A closer look at universality in Efimov physics

Eric Cornell, JILA (NIST and University Colorado Physics) Boulder

What is “universality”?

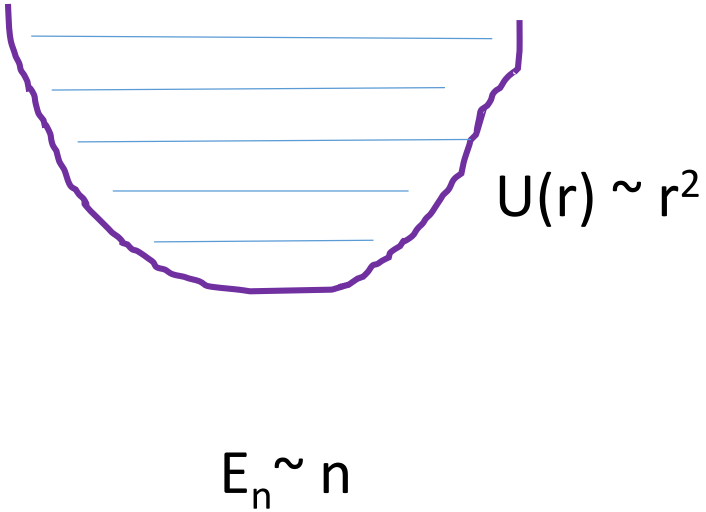
In many-body physics, fairly specific set of ideas, scale invariance, critical exponents etc.

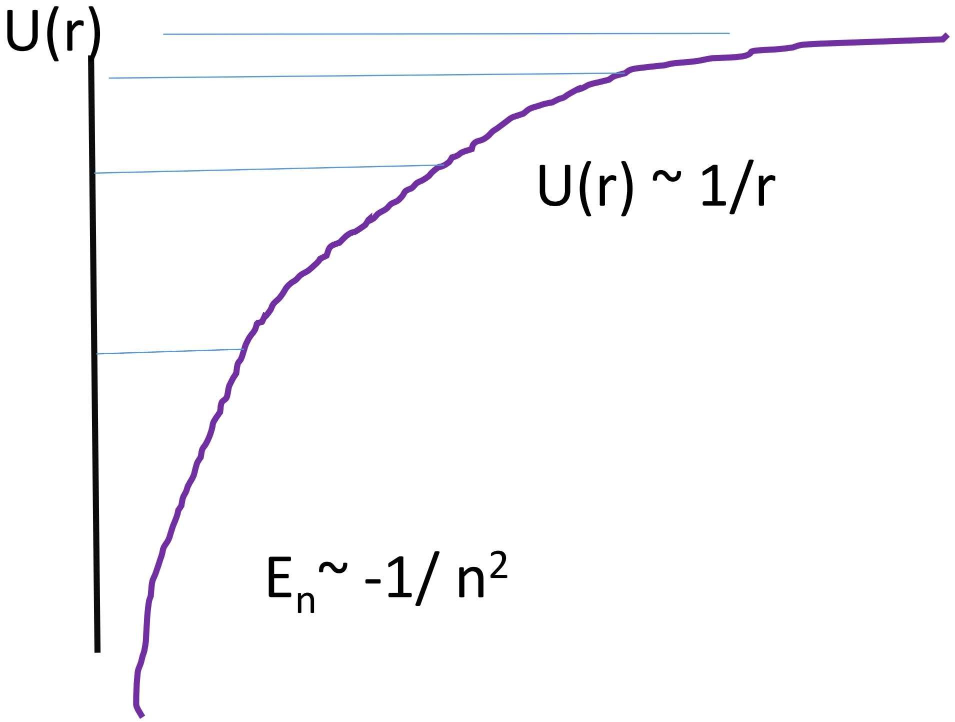
In *few-body physics*, “universality is in the eye of the beholder”

Generically, “universality” means you can “explain a lot with a little” except*

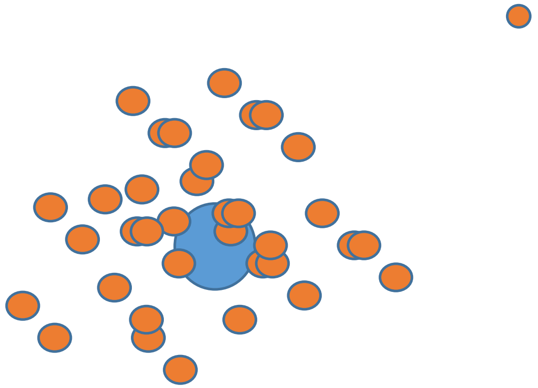
*except “explain a lot with a little” sounds pretty much like the defining feature of ALL physics!!

Few- or even two- body universality?





My own prejudice: this is not what we mean when we say universality…



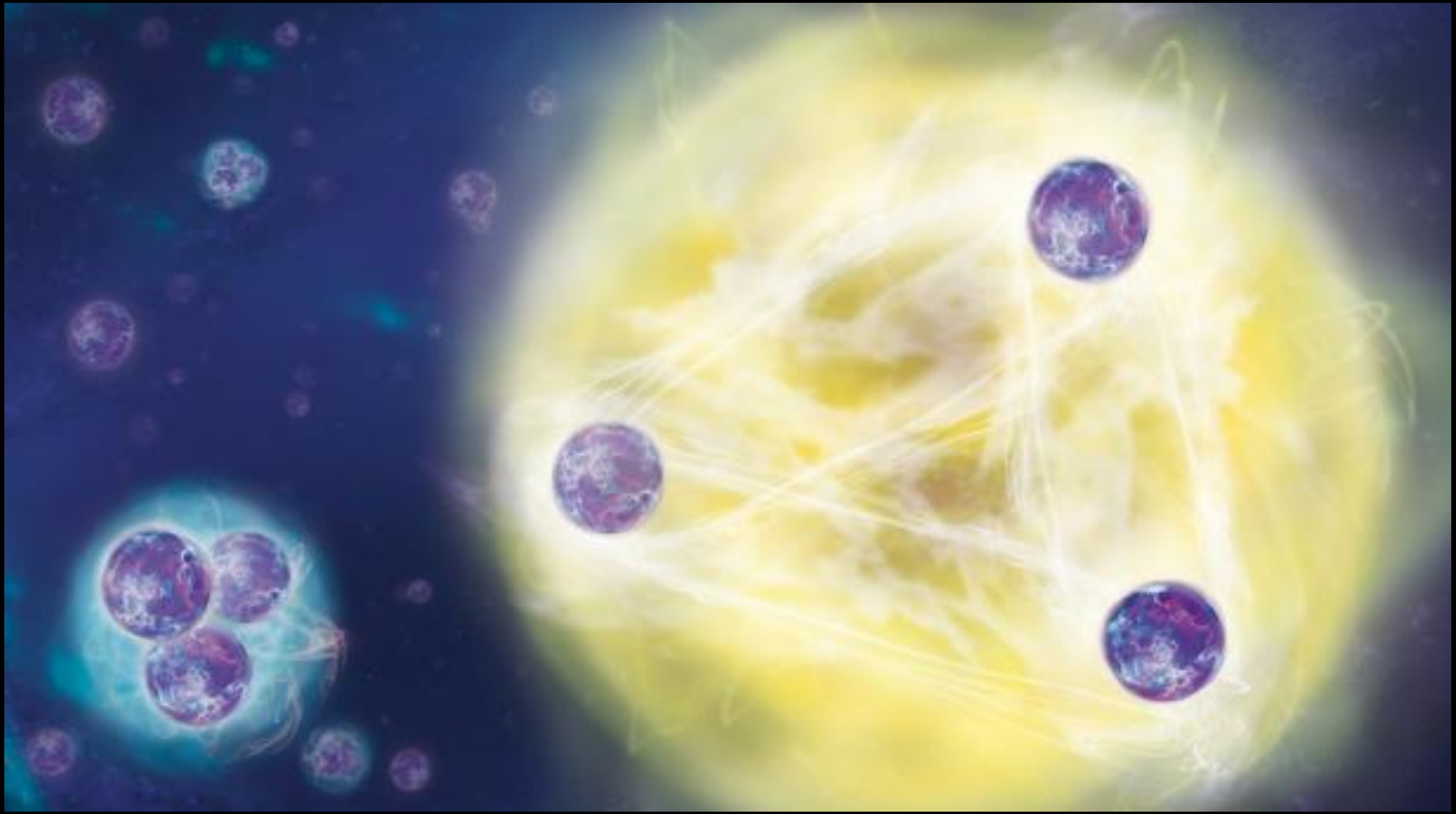
Many-electron atom.
Electron-ion coupling,
Strong electron-electron coupling
Fermi statistics, exchange
Very complicated!!!

$E_{n,l} = - 1/n^2$?   No.

$E_{n,l} = - 1/(n - \delta_l)^2$ Yes!

Old-school atomic physics

My opinion: Quantum defect theory is “few-body universality”



Efimov trimer – typical length ~1000 "conventional" 3-atom molecule.
Typical volume 10^9 larger.
Insensitive to "chemistry" (i.e., short-range inter-atomic physics)

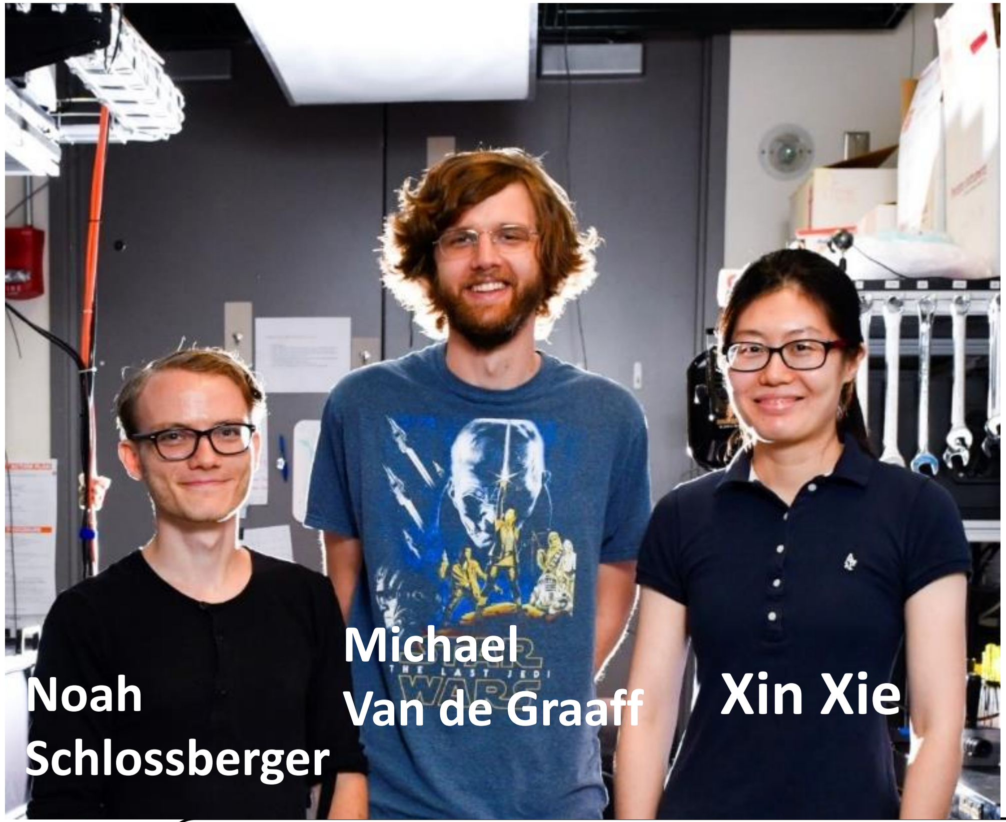
Definitely an example of few-body universality!!

In this talk I will simplify focus on three identical bosons, meaning I will not cover beautiful work of e.g. Cheng Chin

Efimov Structure: Connections Across the Singularity?

Precision studies of universal ratios in three-body physics

JILA, University Colorado/NIST



Theoretical Collaborators

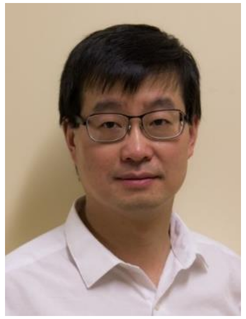
José D'Incao	(JILA)	Jeremy Hutson	(Durham)
Paul Julienne	(JQI)	Matthew Frye	(Durham)

Funding Agencies

Marsico Research Chair

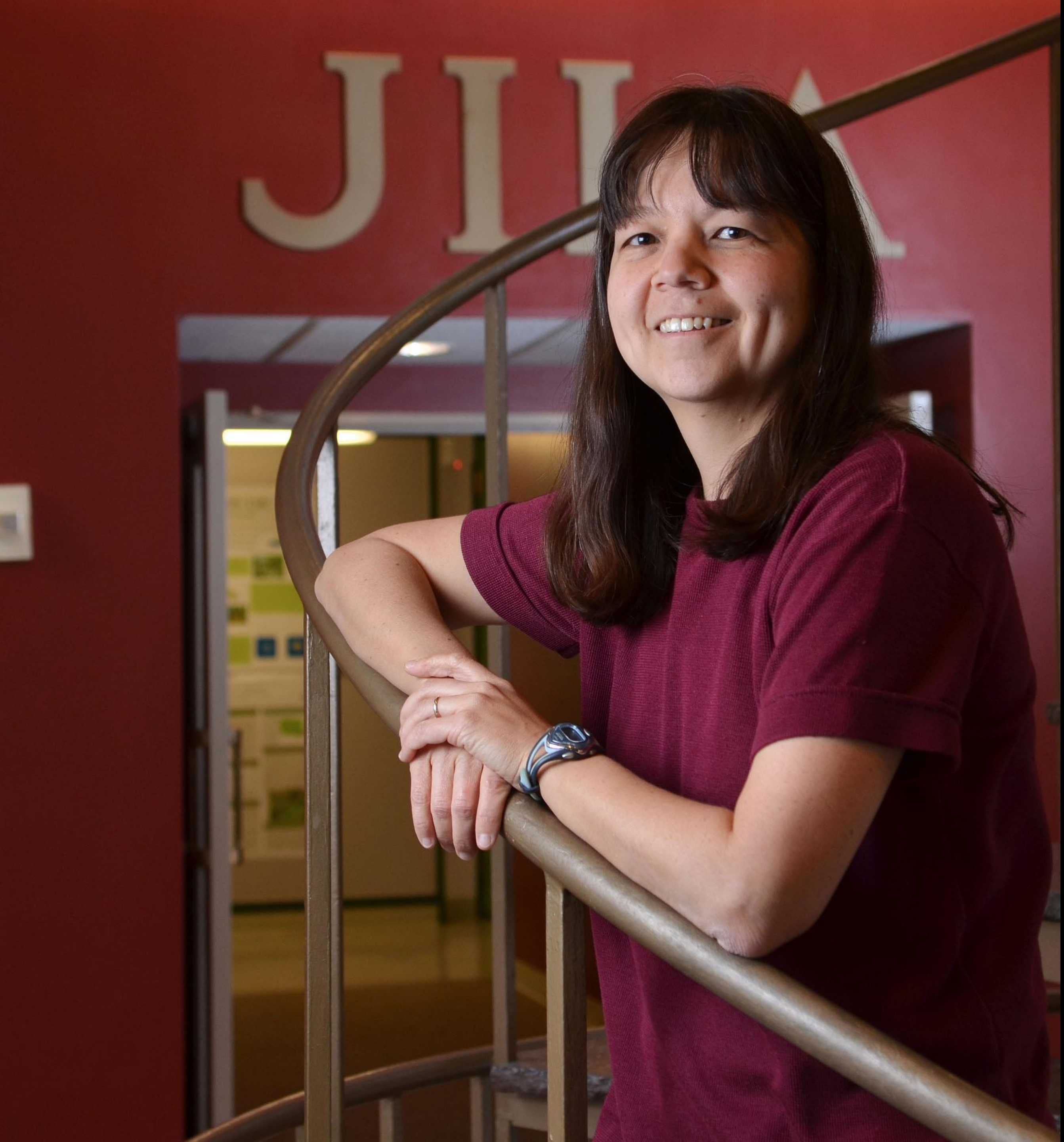

Debbie Jin


Eric Cornell


Jun Ye

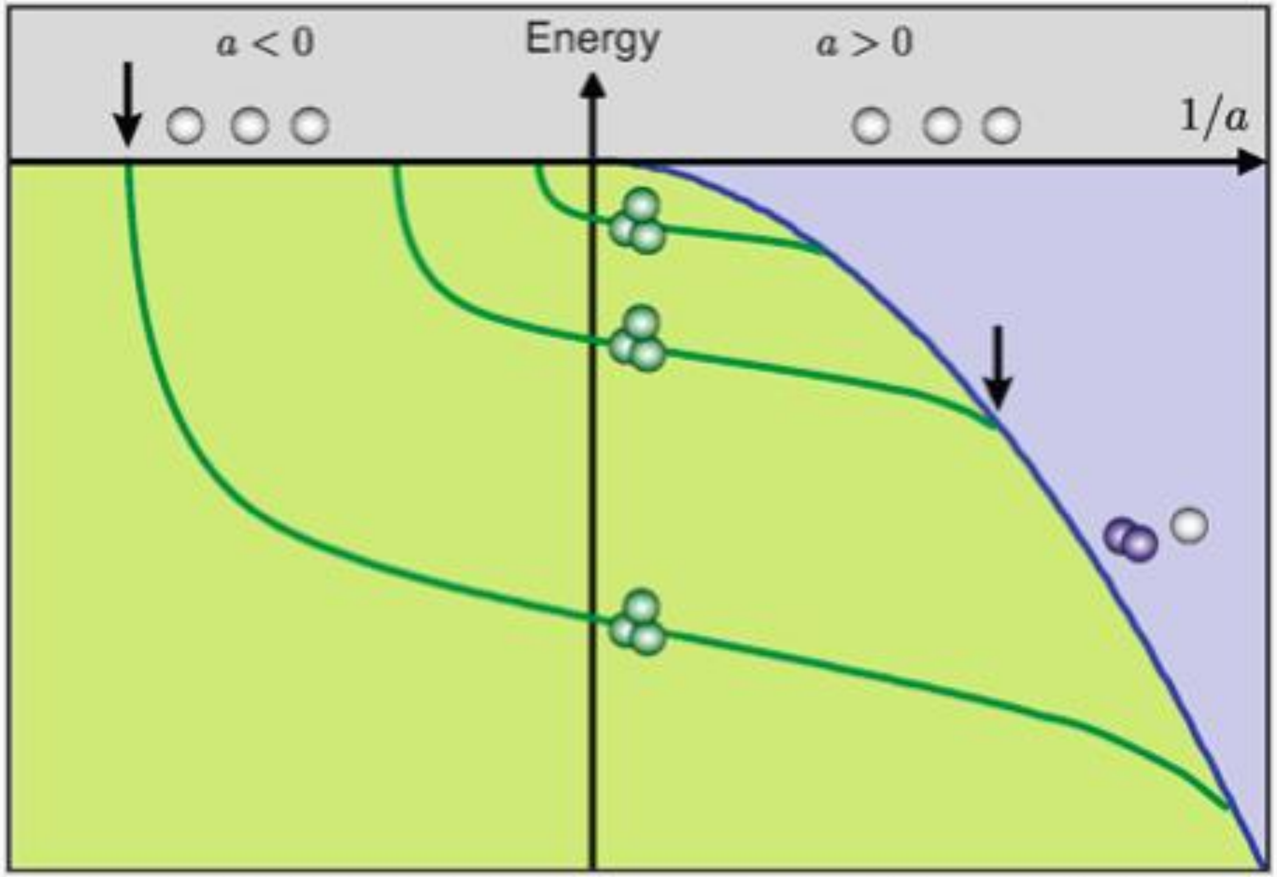

José D'Incao

and Roman Chapurin!!



