Structure of ^{7}H with t+4n cluster model

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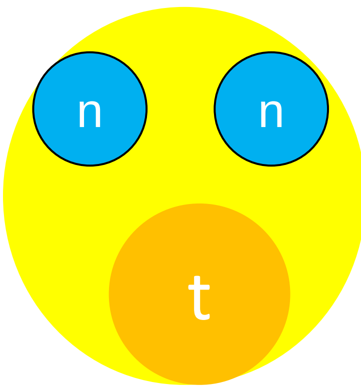
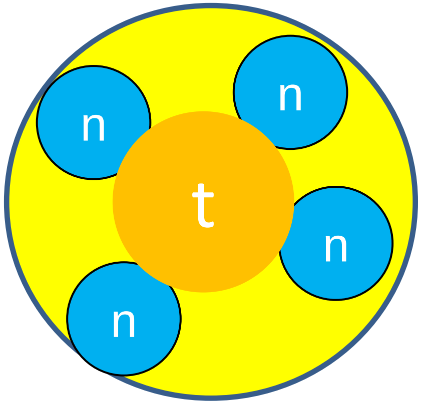
Outline

- Introduction

- ^5H and ^7H



^5H



^7H

Motivation why I study ^7H

Selected for a Viewpoint in *Physics*
PRL **116**, 052501 (2016) PHYSICAL REVIEW LETTERS week ending 5 FEBRUARY 2016

Candidate Resonant Tetraneutron State Populated by the $^4He(^8He,^8Be)$ Reaction

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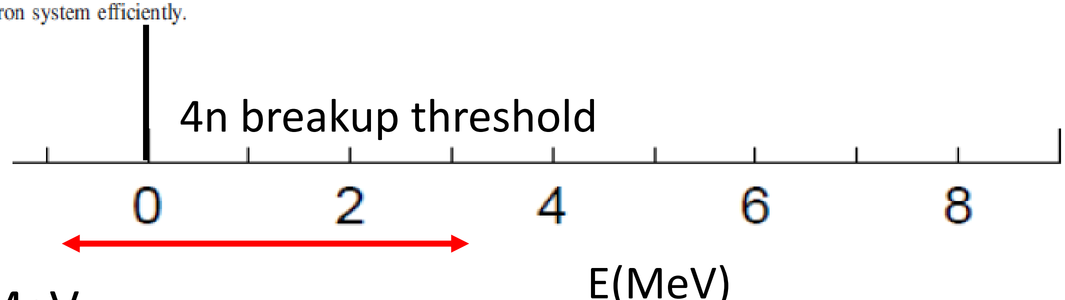
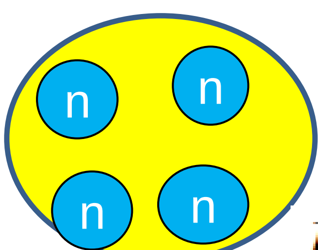
A candidate resonant tetraneutron state is found in the missing-mass spectrum obtained in the double-charge-exchange reaction $^4He(^8He,^8Be)$ at 186 MeV/u. The energy of the state is $0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst})$ MeV above the threshold of four-neutron decay with a significance level of 4.9σ. Utilizing the large positive Q value of the $(^8He,^8Be)$ reaction, an almost recoilless condition of the four-neutron system was achieved so as to obtain a weakly interacting four-neutron system efficiently.

DOI: 10.1103/PhysRevLett.116.052501

Observation of 4n state by RIBF in 2016
If this observation is reliable,
We observe 'no isotope nucleus'.





~-1.0 MeV Exp. ~3 MeV

$$E_R = 0.83 \pm 0.65 \pm 1.25$$

Γ=2.6 MeV (Upper limit)

After observation of 4n at RIBF

PHYSICAL REVIEW C **93**, 044004 (2016)

Possibility of generating a 4-neutron resonance with a $T = 3/2$ isospin 3-neutron force

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(Received 27 December 2015; revised manuscript received 26 February 2016; published 29 April 2016)

Talked at KITP workshop in 2016, International workshop on Universality in Few-body systems, Santa Barbara, Kavli Institute for Theoretical physics, USA, 07 Nov.-16th Dec., 2016.

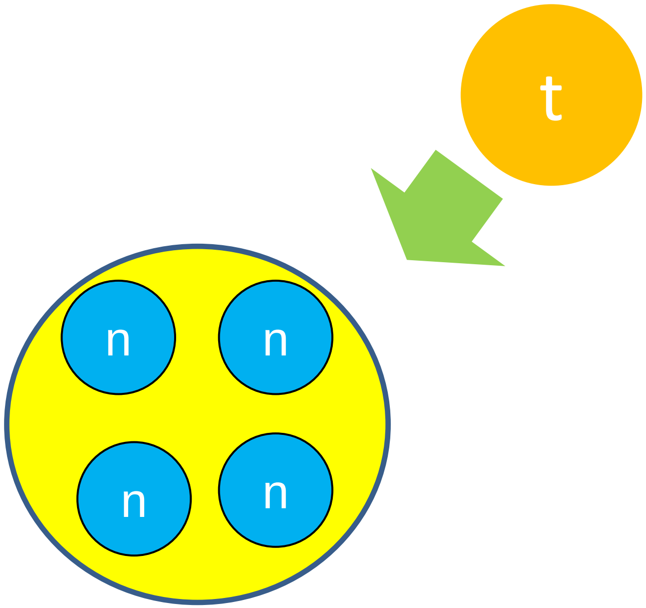
Summary of the 4n calculation, currently

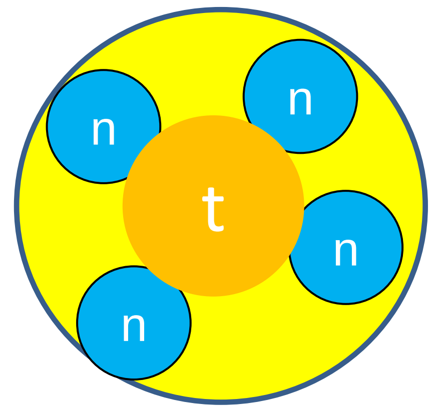
Authors	Method	V_{NN}	resonance
A.M. Shirokov et al.	Non-core shell model + phase shift analysis	JISP16	Er=0.8 MeV Γ=1.4 MeV
S. Gandolfi et al.	Quantum Monte Calro extrapolation	chiral(NNLO)	Er～2.1 MeV
K. Fossez et al.,	no-core Gamow shell model	N3LO, JISP16,	Er～7MeV Γ～3.5MeV
E. Hiyama, R. Lazauskas et al.,	Gaussian Expansion + CSM Faddeev Yakubovsky	AV8	No resonance
Deltuva,	Faddeev Yakbobsky + AGS	SRG(AV18),NLO,	No resonance
M. D. Higgins et al.,	Hypersherical harmonics phase shift analysis	AV8, AV18,	no resonance

In the world, theoretically, we come to negative conclusion, no resonant state for 4n.

How do we understand 4n system?

Let's add one triton to 4n.





^{7}H

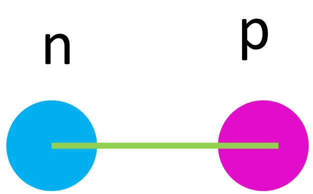
Neutron:6
Proton: 1 → Super heavy hydrogen

^{7}H is bound, resonance, nothing?

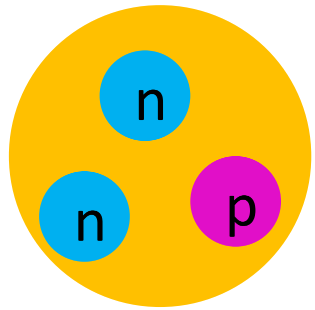
Let's explain about hydrogen Isotope before talking about ^{7}H.

The lightest isotope is Hydrogen (H).

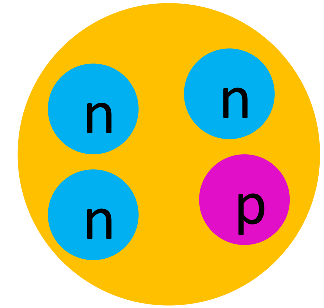
Exp.



^{2}H J=1^{+} -2.22 MeV



^{3}H J=1/2^{+} -8.48 MeV



^{4}H

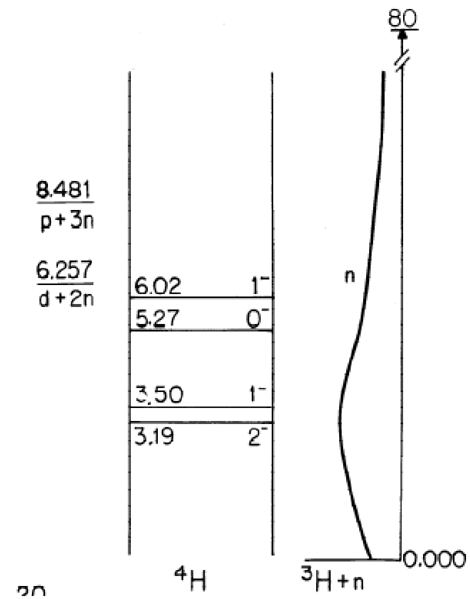


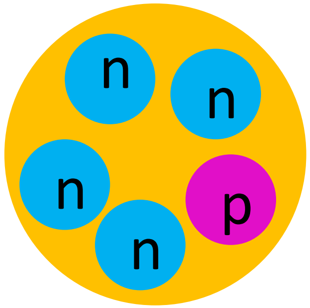
Table 4.1: Energy levels of ^{4}H defined for channel radius $a_n = 4.9$ fm. All energies and widths are in the cm system.

E_x (MeV)	J^{π}	T	Γ (MeV)	Decay	Reactions
g.s. [a]	2^-	1	5.42	n, ^{3}H	1, 11
0.31	1^-	1	6.73 [b]	n, ^{3}H	11, 12
2.08	0^-	1	8.92	n, ^{3}H	
2.83	1^-	1	12.99 [c]	n, ^{3}H	11, 12

[a] 3.19 MeV above the n + ^{3}H mass.

[b] Primarily $^{3}P_1$.

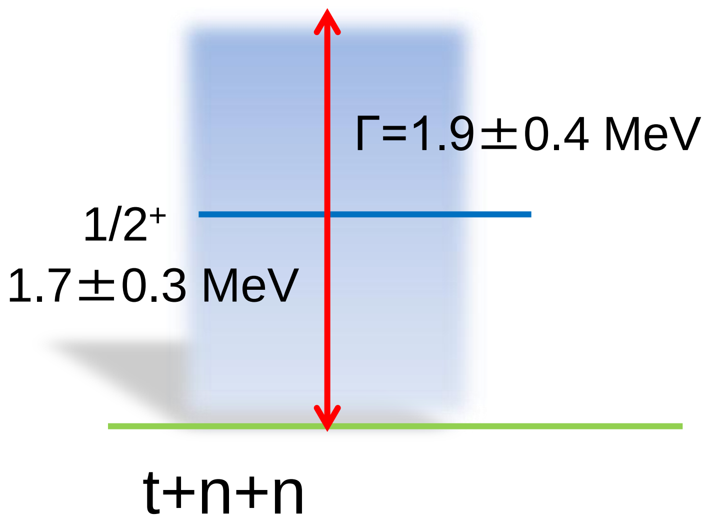
[c] Primarily $^{1}P_1$.



^{5}H

transfer reaction p(^{6}He, ^{2}He)^{5}H

A. A. Korcheninnikov, et al. Phys. Rev. Lett. 87 (2001) 092501.



Superheavy hydrogen

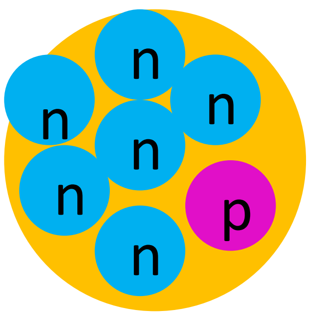
(E_R, Γ_R) (MeV)	
J^π	$1/2^+$
^{5}H (full)	(1.57, 1.53)
^{5}H ($d = 0$)	(1.55, 1.35)
Theor. [16]	(2.26, 2.93)
Theor. [12]	(2.5–3.0, 3–4)
Theor. [13]	(3.0–3.2, 1–4)
Theor. [15]	(1.59, 2.48)
Exp. [3]	(1.7 ± 0.3, 1.9 ± 0.4)
Exp. [8]	(1.8 ± 0.1, < 0.5)
Exp. [4]	(1.8, 1.3)
Exp. [5]	(2, 2.5)
Exp. [6]	(3, 6)
Exp. [9]	(5.5 ± 0.2, 5.4 ± 0.6)

[3] A.A. Korosheninnikov et al., PRL87 (2001) 092501
[8] S.I. Sidorchuk et al., NPA719 (2003) 13
[4] M.S. Golovkov et al. PRC 72 (2005) 064612
[5] G. M. Ter-Akopian et al., Eur. Phys. J A25 (2005) 315.

Energy of 5H is similar. But decay width is dependent on experiment.

In 2017, we have a new data on ^{5}H.
A. H. Wuosmaa, Phys. Rev. C95, 014310 (2017)
$^{6}He\ (d,{}^{3}He)\ {}^{5}H$

$E_r = 2.4 \pm 0.3$ MeV $\Gamma = 5.3 \pm 0.4$ MeV



^{7}H

A. A. Korsheninnikov et al., PRL 90, 082501 (2003)
M. Caamano et al., PRL99, 062502(2007)
PRC 78, 044001 (2008)

$E_r = 0.57^{+0.42}_{-0.21}$ MeV from t+4n threshold

$\Gamma = 0.09^{+0.94}_{-0.06}$ MeV

$^{12}C({}^{8}He,{}^{7}H){}^{13}N$ reaction

If we have narrow
decay at lower energy,
there could exist in
have heavier H-hydrogen isotope
such as ^{9}H.

What is limit for H-isotope? Probably ^{7}H?

Theoretical calculation for ^{5}H and ^{7}H

N. K. Timofeyuk, PRC65, 064306(2002), PRC69 , 034336(2004)
Volkov NN potential, Hyperspherical harmonics method: 5-body and 7-body calculations
^{5}H: about 1 MeV above t+n+n threshold.
^{7}H: about 3MeV above t+4n threshold
She calculated the energies with bound state approximation.
Then, she did not give decay width for these nuclei.

