Deconfined Criticality and `Landau Forbidden' Phase Transitions in Quantum Magnets

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Collaborators

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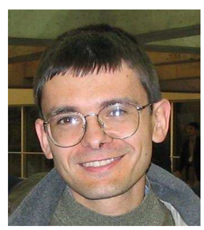
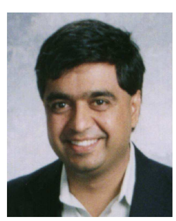
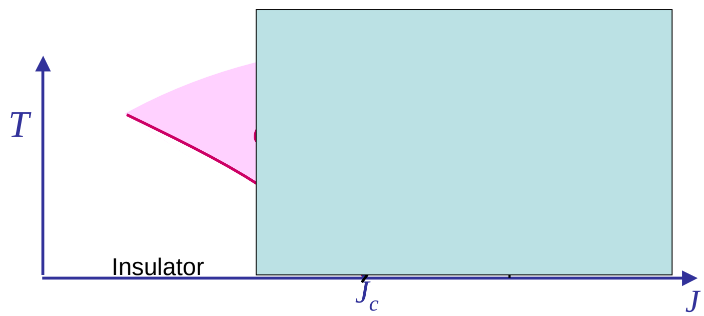
- M. P. A. Fisher (KITP)
  - Science 303,1490, 2004.
  - (cond-mat/0311326).
  - cond-mat/0312617.





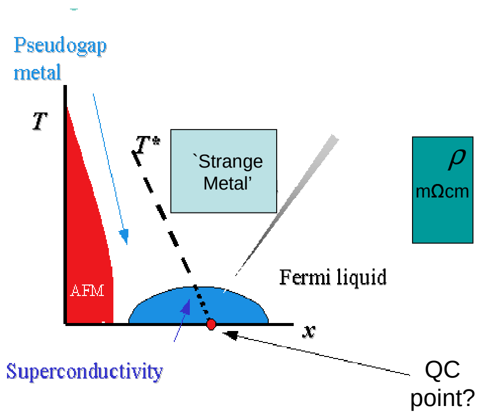
Quantum Phase Transitions

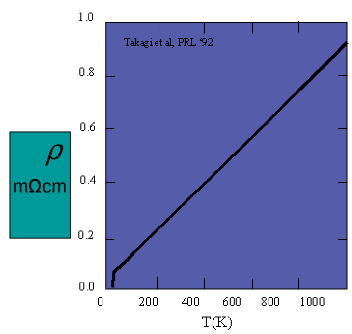
- Universality.
- Continuous quantum phase transitions can control the finite temperature physics in a region.



Puzzles in Strong Correlation Physics

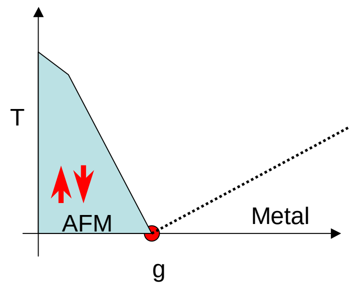
- Cuprate high temperature superconductors





Landau Theory in Quantum Phase Transitions

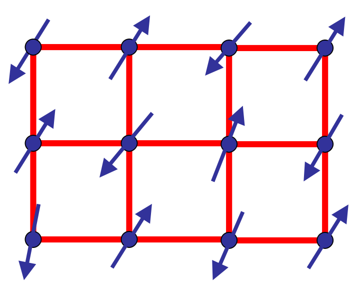
- Magnetic ordering in a metallic environment:
  - **Theory**: Fluctuations of Landau order parameter + damping (Hertz, Millis, Moriya)
  - But several systems (eg. $YbRh_2Si_2$) show fundamental disagreement.



- Physics beyond Landau's paradigm at QPTs?

Phases and Transitions in Quantum Magnets

- S=1/2 on a square lattice (D=2).
  - Realised by the undoped cuprate compounds eg. La_2CuO_4



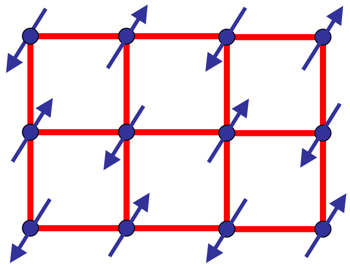
$$H = J\sum_{\langle rr'\rangle} \vec{S}_r \bullet \vec{S}_{r'}$$

$\vec{S}_r$ Spin ½ Operator

- Ground state?

