BOSONS AND MULTI-COMPONENT FERMIONS NEAR UNITARITY

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U.S. DEPARTMENT OF ENERGY
Office of Science

Outline

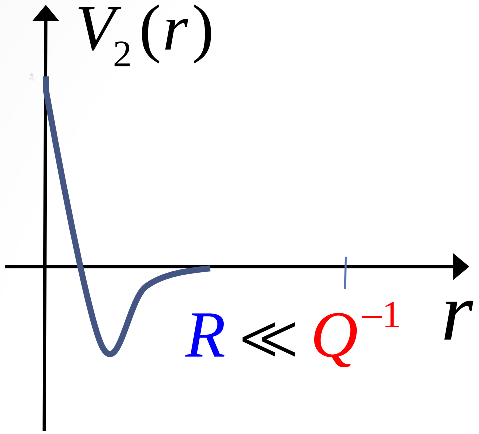
- Unitarity, why?
- Discrete scale invariance
- Bosonic clusters
- Multi-component fermions
- Nucleons
- Conclusion

Vast interdisciplinary subject:
apologies in advance for incomplete acknowledgement

Unitarity, why?

nonrelativistic,
short-range
interactions

$A=2$



singular interactions

regularization

$$V_2(\vec{r}) = \frac{4\pi}{m}\left[C_0(\Lambda)\,\delta_\Lambda^{(3)}(\vec{r}) + C_2(\Lambda)\,R^2\,\nabla^2\delta_\Lambda^{(3)}(\vec{r}) + \ldots\right]$$

quantum
multipole expansion

renormalization:
relative errors in observables $\sim Q/\Lambda \lesssim QR$
for $\Lambda \gtrsim 1/R$

Here:
$\hbar = 1, c = 1$
$[m]=[E]=[p]$
$=[r]^{-1}=[t]^{-1}$

order by order
in QR

vK '97,'99
Kaplan, Savage, Wise '98
...

$$T_2(k \ll R^{-1}) = \frac{4\pi}{m}\left(a_2^{-1} + ik - \frac{r_2}{2}k^2 + \ldots\right)^{-1} \quad [+\, l > 0]$$

Bethe '49

scattering length

effective range

unitarity limit $a_2^{-1} \approx \sqrt{mB_2} \to 0$ $\qquad r_2 \sim \ldots \sim R$ typically

$$\Rightarrow \quad T_2\left(\left|a_2^{-1}\right| \ll k \ll R^{-1}\right) = \frac{4\pi}{m}(ik)^{-1}\left(1 + \mathcal{O}\left(\frac{1}{ka_2}, kR\right)\right)$$

universality

unitarity window

no parameter!

renormalization $\Rightarrow$ $C_0^{(0)}(\Lambda) = -\frac{1}{\theta_0 \Lambda}$ non-trivial fixed point

number depending on specific form of regulator

(continuous) scale invariance

$$S = \int \frac{dt}{2m} \int d^3 r \left\{ \psi^\dagger \left(2m \frac{\partial}{\partial t} - \vec{\nabla}^2 \right) \psi - 4\pi C_0^{(0)} \left(\psi^\dagger \psi \right)^2 + \ldots \right\}$$

invariant

$\updownarrow \quad \alpha \geq 0$

$r \to \alpha r \quad \Leftrightarrow \quad p \to \alpha^{-1} p \quad \Leftrightarrow \quad \Lambda \to \alpha^{-1} \Lambda$

$t/m \to \alpha^2 \, t/m \quad \Leftrightarrow \quad mE \to \alpha^{-2} mE$

$\psi \to \alpha^{-3/2} \psi$

Nucleons

$R \sim m_\pi^{-1}$

$^3S_1$

$\left(a_{2,I=0} m_\pi\right)^{-1} \simeq 0.26$

$r_{2,I=0} m_\pi \simeq 1.2$

$^1S_0$

$\left|a_{2,I=1,I_3=+1} m_\pi\right|^{-1} - \left|a_{2,I=1,I_3=0} m_\pi\right|^{-1} \simeq 0.12$

$\left|a_{2,I=1,I_3=0} m_\pi\right|^{-1} \simeq 0.06$

$\left|a_{2,I=1,I_3=-1} m_\pi\right|^{-1} - \left|a_{2,I=1,I_3=0} m_\pi\right|^{-1} \simeq 0.02$

$r_{2,I=1} m_\pi \simeq 1.9$

Atoms

$R \sim l_{\text{vdW}}$

^{4}He

$l_{\text{vdW}}/a_2 \approx 0.06$

$r_2/l_{\text{vdW}} \approx 1.3$

Near Feshbach resonance

$\left|l_{\text{vdW}}/a_2\right| \to 0$

$r_2/l_{\text{vdW}} \sim 1$

simplify and unify theory with
perturbative expansion around unitarity limit

- *small corrections must be amenable to perturbation theory*
- *focus on essential parameters and symmetries*
- *singular interactions otherwise not renormalizable and model dependent*

More bodies

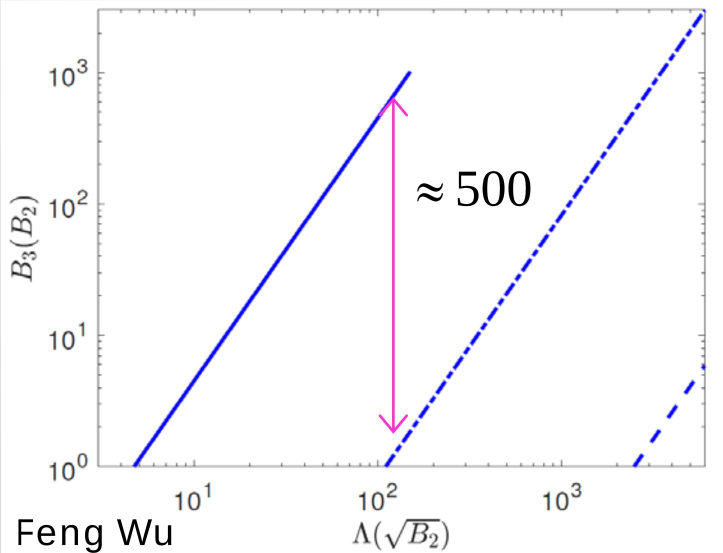
Two-component fermions

no finite energy bound states
unless scale invariance broken by external interaction/trap