Deborah S. Jin
1968 -- 2016

Also
Matthew
Frye
and Jeremy
Hudson!



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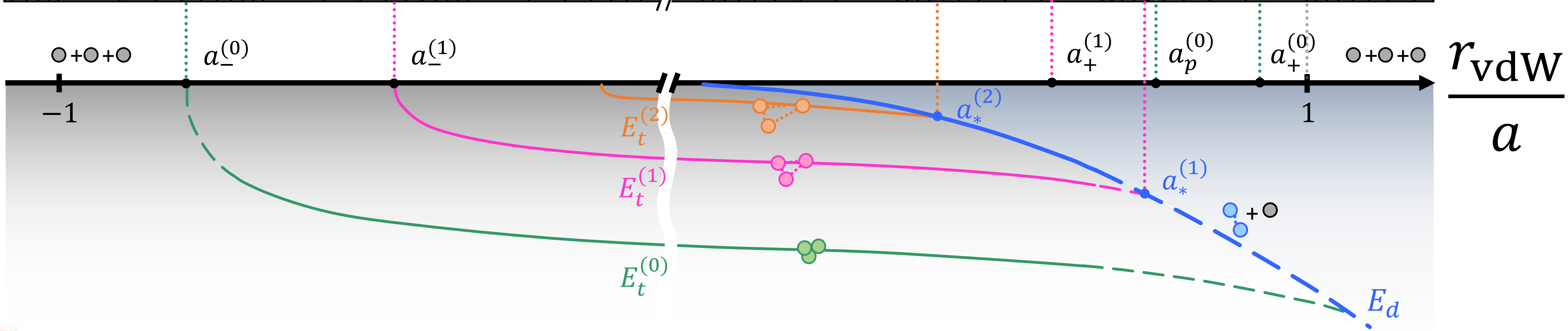
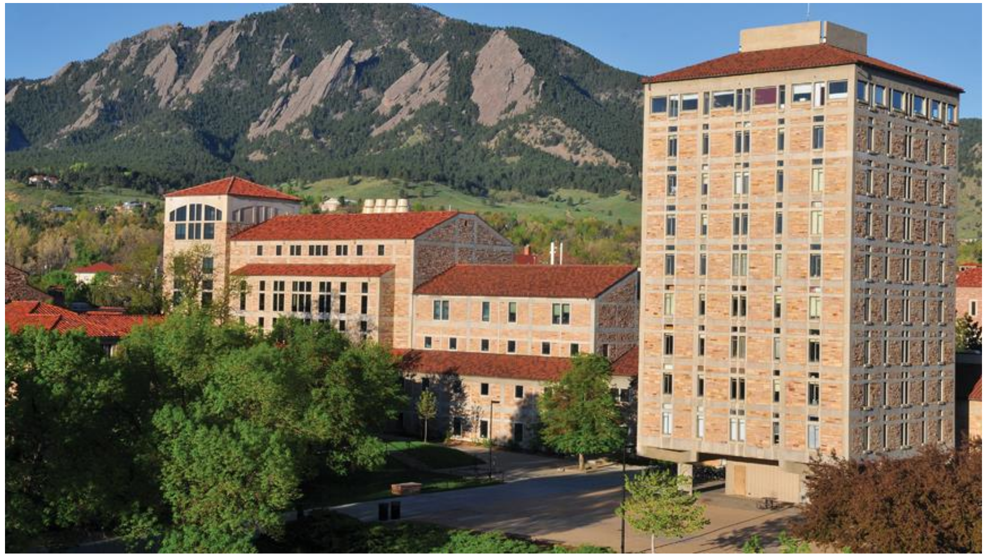
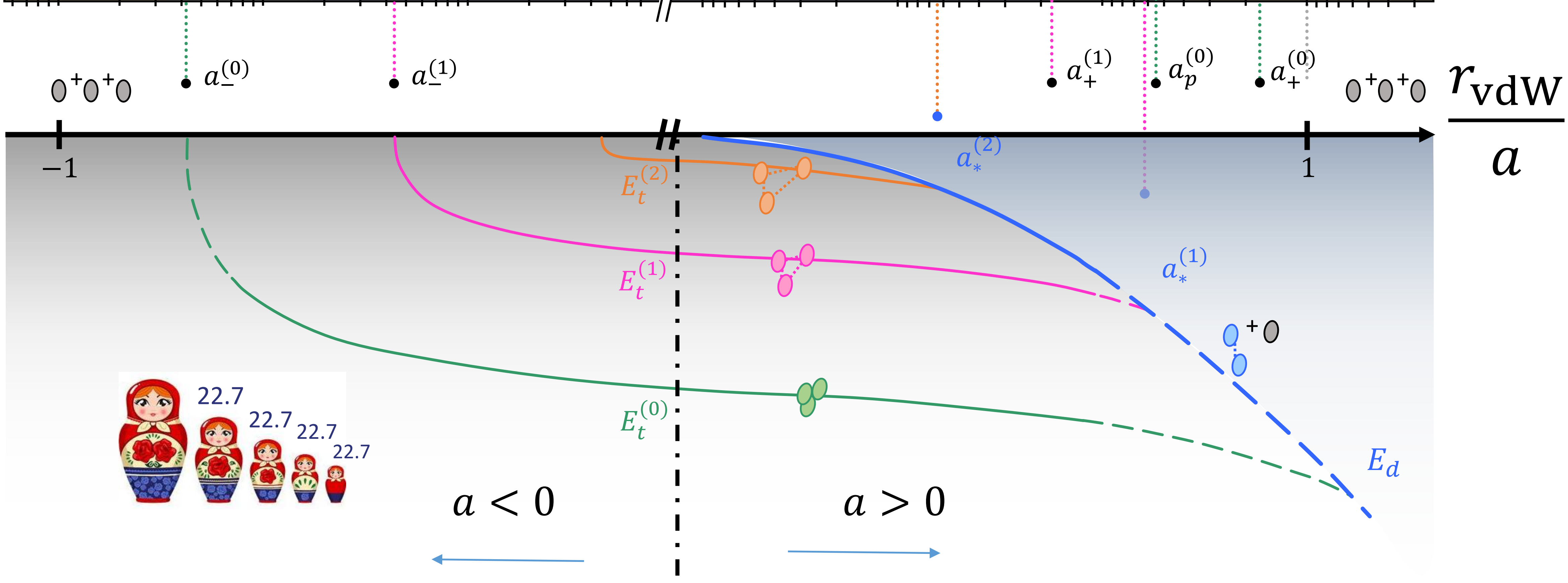
From Ferlaino and Grimm (Phys. Today, 2010)



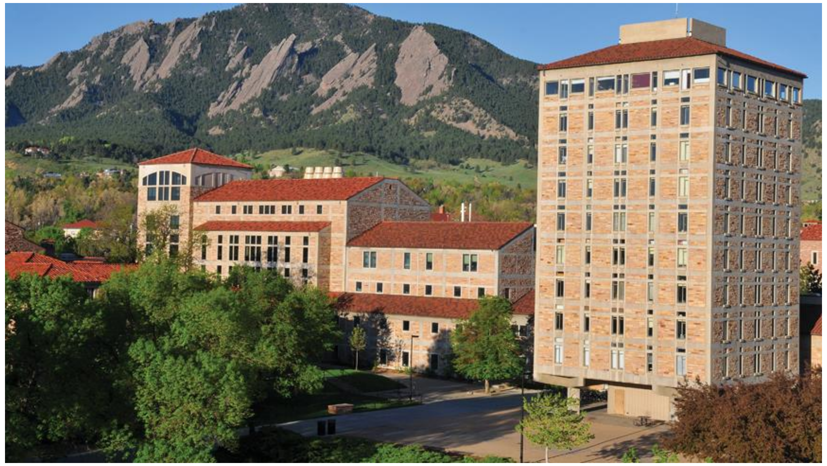
$a < 0$ $a = \pm\infty$ $a > 0$



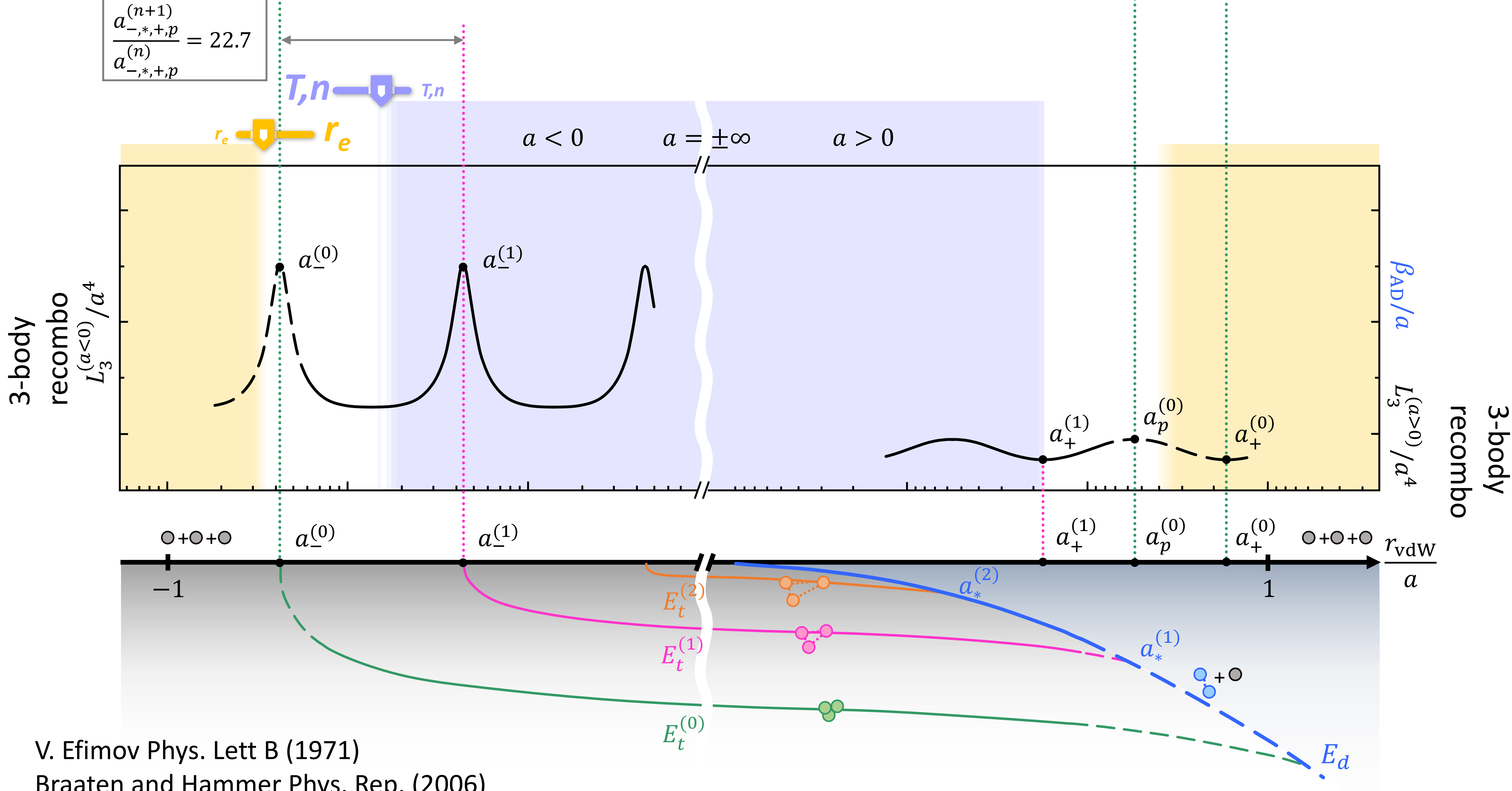


1/a = 0

$a = \pm\infty$

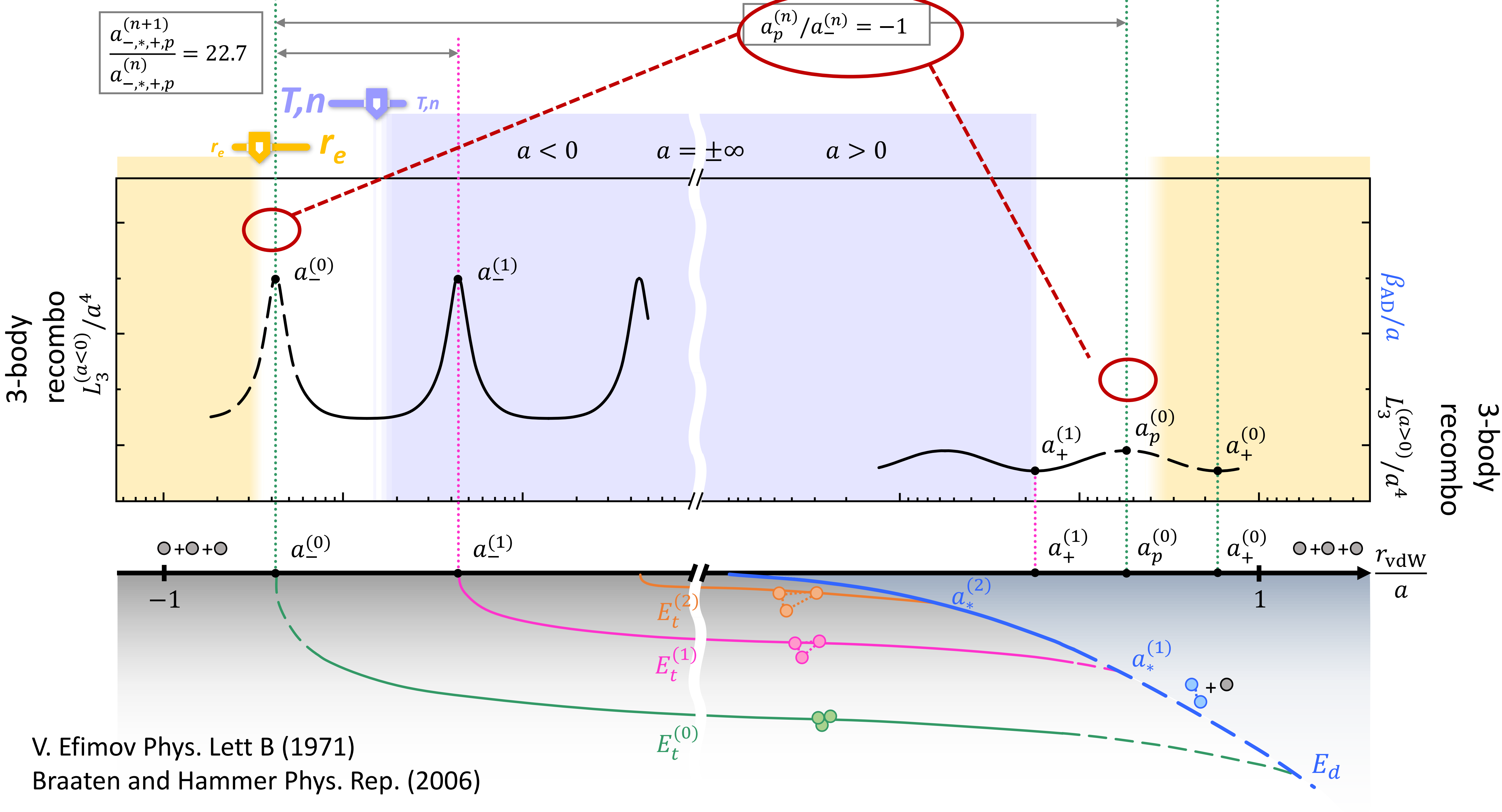


Ferlaino…Grimm
arxiv: 1109.1909



V. Efimov Phys. Lett B (1971)

Braaten and Hammer Phys. Rep. (2006)



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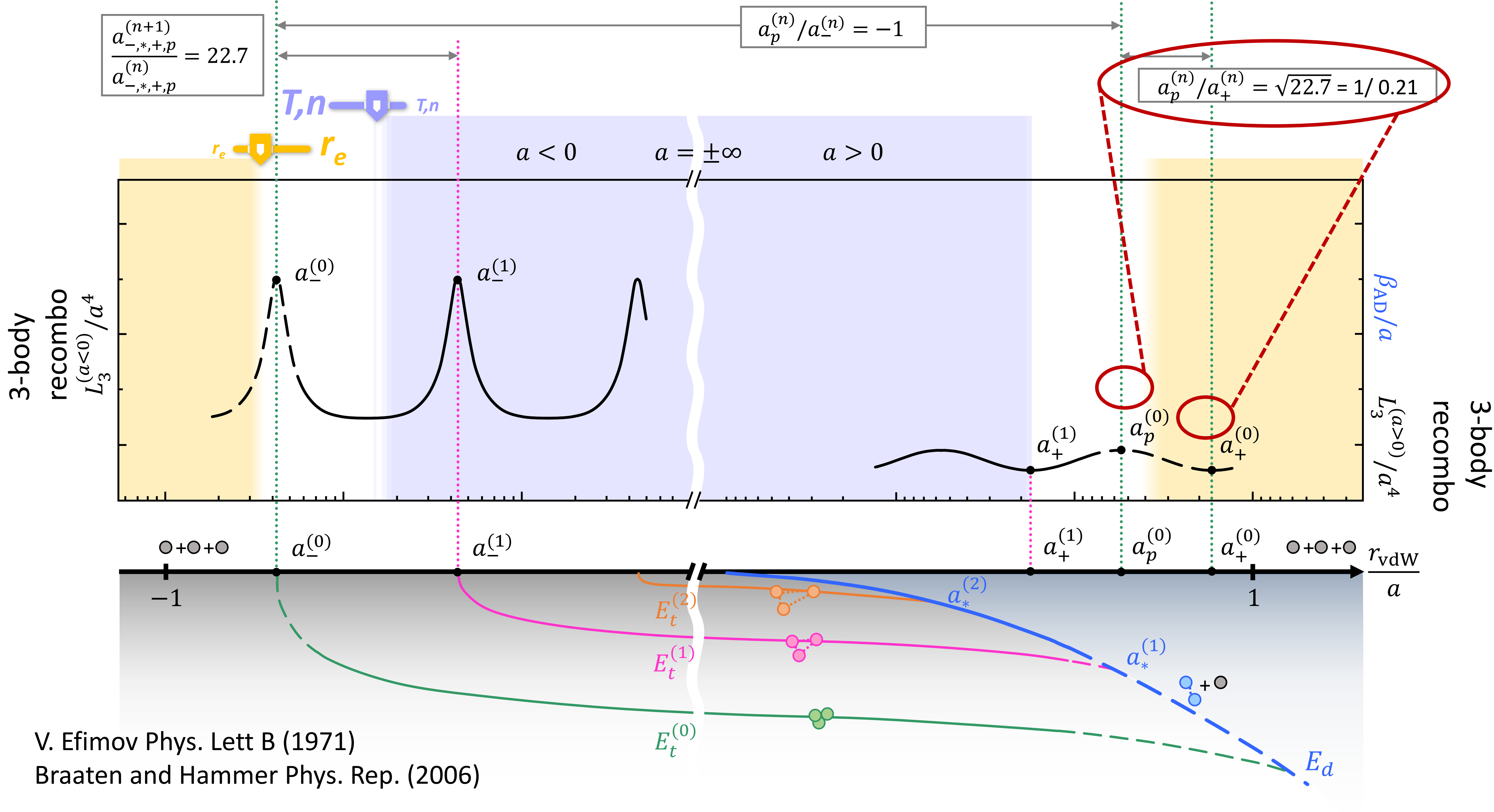

