

Deconfined quantum criticality and the underdoped cuprates

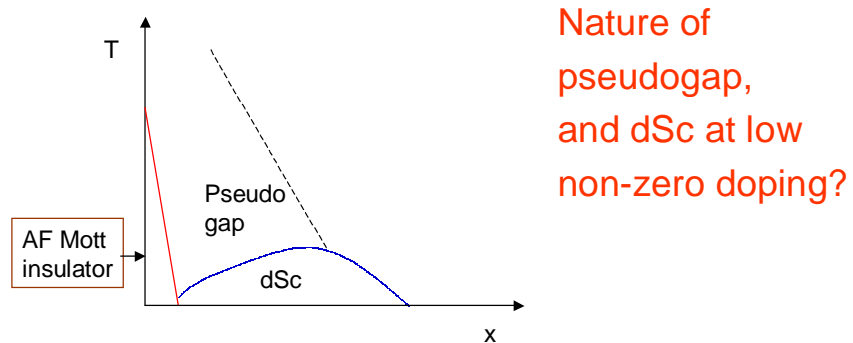
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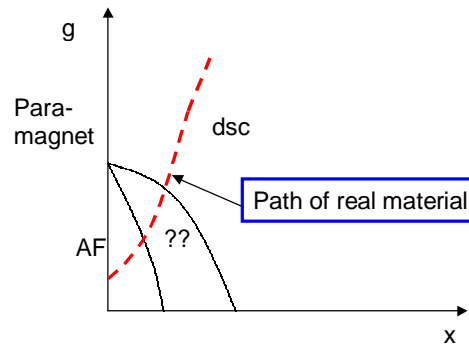
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T.S. Lee, cond-mat/0406066
Levin, T.S, cond-mat/0405702
Hermele et al, cond-mat/0404751

Cuprate phase diagram



Old idea: view as doped paramagnetic Mott insulator



g = frustration/ring exchange,....

What paramagnet? Some hints from experiments

- Softening of neutron resonance mode with decreasing x
 - consider paramagnets proximate to Neel state i.e potentially separated by 2nd order transition.
- Gapless nodal quasiparticles in dSC
 - consider paramagnets with gapless spin excitations.

Candidate states – gapless spin liquids

Rough description: Gapless spin-1/2 nodal spinons coupled to deconfined gauge fields.
(Eg: Z_2 spin liquid with nodal spinons and gapped visons)

Can spin liquid states be reached from conventional collinear Neel by second order transitions?

Orthodox answer: No!

Claim in this talk: Orthodox answer needs to be revisited.

Are the cuprates doped gapless spin liquids?

Natural (old) questions:

- Is the question meaningful?
- How to tell?

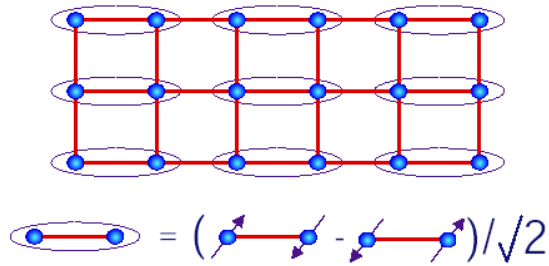
(TS, Lee, condmat/0406066)

Revisit – exploit insights from study of deconfined criticality at Neel-VBS transition.

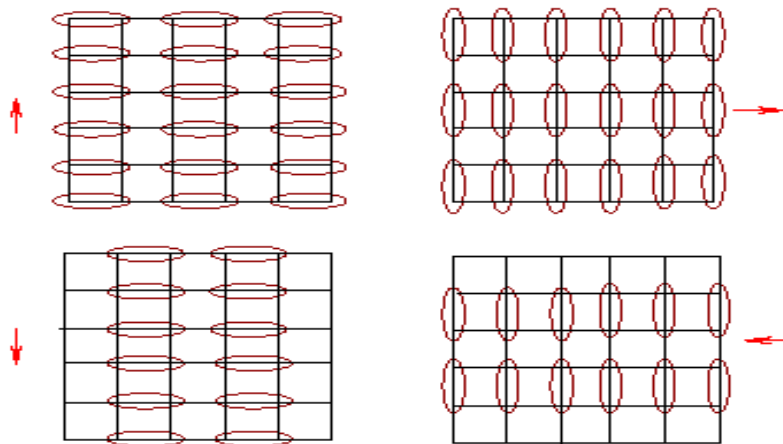
Deconfined criticality again – now from the valence bond solid(VBS) side