e.g. $$\left.\frac{E_N^{(0)}}{N}\right|_{N\to\infty} = \frac{3k_F^2}{10m}\left(\xi + \mathcal{O}\left(\frac{1}{k_F a_2}, k_F r_2\right)\right) \qquad k_F = \left(3\pi^2\rho\right)^{1/3}$$

universal number Bertsch '99

Multi-component fermions, bosons



Feng Wu

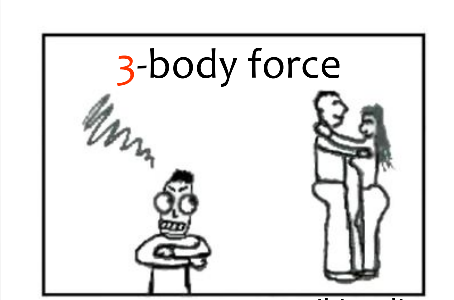
$$\frac{B_3}{3} \propto \frac{\Lambda^2}{m}$$

Thomas collapse

Thomas '35

For LO renormalization

$$V_3^{(0)}(\vec{r}_1-\vec{r}_2,\vec{r}_2-\vec{r}_3) = \frac{(4\pi)^2}{m} D_0^{(0)}(\Lambda)\,\delta_\Lambda^{(3)}(\vec{r}_1-\vec{r}_2)\,\delta_\Lambda^{(3)}(\vec{r}_2-\vec{r}_3)$$



Wikipedia

Bedaque, Hammer + vK '99 '00
...

$A = 3$

$s_0 \simeq 1.00624$

renormalization ⇨ $$D_0^{(0)}(\Lambda) \approx \frac{1}{\Lambda^4} \frac{\sin\left(s_0 \ln(\Lambda_*/\Lambda) - \arctan(1/s_0)\right)}{\sin\left(s_0 \ln(\Lambda_*/\Lambda) + \arctan(1/s_0)\right)}$$

limit cycle

cf. Wilson '71

anomalous breaking of
(continuous)
scale invariance

dimensionful parameter

discrete
scale invariance

$$S = \int \frac{dt}{2m} \int d^3r \left\{ \psi^\dagger \left(2m \frac{\partial}{\partial t} - \vec{\nabla}^2 \right) \psi - 4\pi C_0^{(0)} \left(\psi^\dagger \psi\right)^2 - (4\pi)^2 D_0^{(0)} \left(\psi^\dagger \psi\right)^3 + \dots \right\}$$

invariant

$$\alpha \to \alpha_n = \exp\left(n\pi/s_0\right) = (22.7)^n$$

n integer

Two consequences

1) Towers of excited states

$$mB^{(0)}_{A,n} \to \alpha_l^{-2}\, mB^{(0)}_{A,n} = mB^{(0)}_{A,n+l} \;\Rightarrow\; mB^{(0)}_{A,n}(\Lambda_*) = mB^{(0)}_{A,0}(\Lambda_*)\exp\left(-2n\pi/s_0\right)$$

ground state

fixes tower position

$A=3$ Efimov ’70

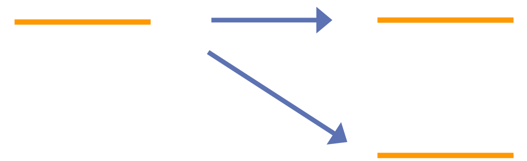
$A=4$ Hammer, Platter, ’07

$A=5,6$ von Stecher ’10’11
Gattobigio, Kievsky, Viviani ’11’12

bosons

doubling

A $\quad$ $A+1$



Efimov state

Efimov descendants

Towers upon towers!

2) Ground-state correlations

single scale → $\frac{B_{A,0}^{(0)}(\Lambda_*)}{A} = \kappa_A \frac{B_{3,0}^{(0)}(\Lambda_*)}{3}$

universal numbers

$\kappa_2 \equiv 0$
$\kappa_3 \equiv 1$
$\kappa_4 \simeq 3.5$ Hammer, Platter '07
$\kappa_{A\geq 5} \simeq ?$ von Stecher '10
...
Carlson, Gandolfi, Vitiello + vK '17