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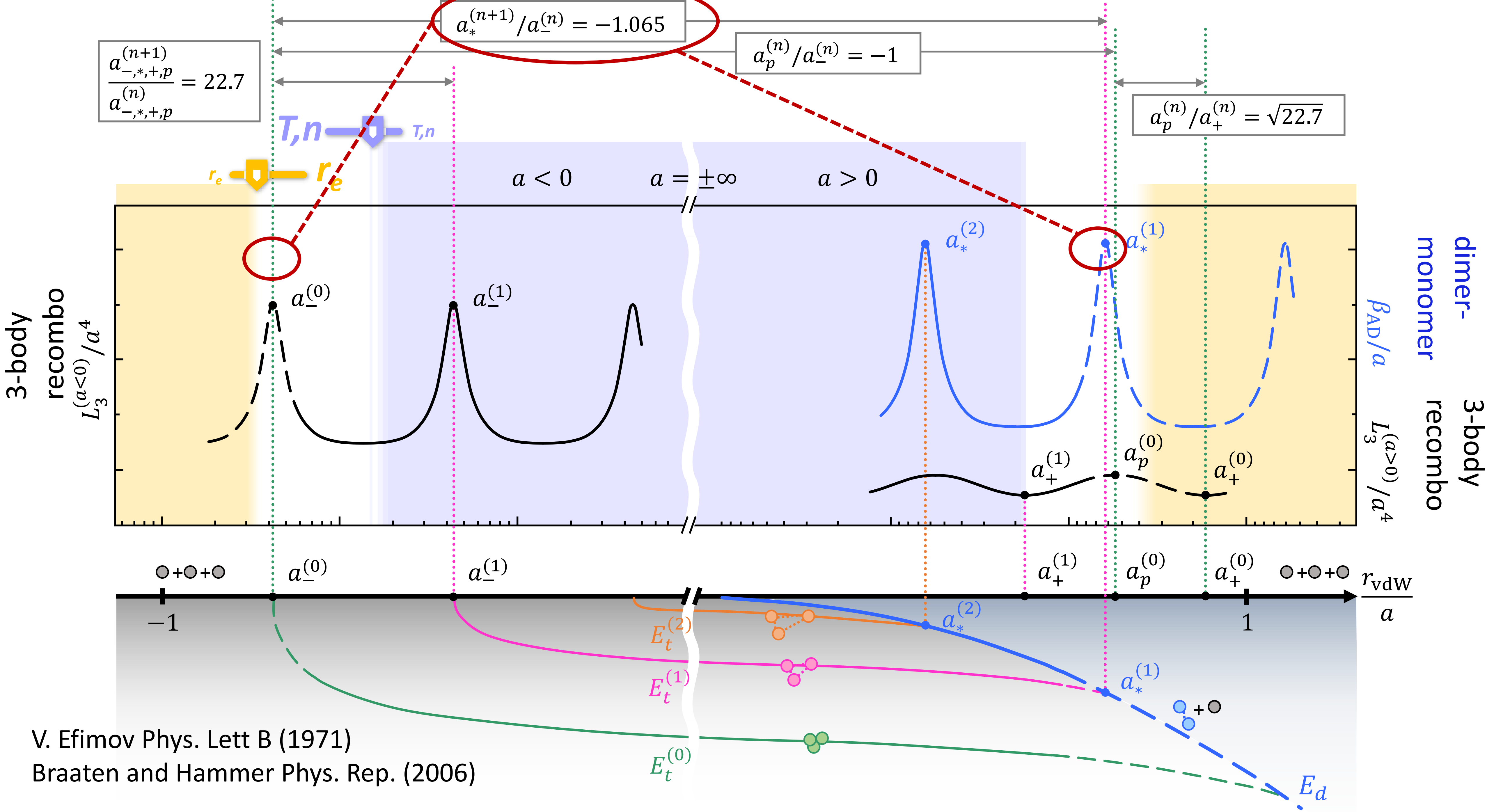

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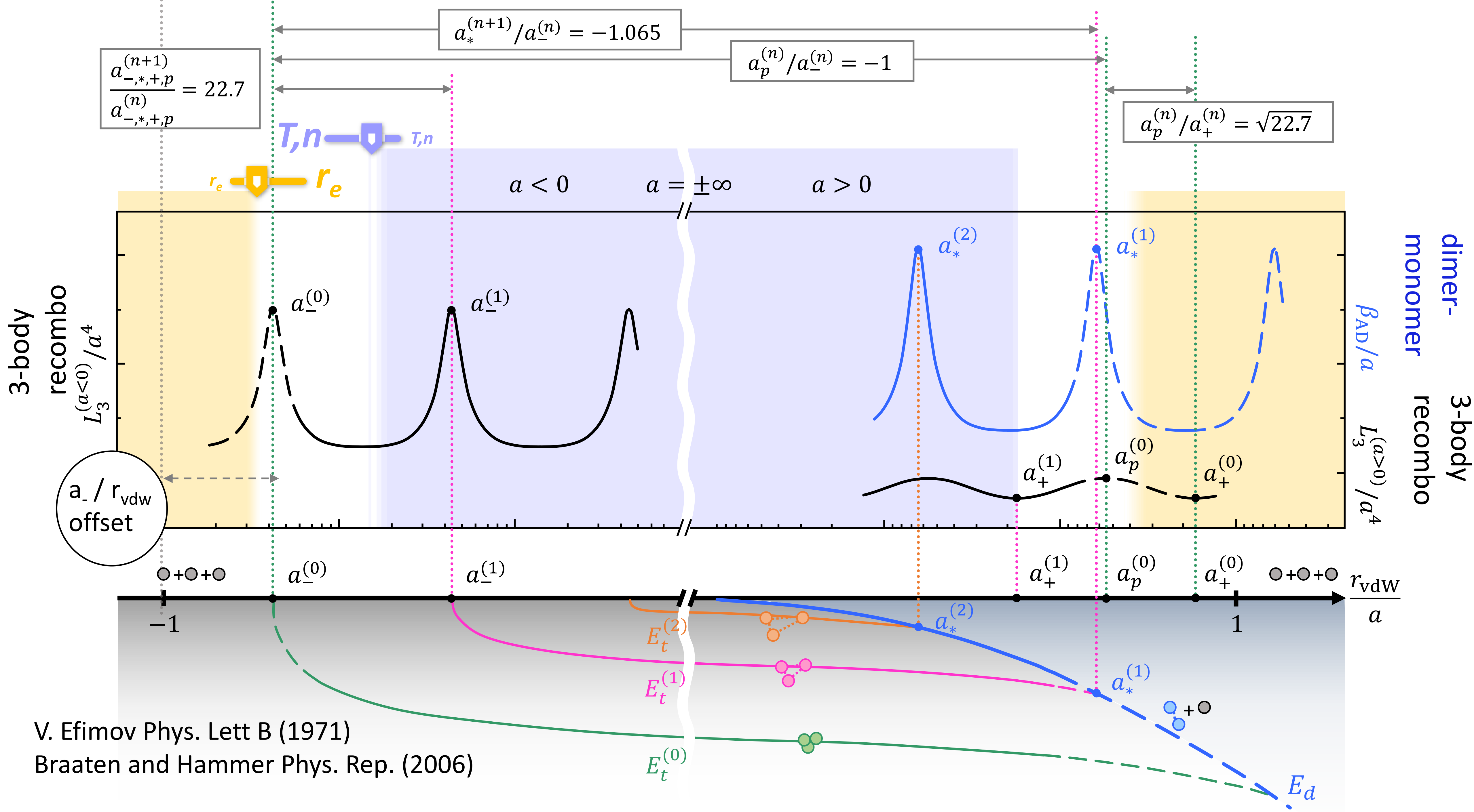

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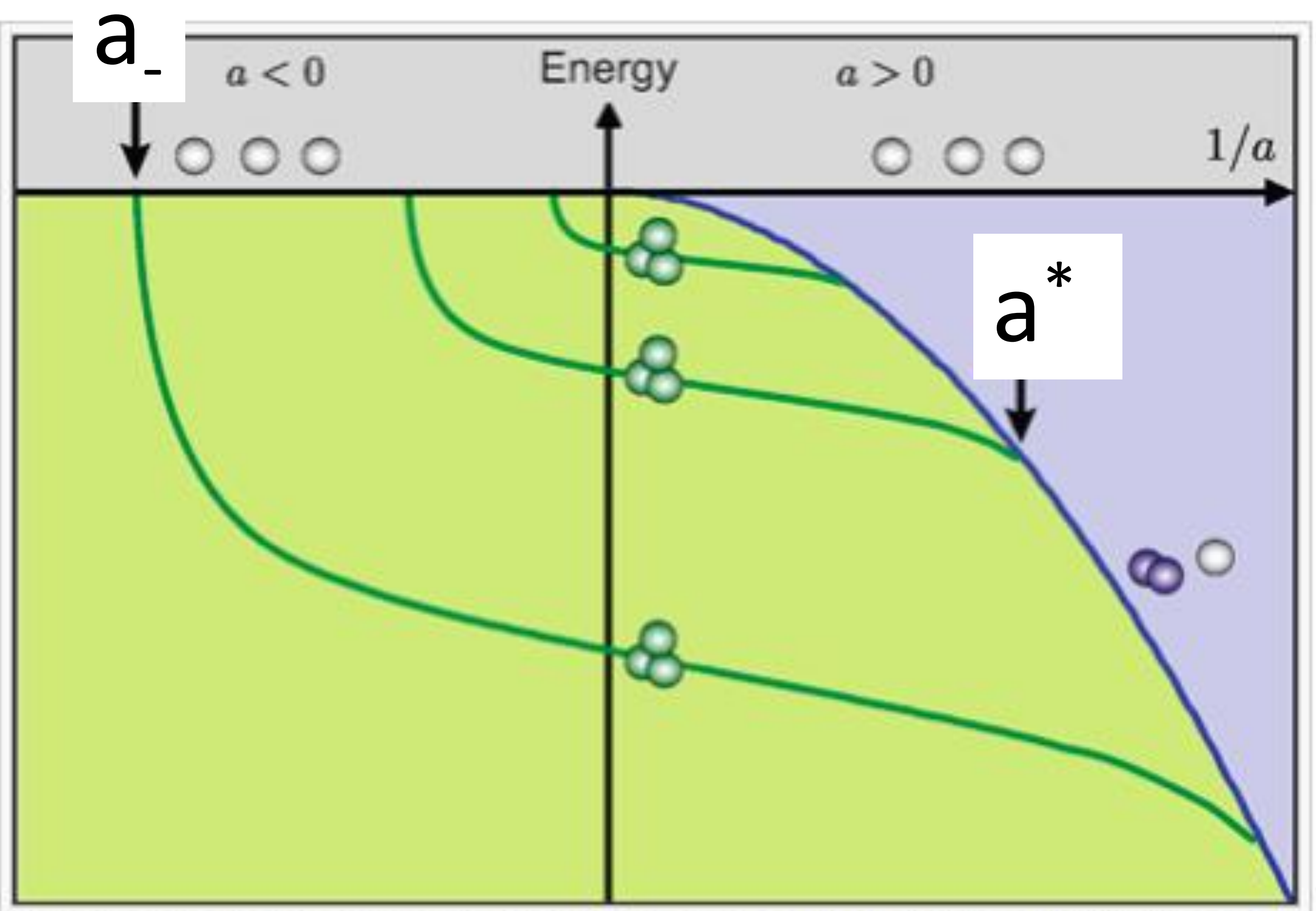
Realistically, how much of this “infinite structure” can one see?



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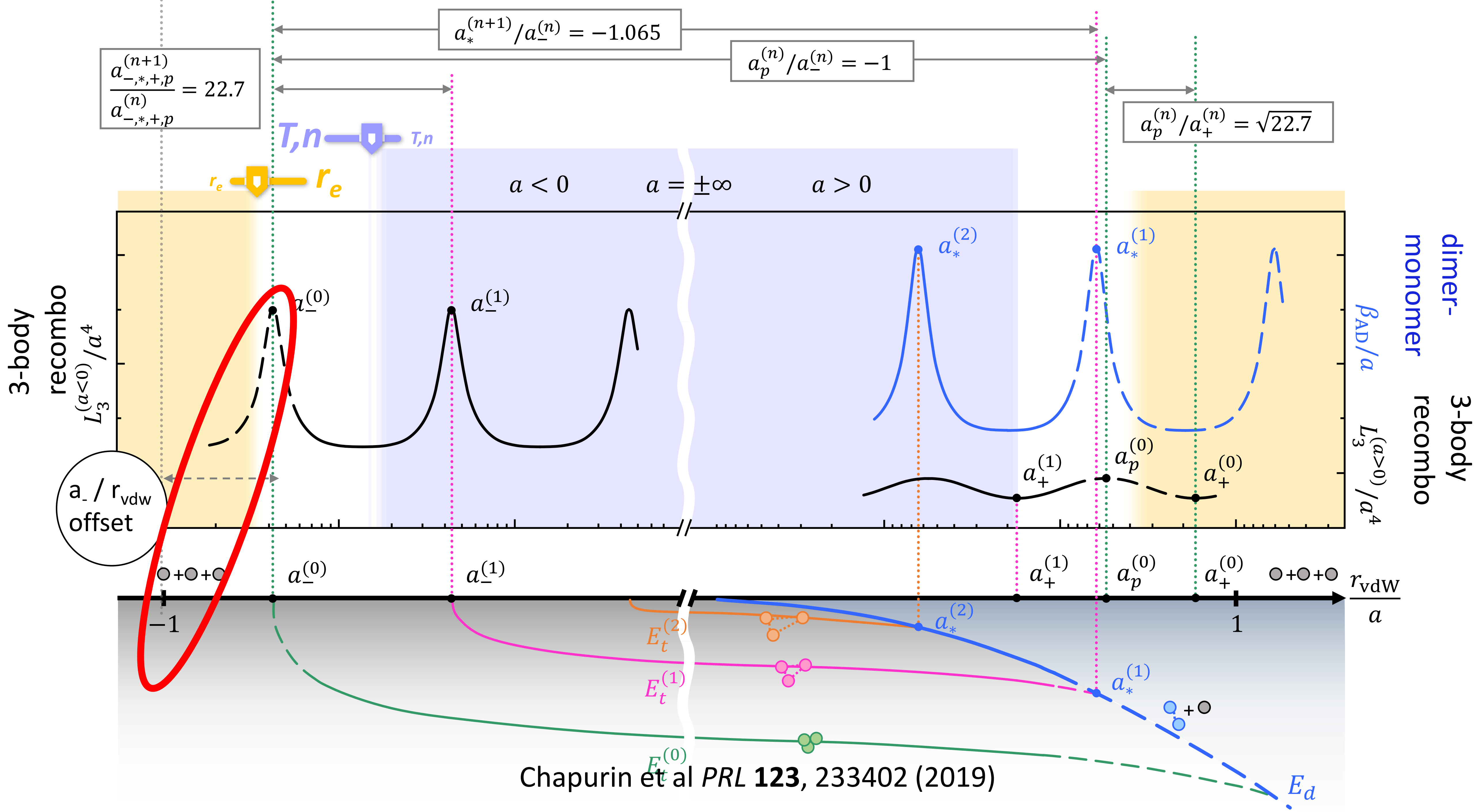
“infrared problem”
(experiment fails)
Higher-order states are too large, too weakly bound, to see in experiment!

