NonRelativistic UnParticles

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research support from U.S. Department of Energy

NonRelativistic UnParticles

Relativistic Unparticles

Georgi

Phys. Rev. Lett. 98, 221601 (2007) [hep-ph/0703260]

NonRelativistic Unparticles

Hammer & Son

Proc.Nat.Acad.Sci. 118 (2021) [arXiv:2103.12610]

Neutrons are Unparticles !

Neutral charm mesons are Unparticles !

Braaten & Hammer

Phys. Rev. Lett. 128, 032002 (2022) [arXiv:2107.03821]

Q. What is an elementary particle?

A. Irreducible representation of the Poincare group

particle is characterized by mass m and spin s

Poincare group

includes 4 spacetime translations
3 rotations
3 Lorentz boosts

---

10 dimensions

Standard Model of Particle Physics: *SU(3)xSU(2)xU(1)* gauge theory

17 elementary particles

$s = 0$: Higgs boson

$s = 1/2$: 6 quarks, 6 leptons

$s = 1$: photon, gluon, $W^{\pm}$, Z^0

Beyond the Standard Model ??

more elementary particles ?

new interactions ?

Hidden Sector ??

with no Standard Model interactions

"Unparticle Physics"

Howard Georgi   hep-ph/0703260

Hidden sector could be
scale invariant theory with conformal symmetry
excitations are "unparticles"

Q. What is an unparticle?
A. Irreducible representation
of the conformal group

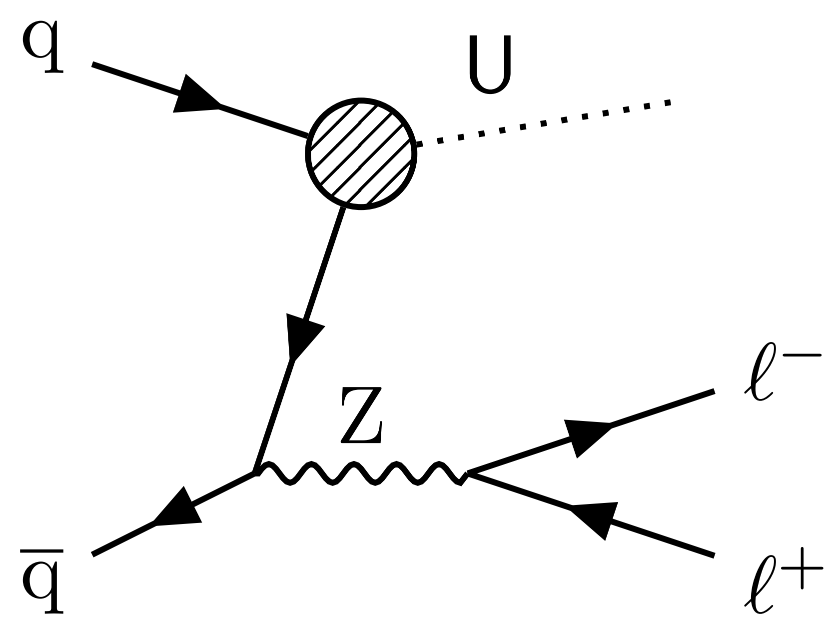
unparticle is characterized by scaling dimension Δ

Conformal group includes 4 spacetime translations
3 rotations
3 Lorentz boosts
1 scale transformation
4 spacetime inversions

15 dimensions

Observation of Unparticles

Unparticle in a hidden sector cannot be observed directly
However it can be observed indirectly
if it is produced in association with a Standard Model particle
(such as Z^0)



Invariant mass-squared P_U^2 of Unparticle
can be measured using recoil momentum of Standard Model particle

$$d\sigma/dP_U^2 \sim (P_U^2)^{\Delta-2}$$

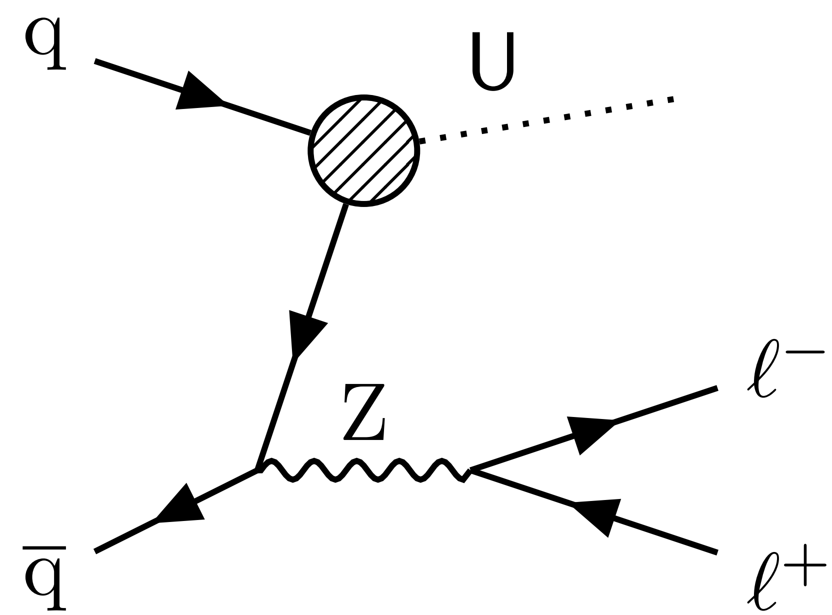
Unparticle signature: power-law dependence on P_U^2
exponent Δ determined by conformal symmetry

For N massless particles, $\Delta = N$.
For Unparticle, Δ can be noninteger

Searches for Unparticles at the LHC

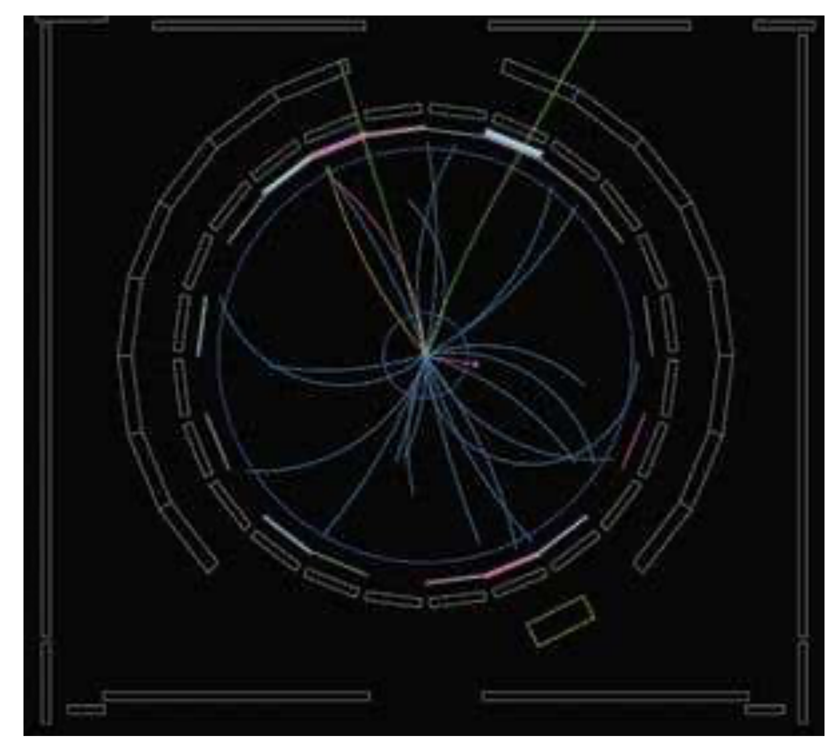
CMS collaboration arXiv:1408.3583, 1511.09375, 1701.02402

production of Unparticle
in association with Z^0



Unparticle invariant mass distribution

$$d\sigma/dP_U^2 \sim (P_U^2)^{\Delta-2}$$



CMS: “95% confidence limits
are obtained on the effective cutoff scale
as a function of the scaling dimension”

NonRelativistic Effective Field Theories

NREFT can have Galilean symmetry

if kinetic mass is conserved in every reaction

Galilean group includes 4 spacetime translations

3 rotations

3 Galilean boosts

1 phase transformation

11 dimensions

phase symmetry guarantees mass conservation

Q. What is a Galilean particle?

A. Irreducible representation

of the Galilean group

characterized by kinetic mass M, spin s

“UnNuclear Physics”