Neel Phase

- Ground state with only nearest neighbour interaction:

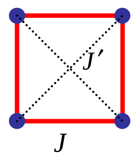


- Order Parameter- staggered spin (Neel vector):

$$\hat{n}_r = (-1)^{x+y} \vec{S}_r$$

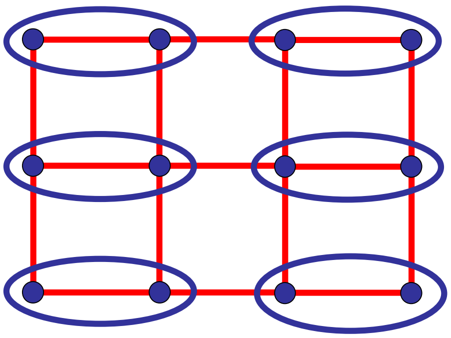
- Spin wave excitations

- However with other interactions (eg. next neighbour)



Valence Bond Solid State (VBS)

- Frozen arrangement of singlets expected.



Columnar State

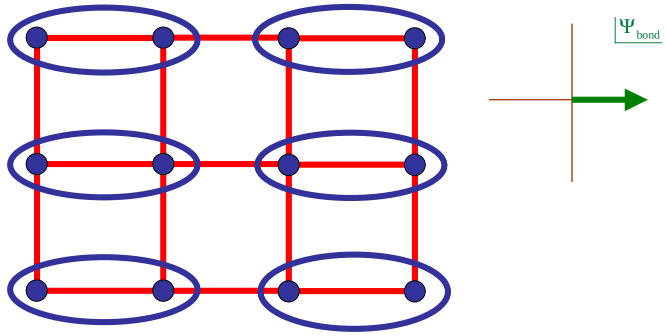
Where:

$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

- Breaks Lattice Translation and Rotation Symmetry
  → Order Parameter Ψ_{bond}

VBS Order Parameter

- Associate a Complex Number Ψ_{bond}



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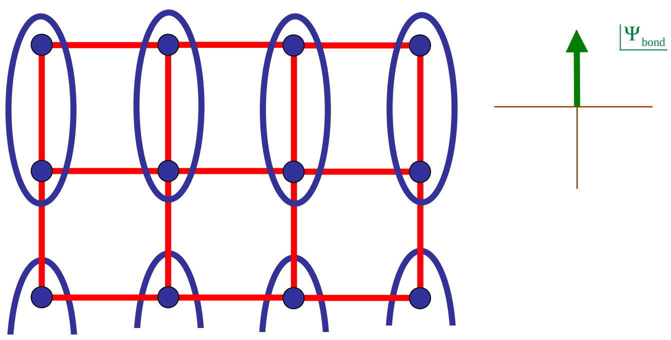

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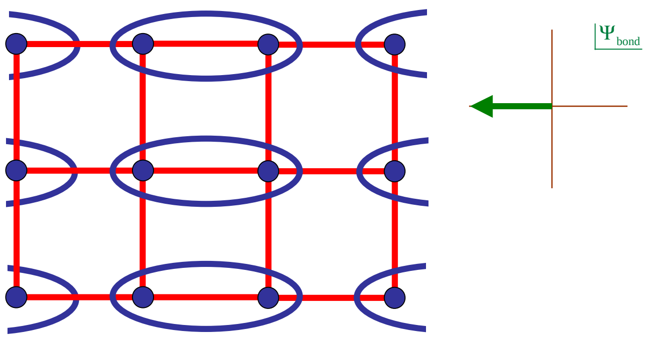

