

Small Atomic Quantum Systems with Large s-Wave Scattering Length

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Collaborators over the years:

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Maksim Kunitski, Reinhard Doerner and group.**

Supported by the NSF.

Helium Droplets: Cold But Not Ultracold Samples

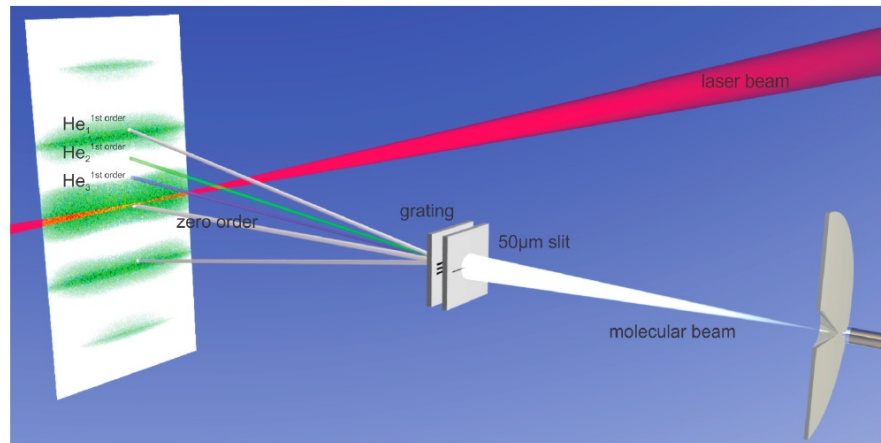
Cold ^4He atoms (sub-Kelvin temperatures):

Three-body (three-body Efimov state)

Larger clusters

Two-body (real-time dynamics)

Size-selected nozzle beam expansion experiments

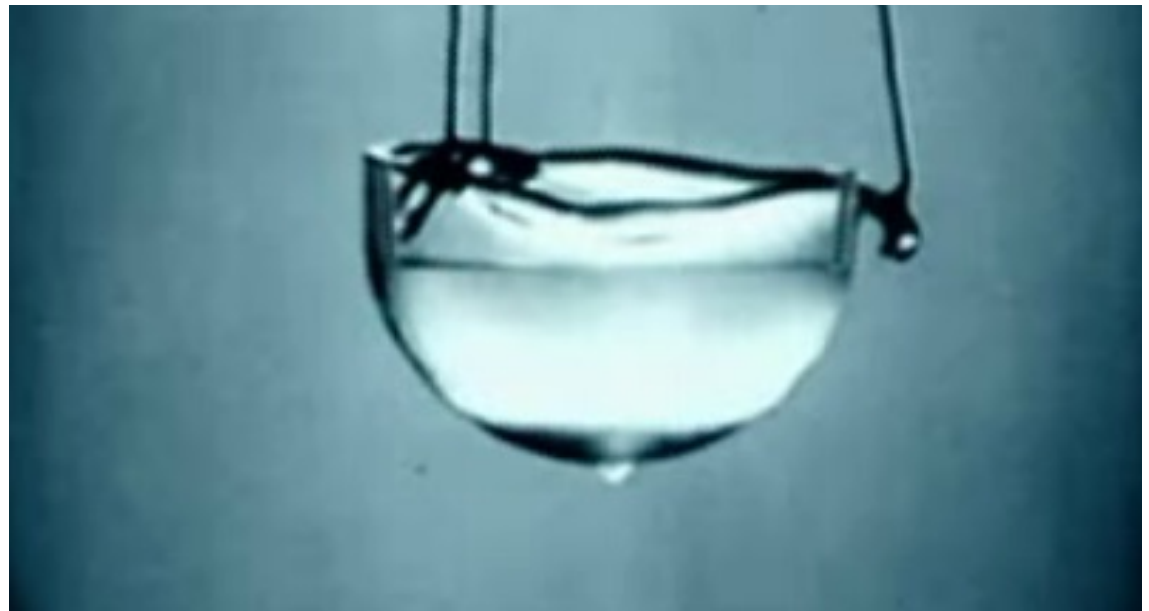


Superfluid Bulk Helium-4

- ^4He is the only substance that remains liquid under normal pressure at zero temperature (superfluid with condensate fraction of around 8%).
- Normal to superfluid transition at 2.17K.