varying Λ_*

$A=4$ Tjon line Platter, Hammer, Meißner '05

Tjon '75
Nakaichi, Akaishi, Tanaka, Lim '78

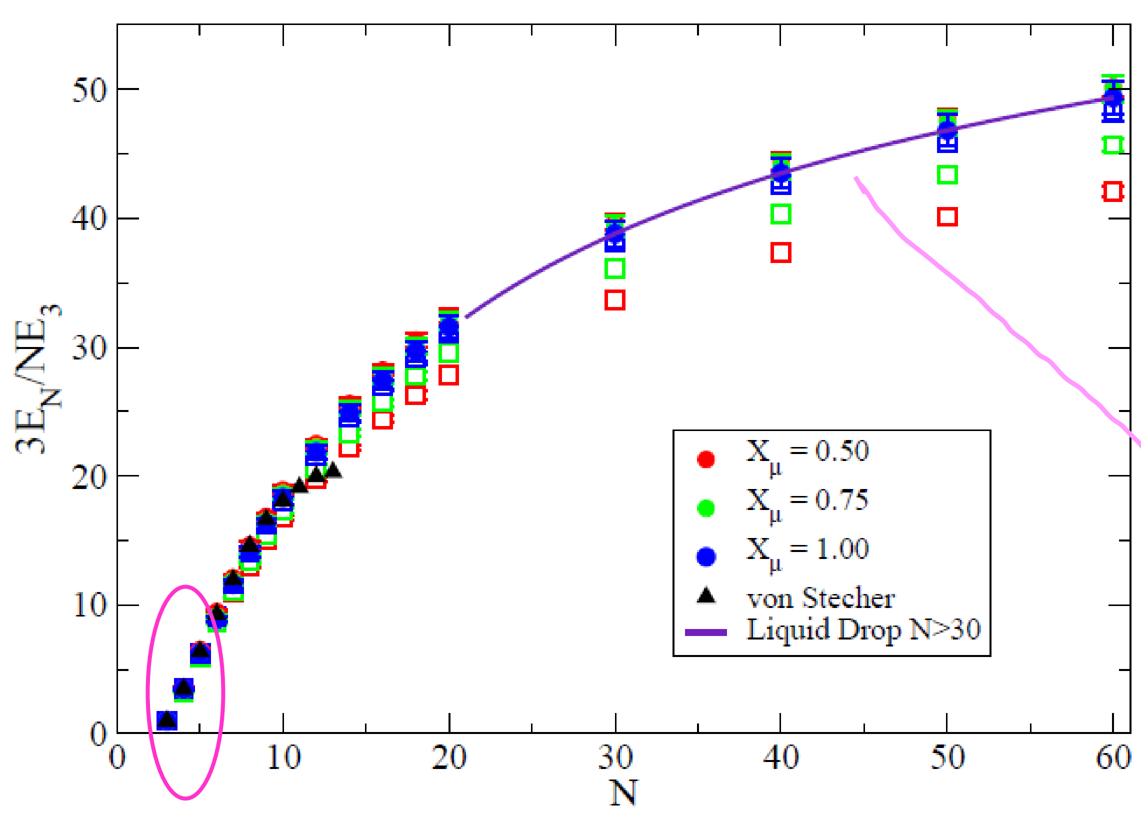
$A=5,6$ bosons Generalized Tjon lines Bazak, Eliyahu + vK '16

Nakaichi, Akaishi, Tanaka, Lim '79'80

BOSONS

Variational and Diffusion Monte Carlo

Carlson, Gandolfi, Vitiello + vK '17



increasing Λ

saturation!

cf. Piatecki, Krauth '14
Comparin, Krauth '16

$$\kappa_N \approx \frac{3}{N}(N-2)^2$$

Bazak, Eliyahu + vK '16

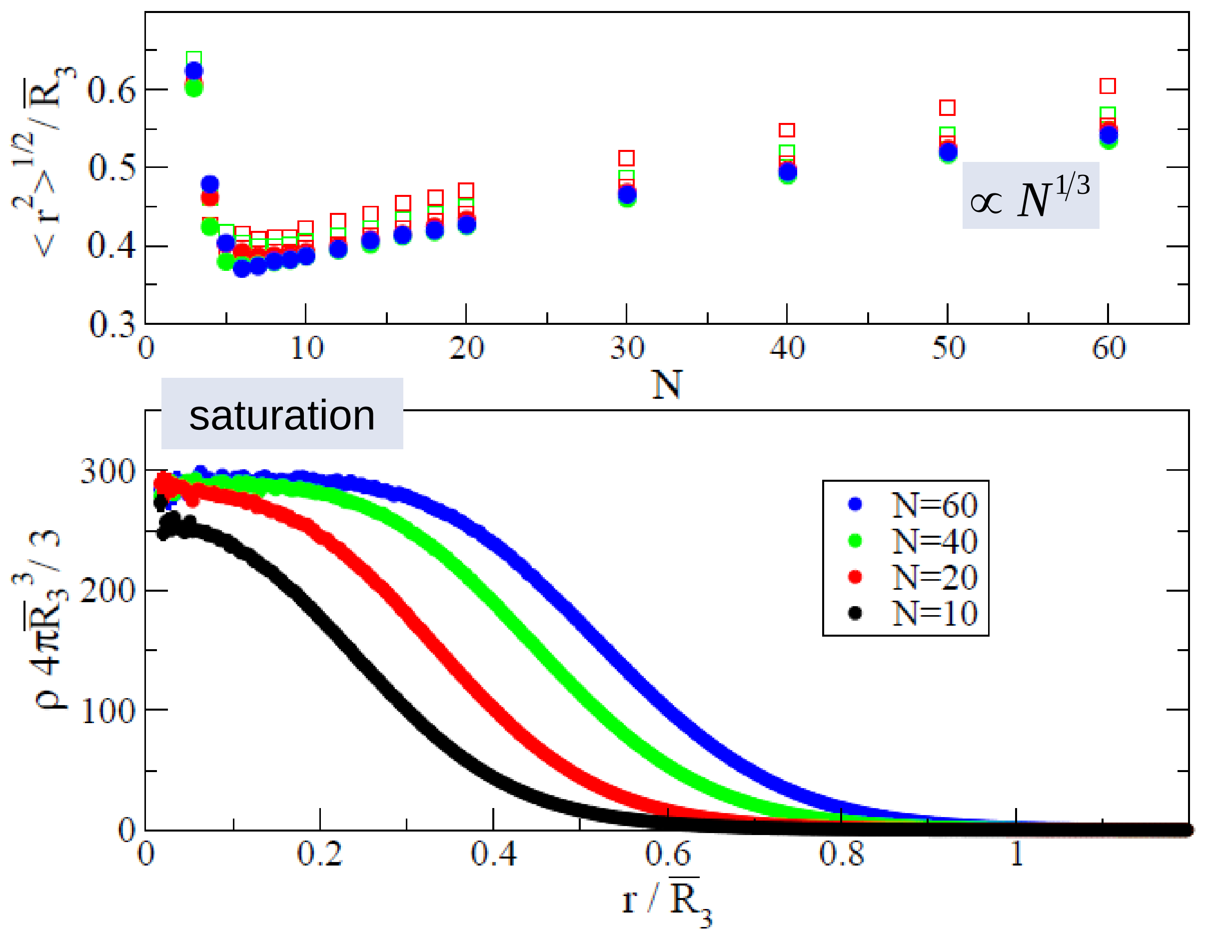
$$\kappa_N = \kappa_\infty\left[1-\eta N^{-1/3} + \mathcal{O}\left(N^{-2/3}\right)\right]$$

$$\kappa_\infty = 90 \pm 10 \quad \eta = 1.7 \pm 0.3$$

cf. ^{4}He $\kappa_\infty \approx 180$ $\eta \approx 2.7$ Pandharipande *et al.* '83

