Review of Higgs Studies at Future Lepton Colliders

Tim Barklow (SLAC) April 13, 2016 Experimental Challenges for the LHC Run II, KITP

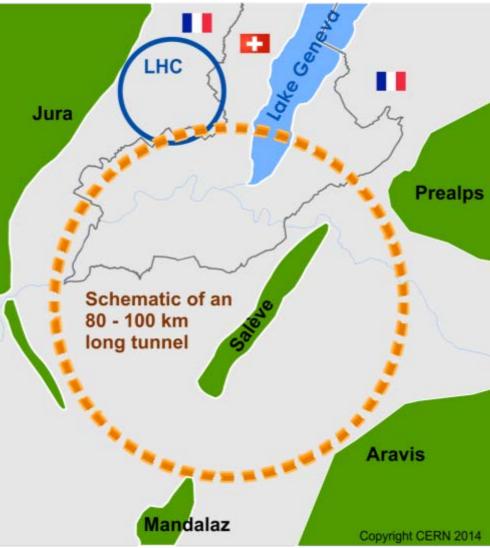
Future Circular Collider Study - SCOPE CDR and cost review for the next ESU (2018)

Intl. collab. to study:

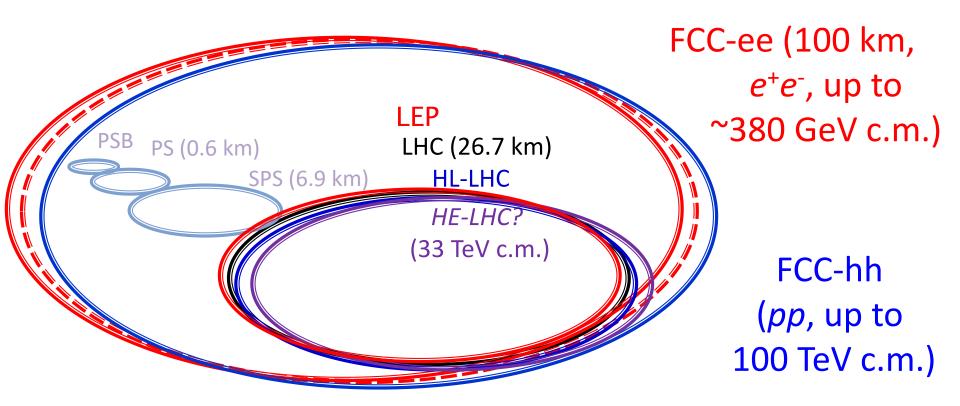
• *pp*-collider (*FCC-hh*) Ultimate goal, defining infrastructure.

~16 T \Rightarrow 100 TeV *pp* in 100 km ~20 T \Rightarrow 100 TeV *pp* in 80 km

- e⁺e⁻ collider (FCC-ee) as potential first step ECM=90-400 GeV
 - p-e (FCC-he) option
- 80-100 km infrastructure in Geneva area



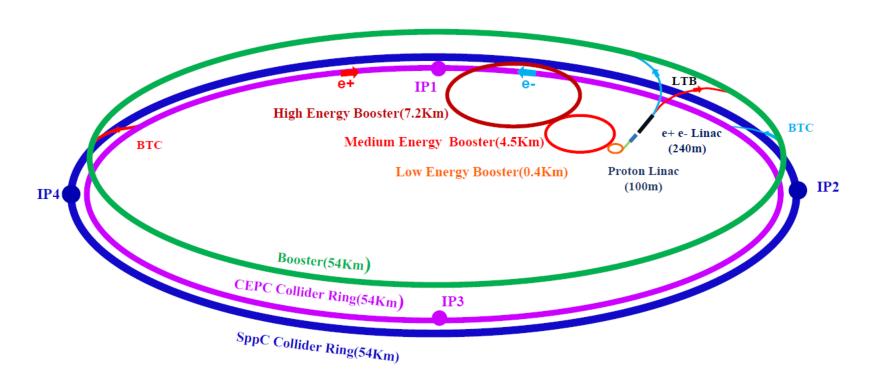
FCC-ee possible long-term strategy



60 years of $e^+e^- pp AA$ (ep) physics @ highest energies

NB Great synergy and complementarity of FCC-ee and FCC-hh!

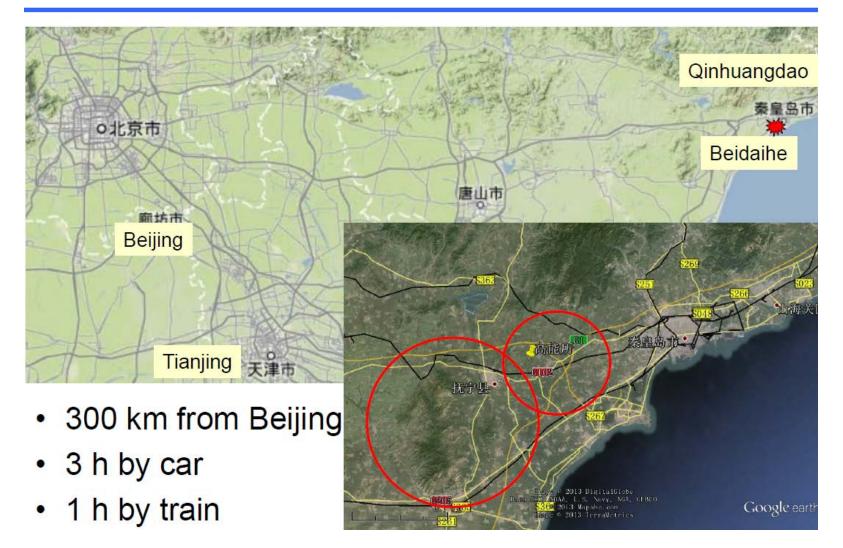
CEPC+SppC



LTB : Linac to Booster

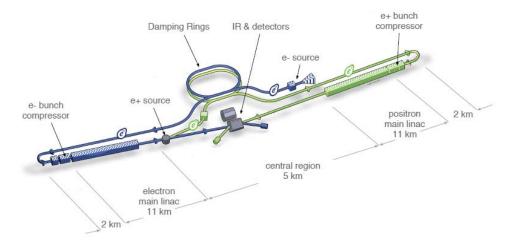
BTC : Booster to Collider Ring

CEPC A Candidate Site

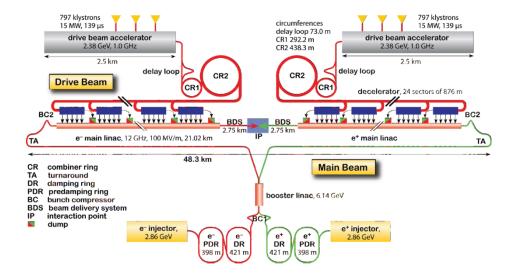


Linear Colliders

ILC International Linear Collider e^+e^- linear collider with SCRF linac $250 \le \sqrt{s} \le 1000 \text{ GeV}$ 31 km length ($\sqrt{s} \le 500 \text{ GeV}$) 49 km length ($\sqrt{s} = 1000 \text{ GeV}$)



CLIC Compact Linear Collider e^+e^- linear collider with X-Band linac RF powered by a 2nd drive beam $350 \le \sqrt{s} \le 3000 \text{ GeV}$ 13 km length ($\sqrt{s} = 500 \text{ GeV}$) 48 km length ($\sqrt{s} = 3000 \text{ GeV}$)



ILC Machine Parameters from TDR

			Baseline	e 500 GeV	Machine	L Upgrade	$E_{\rm CM}$ U	lpgrade
							Α	В
Center-of-mass energy	$E_{\rm CM}$	GeV	250	350	500	500	1000	1000
Collision rate	f_{rep}	Hz	5	5	5	5	4	4
Electron linac rate	f_{linac}	Hz	10	5	5	5	4	4
Number of bunches	$n_{\rm b}$	1010	1312	1312	1312	2625	2450	2450
Bunch population	N	$\times 10^{10}$	2.0	2.0	2.0	2.0	1.74	1.74
Bunch separation	$\Delta t_{\rm b}$	ns	554	554	554	366	366	366
Pulse current	$I_{\rm beam}$	mA	5.8	5.8	5.8	8.8	7.6	7.6
Main linac average gradient	$G_{\mathbf{a}}$	MV m ⁻¹	14.7	21.4	31.5	31.5	38.2	39.2
Average total beam power	P_{beam}	MW	5.9	7.3	10.5	21.0	27.2	27.2
Estimated AC power	P_{AC}	MW	122	121	163	204	300	300
RMS bunch length	$\sigma_{\rm z}$	mm	0.3	0.3	0.3	0.3	0.250	0.225
Electron RMS energy spread	$\frac{\Delta p}{p}$	%	0.190	0.158	0.124	0.124	0.083	0.085
Positron RMS energy spread	$\frac{\Delta p}{p}$	%	0.152	0.100	0.070	0.070	0.043	0.047
Electron polarization	$\overline{P}_{-}^{P/P}$	%	80	80	80	80	80	80
Positron polarization	P_{\pm}	%	30	30	30	30	20	20
Horizontal emittance	$\gamma \epsilon_{\mathbf{x}}$	μm	10	10	10	10	10	10
Vertical emittance	$\gamma \epsilon_y$	nm	35	35	35	35	30	30
IP horizontal beta function	$\beta_{\rm x}^*$	mm	13.0	16.0	11.0	11.0	22.6	11.0
IP vertical beta function	β_x^*	mm	0.41	0.34	0.48	0.48	0.25	0.23
	-							
IP RMS horizontal beam size	$\sigma_{\rm x}^*$	nm	729.0	683.5	474	474	481	335
IP RMS vertical beam size	σ_y^*	nm	7.7	5.9	5.9	5.9	2.8	2.7
Luminosity	L	$\times 10^{34} \text{cm}^{-2} \text{s}^{-1}$	0.75	1.0	1.8	3.6	3.6	4.9
Fraction of luminosity in top 1%	$L_{0.01}/L$		87.1%	77.4%	58.3%	58.3%	59.2%	44.5%
Average energy loss	δ_{BS}		0.97%	1.9%	4.5%	4.5%	5.6%	10.5%
Number of pairs per bunch crossing	N_{pairs}	$\times 10^3$	62.4	93.6	139.0	139.0	200.5	382.6
Total pair energy per bunch crossing	E_{pairs}	TeV	46.5	115.0	344.1	344.1	1338.0	3441.0