(Levin, TS, cond-mat/0405702)

Valence bond solid with spin gap.



Discrete Z_4 broken symmetry



Neel-Valence Bond Solid transition

- Naïve approaches fail

Attack from Neel \neq Usual $O(3)$ fixed point in $D = 3$

Attack from VBS \neq Usual Z_4 fixed point in $D = 3$

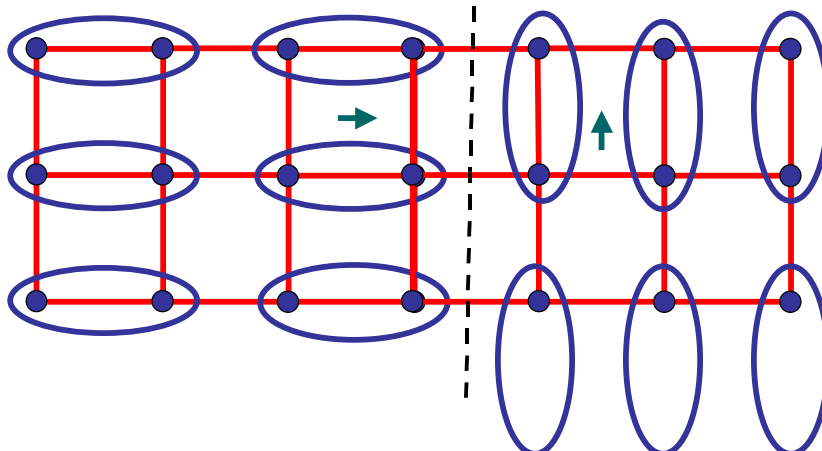
(= XY universality class).

Why do these fail?

Topological defects carry non-trivial quantum numbers!

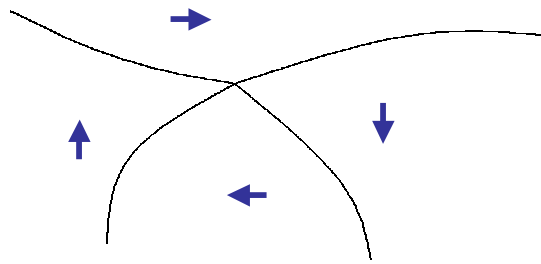
Topological defects in Z_4 order parameter

- Domain walls – elementary wall has $\pi/2$ shift of clock angle



Z_4 domain walls and vortices

- Walls can be oriented; four such walls can end at point.
- End-points are Z_4 vortices.

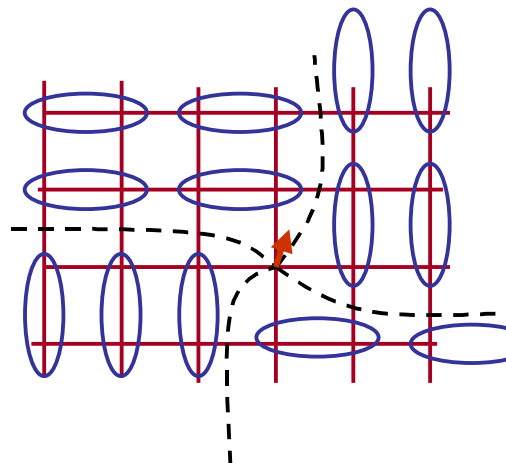


Z_4 vortices in VBS phase

Vortex core has an unpaired spin-1/2 moment!!

Z_4 vortices are "spinons".

Domain wall energy confines them in VBS phase.