S. Aoyama and N. Itagaki, PRC80,021304 (R)
Volkov NN potential, AMD calculation
^{7}H: 4.2 MeV above t+4n threshold, no calculation for decay width
No report for the energy of ^{5}H

H. H. Li et al., PRC 104, L061306 (2021)
Gamow shell model calculation using Minnesota NN potential.

Energy and decay width of ^{5}H is 1.4 MeV and 0.5 MeV, respectively.
Energy and decay width of ^{7}H is about 2-3MeV and about 0.1 MeV, respectively.

They predicted to have very narrow decay width for ^{5}H and ^{7}H.

Experiment situation:

Recently, ^{8}He (p,2p) ^{7}H reaction has been done at RIBF.
RIBF Experimental Proposal NP1512-SAMURAI34.
The analysis is on going.

Then, it is timely to calculate ^{7}H to obtain the energy and width theoretically.

Motivated by this situation, we study ^{7}H structure within the framework of t+4n 5-body problem. We also discuss on the energy and decay width of ^{5}H within t+n+n three-body problem.

^{7}H ground state as a t+4n resonance

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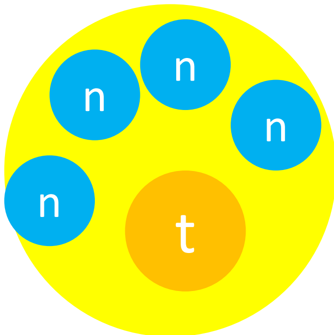
Almost submitted to Physics Letters B

Acknowledgments to KITP workshop

Framework

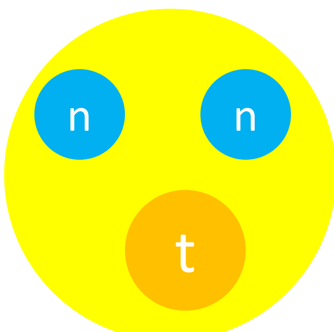


NN: Minnesota potential (central potential)

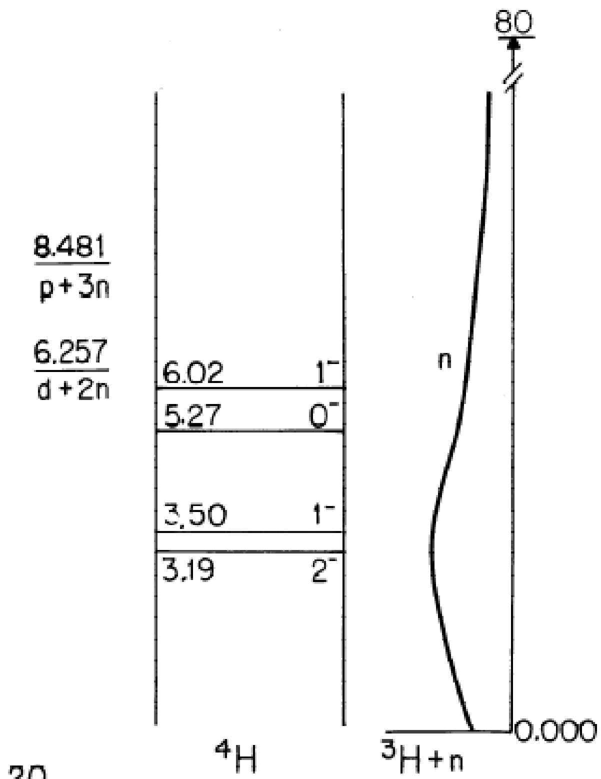
^{7}H=t+4n model

t-n potential => there is a large degree of ambiguity.

Only several data for phase shift of t-n



^{5}H



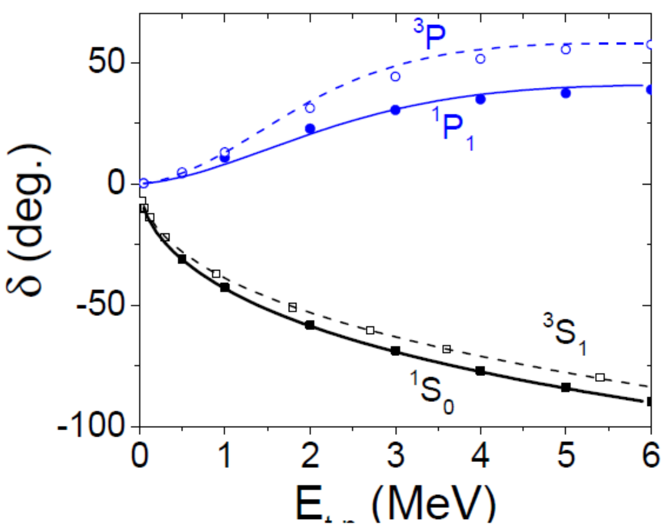
$$V(r,l,s)_{nt} = \delta_{l,0}|\varphi_0\rangle\lambda_\infty\langle\varphi_0| + \sum_{i=1}^{2}\left(v_i^{(c)} + (-)^l v_i^{(P)} + \frac{\hat{s}^2}{2} v_i^{(s)} + (-)^l \frac{\hat{s}^2}{2} v_i^{(SP)}\right)\exp(-\alpha_i r^2)$$

$$|\varphi_0\rangle = \exp(-a_0 r^2)$$

$$\lambda_\infty = \infty$$

i	1	2
$\alpha_i (fm^{-2})$	0.471241	0.0549825
$v_i^{(c)} (MeV)$	−41.3619	1.22768
$v_i^{(P)} (MeV)$	−0.309720	6.89574
$v_i^{(s)} (MeV)$	−28.2483	−0.972465
$v_i^{(SP)} (MeV)$	10.3308	−1.25695

$$a_0 = 0.1979068 \quad fm^{-2}$$



Based on four-body calculation with MT I-III

α_i	V_{nt} (1)	4N [12]
$L = 1^-, S = 0$	1.28-2.61 i	0.88(5)-2.20(5) i
$L = 1^-, S = 1$	1.33-1.84 i	1.08(3)-2.03(3) i

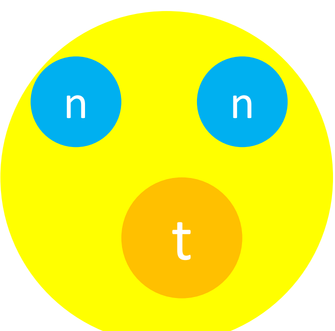
Two-body calculation of t-n is almost consistent with that of 4-body calculation.

+ I introduce a phenomenological three-body t-n-n force to obtain energy trajectory.

$$V_{tnn}(\rho) = -V_0\, e^{-\frac{\rho^2}{b_3^2}} \qquad \rho^2 = \frac{m_n}{M} r_{nn}^2 + \frac{m_t}{M^2} r_{nt}^2 + \frac{m_t}{M^2} r_{nt}^2 \qquad M = 2m_n + m_t$$

V_0, b_3 : parameters. → Fit so as to reproduce the data of ^5H

↓ apply



^5H

Our few-body caluclation method

Gaussian Expansion Method (GEM) , since 1987,

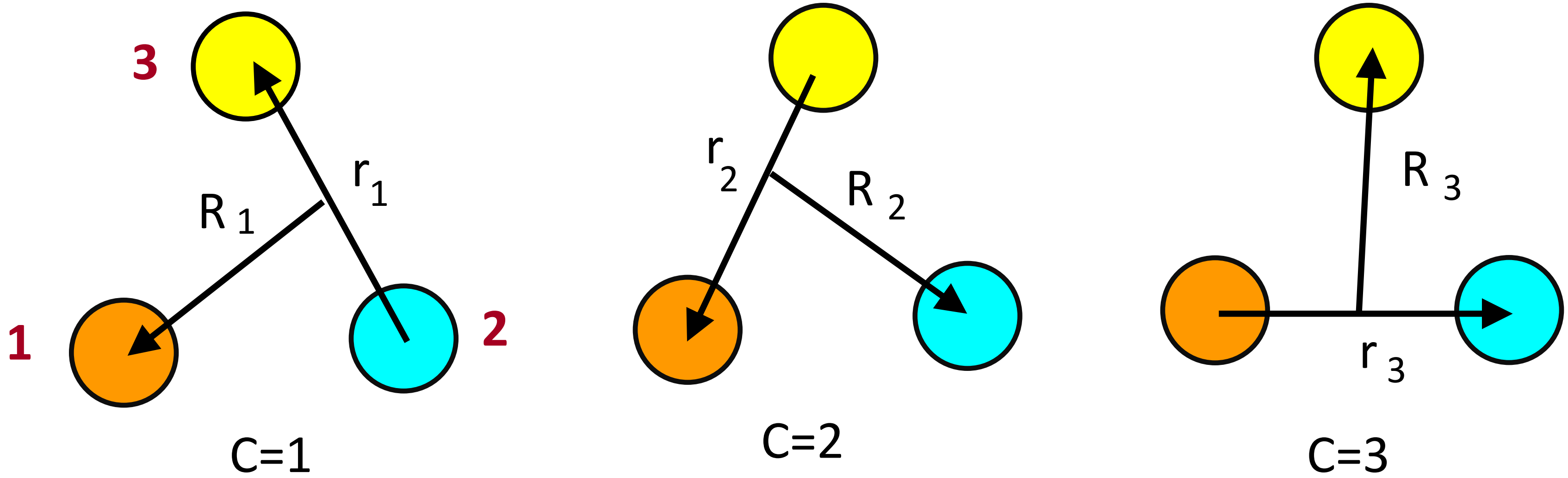
- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group,
Kamimura and his collaborators.

Review article :
E. Hiyama, M. Kamimura and Y. Kino,
Prog. Part. Nucl. Phys. 51 (2003), 223.

High-precision calculations of various 3- and 4-body systems:

Exotic atoms / molecules ,
3- and 4-nucleon systems,
multi-cluster structure of light nuclei,
Light hypernuclei,
3-quark systems,

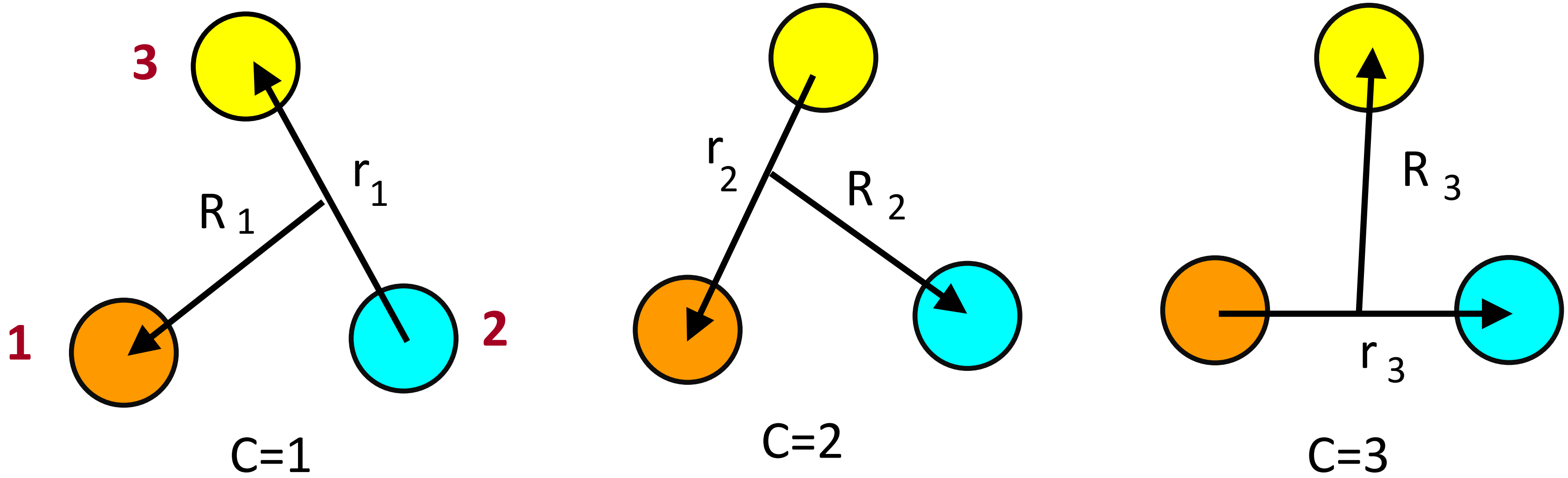


$$(H - E)\Psi_{JM} = 0$$

$$H = T + V_1(r_1) + V_2(r_2) + V_3(r_3)$$

$$T = -\frac{\hbar^2}{2\mu_{r_c}}\nabla^2_{\mathrm{r}_c} - \frac{\hbar^2}{2\mu_{R_c}}\nabla^2_{\mathrm{R}_c} \qquad (c = 1, 2, \text{ or } 3)$$

$$\Psi_{JM} = \Phi^{(1)}_{JM}(\mathrm{r}_1, \mathrm{R}_1) + \Phi^{(2)}_{JM}(\mathrm{r}_2, \mathrm{R}_2) + \Phi^{(3)}_{JM}(\mathrm{r}_3, \mathrm{R}_3)$$



$$\Psi_{JM} = \Phi^{(1)}_{JM}(\mathrm{r}_1, \mathrm{R}_1) + \Phi^{(2)}_{JM}(\mathrm{r}_2, \mathrm{R}_2) + \Phi^{(3)}_{JM}(\mathrm{r}_3, \mathrm{R}_3)$$

Basis functions of each Jacobi coordinate

$$\phi^{(c)}_{nl}(r_c)\, Y_{lm}(\hat{\mathrm{r}}_c), \quad \psi^{(c)}_{NL}(R_c)\, Y_{LM}(\widehat{\mathrm{R}}_c), \quad (c = 1, 2, 3)$$

$\hat{\mathrm{r}}_c \rightarrow (\theta, \phi)$ $\qquad$ $\widehat{\mathrm{R}}_c \rightarrow (\Theta, \Phi)$

$$\Phi^{(c)}_{JM}(\mathrm{r}_c, \mathrm{R}_c) = \sum_{nl,NL} \underline{A^{(c)}_{nl,NL}}\, \phi^{(c)}_{nl}(r_c)\, \psi^{(c)}_{NL}(R_c) \left[Y_l(\hat{\mathrm{r}}_c) \otimes Y_L(\widehat{\mathrm{R}}_c) \right]_{JM}$$

↑ Determined by diagonalizing H

For this purpose, we use the following basis function:

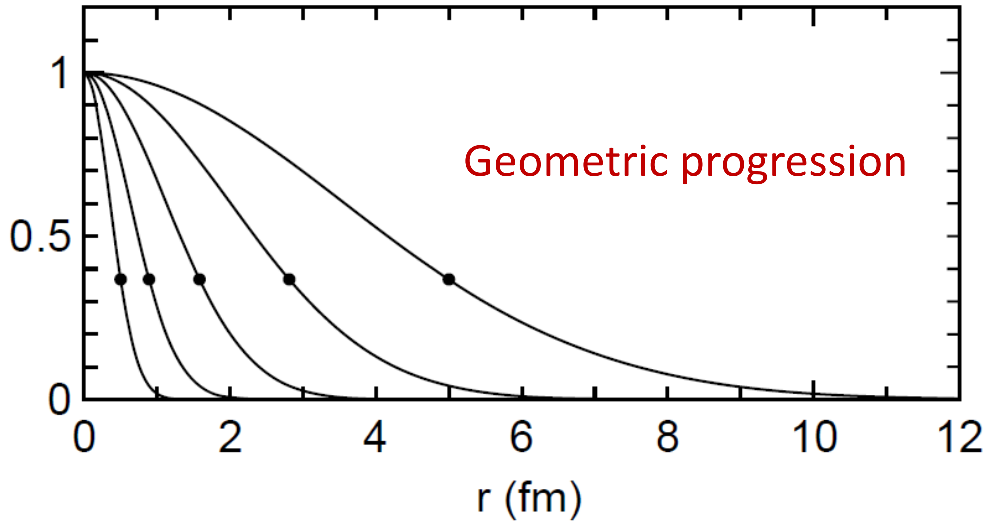
$$\phi_{nlm}(\mathbf{r}) = r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}})$$

$$\nu_n = \frac{1}{r_n^2}$$

Geometric progression

$$r_n = r_1 a^{n-1} \qquad (n = 1, \ldots, n_{\max})$$

The Gaussian basis function is suitable not only for the calculation of the matrix elements but also for describing short-range correlations and long-range tail behaviour.