Note there are two types of upgrades:

Luminosity upgrade: Install extra klystrons and modulators so number of bunches can be doubled; envisioned after 8 years of baseline running

Energy upgrade: Increase accel. gradient, lengthen linac, or both. TDR config assumes 49 km. length; envisioned after 20 years of running

Luminosity Upgrade for Ecm=250 GeV

			Baseline ILC	Lumi Upgrade
Center-of-mass energy	$E_{\rm CM}$	GeV	250	250
Collision rate	$f_{\rm rep}$	Hz	5	10
Electron linac rate	f_{linac}	Hz	10	10
Number of bunches	$n_{ m b}$		1312	2625
Pulse current	$I_{\rm beam}$	mA	5.8	8.75
Average total beam power	$P_{\rm beam}$	MW	5.9	21
Estimated AC power	$P_{\rm AC}$	MW	129	200
Luminosity	L	$ imes 10^{34} { m cm}^{-2} { m s}^{-1}$	0.75	3.0

The $\sqrt{s} = 250$ GeV lumi is quadrupled by doubling the number of bunches *and* the collision rep rate

The 10 Hz operation which in the baseline was split between 5 Hz collision and 5 Hz e^+ production is now 100% collision in the lumi upgrade config. A longer undulator should be ready that can produce sufficient e^+ yield with 125 GeV electrons

Note the AC power is 200 MW, the same as the 5 Hz lumi upgrade power at $\sqrt{s} = 500$ GeV. Also note that ILC produces 3×10^{34} cm⁻²s⁻¹ luminosity with 200 MW total AC power.

ILC Running Scenario

	\sqrt{s}	∫Ldt	Lpeak	Ramp				Т	$T_{\rm tot}$	Comment
	[GeV]	$[fb^{-1}]$	$[fb^{-1}/a]$	1	2	3	4	[a]	[a]	
Physics run	500	500	288	0.1	0.3	0.6	1.0	3.7	3.7	TDR nominal at 5 Hz
Physics run	350	200	160	1.0	1.0	1.0	1.0	1.3	5.0	TDR nominal at 5 Hz
Physics run	250	500	240	0.25	0.75	1.0	1.0	3.1	8.1	operation at 10 Hz
Shutdown								1.5	9.6	Luminosity upgrade
Physics run	500	3500	576	0.1	0.5	1.0	1.0	7.4	17.0	TDR lumi-up at 5 Hz
Physics run	250	1500	480	1.0	1.0	1.0	1.0	3.2	20.2	lumi-up operation at 10 Hz

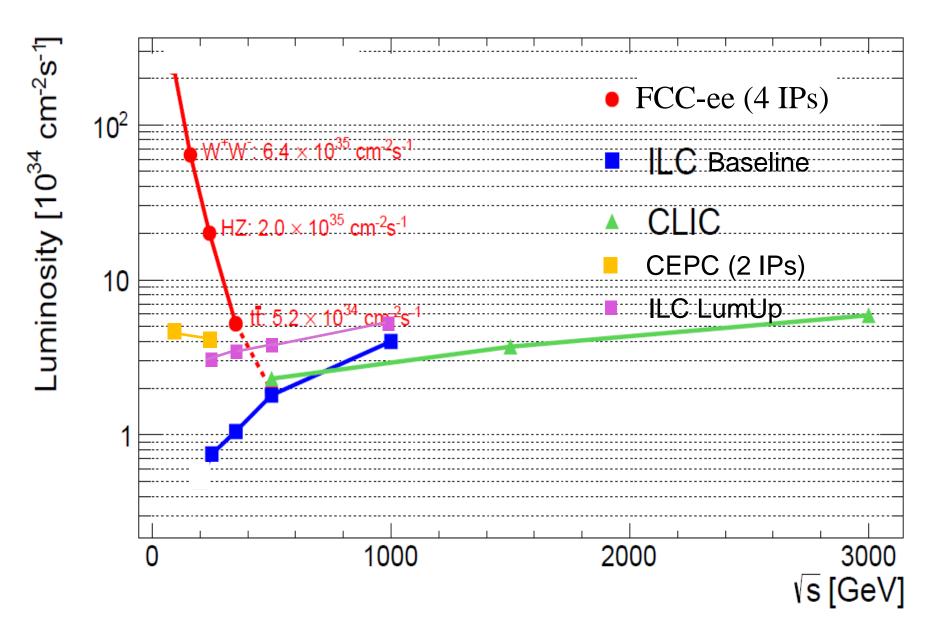
Table 7: Scenario H-20: Sequence of energy stages and their real-time conditions.

H-20

	first phase	lumi upgrade	total	Snowmass Lum-up [†]
250 GeV	500 fb⁻1	1500 fb-1	2 ab⁻1	1.15 ab-1
350 GeV	200 fb-1		0.2 ab-1	
500 GeV	500 fb⁻1	3500 fb-1	4 ab⁻ı	1.6 ab-1
time	8.1 yrs	10.6 yrs	20.2 yrs*	

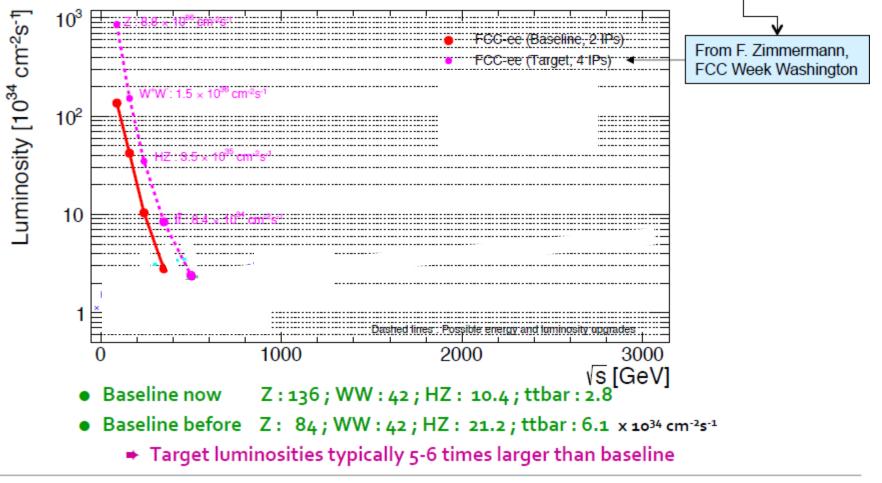
* includes 1.5 years for luminosity upgrade

† ILC Higgs whitepaper: arXiv:1310.0763

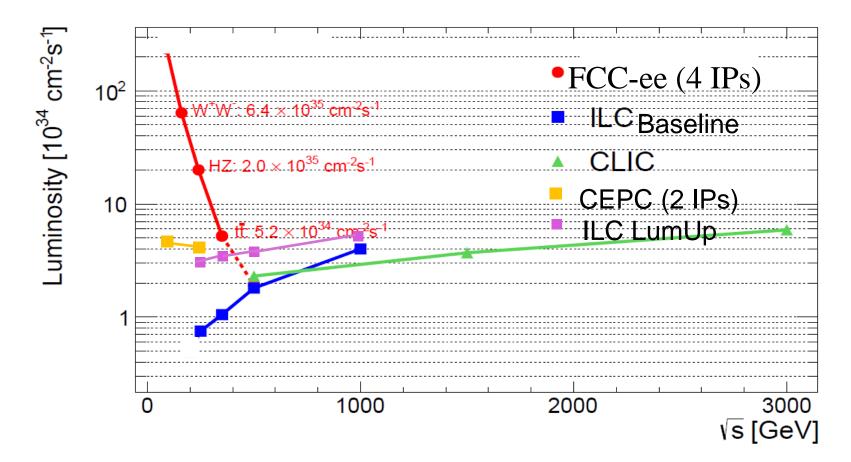


FCC-ee

- We have now coherent and sound optics for the first time !
 - Conservative parameters (see backup slides) but sound base towards the target



 $\sqrt{s} = 90, 160 \, \text{GeV}$



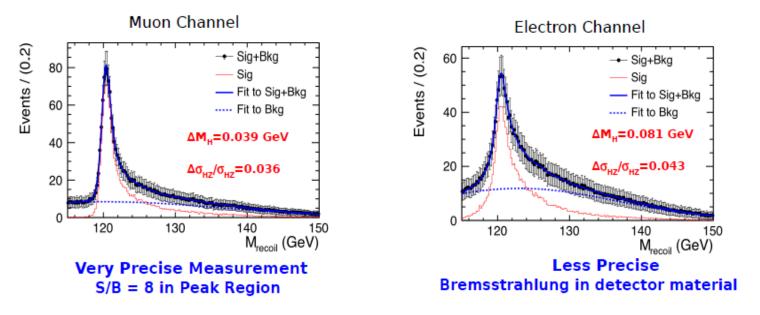
Not easy to run the ILC at these energies.

