Bosons and Fermions: from few to many

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Opportunities and Challenges in Few-Body Physics: Unitarity and Beyond
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Acknowledgments:
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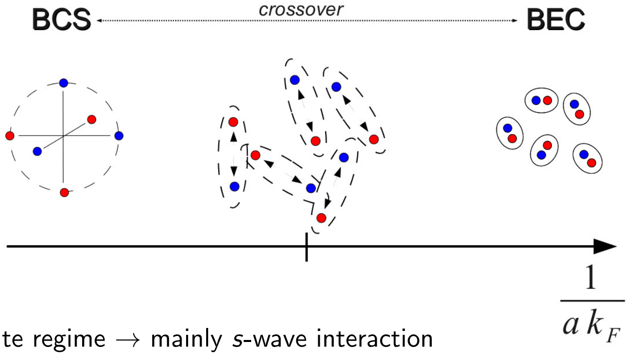
Introduction



- Dilute regime $\rightarrow$ mainly *s*-wave interaction
- T fraction of T_F $\rightarrow$ $T \sim 0$
- Experimentally tunable interaction
- Crossover from weakly interacting Fermions (paired) to weakly repulsive Bosons (molecules)

Cold atoms

Example of Fermionic superfluids:

- Superconductors, $\Delta/E_F \sim 10^{-4}$
- Liquid ^3He, $\Delta/E_F \sim 10^{-3}$
- High-T$_C$ superconductors, $\Delta/E_F \sim 10^{-2}$
- Cold Fermi gases, $\Delta/E_F \sim 0.5$
- Neutron matter, $\Delta/E_F \sim 0.35$

Systems very interesting to study:

- Tunable interaction (Feshbach resonances)
- Universality connecting free Fermions (BCS) to free Bosons (BEC)
- Experiments (EOS, the contact parameter, various responses, ...)
- Very similar to low-density neutron matter

The two topics in this talk:

- What about unitary Bosons? Are they universal?
- Are trapped Fermions useful for nuclear physics?

Unitary Bose gas

- Two-body attractive interaction set to the unitary limit, small effective range.

$$V_{ij} = V_2^0 \frac{\hbar^2}{m} \mu_2^2 \exp[-(\mu_2 r_{ij})^2/2]$$

- Three-body repulsion needed to avoid the system collapsing. Set to have a weakly-bound trimer with a rms radius much larger than the range of the interactions.

$$V_{ijk} = V_3^0 \frac{\hbar^2}{m} \left(\frac{\mu_3}{2}\right)^2 \exp[-(\mu_3 R_{ijk}/2)^2/2]$$

- The energy scale is the trimer binding energy.
- With these conditions, the system is universal, i.e. does not depend on the details of the interactions.

Evolution of Schrodinger equation in imaginary time $\boldsymbol{t}$:

$$\psi(t) = e^{-(H-E_T)t}\psi(0)$$

At $t \to \infty$ we get $\psi(t) \to \phi_0$ if not orthogonal to $\psi(0)$.

Propagation performed by

$$\psi(R,t) = \langle R|\psi(t)\rangle = \int dR' G(R,R',t)\psi(R',0)$$

where $G(R,R',t)$ is an approximate propagator known in the small-time limit:

$$G(R,R',\Delta t) = \langle R|e^{-H\Delta t}|R'\rangle \approx \langle R|e^{-p^2\Delta t}|R'\rangle\langle R|e^{-V\Delta t}|R'\rangle$$

Then we need to iterate the above integral equation many times in the small time-step limit.

Useful to calculate many-particles. Exact for Bosons, good approximation for Fermions.

