Quantum behavior of a heavy impurity in a Bose gas

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FLEET
ARC CENTRE OF EXCELLENCE IN FUTURE LOW-ENERGY ELECTRONICS TECHNOLOGIES

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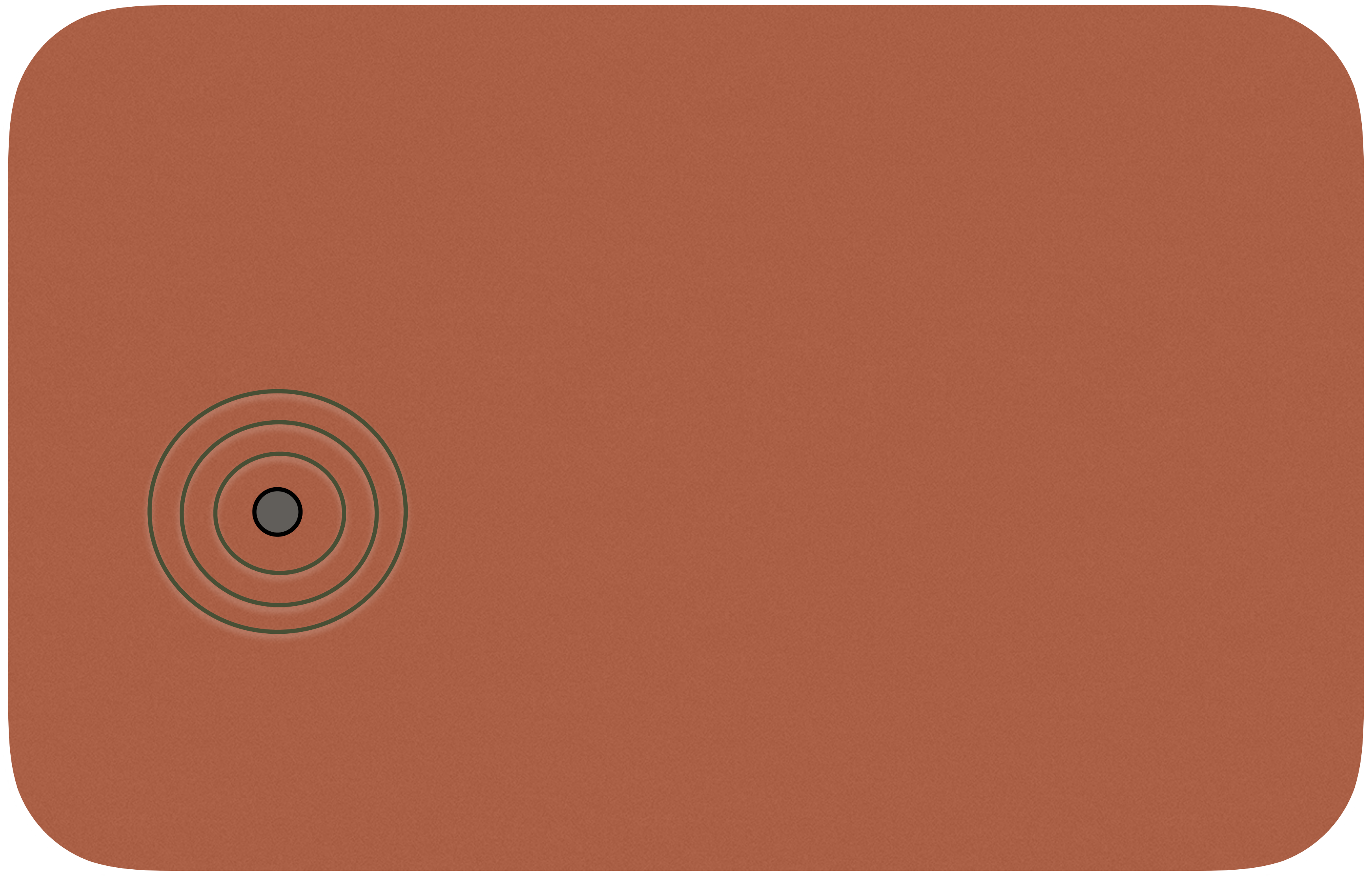
Impurities in a Bose medium

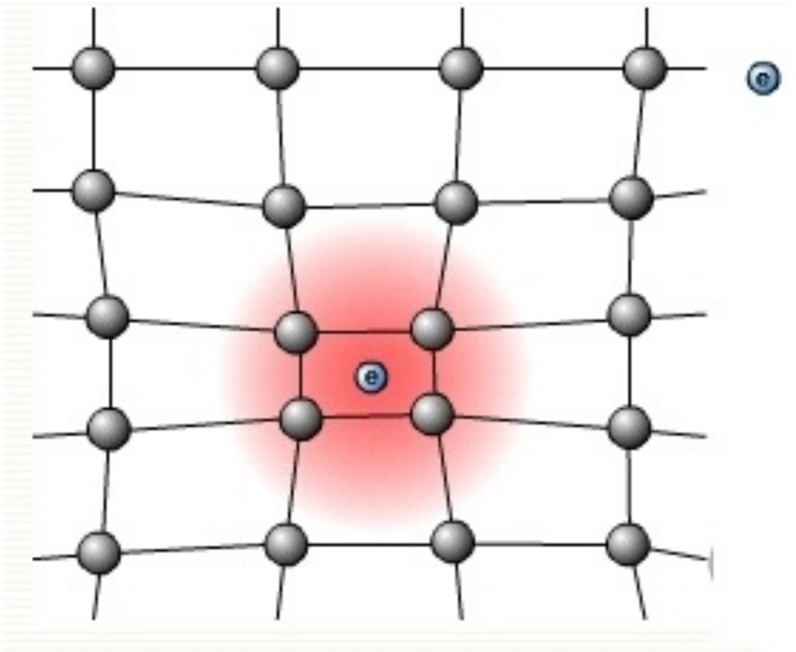


- Fundamental problem in physics

  - Quantum system + environment

    E.g. spin-boson problem - Leggett et al, RMP 1987

  - Electrons in ionic lattice (polarons)



What happens if the medium is a BEC?

Cold-atom experiments: JILA, Aarhus, Kaiserslautern, MIT, …

Outline

- The Bose polaron problem [3D]

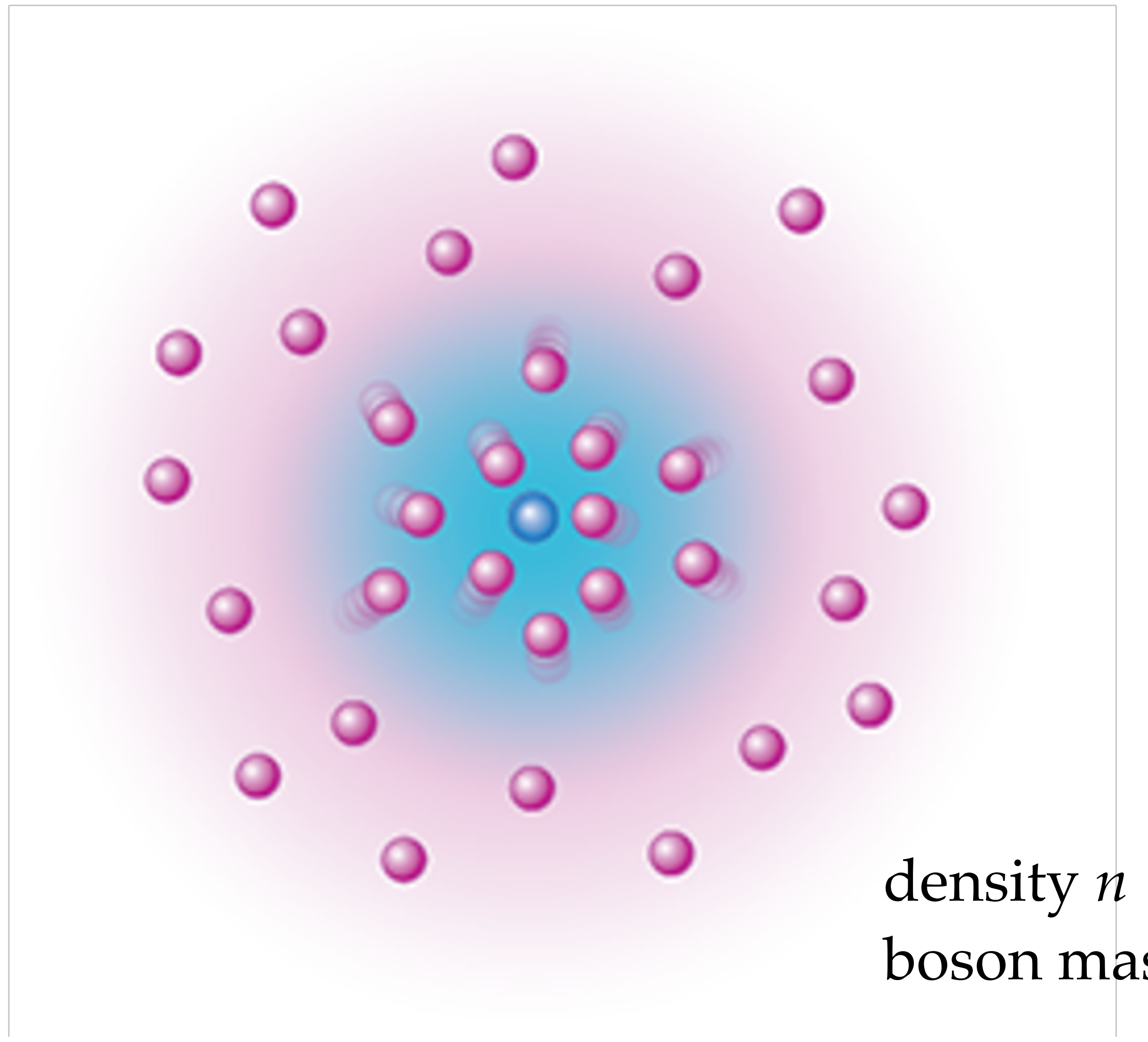
Theory: Tempere, Bruun, Massignan, Enss, Schmidt, Demler, Gurarie, Giorgini …