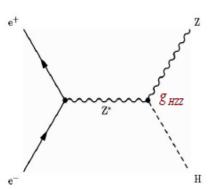
e.g. 150 GeV (125 GeV) e^- beam needed for positron production in baseline (lumi upgrade) design. In the lumi upgrade config L=4×10³³ @ 91 GeV and L=8×10³³ @ 160 GeV. This would provide $\int Ldt = 100 \text{ fb}^{-1}$ @ 91 GeV in 10 mos. and $\int Ldt = 200 \text{ fb}^{-1}$ @ 160 GeV in 10 mos.

Overview of Higgs Physics at e^+e^- **Colliders at** $\sqrt{s} = 250,350,500,1000$ **GeV**

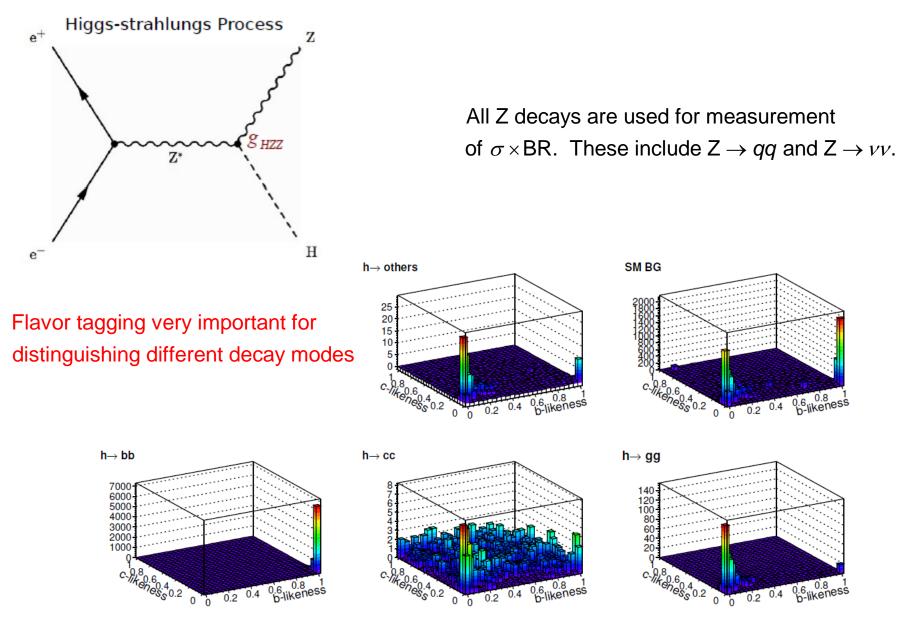
Measurement of $\sigma(e^+e^- \rightarrow ZH)$ $\sqrt{s} = 250 \text{ GeV}$

Higgs Recoil Measurement of Higgs Mass and Higgstrahlung Cross Section -- Key to model independent Higgs coupling measurements

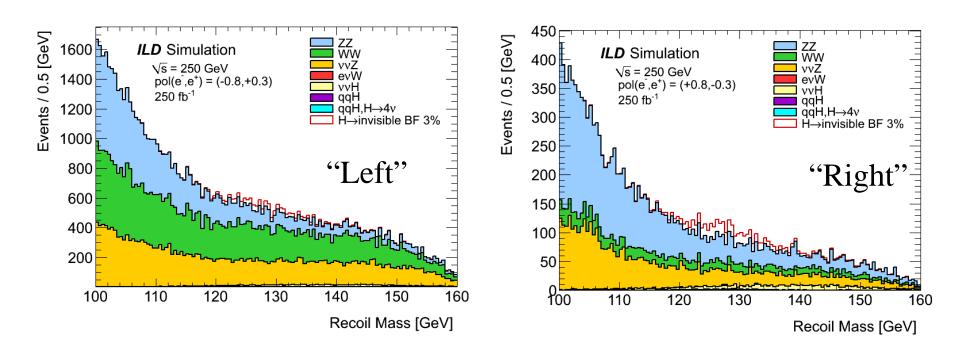




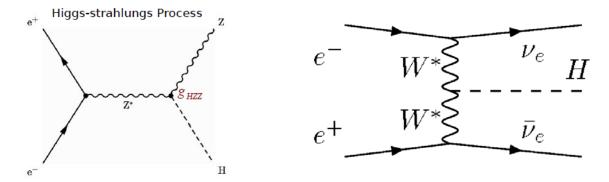
ILC: $\Delta M_H = .032 \text{ GeV}$, $\Delta \sigma_{HZ} / \sigma_{HZ} = 2.5\%$ for L= 250 fb⁻¹ $\Delta M_H = .015 \text{ GeV}$, $\Delta \sigma_{HZ} / \sigma_{HZ} = 1.2\%$ for L=1150 fb⁻¹ $\sigma_{HZ} \sim g_{HZZ}^2$ $\Rightarrow \Delta g_{HZZ} / g_{HZZ} = 1.3\%$ (0.6%) for L=250 (1150) fb⁻¹ $\sigma \times BR$ measurements using $e^+e^- \rightarrow ZH$ $\sqrt{s} = 250 \text{ GeV}$



 $e^+e^- \rightarrow ZH$, $Z \rightarrow qq$, $H \rightarrow invisible$ $\sqrt{s} = 250 \text{ GeV}$



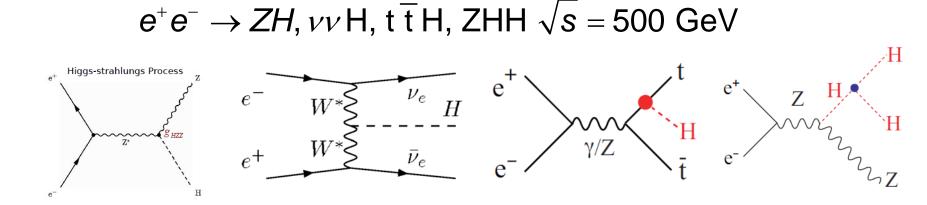
$$e^+e^- \rightarrow ZH$$
, $\nu\nu H \sqrt{s} = 350 \text{ GeV}$



All of the Higgstrahlung studies that were done at $\sqrt{s} = 250$ GeV can also be done at $\sqrt{s} = 350$ GeV. Precisions for $\sigma \cdot BR$ are comparable, as is the precision for $\sigma(ZH)$ once $Z \rightarrow q \bar{q}$ decays are included.

WW fusion production of the Higgs at $\sqrt{s} = 350$ GeV provides a much better measurement of g_{HWW} compared to $\sqrt{s} = 250$ GeV. This gives a much improved estimate of the total Higgs width Γ_H which in turn significantly improves the coupling errors obtained from $\sigma \cdot BR$ measurements made at $\sqrt{s} = 250$ GeV.

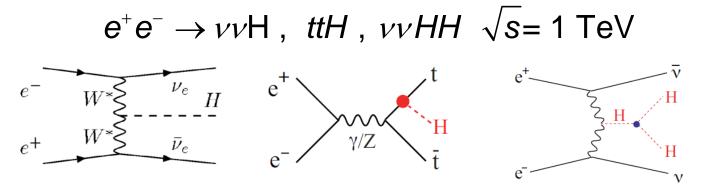
The recoil Higgs mass measurement is significantly worse at $\sqrt{s} = 350$ GeV with respect to $\sqrt{s} = 250$ GeV. However, there is hope that direct calorimeter Higgs mass measurements using $e^+e^- \rightarrow vvH$ will recover the precision.



The g_{HWW} coupling can also be measured well at $\sqrt{s} = 500$ GeV through WW fusion production of the Higgs. Also the measurement of $\sigma(e^+e^- \rightarrow vvH) \times BR(H \rightarrow X)$ can be made for many Higgs decay modes $H \rightarrow X$.

Through $e^+e^- \rightarrow ttH$ the top Yukawa coupling can be measured to $\Delta y_t / y_t = 16.6\%$ with 500 fb⁻¹ at $\sqrt{s} = 500$ GeV. With same luminosity at $\sqrt{s} = 550$ GeV the precision is $\Delta y_t / y_t = 6.73 \implies$ strong motivation to increase nominal energy to $\sqrt{s} = 550$ GeV

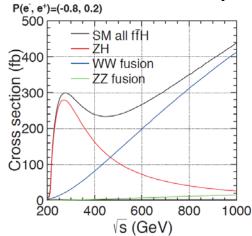
The ZHH channel is open at $\sqrt{s} = 500$ GeV. The Higgs self coupling can be measured to 27% with 4 ab⁻¹ assuming the true value is the SM value.