VBS Order Parameter

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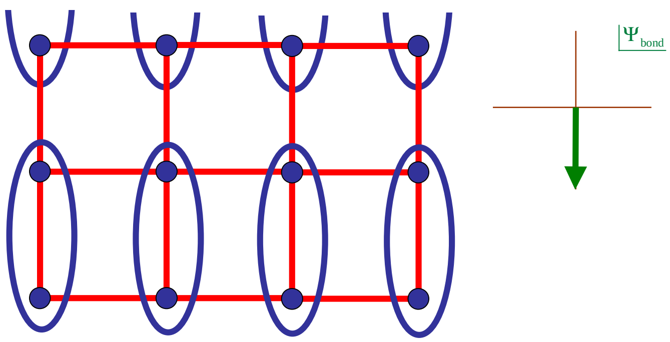
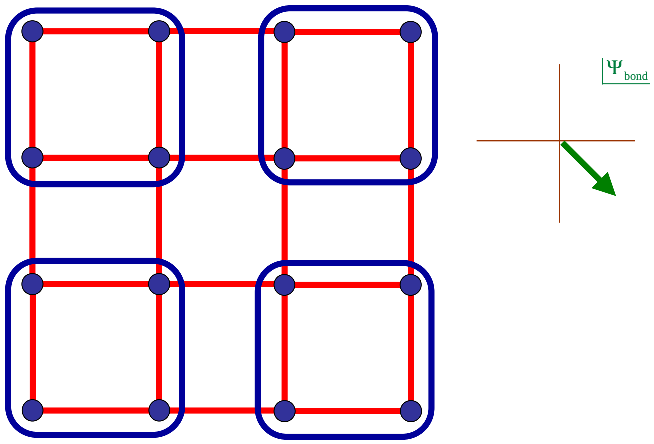