Carlson, Gandolfi, Vitiello + vK '17

A liquid indeed...



increasing Λ

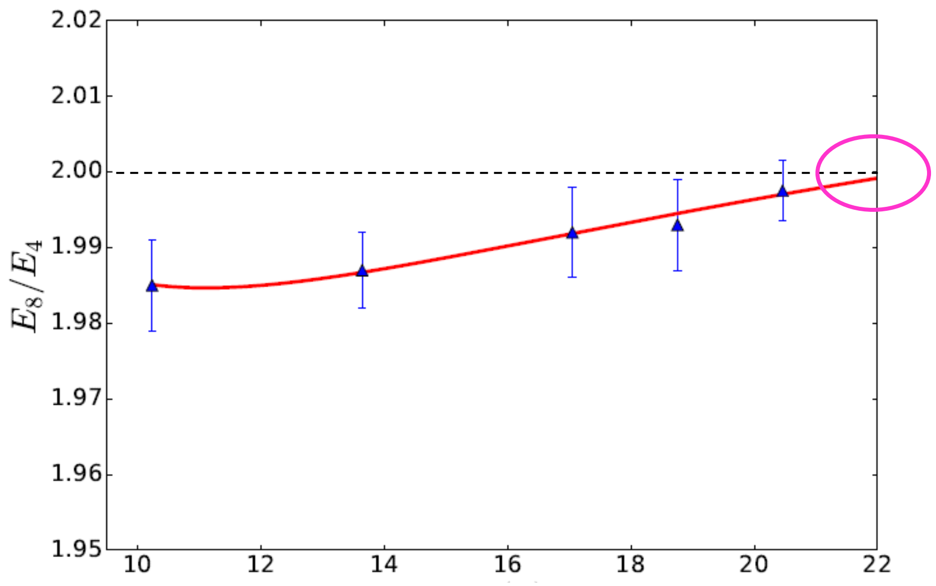
$$\bar{R}_3 \equiv \left(2mB_3\right)^{-1/2}$$

Dawkins, Carlson, Gezerlis + vK ’20

FOUR-COMPONENT FERMIONS

$A = 8$

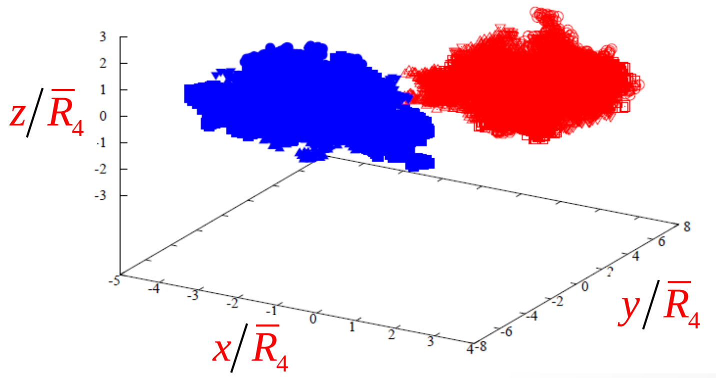
Variational and Diffusion Monte Carlo



$\overline{R}_4 \Lambda$

$\overline{R}_4 \equiv (2mB_4)^{-1/2}$

$\kappa_8 \simeq \kappa_4$?



consistent with ^{8}Be

Clustering a universal property of multi-component unitary fermions?

See also Schäefer, Contessi, Kirscher, Mareš ‘20

A lot of structure at unitarity!

How much of the physical world is *perturbatively* close to it?

Distorted-wave expansion around unitarity

NLO (first-order perturbation theory)

kR

$(ka_2)^{-1}$ — two-body force corrections — König, Grießhammer, Hammer + vK '16'17

$\alpha_e m k^{-1}$ — Coulomb force (if present)

$\left(\frac{B_{4,0}}{B_{4,0}^{(0)}}\right)^{1/2} - 1$ — four-body force — Bazak, Kirscher, König, Pavón, Barnea + vK '19

⇧

renormalization

cf. Hadizadeh *et al.* '11

etc.

^{4}He atoms

potentials

Aziz, Slaman '91 — Przybytek *et al.* '10

	(in mK)	LM2M2	PCKLJS	experiment	
LO: $C_0^{(0)}$	B_2	1.3094	1.6154	$1.3^{+0.25}_{-0.19}$, 1.76(15)	Grisenti *et al.* '00 (+ Cencek *et al.* '12); Zeller *et al.* '16
$D_0^{(0,1)}$	B_3^*	2.2779	2.6502		
	$B_3^* - B_2$	0.9685	1.0348	0.98(2)	Kunitski *et al.* '15
predictions	B_3	126.50	131.84		
	B_4^*	127.42	132.70		
$E_0^{(1)}$	B_4	559.22	573.90		