Where the energy and overlap matrix elements are given by

$H_{in} = \langle \Phi_i \mid H \mid \Phi_n \rangle \quad (i, n = 1,\ldots,N)$

$N_{in} = \langle \Phi_i \mid 1 \mid \Phi_n \rangle$ --- non-orthogonal basis

Next, we get eigenenergy E and coefficients C_n

by solving generalized matrix eigenvalue problem,

$$(\mathbf{H} - \mathbf{E})\Psi = 0 \qquad \Psi = \sum_{n=1}^{N} C_n \Phi_n$$

$$\downarrow$$

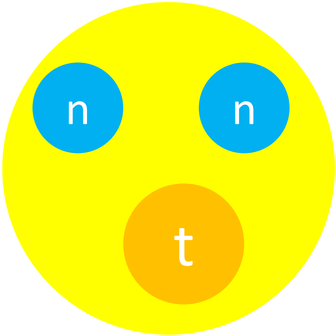
$$\left[(H_{in}) - E(N_{in}) \right] \left[C_n \right] = 0$$

solution $\Psi = \Psi_0, \Psi_1, \Psi_2, \ldots, \Psi_N$

$E = E_0, E_1, E_2, \ldots, E_N$

The calculation is for the bound states.

Observed data of ^5H is resonant state.

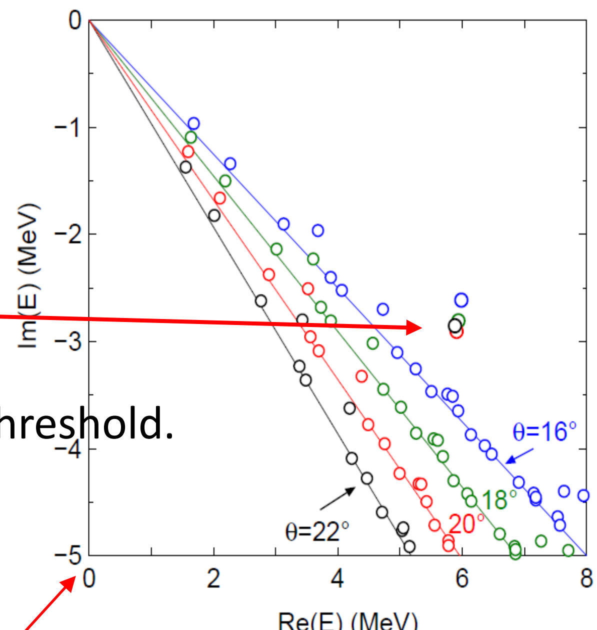


^5H

To obtain resonant state of ^5H,
we use complex scaling method.

$$r_c \to r_c e^{i\theta},\ R_c \to R_c e^{i\theta}$$

The energy pole is stable
with respect to θ.
Re(E) corresponds to energy
With respect to 4n breakup threshold.
Im(E) corresponds to Γ/2.

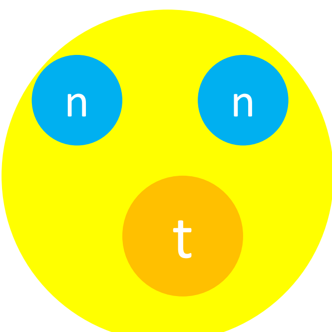


t+n+n breakup threshold

+ I introduce a phenomenological three-body t-n-n force to obtain energy trajectory.

$$V_{tnn}(\rho) = -V_0\, e^{-\frac{\rho^2}{b_3^2}} \qquad \rho^2 = \frac{m_n}{M} r_{nn}^2 + \frac{m_t}{M^2} r_{nt}^2 + \frac{m_t}{M^2} r_{nt}^2 \qquad M = 2m_n + m_t$$

V_0, b_3 : parameters. → Fit so as to reproduce the data of ^5H



^5H

Question: Which experimental data of ^5H should we fit?

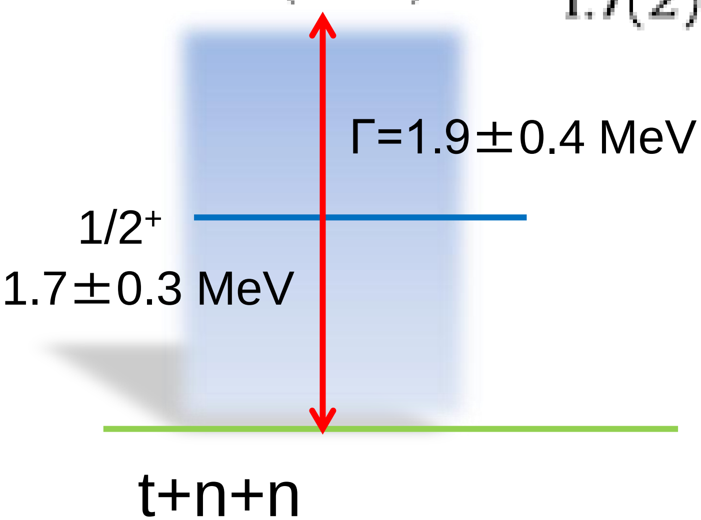
(E_R, Γ_R) (MeV)	
J^π	$1/2^+$
^{5}H (full)	(1.57, 1.53)
^{5}H ($d = 0$)	(1.55, 1.35)
Theor. [16]	(2.26, 2.93)
Theor. [12]	(2.5–3.0, 3–4)
Theor. [13]	(3.0–3.2, 1–4)
Theor. [15]	(1.59, 2.48)
Exp. [3]	$(1.7 \pm 0.3, 1.9 \pm 0.4)$
Exp. [8]	$(1.8 \pm 0.1, < 0.5)$
Exp. [4]	(1.8, 1.3)
Exp. [5]	(2, 2.5)
Exp. [6]	(3, 6)
Exp. [9]	$(5.5 \pm 0.2, 5.4 \pm 0.6)$

[3] A.A. Korosheninnikov et al., PRL87 (2001) 092501
[8] S.I. Sidorchuk et al., NPA719 (2003) 13
[4] M.S. Golovkov et al. PRC 72 (2005) 064612
[5] G. M. Ter-Akopian et al., Eur. Phys. J A25 (2005) 315.

Energy of ^{5}H is similar. But decay width is dependent on experiment.

R. Lazauskas, E. Hiyama, J. Carbonell, PRB 791 335 (2019)
Fadeev-Yakubovsky method calculation of ^5H

		J=1/2$^+$	
		E_R	Γ
N3LO	$^{(ACCC)}$	1.8(1)	2.4(2)
	$^{(SECS)}$	1.9(2)	2.4(2)
INOY	$^{(ACCC)}$	1.7(1)	2.4(2)
	$^{(SECS)}$	1.8(1)	2.4(2)
MT13	$^{(ACCC)}$	1.4(1)	2.4(2)
	$^{(SECS)}$	1.7(2)	2.4(2)



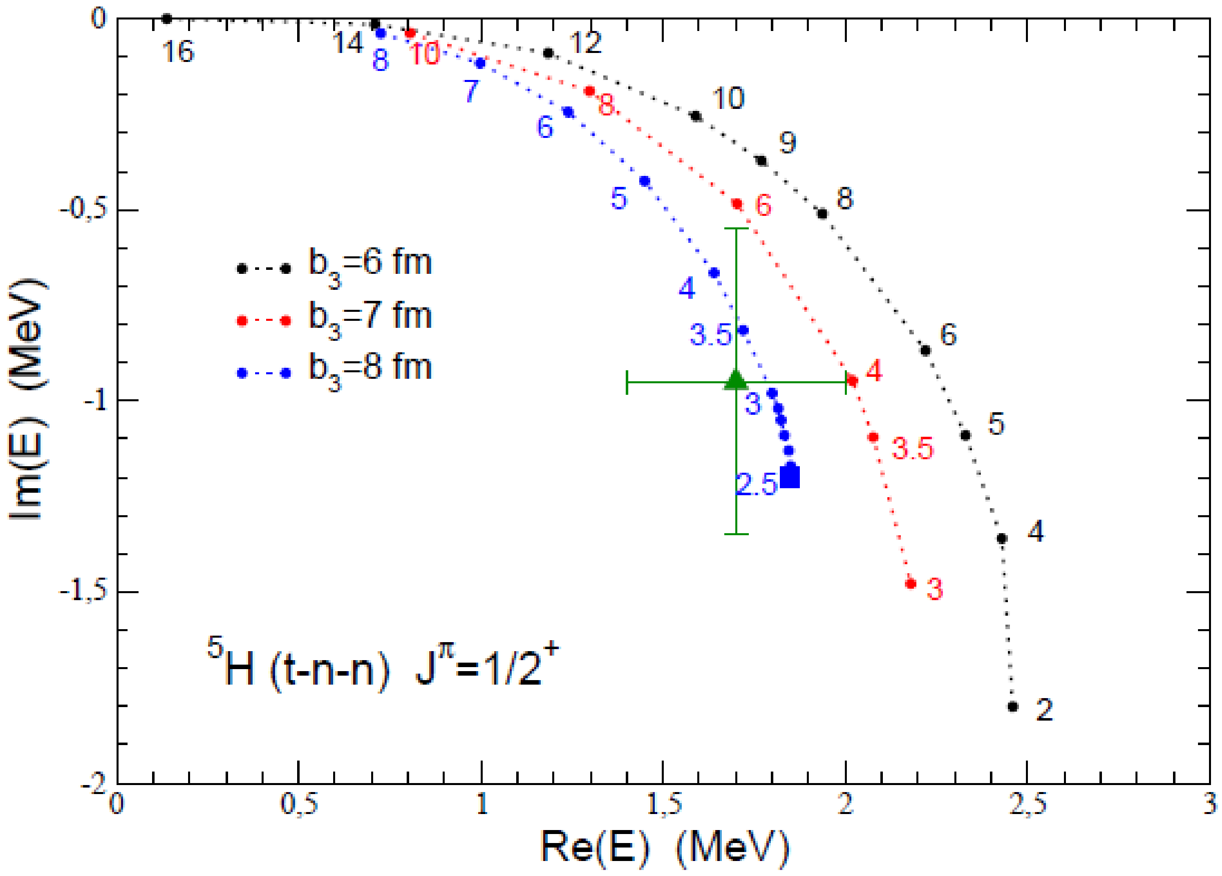
We take this result as 'exp.' data.

Close to the below exp.data

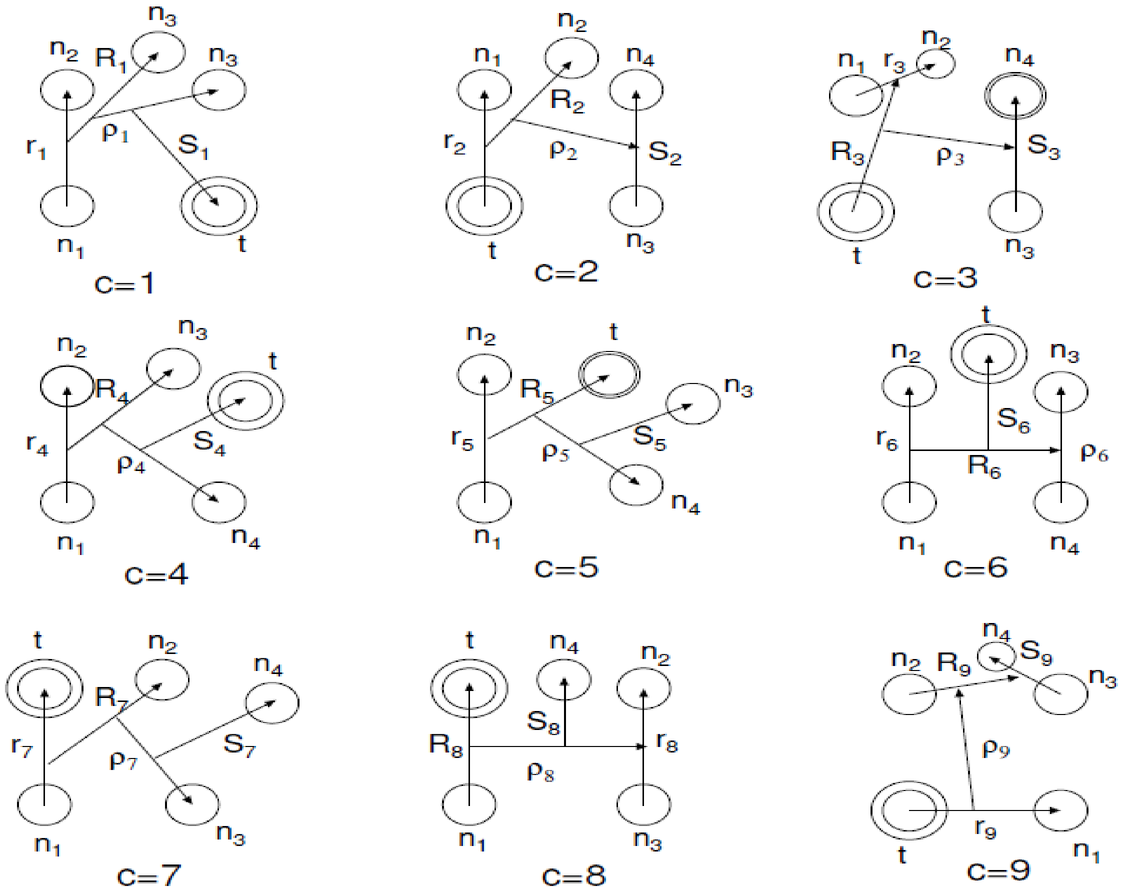
A. A. Korcheninnikov, et al. Phys. Rev. Lett. 87 (2001) 092501.

$$V_{tnn}(\rho) = -V_0 \, e^{-\frac{\rho^2}{b_3^2}} \qquad \rho^2 = \frac{m_n}{M} r_{nn}^2 + \frac{m_t}{M^2} r_{nt}^2 + \frac{m_t}{M^2} r_{nt}^2 \qquad M = 2m_n + m_t$$

When b_3=8 fm and V_0=3 to 2.5 MeV, the energy pole of ^5H is close to exp. data. If we have this potential parameter, what is energy pole of ^7H?