VBS Order Parameter

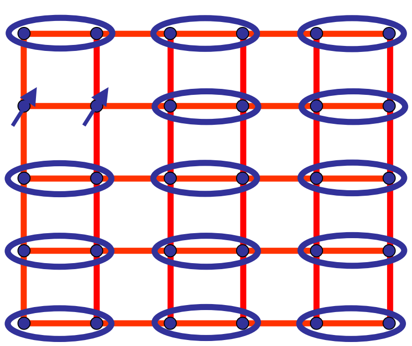
- Associate a Complex Number Ψ_{bond}

Plaquette Order

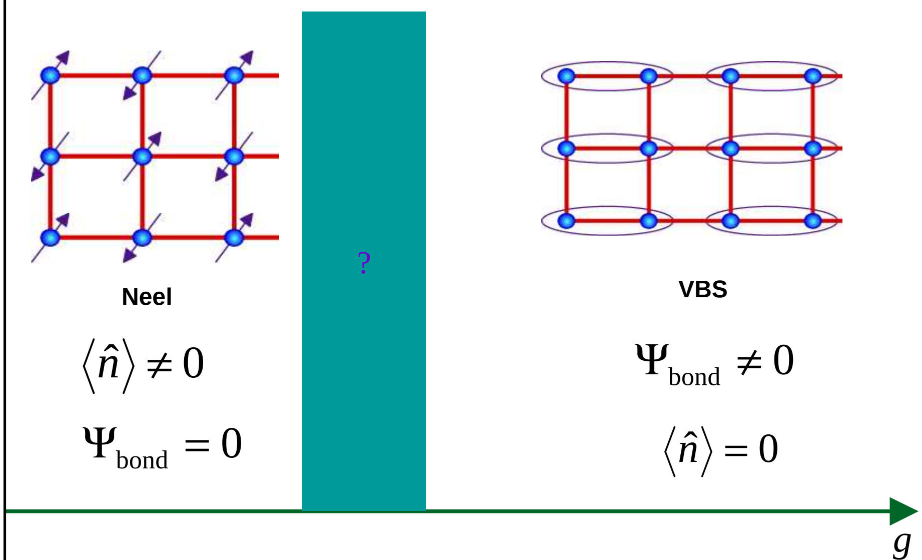


Excitations of the VBS

- Gapped S=1 excitation

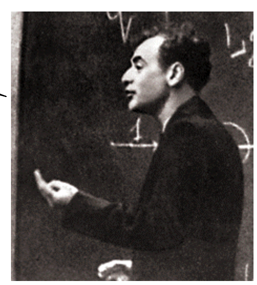


Phase Diagram…

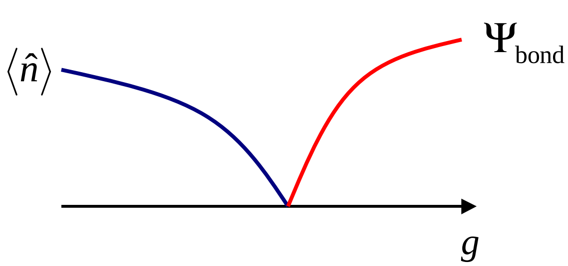


Constraints From Landau's Theory

No direct transition that is continuous



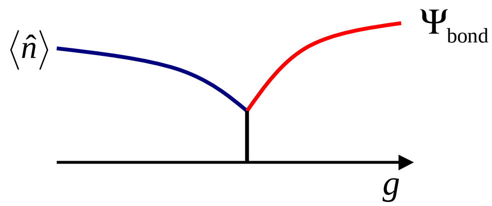
Two unrelated orders – Neel and VBS



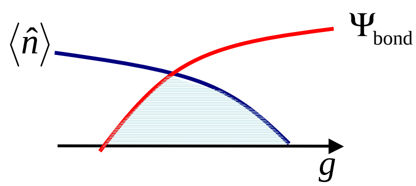
Needs special fine tuning

Generic Possibilities

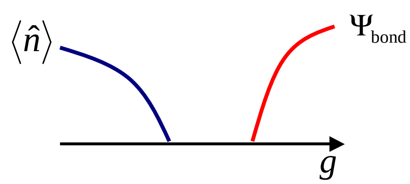
- First Order



- Coexistence

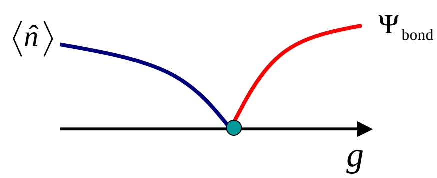


- Or:



Not Always True for Quantum Phase Transitions!

- Continuous transition directly between Neel and VBS is a generic possibility
  - **no** fine tuning required.
  - `Landau forbidden' transition
  - Berry Phases



- Critical point **not** described by fluctuations of the order parameters.
  - Theory of the critical point:
    - emergent `photons'
    - fractionalized excitations
    - Topological `order'.
- But phases - Neel and VBS - are conventional.

Describing Quantum Magnets

Quantum Partition Function

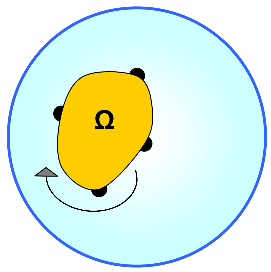
$$Z = \mathrm{Tr}\, e^{-\beta H} \qquad \left(H = J \sum_{\langle \mathrm{rr}' \rangle} \vec{S}_{\mathrm{r}} \bullet \vec{S}_{\mathrm{r}'} + \ldots \right)$$

Staggered Spin

$$\hat{n}_{\mathrm{r}} = \eta_{\mathrm{r}} \frac{\vec{S}_{\mathrm{r}}}{S} \qquad \left(\eta_{\mathrm{r}} = (-1)^{x+y} \right)$$

Path Integral Representation

$$Z = \int D\hat{n}\, e^{is \sum_{\mathrm{r}} \eta_{\mathrm{r}} \Omega(\hat{n}_{\mathrm{r}})}\, e^{-\int d\tau H(\hat{n})}$$



Describing Quantum Magnets

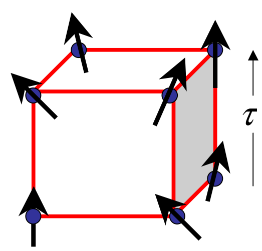
- Focus on the local Neel vector $\hat{n}$ (*F. D. M. Haldane 1988*).
- Obtain amplitude for a given space-time configuration $\{\hat{n}(r,\tau)\}$

$$Z = \int D\hat{n}\, e^{-S_E[\hat{n}]}$$

$$S_E = -\mathrm{J} \int d\tau \sum_{\langle \mathrm{ij} \rangle} \hat{n}_i \bullet \hat{n}_j + S_B[\hat{n}]$$

Aligns Neel field.

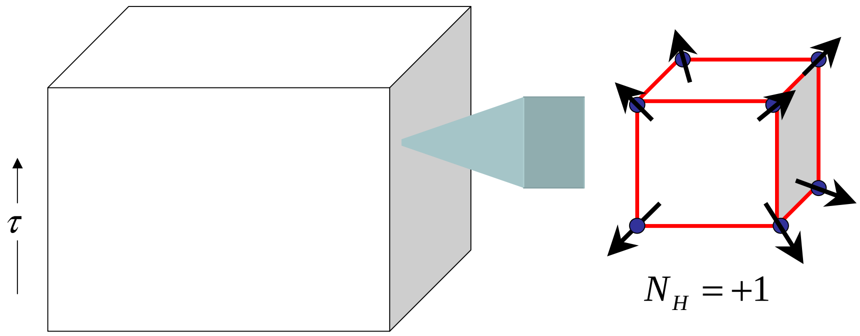
Berry phase. Active only for *hedgehogs*