Hiyama, Kamimura '12

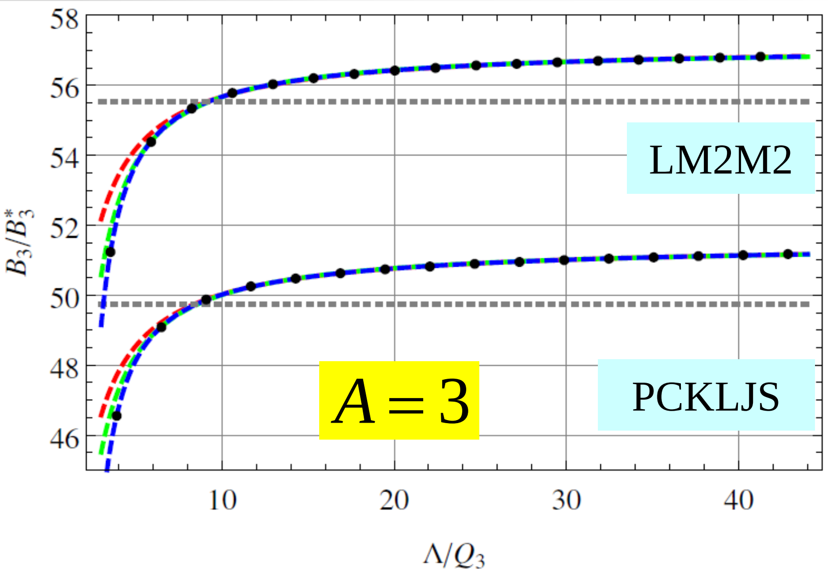
fit: experimental data (LO only); potential results (LO + NLO)

	(in Å)	LM2M2	PCKLJS	
NLO: $C_{0,2}^{(1)}$	a_2	100.23	90.42(92)	Janzen, Aziz '95; Kolganova *et al.* '04; Przybytek *et al.* '10
	r_2	7.326	7.27	
	r_{vdW}	5.378	5.378	Yan *et al.* '96; Zhang *et al.* '06