- Few-body bound states
    - Impurity + *N* bosons
- Many-body limit
- Conclusion

The Bose polaron

– Infinitely heavy case

- Fixed impurity in a weakly repulsive Bose gas
- Tunable short-range *attractive* interactions between impurity and bosons



density n
boson mass m

impurity-boson scattering length a

$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} b^\dagger_{\mathbf{k}} b_{\mathbf{k}} + \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \frac{V(\mathbf{q})}{2} b^\dagger_{\mathbf{k}} b^\dagger_{\mathbf{k}'} b_{\mathbf{k}'+\mathbf{q}} b_{\mathbf{k}-\mathbf{q}} + g \sum_{\mathbf{k}\mathbf{k}'} b^\dagger_{\mathbf{k}} b_{\mathbf{k}'}$$

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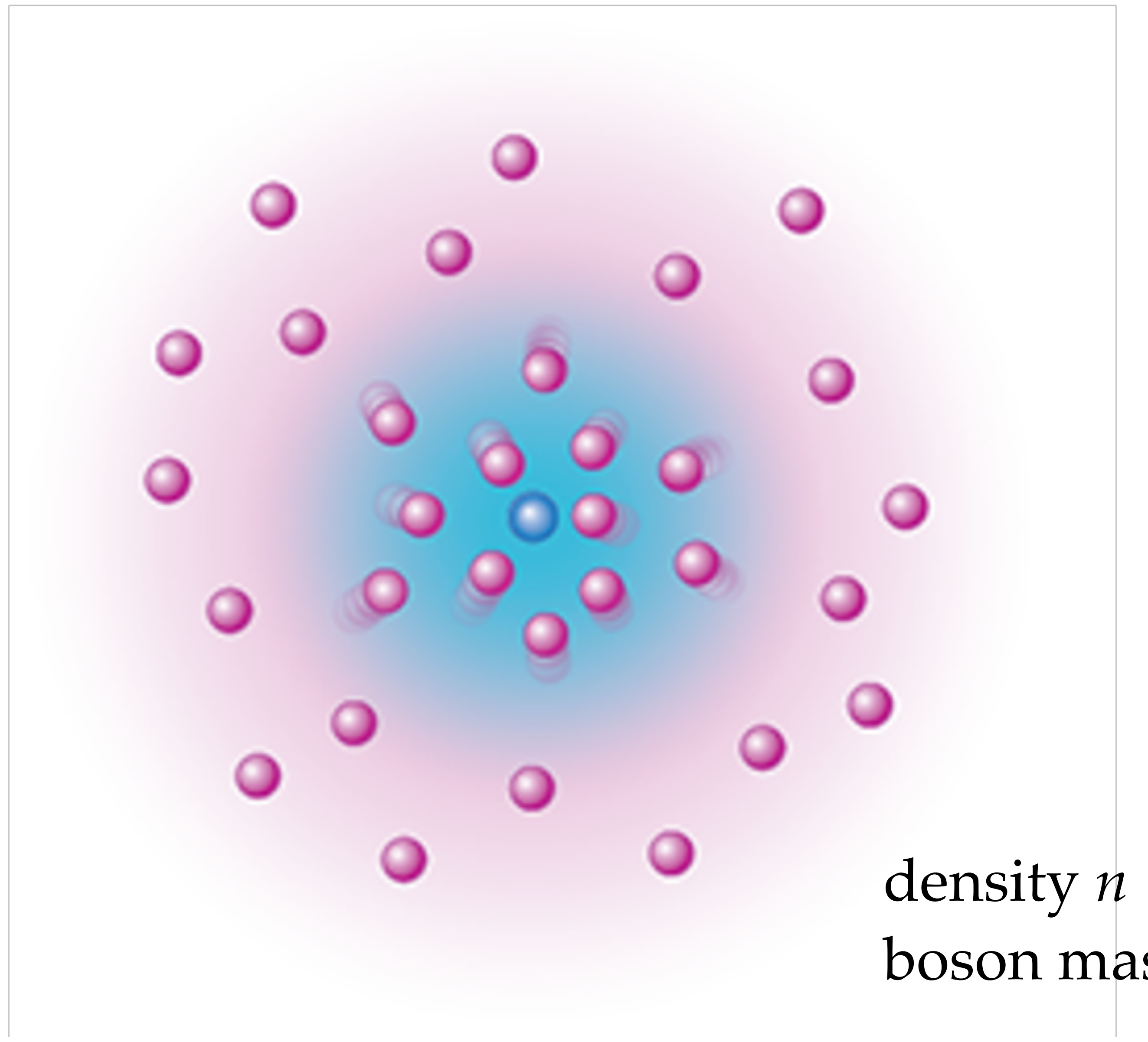

$$\hat{H} = \sum_{\mathbf{k}} \frac{k^2}{2m} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \frac{V(\mathbf{q})}{2} b_{\mathbf{k}}^\dagger b_{\mathbf{k}'}^\dagger b_{\mathbf{k}'+\mathbf{q}} b_{\mathbf{k}-\mathbf{q}} + g \sum_{\mathbf{k}\mathbf{k}'} b_{\mathbf{k}}^\dagger b_{\mathbf{k}'}$$

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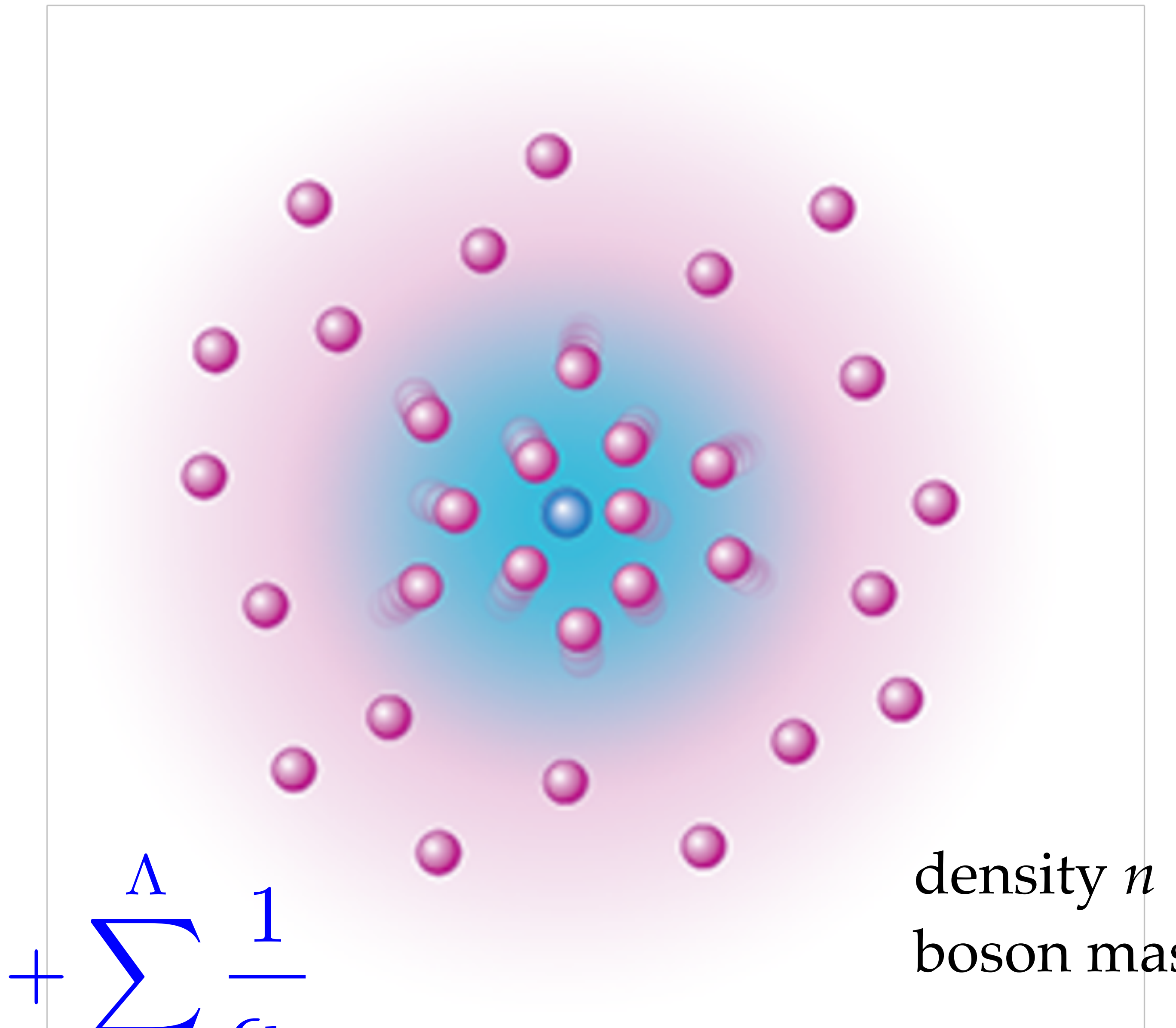

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$V(\mathbf{q}) \rightarrow a_B$

$g \rightarrow$ $$\frac{m}{2\pi a} = \frac{1}{g} + \sum_{\mathbf{k}}^{\Lambda} \frac{1}{\epsilon_{\mathbf{k}}}$$