Framework of ^{7}H   The Hamiltonian is the same as the case of ^{5}H.



Totally 120 Jacobi coordinates

$$
\begin{aligned}
\Psi_{JM}(^7\mathrm{H}) \;=\; & \left[\left[[[\eta_{\frac{1}{2}}(n)\eta_{\frac{1}{2}}(n)]_t\eta_{\frac{1}{2}}(n)]_{T_0}\eta_{\frac{1}{2}}(n)\right]_{T_4}\eta_{\frac{1}{2}}(t)\right]_{TT_z} \\
\times\; & \left[\left[\left[[[\chi_{\frac{1}{2}}(n)\chi_{\frac{1}{2}}(n)]_t\chi_{\frac{1}{2}}(n)]_{\Sigma}\chi_{\frac{1}{2}}(n)\right]_{S_4}\chi_{\frac{1}{2}}(t)\right]_S \\
\times\; & [[[\phi_\ell(r_c)\psi_L(R_c)]_\Lambda\phi_\lambda(\rho_c)]_I\phi_\xi(s_c)]_K\Big]_{JM}
\end{aligned}
$$

Form of each basis function

5-body spatial function

$$\Big[\, \Big[\, \big[\phi^{(c)}_{nl}(\mathbf{r}_c)\, \psi^{(c)}_{NL}(\mathbf{R}_c)\big]_I\, \varphi^{(c)}_{n'l'}(\boldsymbol{\rho}_c)\Big]_K\, \Phi^{(c)}_{N'L'}(\mathbf{S}_c)\Big]_L$$

Gaussian for radial part :

$$\phi_{nlm}(\mathbf{r}) = r^l\, e^{-(r/r_n)^2} Y_{lm}(\hat{\mathbf{r}})$$

geometric progression for Gaussian ranges :

$$r_n = r_1 a^{n-1} \qquad (n = 1 - n_{\rm max})$$

Similarly for the other basis :

$$\psi^{(c)}_{NLM}(\mathbf{R}_c) \quad \varphi^{(c)}_{n'l'm'}(\boldsymbol{\rho}_c) \quad \Phi^{(c)}_{N'L'M'}(\mathbf{S}_c)$$

Use of this type gaussian basis is known to be very suitable for describing simultaneously both the short-range correlations and long-range tail behaviour of few-body systems;

This is precisely shown in →

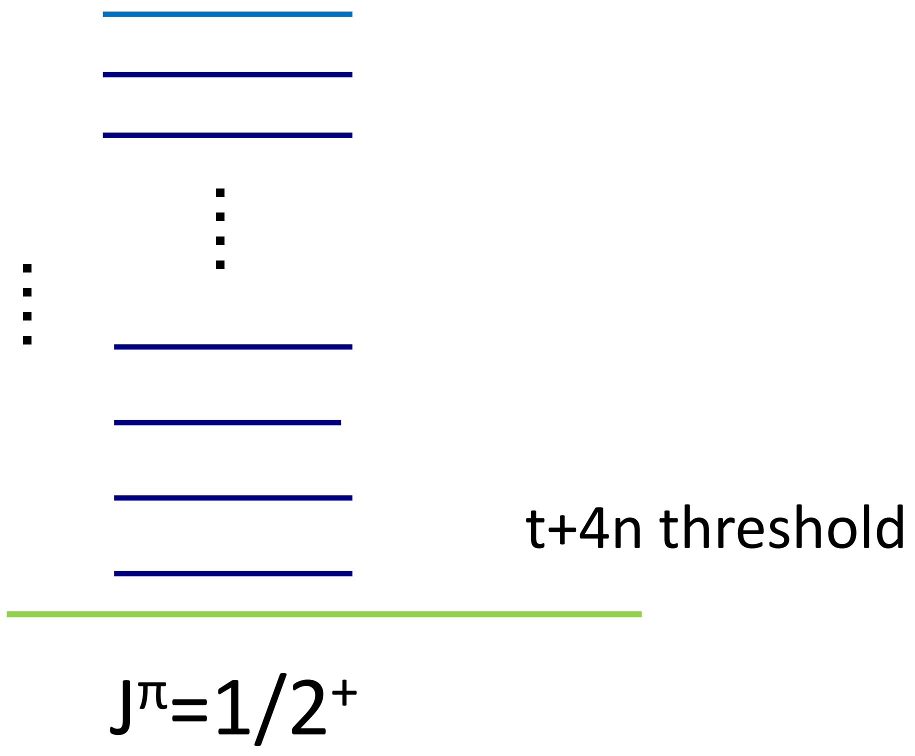
Gaussian Expansion Method (GEM)
(review paper) E. H., Y. Kino and M. Kamimura, Prog. Part. Nucl. Phys., 51 (2003) 223.

$(H-E)\Psi=0$

By the diagonalization of Hamiltonian, we obtain N eigenstates for each J^{π}.

Here, we use about 56,000 basis functions.

Then, we obtained 56,000 eigenfunctions for $J^{\pi}=1/2^{+}$.



For the calculation of ^{7}H, it would be difficult to apply complex scaling method for 5-body calculation. Then, for this calculation, I used real scaling method.

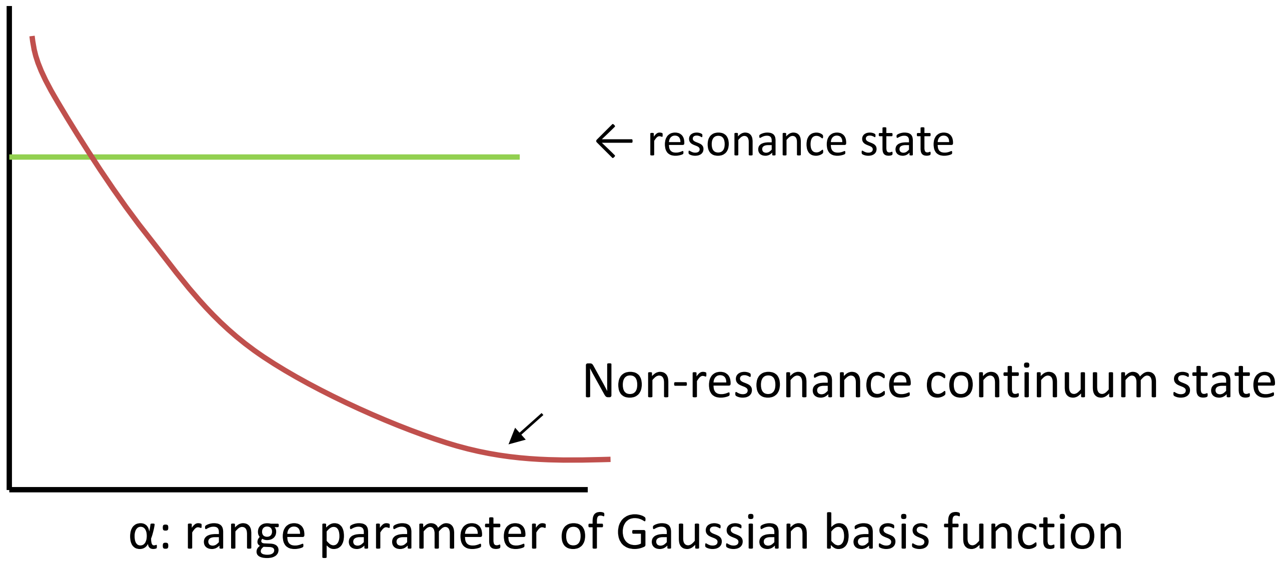
useful method: real scaling method

often used in atomic physics

In this method, we artificially scale the range parameters of our Gaussian basis functions by multiplying a factor α:

$r_n \rightarrow \alpha r_n$ in $r^l \exp^{(-r/r_n)^2}$ for exmple $0.8 < \alpha < 1.5$

and repeat the diagonalization of Hamiltonian for many value of α.

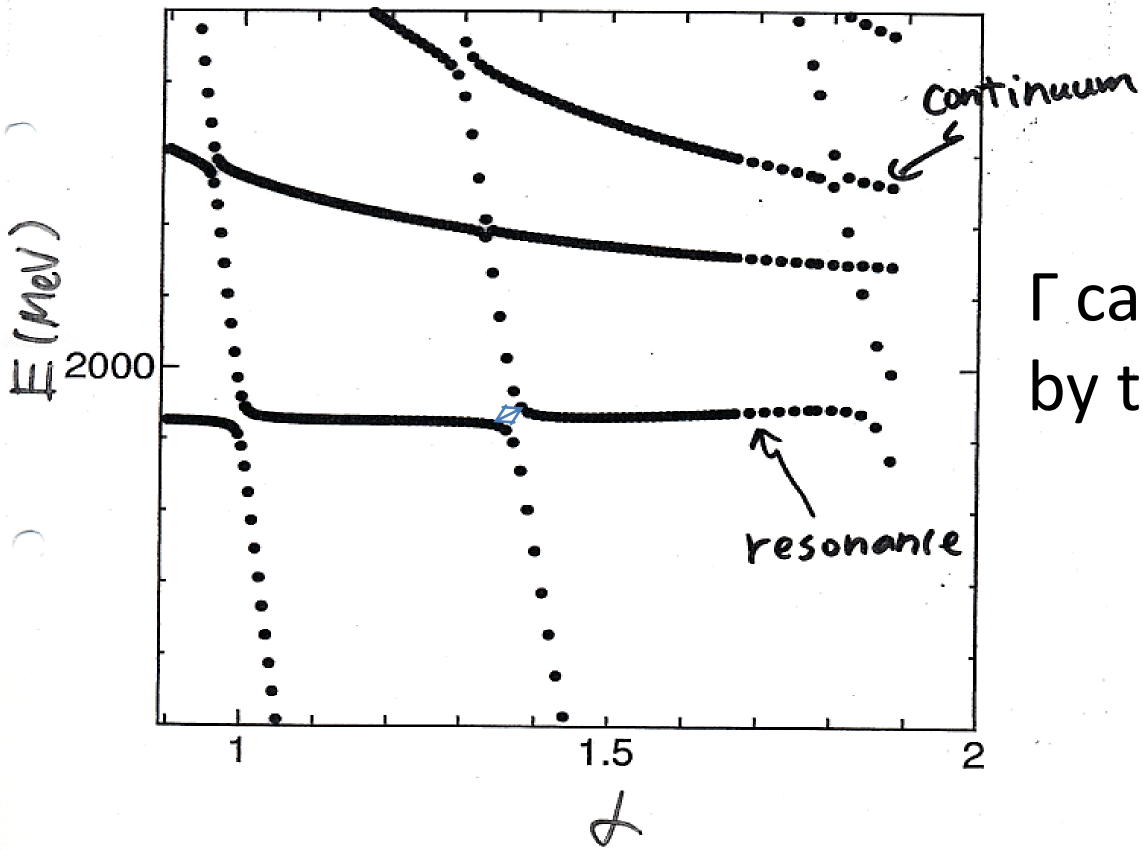


[schematic illustration of the real scaling]

What is the result in our pentaquark calculation?

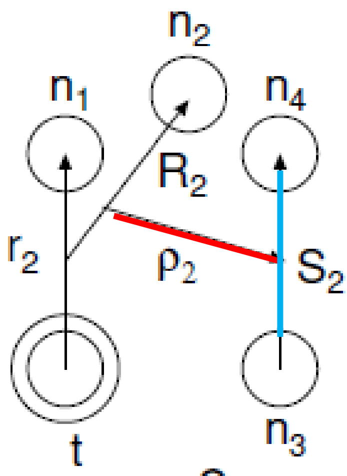
Example of real scaling

Not result of penta quark system

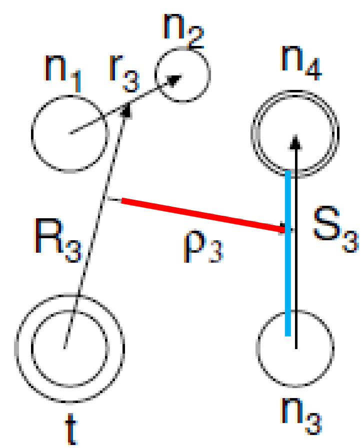


Γ can be estimated by the ΔE.