$Q_3 r_{\mathrm{vdW}} \simeq 0.4$

Bazak, Eliyahu + vK ’16

Bazak, Kirscher, König, Pavón, Barnea + vK ‘18

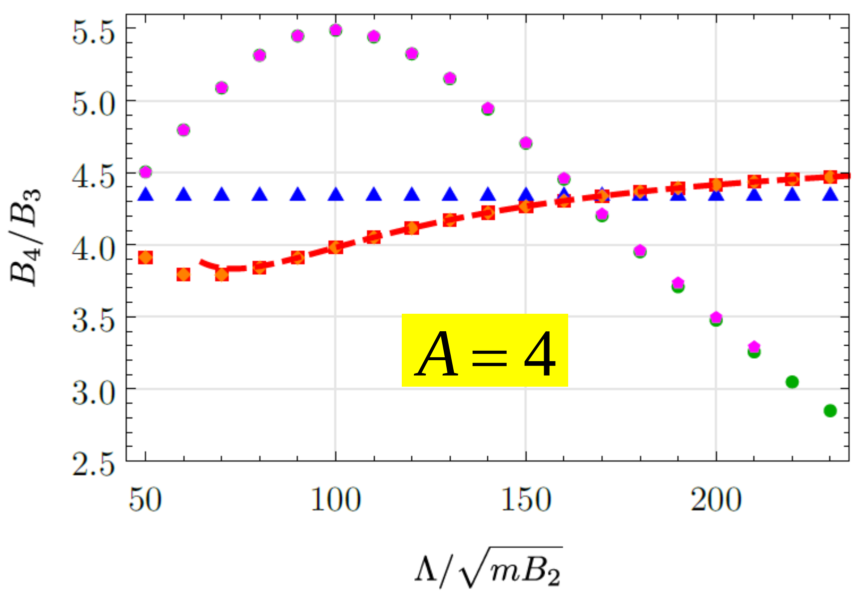


LO

Hiyama, Kamimura ‘12

LO

Hiyama, Kamimura ‘12

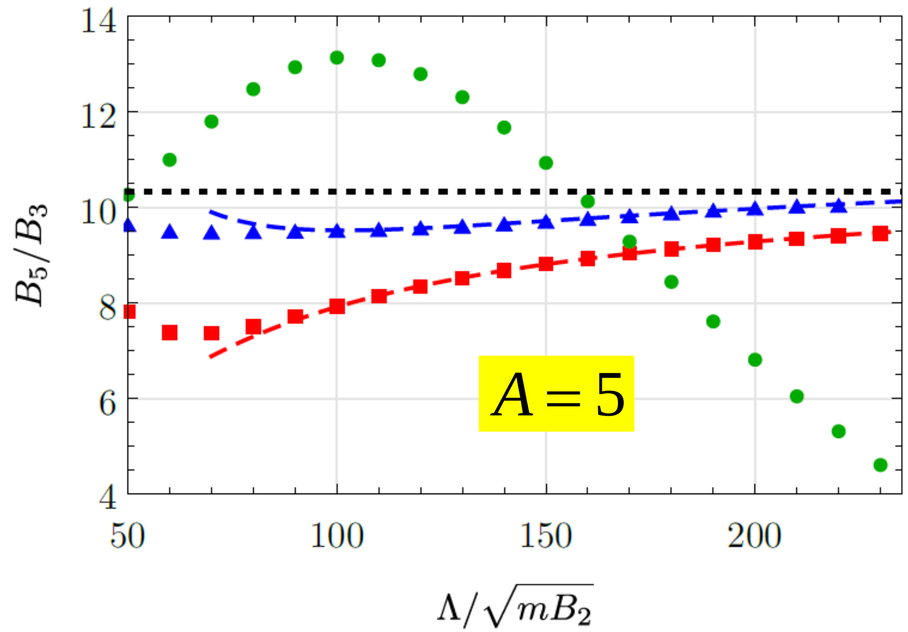


LO

PCKLJS → NLO

Hiyama, Kamimura ‘12

LO + two-body eff range



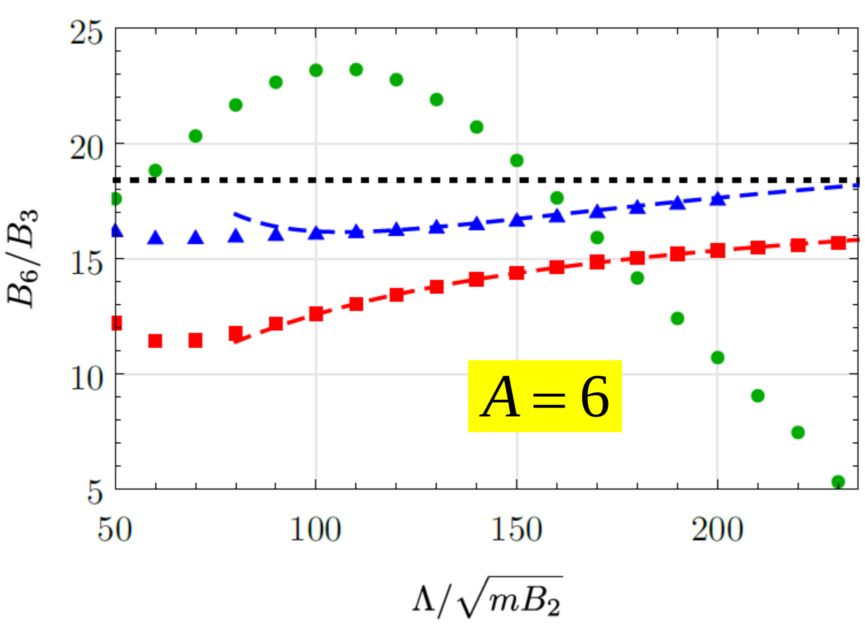
Blume, Greene ‘00

LM2M2

NLO

LO

