האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Studies in Baryonic EFT

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Living near unitarity, KITP UCSB

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Collaboration

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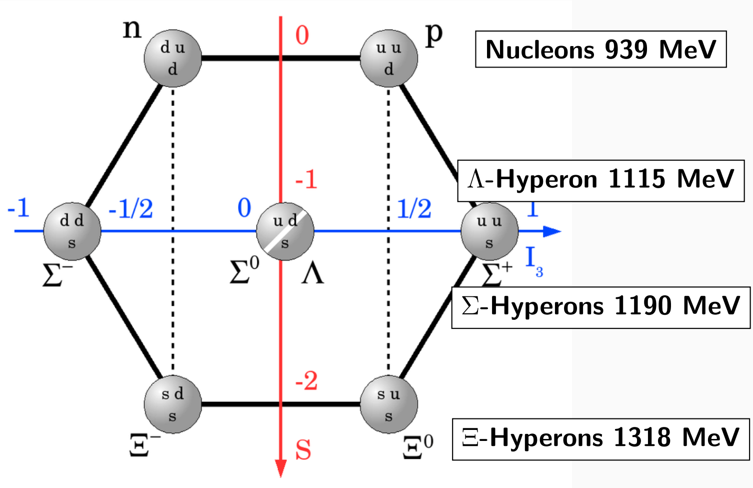
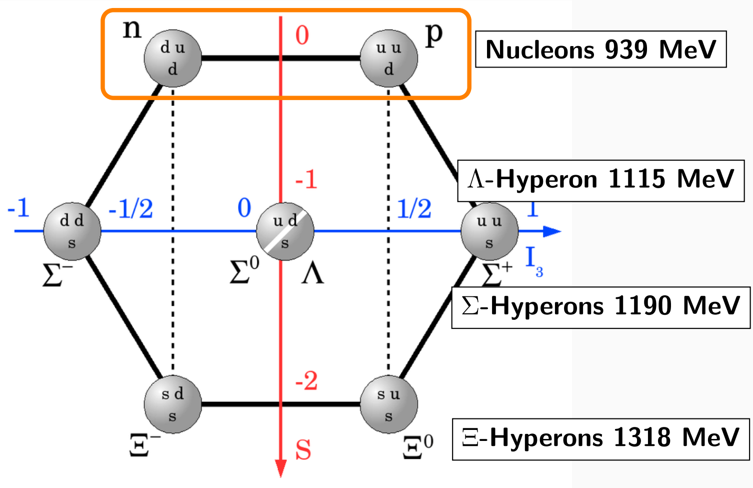
Introduction - the baryon octet



Hyperon

A hyperon is a baryon containing one or more strange quarks



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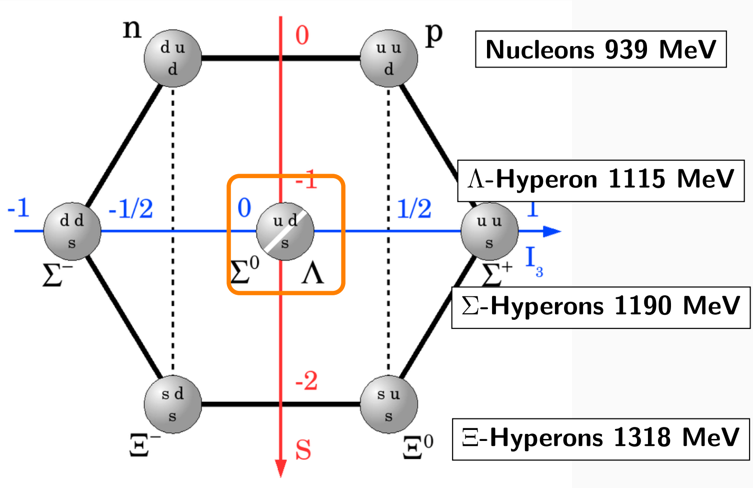

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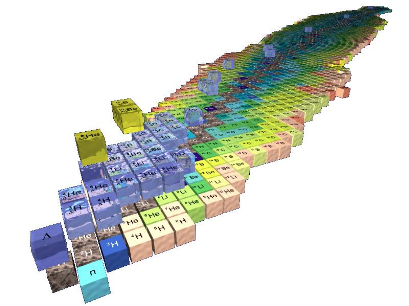
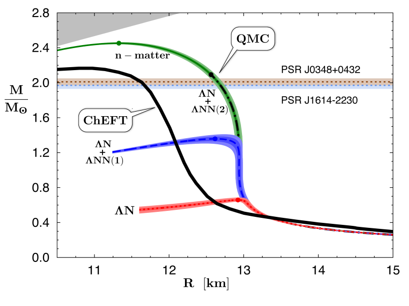