Hammer & Son   arXiv:2103.12610

Unparticles can rise in nonrelativistic effective field theories

Q. What is a nonrelativistic unparticle?

A. Irreducible representation
of nonrelativistic conformal group

unparticle is characterized by kinetic mass M, scaling dimension Δ

nonrelativistic conformal (Schroedinger) group

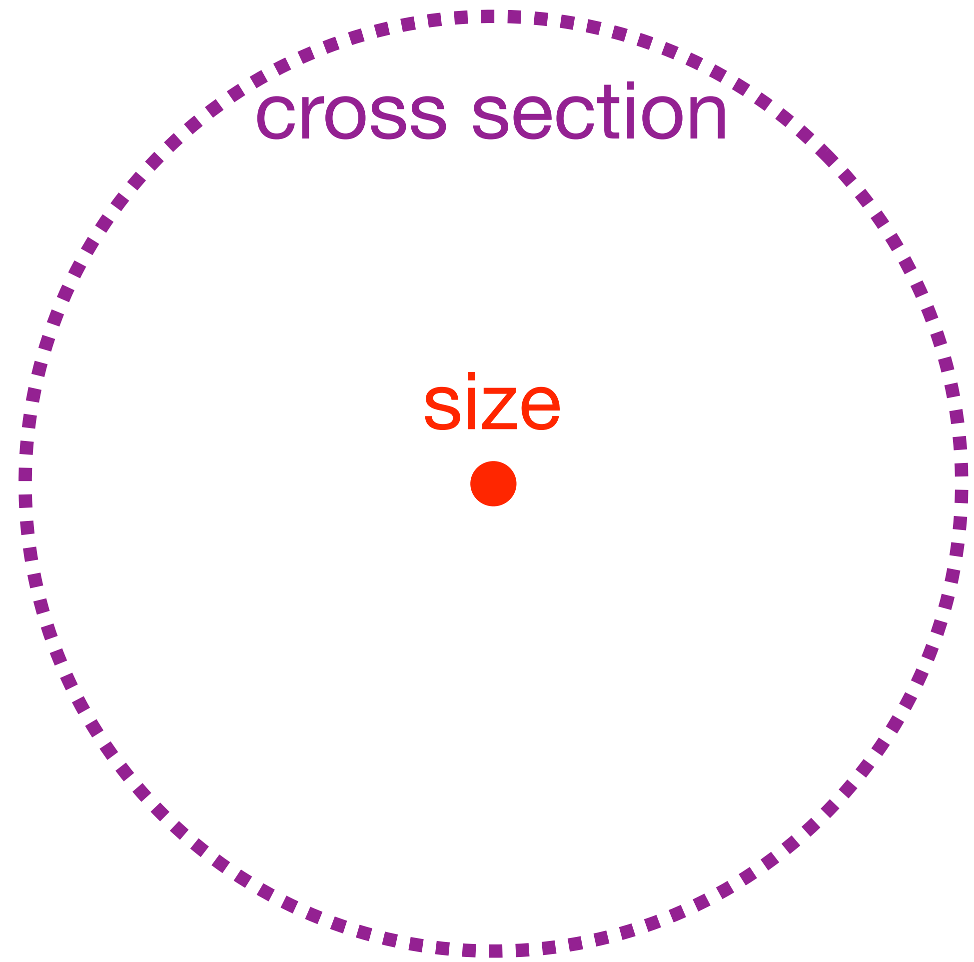
includes 4 spacetime translations
3 rotations
3 Galilean boosts
1 phase transformation
1 scale transformation
1 time inversion

13 dimensions

“UnNuclear Physics”

Hammer & Son arXiv:2103.12610

Neutrons with opposite spins
have large scattering length $a = -19$ fm
and enormous cross section at low energy
(accidental fine tuning of QCD makes dineutron almost bound)



radius of neutron: 0.8 fm

radius of cross section: 20 fm

Interactions between low-energy neutrons
are approximately scale invariant !

"UnNuclear Physics"

Hammer & Son   arXiv:2103.12610

Low-energy neutrons can be described by
nonrelativistic conformal field theory

Neutrons are Unparticles !

For *N* weakly interacting particles, $\Delta = (3/2)N$.
For Unparticle, Δ can be noninteger

	mass	scaling dimension
1 neutron:	m_n	$\Delta_1 = 3/2$
2 neutrons:	$2m_n$	$\Delta_2 = 2$
3 neutrons:	$3m_n$	$\Delta_3 = 4.27272$
4 neutrons:	$4m_n$	$\Delta_4 \approx 5.1$

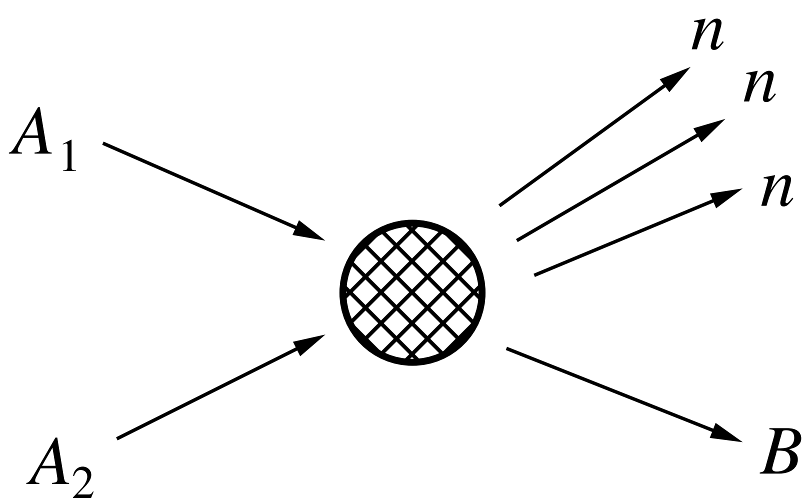
Cold Atom analog:
fermionic atoms with 2 spin states tuned to Feshbach resonance

“UnNuclear Physics”

Hammer & Son   arXiv:2103.12610

Nuclear reaction $A_1\ A_2 \rightarrow B + (N\ n)$

creates N neutrons near their threshold with invariant mass $N\ m_n + E$
and substantial recoil momentum from nucleus B



Energy distribution of neutrons
can be measured using
recoil momentum of nucleus B

$$\frac{d\sigma}{dE} \sim E^{\Delta_N - 5/2}$$

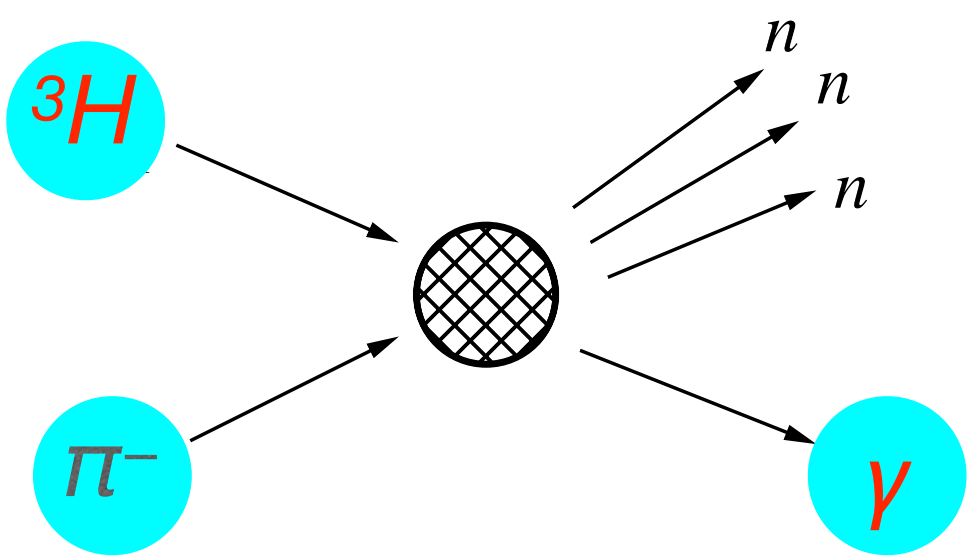
Unparticle signature: power-law behavior with
exponent Δ_N determined by conformal symmetry

"UnNuclear Physics"