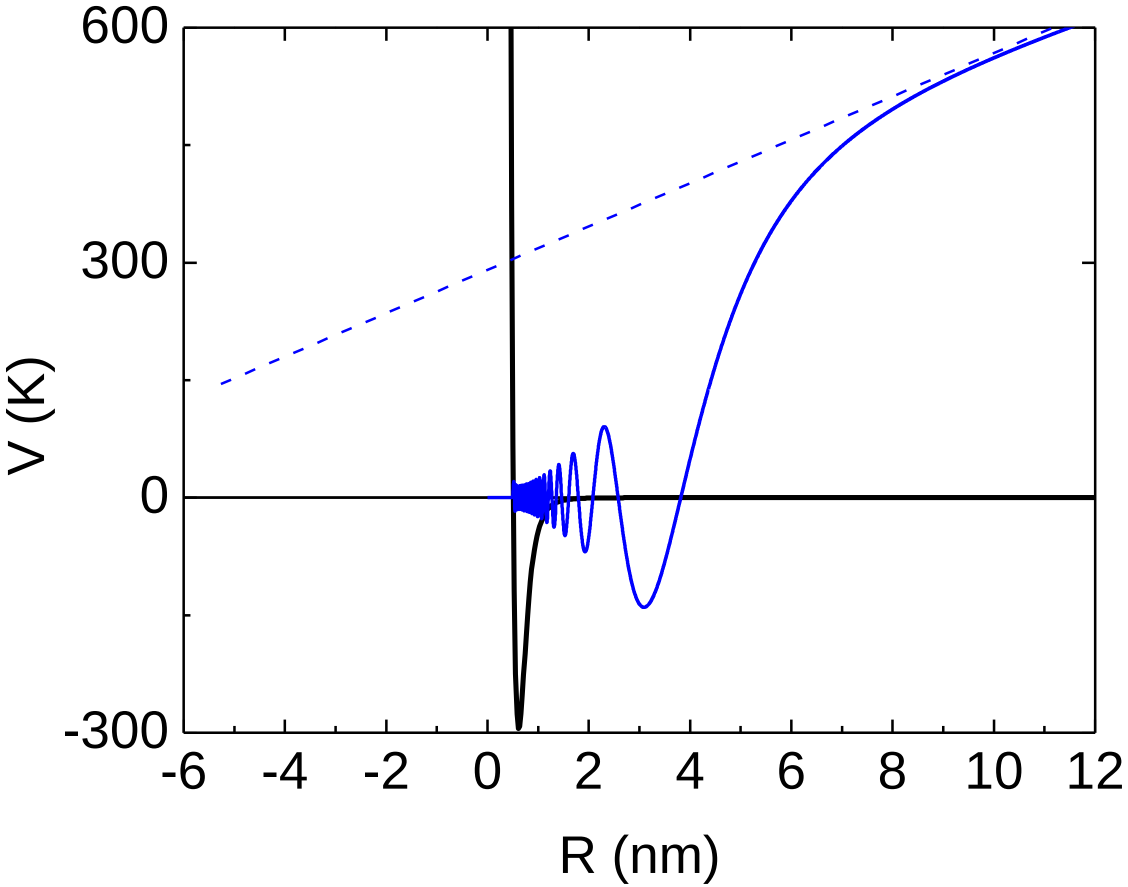
“ultraviolet problem”
(theory model fails)
Short length scales, high binding energies probe system-dependent short-range physics



Chapurin et al *PRL* **123**, 233402 (2019)

What is r_{vdw}?

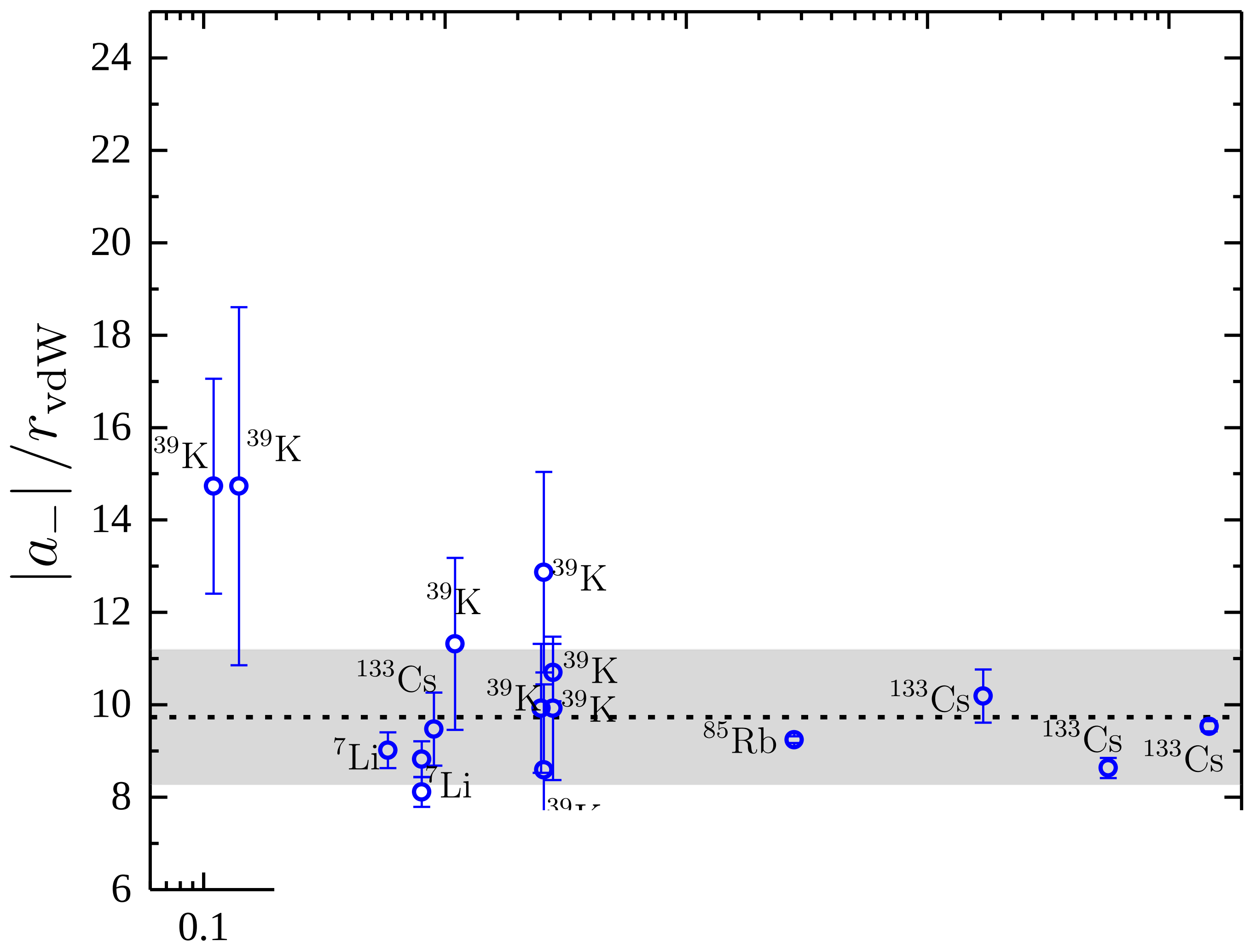
$r_{vdw} \sim (m\, C_6 / \text{hbar}^2)^{1/4}$



The longest-range interaction is a $1/r^6$ “van der waals” so weak that it is not visible in this plot. The “van der waals length” characterizes its strength.

Roughly speaking, the scattering length “a” is where, from a distance, it *looks like the* w.f. will cross zero. r_{vdw} is where it *really does* cross zero

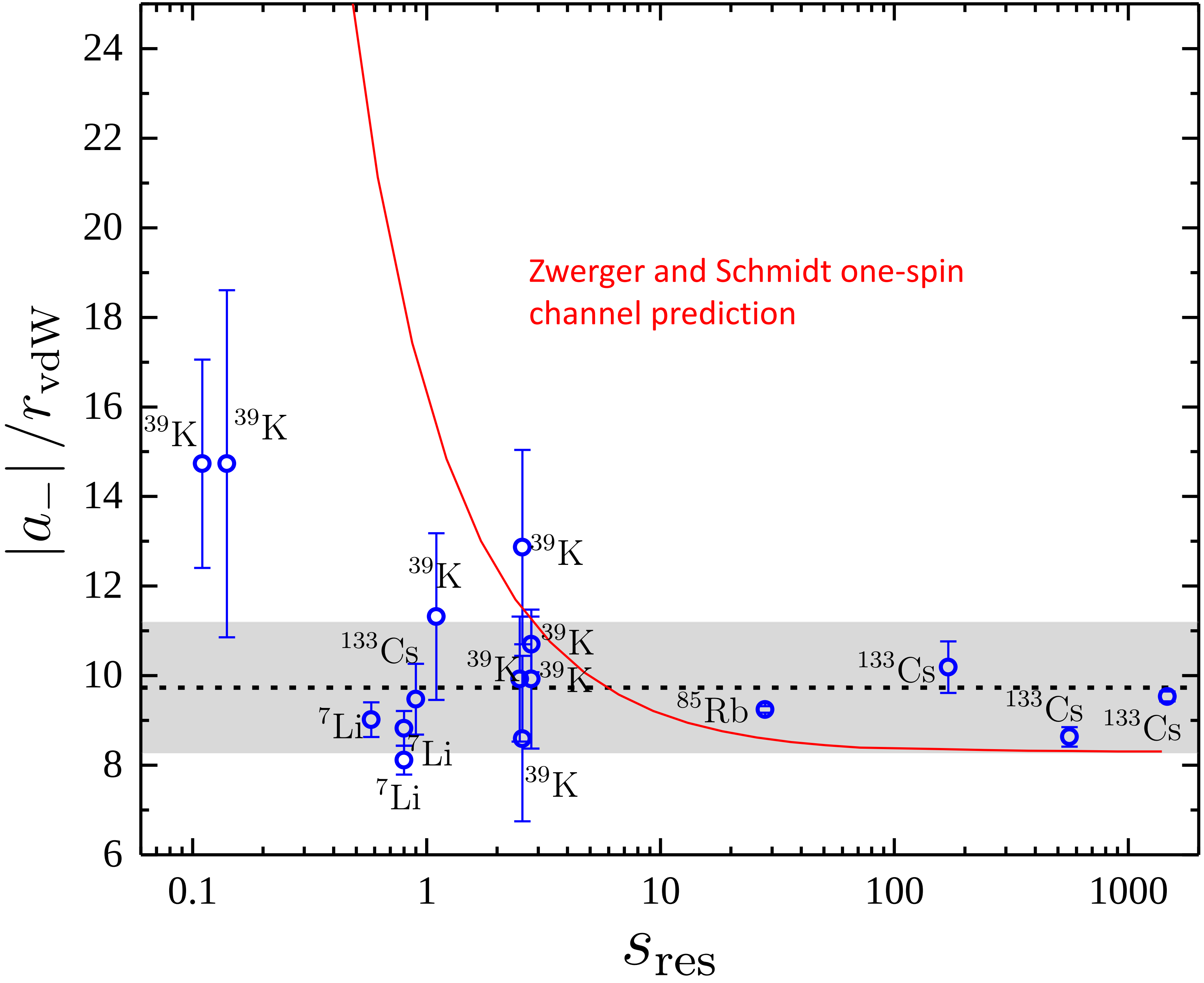
VdW Universality



Data:
Innsbruck,
LENS
Rice
JILA
Bar-Ilan Univ.
Aarhus

Figure style follows Chin group, Chicago.

Theory ideas from Chris Greene, Cheng Chin, Jose d'Incao…



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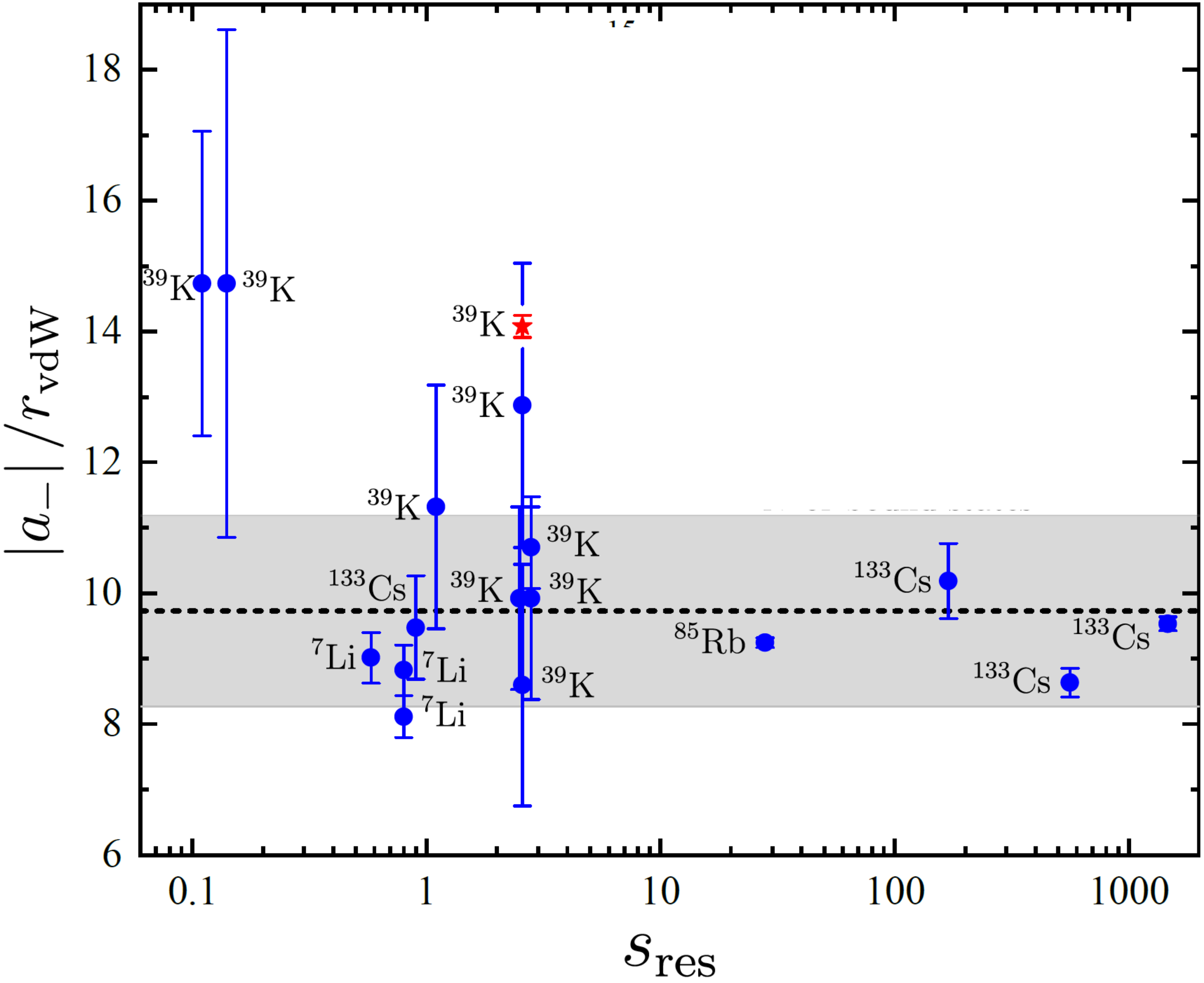
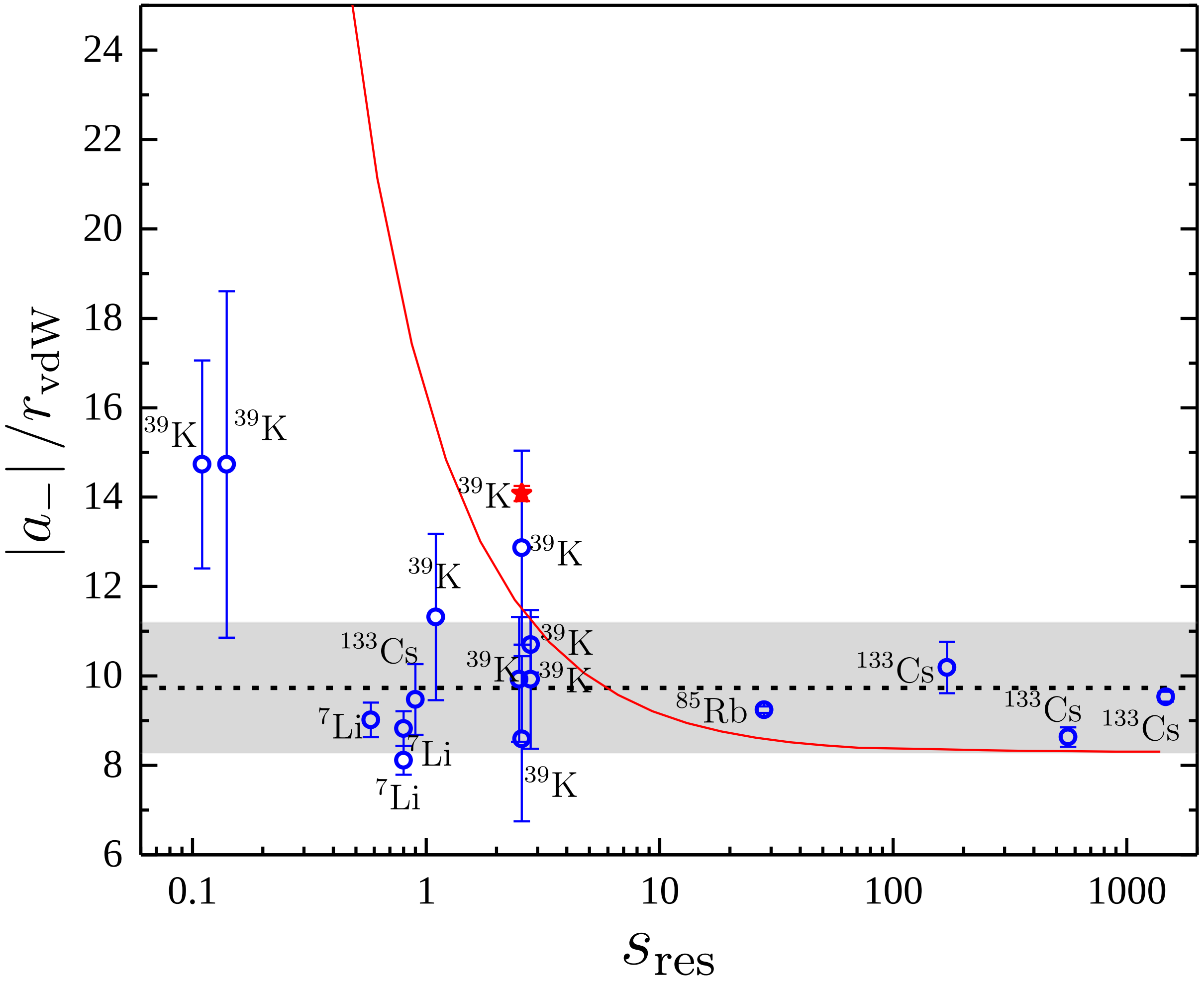

$|a_-|/r_{\mathrm{vdW}}$
s_{res}
6
8
10
12
14
16
18
0.1
1
10
100
1000
^{39}K
^{39}K
^{39}K
^{39}K
^{39}K
^{39}K
^{39}K
^{39}K
^{39}K
^{133}Cs
^{7}Li
^{7}Li
^{7}Li
^{85}Rb
^{133}Cs
^{133}Cs
^{133}Cs

In 2017 we set out to bring a “precision metrology” mind-set to few-body physics, emphasis on

“precision” = small error bars

“accuracy” = making sure the real answer is inside those error bars!