Nuclei & Hypernuclei

≈3300 nuclear isotopes

≈40 single Lambda hypernuclei

3 double Lambda hypernuclei

In neutron stars

Hyperons softens the EOS

Adding ΛNN-force stiffens the EOS

`Lonardoni et al. PRL 114 (2015) 092301`

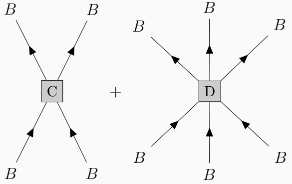
Baryonic EFT

Few-body baryonic states

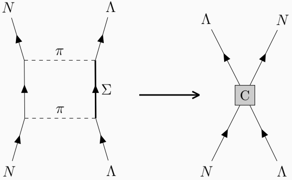
Nuclei	Λ-Hypernuclei
Deuteron - NN $B/A \approx 1$ MeV	$N\Lambda$,$\Lambda\Lambda$ not bound
Triton - $3N$ $B/A \approx 3$ MeV	${}^{3}_{\Lambda}$H $B_\Lambda \approx 0.1$ MeV
Alpha - $4N$ $B/A \approx 7$ MeV	${}^{4}_{\Lambda}$He,${}^{4}_{\Lambda}$H $B_\Lambda \approx 3$ MeV

- $B = n, p, \Lambda$ are the only dof.

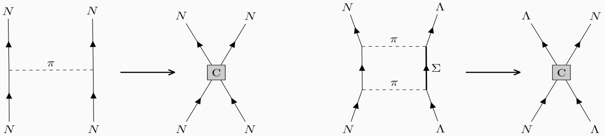
$$\mathcal{L}_{QCD}(q,G) \longrightarrow \mathcal{L}_{\chi EFT}(B,\pi,K) \longrightarrow \mathcal{L}(B)$$



- $\mathcal{L}$ is expanded in powers of Q/M_h.
- Include contact terms and derivatives.
- Not too many parameters



$$\mathcal{L} = N^\dagger\left(i\partial_0 + \frac{\nabla^2}{2m}\right)N + \Lambda^\dagger\left(i\partial_0 + \frac{\nabla^2}{2m}\right)\Lambda + \mathcal{L}_{2B} + \mathcal{L}_{3B} + \ldots$$



Accuracy for light nuclei

Nuclei The pion mass is our breaking scale M_h

$$\left(\frac{Q}{M_h}\right) = \frac{\sqrt{2B_N M_N}}{m_\pi} \approx 0.5 - 0.8$$

Seems to work better in practice as $\Delta B(^4\text{He}) \approx 10\%$

Hypernuclei No OPE therefore breaking scale is $2m_\pi$

$$\left(\frac{Q}{M_h}\right) = \frac{\sqrt{2B_\Lambda M_\Lambda}}{2m_\pi} \approx 0.3$$

At LO accuracy goes as $(Q/M_h)^2$

1 The Wigner Bound Phillips, Beane and Cohen (1997-1998)

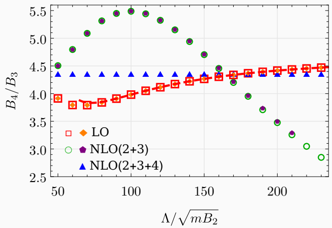
- The effective range is bounded by the cutoff $r_{\text{eff}} \leq W/\lambda$
- All orders but LO are perturbation (Kaplan, van Kolck, ...).

2 The Thomas collapse Bedaque, Hammer, and van Kolck (1999)

- With LO 2-body interaction $B_3 \propto \hbar\lambda^2/m.$
- A 3-body counter term must be introduced at LO.

3 NLO - no end to surprises Bazak et al. (2019)

- At NLO the 4-body system is unstable.
- **Conclusion:** the 4-body force is promoted to NLO.



What do we have?

- LO and NLO $\not\pi$EFT fitted to low-energy experimental constraints
- No Coulomb
- **Schrödinger equation**

$$[T_k + V_{LO} + V_{NLO}] |\Psi\rangle = E |\Psi\rangle$$

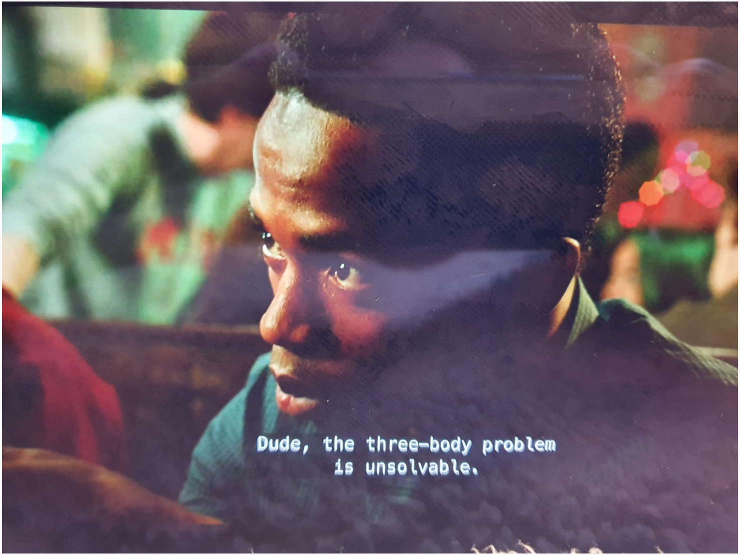
What do we want to know?