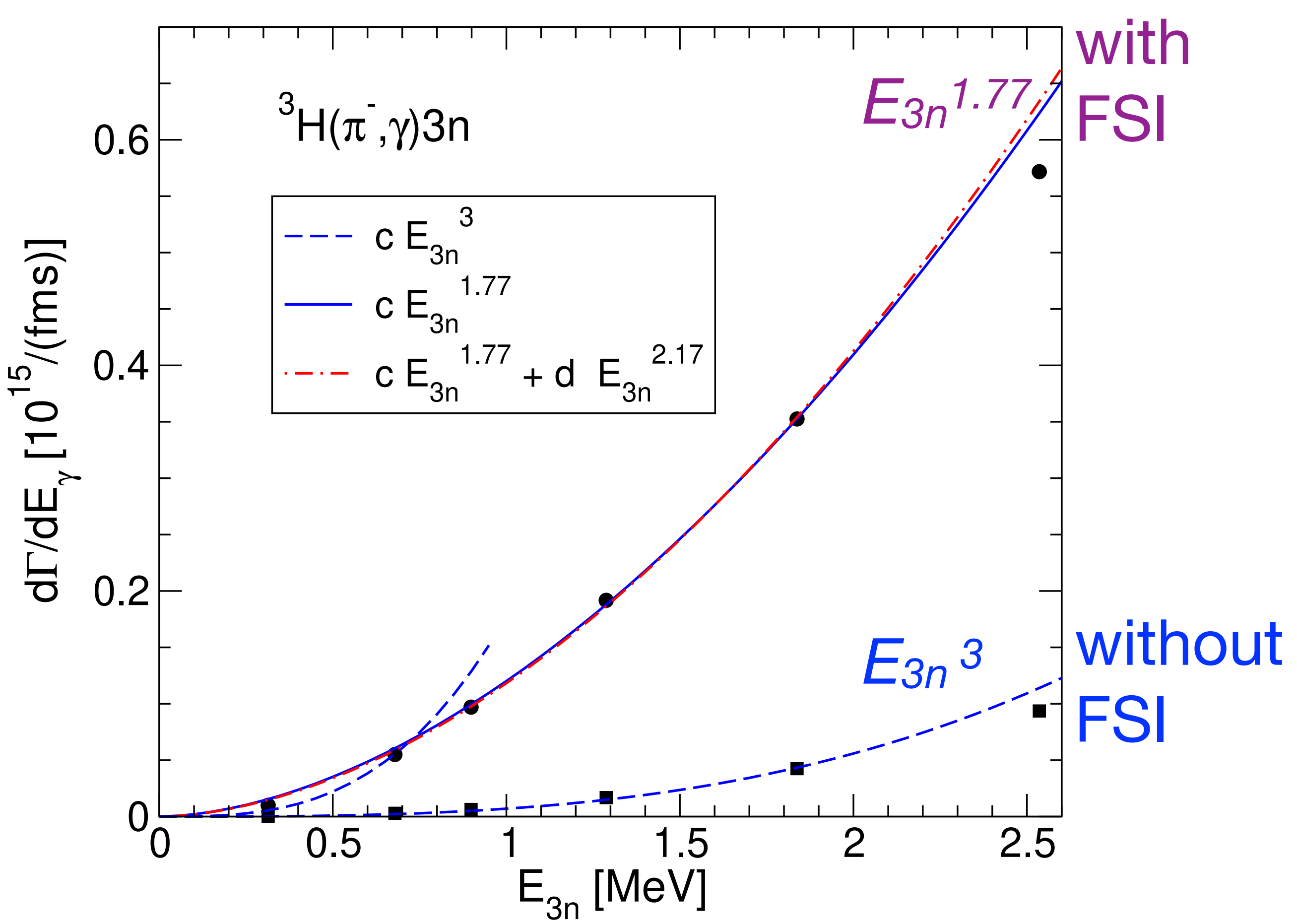
Hammer & Son   arXiv:2103.12610

Nuclear reaction $\pi^- \, {}^3H \rightarrow \gamma + (3\,n)$

creates 3 neutrons with invariant mass $3\,m_n + E$ with $E \ll m_\pi^2/2M_n$



$$\frac{d\sigma}{dE_{3n}} \sim (E_{3n})^{1.77272}$$



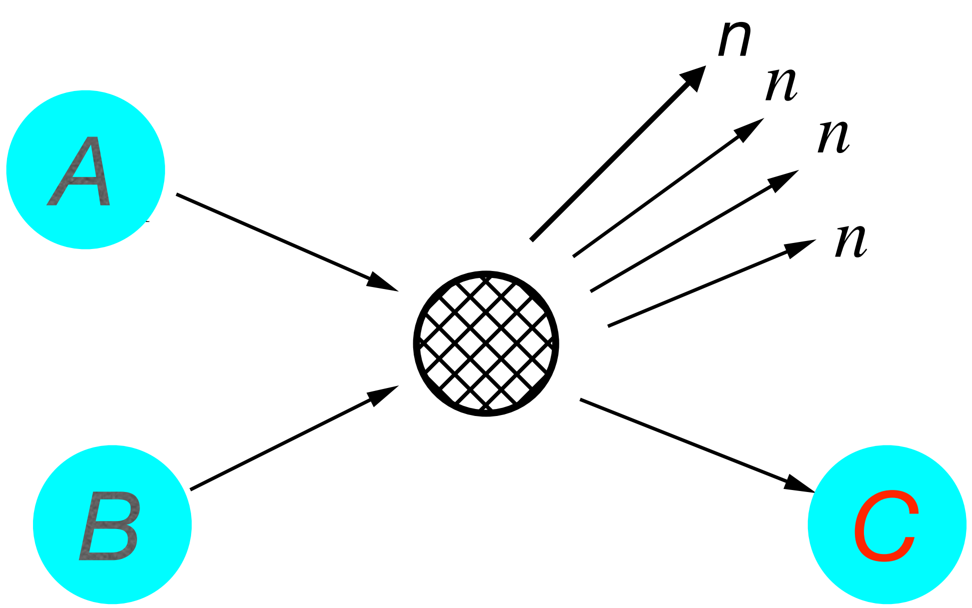
Signature for 3-neutron unparticle: power-law behavior with exponent 1.77272 (naive exponent is 3.0)

"UnNuclear Physics"

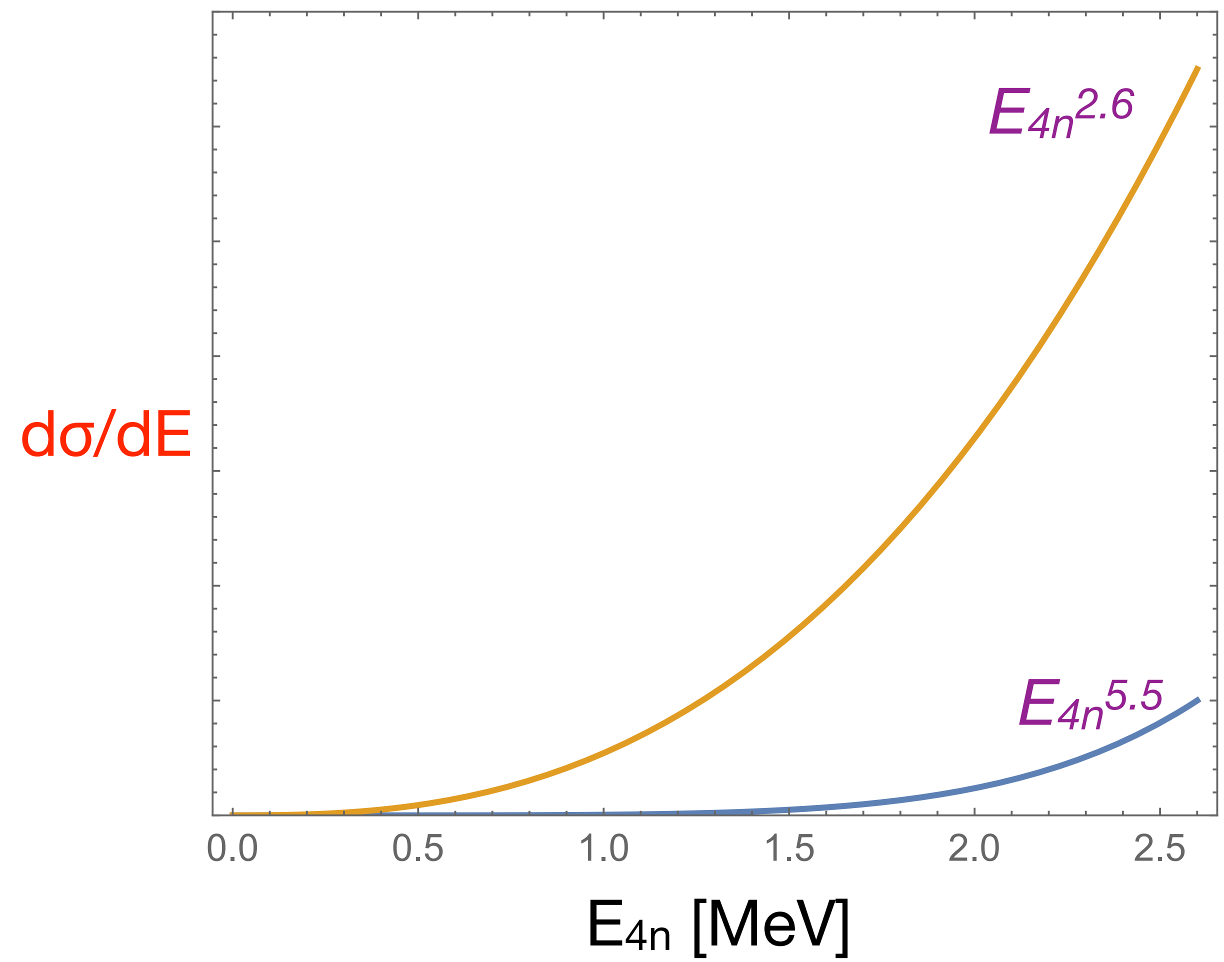
Nuclear reaction *A B* → *C* + (*4 n*)

creates 4 neutrons with small invariant mass $4\,m_n + E$ with $E \ll m_\pi^2/2M_n$

and large recoil momentum



$$\frac{d\sigma}{dE_{4n}} \sim (E_{4n})^{2.6}$$



Signature for 4-neutron unparticle: power-law behavior

with exponent 2.6

(naive exponent is 5.5)

Low-energy neutral charm mesons are Unparticles !

arXiv:2107.03821
with H.-W. Hammer

Neutral charm mesons

spin 0: $D^0 = c\bar{u}$, $\bar{D}^0 = \bar{c}u$ mass: M = 1865 MeV

spin 1: $D^{*0} = c\bar{u}$, $\bar{D}^{*0} = \bar{c}u$ mass: M_* = 2007 MeV

X(3872) resonance in $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$ channel

$\Longrightarrow$ bosons with large positive scattering length but no Efimov effect

$\Longrightarrow$ neutral charm mesons are Unparticles!

reaction rates with power-law behavior

- for multiple charm mesons (different scaling dimensions)
- for *X(3872)* + charm mesons (new reactions)

exponents determined by conformal symmetry !!

Cold Atom analog:
bosonic atoms with 2 spin states *A*, *A'*
and Feshbach resonance in channel *A'A* + A'A

$X(3872) \equiv \chi_{c1}(3872)$

discovered at e^+e^- collider Belle 2003

$B^\pm \to K^\pm X, \quad X \to J/\psi\, \pi^+\pi^-$

- quantum numbers $J^{PC} = 1^{++}$ LHCb 2013

- mass is extremely close to $D^{*0}\bar{D}^0$ threshold

  $E_X \equiv M_X - (M_{D^{*0}} + M_{D^0}) = (-0.07 \pm 0.12)$ MeV LHCb 2020

  $|E_X| < 0.22$ MeV at 90% CL

- width is extremely narrow

  $\Gamma_X = (0.22 \pm 0.14)$ MeV LHCb 2020

- 7 observed decay modes

  $J/\psi\, \pi^+\pi^-$, $J/\psi\, \pi^+\pi^-\pi^0$, $J/\psi\, \gamma$, $\psi(2S)\, \gamma$, $\chi_{c1}\, \pi^0$, $D^0\bar{D}^0\gamma$, $D^0\bar{D}^0\pi^0$

first of dozens of exotic heavy hadrons

that have been discovered since 2003 !