At $\sqrt{s} = 1$ TeV the ILC provides better measurements of the top Yukawa coupling and Higgs self coupling. For example the Higgs self coupling can be measured to an accuracy of 10% with 4 ab⁻¹ at $\sqrt{s} = 1$ TeV (again, assuming the true value is the SM value).

Search for additional Higgs bosons that might have been missed at LHC, and study new resonances below $\sqrt{s} = 1$ TeV that might be seen at LHC

In addition, the ILC becomes a Higgs factory again since the total Higgs cross section is larger than the total cross sections at 250 GeV, especially if polarized beams are used:



Summary of ILC Higgs Measurement Precisions

From "500 GeV ILC Operating Scenarios" arXiv:1506.07830

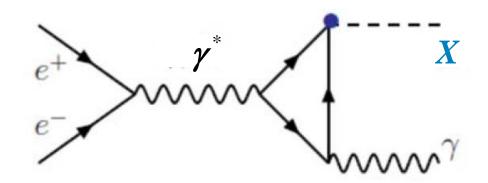
$\int \mathcal{L} dt$ at \sqrt{s}	$250 {\rm fb}^{-1}$ at (250 GeV	330fb^{-1} at 3	350 GeV	500fb^{-1} at 500GeV		
$P(e^{-}, e^{+})$			(-80%	6,+30%)			
production	Zh	$v\bar{v}h$	Zh	$v\bar{v}h$	Zh	vvh	tīh
$\Delta\sigma/\sigma$	[39] 2.0%	-	[10,40] 1.6%	-	3.0	-	-
BR(invis.) [41]	< 0.9%	-	< 1.2%	-	< 2.4%	-	-
decay		$\Delta(\sigma \cdot BR)/(\sigma \cdot BR)$					
$h \rightarrow b\bar{b}$	1.2%	10.5%	1.3%	1.3%	1.8%	0.7%	28%
$h \rightarrow c\bar{c}$	8.3%	-	9.9%	13%	13%	6.2%	-
$h \rightarrow gg$	7.0%	-	7.3%	8.6%	11%	4.1%	-
$h \rightarrow WW^*$	6.4%	-	6.8%	5.0%	9.2%	2.4%	-
$h ightarrow au^+ au^-$	[42] 3.2%	-	[43] 3.5%	19%	5.4%	9.0%	-
$h \rightarrow ZZ^*$	19%	-	22%	17%	25%	8.2%	-
$h ightarrow \gamma \gamma$	34%	-	34%	[44] 39%	34%	[44] 19%	-
$h \rightarrow \mu^+ \mu^-$ [45]	72%	-	76%	140%	88%	72%	-

ILC Model Independent Higgs Coupling Precisions

		H20 @ 8yrs	H20 @ 20yrs		
Topic	Parameter	Initial Phase	Full Data Set	units	ref.
Higgs	m_h	25	15	MeV	[51]
	g(hZZ)	0.58	0.31	%	[8]
	g(hWW)	0.81	0.42	%	[8]
	$g(hb\bar{b})$	1.5	0.7	%	[8]
	g(hgg)	2.3	1.0	%	[8]
	$g(h\gamma\gamma)$	7.8	3.4	%	[8]
		1.2	1.0	%, w. LHC results	[52]
	g(h au au)	1.9	0.9	%	[8]
	$g(hc\bar{c})$	2.7	1.2	%	[8]
	$g(ht\bar{t})$	18	6.3	%, direct	[8]
		20	20	%, <i>tī</i> threshold	[53]
	$g(h\mu\mu)$	20	9.2	%	[8]
	g(hhh)	77	27	%	[8]
	Γ_{tot}	3.8	1.8	%	[8]
	Γ_{invis}	0.54	0.29	%, 95% conf. limit	[8]

- [8] D. M. Asner et al., "ILC Higgs White Paper," arXiv:1310.0763 [hep-ph].
- [51] H. Li, arXiv:1007.2999 [hep-ex].
- [52] M. E. Peskin, in the Proceedings of the APS DPF Community Summer Study (Snowmass 2013), arXiv:1312.4974 [hep-ph].
- [53] T. Horiguchi, A. Ishikawa, T. Suehara, K. Fujii, Y. Sumino, Y. Kiyo and H. Yamamoto, arXiv:1310.0563 [hep-ex].

750 GeV DiPhoton Resonance at 1 TeV ILC



$$\Gamma_{\gamma\gamma} (X_{750}) \approx 10^3 \times SM \ \Gamma_{\gamma\gamma} (H_{125})$$

$$\sigma(e^+e^- \rightarrow \gamma X_{750}) \approx 1 - 100 \ \text{fb} \quad \text{at } \sqrt{s} = 1 \ \text{TeV}$$

In principle this reaction has a threshold near 750 GeV, however the threshold dependence is given by $(1-M_x^2/s)^3$, meaning that in practice one should operate at 1 TeV

At 1 TeV, X is accompanied by a monochromatic photon with E $\gamma \sim$ 220GeV, which provides a very clean signature. This allows to measure the total cross section and therefore $\Gamma \gamma \gamma$ at the % level. The Z γ contribution can be separated from $\gamma \gamma$ by using an electron beam with longitudinal polarisation giving ALR.

CEPC 5 ab ⁻¹

	PreCDR	Now
σ(ZH)	0.51%	0.50%
σ(ZH)*Br(H→bb)	0.28%	0.21%
σ(ZH)*Br(H→cc)	2.1%	2.5%
σ(ZH)*Br(H→gg)	1.6%	1.7%
$\sigma(ZH)^*Br(H\rightarrow WW)$	1.5%	1.2%
$\sigma(ZH)^*Br(H \rightarrow ZZ)$	4.3%	4%
σ(ZH)*Br(H→ττ)	1.2%	1.0%
σ(ZH)*Br(H→γγ)	9.0%	9.0%
σ(ZH)*Br(H→μμ)	17%	17%
$\sigma(vvH)^*Br(H\rightarrow Z\gamma)$	-	-
σ(vvH)*Br(H→bb)	2.8%	2.8%
Higgs Mass/MeV	5.9	5.0
σ(ZH)*Br(H→inv)		
Br(H→ee)		
Br(H→bbχχ, 4b)	<10 ⁻³	95%. CL = 3e-4

FCC-ee

Errors on σ and σ -BR from arXiv:1308.6176 assuming 240+350 GeV with 10.0 + 2.6 ab⁻¹ :

	TLEP 240
$\sigma_{ m HZ}$	0.4%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to {\rm b}\bar{\rm b})$	0.2%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to {\rm c}\bar{\rm c})$	1.2%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to {\rm gg})$	1.4%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to {\rm WW})$	0.9%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to \tau \tau)$	0.7%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to {\rm ZZ})$	3.1%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to \gamma \gamma)$	3.0%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to \mu\mu)$	13%

$$\sigma_{WW \to H} \times BR(H \to b\bar{b})$$

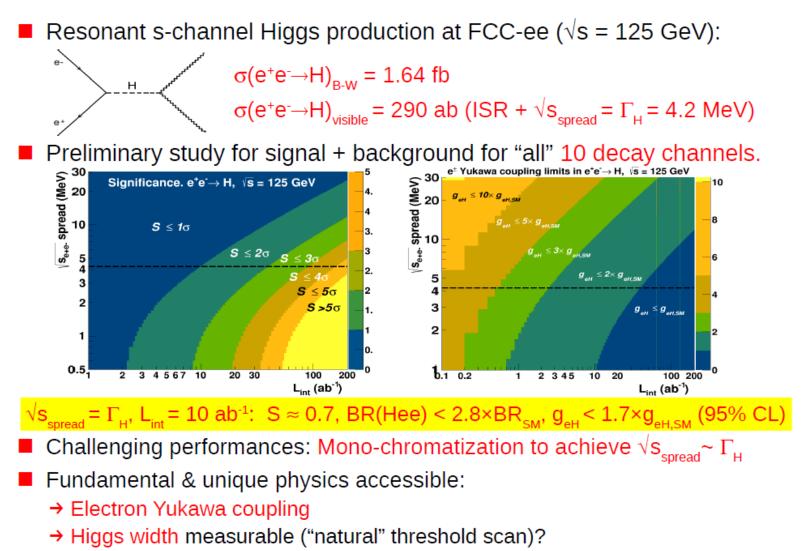
$$\boxed{\sqrt{s} (GeV) \quad TLEP}$$

$$\boxed{240 - 250 \quad 2.2\%}$$

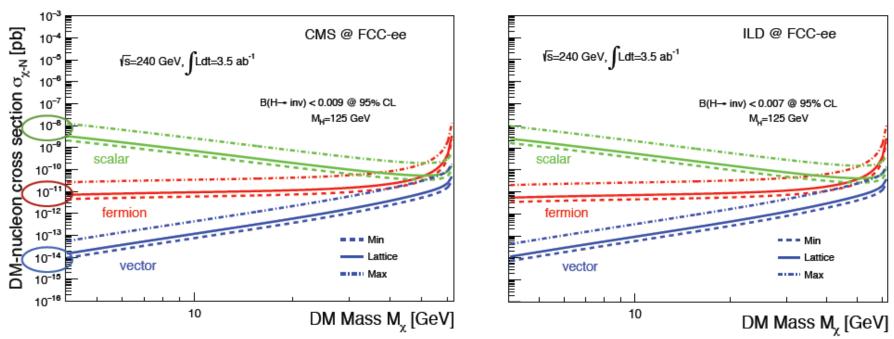
$$\boxed{350 \quad 0.6\%}$$

The additional events from the Higgs-strahlung process at 350 GeV allow the statistical precision for all the aforementioned measurements to be improved by typically 5% for TLEP with respect to the sole 240 GeV data.