- Bound states, resonances, scattering

How do we get there?

- Gaussian basis functions
- Few-body bound states ⇒ SVM
- Scattering ⇒ Busch formula
- Complex rotation, analytic continuation ⇒ Resonances

Just got this morning...



- **Nuclear scattering**

  Elastic s-wave scattering @NLO for $A \leq 5$

- **Λ hypernuclei ($^{A}_{\Lambda}Z$)**

  s-shell hypernuclei - overbinding of $^{5}_{\Lambda}$He

  Hypernuclear resonances

- **$\Lambda\Lambda$ hypernuclei ($^{A}_{\Lambda\Lambda}Z$)**

  Onset of binding, A=4 or 5?

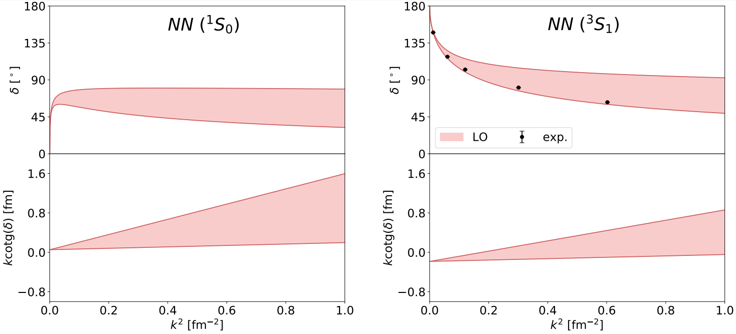
- **Charge symmetry breaking**

  The Dalitz von Hippel paramaters from SU(3) symmetry.

- **Nucleons in a box**

  EFT matching of LQCD calcs.

The nuclear sector



Leading order (LO):

(exp. constraints)

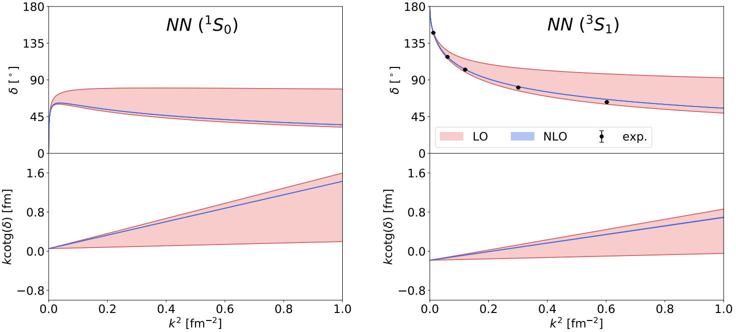
$$a_0^{nn} = \ -18.95(40)\ \text{fm}$$

$$a_1^{np} = \quad 5.419(7)\ \text{fm}$$

$$B(^3\text{H}) = 8.482\ \text{MeV}$$

Effective range expansion :

$$k\mathrm{cotg}(\delta) = -\frac{1}{a} + \frac{1}{2} r k^2 + \ldots$$



Leading order (LO):
(exp. constraints)

$a_0^{nn} = \; -18.95(40)$ fm

$a_1^{np} = \quad 5.419(7)$ fm

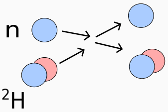
$B(^3\mathrm{H}) = 8.482$ MeV

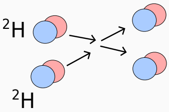
Next-to-leading order (NLO):
(exp. constraints)

$r_0^{nn} = \; 2.75(11)$ fm

$r_1^{np} = \; 1.753(8)$ fm

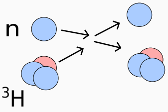
$B(^4\mathrm{He}) = 28.296$ MeV

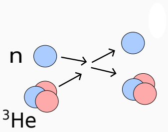




Few-body s-wave scattering

(Schafer, Bazak, Bagnarol)





Universal fermionic relations (STM, Petrov, Deltuva,...)

Atom-Dimer scattering

$$\frac{a_{ad}}{a_{aa}} = 1.1791 + 0.553\frac{r_{aa}}{a_{aa}} \quad ; \quad \frac{r_{ad}}{a_{aa}} = -0.038 + 1.04\frac{r_{aa}}{a_{aa}}$$

Dimer-Dimer scattering

$$\frac{a_{dd}}{a_{aa}} = 0.5986 + 0.105\frac{r_{aa}}{a_{aa}} \quad ; \quad \frac{r_{dd}}{a_{aa}} = 0.133 + 0.51\frac{r_{aa}}{a_{aa}}$$

These results are reproduced for spin saturated system:

- Neutron-Deuteron $S = \frac{3}{2}$ scattering
- Deuteron-Deuteron $S = 2$ scattering.

