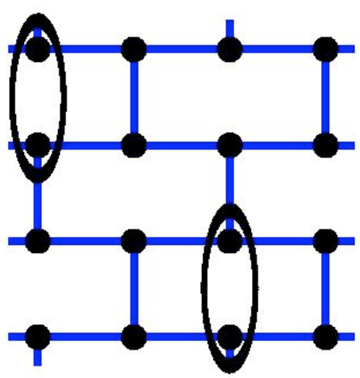


$g = J_2/J_1$



Dimers form triangular lattice; p*=1/2

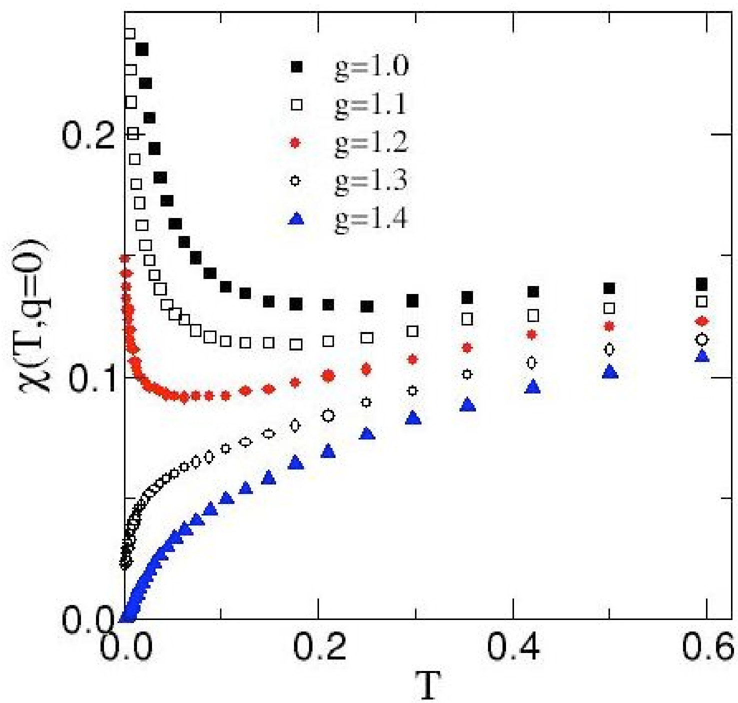
Brick-wall lattice when g=0; different percolation point

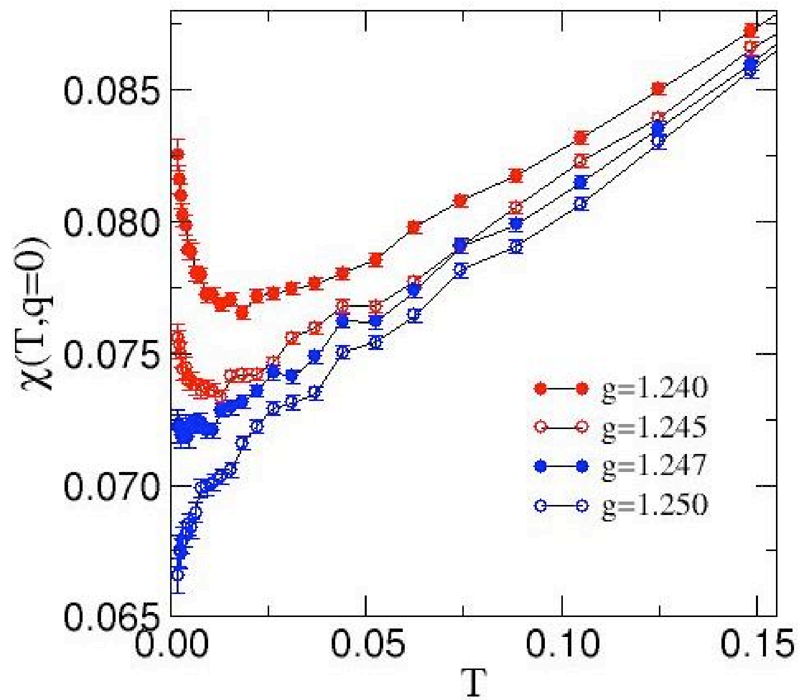


$$P^*(g=0) \approx 0.29$$

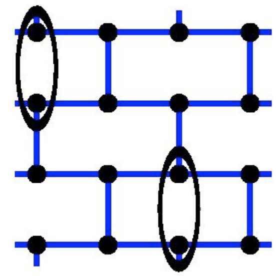
Uniform susceptibility at p*(g>0)