What is the $X(3872)$?

experimental inputs: $J^{PC} = 1^{++}$ and $|E_X| < 0.22$ MeV

resonant S-wave interactions
with pairs of neutral charm mesons
transform X into loosely bound molecule !!

$$X(3872) = (D^{*0}\bar{D}^0 + D^0\bar{D}^{*0})/\sqrt{2}$$

<u>small additional components</u>

at long distances: $D^0\bar{D}^0\pi^0$

at short distances:

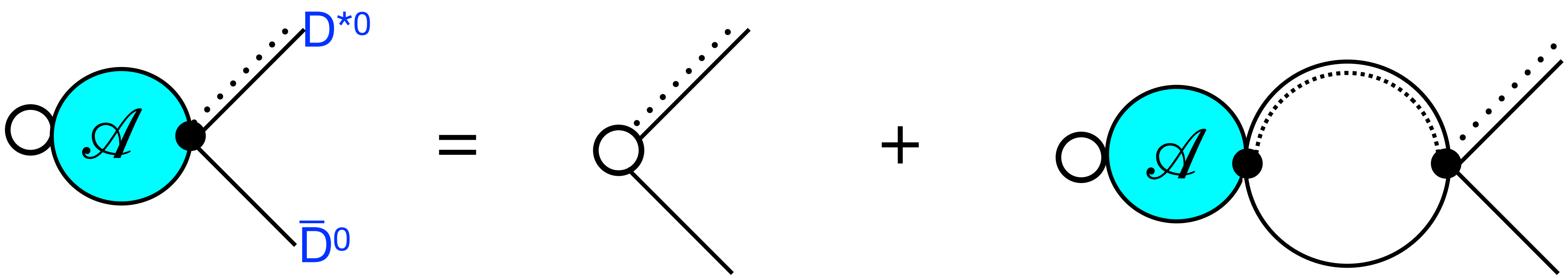
charged charm mesons $D^{*+}D^- + D^+D^{*-}$

P-wave charmonium $\chi_{c1}(2P)$??

compact tetraquark $[cq][\bar{c}\bar{q}]$??

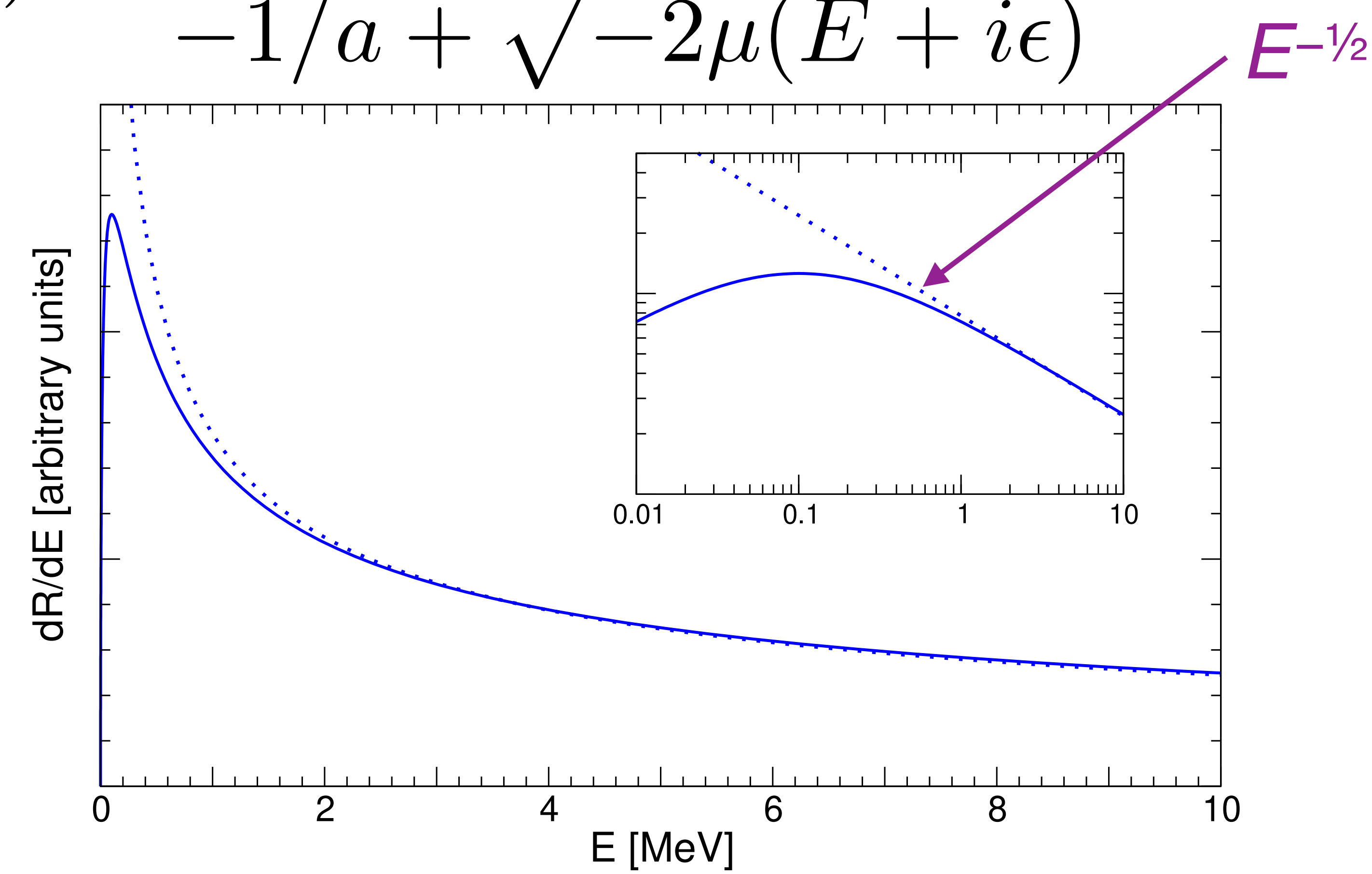
Point Production of $D^{*0}\bar{D}^0$

integral equation for production amplitude for $D^{*0}D^0$
from creation of charm mesons at a point



solve algebraically:
$$\mathcal{A}(E) = \frac{1}{-1/a + \sqrt{-2\mu(E + i\epsilon)}}$$

Production rate
as function of the energy E:
$dR/dE \sim |\mathscr{A}(E)|^2 \sqrt{E}$

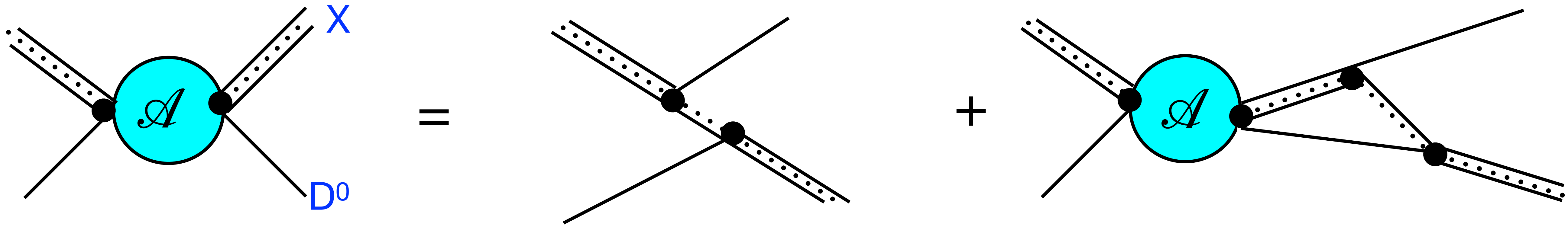


high energy scaling behavior: $dR/dE \sim E^{-1/2}$

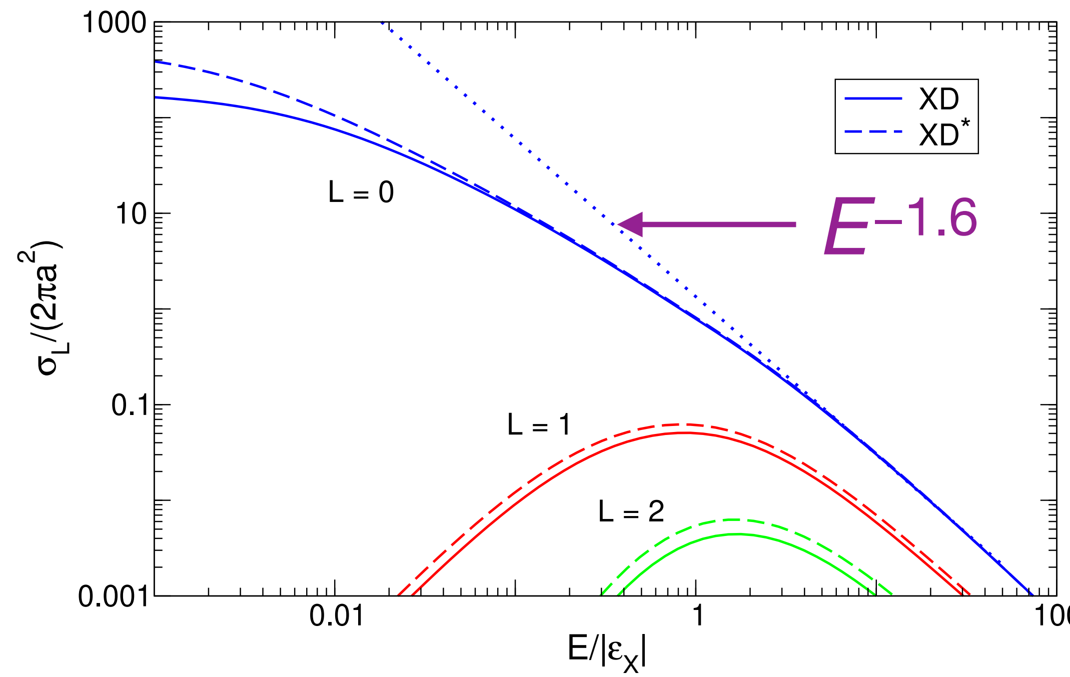
Scattering of X(3872) and Charm Meson

Canham, Hammer & Springer arXiv:0906.1263

numerical solution of STM integral equation for $X\,D \to X\,D$ ($D = D^0$ or D^{*0})



cross sections
as functions of energy E

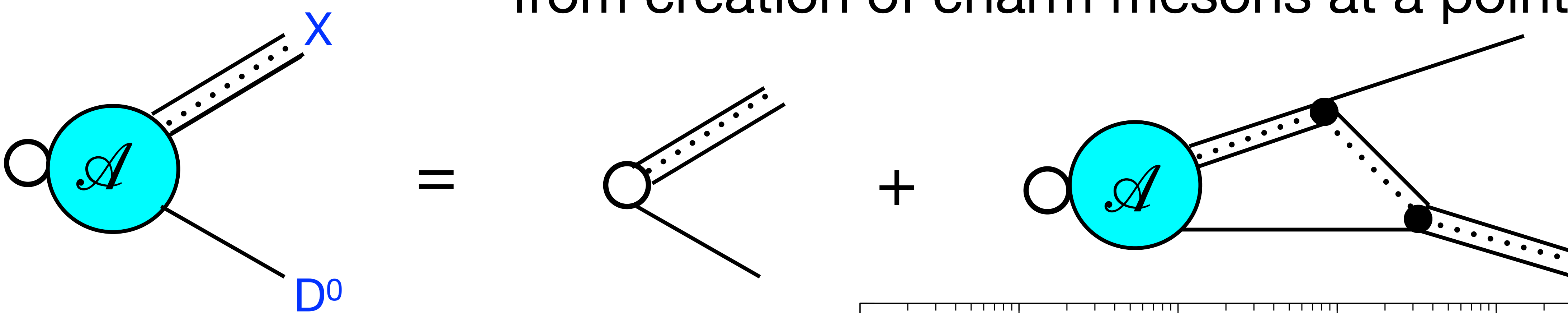


low energy: $\sigma(E=0) = 4\pi\, a_{DX}^2$
huge scattering lengths: $a_{D^0X} = -9.7\,a,\ a_{D^{*0}X} = -16.6\,a$