Neel Field in spacetime

Hedgehogs and Berry Phases

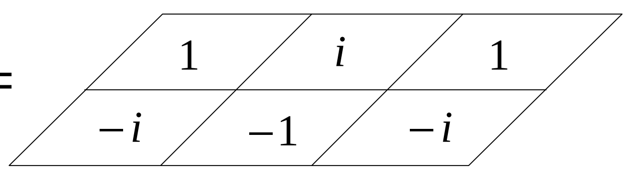
- Integer Hedgehog number defined at space-time points (events)



Berry Phase

- Each Hedgehog associated with a spatial plaquette

$\xi_r =$



- Berry Phase = $\xi_r^{\,N_H}$ $\left(\xi_r^{\,2SN_H}\right)$
- Note $\xi^4 = 1$

Obtaining the Neel and VBS phases

$$S_E = -\mathrm{J}\int d\tau \sum_{\langle \mathrm{ij} \rangle} \hat{n}_i \bullet \hat{n}_j + S_B[\hat{n}]$$

- Large J, spins locked;
  - Neel phase.
  - Few hedgehogs
- Small J, large fluctuations
  - Hedgehogs proliferate
  - VBS phase results (Read and Sachdev, 1989).
    - Due to Berry phases attached to hedgehogs.
- To understand region in between→ hedgehog free model.

Hedgehog Proliferation => VBS

- Action of Symmetries on Hedgehog Insertion Operator V_{r}^{+}
  - Spin singlet
  - Lattice Translations

    $\mathbf{T_x} V_{\mathrm{r}}^{+} = -\mathrm{i}\, V_{\mathrm{r}+\hat{\mathrm{x}}}$

    $\mathbf{T_y} V_{\mathrm{r}}^{+} = \mathrm{i}\, V_{\mathrm{r}\hat{+}\hat{\mathrm{y}}}$
  - Lattice Rotations (about lattice point)

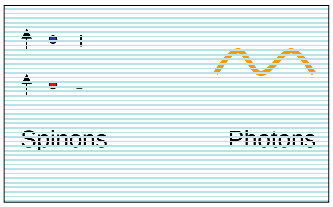
    $\mathbf{R}_{\pi/2} V_{\mathrm{r}}^{+} = \mathrm{i}\, V_{\mathrm{R}_{\pi/2}\mathrm{r}}^{+}$

- Exactly the same as $\mathrm{e}^{\mathrm{i}\pi/4}\Psi_{\mathrm{bond}}$

$$\Psi_{\mathrm{bond}} = (-1)^{x}\,\vec{\mathrm{S}}_{\mathrm{r}} \bullet \vec{\mathrm{S}}_{\mathrm{r}+\hat{\mathrm{x}}} + \mathbf{i}\,(-1)^{y}\,\vec{\mathrm{S}}_{\mathrm{r}} \bullet \vec{\mathrm{S}}_{\mathrm{r}+\hat{\mathrm{x}}}$$

Hedgehog Free Model

Natural Variables to describe hedgehog free model



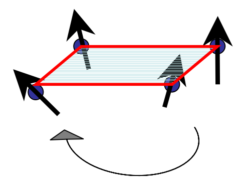
Spin ½ fields

Neel vector

$$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \Rightarrow \hat{n} = z^{+}\vec{\sigma} z$$

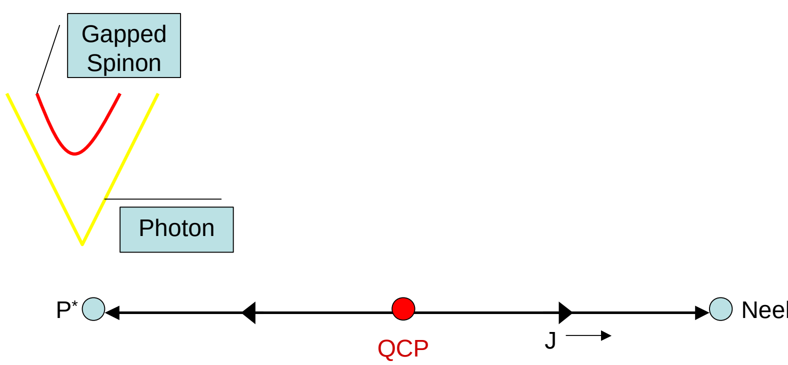
Photon

Neel Vector Chirality $\Rightarrow$



$$E = -J'\sum_{\langle ij\rangle}(z_i^{+} e^{ia_{ij}} z_j + h.c.) + K\sum_{\otimes}(\nabla\times a)^2$$