Keep track of effects of finite T, finite n
avoid misidentified peaks through absolute density metrology
calibrate a(B)



Data:
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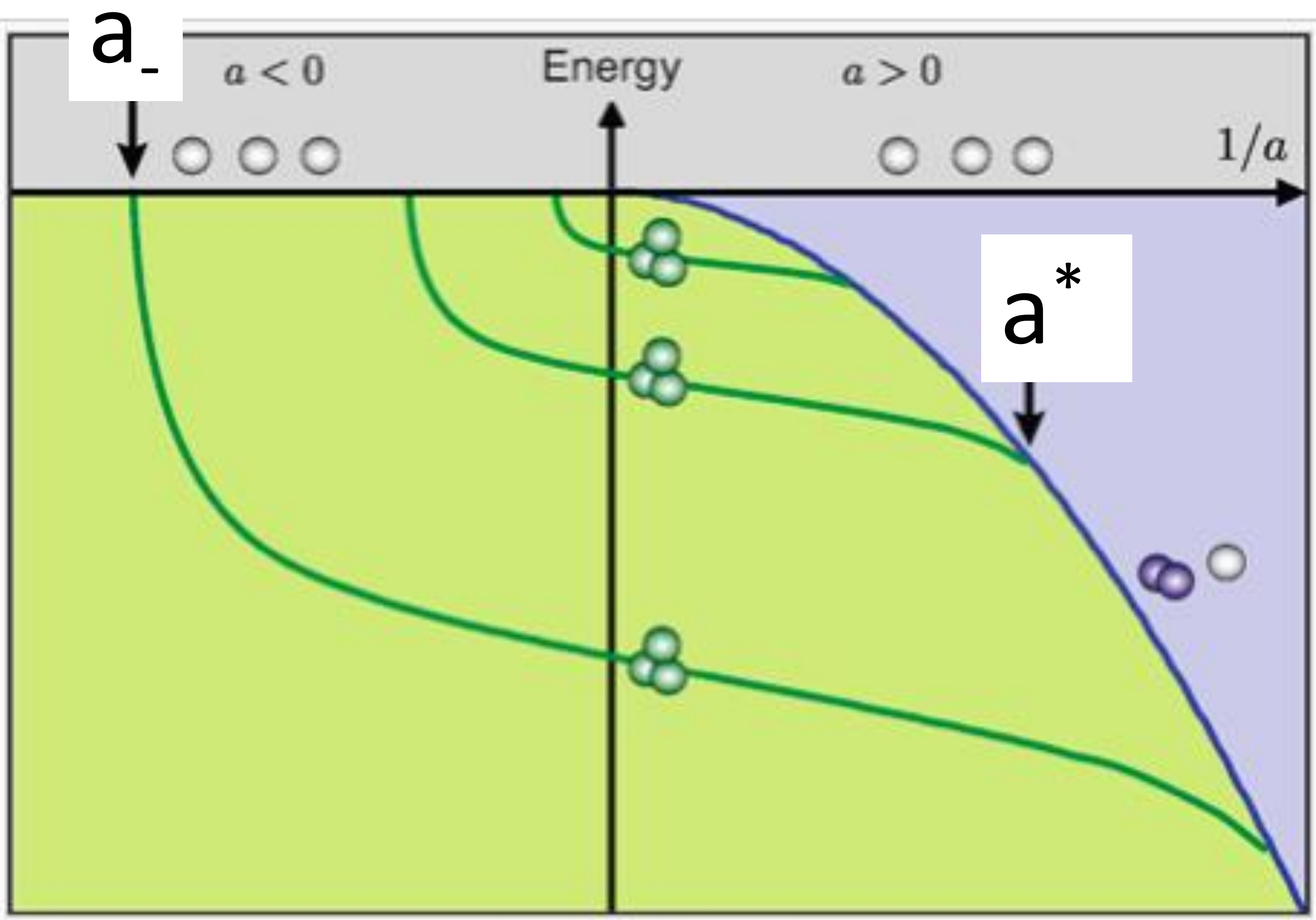
Figure style follows Chin group, Chicago.

Zwerger/Schmidt model in red

2 complications: short-range ambiguity.

Definition of S_{res}

Realistically, how much of this “infinite structure” can one see?



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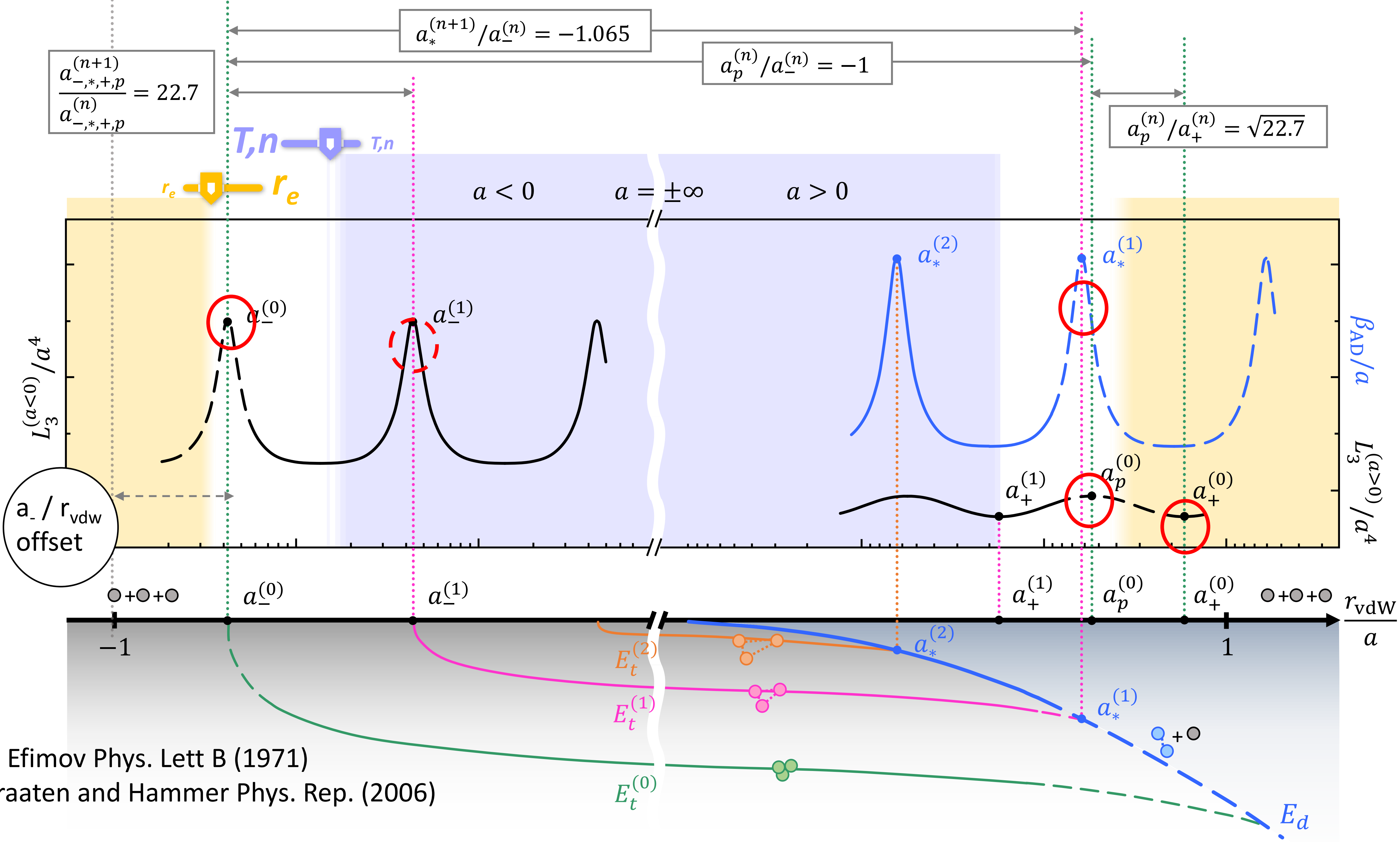
In mixed-atom efimov physics, Cheng Chin saw multiple cycles of efimov levels.

In the three-identical-boson situation…..



$$a_*^{(n+1)}/a_-^{(n)} = -1.065$$

$$\frac{a_{-,*,+,p}^{(n+1)}}{a_{-,*,+,p}^{(n)}} = 22.7$$

$$a_p^{(n)}/a_-^{(n)} = -1$$

$$a_p^{(n)}/a_+^{(n)} = \sqrt{22.7}$$

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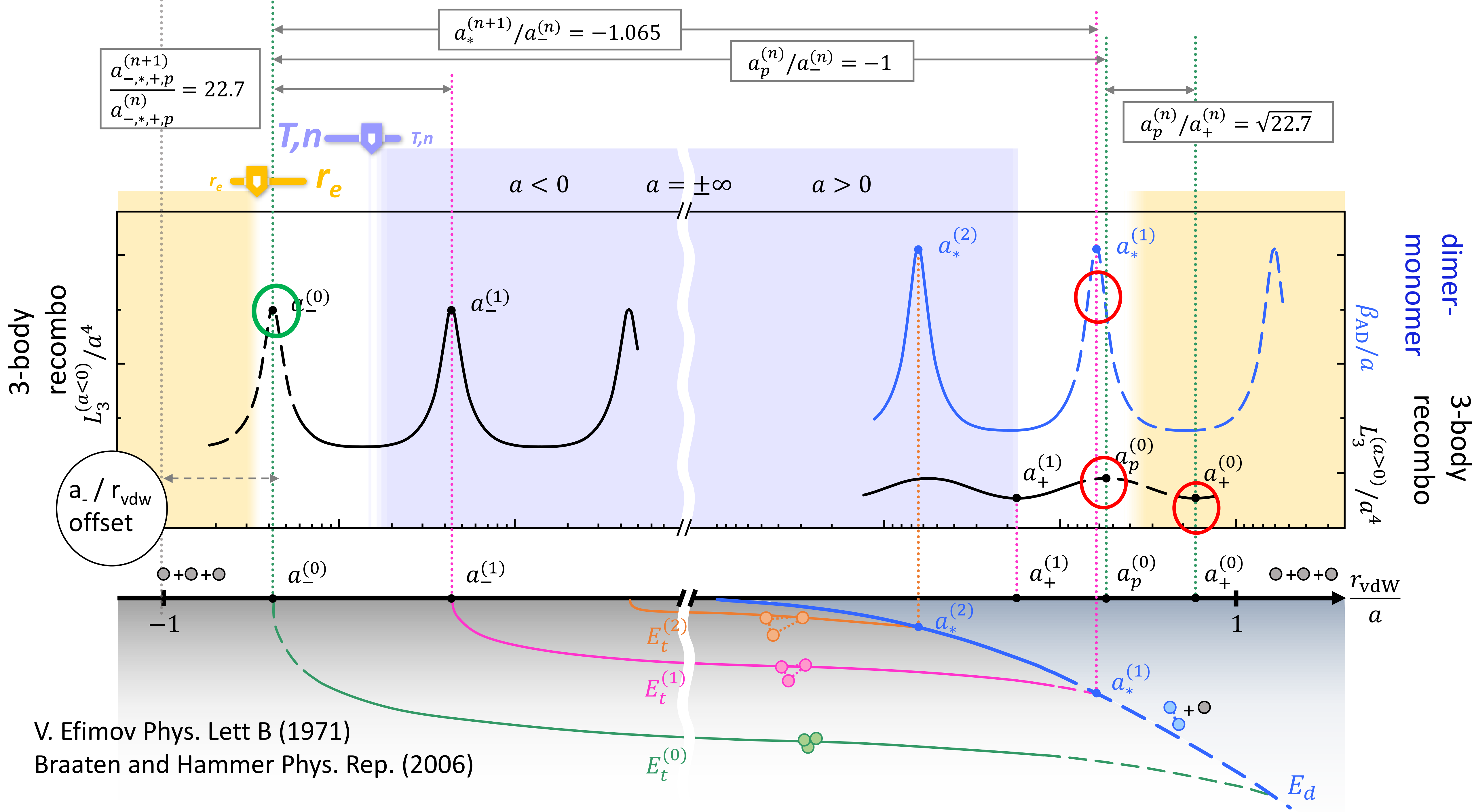
Braaten and Hammer Phys. Rep. (2006)



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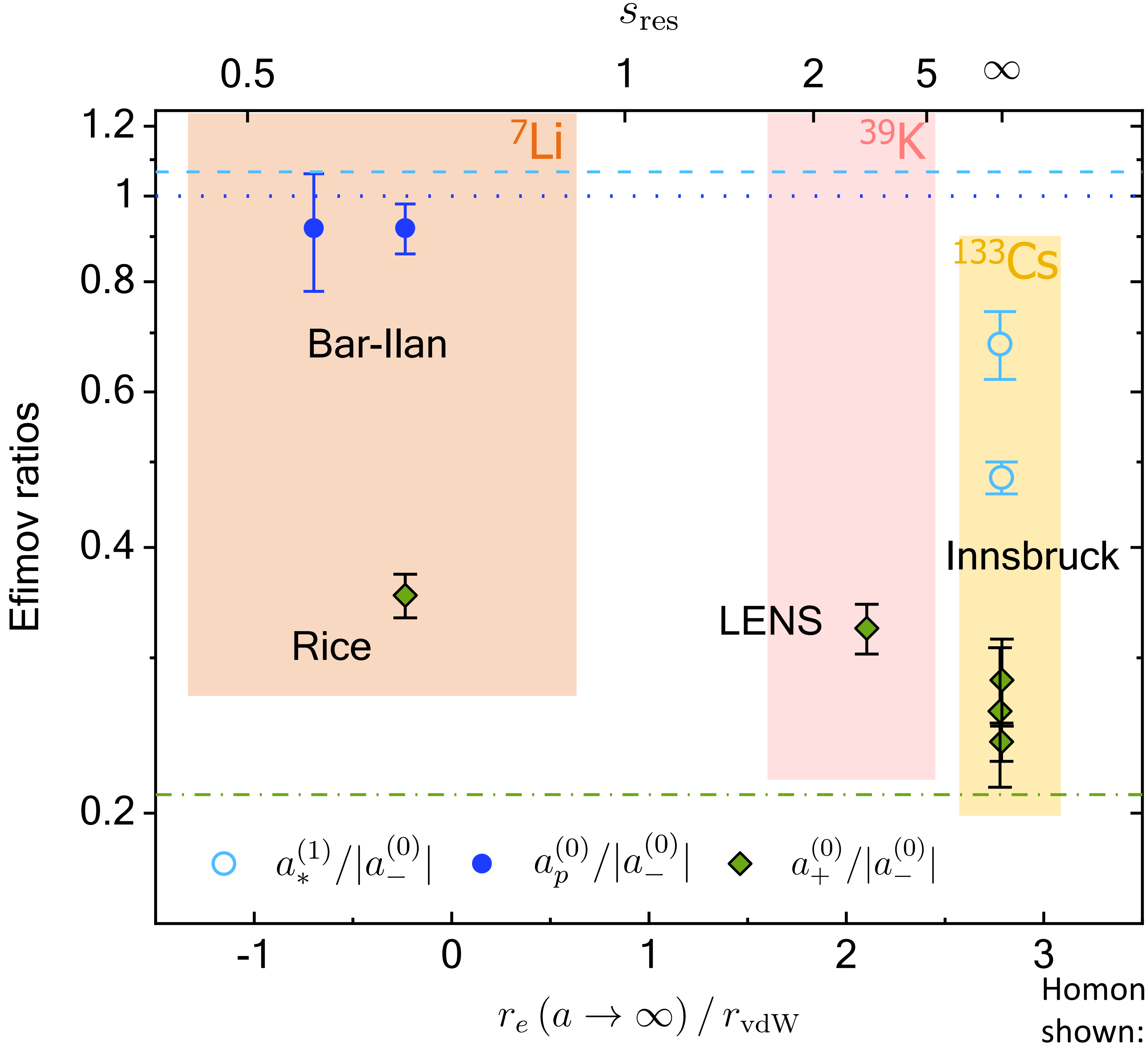


(atom-dimer)

○ $a_*^{(1)} / |a_-^{(0)}|$

● $a_p^{(0)} / |a_-^{(0)}|$

(a>0 3-body peak)

$(22.7)^{1/2} = 4.8$

$(22.7)^{1/4} = 2.2$ is a no-signal result

◆ $a_+^{(0)} / |a_-^{(0)}|$

(a>0 3-body minimum)

Homonuclear, s-wave results only. Not shown: intergenerational a^- result from Innsbruck.

How do we see “three-body recombination”?

1. Cool ^{39}K atoms with lasers.
2. Evaporatively cool them in an optical trap.
3. Apply a precisely controlled magnetic field. Wait for some of the atoms to vanish

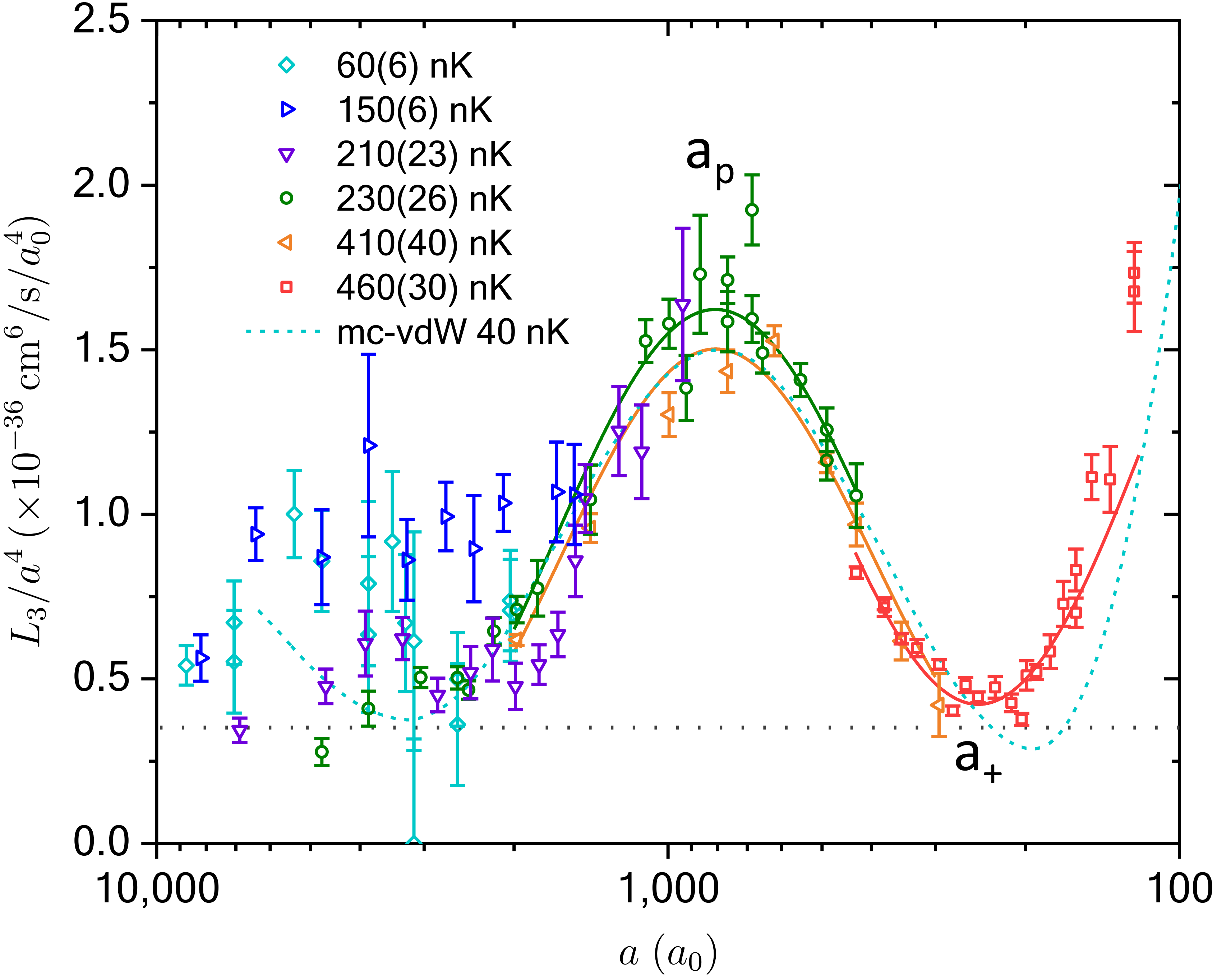
How do we see “three-body recombination”?