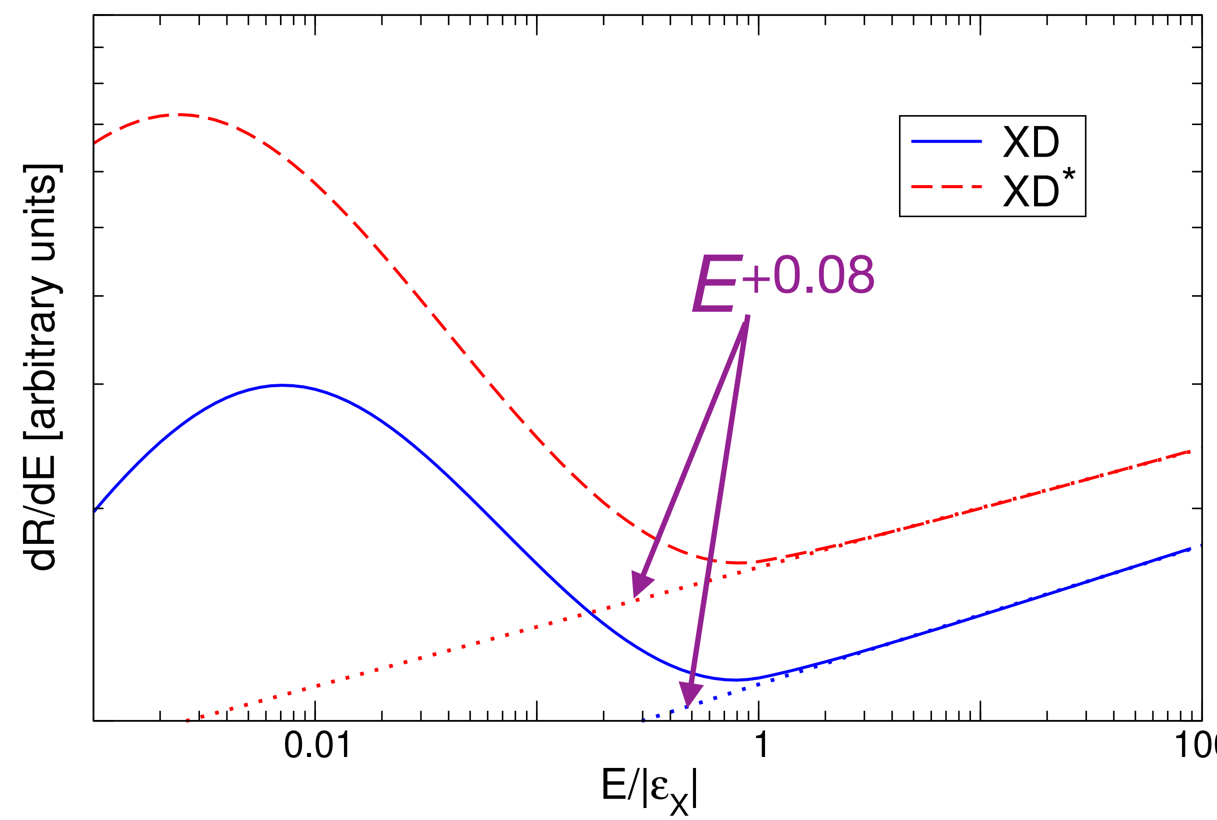
high energy scaling behavior: $\sigma(E) \sim E^{-1.6}$
power-law with approximately same exponent for $X\,D^0$, $X\,D^{*0}$

Point Production of *X(3872)* + Charm Meson

STM integral equation for production of *X D* (*D* = D^0 or D^{*0})
from creation of charm mesons at a point



production rates *dR/dE*
as function of energy *E*



<u>low energy</u>: determined by huge scattering length a_{DX}

<u>high energy scaling behavior</u>: $dR/dE \sim E^{+0.08}$
power-law with approximately same exponent for XD^0, XD^{*0}

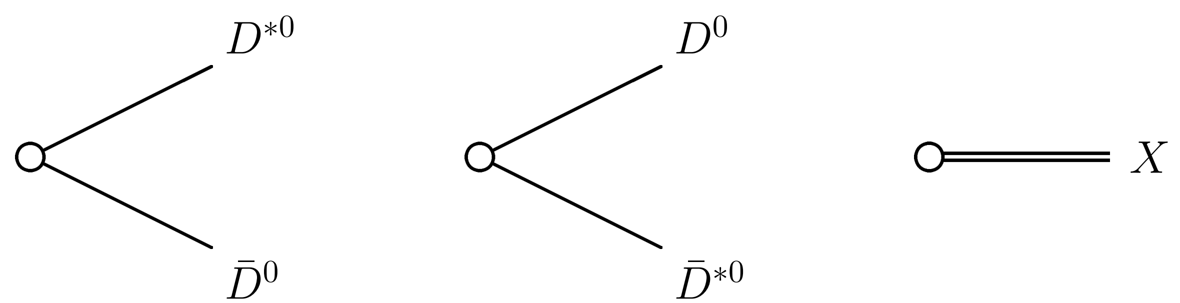
Low-energy neutral charm mesons are Unparticles !

arXiv:2107.03821
with H.-W. Hammer

in the limit $a \to \infty$ ($\varepsilon_X \to 0$)
Effective Field Theory for neutral charm mesons is scale invariant
⇒ Nonrelativistic Conformal Field Theory !

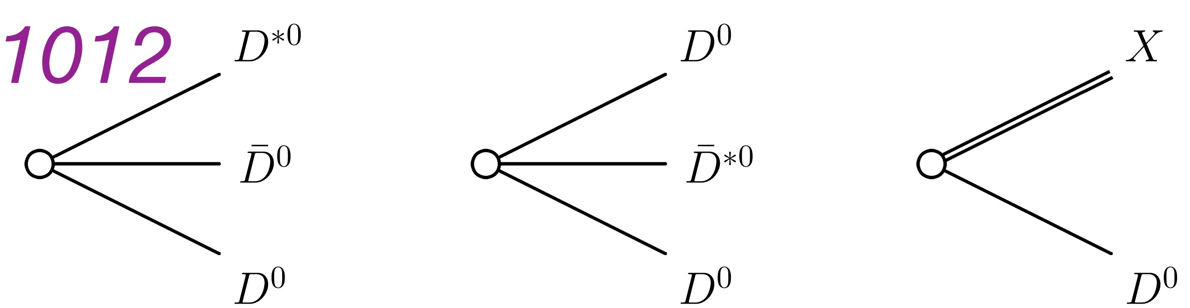
X unparticle

operator with scaling dimension $\Delta_2 = 2$
creates $D^{*0}+\bar{D}^0$, $D^0+\bar{D}^{*0}$, *and* $X(3872)$



XD unparticle

operator with scaling dimension $\Delta_3 = 3.1012$
creates $D^{*0}+\bar{D}^0+D^0$, $D^0+\bar{D}^{*0}+D^0$,
and $X(3872)+D^0$

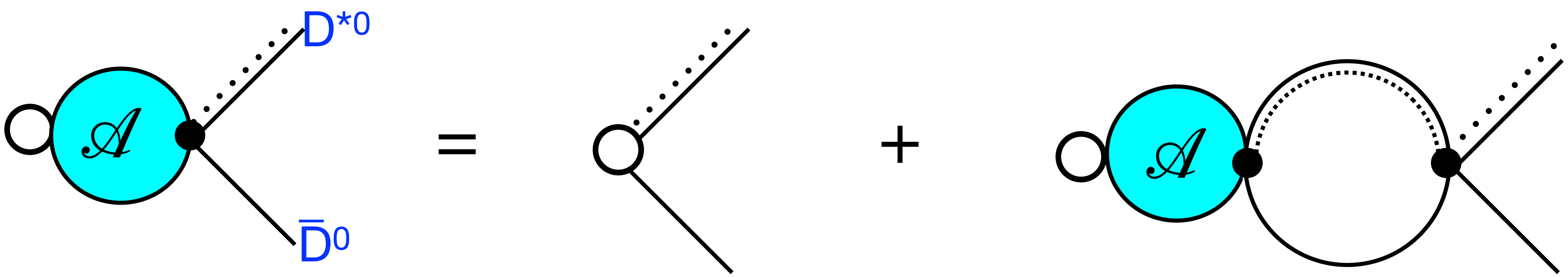


*XD** unparticle

operator with scaling dimension $\Delta_{3*} = 3.0870$
creates $D^{*0}+\bar{D}^0+D^{*0}$, $D^0+\bar{D}^{*0}+D^{*0}$, *and* $X(3872)+D^{*0}$

Scaling Dimension: *X* Unparticle

integral equation for production amplitude for $D^{*0}\bar{D}^0$
from creation of charm mesons at a point



solve algebraically: $$\mathcal{A}(E) = \frac{1}{-1/a + \sqrt{-2\mu(E+i\epsilon)}}$$