LO + two-body eff range



Blume, Greene ‘00

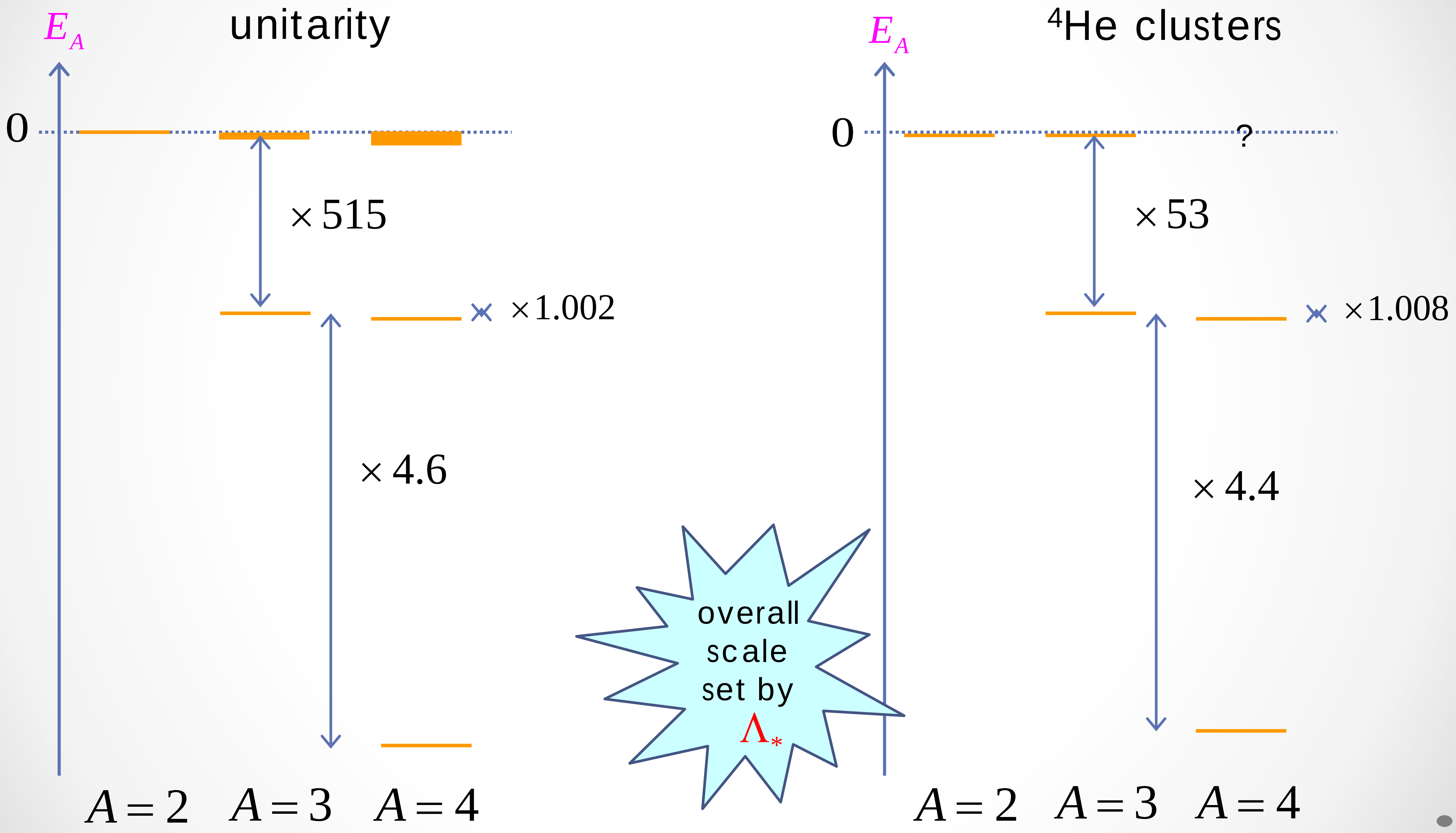
LM2M2

NLO

LO

LO + two-body eff range

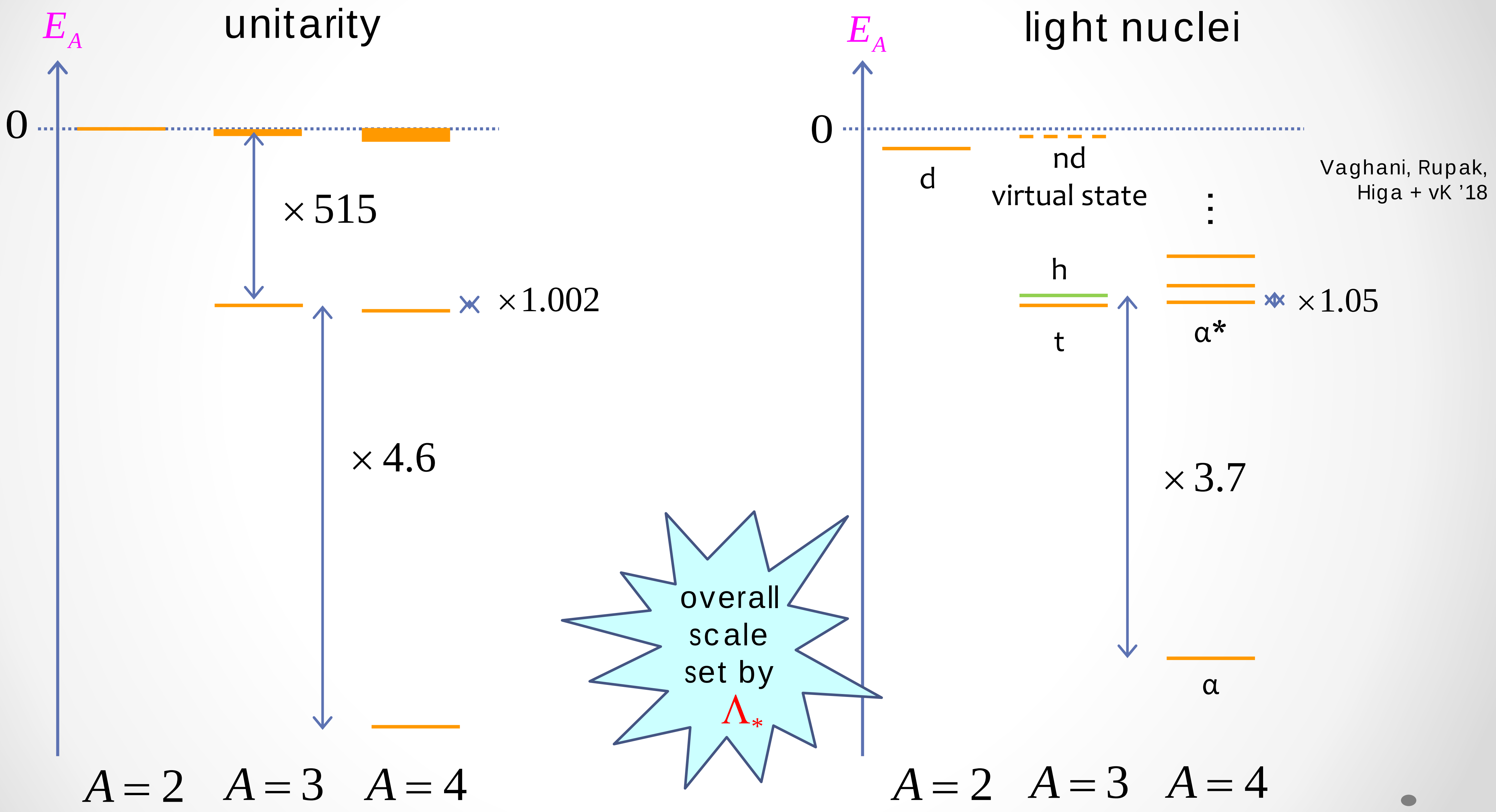
Schematically

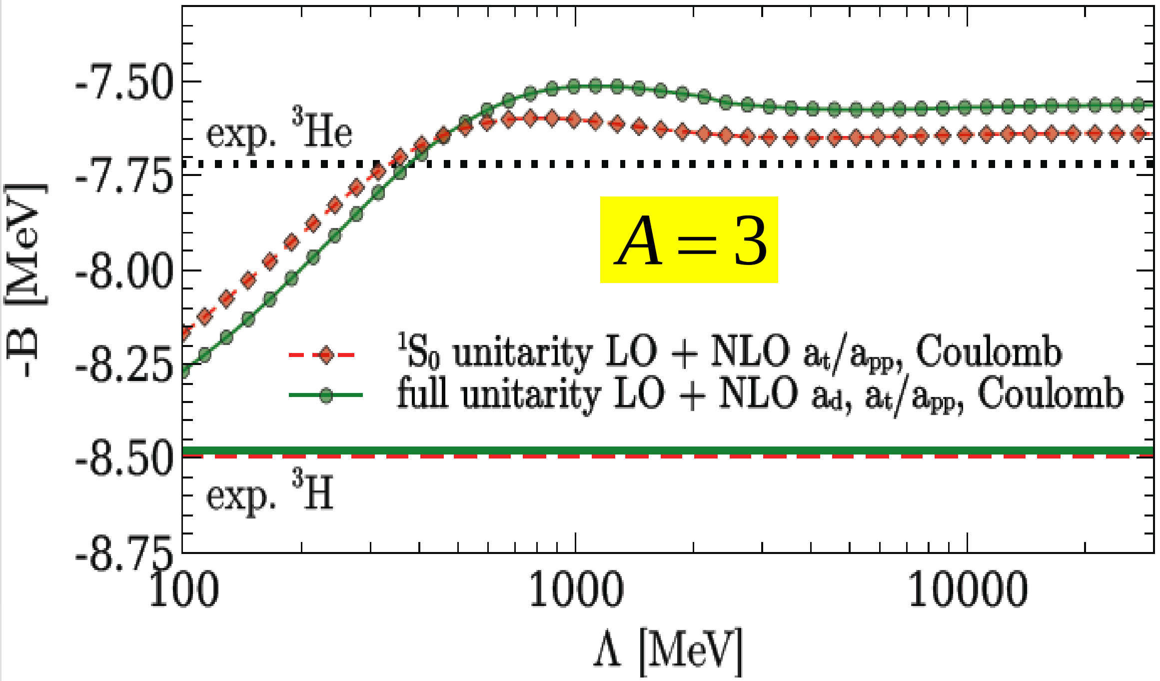


Expansion around unitarity

$A \leq 4$	full NLO for ^{4}He	Wu, König + vK, in progress
$A \leq 60$	$1/a$ corrections	Contessi, Gandolfi, Carlson + vK, in progress