What is the result of our pentquark calculation?



c=2



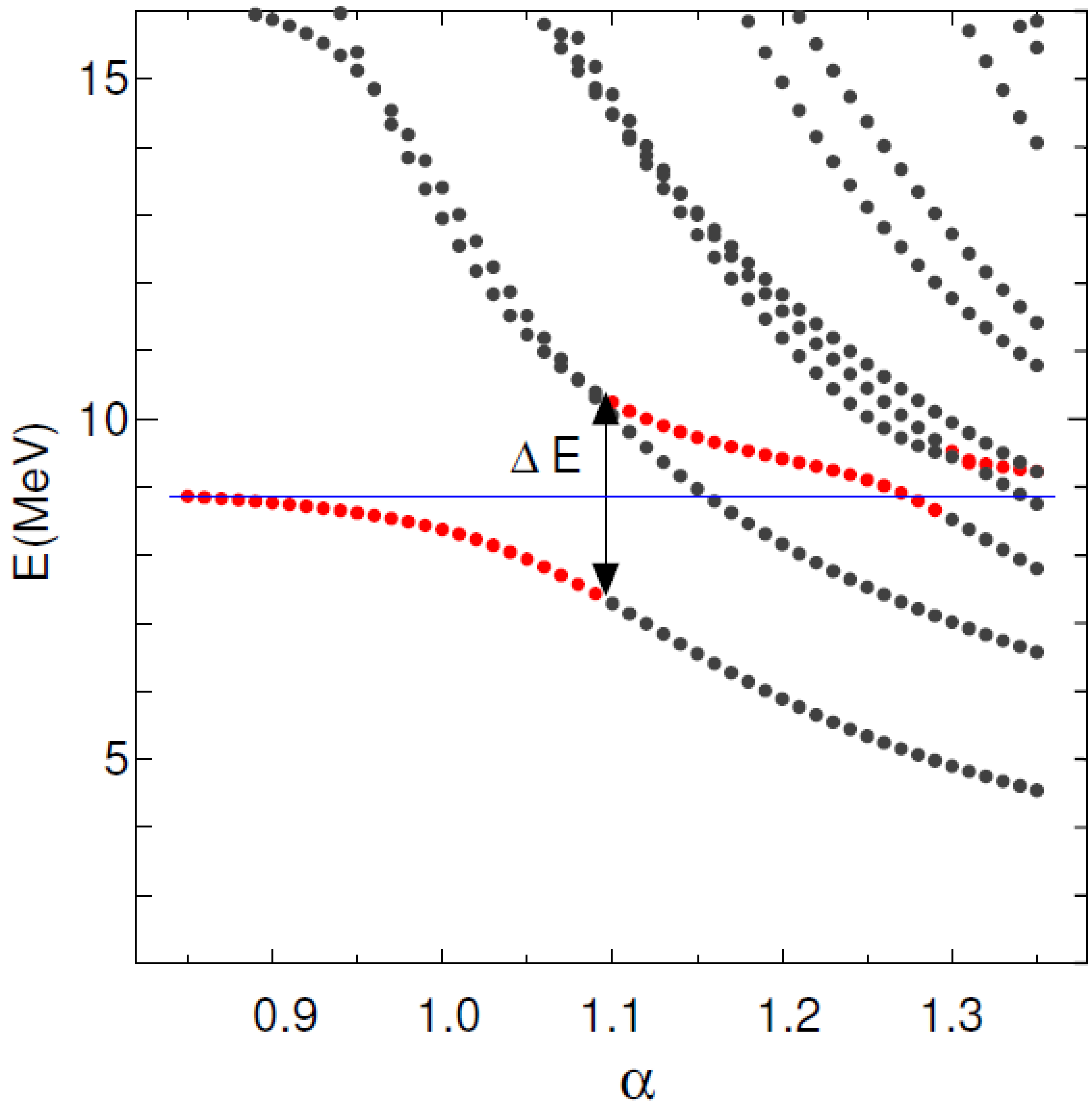
c=3

$\rho_n \Rightarrow \alpha \rho_n$

$s_n \Rightarrow \alpha s_n$

$$V_{tnn}(\rho) = -V_0\, e^{-\frac{\rho^2}{b_3^2}}$$

b_3=8.0fm V_0=-3 MeV

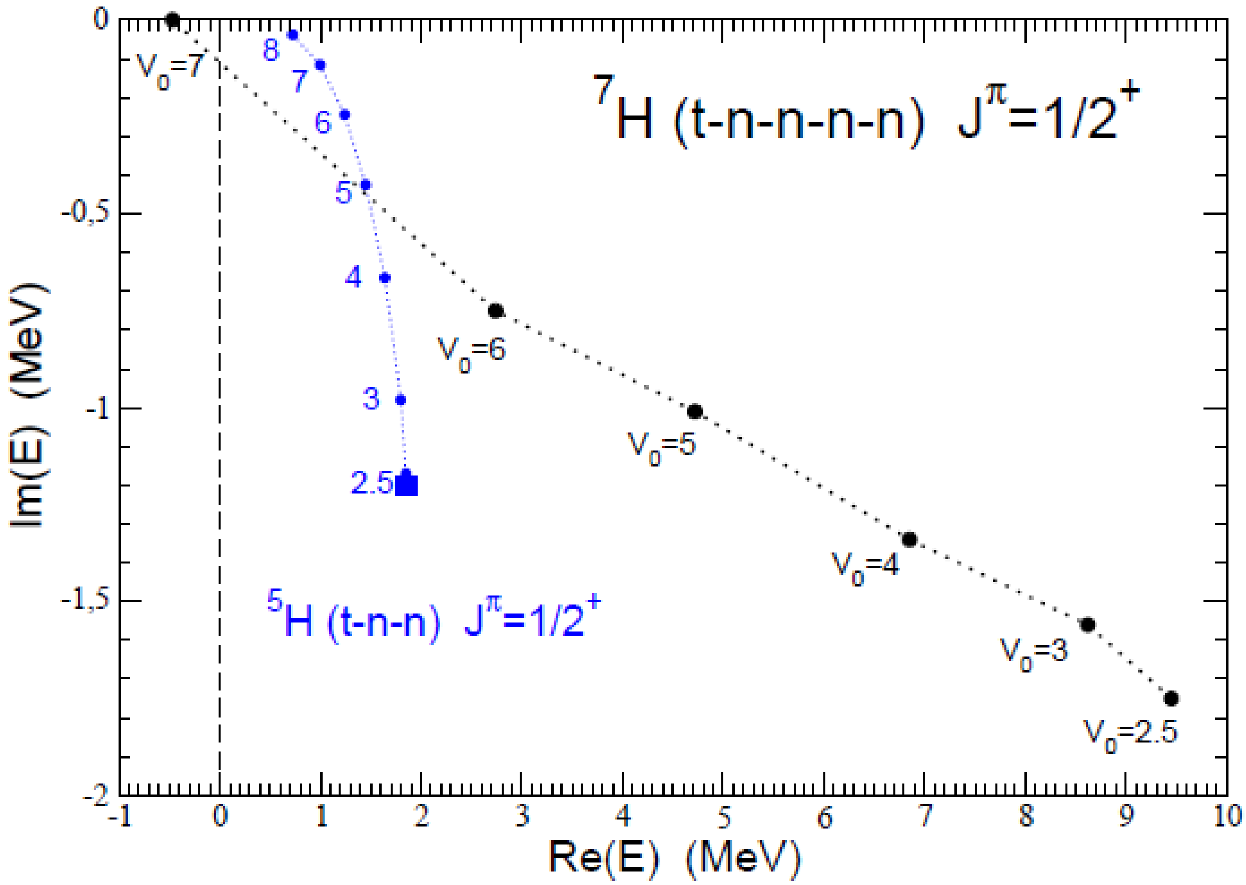


Er∼8.8 MeV

Γ∼ 3.1 MeV

5H: close to Exp. data

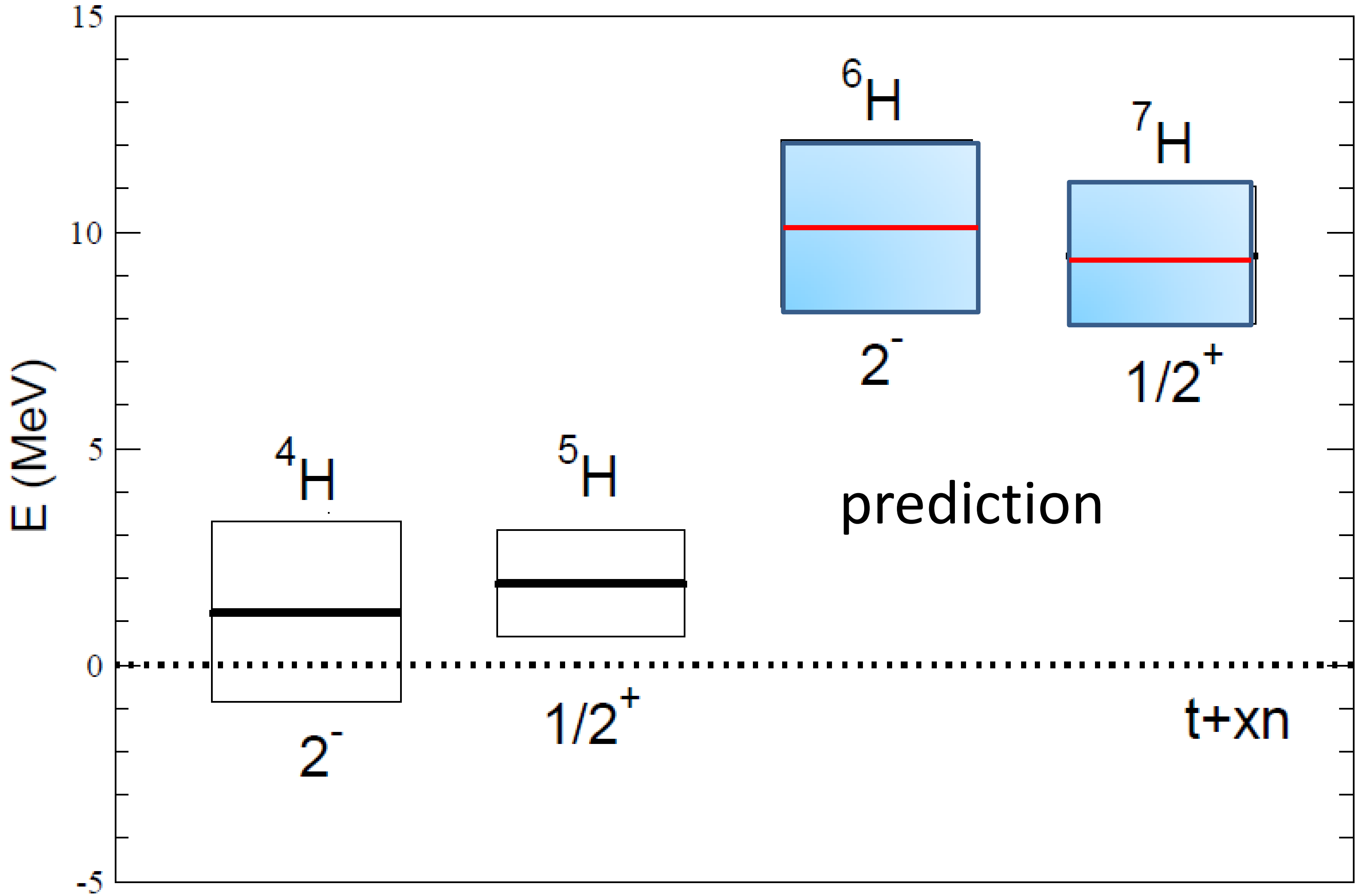
Im (E)=Γ/2



For V_0=2.5, we reproduce the data of ^5H accurately.
In this case, the energy pole of ^7H, E=9.5 MeV, Γ～3.5 MeV.
Our energy of ^7H is much higher and broad decay width.

Summary of H-isotope (according to our calculation)

End of H-isotope



Summary

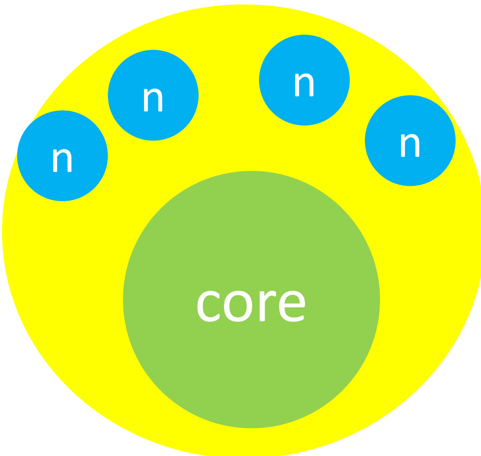
Assuming Er～1.9 MeV and Γ～2.4 MeV for ^{5}H,
Our calculated energy and decay width of ^{7}H are about Er～ 8 to 9 MeV, and Γ～ 3 MeV.
That is much higher than ^{5}H+n+n threshold, broad decay width.

^{8}He (p,2p) ^{7}H reaction was done at RIBF,recently.
RIBF Experimental Proposal NP1512-SAMURAI34.
The analysis is on going.

I am waiting for future experimental result.
If our result of ^{7}H is in good agreement with the data.

Thank you!

Future prospect:



We have a code to calculate core+4n.

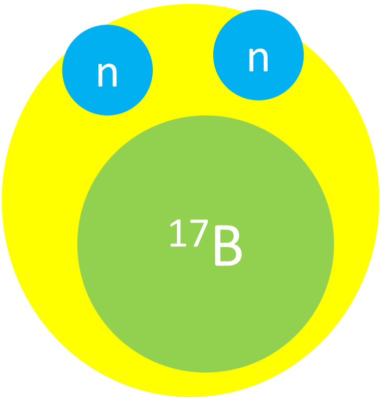
↓

We could apply the method to many neutron-rich nuclei.

Example: $^{19}B = {}^{15}B + 4n$

Recent measurement of ^{19}B(PRL 124, 212503 (2020)

At that time, E. Hiyama, R. Lazauskas, F.M. Marqu´es, and J. Carbonell, Phys. Rev. C 100, 011603(R) (2019).



Next, we plan to study $^{17}B+4n$.

In order to solve few-body problem accurately,

Gaussian Expansion Method (GEM) , since 1987

- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group,
Kamimura and his collaborators.
Review article :
E. Hiyama, M. Kamimura and Y. Kino,
Prog. Part. Nucl. Phys. 51 (2003), 223.