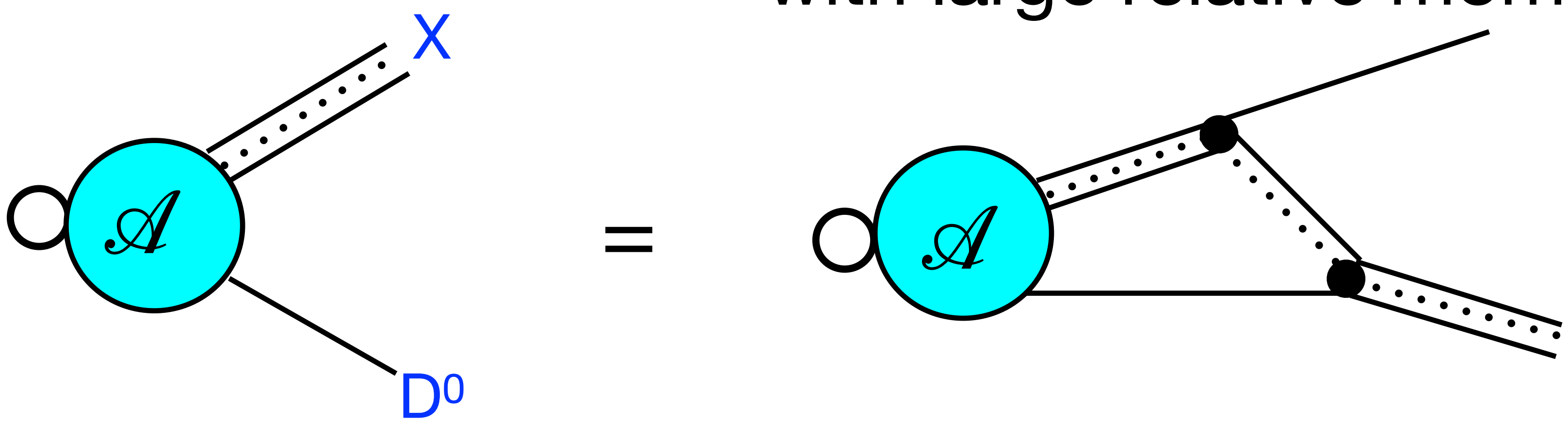
scale-invariant limit: $a \to \infty \quad \Rightarrow \quad \mathscr{A}(E) \sim |E|^{-1/2}$

$$\mathscr{A}(E) \sim |E|^{\Delta-5/2} \quad \Rightarrow \quad \Delta = 2$$

X Unparticle has scaling dimension $\Delta_2 = 2$

Scaling Dimensions: *XD, XD** Unparticles

homogeneous STM integral equation for production of *XD* (*D* = D^0 or D^{*0})
with large relative momentum $p \gg \sqrt{2\mu E}$



$$\mathcal{A}(p) = \frac{1}{4\pi r\sqrt{1-r^2}\,p}\int_0^\infty dq\,\mathcal{A}(q)\log\frac{p^2+q^2+2rpq}{p^2+q^2-2rpq} \qquad r = M_D/M_X$$

look for power-law solution: $\mathscr{A}(p) = p^{s-1}$

$$1 = \frac{\sin\big(s\arcsin(r)\big)}{2r\sqrt{1-r^2}\,s\cos(s\pi/2)}$$

smallest positive solution: $s = 0.60119$ if $D = D^0$
$s = 0.51834$ if $D = D^{*0}$

operator dimension: $\Delta = 5/2+s$

XD Unparticle has scaling dimension $\Delta_3 = 3.10119$
*XD** Unparticle has scaling dimension $\Delta_{3*} = 3.08697$

Propagators for Unparticles

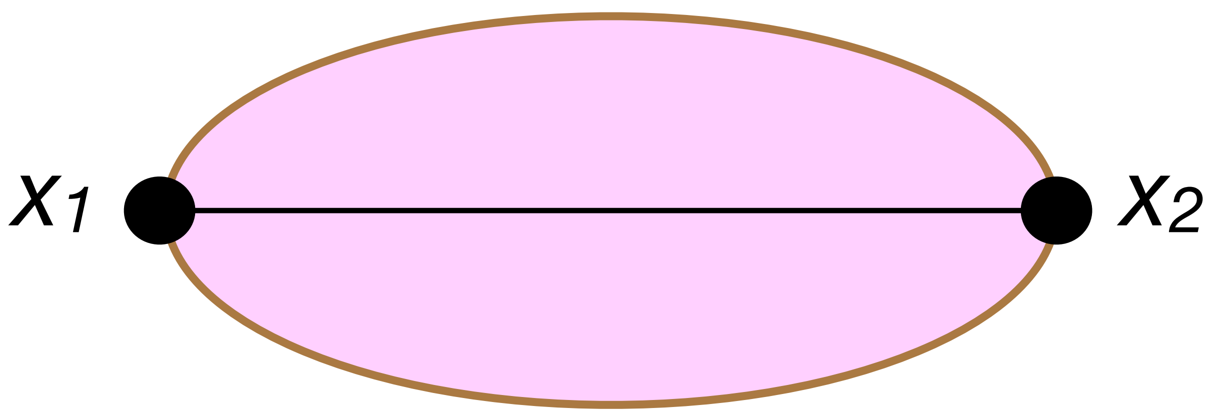
primary operator $\phi_n(x)$: scaling dimension Δ_n, mass M_n

propagator is determined by conformal symmetry

spacetime propagator: $\langle \phi_n(x_1)\phi_n^\dagger(x_2)\rangle = C_n\,\theta(t_{12})\,(t_{12})^{-\Delta_n}\ \exp(iM_n r_{12}^2/2t_{12})$

Fourier transform has branch cut: $D_n(E,p) = C_n' \left(\frac{p^2}{2M_n} - E - i\epsilon\right)^{\Delta_n - 5/2}$

AdS/CFT correspondence: spacetime is boundary of anti-deSitter space



Unparticle propagator is free-field propagator in AdS space

Correlator for Three Operators

primary operators $\phi_1(x)$, $\phi_2(x)$, $\phi_3(x)$: scaling dimensions Δ_1, Δ_2, Δ_3
masses M_1, M_2, $M_3 = M_1+M_2$

3-point function is determined by conformal symmetry!

Henkel and Unterberger 1993
Fuertes and Morozov 2009
Volovich and Wen 2009

$$\begin{aligned}\langle \phi_1(x_1)\phi_2(x_2)\phi_3^\dagger(x_3)\rangle = C_{12,3}\,\theta(t_{13})\,(t_{13})^{-\Delta_{13,2}/2}\ \exp(iM_n r_{13}^2/2t_{13}) \\ \times\,\theta(t_{23})\,(t_{23})^{-\Delta_{23,1}/2}\ \exp(iM_n r_{23}^2/2t_{23}) \\ \times\,(t_{12})^{-\Delta_{13,2}/2}\,\Phi(w)\end{aligned}$$

$$\Delta_{ij,k} = \Delta_i + \Delta_j - \Delta_k$$

$\Phi(\omega)$ is function of single scaling variable $w = \dfrac{r_{12}^2}{2t_{12}} - \dfrac{r_{13}^2}{2t_{13}} + \dfrac{r_{23}^2}{2t_{23}}$

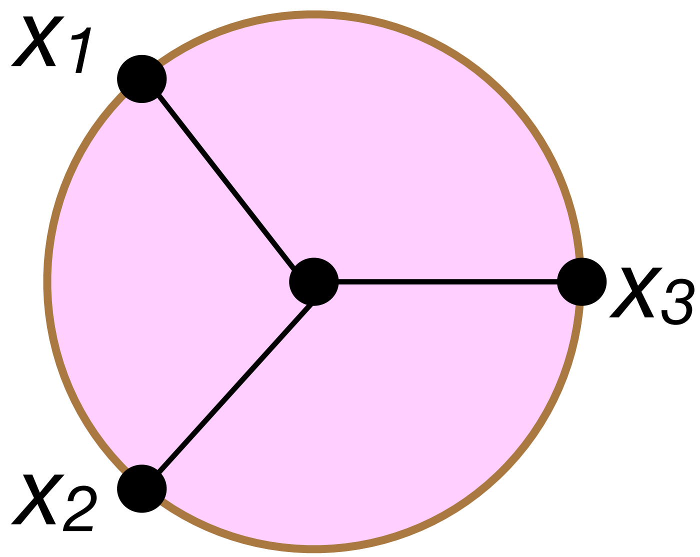
2-dimensional integral representation

$$\begin{aligned}\Phi(w) = &\int_{-\infty}^{+\infty} du(u+i\epsilon)^{-\Delta_{13,2}/2}e^{-iM_1 u} \\ &\times\int_{-\infty}^{+\infty} dv(v+i\epsilon)^{-\Delta_{23,1}/2}e^{-iM_2 v}\left[u-v+(1+i\epsilon)w\right]^{-\Delta_{12,3}/2}\end{aligned}$$

Correlator for Three Operators

$\langle \phi_1(x_1)\phi_2(x_2)\phi_3^\dagger(x_3)\rangle$ is determined by conformal symmetry!