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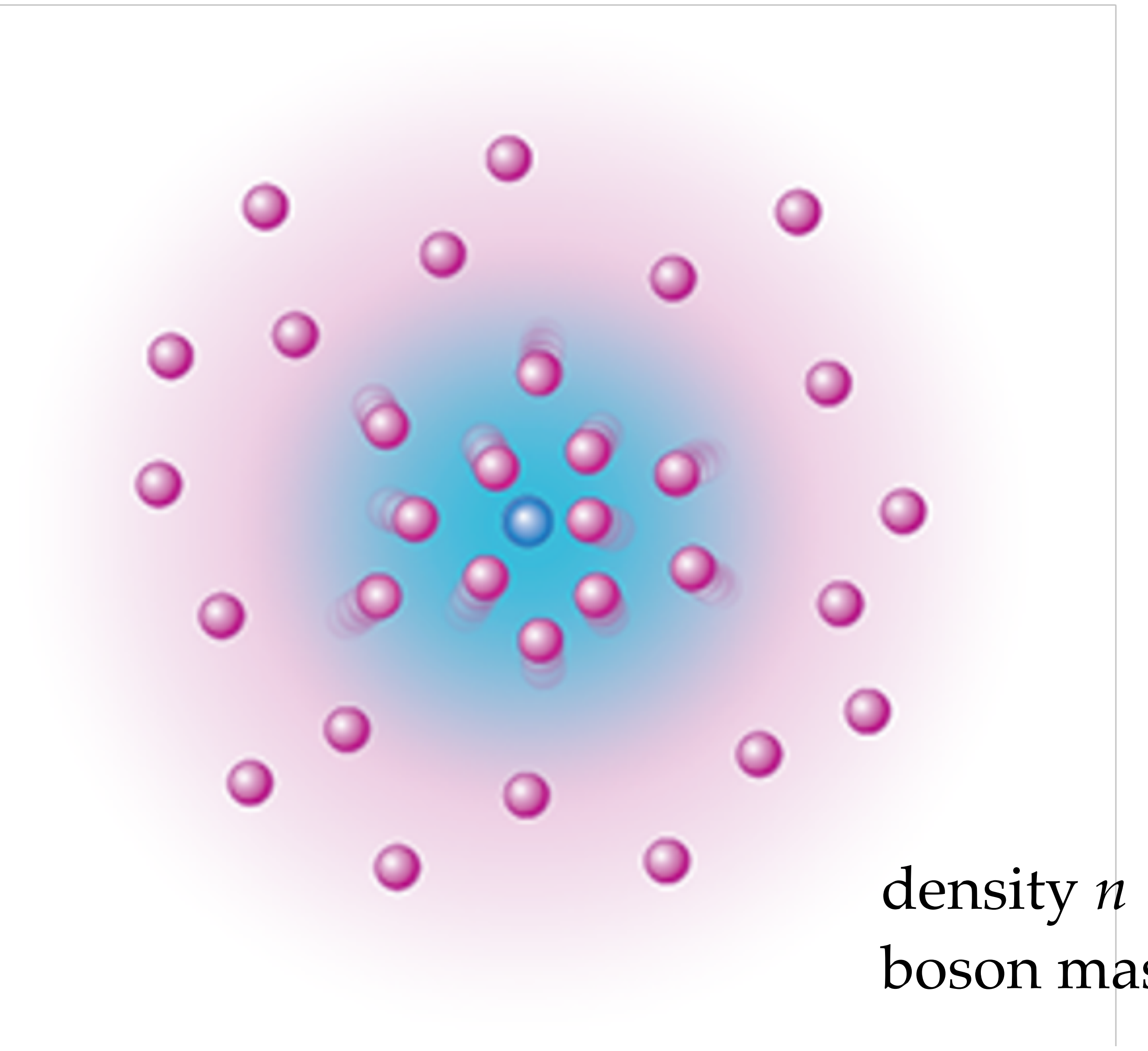

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Guenther, Schmidt, Bruun, Gurarie & Massignan, PRA 2021;
Drescher, Salmhofer & Enss, PRA 2021

The Bose polaron

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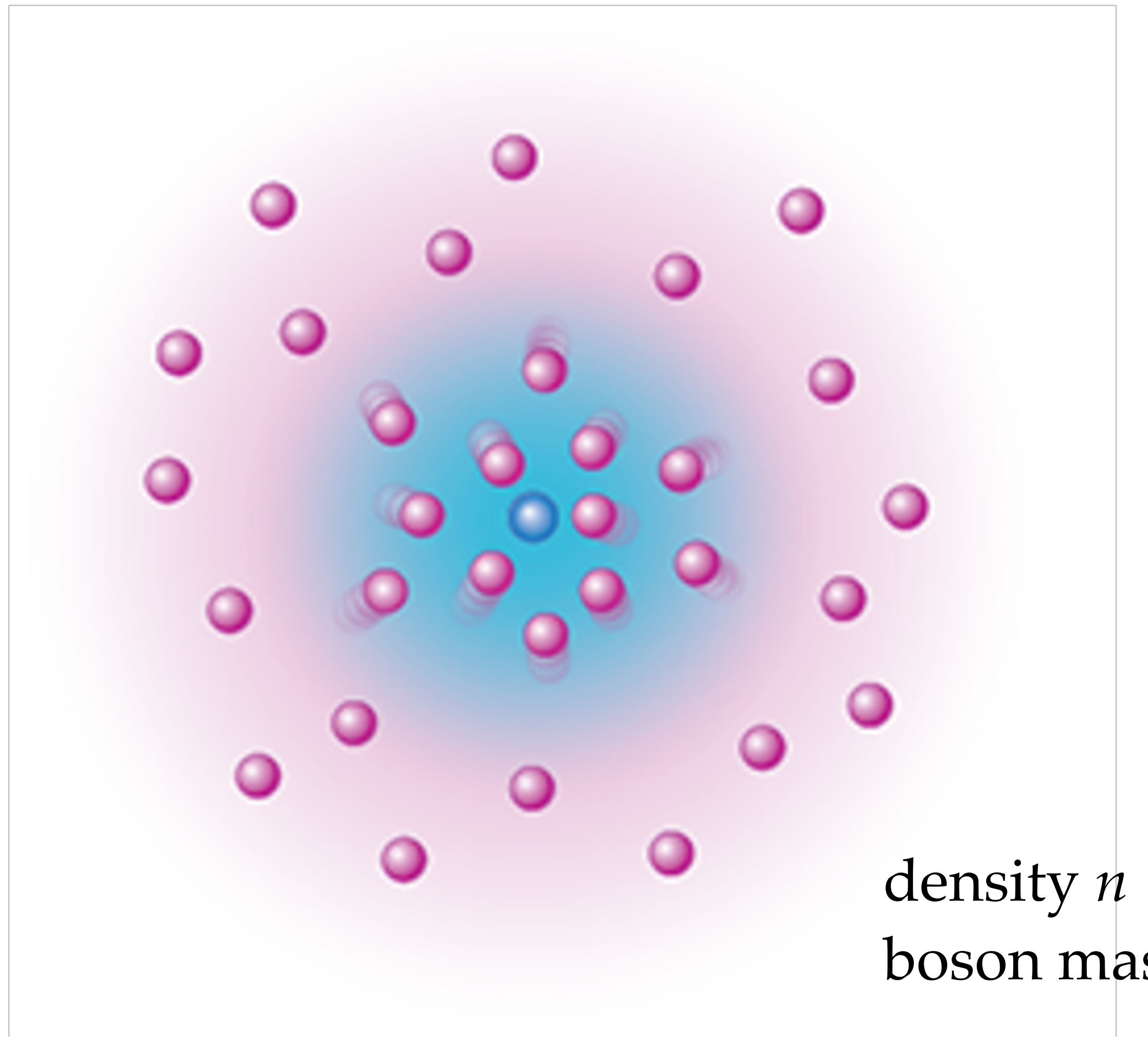

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- **Exactly solvable** for ideal Bose gas
  - Polaron energy: $E = \dfrac{2\pi a n}{m}$

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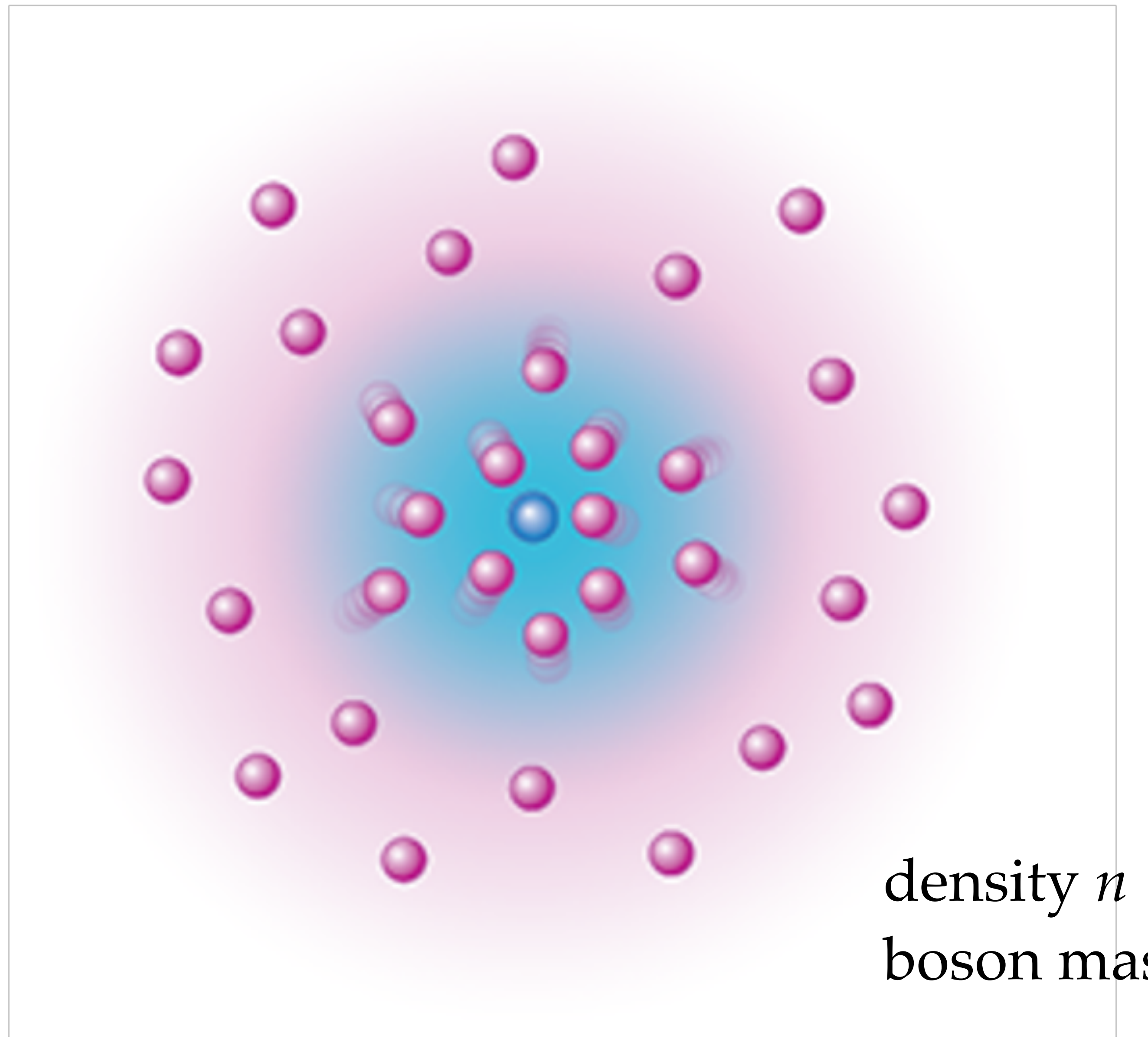