Phase Structure of Hedgehog Free Model



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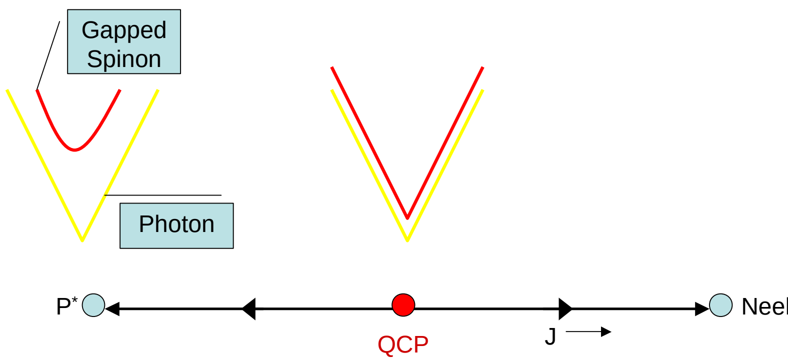

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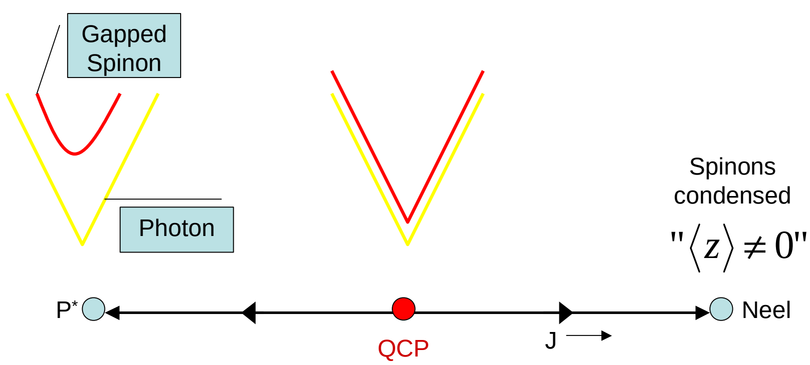
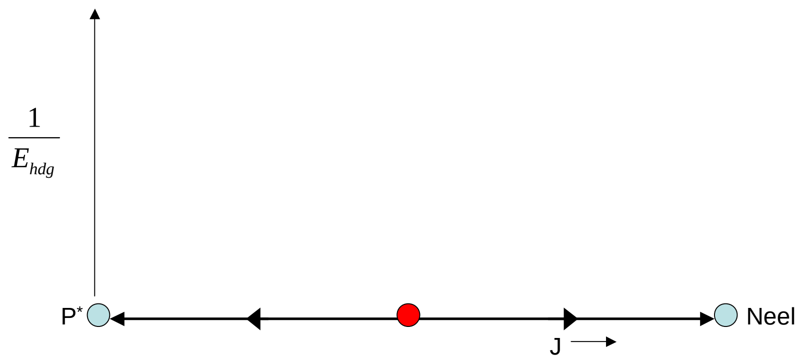
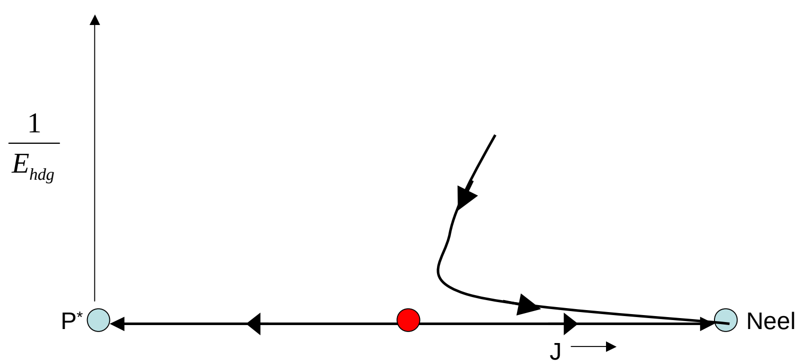