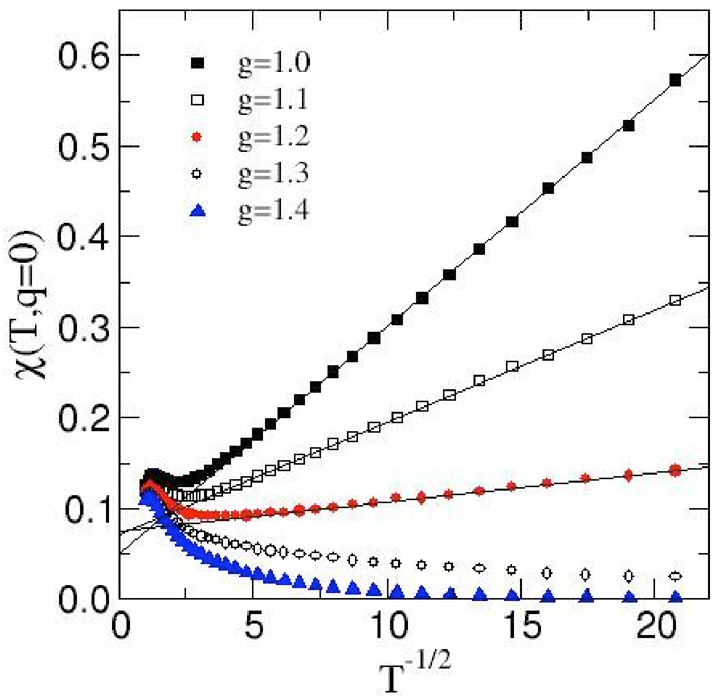
- Indicates critical point; $g_c \approx 1.247$
- Finite susceptibility at critical point
- Divergent susceptibility for $g < g_c$

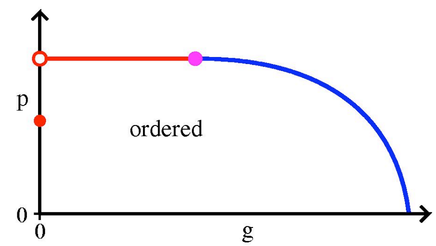


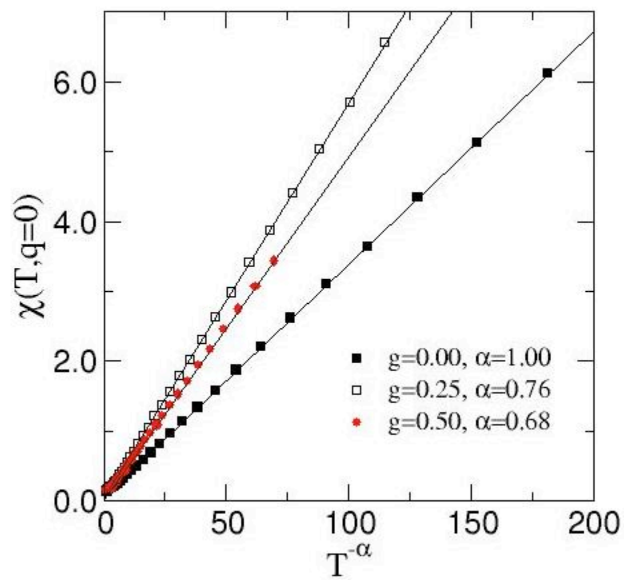


- Divergence follows closely $T^{-1/2}$ for g > 1
- Faster divergence for smaller g
- Curie form for g=0, due to "broken clusters"