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… but singular

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Few-body bound states
— Exact calculations

- Impurity + N non-interacting bosons
  - All bosons occupy bound state once $a > 0$

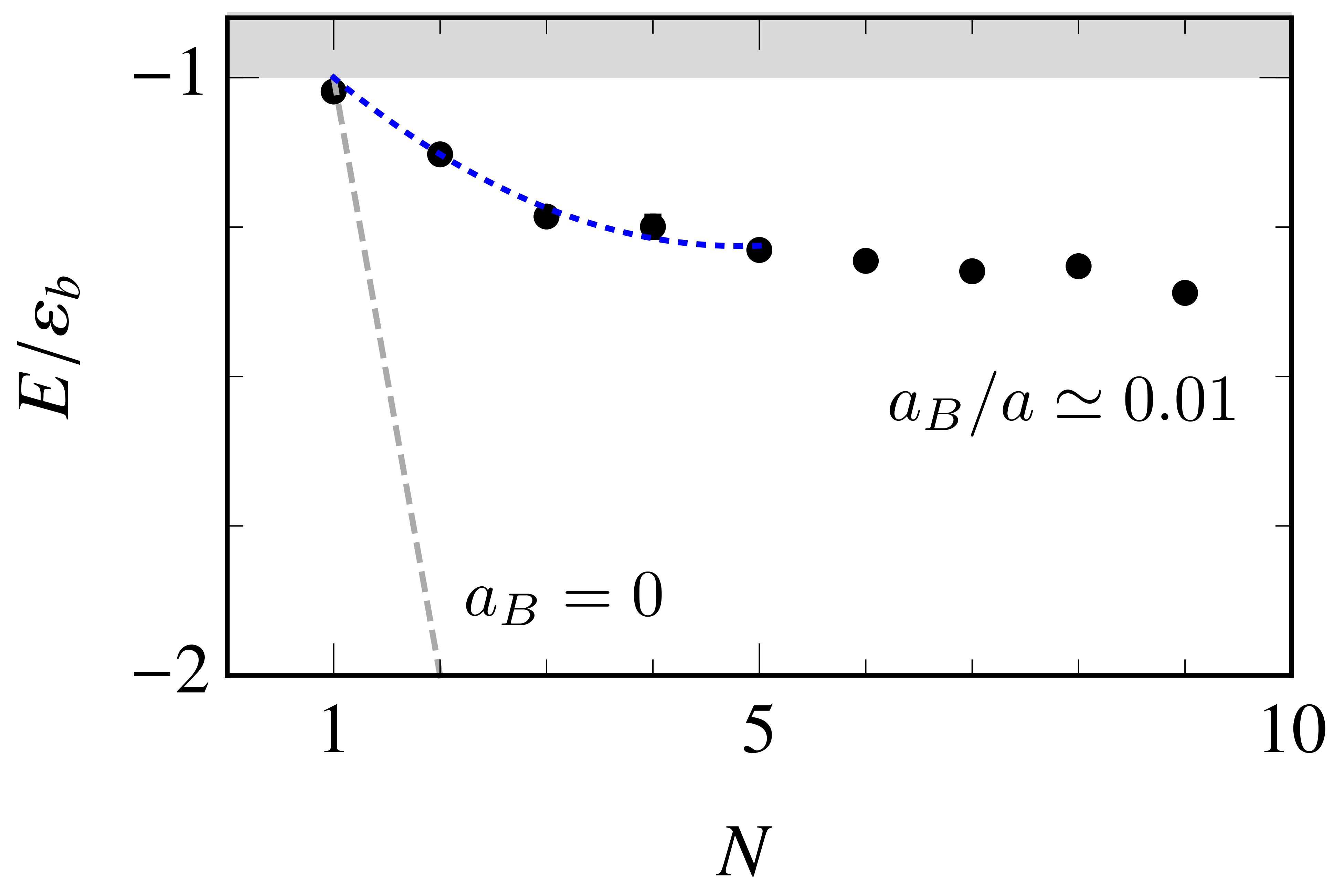
$$E = -N\varepsilon_b$$

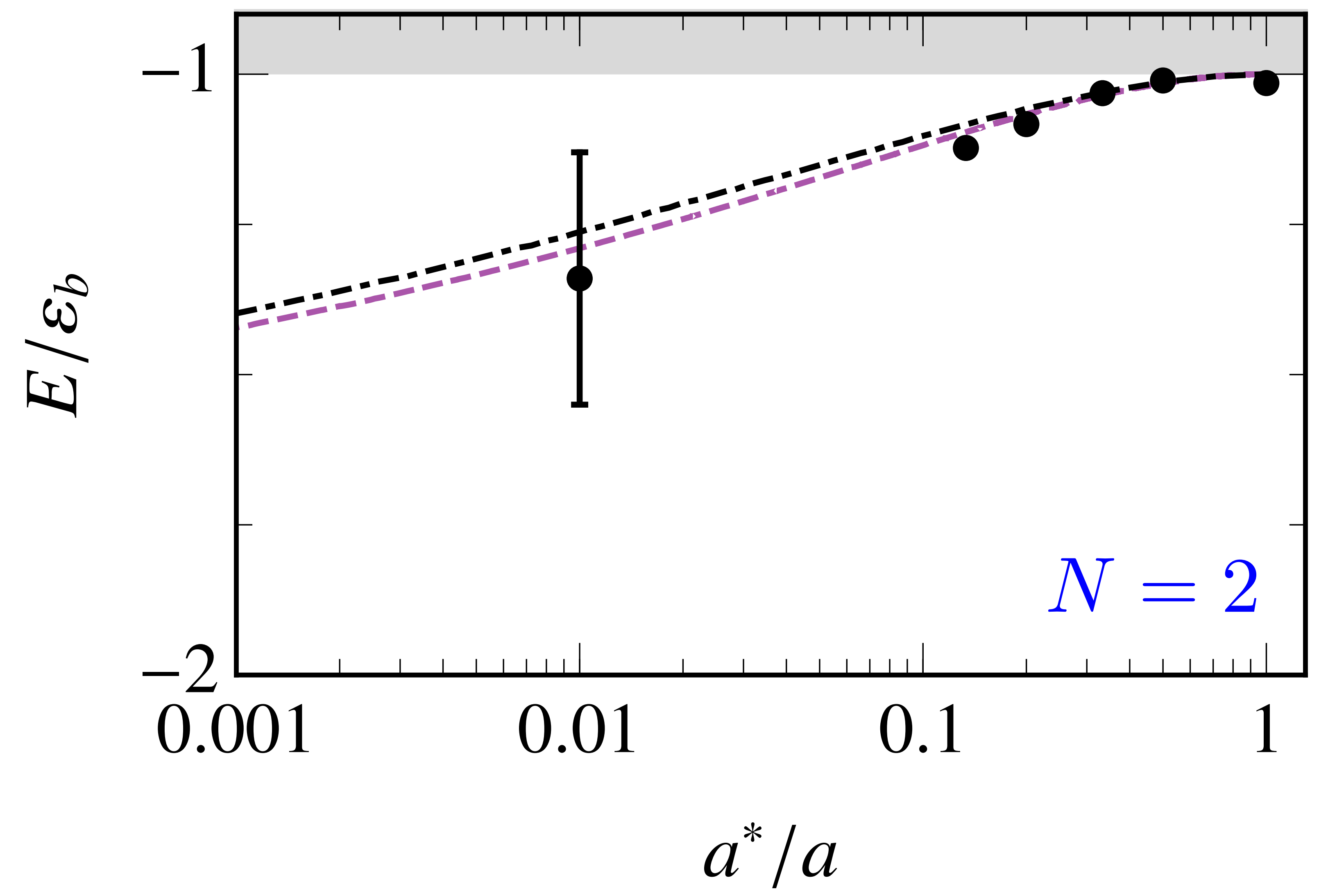
- Effect of boson-boson repulsion?
  (i.e., non-zero a_B)

- Trimer <u>unbinds</u> at critical interaction a^*

$$\Delta E = -\varepsilon_b + U_{int} \approx -\frac{1}{2ma^2} + \frac{4\pi a_B}{ma^3} \Rightarrow a^* \approx \quad a_B$$

Levinsen, Peña Ardila, Yoshida & MMP, PRL 2021;
Shi, Yoshida, MMP & Levinsen, PRL 2018