High-precision calculations of various 3- and 4-body systems:

Exotic atoms / molecules ,	Light hypernuclei,
3- and 4-nucleon systems,	3-quark systems,
multi-cluster structure of light nuclei,	^{4}He-atom tetramer

Benchmark-test 4-body calculation ： Phys. Rev. C64 (2001), 044001

Benchmark test calculation of a four-nucleon bound state

by 7 groups

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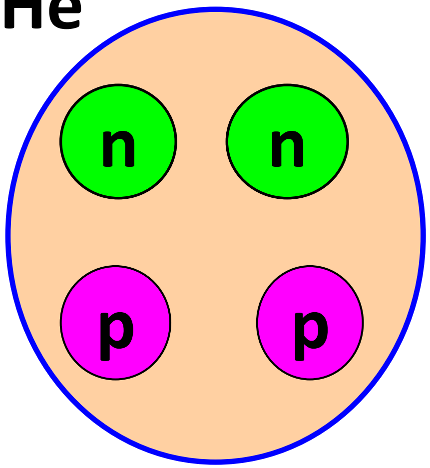
^{4}He



4 nucleon bound state

Realistic NN force: AV8’

Benchmark-test calculation of the 4-nucleon bound state

Good agreement among 7different methods

In the binding energy, r.m.s. radius and wavefunction density

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
GEM	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

ours

very different techniques and the complexity of the nuclear force chosen. Except for NCSM and EIHH, the expectation values of T and V also agree within three digits. The NCSM results are, however, still within 1% and EIHH within 1.5% of the others, but note that the EIHH results for T and V are

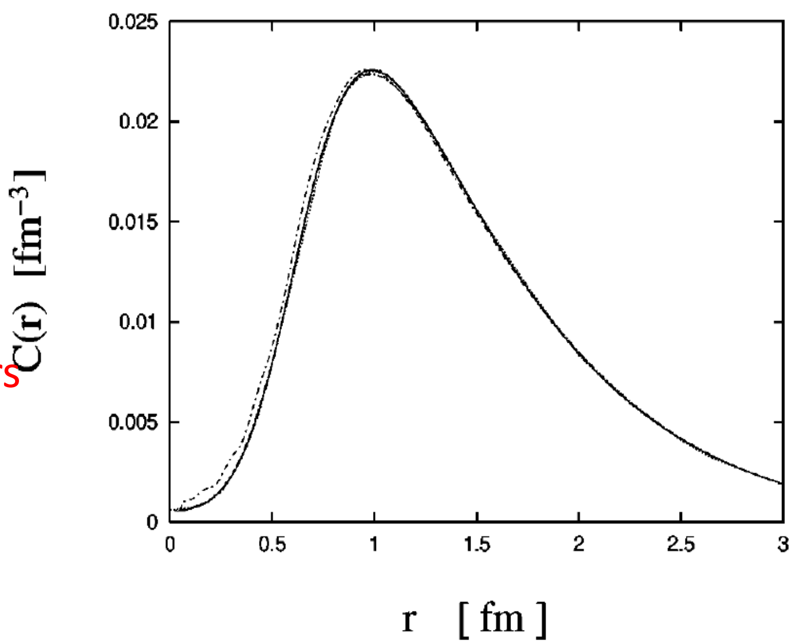


FIG. 1. Correlation functions in the different calculational schemes: EIHH (dashed-dotted curves), FY, CRCGV, SVM, HH, and NCSM (overlapping curves).

After the observed data, there have been positive and negative theoretical results.

Positive result:

A.M. Shirokov et al., PRL117, 182502 (2016).

Non-core shell model calculation+JISP16 NN interaction

Er=0.8 MeV with Γ=1.4 MeV

S. Gandolfi et al., PRL118, 232501 (2017)

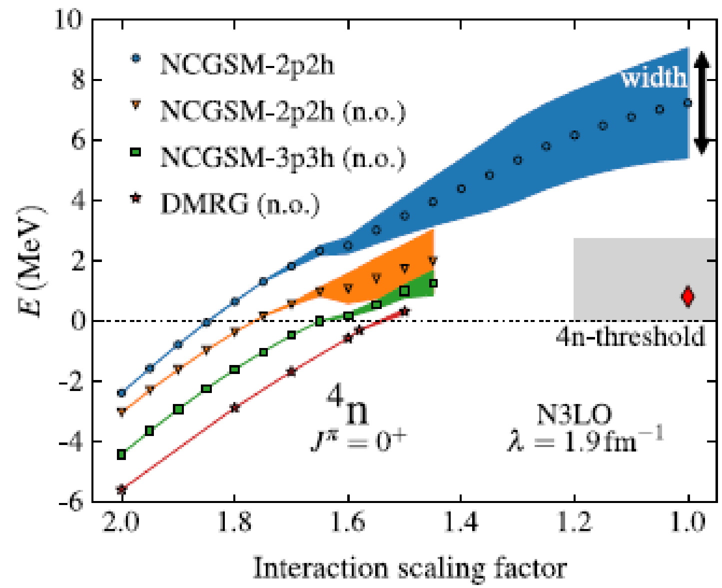
Quantum Monte Calro Method +Chrial (NNLO) interaction+Woods Saxon-well : extrapolation

Er=1.84 MeV with Γ=0.282 MeV

Not positive , not negative result

K. Fossez et al., PRL119, 032501 (2017)
no-core Gamow shell model+ N3LO, JISP16
Er～ 7 MeV, Γ ～3.5 MeV to 3.7 MeV

Much higher energy and
broader width than observed data



$f \times V_{NN}$

Scaling factor

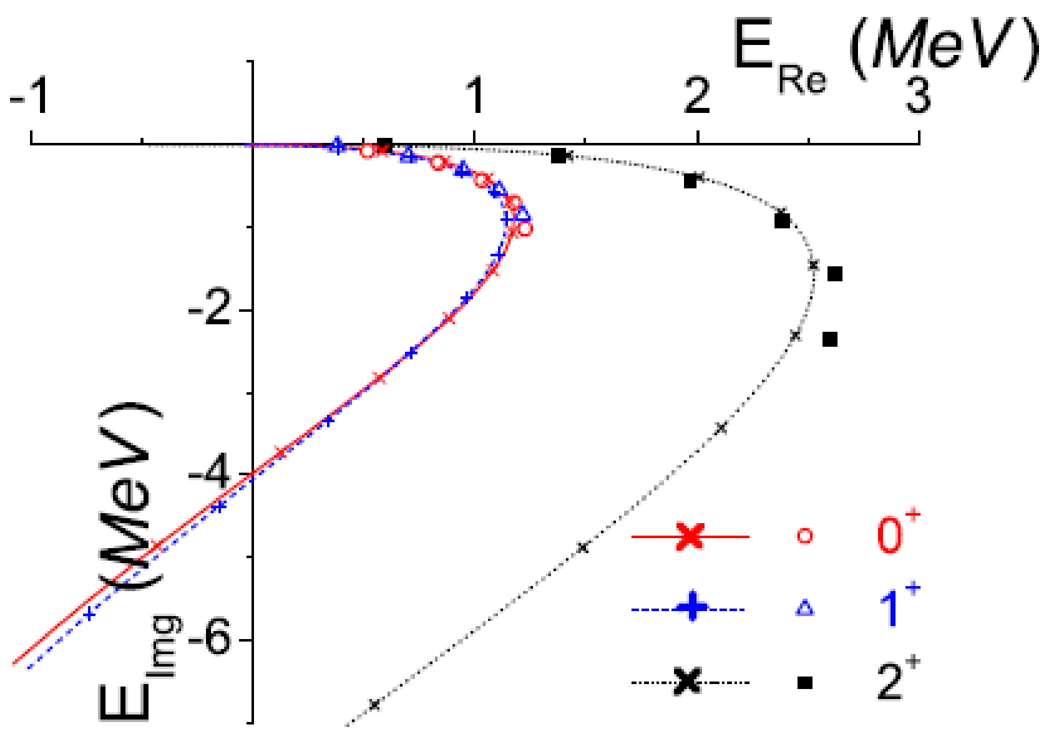
Negative results

R. Lazauskas, and J. Carbonell, Phys. Rev. C72, 034003 (2005).

Before measurement of tetra-neutron system at RIBF

Charge-symmetry-breaking Reid93 nn potential +a phenomenological 4N force

$$V_{4n} = -W\rho e^{-\frac{\rho}{\rho_0}}, \qquad \text{hyperradius } \rho = \sqrt{x^2 + y^2 + z^2}$$



In the case of W=0, energy pole goes to the third quadrant. This means that two-body NN interaction does not produce any resonant state of 4n.

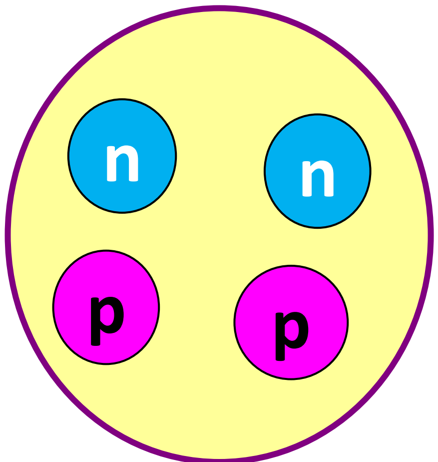
A phenomenological three-body force

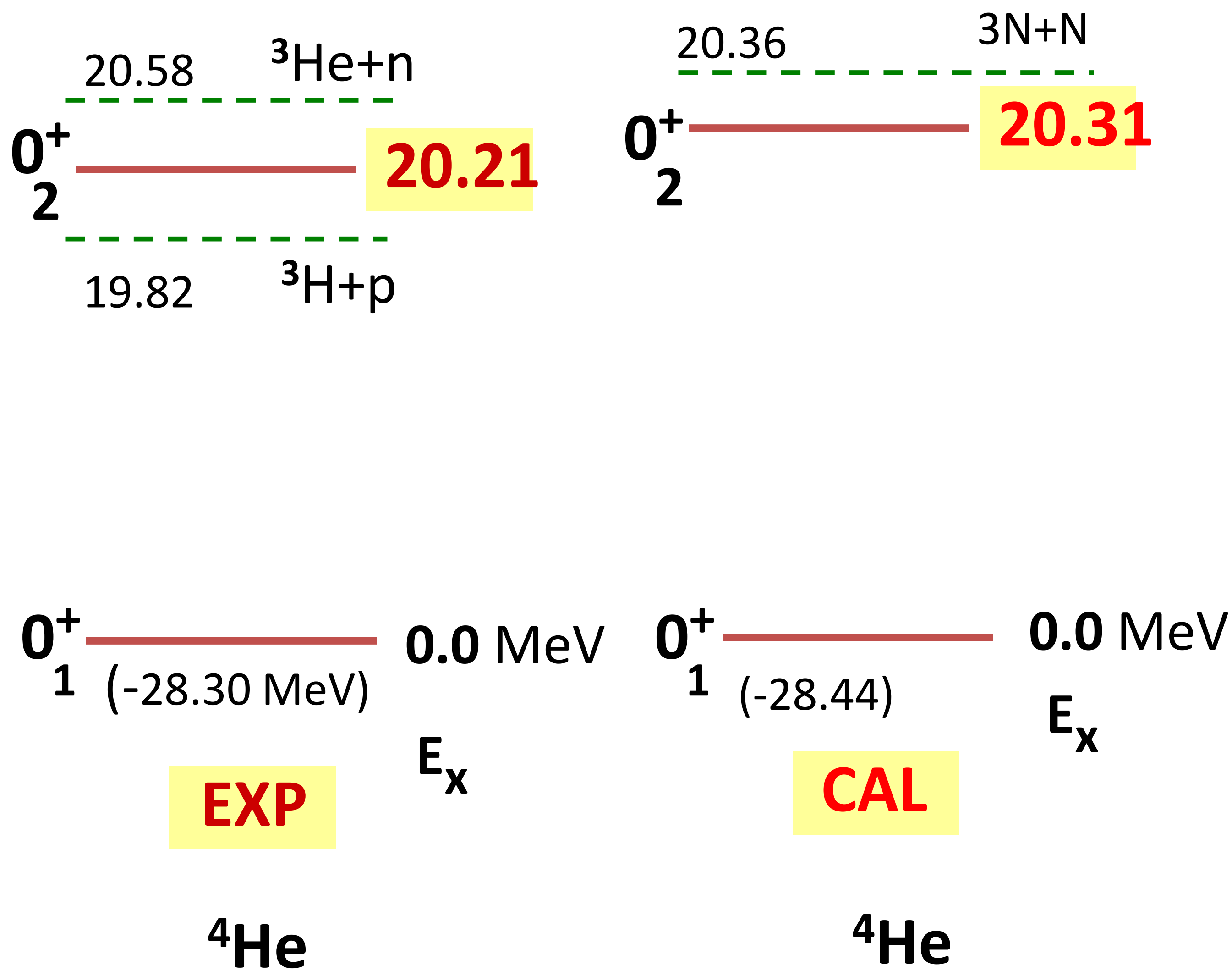
$$V_{ijk}^{3N} = \sum_{T=1/2} \sum_{n=1}^{2} W_n(T) e^{-(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T)$$

$$\begin{aligned} W_1(T=1/2) &= -2.04 \text{ MeV} \quad & b_1 = 4.0 \ \text{ fm} \\ W_2(T=1/2) &= +35.0 \text{ MeV} \quad & b_2 = 0.75 \text{ fm} \end{aligned}$$

This potential has been applied to ^{4}He.

E. Hiyama, B.F. Gibson and M. Kamimura, Phys. Rev. C **70** (2004) 031001(R)

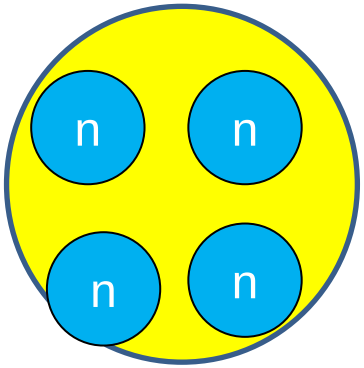




AV8 NN +Coulomb potential + three-body force reproduce the data.

$$V_{ijk}^{3N} = \sum_{T=1/2} \sum_{n=1}^{2} W_n(T) e^{-(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T)$$

$$W_1(T=1/2) = -2.04 \text{ MeV} \quad b_1 = 4.0 \text{ fm}$$
$$W_2(T=1/2) = +35.0 \text{ MeV} \quad b_2 = 0.75 \text{ fm}$$



For 4n system, we need T=3/2 three-body force. We use the same potential with T=1/2, but, different parameter of W_1.