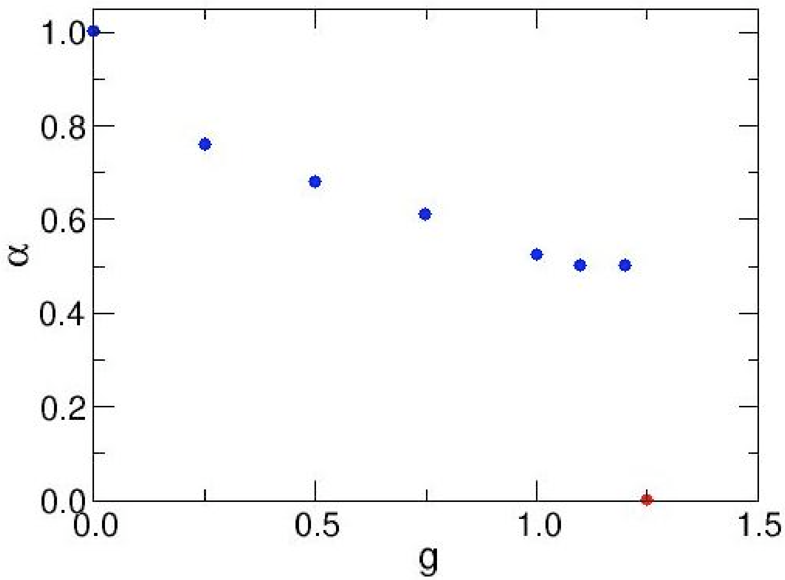


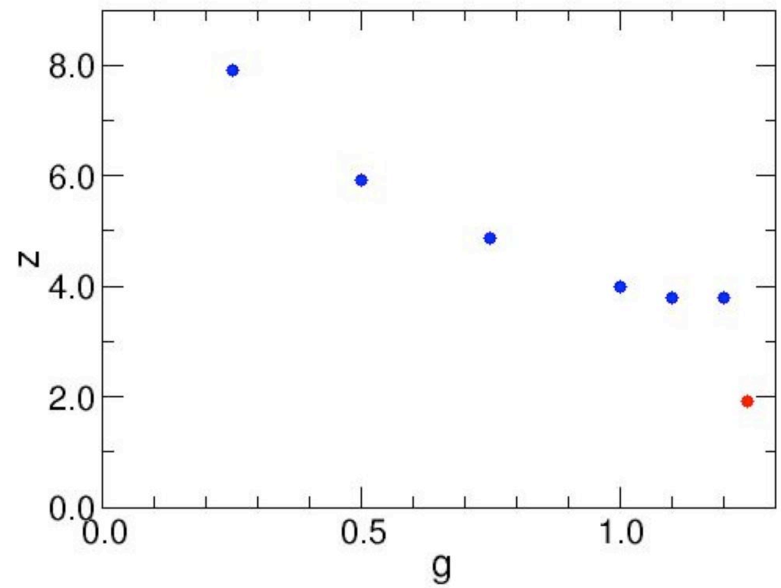


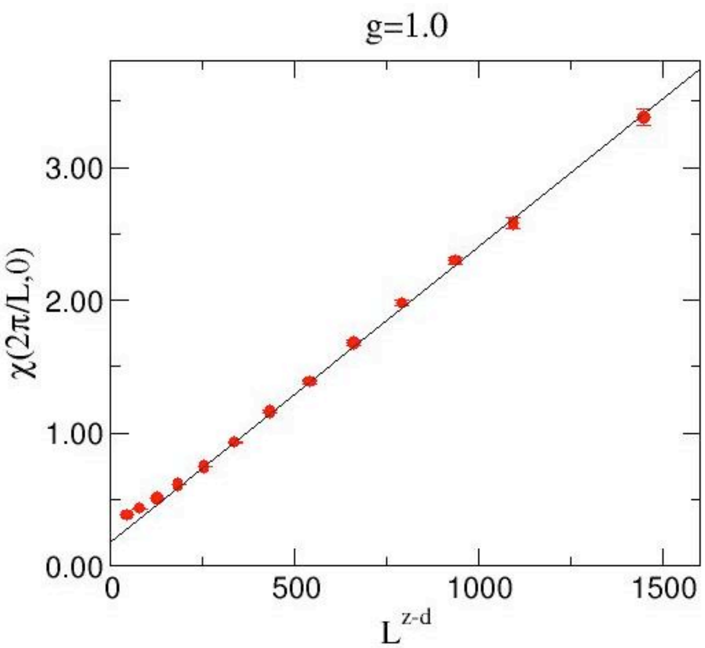




Extracting z using the form $\chi = aT^{-\alpha} = aT^{d/z-1}$

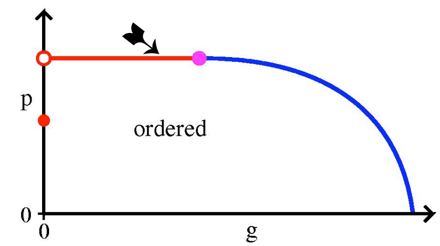




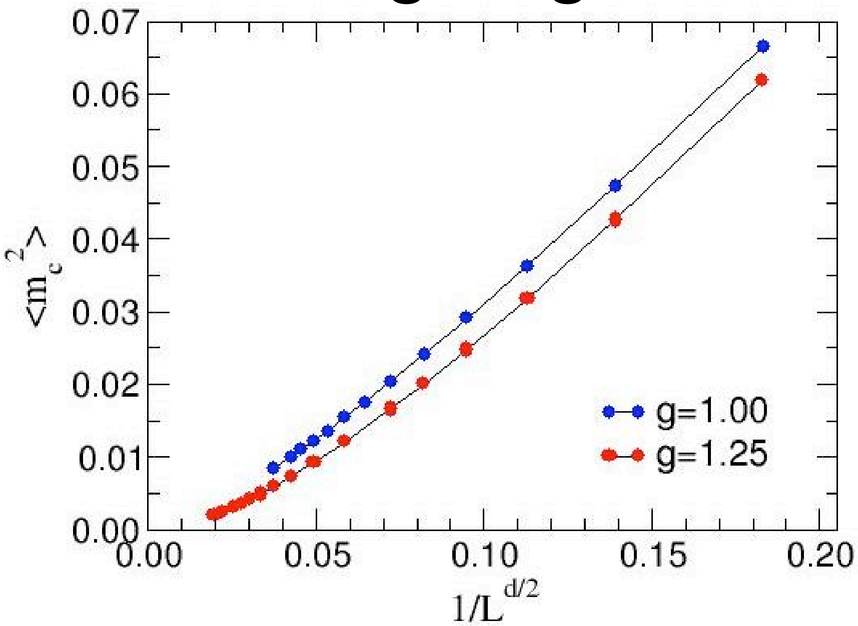


T=0 scaling consistent with T>0 scaling

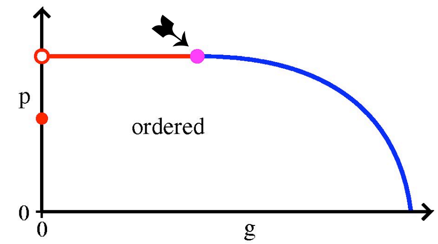
$$\chi(q \to 0^+) \sim L^{z-d}$$

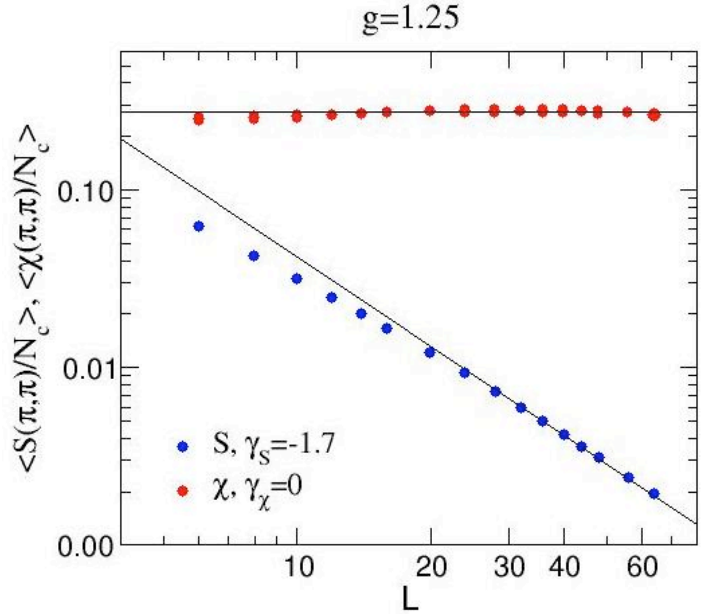


No long-range order