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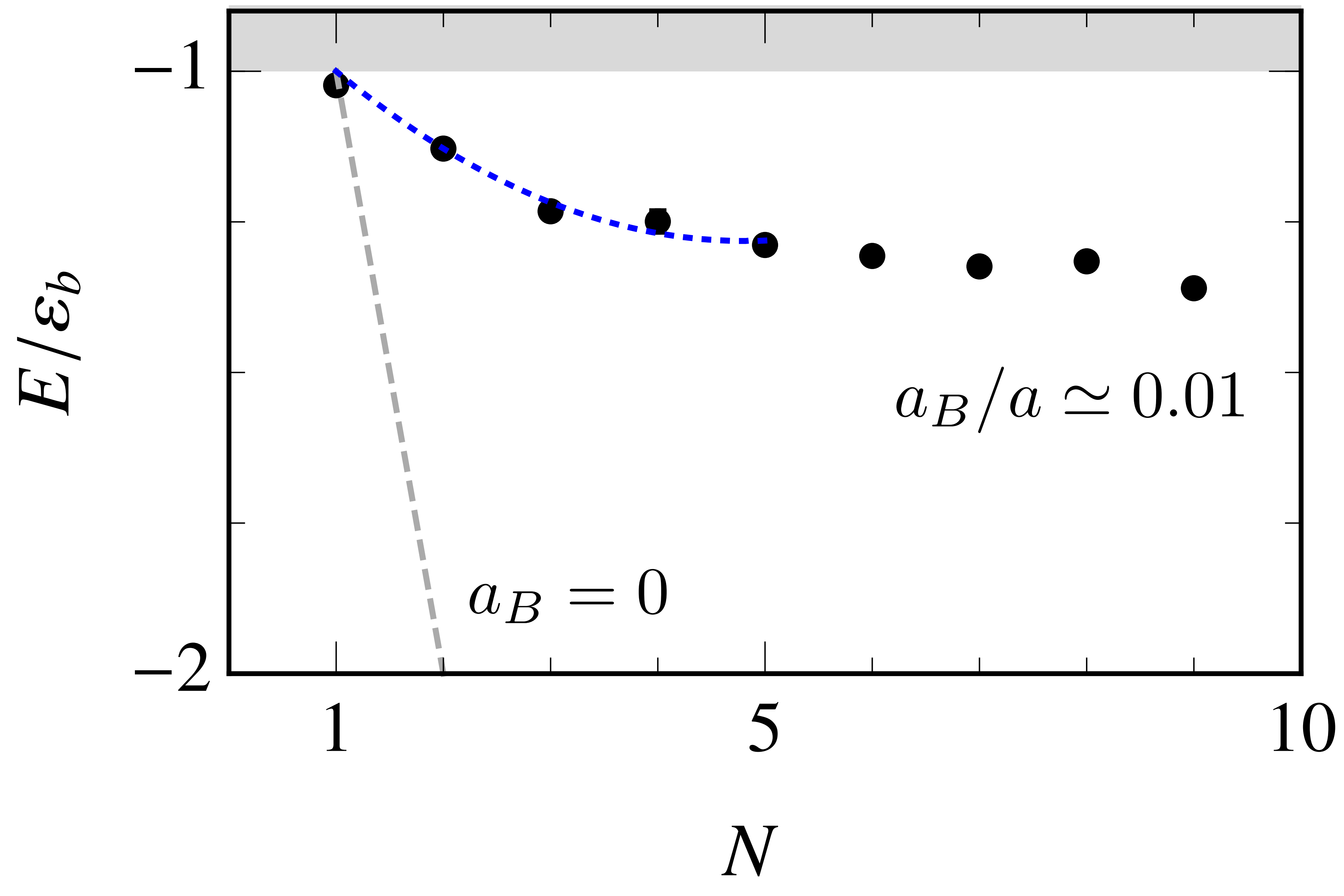

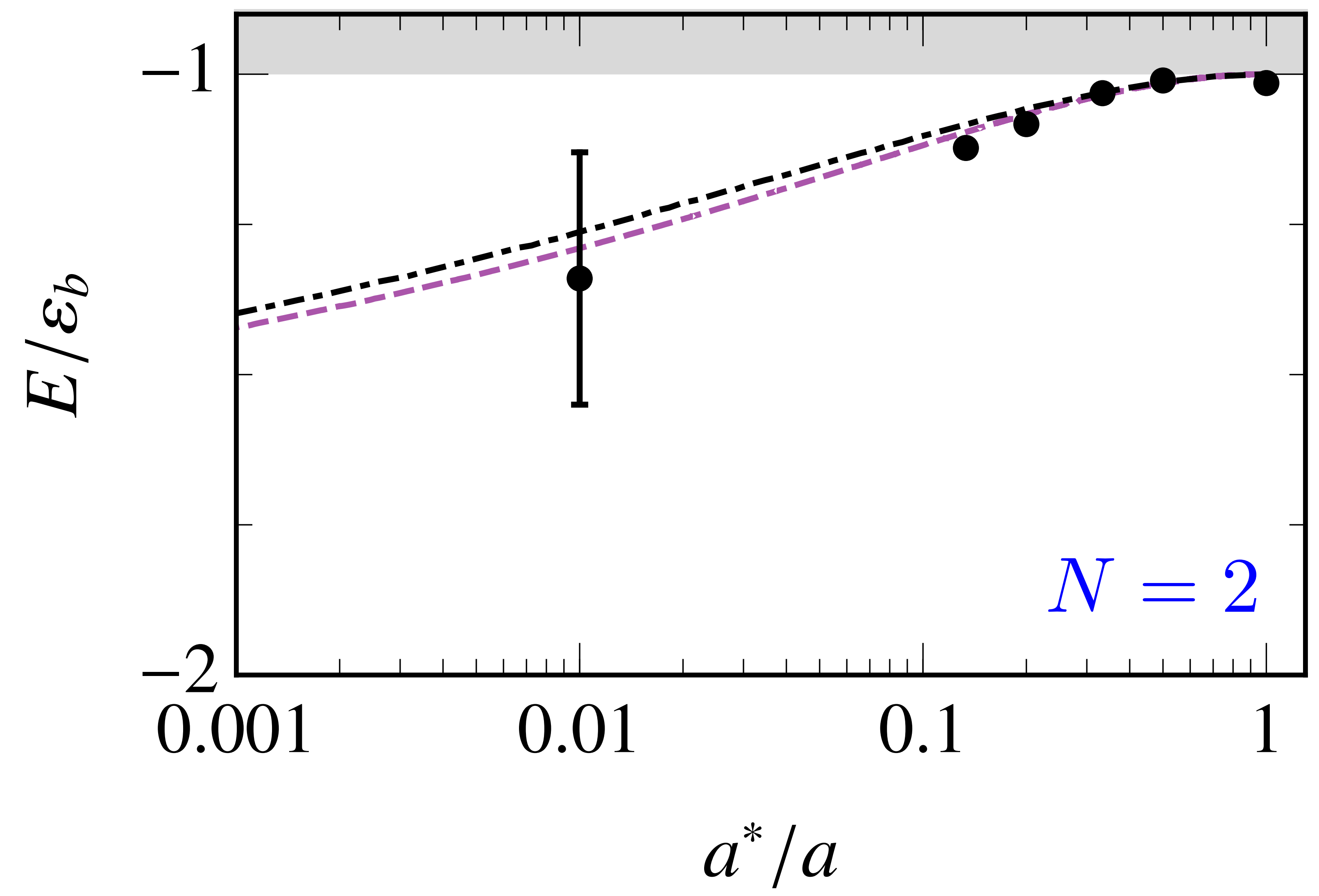

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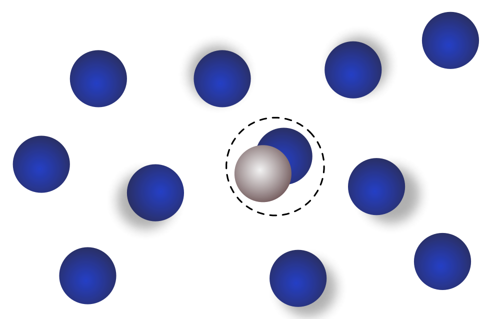
Few-body bound states