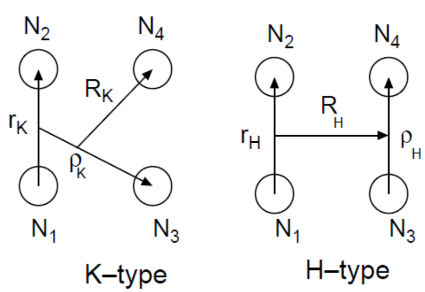
$W_1(T=3/2)$= free b_1=4.0fm => W_1 should be adjusted so as to reproduce the observed 4n system

$W_2(T=3/2)$ = +35 MeV b_2=0.75

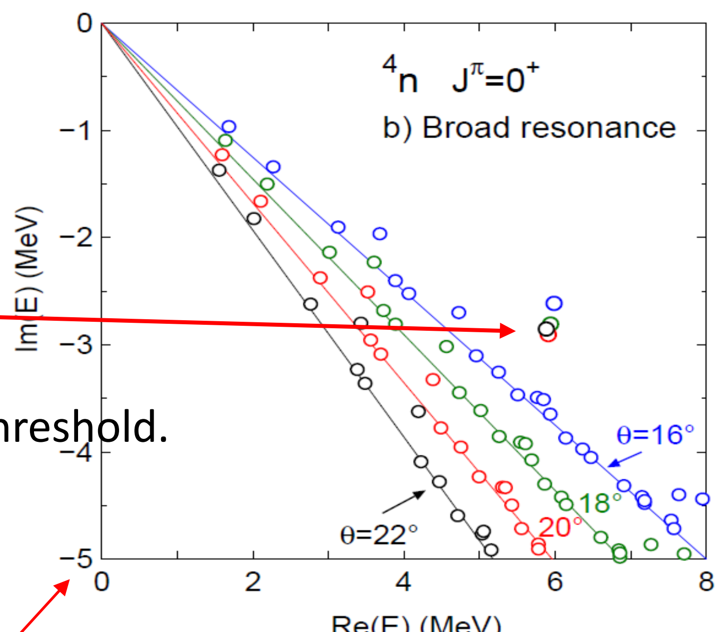
The observed 4n system was reported from the bound region to resonant region. In order to obtain energy position (E_r) and decay width (Γ), we use complex scaling method.

$$[H(\theta) - E(\theta)]\Psi_{JM,TT_z}(\theta) = 0$$

$$\Psi_{JM,TT_z}(\theta) = \sum_{\alpha} C_{\alpha}^{(\mathrm{K})}(\theta)\Phi_{\alpha}^{(\mathrm{K})} + \sum_{\alpha} C_{\alpha}^{(\mathrm{H})}(\theta)\Phi_{\alpha}^{(\mathrm{H})}$$



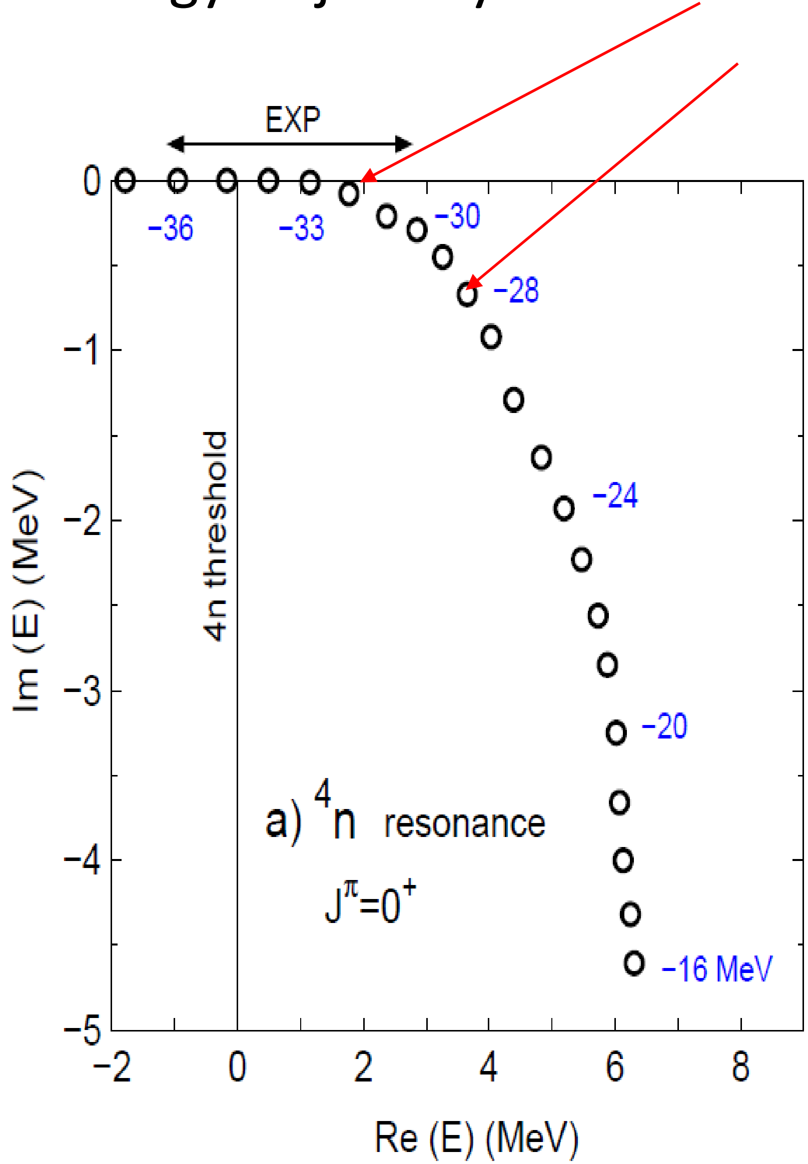
$$r_{\mathrm{c}} \to r_{\mathrm{c}}\, e^{i\theta},\; R_{\mathrm{c}} \to R_{\mathrm{c}}\, e^{i\theta},\; \rho_{\mathrm{c}} \to \rho_{\mathrm{c}}\, e^{i\theta} \quad (\mathrm{c} = \mathrm{K}, \mathrm{H})$$

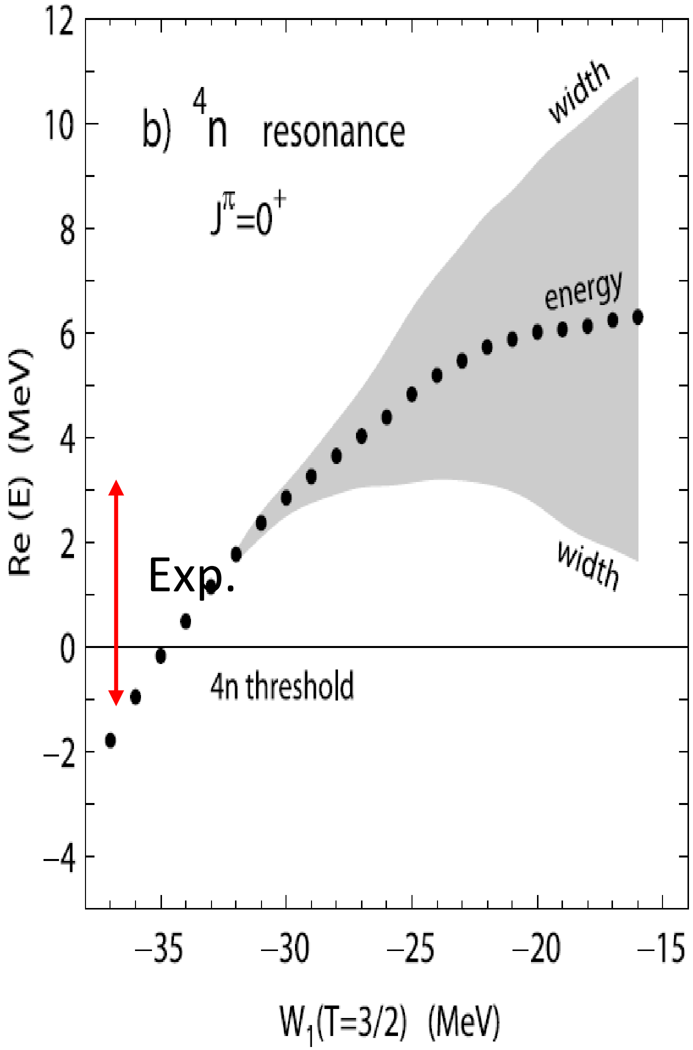


The energy pole is stable with respect to θ.
Re(E) corresponds to energy With respect to 4n breakup threshold.
Im(E) corresponds to Γ/2.

4n breakup threshold

energy trajectory of J=0+ state changing W_1





In order to reproduce the data of 4n system, We need W_1(T=3/2)= -36 MeV～-30MeV.

Attraction is 15 times Stronger.

It should be noted that W_1(T=1/2)=-2.04 MeV to reproduce the observed binding energy of ^{4}He, ^{3}He and ^{3}H.

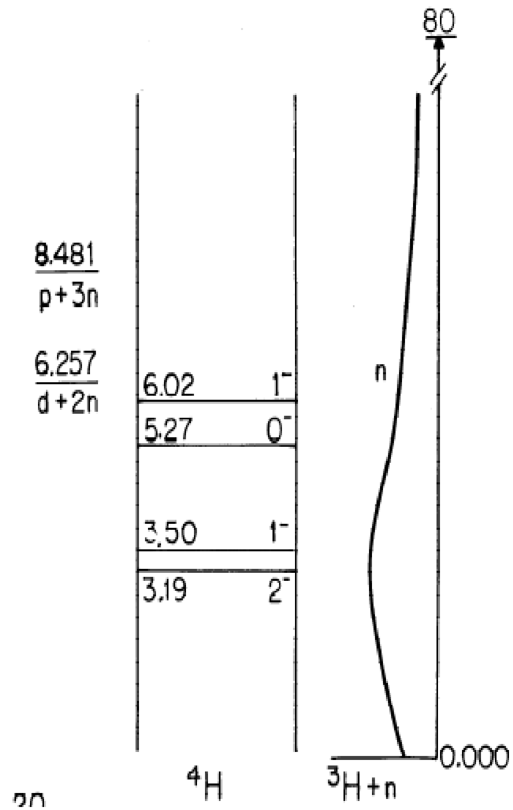
$$V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^{2} W_n(T) e^{-(r_{ij}^2+r_{jk}^2+r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T)$$

W_1(T=3/2)= free b_1=4.0fm
W_2(T=3/2) = +35 MeV b_2=0.75 fm

Question: W_1 value for T=3/2 is reasonable?

To check the validity of three-body force, we calculate the energies of ^{4}H, ^{4}He(T=1), ^{4}Li.

It is noted that I took benchmark test of 4n with Faddeev-Yakubovsky method by Lazauskas. My result is the same as that by FY.



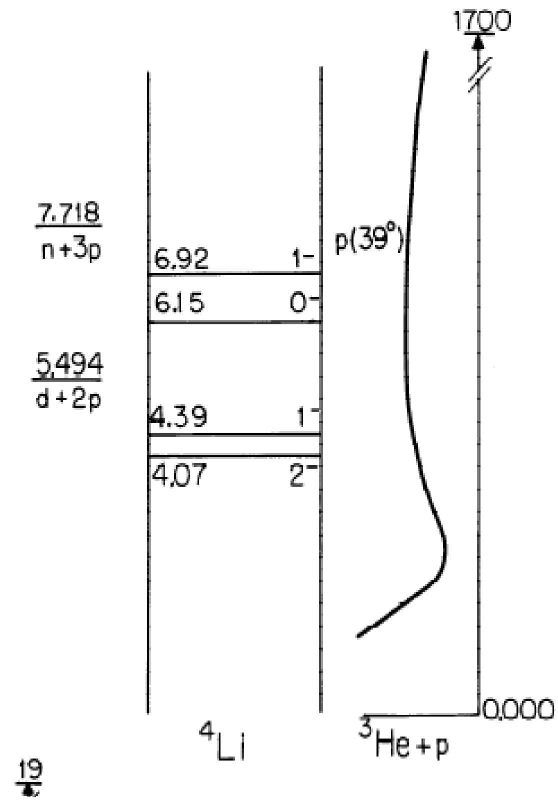


Table 4.1: Energy levels of ^{4}H defined for channel radius $a_n = 4.9$ fm. All energies and widths are in the cm system.

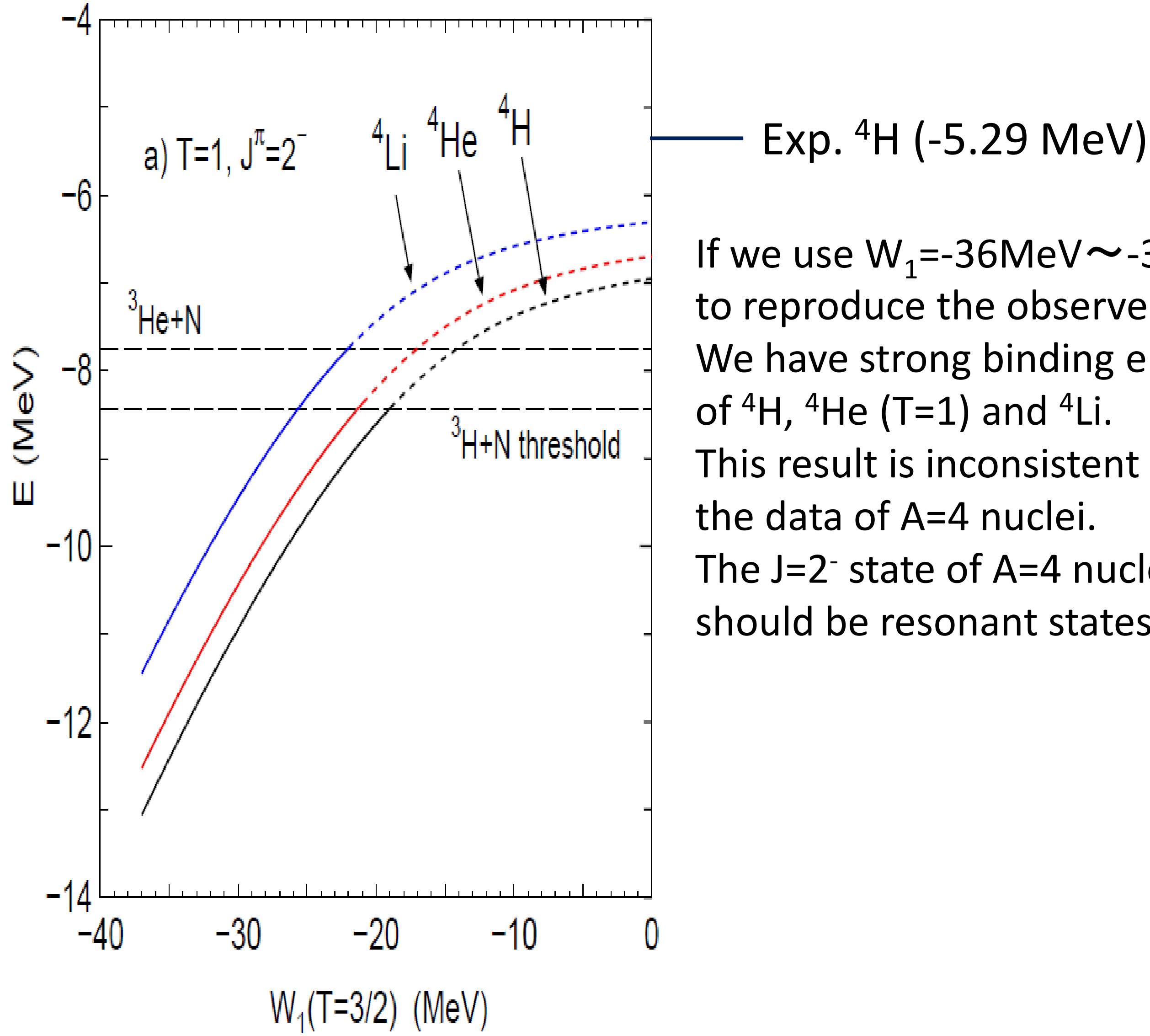
E_x (MeV)	J^π	T	Γ (MeV)	Decay	Reactions
g.s. [a]	2^-	1	5.42	n, ^{3}H	1, 11
0.31	1^-	1	6.73 [b]	n, ^{3}H	11, 12
2.08	0^-	1	8.92	n, ^{3}H	
2.83	1^-	1	12.99 [c]	n, ^{3}H	11, 12

[a] 3.19 MeV above the n + ^{3}H mass.
[b] Primarily $^3P_1$.
[c] Primarily $^1P_1$.