1. Cool ^{39}K atoms with lasers.
2. Evaporatively cool them in an optical trap.
3. Apply a precisely controlled magnetic field – to specify a certain value of a;
wait for some of the atoms to vansh

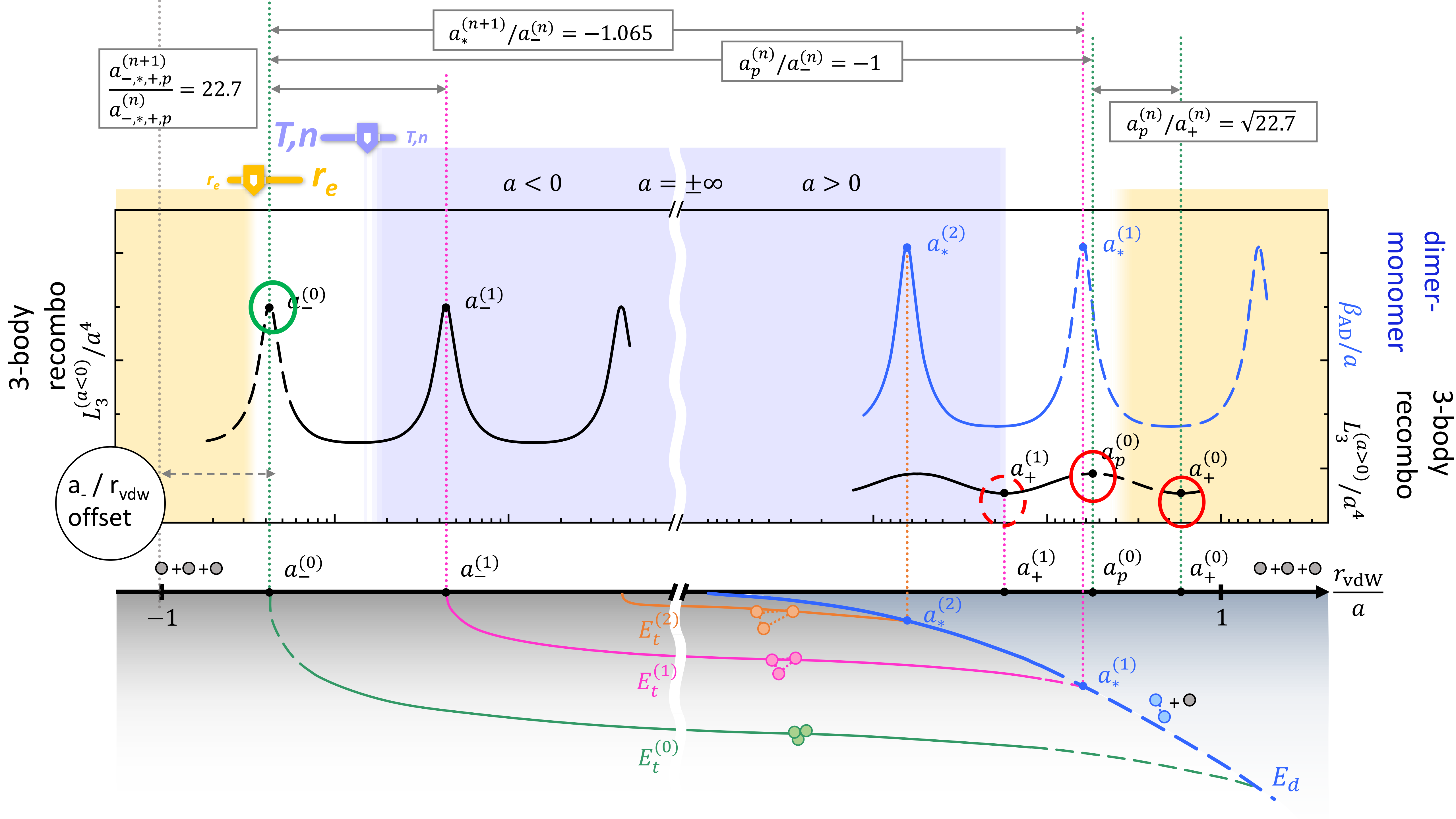
4. Vary the amount of time one waits – see differing amounts of loss.
Plot loss rate versus n, T

5. Extrapolate data to T=0

6. Repeat for different values of a, identify peak loss – a feature in the efimov diagram!



60(6) nK
150(6) nK
210(23) nK
230(26) nK
410(40) nK
460(30) nK
mc-vdW 40 nK
a_p
a_+
$L_3/a^4\,(\times 10^{-36}\,\mathrm{cm}^6/\mathrm{s}/a_0^4)$
$a\ (a_0)$
0.0
0.5
1.0
1.5
2.0
2.5
10,000
1,000
100



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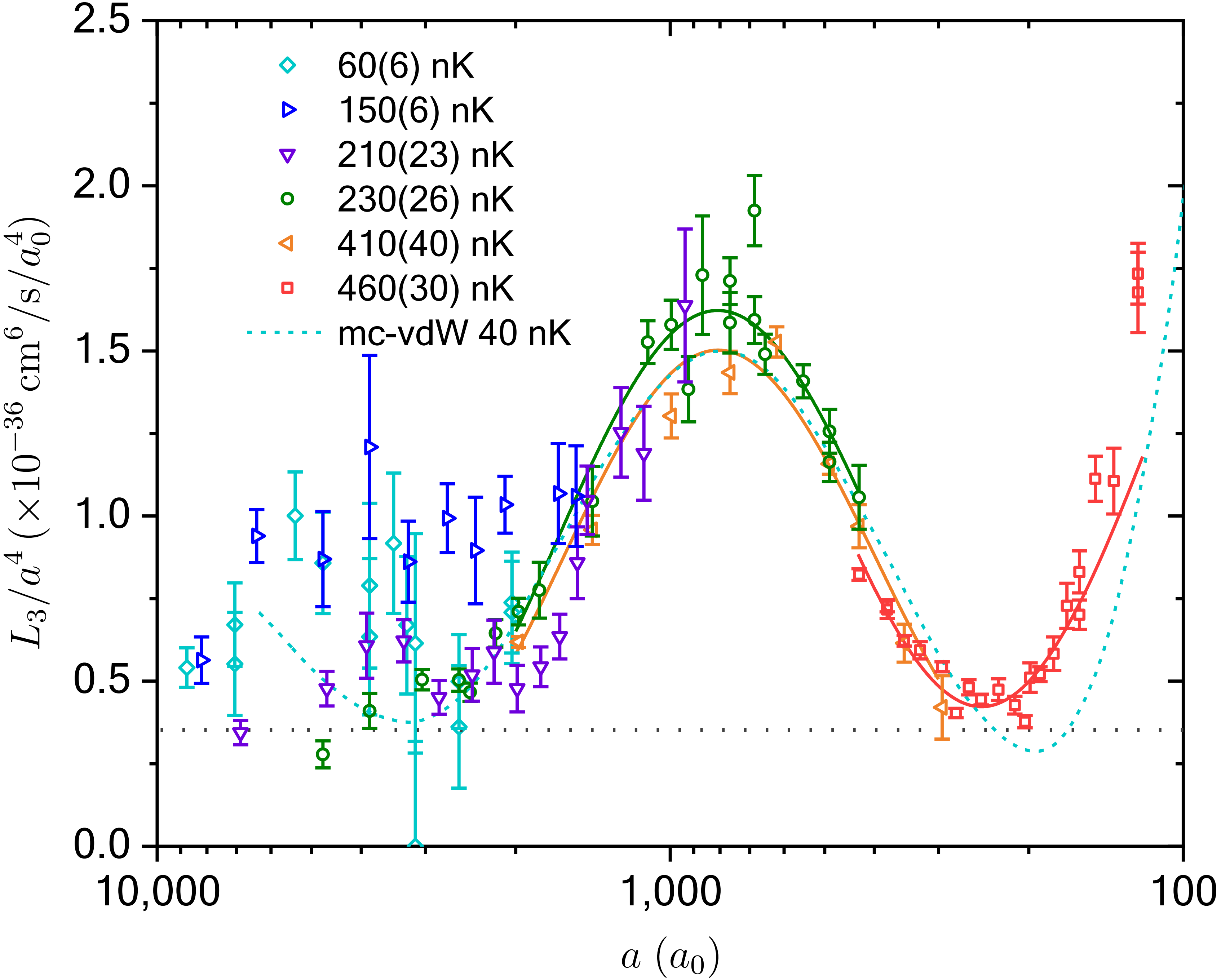
T,n
T,n
r_e
r_e
$a < 0$
$a = \pm\infty$
$a > 0$
3-body recombo $L_3^{(a<0)}/a^4$
dimer-monomer β_{AD}/a
3-body recombo $L_3^{(a>0)}/a^4$
$a_-^{(0)}$
$a_-^{(1)}$
$a_*^{(2)}$
$a_*^{(1)}$
$a_+^{(1)}$
$a_p^{(0)}$
$a_+^{(0)}$
a- / rvdw offset
+ +
+ +
r_{vdW}/a
−1
1
$E_t^{(2)}$
$E_t^{(1)}$
$E_t^{(0)}$
+
E_d

Problems at high scattering length

a) p_{th} not << κ_{efimov}
b) kT not << binding energies
c) n a_{eff}^3 not << 4 π note a_{eff} can be > a
d) cloud collisionally thick to ejected decay products
e) trap depth not << decay-product energies
f) error in location of feshbach pole
g) other unmodeled two-body physics


2.5
2.0
1.5
1.0
0.5
0.0
$L_3/a^4\,(\times10^{-36}\,\mathrm{cm}^6/\mathrm{s}/a_0^4)$
10,000
1,000
100
$a\ (a_0)$
60(6) nK
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210(23) nK
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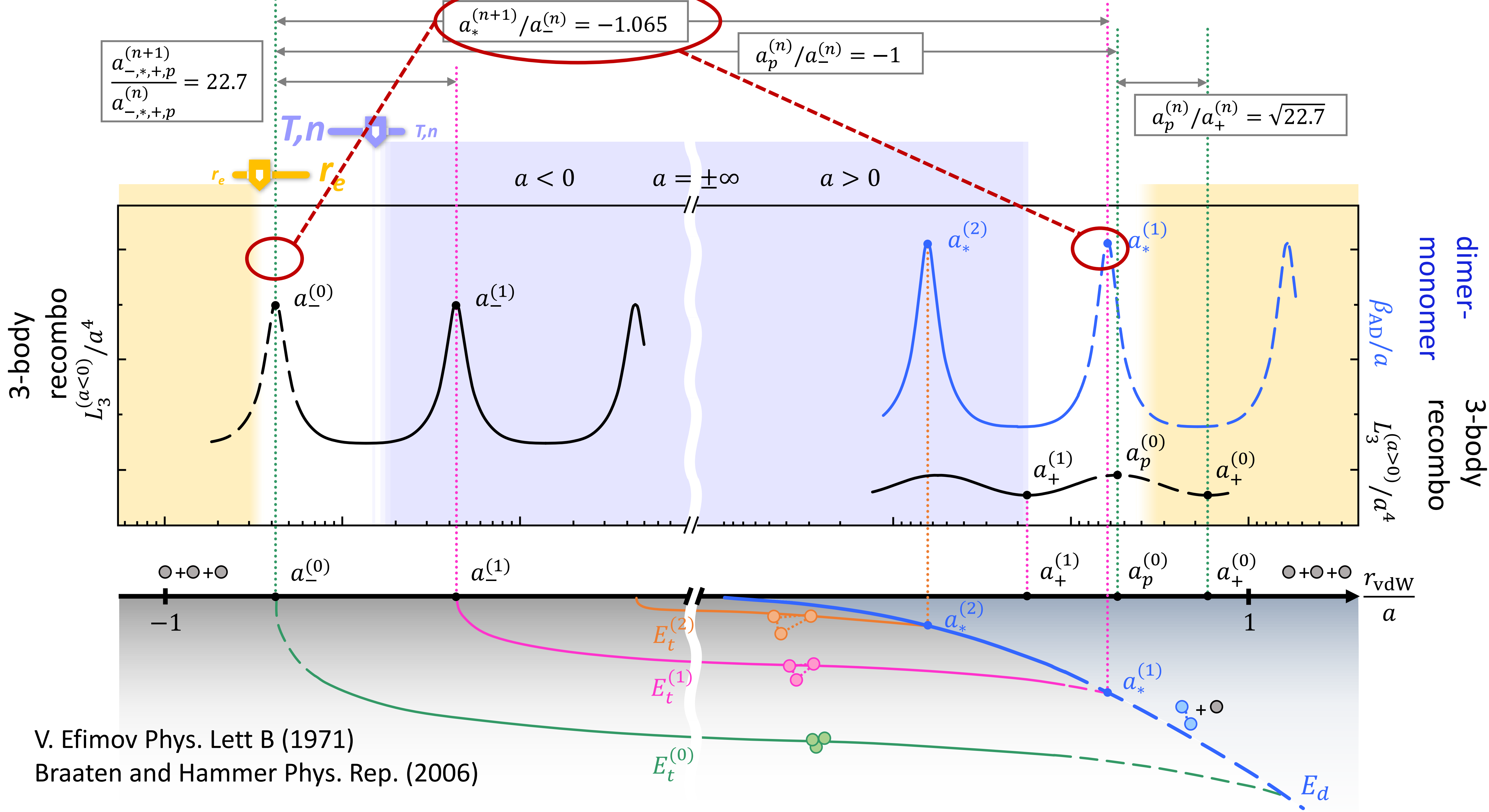
mc-vdW 40 nK



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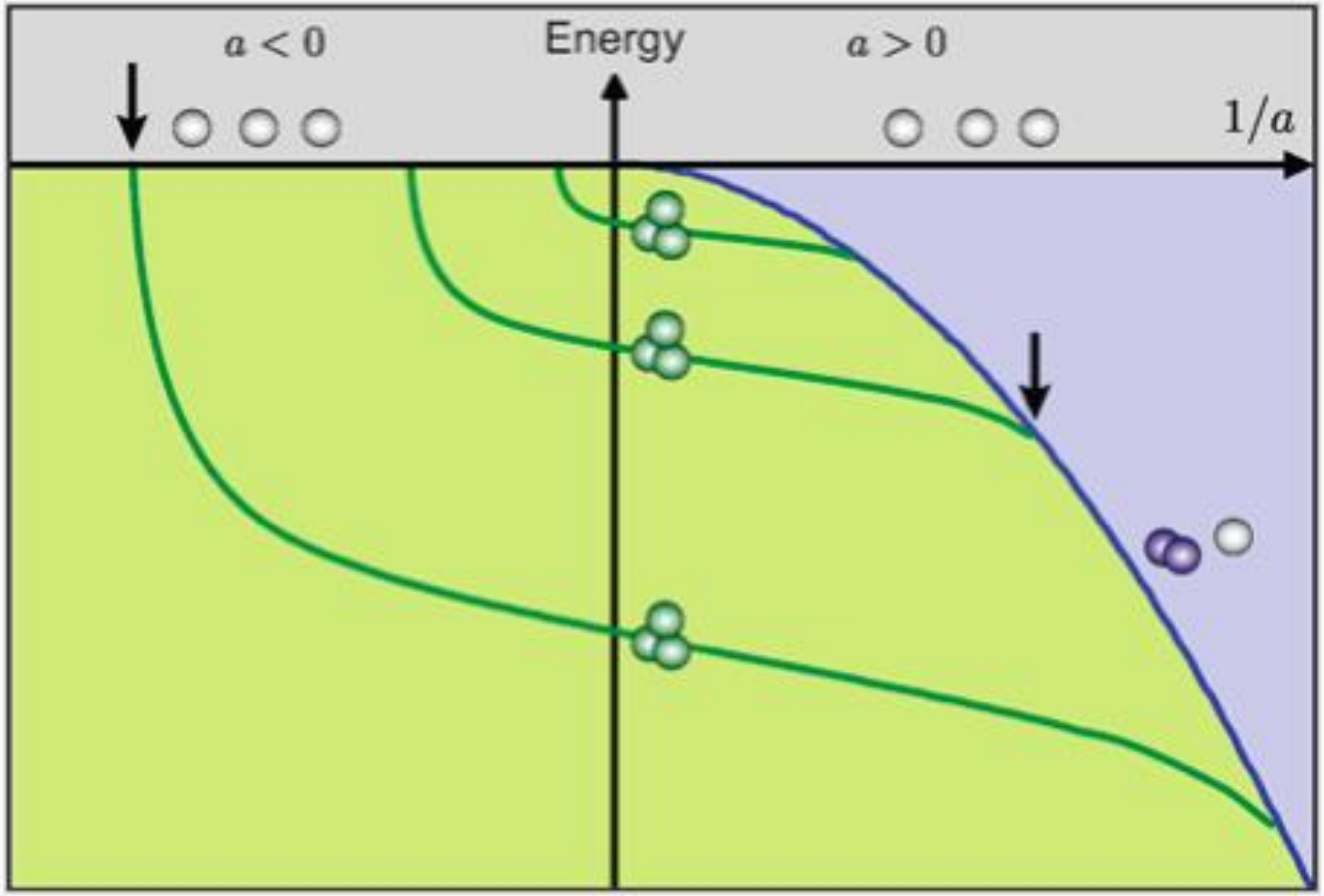
V. Efimov Phys. Lett B (1971)

Braaten and Hammer Phys. Rep. (2006)

How do we measure atom-dimer collision rate?