FCC-ee



FCC-ee@3.5ab⁻¹



Limits on Dark Matter models

Limits on Dark Matter models

 We can exclude wrt to current CMS more ~2 order of magnitude lower DM-nucleon cross section

ILC + CEPC Model Independent Higgs Coupling Precision

Take CEPC errors on σ and σ -BR from pre Conceptual Design Report assuming 240 GeV with 5 ab⁻¹ :

ΔM_H	Γ_H	$\sigma(ZH)$
5.9 MeV	2.8%	0.51%
Decay mode		$\sigma(ZH) \times BR$
$H \rightarrow bb$		0.28%
$H \rightarrow cc$		2.2%
$H \rightarrow gg$		1.6%
$H\to\tau\tau$		1.2%
$H \to WW$		1.5%
$H \rightarrow ZZ$		4.3%
$H \to \gamma \gamma$		9.0%
$H \to \mu \mu$		17%
$H \to \mathrm{inv}$		-

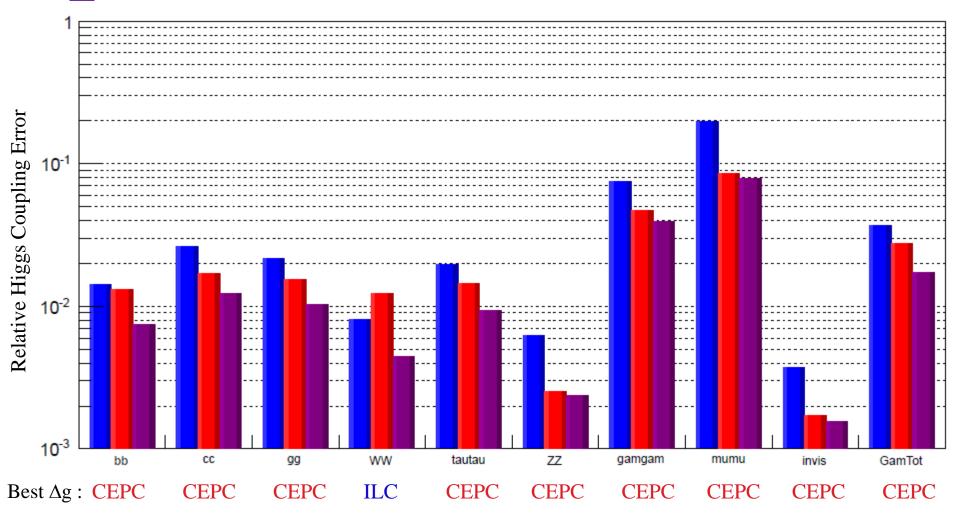
Take ILC errors on σ and σ -BR from arXiv:1506.07830 assuming 250+350+500 GeV with 2.0+0.2+4.0 ab⁻¹ (H-20 scenario)

Perform model independent fit of b,c,g,W, τ ,Z, γ , μ ,invis Higgs couplings and total width using standard program (from Michael Peskin) for ILC & CEPC separately and combined.

ILC 250+350+500 GeV with 500+200+500 fb⁻¹ (H-20 scenario at 8.1 yrs)

CEPC 250 GeV with 5000 fb^{-1}

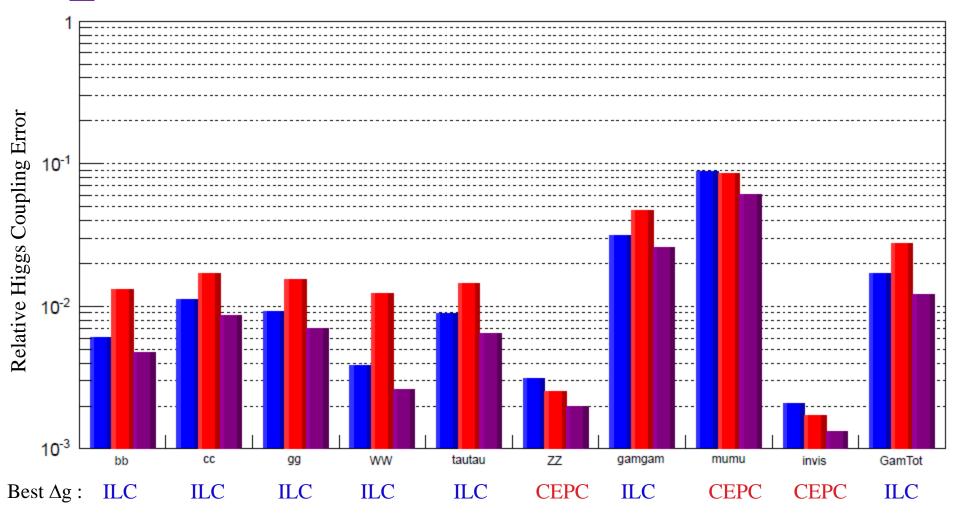
ILC + CEPC under the conditions listed above

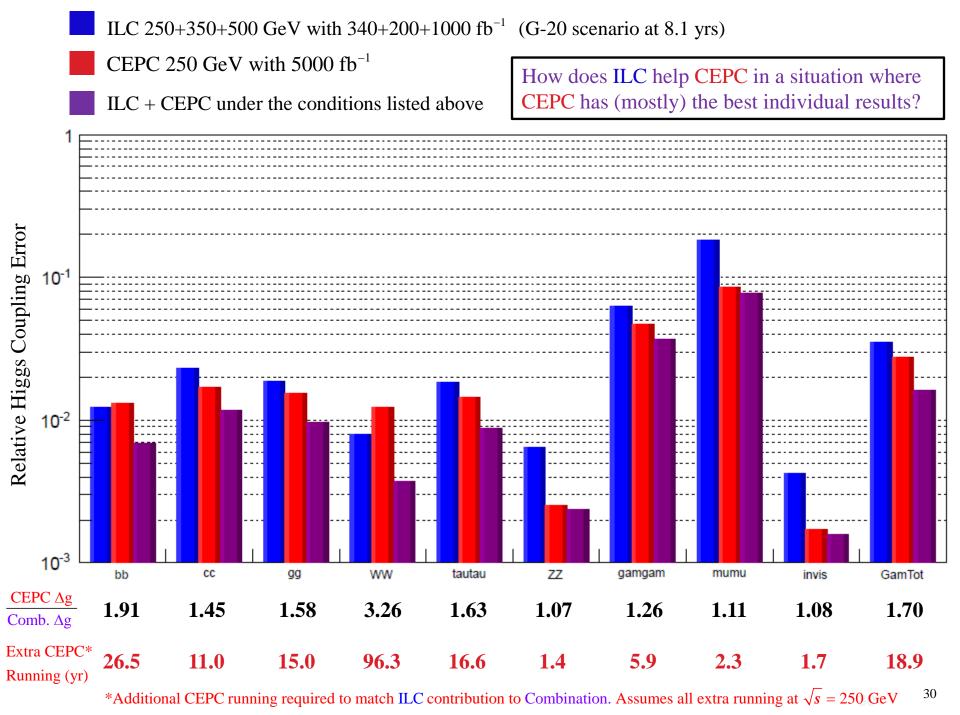


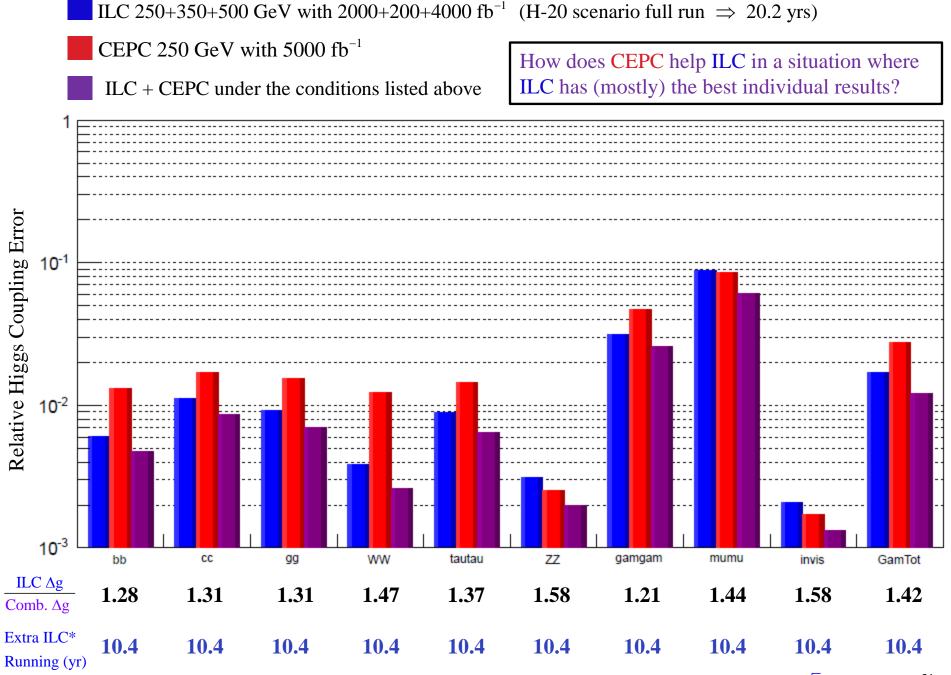
ILC 250+350+500 GeV with 2000+200+4000 fb⁻¹ (H-20 scenario full run \Rightarrow 20.2 yrs)