$n + d\ (S = \frac{1}{2}, T = \frac{1}{2})$ scattering

1. **Near-threshold $^3\mathrm{H}^*$ virtual state**

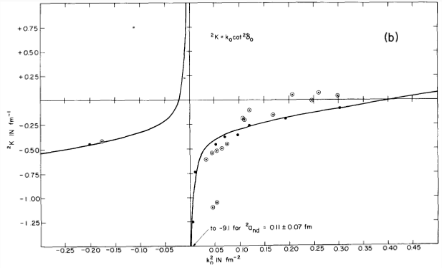
   $\Rightarrow$ pole of S-matrix

2. **Near-threshold zero in S-matrix**

$$\frac{1}{k\mathrm{cotg}(\delta) - ik} = 0$$

$$\lim_{k \to k_0} k\mathrm{cotg}(\delta) = \pm\infty$$

$\Rightarrow$ modified ERE

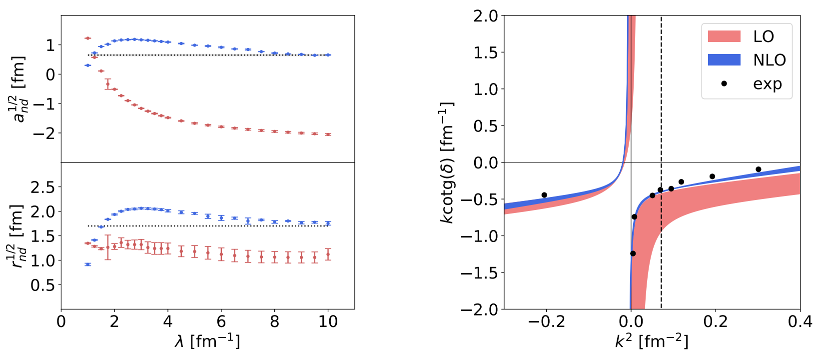


(Oers and Seagrave, PLB 24, 11 (1967))

$$a^{1/2}_{n^2\mathrm{H}} = 0.29 \text{ fm}$$
$$r^{1/2}_{n^2\mathrm{H}} = 1.70 \text{ fm}$$

$$k\mathrm{cotg}(\delta) = A + B\,k^2 + \frac{C}{(1 + D\,k^2)} \quad ; \quad a = -\frac{1}{A + C} \quad \text{and} \quad r = 2B$$

$n + d\ (S = \frac{1}{2}, T = \frac{1}{2})$ scattering



Pole of S-matrix :

$$A + B\,k^2 + \frac{C}{(1 + D\ k^2)} - \mathrm{i}k = 0$$

LO :

$$\mathrm{Im}[\gamma_{nd}^{1/2}] = -0.117(19)\ \mathrm{fm}^{-1}$$

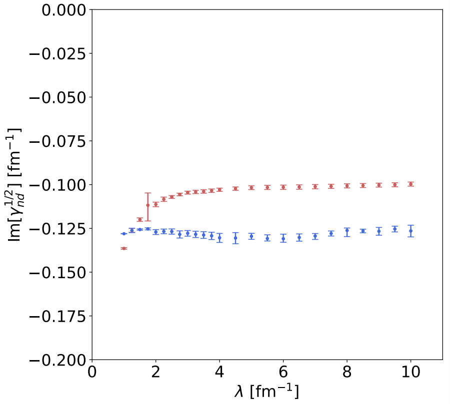
$$E_{nd}^{1/2} = -0.43(14)\ \mathrm{MeV}$$

NLO :

$$\mathrm{Im}[\gamma_{nd}^{1/2}] = -0.1271(39)\ \mathrm{fm}^{-1}$$

$$E_{nd}^{1/2} = -0.503(31)\ \mathrm{MeV}$$

$\Rightarrow$ **Effimov virtual state**



Adhikari et al., Phys. Rev. C 26 (1982) 77

Higa et al., Phys. Lett. B 791 (2019) 414

Zero energy $n + {}^2\mathrm{H}$ scattering experiments

Coherent scattering length: $a^c_{n^2\mathrm{H}} = \frac{1}{3} a^{1/2}_{n^2\mathrm{H}} + \frac{2}{3} a^{3/2}_{n^2\mathrm{H}}$