Line of critical points with continuously varying exponents





$S(\pi,\pi)$, $\chi(\pi,\pi)$

Finite-size scaling at g*

Z ≈ 1.7

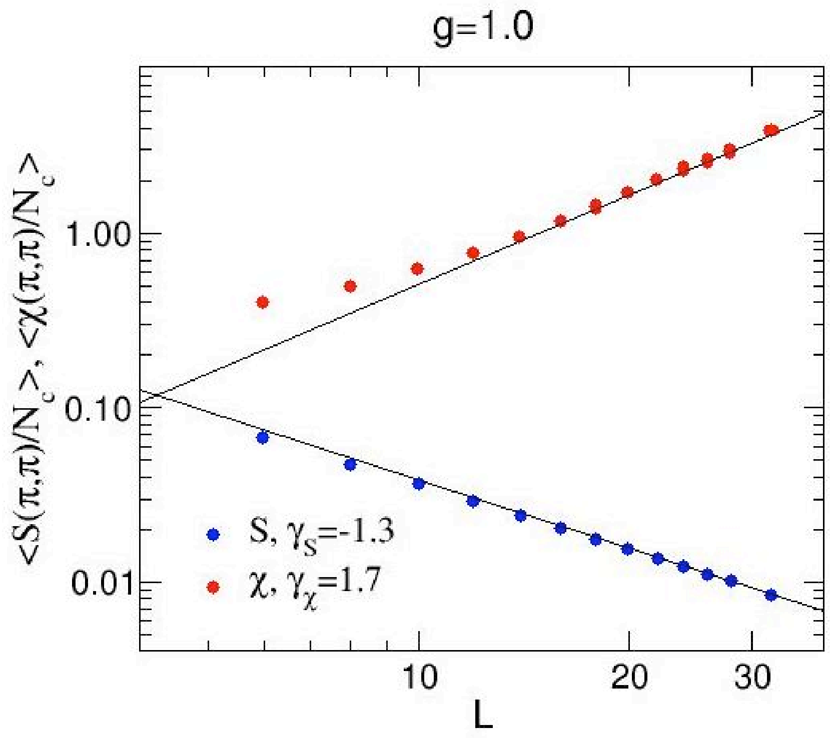
$S(\pi,\pi)$, $\chi(\pi,\pi)$ Finite-size scaling at g=1.0

Z ≈ 3.0

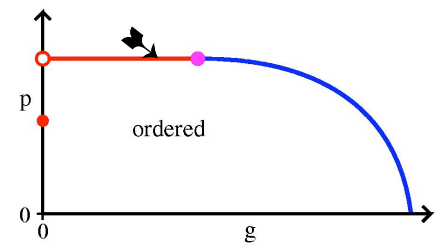
Disagrees with Z≈4.0 from uniform susceptibility

None-asymptotic behavior?

Or problems related to disorder averaging?

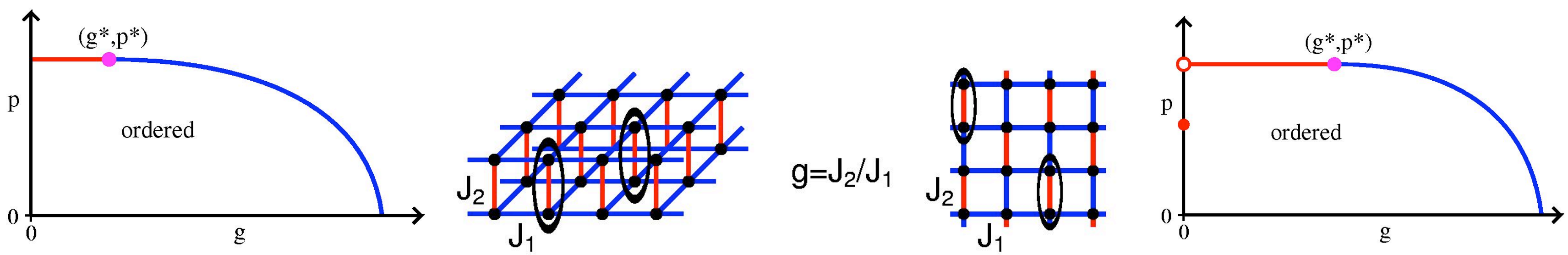


$$z \text{ from } \frac{\langle S\rangle}{\langle \chi\rangle} \text{ or } \left\langle \frac{S}{\chi} \right\rangle ?$$



Conclusions

Novel quantum-critical behavior discovered numerically in dimer-diluted systems



percolation transition ending in multi-critical point

Line of critical points at p*

Conventional scaling for the p<p* transition; $z \approx 1.31$

Why does the single layer have a line of critical points?

Conditions under which the percolating cluster orders?