Unitary Bosons, variational wave function:

To simulate unitary Bosons, we use a variational wave function with the form:

$$\psi_T(R) = \prod_i f^{(1)}(r_i) \prod_{i<j} f^{(2)}(r_{ij}) \prod_{i<j<k} f^{(3)}(R_{ijk}) \,,$$

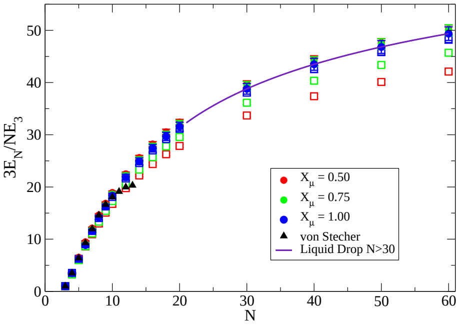
where

$$f^{(1)}(r) = \exp(-\alpha r^2) \,,$$
$$f^{(2)}(r) = \exp(K \tanh(\mu_J r) \cosh(\gamma r)/r),$$
$$f^{(3)}(R) = \exp[u_0 \exp(-R^2/(2r_0^2))].$$

All the parameters are variationally optimized.

For homogeneous matter, $f^{(1)} = 1$.

Binding energy as a function of the number of Bosons:

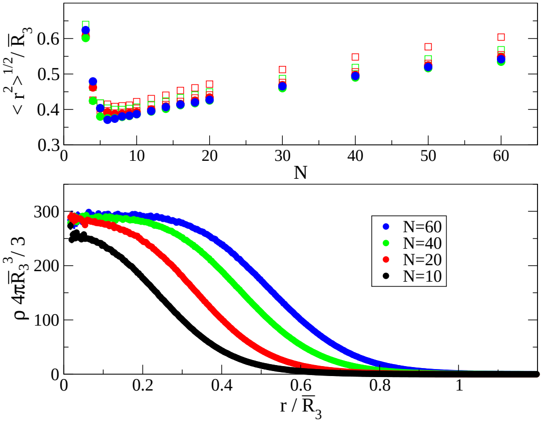


$X_\mu = \mu_3/\mu_2$, filled symbols correspond to weakly-bound trimers, open symbols to more bound trimers.

Universality provided for weakly-bound trimers!

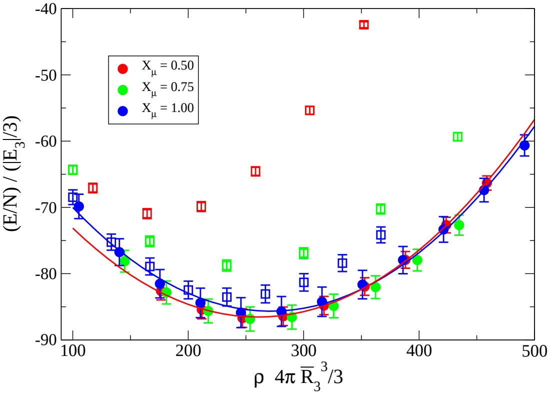
Carlson, Gandolfi, van Kolck, Vitiello, PRL 119, 223002 (2017).

Radii and densities:



$$\bar{R}_3 = 1/\sqrt{\frac{\hbar^2}{-2mE_3}}$$

Equation of state of the unitary Bose gas:



Saturation point well reproduced.

Universality restored for interactions corresponding to weakly-interacting trimers.

Now let's talk about Fermi systems in an external trap (two-components Fermi gas).

Fermi gas in a trap

Let's consider two-components (spin-up and spin-down) Fermions, interacting in s-wave, in a harmonic trap:

$$H=\sum_{i=1}^{N}\left(\frac{p_i^2}{2m}+\frac{1}{2}m\omega^2 r_i^2\right)-v_0\frac{8\hbar^2}{mr_e^2}\sum_{i\uparrow,j\downarrow}\frac{1}{\cosh^2(2r_{ij}/r_e)}\,,$$

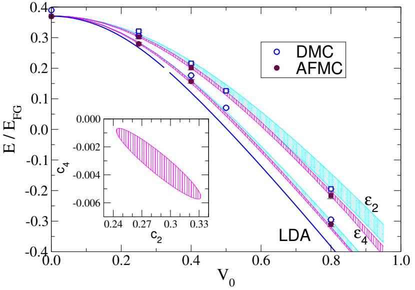
Variational wave-function:

$$\Psi_T=\prod_{ij}f_J(r_{ij'})\;\Phi_{\rm BCS}\,,\qquad \Phi_{\rm BCS}=\mathcal{A}[\phi(r_{11'})\phi(r_{22'})...\phi(r_{nn'})]\,,$$

$$\phi(r_{12})=\sum_{k=1}^{N_c}c_k\sum_{m=-l_k}^{l_k}\frac{(-1)^{l_k+m}}{\sqrt{2l_k+1}}\times\psi_{n_k l_k m}(\alpha_k\mathbf{r}_1)\psi^*_{n_k l_k -m}(\alpha_k\mathbf{r}_2)\,,$$

H scale invariant, functional should work for any external potential.