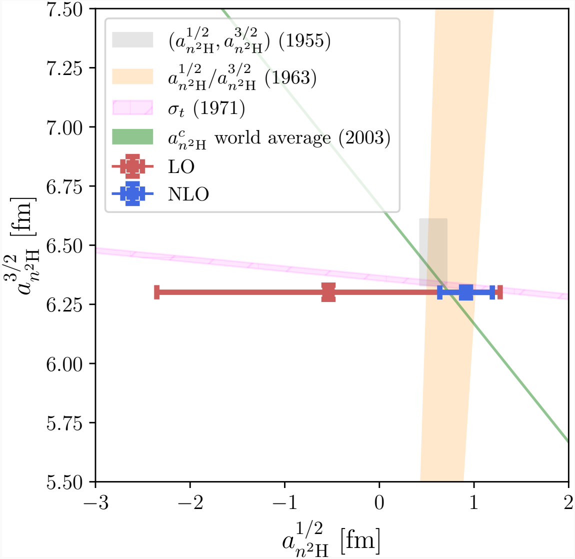
Incoherent scattering length: $\left(a^{inc}_{n^2\mathrm{H}}\right)^2 = \frac{1}{3}\left(a^{1/2}_{n^2\mathrm{H}}\right)^2 + \frac{2}{3}\left(a^{3/2}_{n^2\mathrm{H}}\right)^2$

Total cross section ($k=0$): $\sigma_t = 4\pi\left[\left(a^c_{n^2\mathrm{H}}\right)^2 + \left(a^{inc}_{n^2\mathrm{H}}\right)^2\right]$

$n + {}^2\mathrm{H}$

- **S. J. Nikitin et al.** (First Geneva Conf. 2 (1955) 81)
  $a^{1/2}_{n^2\mathrm{H}}$ and $a^{3/2}_{n^2\mathrm{H}}$ limits (ortho/para hydrogen)
- **W. Gissler** (Z. Kristallographie 118 (1963) 149)
  $a^{1/2}_{n^2\mathrm{H}}/a^{3/2}_{n^2\mathrm{H}}$ ratio
- **W. Dilg et al.** (Phys. Lett. B 36 (1971) 208)
  total crosssection
- **K. Schoen et al.** (Phys. Rev. C 67 (2003) 044005)
  coherent scattering length (the most recent world average)

$n + {}^2\mathrm{H}$ scattering lengths



$n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering

- Four different 4-body channels

$$S, T = (0,1), (0,0), (1,1), (1,0)$$

- No isospin breaking terms
- We do not distinguish between different 4-body T_z

A. $n +^3 \mathrm{H}\ (T_z = -1)$ **:**

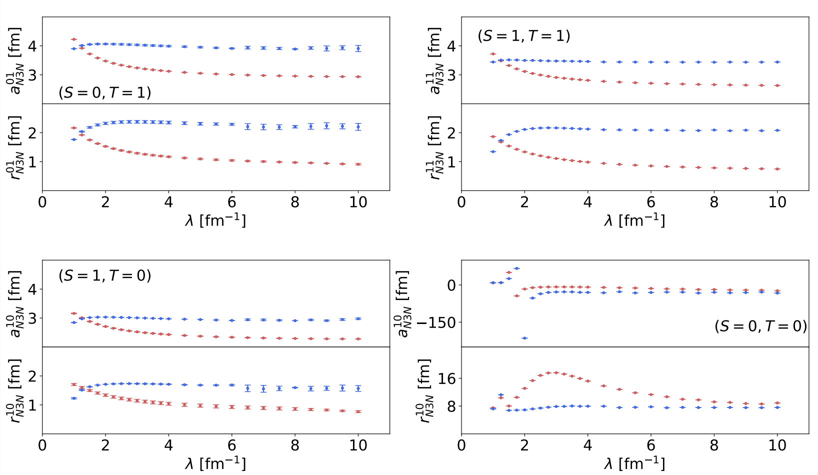
$$S = 0 \longrightarrow (S = 0, T = 1)$$
$$S = 1 \longrightarrow (S = 1, T = 1)$$

B. $n +^3 \mathrm{He}\ (T_z = \ 0)$ **:**

$$S = 0 \longrightarrow (S = 0, T = 0) + (S = 0, T = 1) \quad {}^4\mathrm{He}(0_2^+)\ \ \mathrm{resonance}$$
$$S = 1 \longrightarrow (S = 1, T = 0) + (S = 1, T = 1)$$

- for $n +^3 \mathrm{He}$ scattering we must include two different isospin channels