Schematically





$$B_h^{(1)} - B_t \simeq -(0.92 \pm 0.18)\ \text{MeV}$$

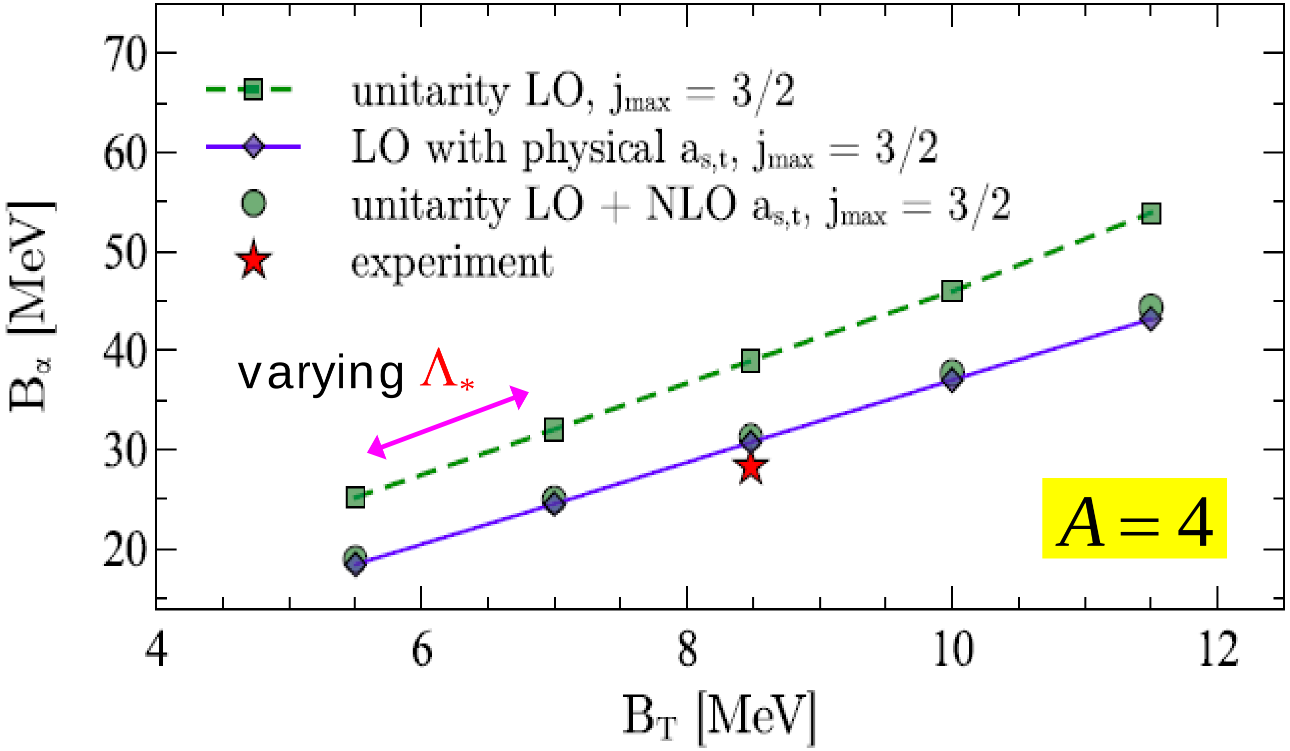
vs.

$$-0.764\ \text{MeV (exp)}$$

$$B_t = 8.48\ \text{MeV} \Rightarrow D_0^{(0)}(\Lambda)$$

König, Grießhammer, Hammer + vK ’16

nucleons



König, Grießhammer, Hammer + vK ’17

König '20

$A \leq 4$

Binding energies

state	$E_B^{\mathrm{LO}}/\mathrm{MeV}$	$E_B^{\mathrm{NLO}}/\mathrm{MeV}$	$E_B^{\mathrm{N^2LO}}/\mathrm{MeV}$	$E_B^{\mathrm{exp.}}/\mathrm{MeV}$
$^2\mathrm{H}$	0	0	1.41 ± 1.12	2.22
$^3\mathrm{H}$	$\underline{8.48}$	$\underline{8.48}$	$\underline{8.48}$	8.48
$^3\mathrm{He}$	8.5 ± 2.5	7.6 ± 0.2	$\underline{7.72}$	7.72
$^4\mathrm{He}$	39 ± 12	$30 \pm 9^*$		28.3

Point-charge radii

$\langle r_0^2\rangle_{^3\mathrm{H}} = \langle r^2\rangle_{^3\mathrm{H}} - \langle r^2\rangle_p - 2\langle r^2\rangle_n$	1.04(31) fm	1.10(33) fm*	1.59 fm
$\langle r_0^2\rangle_{^4\mathrm{He}} = \langle r^2\rangle_{^4\mathrm{He}} - 2\langle r^2\rangle_p - 2\langle r^2\rangle_n$	1.49(45) fm	1.73(52) fm*	1.72 fm

* incomplete NLO

full NLO Wu, König + vK, in progress

$A = 5, 6$ Contessi + vK, in progress

Conclusions

Systems near unitarity can be described by essentially one parameter Λ_*

Renormalization leads to discrete scale invariance

Bosons saturate and form a quantum liquid

Multi-component fermions tend to clusterize

Expansion around unitarity works for light nuclei.

How far can we go?