Disordering VBS order

- If Z_4 vortices proliferate and condense, cannot sustain VBS order.
- Vortices carry spin => develop Neel order

Z_4 disordering transition to Neel state

- As for usual (quantum) Z_4 transition, expect clock anisotropy is irrelevant.
(confirm in various limits).

Critical theory: (Quantum) XY but with vortices that carry physical spin-1/2 (= spinons).

Alternate (dual) view

- Duality for usual XY model (Dasgupta-Halperin)
Phase mode - ``photon''

Vortices – gauge charges coupled to photon.

Neel-VBS transition: Vortices are spinons

=> Critical spinons minimally coupled to
fluctuating non-compact U(1) gauge field.

Proposed critical theory ``Non-compact CP₁ model''

$$S = \int d^2x d\tau |(\partial_\mu - ia_\mu)z|^2 + r|z|^2 + u|z|^4 + (\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2$$

z = two-component spin-1/2 spinon field

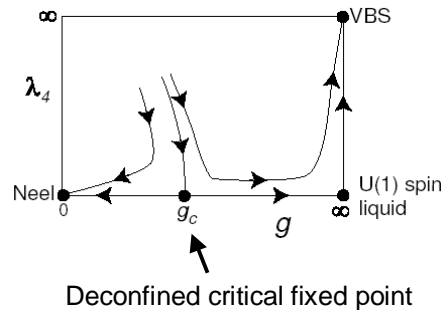
a_μ = non-compact U(1) gauge field.

Distinct from usual O(3) or Z₄ critical theories.

Reobtain same result as by attack from Neel state!

Renormalization group flows

Clock anisotropy
= quadrupled monopole
fugacity



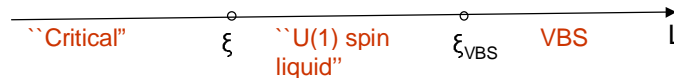
Monopoles are "dangerously irrelevant".

Precise meaning of deconfinement:

Conservation of gauge flux \Leftrightarrow

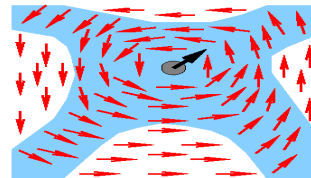
Extra emergent global (topological) U(1) symmetry associated with skyrmion conservation

Two diverging length scales in paramagnet



$\xi_{VBS} \sim \xi^k$ diverges faster than ξ

Spinons confined in either phase but 'confinement scale' diverges at transition.



Pertinent lessons

- **Lesson 1:** Gapless spinons may kill confinement in U(1) gauge theories in $d = 2$.
- **Lesson 2:** Even unstable spin liquids may control broad intermediate regime near certain quantum transitions.

Application to cuprates: theory of gapless U(1) spin liquids

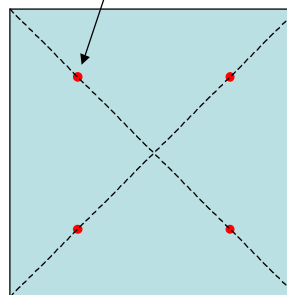
- Affleck-Marston '88, Marston'91: Flux phase of large-N spin models.

Mean field: Half-filled tight binding band of fermionic spinons with π -flux through each plaquette.

π	π	π
π	π	π
π	π	π

Low energies: massless Dirac theory in $D = 2+1$.

Band structure: four gapless Fermi points



Beyond mean field

Describe by fermionic massless Dirac spinons
coupled to compact U(1) gauge field

Ultimate fate?? Confinement??

- Doped versions: Lee, Nagaosa, Wen,
(1996 -)

Mostly ignore compactness (and hence possibility of confinement).

Stability of gapless U(1) spin liquids (Affleck-Marston pi-flux phase)

Hermele et al, [cond-mat/0404751](#)

- Analyse in limit of large number $2N$ of Dirac spinons (appropriate for SU(N) spin model).
- First ignore monopole events in space-time
⇒ Gauge flux exactly conserved.