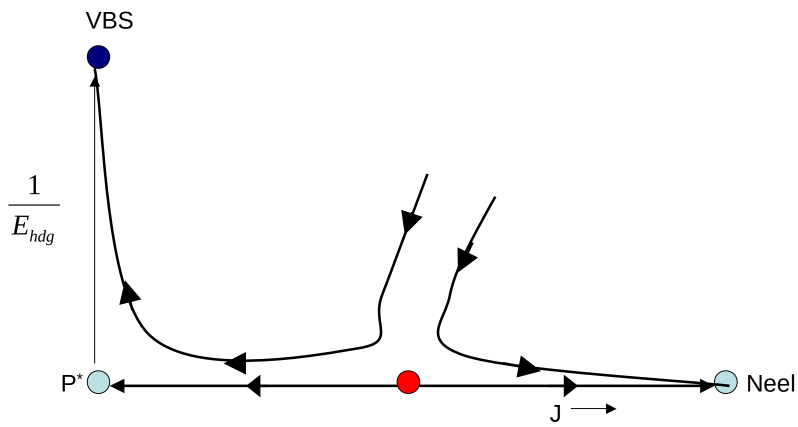
Reintroducing Hedgehogs



Reintroducing Hedgehogs



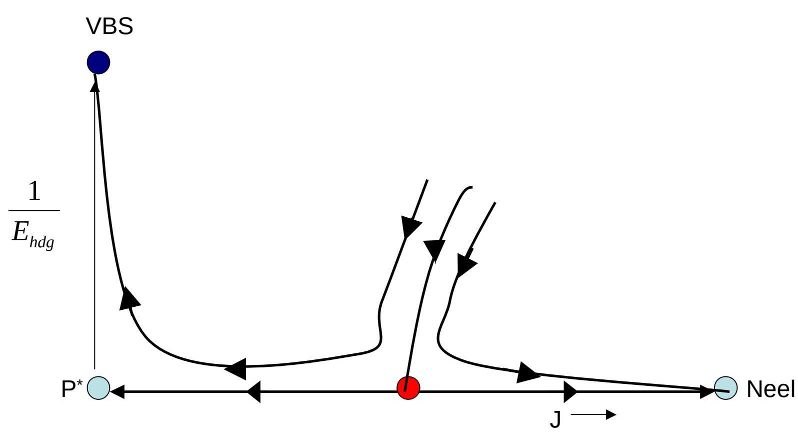
Reintroducing Hedgehogs



A. Polyakov

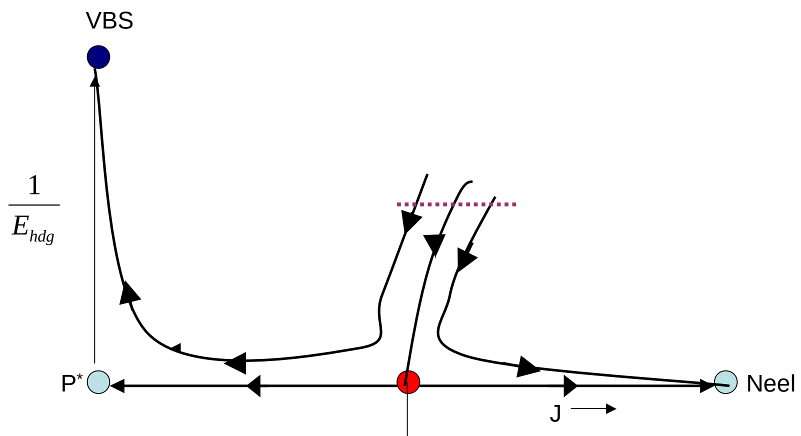
N. Read and S. Sachdev

Reintroducing Hedgehogs



Quadrupled Hedgehogs
Irrelevant at Transition.
T. Senthil et al.

Direct Neel – VBS Transition



$$L = \left|(\partial_\mu - i a_\mu) z\right|^2 + K(\nabla \times a)^2 + V(|z|^2)$$

Irrelevance of 4 Hedgehog Tunneling at QCP

- Consider general family of models (Sachdev-Jalabert)

  CP_{N-1}+ Berry Phases for monopoles. (N=2 is case of interest)

  $$(z_1\, z_2) \rightarrow (z_1\, z_2 \ldots z_N)$$

  - 4 Hedgehogs <u>Irrelevant</u> at criticality
    - For $N \rightarrow \infty$

    AND
    - For $N = 1$

- Easy Plane Limit
  - Explicit form of 4 hedgehog tunneling term in dual theory. Argued to be irrelevant at QCP.