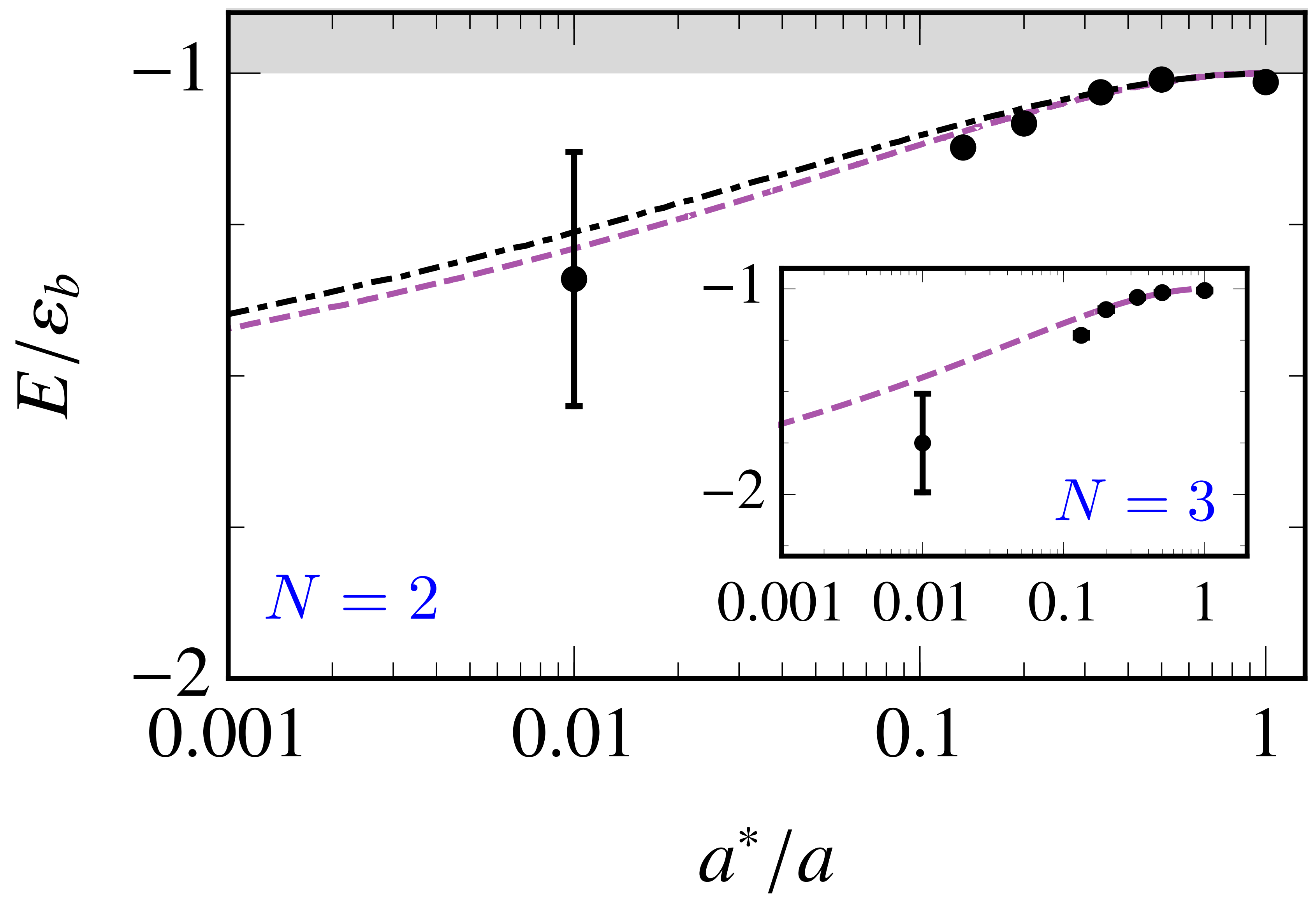
— Exact calculations

- *All N*>1 bound states unbind at *a**
  - Unitarity point for dimer impurity



- *Universal* bound states near *a**

- Ground-state energy is bounded from below for any boson number *N*



$$E(1/a) \geq E(1/a^*) = -\frac{1}{2m(a^*)^2}\ , \qquad 1/a \leq 1/a^*$$

since $\frac{\partial E}{\partial(-1/a)} > 0$

Levinsen, Peña Ardila, Yoshida & MMP, PRL 2021;
Shi, Yoshida, MMP & Levinsen, PRL 2018

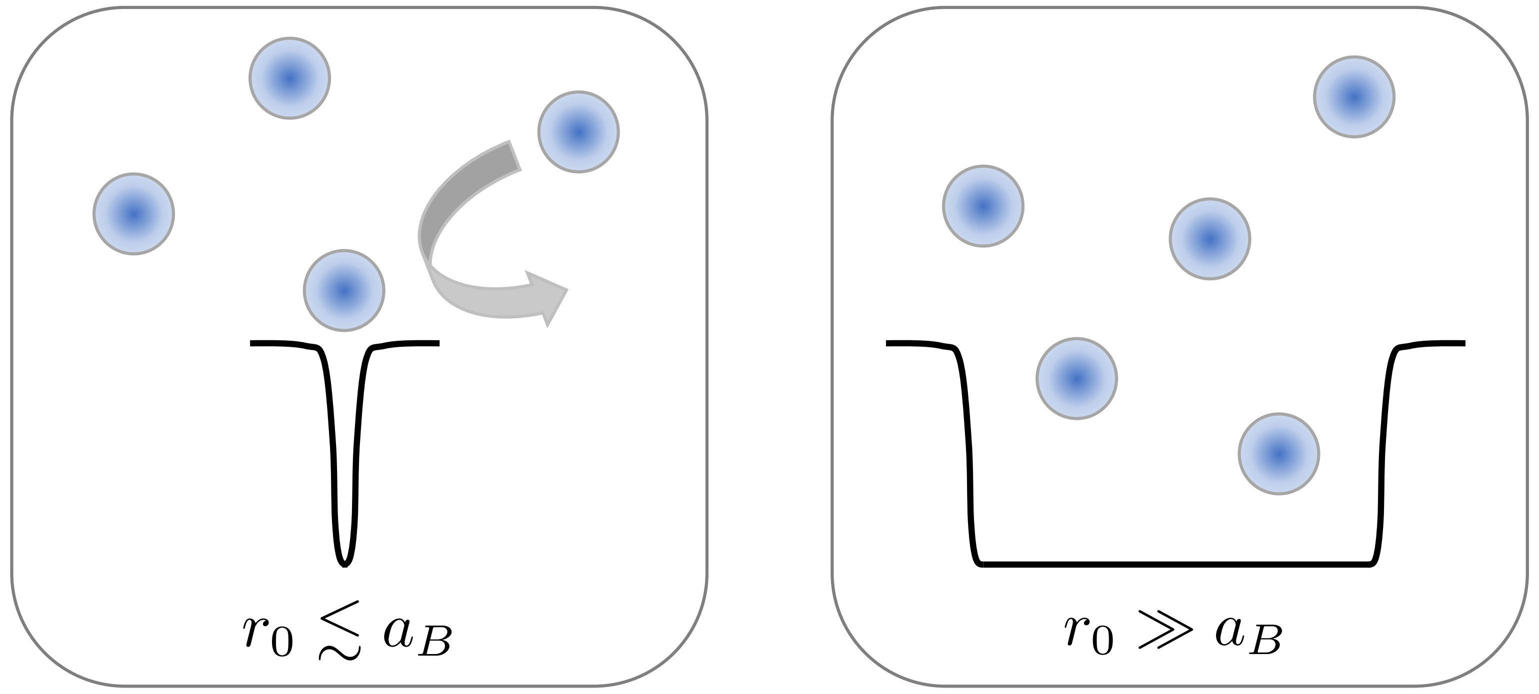
Polaron ground state
— Many-body limit

- Popular approach: treat BEC as a classical field

Shchadilova, Schmidt, Grusdt & Demler, PRL 2016;
Massignan, Yegovtsev & Gurarie, PRL 2021

Ignores quantum "granular" nature of gas

Crucial for short-range impurity potential!



"quantum blockade"

Classical-field approach requires conditions:

$$n_l a_B^3 \ll 1$$

$$n_l r_0^3 \gg 1$$

See also: Chen, Prokof'ev & Svistunov, PRA 2018

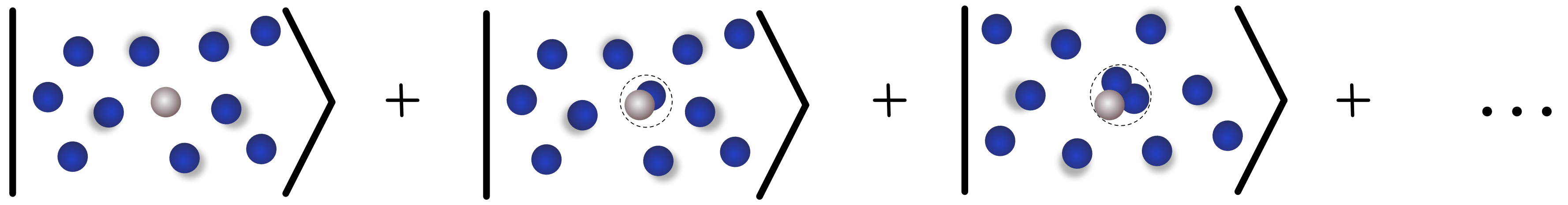
Polaron ground state
— Many-body limit

- Consider general (correlated) state:

$$|\Psi\rangle = \Big(\alpha_0 + \sum_{\mathbf{k}\neq\mathbf{0}} \alpha_{\mathbf{k}} b^\dagger_{\mathbf{k}} + \frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2\neq\mathbf{0}} \alpha_{\mathbf{k}_1\mathbf{k}_2} b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} \ldots\Big)|\Phi\rangle$$

very dilute BEC

$$e^{\sqrt{n}(b^\dagger_{\mathbf{0}} - b_{\mathbf{0}})}|0\rangle$$



➡ Polaron ground-state energy:

$$E = n\left[\frac{m}{2\pi a} + \sum_{\mathbf{k}}\left(\frac{1}{\epsilon_{\mathbf{k}} + G_{\mathbf{k}}} - \frac{1}{\epsilon_{\mathbf{k}}}\right)\right]^{-1}$$

- Boson-boson interactions encoded in:

$$G_{\mathbf{k}} = g\sqrt{n}\left(\sum_{\mathbf{k}'} \alpha_{\mathbf{k}\mathbf{k}'}/\alpha_{\mathbf{k}} - \sum_{\mathbf{k}'} \alpha_{\mathbf{k}'}/\alpha_0\right)$$

- Ideal gas: $G_{\mathbf{k}} = 0$
- Coherent state: $G_{\mathbf{k}} = 8\pi a_B n/m$

Shchadilova et al, PRL 2016

- "Chevy-type" ansatz: $G_{\mathbf{k}} = -E$

Levinsen, Peña Ardila, Yoshida & MMP, PRL 2021

Polaron energy

— Variational vs exact calculations

$n^{1/3}a_B \simeq 2.15 \times 10^{-2}$

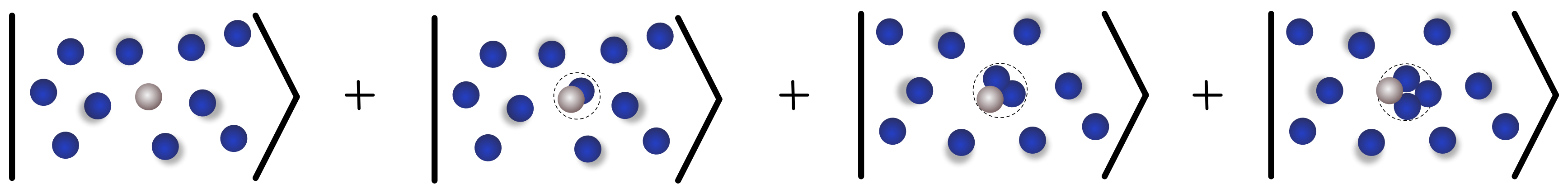
2.15×10^{-4}

- Bosonic Anderson model:

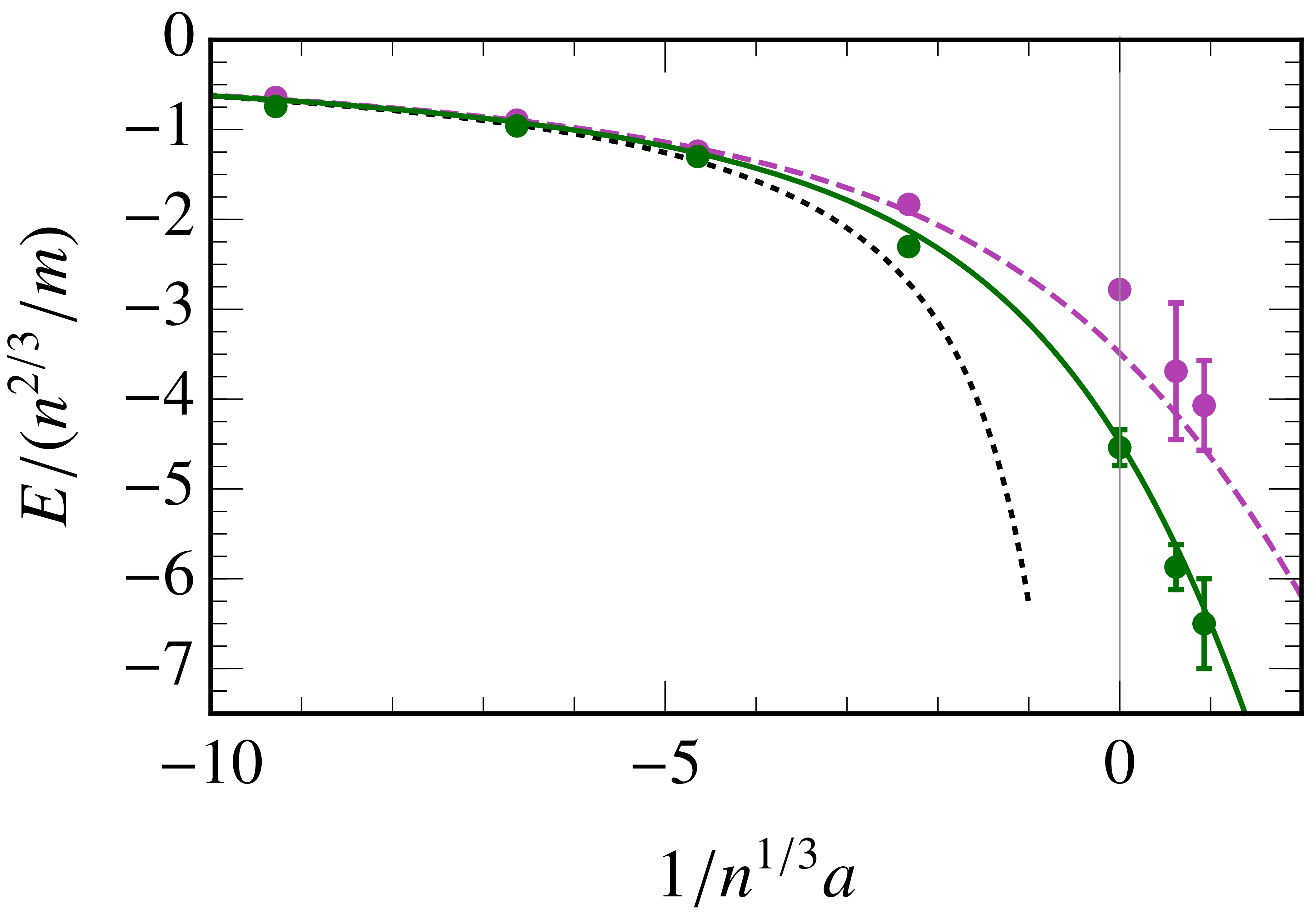
$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} b^\dagger_{\mathbf{k}} b_{\mathbf{k}} + \nu_0 d^\dagger d + \lambda \sum_{\mathbf{k}} (d^\dagger b_{\mathbf{k}} + b^\dagger_{\mathbf{k}} d) + \frac{U}{2} d^\dagger d^\dagger d d, \qquad U \to +\infty$$

  - Mimics quantum blockade

- Truncated basis approach
  - Up to 3 excitations



  - Compare with QMC for same a*



- Boson repulsion suppresses excitations
- Insensitive to microscopic details

Levinsen, Peña Ardila, Yoshida & MMP, PRL 2021

Polaron energy

— Unitarity limit $1/a = 0$

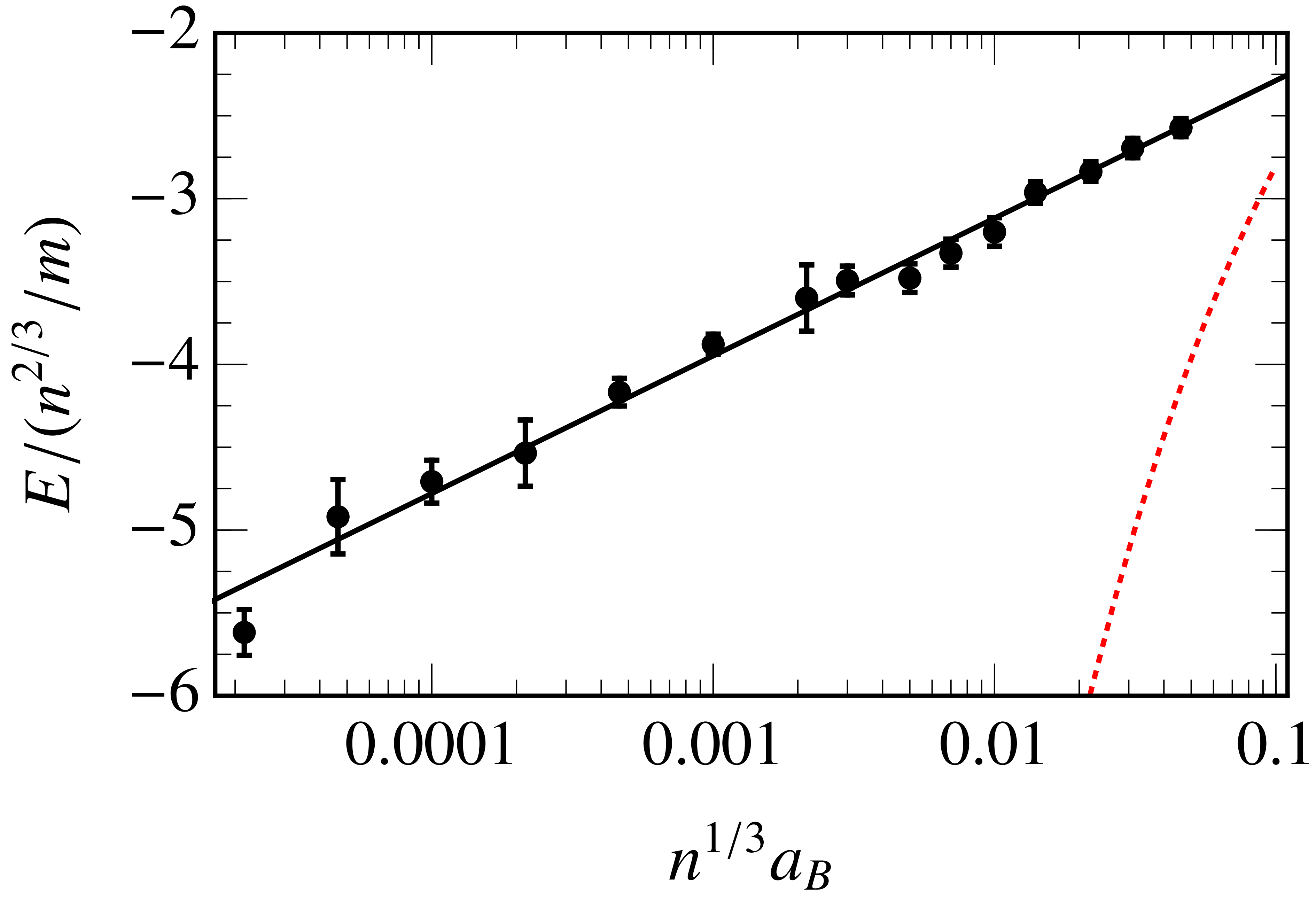
$$E = -f(n^{1/3}a_B)n^{2/3}/m$$

$$\to -\infty, \qquad a_B \to 0$$

$$\to 0 \qquad n \to 0$$

- Thus require:

$$f(x) \to \infty \text{ slower than } \sim 1/x^2$$



➡ Logarithmically slow dependence!