Static response to an external potential $V = -V_0 \sum \cos(\mathbf{q} \cdot \mathbf{r})$

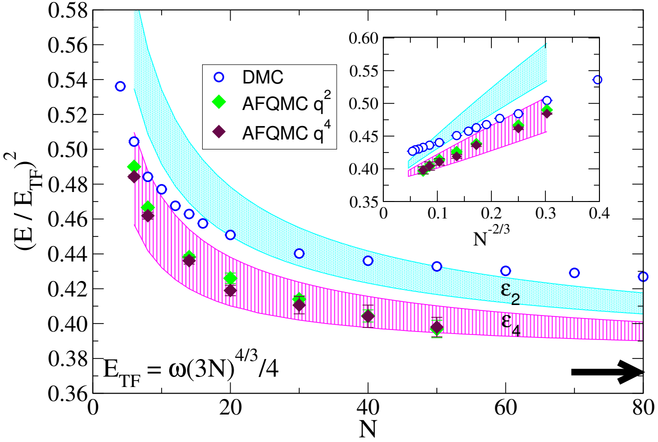


Carlson, Gandolfi, PRA 90, 011601(R) (2014).

Simple density functional:

$$\mathcal{E}_g = \int V_{ext}(r)\rho(r) + \xi \frac{3}{5}(3\pi^2)^{2/3}\rho^{5/3} + c_2 \nabla\rho^{1/2} \cdot \nabla\rho^{1/2} - c_4 \frac{\nabla^2\rho^{1/2}\nabla^2\rho^{1/2}}{\rho^{2/3}}$$

Unitary gas in a trap:



Carlson, Gandolfi, PRA 90, 011601(R) (2014).

Bands: predictions from the energy density functional.

Very good agreement even for few particles.

See also Forbes, Gandolfi, Gezerlis, PRA 86, 053603 (2012).

Can two-components Fermi gas in a trap be useful for nuclear physics?

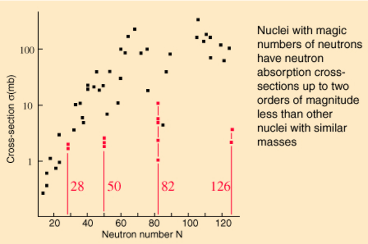
The nuclear shell model

Clear experimental evidence of magic numbers.

N or $Z = 2, 8, 20, 28, 50, 82, 126$

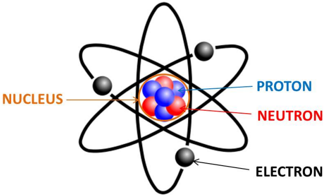
Signatures (incomplete list) of properties of magic nuclei:

- Nuclei very stable (long lasting)
- Large separation energy (energy needed to extract a nucleon)
- Neutron-capture cross-sections very low (nuclei like to stay in those configurations)



hyperphysics.phy-astr.gsu.edu

Another similarity in nature: atoms!



Magic numbers explain a lot of stable configurations, high separation energies, low cross-sections, quadrupole deformations, etc.

End of the story??? Of course not!

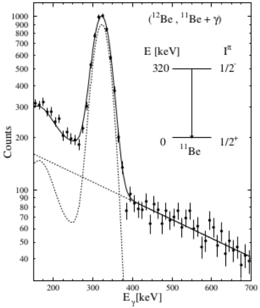
There is experimental evidence suggesting that magic numbers can disappear for some particular nucleus!