CEPC 250 GeV with 5000 fb^{-1}

ILC + CEPC under the conditions listed above







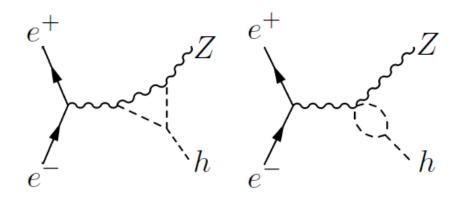
*Additional ILC running required to match CEPC contribution to Combination. Assumes all extra running at $\sqrt{s} = 250 \text{ GeV}$ ³¹

Highlights of Combination of CEPC with ILC H-20 @ 20 yrs

	CEPC		ILC+CEPC
Δg_{HZZ}	0.26%	\Rightarrow	0.20%
$\Delta g_{\scriptscriptstyle HWW}$	1.22%	\Rightarrow	0.26% *
$\Delta g_{\scriptscriptstyle Hbb}$	1.30%	\Rightarrow	0.47%
$\Delta g_{_{H au au}}$	1.44%	\Rightarrow	0.65%
$\Delta g_{_{Hgg}}$	1.53%	\Rightarrow	0.70%

* Might be interesting to include $\sigma(WW \rightarrow H)$ in precision Higgs analyses

CEPC Higgs Self Coupling Measurement at Ecm=240 GeV



M. McCullough, arXiv:1312.3322

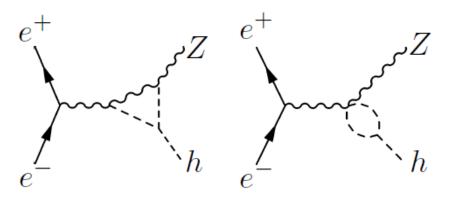
 $\delta_{\sigma}^{240} = 100 \left(2\delta_Z + 0.014\delta_h \right) \%$

 g_{hZZ} fixed to SM value ($\delta_z = 0$) g_{hhZZ} fixed to SM value

$$\Rightarrow \delta_H = \frac{\delta_{\sigma}^{240}}{0.014} = \frac{0.0051}{0.014} = 36\%$$

Note : Oft quoted 30% error comes from combining CEPC with 50% HL-LHC meas.

CEPC Higgs Self Coupling Measurement at Ecm=240 GeV



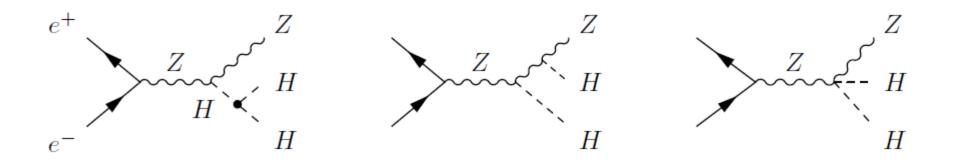
M. McCullough, arXiv:1312.3322

 g_{hZZ} fixed to SM value ($\delta_z = 0$) g_{hhZZ} fixed to SM value

$$\Rightarrow \delta_H = \frac{\delta_{\sigma}^{240}}{0.014} = \frac{0.0051}{0.014} = 36\%$$

Note : Oft quoted 30% error comes from combining CEPC with 50% HL-LHC meas.

ILC Higgs Self Coupling Measurement at Ecm=500 GeV



 g_{hZZ} fixed to value from $\sigma(ZH)$ measurement

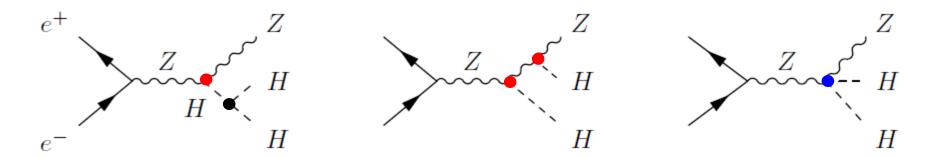
 g_{hhZZ} fixed to SM value \leftarrow See following slides

Extract g_{hhh} from measurement of $\sigma(ZHH)$ using HH $\rightarrow b\overline{b}b\overline{b} \& b\overline{b}W^+W^-$

 $\frac{\Delta\sigma(ZHH)}{\sigma(ZHH)} = 16\% \implies \frac{\Delta g_{hhh}}{g_{hhh}} = 27\% \text{ for ILC scenario H-20 @ 20 years.}$

Note: This assumes SM g_{HHH} . If $g_{HHH} = 2 \times SM$ then $\frac{\Delta g_{hhh}}{g_{hhh}} = 27 \% \implies \frac{\Delta g_{hhh}}{g_{hhh}} = 14 \%$.

ILC Higgs Self Coupling Systematic Error Uncertainties for g_{ZZH} g_{ZZHH} in $\sigma(e^+e^- \rightarrow HHZ)$



We assume that $\sigma(e^+e^- \rightarrow HHZ)$ can be described by an effective field theory (EFT) containing a general $SU(2) \times U(1)$ gauge invariant Lagrangian with dimension-6 operators in addition to the SM.

Using the convention of arXiv:1310.5150 we have, before EWSB, the following dim-6 operators:

$$\mathcal{L}_{\text{SILH}} = \frac{\bar{c}_{H}}{2v^{2}} \partial^{\mu} \left[\Phi^{\dagger} \Phi \right] \partial_{\mu} \left[\Phi^{\dagger} \Phi \right] + \frac{\bar{c}_{T}}{2v^{2}} \left[\Phi^{\dagger} \overleftrightarrow{D}^{\mu} \Phi \right] \left[\Phi^{\dagger} \overleftrightarrow{D}_{\mu} \Phi \right] - \frac{\bar{c}_{6} \lambda}{v^{2}} \left[\Phi^{\dagger} \Phi \right]^{3} \\ + \frac{ig \ \bar{c}_{W}}{m_{W}^{2}} \left[\Phi^{\dagger} T_{2k} \overleftrightarrow{D}^{\mu} \Phi \right] D^{\nu} W_{\mu\nu}^{k} + \frac{ig' \ \bar{c}_{B}}{2m_{W}^{2}} \left[\Phi^{\dagger} \overleftrightarrow{D}^{\mu} \Phi \right] \partial^{\nu} B_{\mu\nu} \\ + \frac{2ig \ \bar{c}_{HW}}{m_{W}^{2}} \left[D^{\mu} \Phi^{\dagger} T_{2k} D^{\nu} \Phi \right] W_{\mu\nu}^{k} + \frac{ig' \ \bar{c}_{HB}}{m_{W}^{2}} \left[D^{\mu} \Phi^{\dagger} D^{\nu} \Phi \right] B_{\mu\nu} \\ + \frac{g'^{2} \ \bar{c}_{\gamma}}{m_{W}^{2}} \Phi^{\dagger} \Phi B_{\mu\nu} B^{\mu\nu} + \text{ other operators that are not relevant or that violate CP$$

Note there are only 8 EFT parameters: $\overline{c}_H = \overline{c}_T = \overline{c}_6 = \overline{c}_W = \overline{c}_{HW} = \overline{c}_{HB} = \overline{c}_{\gamma}$

The couplings $g_{xxx}^{(j)}$ and $g_{xxxx}^{(j)}$ take the following form in our EFT:

 $g^{(1)}_{hhh}$ $g^{(2)}_{hhh}$

 $1 + \frac{7}{8}\bar{c}_6 - \frac{1}{2}\bar{c}_H$ $\frac{g}{m_W}\bar{c}_H$

$$g_{hzz}^{(1)} = \frac{2g}{c_W^2 m_W} \Big[\bar{c}_{HB} s_W^2 - 4 \bar{c}_\gamma s_W^4 + c_W^2 \bar{c}_{HW} \Big] \\g_{hzz}^{(2)} = \frac{g}{c_W^2 m_W} \Big[(\bar{c}_{HW} + \bar{c}_W) c_W^2 + (\bar{c}_B + \bar{c}_{HB}) s_W^2 \Big] \\g_{hzz}^{(3)} = \frac{gm_W}{c_W^2} \Big[1 - \frac{1}{2} \bar{c}_H - 2 \bar{c}_T + 8 \bar{c}_\gamma \frac{s_W^4}{c_W^2} \Big]$$

 \overline{c}_6 is uniquely accessible through the Higgs self coupling measurement. The other 7 EFT parameters appear in several places.

$$g_{hww}^{(1)} \qquad \qquad \frac{2g}{m_W} \bar{c}_{HW}$$

$$g_{hww}^{(2)} \qquad \qquad \frac{g}{m_W} \Big[\bar{c}_W + \bar{c}_{HW} \Big]$$

$$\begin{cases} g_{hhzz}^{(1)}, g_{hhzz}^{(2)}, g_{hhaz}^{(1)}, g_{hhaz}^{(2)}, g_{hhww}^{(1)}, g_{hhww}^{(2)} \end{cases} & \frac{g}{2m_W} \left\{ g_{hzz}^{(1)}, g_{hzz}^{(2)}, g_{haz}^{(1)}, g_{haz}^{(2)}, g_{hww}^{(1)}, g_{hww}^{(2)} \right\} \\ g_{hhzz}^{(3)} & \frac{g^2}{2c_W^2} \left[1 - 6\bar{c}_T - \bar{c}_H + 8\bar{c}_\gamma \frac{s_W^4}{c_W^2} \right] \end{cases}$$

Note the relationships between the *hzz* & *hhzz* couplings.