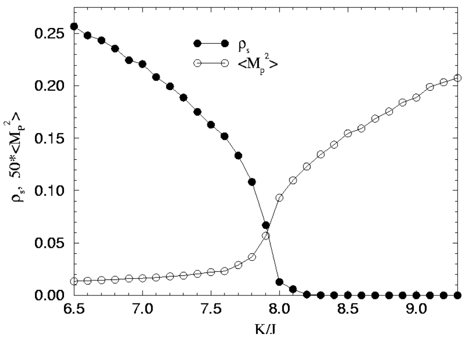
$$\mathcal{L}_{\rm dual} = \sum_{a=1,2} |(\partial_\mu - iA_\mu)\psi_a|^2 + s_d|\psi|^2 + u_d(|\psi|^2)^2 + w_d|\psi_1|^2|\psi_2|^2 + \kappa_d(\epsilon_{\mu\nu\kappa}\partial_\nu A_\kappa)^2 - \lambda{\rm Re}[(\psi_1^*\psi_2)^4]. \quad (5.6)$$

Results from Large Scale Numerics

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett*. **89**, 247201 (2002).

First *large scale* (> 8000 spins) numerical study of disordering Neel state in a *S=1/2* antiferromagnet with square lattice symmetry. Easy plane spins.

$$H = 2J\sum_{\langle ij\rangle}\left(S_i^xS_j^x + S_i^yS_j^y\right) - K\sum_{\langle ijkl\rangle\subset\square}\left(S_i^+S_j^-S_k^+S_l^- + S_i^-S_j^+S_k^-S_l^+\right)$$



Direct Continuous Transition??

Properties of the Transition I

- Emergent Conserved Flux
  - Absence of Hedgehogs at the transition.
    - Flux correlations $\langle f_R f_0 \rangle \approx 1/R^4$ (f = Neel vector chirality)
    - Plaquette order and Columnar-dimer order correlators power law with equal exponents (independent of columnar/plaquette nature of VBS phase).

- `Deconfined Spinons'

  Scaling Dim of Neel field

  $$[\hat{n}] = L^{-\frac{1+\eta}{2}}$$

  with η=0.6 (large! Compare η(Heisenberg)=0.03)

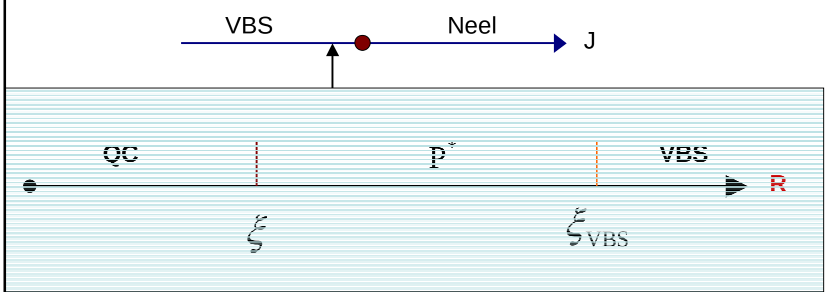
  Because $\hat{n} = z^+\vec{\sigma}z$

  For free spinons => η=1.0

Properties of the Transition II

- Structure of Correlations

  Approaching the QCP from (say) the VBS side.
  - Two diverging length scales. (dangerous irrelevance)
  - Intermediate scales - P^* (free photon) phase



$\langle \hat{n}_R \hat{n}_0 \rangle \approx 1/R^{1+\eta}$

$\langle f_R f_0 \rangle \approx 1/R^4$

$\langle f_R f_0 \rangle \approx 1/R^3$

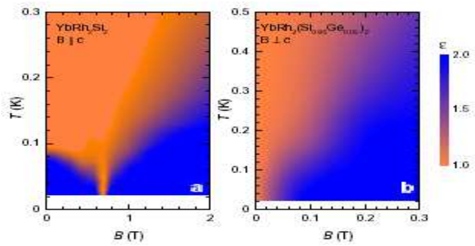
Summary

- Landau Forbidden Transitions possible for Quantum Phase Transitions
  - Disorder operator (hedgehog insertion) carries quantum numbers.

- Transition is ordered topologically (no Hedgehogs).
  - Emergent conservation law - `flux' conserved.
  - Critical fields – photons and spinons.

Future Directions

- Relation to strongly correlated materials?
  - Need charge degrees of freedom & fermions.
  - Extensions to 3D?



Key Player:

Hedgehogs