Example: ^{12}Be

$Z = 4$, $N = 8$ $(1s_{1/2}, 1p_{3/2}, 1p_{1/2})$

Experimentally it has been demonstrated that there is a strong $2s_{1/2}$ component in the ground state *and hence the breakdown of the N=8 shell closure*.

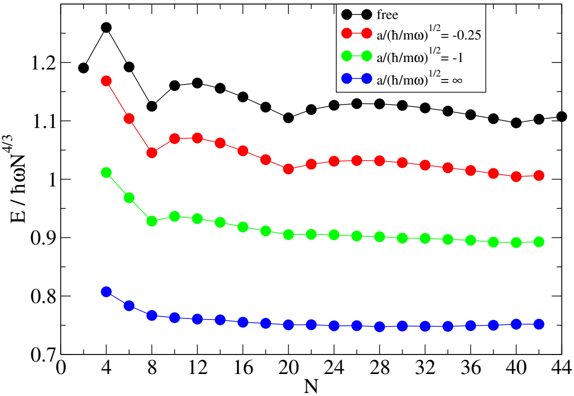
Navin et al., PRL 85, 266 (2000).



There are many other examples where magic numbers disappear for particular Z or (large) N. Also some evidence of new magic numbers!

Fermi gas in a trap

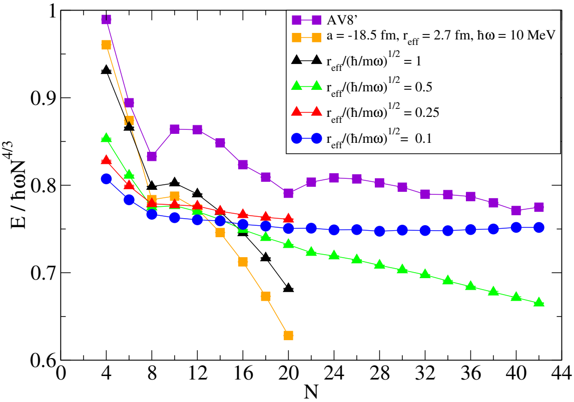
Effect of the scattering length (with a small effective range):



$|a| \to 0$ (in the BCS side) restores shell effects

Gandolfi, Carlson, unpublished.

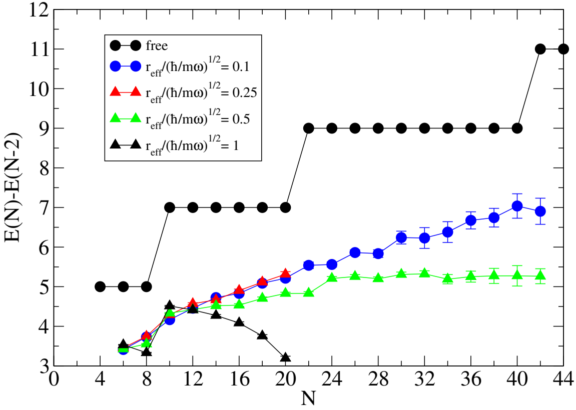
Effect of the effective range (with scattering length $|a| = \infty$):



Large effective range (or large densities) restores shell effects.

Fermi gas in a trap

Two-particles energy separation:



Shell effects even more evident here.

Connection with nuclear physics

Neutron-neutron interactions have a “large” scattering length $a \sim 19fm$, and a “large” effective range $r_{eff} \sim 2.8fm$.

- Can these effects explain why nuclei with many N (neutrons) do not show shell effects (for example Calcium isotopes)?
  Think about neutrons in the core, i.e. “large” density vs. neutrons on the surface, i.e. “low” density.
- Model dependence? (expected in particular for large effective range)

Stay tuned...

Conclusions

- Unitary Bose systems are universal for weakly three-body interactions.
- Bosonic clusters saturate (as liquid ^4He). Consistency with homogeneous matter.
- Relatively simple Bosonic density functionals can be very useful to study small Fermionic systems (at least at unitarity).
- Fermi gas in a trap *may* provide an explanation of nuclear shell effects.
- Can we use ultra-cold Fermi gas experiments to understand nuclear physics?

Thanks!