Low energy theory is critical with no relevant perturbations (non-compact QED₃) :
conformally invariant with power law spin correlations.

$$\mathcal{L} = \bar{\psi}_j \gamma^\mu (\partial_\mu + i a_\mu) \psi_j + \frac{1}{8\pi e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2.$$

Monopoles

- Break flux conservation symmetry

Careful consideration: monopoles irrelevant at low energy critical fixed point for large enough N .

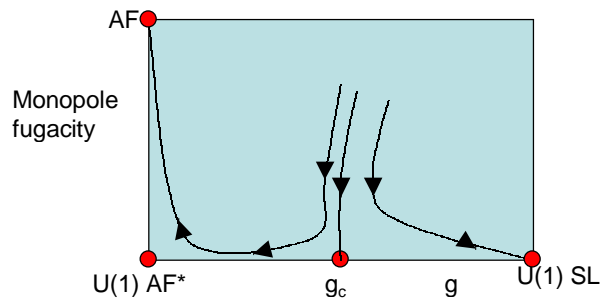
⇒ Deconfined critical phase

Precise meaning of "deconfinement": extra global topological $U(1)$ symmetry associated with gauge flux conservation.

Second order transition to Neel

(induce by increasing strength of quartic spinon interaction)

- Spin density wave of spinons
- Monopoles continue to be irrelevant at critical point to Neel.
- Spinons gapped in Neel phase => monopoles no longer irrelevant, cause confinement to yield conventional Neel state.
- Deconfined critical point with dangerous irrelevant monopoles, 2 diverging length scales, etc.



General lesson I

- Stable gapless U(1) spin liquids exist in $D = 2+1$ (at least for SU(N) models and $N > \text{some } N_{c1}$).

N_{c1} possibly smaller than 2, not known at present*.

$N_{c1} < 2 \Rightarrow$ appealing description of cuprates as doped U(1) spin liquids.

*Indications from numerics: $N_{c1} < 4$ (Assaad, cond-mat/0406...)

Alternate possibility

(or how Z₂ and U(1) spin liquids may give each other 2nd lives)

Z₂ spin liquid with nodal spinons and gapped Z₂ vortices (visons) – clearly stable even for SU(2) spin models.

?? 2nd order transition to conventional collinear Neel state
??

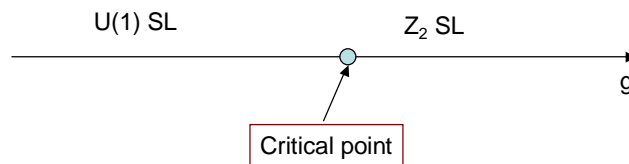
Z₂ state: Higgs phase of compact U(1) gauge theory coupled to charge-2 boson (spinon pair) field.

Neel: some confined phase of same theory.

Confinement transition from gapless Z_2 SL

Transition where spinon pair field gets gapped.

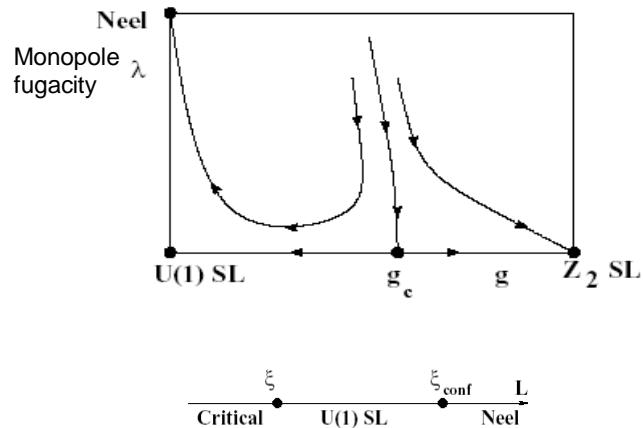
First ignore monopoles: Phase with gapped spinon pair described by non-compact QED_3 for spinons (i.e the U(1) SL)



Z_2 SL –Neel transition

- Expect monopole scaling dim at critical point > at U(1) SL fixed point
- ⇒ Can get situation where monopoles are irrelevant at critical point but relevant at U(1) SL fixed point (for $N_{c2} < N < N_{c1}$)
- ⇒ Possibility of direct 2nd order transition from Z_2 SL to conventional collinear Neel.