Henkel and Unterberger 1993
Fuertes and Morozov 2009
Volovich and Wen 2009



- Fourier transform in time
- Fourier transform in space
- isolate pole from particle propagator $D_1(E_1,p_1) = i/(E_1 - p_1^2/2M_1)$
- isolate branch cut from 2-Unparticle propagator

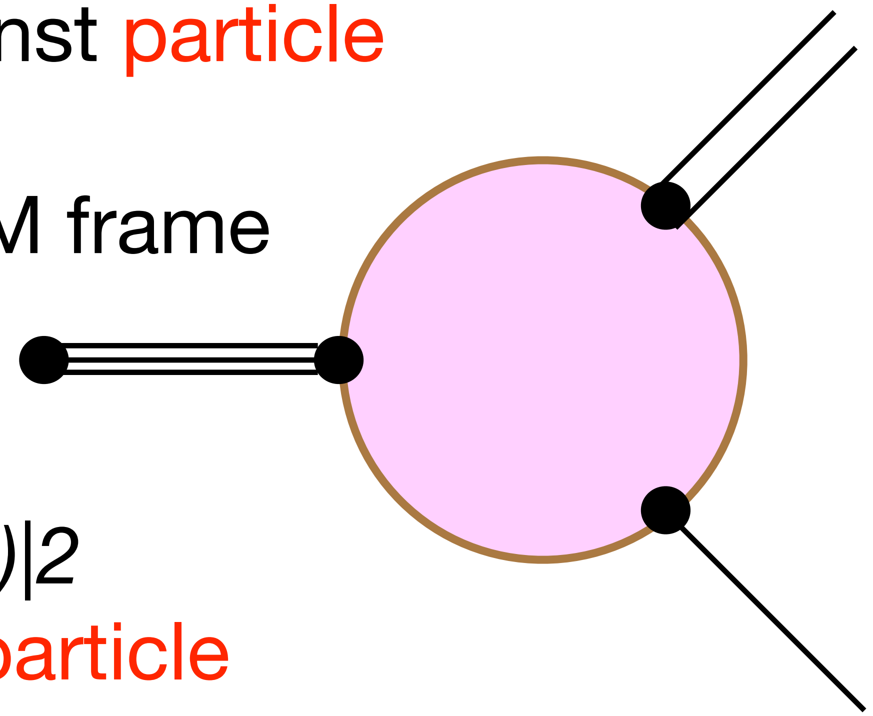
$$D_2(E_2,p_2) = C_2\left(\frac{p_2^2}{2M_2} - E_2 - i\epsilon\right)^{\Delta_2 - 5/2}$$

3-point function at large momentum $p_1 = p_2 = p$

$$G(E_1,E_2,p) \longrightarrow (\text{constant})\, D_1(E_1,p_1)\, D_2(E_2,p_2)\, p^{\Delta_3-\Delta_1-\Delta_2}$$

Point Production of Unparticle

production rate of 2-Unparticle recoiling against particle
from the creation of a 3-Unparticle at a point
with energy E_3 in its CM frame



amputated 3-point function $G_{amp}(E_2,p)$

- multiply by complex conjugate: $|G_{amp}(E_2,p)|2$
- multiply by discontinuity in propagator of particle

$$D_1(E_1 + i\epsilon, p) - D_1(E_1 - i\epsilon, p) = 2\pi\delta\big(E_2 - p^2/2M_2\big)$$

- multiply by discontinuity in propagator of 2-Unparticle

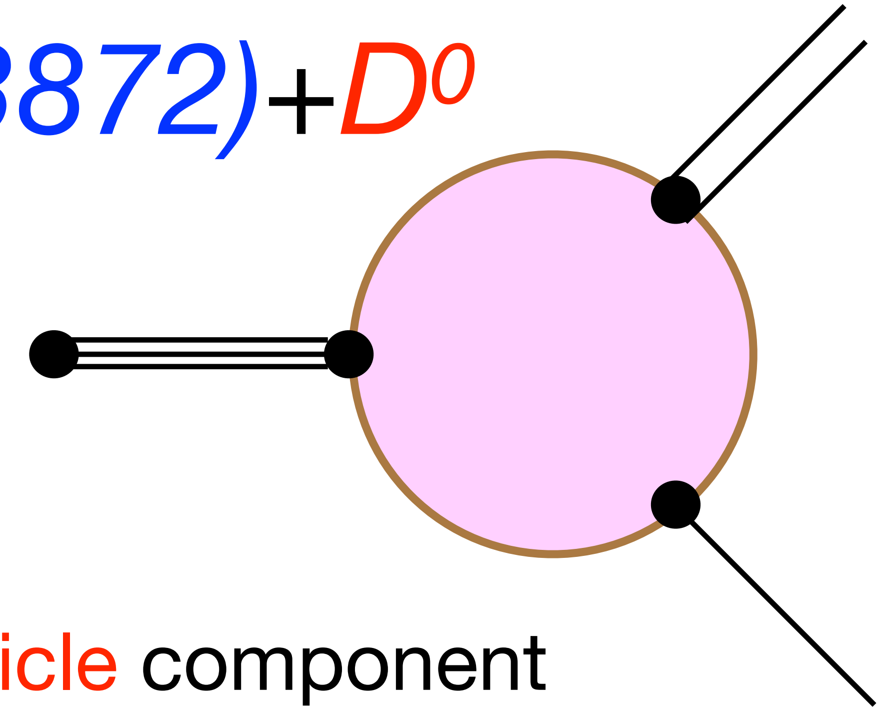
$$D_2(E_2 + i\epsilon, p) - D_2(E_2 - i\epsilon, p) = \frac{1}{\sqrt{E_2 - p^2/2M_2}}\theta\left(E_2 - \frac{p^2}{2M_2}\right)$$

- integrate over phase space: $\int dE_1 d^3p/(2\pi)^4$

behavior near the threshold $E_3 > p^2/2M_{12}$

$$dR \longrightarrow C \left(E_3 - \frac{p^2}{2M_{12}}\right)^{\Delta_2 - 5/2} \left(\frac{p^2}{2M_{12}}\right)^{\Delta_3 - \Delta_1 - \Delta_2} \frac{d^3p}{(2\pi)^3}$$

Point Production of X(3872)+D^0



finite charm-meson pair scattering length

- breaks conformal symmetry
- 3-Unparticle develops bound state + particle component
- 2-Unparticle propagator is modified by bound-state energy ε_X

discontinuity in 2-Unparticle propagator

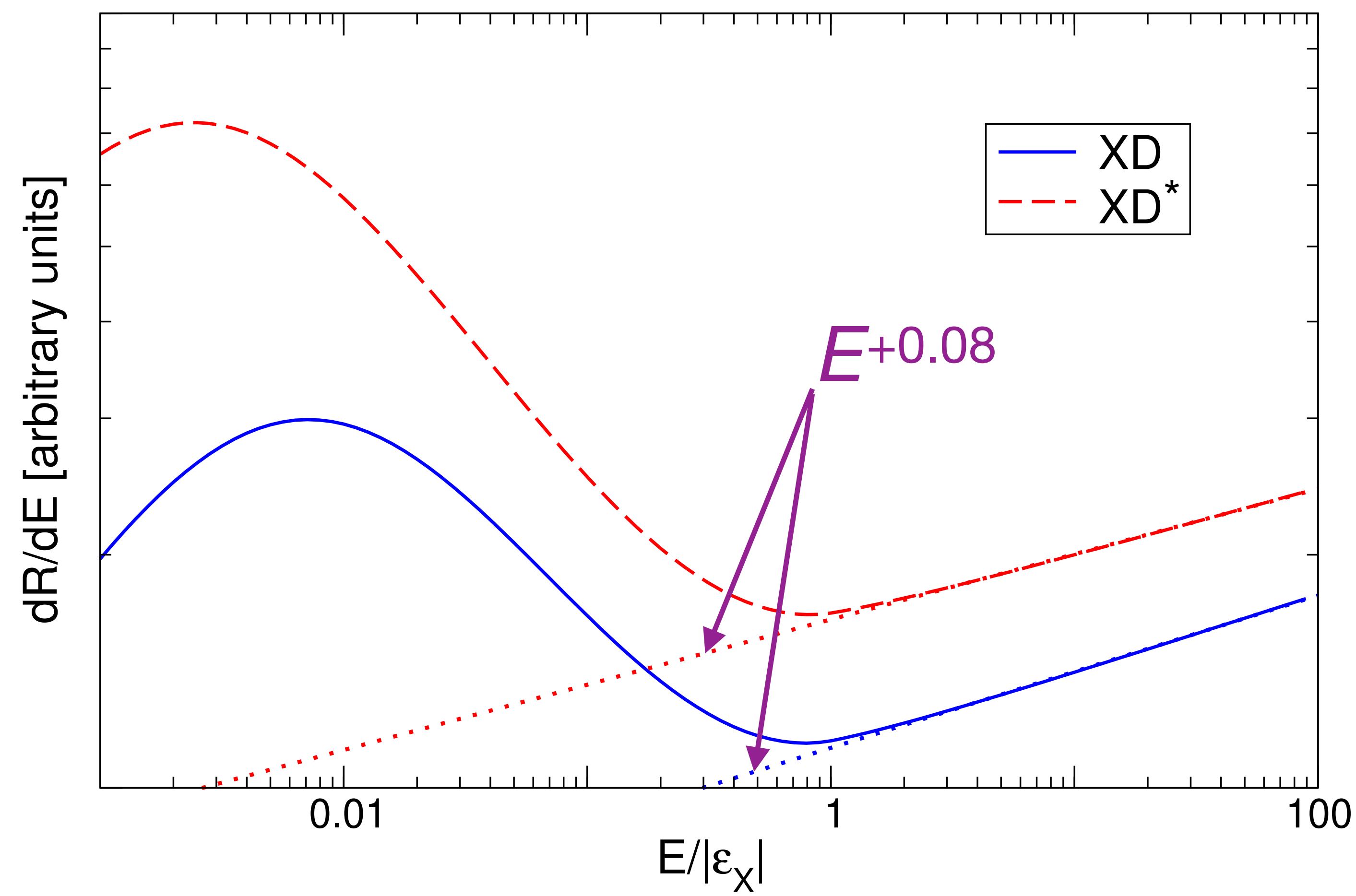
$$D_2(E_2 + i\epsilon, p) - D_2(E_2 - i\epsilon, p) = \frac{\sqrt{E_2 - p^2/2M_2}}{E_2 - p^2/2M_2 + |\varepsilon_X|}\theta\left(E_2 - \frac{p^2}{2M_2}\right) + 2\pi\sqrt{|\varepsilon_X|}\delta\left(E_2 - \frac{p^2}{2M_2} + |\varepsilon_X|\right)$$

rate for producing bound state + particle: keep δ-function term

$$dR \longrightarrow (\text{constant})\,\sqrt{|\varepsilon_X|}\,E_3^{\Delta_3-\Delta_2-\Delta_1-1/2}\,dE_3$$