$n + {}^3\text{H}$ and $n + {}^3\text{He}$ scattering



- four-body force needed only in $(S = 0, T = 0)$ channel

Zero energy $n + {}^3\text{H}$ and $n + {}^3\text{He}$ scattering experim

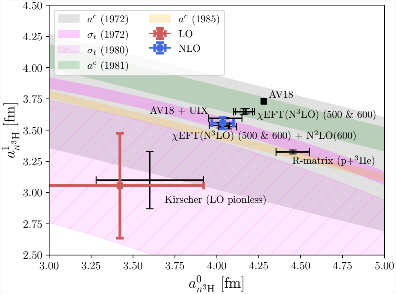
$n + {}^3\text{H}$

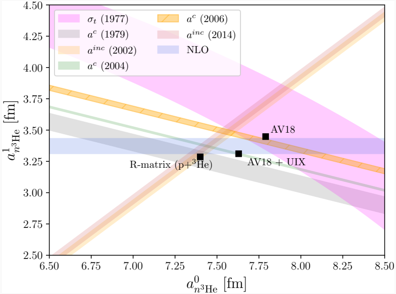
- (Phys. Rev. C (1972) 1952)
  coherent scatetring length,
  total crosssection
- **T. W. Phillips et al.**
  (Phys. Rev. C 22 (1980) 384)
  total crosssection
- **S. Hammerschmied et al.**
  (Z. Phys. A 302 (1981) 323)
  coherent scattering length
- **H. Rauch et al.**
  (Phys. Lett. B 165 (1985) 39)
  coherent scattering length

$n + {}^3\text{He}$

- **V. P. Alfimenkov**
  (Sov. J. Nucl. Phys 25 (1977) 607)
  total crosssection
- **H. Kaiser et al.**
  (Z. Phys. A 291 (1979) 231)
  cohherent scattering length
- **O. Zimmer et al.**
  (Eur. Phys. J. Direct A 1 (2002) 1)
  incoherent scattering length
- **P.R. Huffman et al.**
  (Phys. Rev. C 70 (2004) 014004)
  coherent scattering length
- **W. Ketter et al.**
  (Eur. Phys. J. A 27 (2006) 243)
  coherent scattering length
- **M.G. Huber et al.**
  (Phys. Rev. C 90 (2014) 064004)
  incoherent scattering length

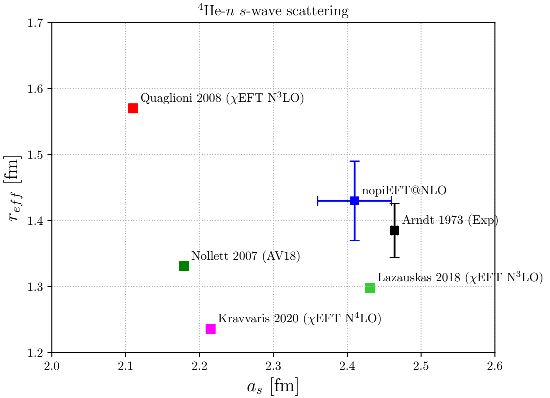
Experiment & Theory : $n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering lengths





Phys. Rev. C 42 (1990) 438; Phys. Rev. C 102 (2020) 034007; Few-Body Syst. 34 (2004) 105; Phys. Lett B 721 (2013) 355; Phys. Rev. C 68(R) (2003) 021002

$n + {}^4\text{He}$ s-wave scattering



Exp	a_0 [fm]	r_{eff} [fm]
Arndt 1973	2.4641 ± 0.0037	1.385 ± 0.041
Haun 2020	$2.4746^{\pm 0.0017\ [\text{stat}]}_{\pm 0.0011\ [\text{syst}]}$	-

EFT for Λ hyperons

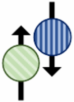
Data shrotage

- Limited scattering data
- No real low energy data
- Only 5 known bound states for $A \leq 6$ (+mirror nuclei)
- No resonance data
- ...

Issues

- Contradicting results for life time and binding energy of ${}^{3}_{\Lambda}$H
- Charge symmetry breaking
- What is the onset of double Lambda binding?
- Stability of the Λnn system
- Overbinding of ^{5}He
- ...

Two body



$2 \times NN$ — S = 0, S = 1 → a_{NN}

$2 \times N\Lambda$ — S = 0, S = 1 → $a_{N\Lambda}$

$\Lambda\Lambda$ → $a_{\Lambda\Lambda}$

No precise Experimental data ($a_{N\Lambda}$, $a_{\Lambda\Lambda}$)

NNN — $^{3}\mathrm{H}$

$3 \times NN\Lambda$:
- $^{3}_{\Lambda}\mathrm{H}$
- ~~$^{3}_{\Lambda}\mathrm{H}$~~$(S = 2/3)$ → $^{4}_{\Lambda}\mathrm{H}(S = 0, I = 1/2)$
- ~~$^{3}_{\Lambda}\mathrm{He}$~~ → $^{4}_{\Lambda}\mathrm{H}(S = 1, I = 1/2)$

$N\Lambda\Lambda$ — ~~$^{3}_{\Lambda\Lambda}\mathrm{H}$~~ → $^{6}_{\Lambda\Lambda}\mathrm{He}$

Three body

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In a nutshell

${}^{3}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{H}^*$, ${}^{5}_{\Lambda}\mathrm{He}$

It is possible to describe them all together.
(No overbinding problem!)