A deconfined critical point again



General lesson II

- Can possibly reach Z_2 spin liquid with nodal spinons by direct second order transition from conventional Neel state.
- The (unstable) gapless U(1) spin liquid controls a large intermediate length scale regime in Neel state near the transition.

Summary, conclusions, etc - I

- Gapless spin liquids exist as stable phases in $D = 2+1$.

They may be accessed from conventional Neel by second order transitions.

Needed: Numerics to determine N_{c1} , N_{c2}

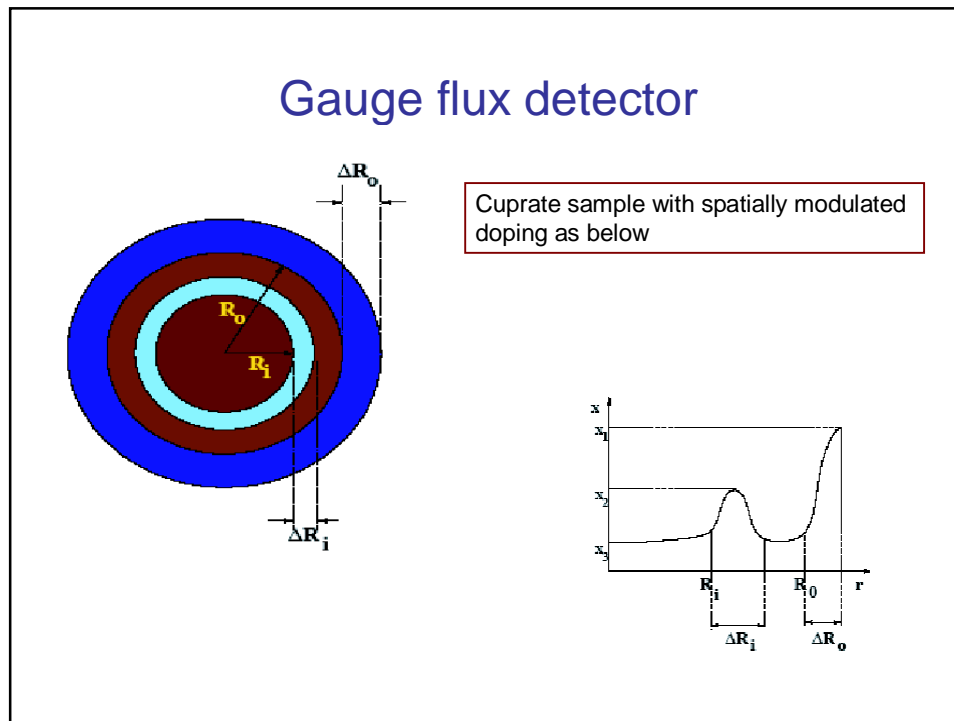
Summary, conclusions -II

- U(1) SL with “gapless Dirac spinons” apparently plays an important role whether it is stable or not.

Are the cuprates doped U(1) spin liquids?

How to tell?

Detect conserved U(1) gauge flux!



- ### Gauge flux detection
- Start with outer ring superconducting and trap an odd number of $hc/2e$ vortices
(choose thin enough so that there is no physical flux).
 - Cool further till inner annulus goes superconducting.
 - For carefully constructed device will spontaneously trap $hc/2e$ vortex of either sign in inner annulus.

How does it work?

- Odd $hc/2e$ vortex inside outer ring => π flux of internal gauge field spread over the inner radius.
(Lee, Wen, 2001)
- If inner annulus sees major part of this internal flux, when it cools into SC, it prefers to form a physical vortex.
- For best chance, make both SC rings thinner than penetration depth and device smaller than roughly a micron.