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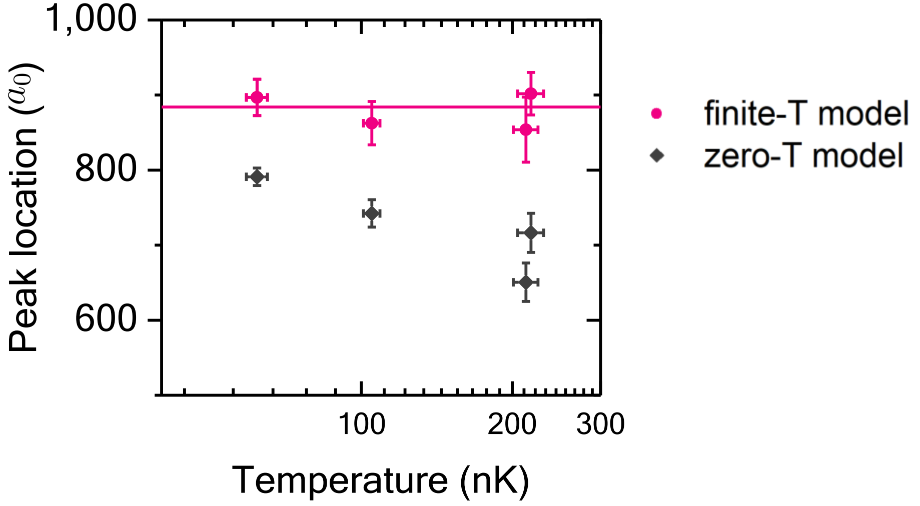
1. Start with cold free atoms.
2. Ramp scattering length to resonance (atoms, moecules are degenerate)
3. Pause for many-body mixing
4. Ramp out to desired scattering length, creating mixture of free atoms and two-atom molecules

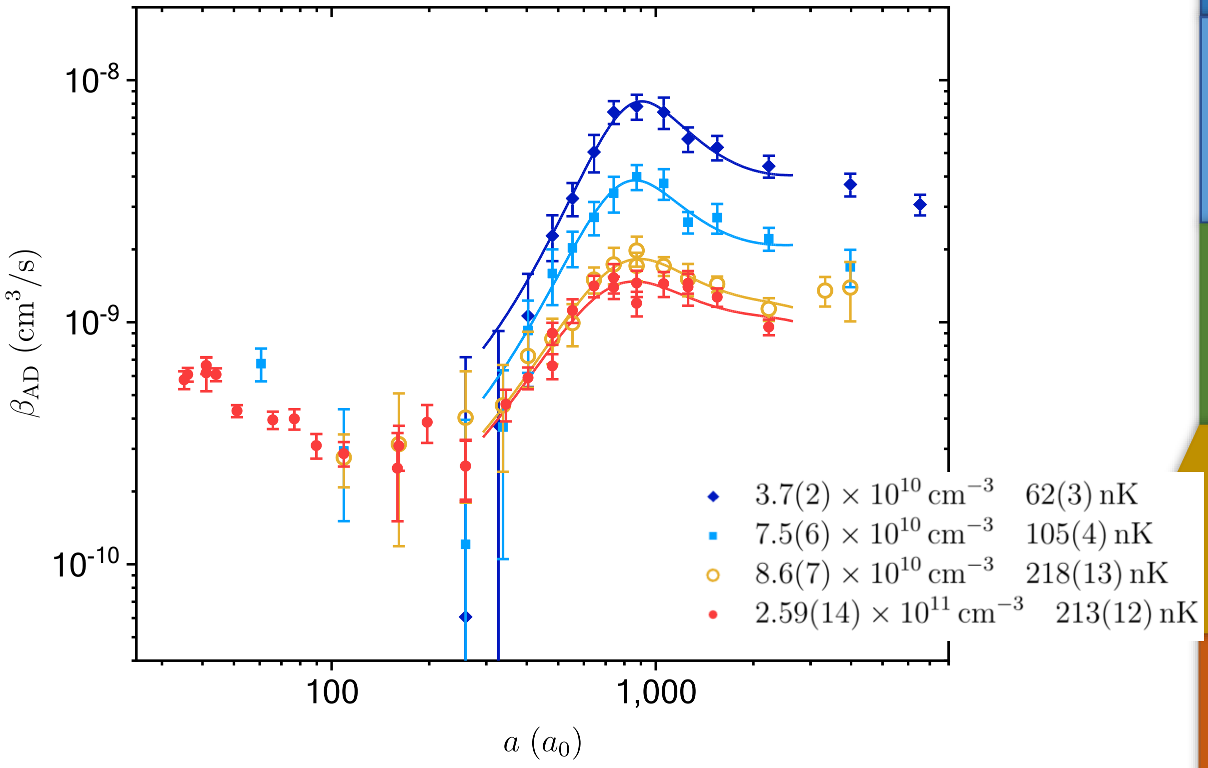
Finite Temperature Effects

Contrast zero-T and finite-T models

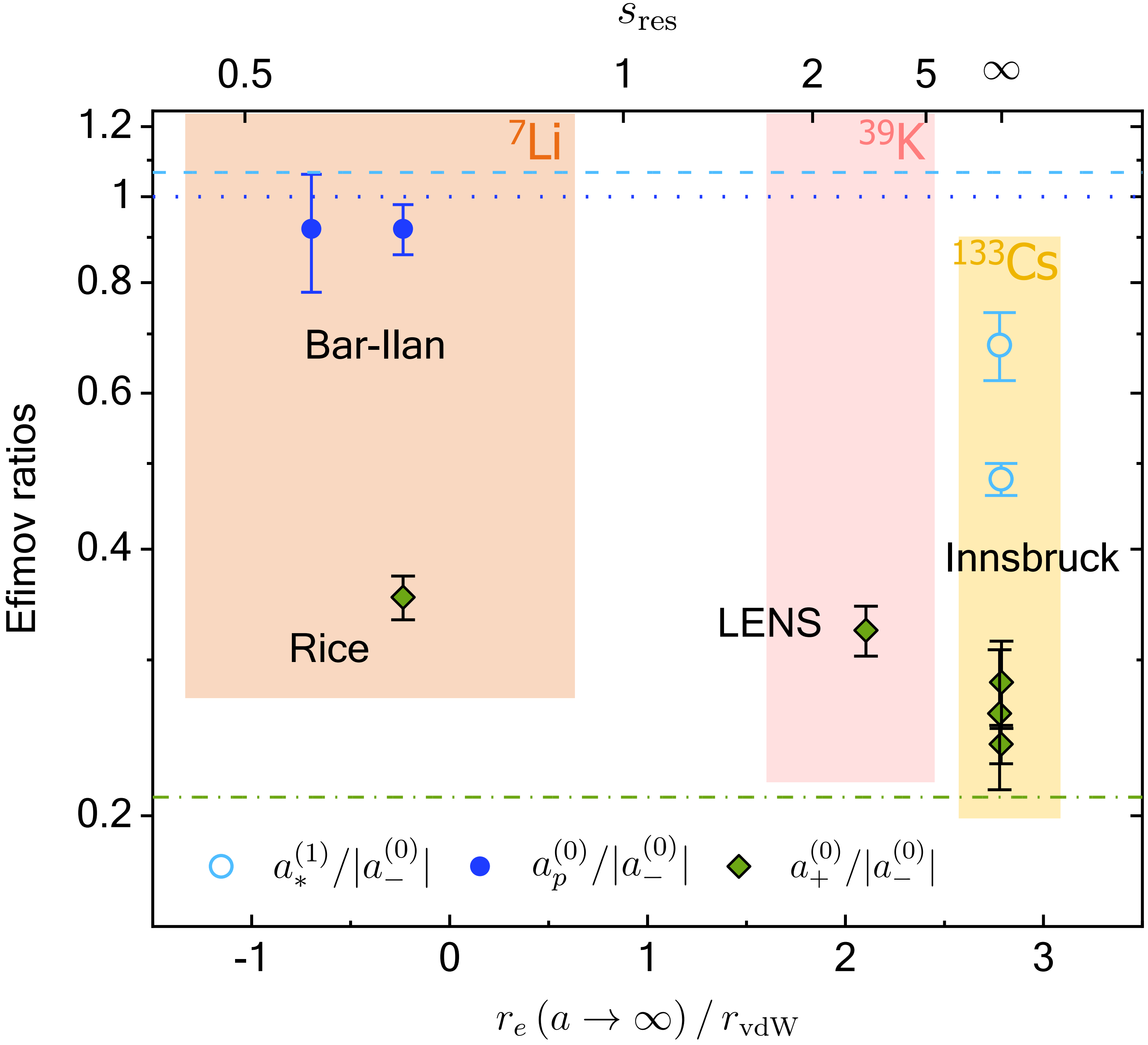


We use model from *Helfrich et al PRL (2009)*

Temperature dependence of atom-dimer resonance



Temperature shifts in monomer-dimer inelastic decay not previously reported in homonuclear or heteronuclear systems.



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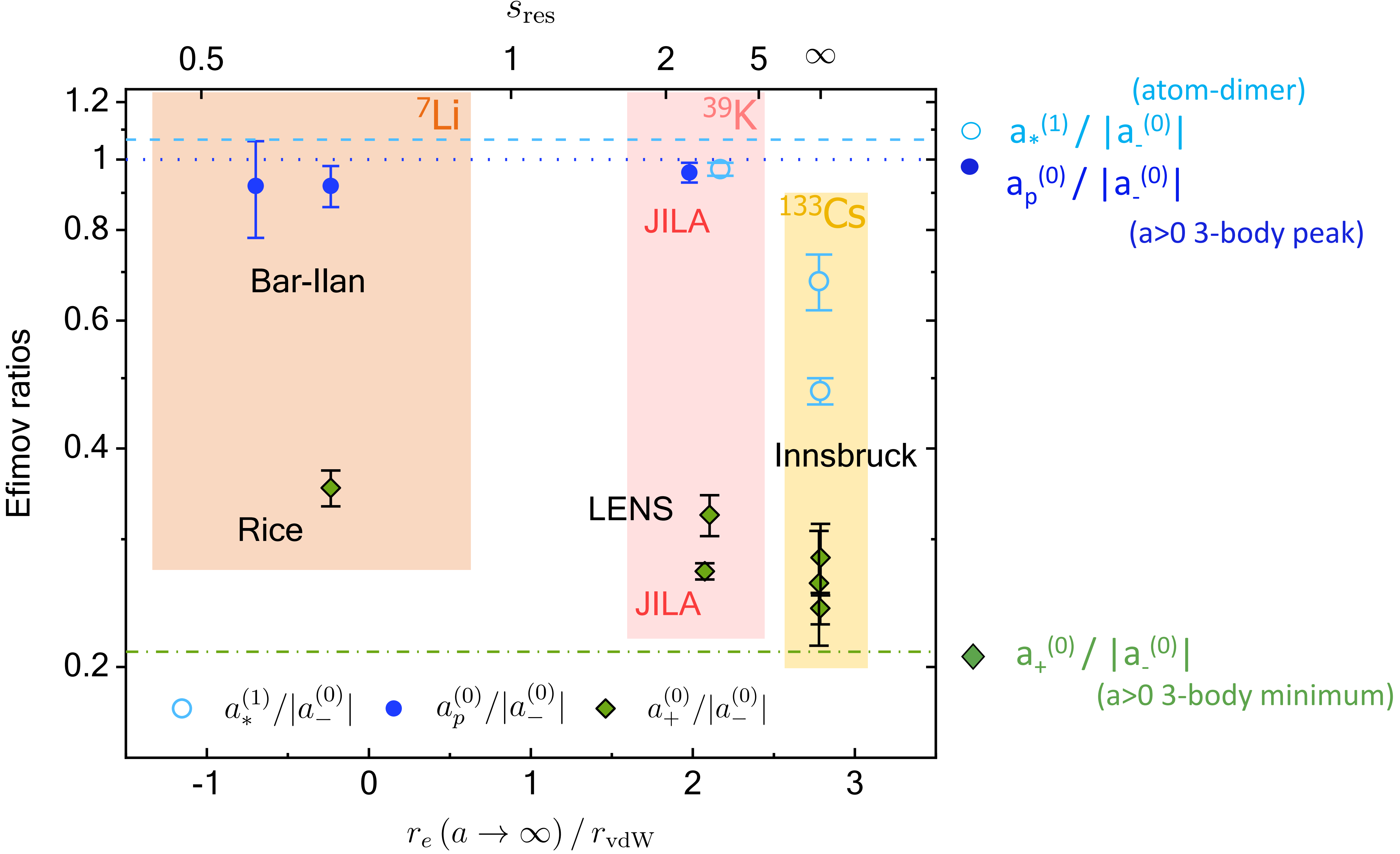
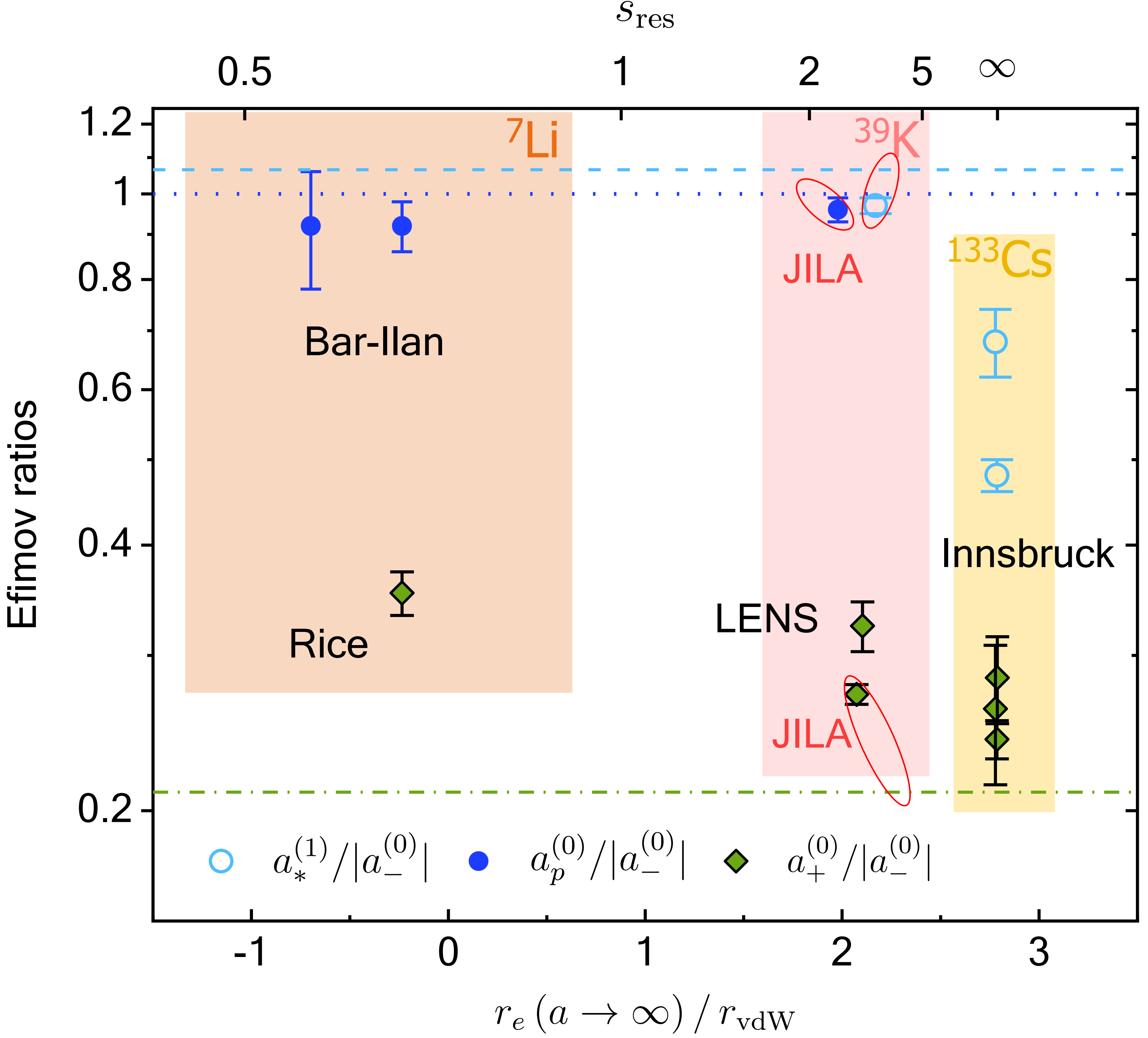
(a>0 3-body peak)

◆ $a_+^{(0)} / |a_-^{(0)}|$

(a>0 3-body minimum)


$s_{\rm res}$
0.5
1
2
5
∞
1.2
1
0.8
0.6
0.4
0.2
Efimov ratios
^{7}Li
Bar-Ilan
Rice
^{39}K
JILA
LENS
JILA
^{133}Cs
Innsbruck
$a_*^{(1)}/|a_-^{(0)}|$
$a_p^{(0)}/|a_-^{(0)}|$
$a_+^{(0)}/|a_-^{(0)}|$
-1
0
1
2
3
$r_e\,(a\to\infty)\,/\,r_{\rm vdW}$
(atom-dimer)
$a_*^{(1)}$ / $|a_-^{(0)}|$
$a_p^{(0)}$ / $|a_-^{(0)}|$
(a>0 3-body peak)
$a_+^{(0)}$ / $|a_-^{(0)}|$
(a>0 3-body minimum)



(atom-dimer)