Point Production of Bound State + Particle

production rate *dR/dE*
as function of energy *E*



dR/dE has power-law behavior at high energy

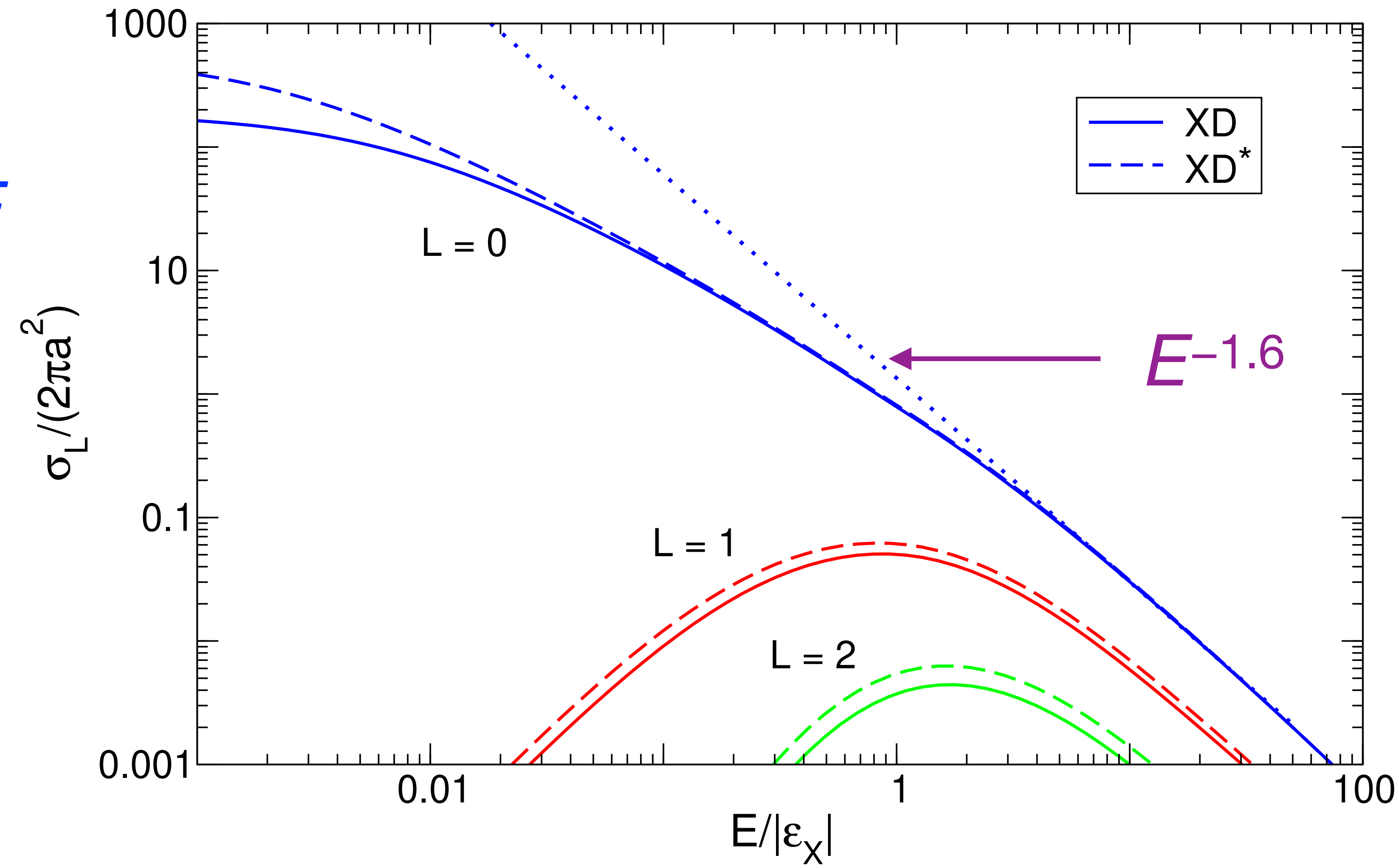
exponent is determined by conformal symmetry !!

$\Delta_3 - \Delta_2 - \Delta_1 + ½ = +0.1012$ for XD^0

$\Delta_{3*} - \Delta_2 - \Delta_1 + ½ = +0.0870$ for XD^{*0}

Scattering of *X(3872)* and Charm Meson

cross sections
as functions of energy *E*



σ(E) has power-law behavior at high energy

is exponent determined by conformal symmetry?

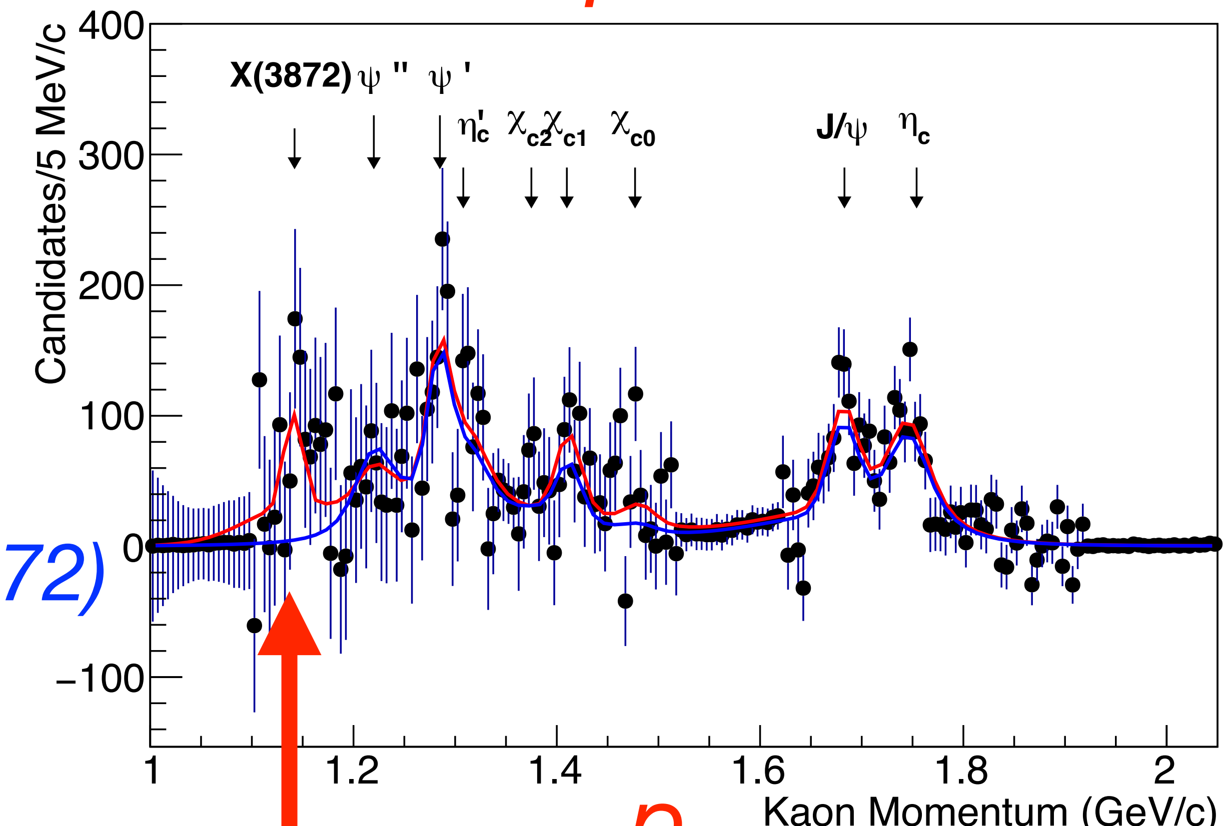
can it be derived from 4-point function $\langle \phi_1(x_1)\phi_2(x_2)\phi_1^\dagger(x_3)\phi_2^\dagger(x_4)\rangle$
in nonrelativistic conformal field theory ?

X Unparticle

Babar collaboration arXiv:1911.11740

inclusive decays $B^\pm \rightarrow K^\pm$ + (anything)

measure distribution of K recoil momentum p



p

transition $B^\pm \rightarrow K^\pm$

creates *X* unparticle

produces D^{*0}+D^0, D^0+D^{*0}, *and X(3872)*

peak from *X(3872)*

above p_{max} = 1141 MeV

peak from D^{*0}+D^0, D^0+D^{*0} below p_{max} = 1141 MeV

power-law behavior determined by $\Delta_2 = 2$ $d\Gamma/dp \sim (p_{\max} - p)^{-1/2}$

can it be observed by Belle II collaboration ?

will collect larger data set by factor of 40

XD and *XD** Unparticles

production of *XD* or *XD** unparticles
requires creation of two $c\bar{c}$ pairs
sufficient rate only for prompt production at Large Hadron Collider

no trigger for events with 3 charm mesons
need $X(3872) \to J/\psi\, \pi^+\pi^-$, $J/\psi \to \mu^+\mu^-$ to provide trigger

<u>*XD* unparticle</u>
observe through $X(3872)$+D^0 component with decay $D^0 \to K^-\pi^+$
production rate with energy dependence $E^{+0.1012}$

can it be observed by LHCb collaboration ?

Summary

Nonrelativistic Unparticle

excitation created by an operator with definite scaling dimension
in a Nonrelativistic Conformal Field Theory

Low-energy neutrons are Unparticles !

because dineutron is almost bound   Hammer & Son arXiv:2103.06290

Low-energy neutral charm mesons are Unparticles !

because of *X(3872)* resonance   Braaten & Hammer arXiv:2107.03821

X unparticle: can be observed through *K* recoil momentum distribution
in inclusive decays $B^{\pm} \to K^{\pm}$ + (anything)

XD unparticle: may be observable
in prompt production of *X(3872)* D^0 at LHC

Can Unparticle behavior be observed in ultracold atoms ??