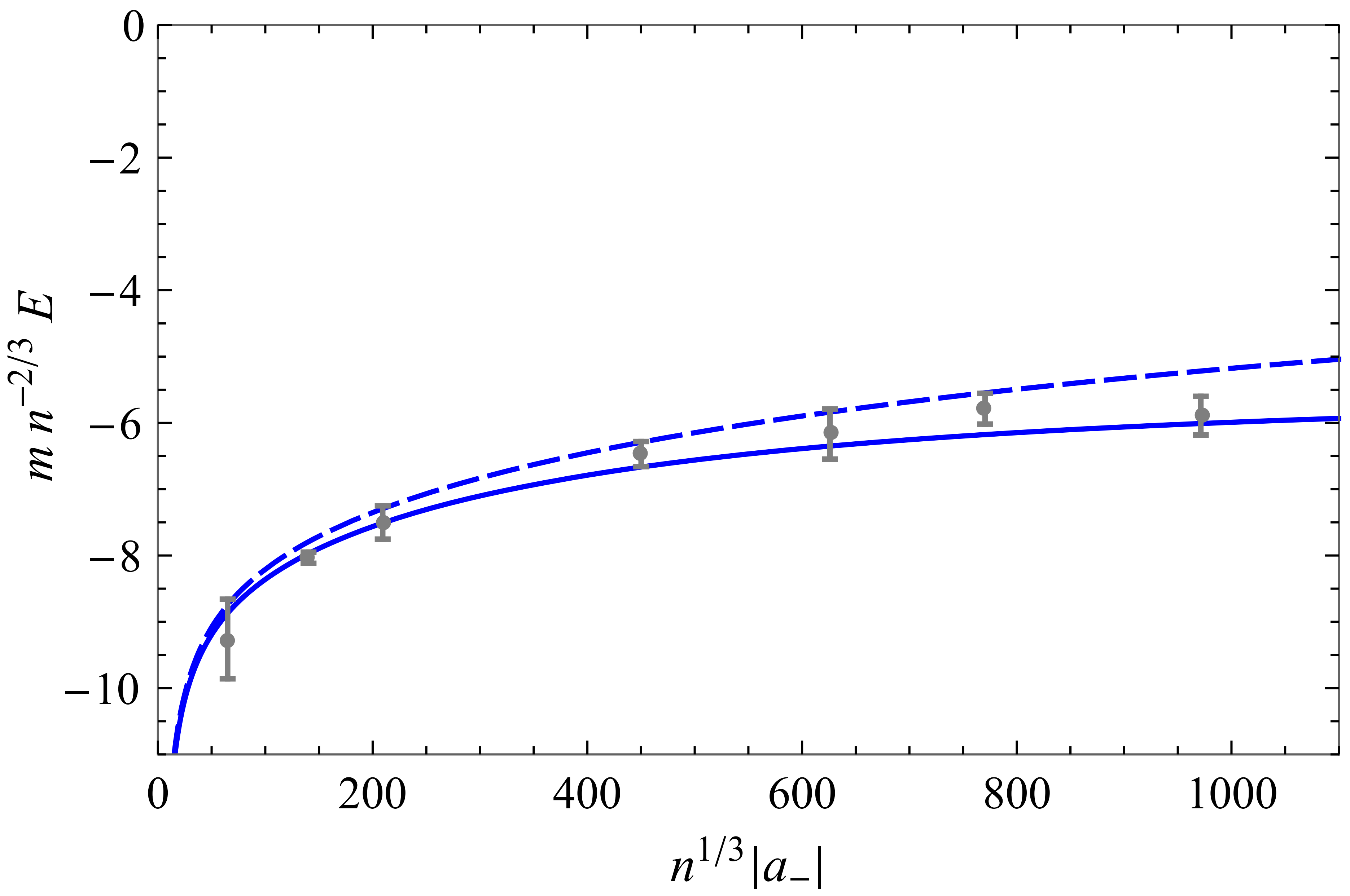
$$f(x) \sim -\ln(x)$$

Levinsen, Peña Ardila, Yoshida & MMP, PRL 2021

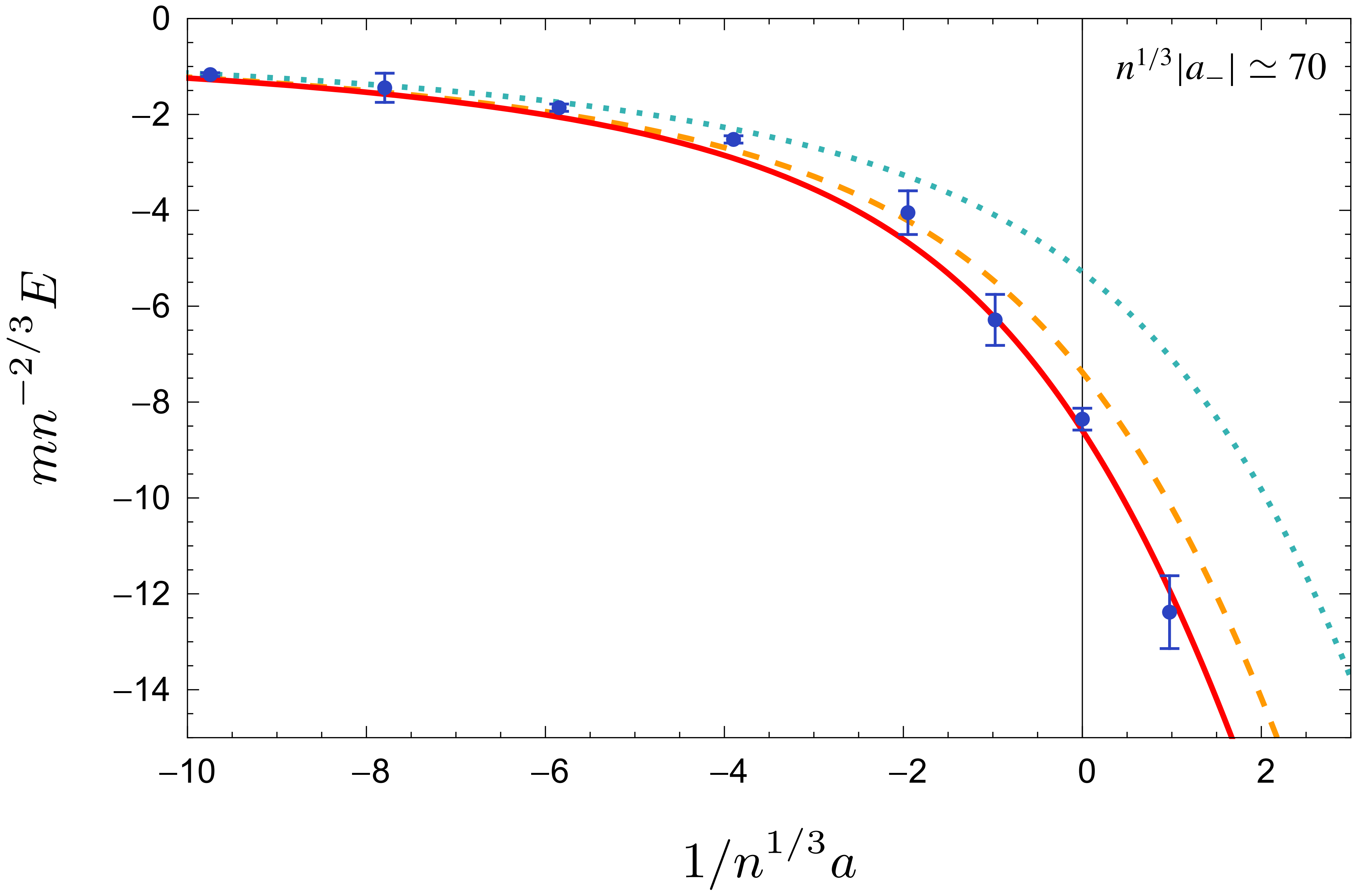
Yoshida, Endo, Levinsen & MMP, PRX 2018;
Field, Levinsen & MMP, PRA 2020

Finite mass?

- Equal-mass case



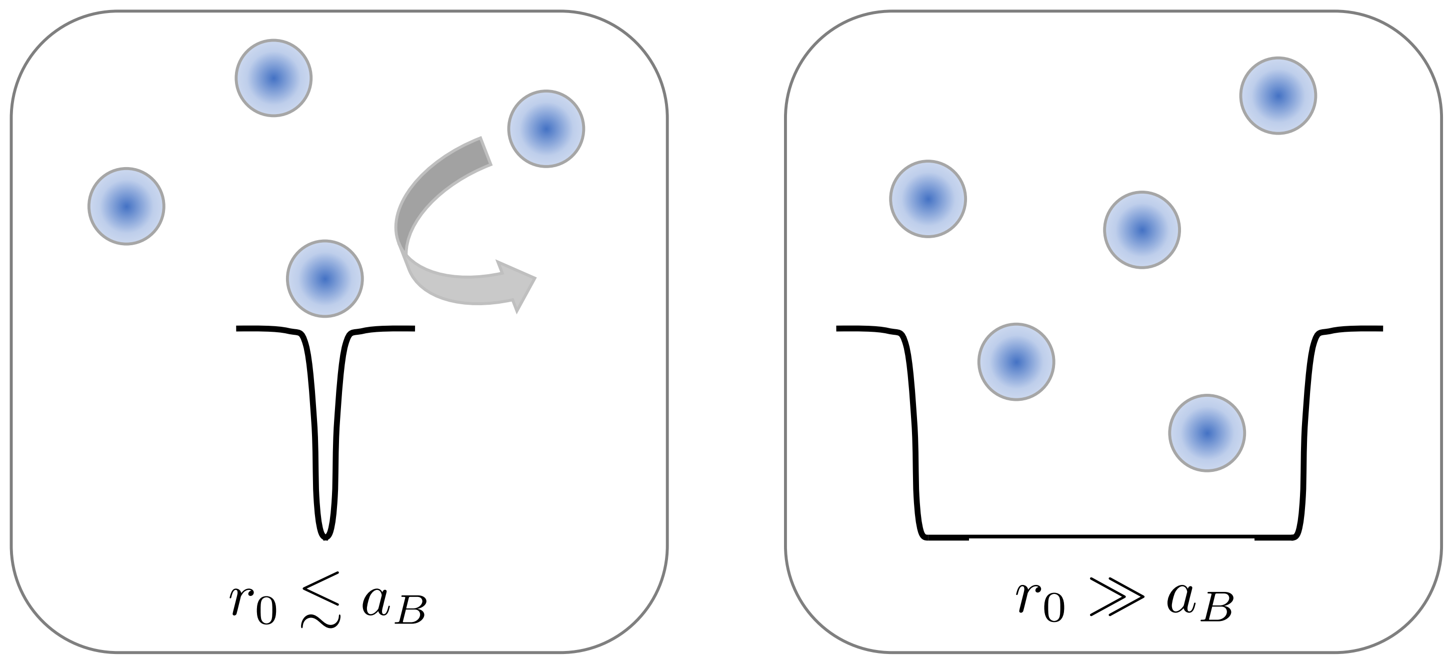
Efimov effect ➡ Power-law dependence



- Also obtain *universal* quantum blockade due to boson repulsion

Conclusion

- **Quantum behavior when impurity potential is short-ranged**
  - "Quantum blockade" at position of impurity
  - Universal logarithmic behavior at unitarity
  - Universal few-body bound states



- **Outlook**
  - Directly applicable to cold-atom experiments
  - Route to enhancing quantum correlations in photonic systems?