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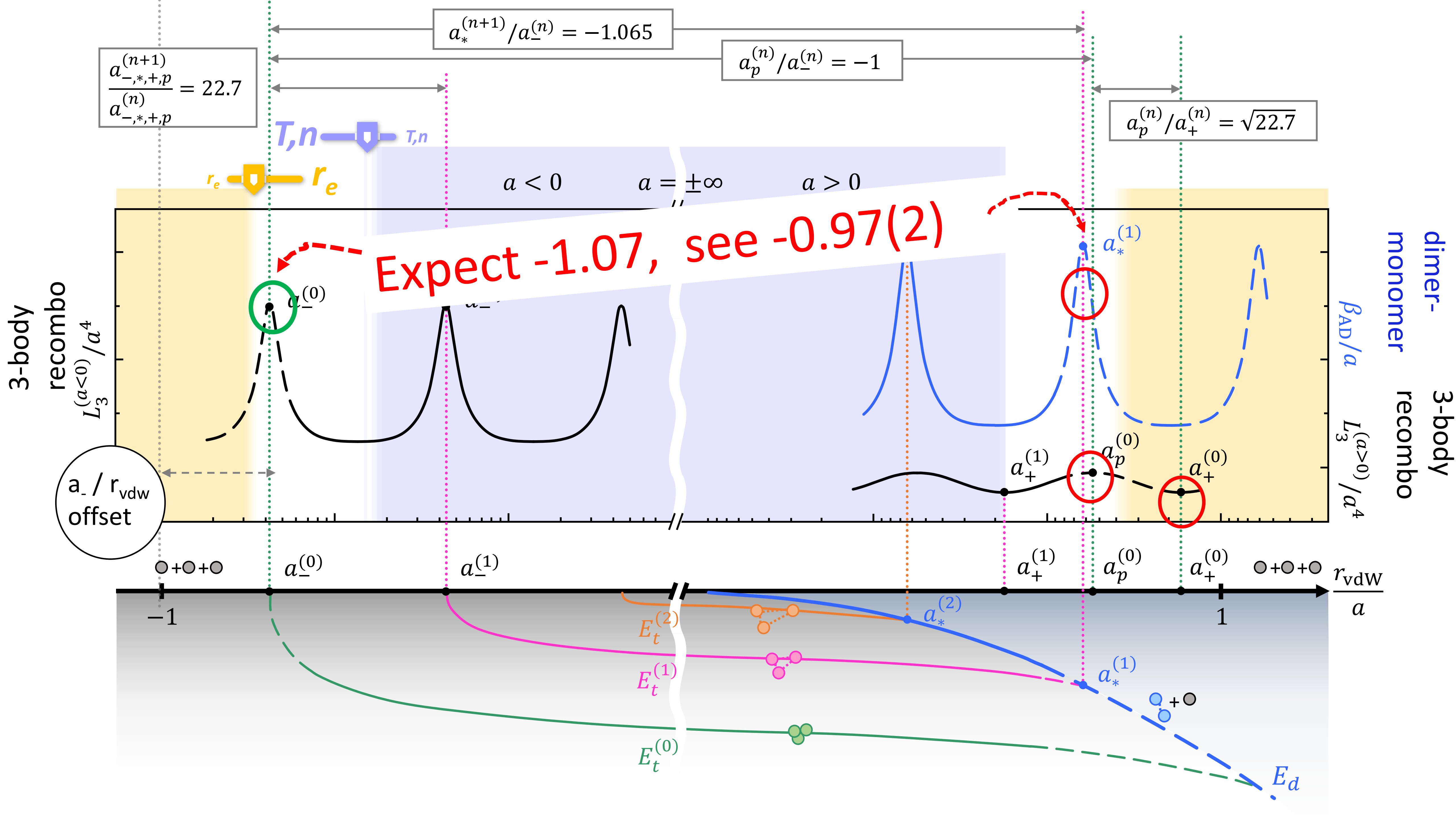
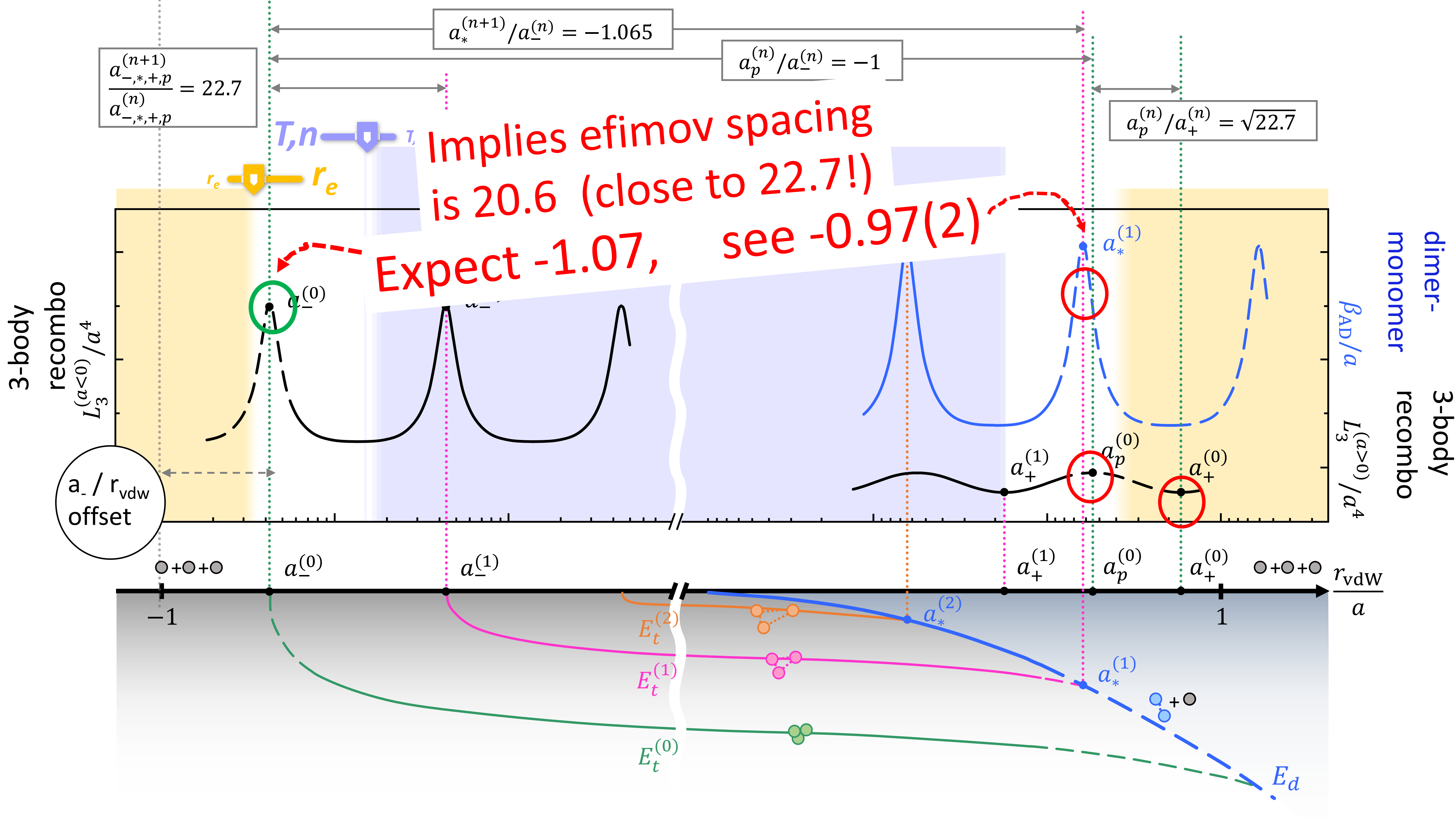


Expect -1.07, see -0.97(2)



Implies efimov spacing is 20.6 (close to 22.7!)

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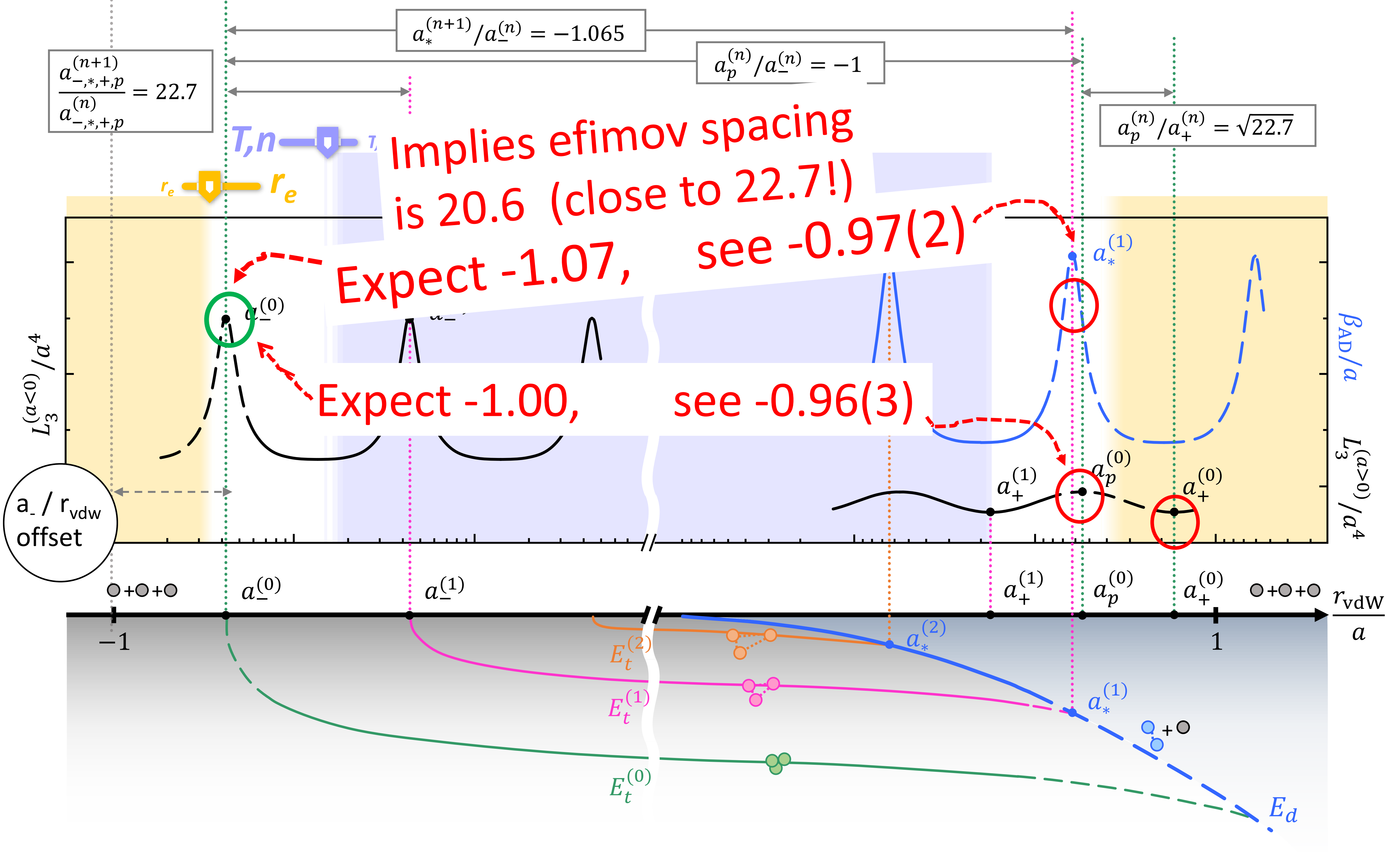
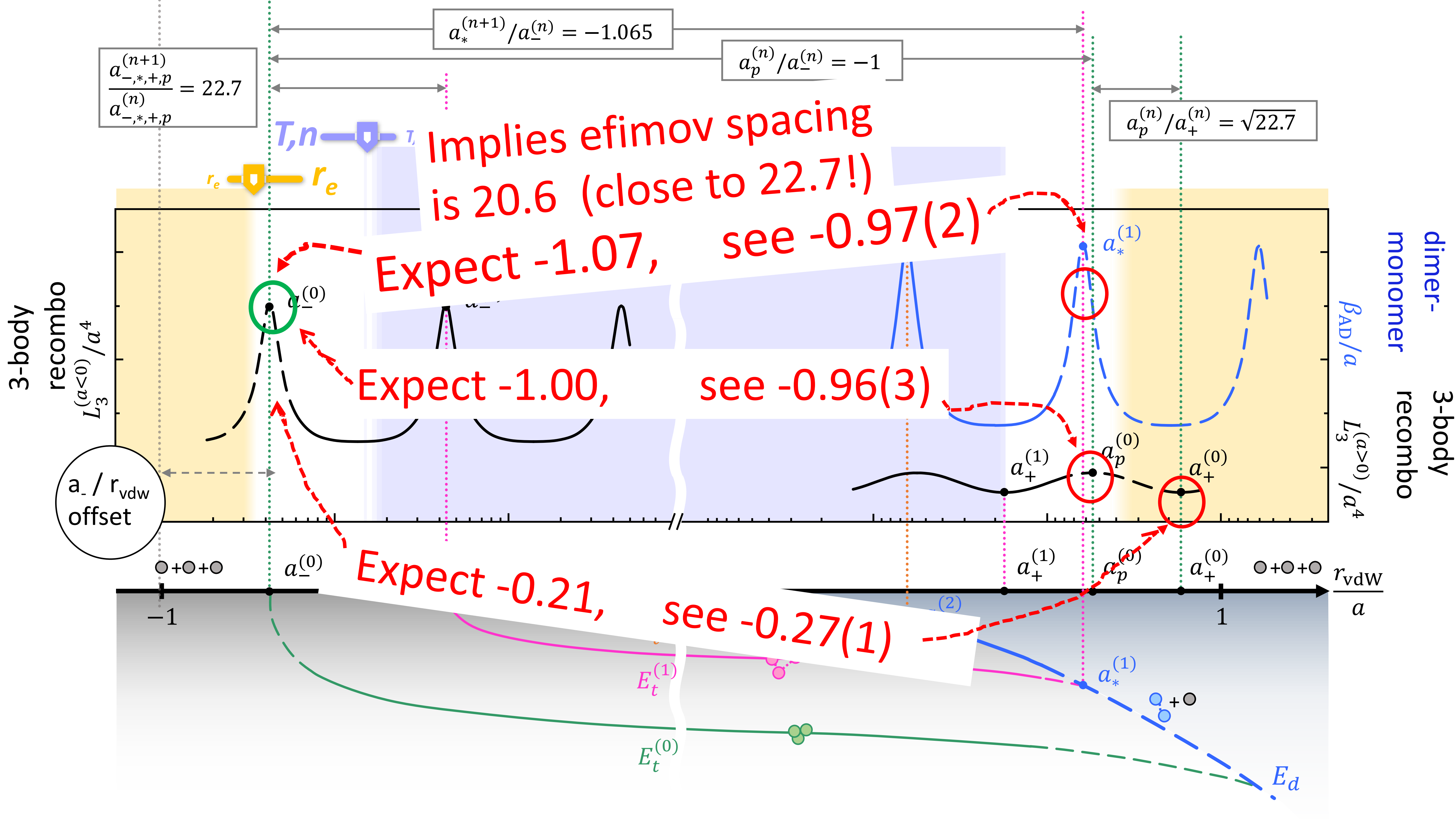


Implies efimov spacing is 20.6 (close to 22.7!)

Expect -1.07, see -0.97(2)

Expect -1.00, see -0.96(3)

a_ / r_vdw offset



Implies efimov spacing is 20.6 (close to 22.7!)

Expect -1.07, see -0.97(2)

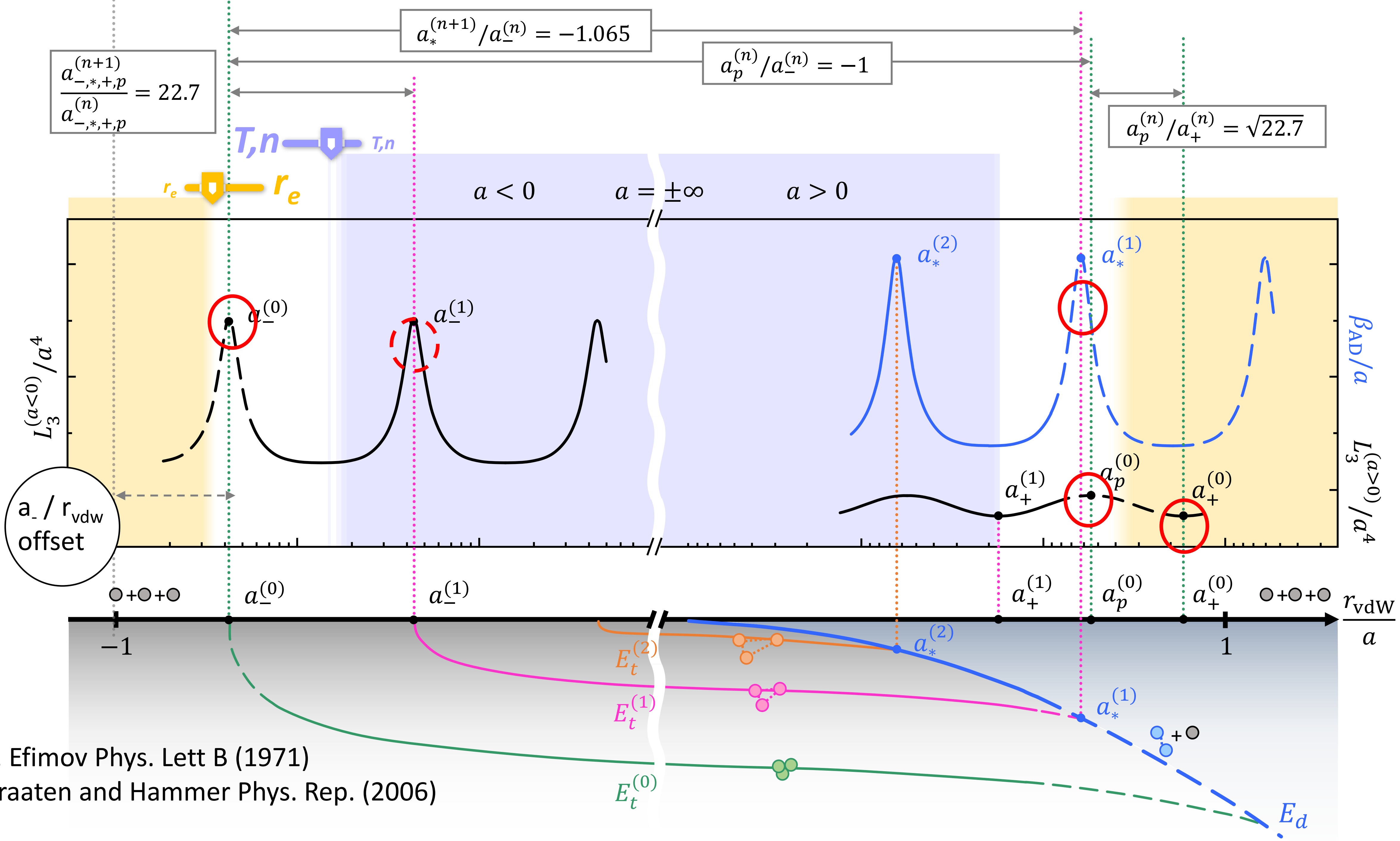
Expect -1.00, see -0.96(3)

Expect -0.21, see -0.27(1)

a₋ / r_vdw offset

	monomer-dimer $a^{*(1)} / a_0^{(0)}$	peak 3-body, a>0 $a_p^{(0)} / a_0^{(0)}$	min. 3-body, a>0 $a_+^{(0)} / a_0^{(0)}$	
Efimov universality	1.07	1.00	0.21	
JILA experimental	0.97(2)	0.96 (3)	0.27(1)	
$\log_{22.7}$ fractional agreement	0.03	0.013	0.08	(Recall, 0.25 is “no-signal result”)
multi-channel vdW	0.95(2)	0.96 (2)	0.24(1)	

Bonus: Efimov widths (ie “η”) are measured to be the same within +/- 20% for three different features



V. Efimov Phys. Lett B (1971)

Braaten and Hammer Phys. Rep. (2006)