${}^{5}_{\Lambda\Lambda}\mathrm{He}$

Solidly bound!

$$B_{\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{He}) = 1.14(1)^{+(44)}_{-(26)}$$

$np\Lambda\Lambda$

Bound?
Need more precise
ΛΛ scattering data!

$nn\Lambda$, $nn\Lambda\Lambda$, $n\Lambda\Lambda$

Unbound

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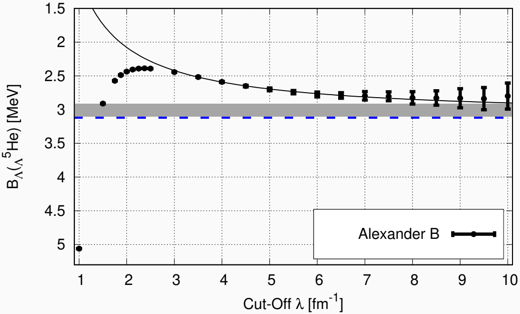
B$_\Lambda$($^5_\Lambda$He) vs. cut-off λ in LO $\not\pi$EFT



L.Contessi N.Barnea A.Gal, PRL (2018)

With Alexander B scattering lengths a_s, a_t $\not\pi$EFT reproduces B$_\Lambda$($^5_\Lambda$He)

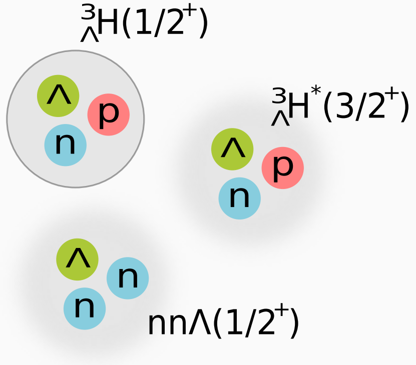
Cut-off dependence
$$\frac{B_\Lambda(\lambda)}{B_\Lambda(\infty)} = 1 + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \ldots$$

${}^3_\Lambda\mathrm{H}^*(3/2^+)$

- no experimental evidence
- JLab C12-19-002 proposal

$\Lambda nn(1/2^+)$

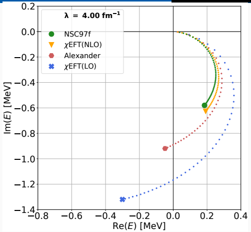
- experiment (HypHI)
- JLab E12-17-003 experiment



Continuum states

$\Lambda nn(\frac{1}{2}^+)$ - **resonant state**

- Question observable Λnn resonance (physical Riemann sheet)
- Bound $\Lambda nn \Rightarrow$ serious disagreement with 4,5-body hypernuclei.

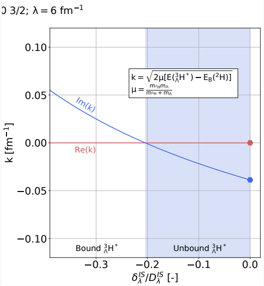


${}^3_\Lambda H^*(\frac{3}{2}^+)$ - **virtual state**

- Virtual state from 0.02-0.25 MeV below the ${}^2H + \Lambda$ threshold
- If bound $\Rightarrow$ 4,5-body hypernuclei do not change much
- Pole sensitive to ΛN spin-singlet strength.

IACCC

$$\Rightarrow \delta(\kappa),\ \kappa = -ik = -i\sqrt{E}$$



Charge symmetry breaking

$A = 4$ hypernuclear level scheme

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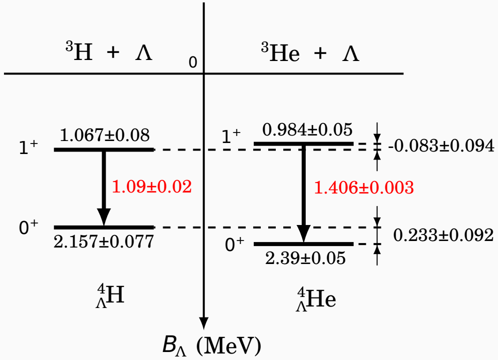


- **Charge symmetry:** invarince under $n \leftrightarrow p$, e.g. $^3\mathrm{H} \leftrightarrow {}^3\mathrm{He}$
- **Nuclei:** for ^{3}He - ^{3}H, ΔE_{CSB} without Coulomb is about 70 keV
- For ^{3}He - ^{3}H: $\Delta E_{CSB}/\Delta E \approx 0.01$
- **Hypernuclei:** CSB in $^4_\Lambda$He-$^4_\Lambda$H: $\Delta E_{CSB}/\Delta E \approx 0.22$

Charge symmetry breaking - χEFT

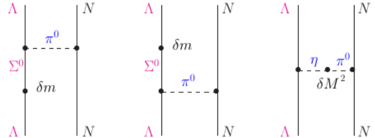


Fig. 1 CSB contributions involving pion exchange, according to Dalitz and von Hippel [1], due to $\Lambda - \Sigma^0$ mixing (left two diagrams) and $\pi^0 - \eta$ mixing (right diagram).