Table 4.24: Energy levels of ^{4}Li defined for channel radius $a_p = 4.9$ fm. All energies and widths are in the c.m. system.

E_x (MeV)	J^π	T	Γ (MeV)	Decay	Reactions
g.s. [a]	2^-	1	6.03	p, ^{3}He	3
0.32	1^-	1	7.35 [b]	p, ^{3}He	3
2.08	0^-	1	9.35	p, ^{3}He	3
2.85	1^-	1	13.51 [c]	p, ^{3}He	3

[a] 4.07 MeV above the p + ^{3}He mass.
[b] Primarily $^3P_1$.
[c] Primarily $^1P_1$.

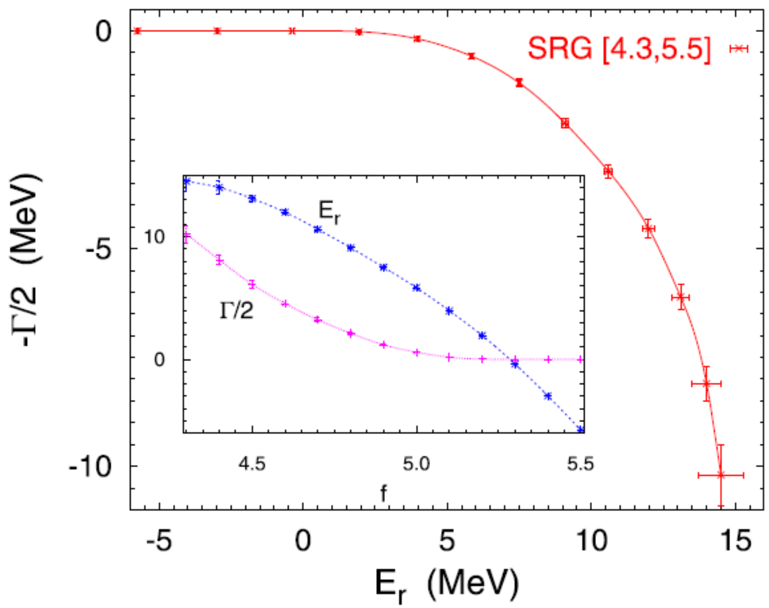


If we use W_1=-36MeV～-30 MeV
to reproduce the observed data of 4n,
We have strong binding energies
of ^{4}H, ^{4}He (T=1) and ^{4}Li.
This result is inconsistent with
the data of A=4 nuclei.
The $J=2^-$ state of A=4 nuclei
should be resonant states.

Conclusion: to reproduce the data of 4n, unlikely attractive three-body force is required.

A. Deltuva, Physics Letters B 782, 238 (2019).

Faddeev Yakubovsky method+SRG potential (based on AV18 potential)



fxV($^1s_0$)

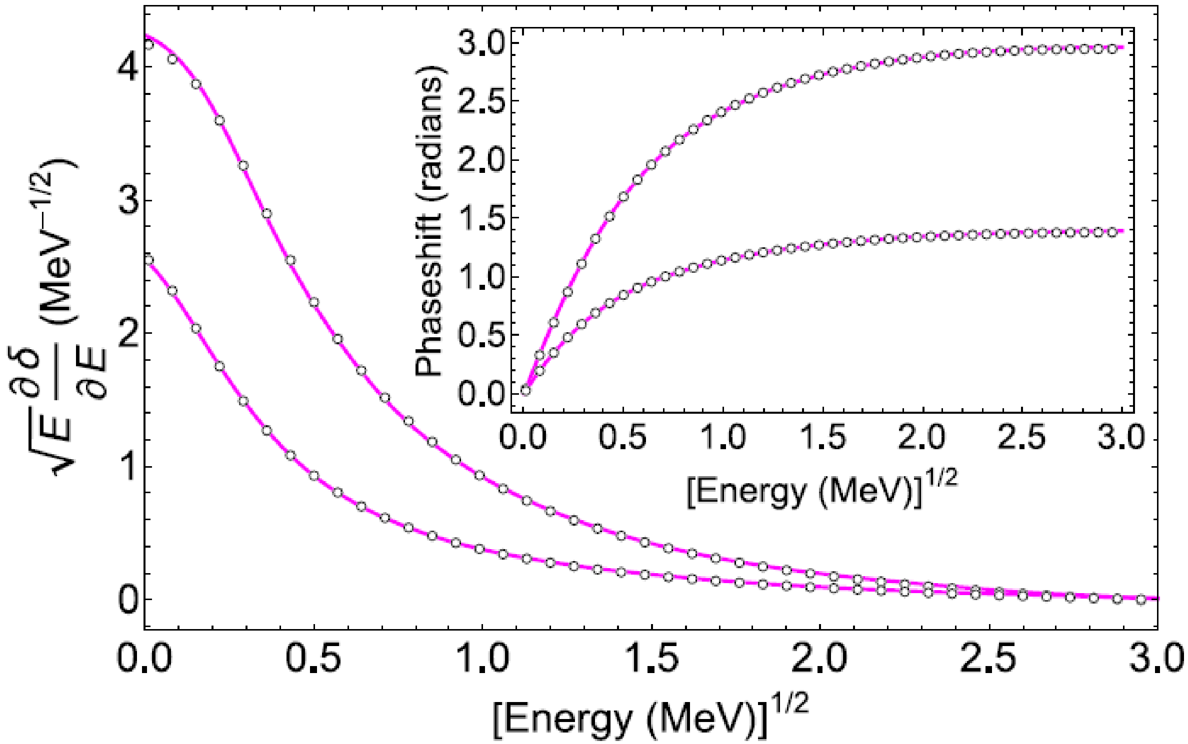
↑

Scaling factor: f<4.3=> no resonant state

M. D. Higgins, C.H. Greene, A. Kievsky, M. Vivianni,
Phys. Rev. Lett. 125, 052501(2020)
Phys. Rev. C 103, 024004 (2021)

Hypersherical harmonics Method

AV8 potential which is the same one used by me.



No resonant state

They used AV18 potential and they had no resonant state.

Summary of the 4n calculation

Author	Method	How to obtain resonant state	V_{NN}	resonance
A.M. Shirokov et al.	Non-core shell model	+ phase shift analysis	JISP16	Er=0.8 MeV Γ=1.4 MeV
S. Gandolfi et al.	Quantum Monte Calro	extrapolation	chiral(NNLO)	Er=1.84 MeV Γ=0.282 MeV
K. Fossez et al.,	no-core Gamow shell model		N3LO, JISP16,	Er～7MeV Γ～3.5MeV
E. Hiyama, R. Lazauskas et al.,	Gaussian Expansion + CSM Faddeev Yakubovsky		AV8	No resonance
Deltuva,	Faddeev Yakbobsky	+ AGS	SRG(AV18),NLO,	No resonance
M. D. Higgins et al.,	Hypersherical harmonics	phase shift analysis	AV8, AV18,	no resonance

AV8 and AV18 potentials give negative result: no resonance.
This conclusion is not dependent on the method employed.

Chiral NN interaction gives different conclusion, which is dependent on method.
Question: each method can be treated continuum states explicitly?
For 4n state, we have only 4n breakup threshold. We should treat 4n breakup threshold explicitly.

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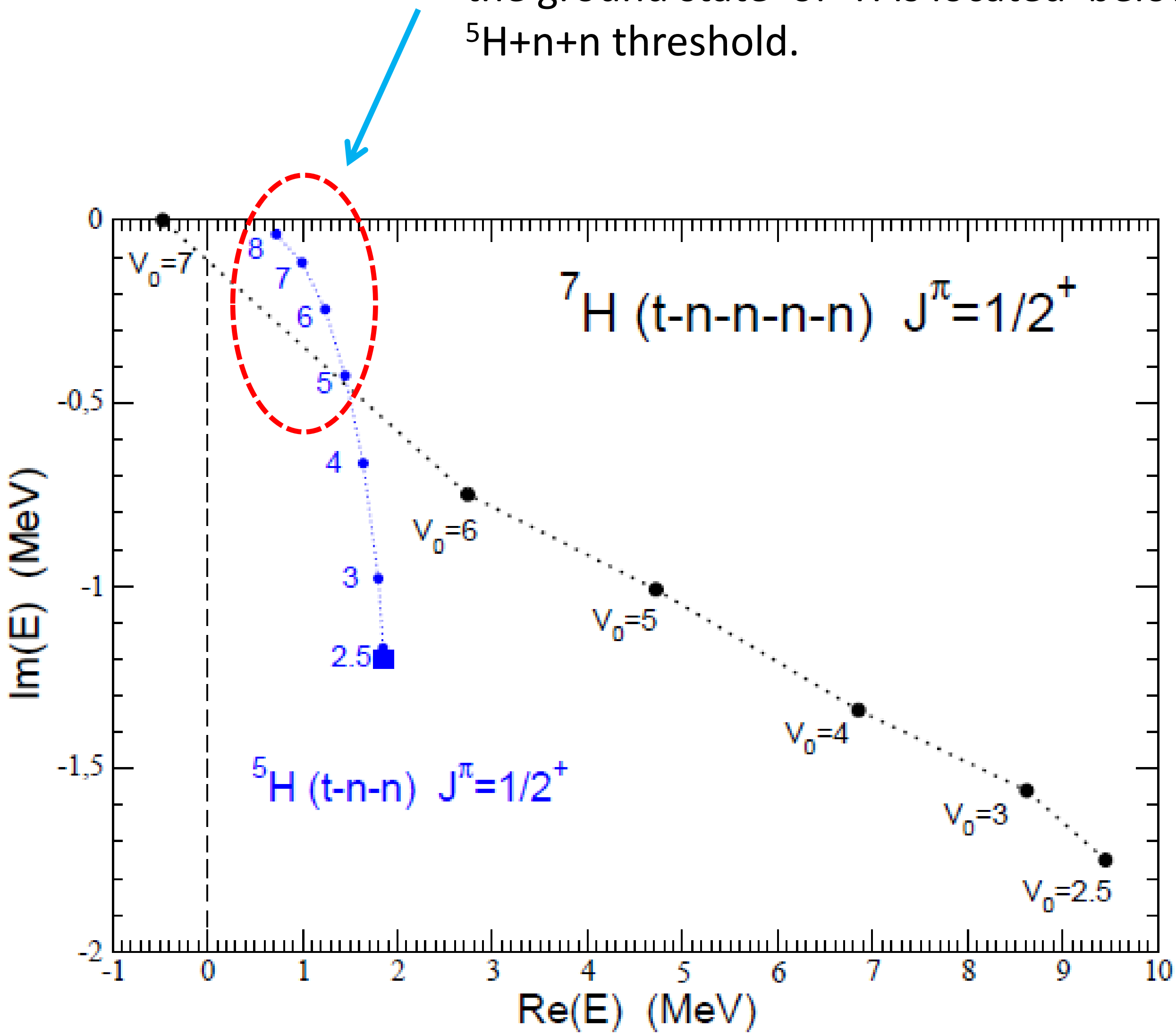
In the world, theoretically, we come to negative conclusion, no resonant state for 4n.

How do we produce 4n resonant state?

Im (E)=Γ/2

When the decay width of ^5H is narrower, the ground state of ^7H is located below the ^5H+n+n threshold.



(E_R, Γ_R) (MeV)	
J^π	$1/2^+$
^{5}H (full)	(1.57, 1.53)
^{5}H ($d = 0$)	(1.55, 1.35)
Theor. [16]	(2.26, 2.93)
Theor. [12]	(2.5–3.0, 3–4)
Theor. [13]	(3.0–3.2, 1–4)
Theor. [15]	(1.59, 2.48)
Exp. [3]	$(1.7 \pm 0.3, 1.9 \pm 0.4)$
Exp. [8]	$(1.8 \pm 0.1, < 0.5)$
Exp. [4]	(1.8, 1.3)
Exp. [5]	(2, 2.5)
Exp. [6]	(3, 6)
Exp. [9]	$(5.5 \pm 0.2, 5.4 \pm 0.6)$

[3] A.A. Korosheninnikov et al., PRL87 (2001) 092501
[8] S.I. Sidorchuk et al., NPA719 (2003) 13
[4] M.S. Golovkov et al. PRC 72 (2005) 064612
[5] G. M. Ter-Akopian et al., Eur. Phys. J A25 (2005) 315.

Energy of 5H is similar. But decay width is dependent on experiment.