 $g_{aww}^{(1)} = e \left[1 - 2\bar{c}_W \right]$ $g_{aww}^{(2)} = e \left[1 - 2\bar{c}_W - \bar{c}_{HB} - \bar{c}_{HW} \right]$ $g_{zww}^{(1)} = \frac{g}{c_W} \left[c_W^2 - \bar{c}_{HW} + (2s_W^2 - 3)\bar{c}_W \right]$ $g_{zww}^{(2)} = \frac{g}{c_W} \left[c_W^2 (1 - \bar{c}_{HW}) + s_W^2 \bar{c}_{HB} + (2s_W^2 - 3)\bar{c}_W \right]$

The TGC's depend on \overline{c}_W , \overline{c}_{HW} , \overline{c}_{HB} through the Goldstone bosons that are eaten by the *B* and *W* fields

The parameters \overline{c}_W , \overline{c}_B , \overline{c}_T are constrained by electroweak precision tests. From arXiv:1410.7703 :

Operator	Coefficient	LEP Constraints	
Operator		Individual	Marginalized
$\mathcal{O}_W = \frac{ig}{2} \left(H^{\dagger} \sigma^a \overset{\leftrightarrow}{D^{\mu}} H \right) D^{\nu} W^a_{\mu\nu}$ $\mathcal{O}_B = \frac{ig'}{2} \left(H^{\dagger} \overset{\leftrightarrow}{D^{\mu}} H \right) \partial^{\nu} B_{\mu\nu}$	$\frac{m_W^2}{\Lambda^2}(c_W + c_B)$	(-0.00055, 0.0005)	(-0.0033, 0.0018)
$\mathcal{O}_T = \frac{1}{2} \left(H^{\dagger} \overset{\leftrightarrow}{D}_{\mu} H \right)^2$	$\frac{v^2}{\Lambda^2} c_T$	(0, 0.001)	(-0.0043, 0.0033)

By definition
$$\overline{c}_W \equiv \frac{M_W^2}{\Lambda^2} c_W$$
, $\overline{c}_T \equiv \frac{v^2}{\Lambda^2}$, etc.

We will assume in the following that, at the 10^{-3} level, $\overline{c}_W = -\overline{c}_B$ $\overline{c}_T = 0$

The TGC's $\Delta \kappa_{\gamma}$ and Δg_1^Z are related to EFT parameters by

$$\Delta \kappa_{\gamma} = \overline{c}_{HW} + \overline{c}_{HB}$$
$$\Delta g_1^Z = \overline{c}_W + \overline{c}_{HW}$$

At ILC with the full H-20 scenario the error on the TGC's are $\Delta(\Delta \kappa_{\gamma}) = 2 \times 10^{-4}$ $\Delta(\Delta g_{1}^{Z}) = 8 \times 10^{-4}$

We can then assume at the 10^{-3} level that $\overline{c}_W = -\overline{c}_{HW} = \overline{c}_{HB}$

so that we can write $g_{hzz}^{(2)}$ and $g_{hww}^{(2)}$ as

$$g_{hzz}^{(2)} = \frac{g}{c_{\theta_W}^2 M_W} \left[\Delta g_1^Z c_{\theta_W}^2 + \Delta \kappa_\gamma s_{\theta_W}^2 \right] \approx 0$$
$$g_{hww}^{(2)} = \frac{g}{M_W} \Delta g_1^Z \approx 0$$

Through EWPT's and ILC measurements of TGC's the number of independent EFT parameters has been reduced from 8 to just 4: \overline{c}_H , \overline{c}_γ , \overline{c}_{HW} , \overline{c}_6

The first two parameters are determined by $g_{hww}^{(3)}$ and $g_{hzz}^{(3)}$, the usual Higgs couplings to W and Z that are measured assuming the SM Lorentz structure. The relationship between the W coupling $g_{hww}^{(3)}$ and \overline{c}_{H} is simply

$$g_{hww}^{(3)} \equiv gM_W(1 - \frac{1}{2}\overline{c}_H)$$

The parameter \overline{c}_{γ} is given by

$$8\frac{s_{\theta_{W}}^{4}}{c_{\theta_{W}}^{2}}\overline{c}_{\gamma} = \frac{1}{gM_{W}}\left[c_{\theta_{W}}^{2}g_{hzz}^{(3)} - g_{hww}^{(3)}\right]$$

The EFT parameter \overline{c}_{HW} is related to the "*b*" parameter of the *HVV* Lorentz structure studies by Fujii, Tian, Ogawa and others done here at KEK:

HWW arXiv:1011.5805

HZZ Talk by T. Ogawa at LCWS15 http://agenda.linearcollider.org/event/6662/

Let $b_w \& b_z$ be the "b" parameters of the HWW & HZZ analyses respectively.

$$g_{hww}^{(1)} = -2\frac{b_w}{\Lambda} \qquad g_{hzz}^{(1)} = -4\frac{b_z}{\Lambda}$$

$$\overline{c}_{HW} = -\frac{v}{2\Lambda}b_w \approx \frac{b_w}{8} \quad \text{for } \Lambda = 1 \text{ TeV assumed in the analysis}$$

$$\overline{c}_{HW} = \frac{1}{1 - t_{\theta_w}^2} \left[\frac{1}{2gM_w} (g_{hzz}^{(3)} - \frac{1}{c_{\theta_w}^2} g_{hww}^{(3)}) - \frac{v}{\Lambda} b_z \right] \approx \frac{1}{1 - t_{\theta_w}^2} \left[\frac{1}{2gM_w} (g_{hzz}^{(3)} - \frac{1}{c_{\theta_w}^2} g_{hww}^{(3)}) - \frac{b_z}{4} \right]$$
For full H20 $\Delta b_w = 0.18 \quad \Delta b_z = 0.004$. Since Δb_z is much better right now we ignore the b_w measurement and use $g_{hzz}^{(1)} = -4\frac{b_z}{\Lambda} = -\frac{b_z}{v}$

Let's now rewrite the Lagrangian using our measured variables $g_{hzz}^{(3)} g_{hww}^{(3)} g_{hzz}^{(1)}$ and the one remaining unconstrained EFT parameter \overline{c}_6

$$\begin{split} \mathcal{L} &= -\frac{M_{H}^{2}}{2v} \left(\frac{g_{\text{hww}}^{(3)}}{gM_{W}} + \frac{7}{8} \overline{c}_{6} \right) h^{3} + \frac{g}{M_{W}} \left(1 - \frac{g_{\text{hww}}^{(3)}}{gM_{W}} \right) h \partial_{\mu} h \partial^{\mu} h \\ &+ \frac{b_{z}}{4v} Z_{\mu\nu} Z^{\mu\nu} h + \frac{1}{2} g_{\text{hzz}}^{(3)} Z_{\mu} Z^{\mu} h \\ &+ \frac{b_{z}}{8v} Z_{\mu\nu} Z^{\mu\nu} h^{2} + \frac{g^{2}}{8c_{\theta_{W}}^{2}} \left(\frac{g_{\text{hww}}^{(3)}}{gM_{W}} + c_{\theta_{W}}^{2} \frac{g_{\text{hzz}}^{(3)}}{gM_{W}} - 1 \right) Z_{\mu} Z^{\mu} h^{2} \end{split}$$

In this EFT approach all of the couplings in the calculation of $\sigma(e^+e^- \rightarrow HHZ)$ are tightly constrained by the other Higgs coupling measurements, TGC measurements, and EWPT's. The only unconstrained parameter is $\overline{c_6}$. If the best match to the measured $\sigma(e^+e^- \rightarrow HHZ)$ is $\overline{c_6} = 0$ within sys+stat errors then we have observed SM Higgs self coupling.