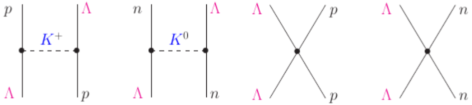
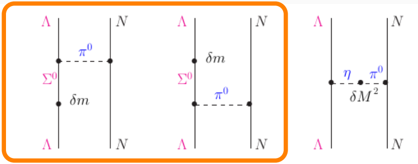


Fig. 2 CSB contributions from $K^{\pm}/K^0$ exchange (left) and from contact terms (right).

Λ	NLO13		NLO19	
	C_s^{CSB}[MeV^{-2}]	C_t^{CSB}[MeV^{-2}]	C_s^{CSB}[MeV^{-2}]	C_t^{CSB}[MeV^{-2}]
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}

Haidenbauer et al., Few-body sys. (2019)



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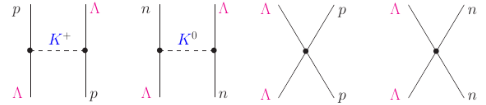

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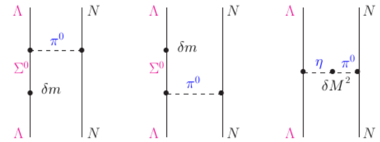
Haidenbauer et al., Few-body sys. (2019)

Charge symmetry breaking - χEFT



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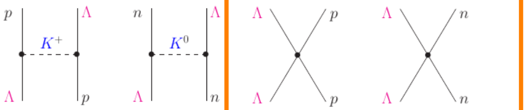

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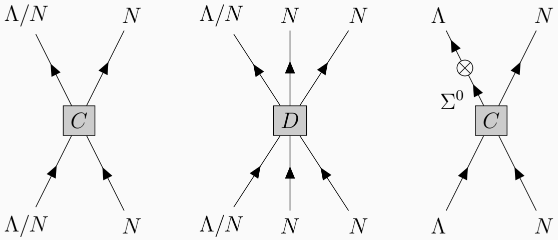
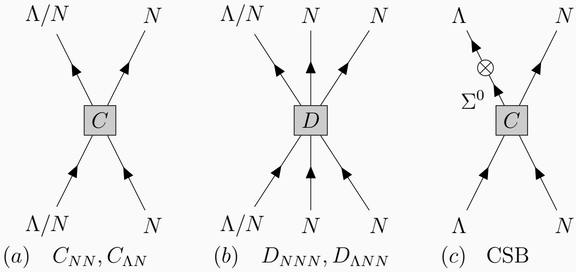


(a) $C_{NN}, C_{\Lambda N}$ (b) $D_{NNN}, D_{\Lambda NN}$ (c) CSB

$$\langle \Lambda N | V_{\rm CSB} | \Lambda N \rangle = -\frac{2}{\sqrt{3}} \, \mathcal{A}^{(0)}_{I=1} \, \langle \Sigma N | V_{\rm CS} | \Lambda N \rangle \, \tau_z .$$

We use $\mathrm{SU(3)_f}$ to relate $C^S_{\Lambda N,\Sigma N}$ to the NN and ΛN LECs:

$$C^0_{\Lambda N,\Sigma N} = -3(C^0_{NN} - C^0_{\Lambda N}),$$
$$C^1_{\Lambda N,\Sigma N} = \quad (C^1_{NN} - C^1_{\Lambda N}).$$



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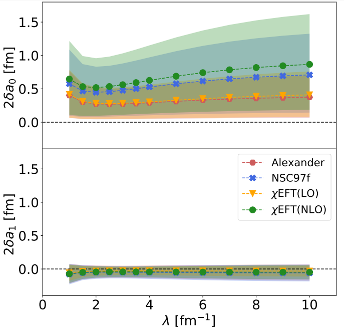
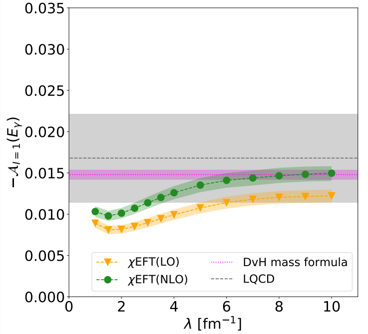
Extracting the DvH parameter





Method/Input	$-\mathcal{A}_{I=1}$
SU(3)$_f$ [DvH64]	0.0148 ± 0.0006
LQCD [LQCD20]	0.0168 ± 0.0054
$\not\pi$EFT(LO)/χEFT(LO) [Polinder06]	0.0139 ± 0.0013
$\not\pi$EFT(LO)/χEFT(NLO) [Haidenbauer13]	0.0168 ± 0.0014

Conclusions

STRONG@HUJI