Other Higgs Measurements with CEPC & ILC H-20 at 20 yrs

	CEPC	ILC	
	250 GeV	250 + 350 + 500 GeV	Combined
	5000 fb^{-1}	$2000 + 250 + 4000 \text{ fb}^{-1}$	
Δm_{H}	5.9 MeV	12.5 MeV	5.3 MeV
Δg_{HHH}	36 %	27 %	22 %
g_{HHH} Δg_{ttH}	_	5.9 %	5.9 %
$\begin{array}{c} \boldsymbol{g}_{ttH} \\ \underline{\Delta \boldsymbol{g}_{ttH}}^{(*)} \end{array}$			
$\underline{g_{ttH}}$		2.4 %	2.4 %

^(*) Assumes ILC 500 GeV running actually takes place at $\sqrt{s} = 550$ GeV

ILC + FCC-ee Model Independent Higgs Coupling Precision

Take FCC-ee errors on σ and σ -BR from arXiv:1308.6176 assuming 240+350 GeV with 10.0 + 2.6 ab⁻¹ :

	TLEP 240
$\sigma_{ m HZ}$	0.4%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to {\rm b}\bar{\rm b})$	0.2%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to {\rm c}\bar{\rm c})$	1.2%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to {\rm gg})$	1.4%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to {\rm WW})$	0.9%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to \tau \tau)$	0.7%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to {\rm ZZ})$	3.1%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to \gamma \gamma)$	3.0%
$\sigma_{\rm HZ} \times {\rm BR}({\rm H} \to \mu\mu)$	13%

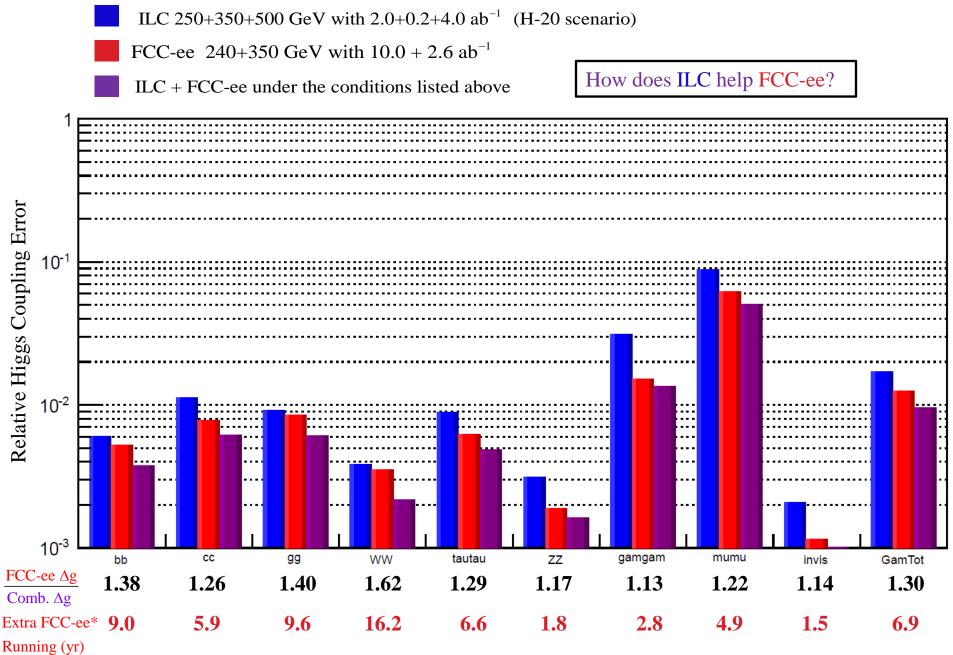
$\sigma_{\rm WW \to H} \times {\rm BR}({\rm H} \to {\rm b}\bar{\rm b})$			
\sqrt{s} (GeV)	TLEP		
240 - 250	2.2%		
350	0.6%		

The additional events from the Higgs-strahlung process at 350 GeV allow the statistical precision for all the aforementioned measurements to be improved by typically 5% for TLEP with respect to the sole 240 GeV data.

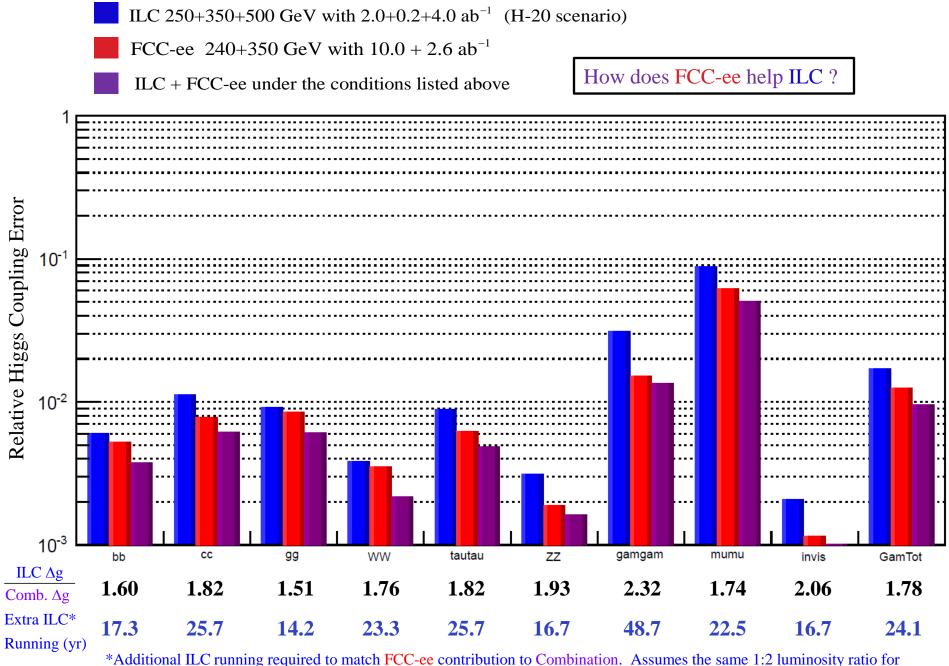
 Branching fraction to invisible tested directly to 0.19% @ 95% CL

Take ILC errors on σ and σ -BR from arXiv:1506.07830 assuming 250+350+500 GeV with 2.0+0.2+4.0 ab⁻¹ (H-20 scenario)

Perform model independent fit of b,c,g,W, τ ,Z, γ , μ ,invis Higgs couplings and total width using standard program (from Michael Peskin) for ILC & FCC-ee separately and combined. ₄₄



*Additional FCC-ee running required to match ILC contribution to Combination. Assumes the same 10:2.6 luminosity ratio for 240:350 GeV except ZZ & invis which assume that all extra running is at 240 GeV



250:500 GeV except ZZ & invis which assumes all extra running at 250 GeV.

Highlights of Combination of FCC-ee with ILC H-20

	ILC	FCC-ee	ILC+FCC-ee
Δg_{HZZ}	0.31%	0.19%	0.16%
$\Delta g_{\scriptscriptstyle HWW}$	0.38%	0.35%	0.22%
$\Delta g_{\scriptscriptstyle Hbb}$	0.60%	0.52%	0.38%
$\Delta g_{H\tau\tau}$	0.89%	0.63%	0.49%
$\Delta g_{_{Hgg}}$	0.92%	0.85%	0.61%

Other Higgs Measurements with FCC-ee & ILC H-20

	FCC-ee	ILC	
	240 +350 GeV	250 + 350 + 500(550) GeV	Combined
	$10.0+2.6 \text{ ab}^{-1}$	$2.0 + 0.25 + 4.0 \text{ fb}^{-1}$	
Δm_{H}	11 MeV	12.5 MeV	8.3 MeV
$\frac{\Delta g_{HHH}}{g_{HHH}}$	29 % *	27 %*	20 %
$rac{\Delta m{g}_{ttH}}{m{g}_{ttH}}$	13%	5.9 (2.4) %	5.4 (2.4) %
$\frac{g_{eeH}}{g_{eeH}^{SM}}$	< 2.2 @ 3 σ	_	< 2.2 @ 3 σ

- * Loop contribution to σ (ZH)
- * Tree-level contribution to σ (ZHH)

ILC + FCC-ee Summary (Serves Also as Talk Summary)

ILC helps FCC-ee:

- The 0.25% measurement of $\sigma(vvh)XBR(H \rightarrow bb)$ reduces errors on all Higgs couplings
- The 2.4% Top Yukawa coupling measurement from ttH production improves upon the 13% measurement from the tt threshold scan.
- ILC σ (ZHH) measurement provides a 27% tree-level determination of the Higgs selfcoupling, and could help clarify a Higgs self-coupling interpretation of the precision FCC-ee σ (ZH) measurement.

FCC-ee helps ILC:

- Precision measurement of g_{HZZ} and various σXBR at 240 GeV help turn the ILC 0.25% measurement of $\sigma(vvh)XBR(H\rightarrow bb)$ into $\Delta g_{WW} = 0.22\%$
- Much better meas. of Higgs invisible width, BSM decays, rare decays such as $\gamma\gamma$ and μμ Note: $\sum BR_i = 1$ can be used to improve all coupling errors if $\Delta BR(H \rightarrow BSM) < 1\%$
- Unique access to Higgs coupling to 1st generation fermions.
- FCC-ee+ILC combination helps the particle physics community:
 - Higgs Z coupling error $\Delta g_{HZ} = 0.16\%$
 - $\,\circ\,\,$ Higgs W coupling error $\Delta g_{WW}=\,0.22\%$
 - Higgs b coupling error $\Delta g_{bb} = 0.38\%$
 - Higgs self coupling error $\Delta g_{HHH} = 20\%$