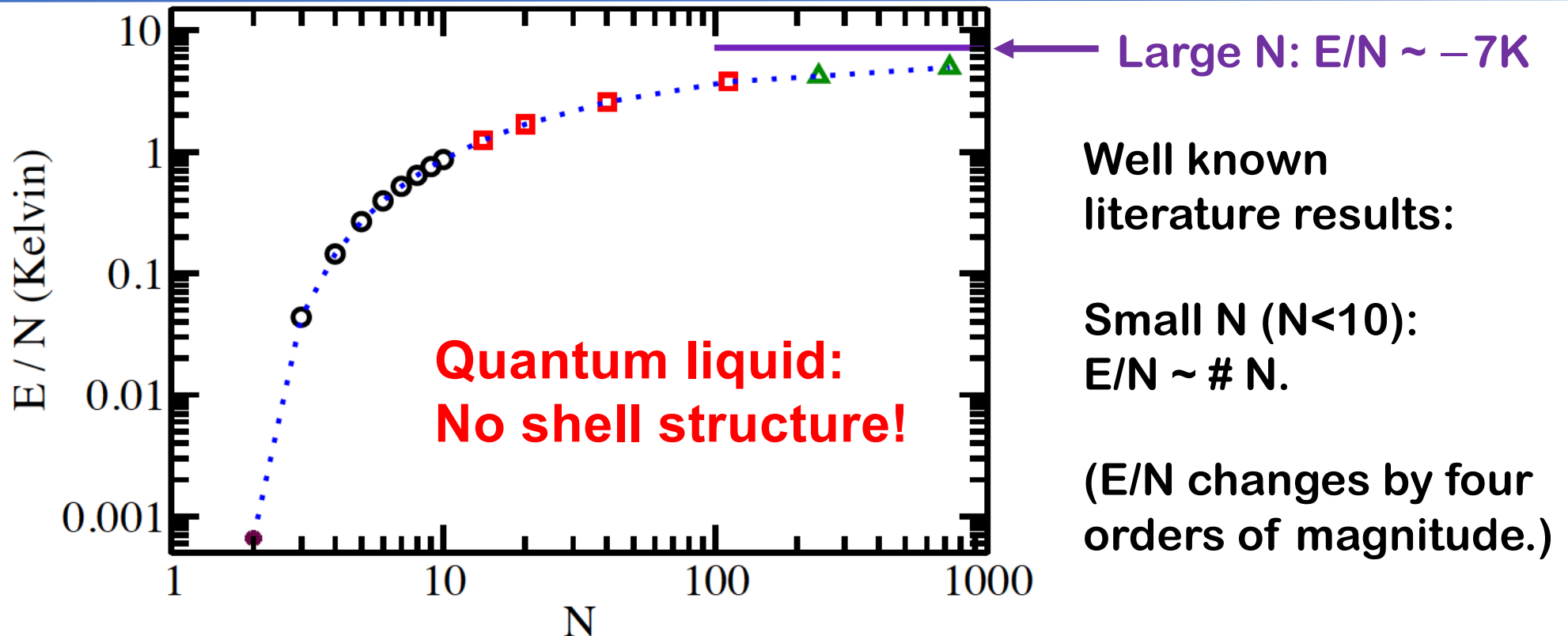
Helium named after
the sun (greek “helios”).
Discovered in 1868.

Bulk liquid helium-4:
Binding energy per
particle $E/N = -7\text{K}$
(1 K = 8.6×10^{-5} eV).



From Wikipedia

Helium Droplets = Quantum Liquid

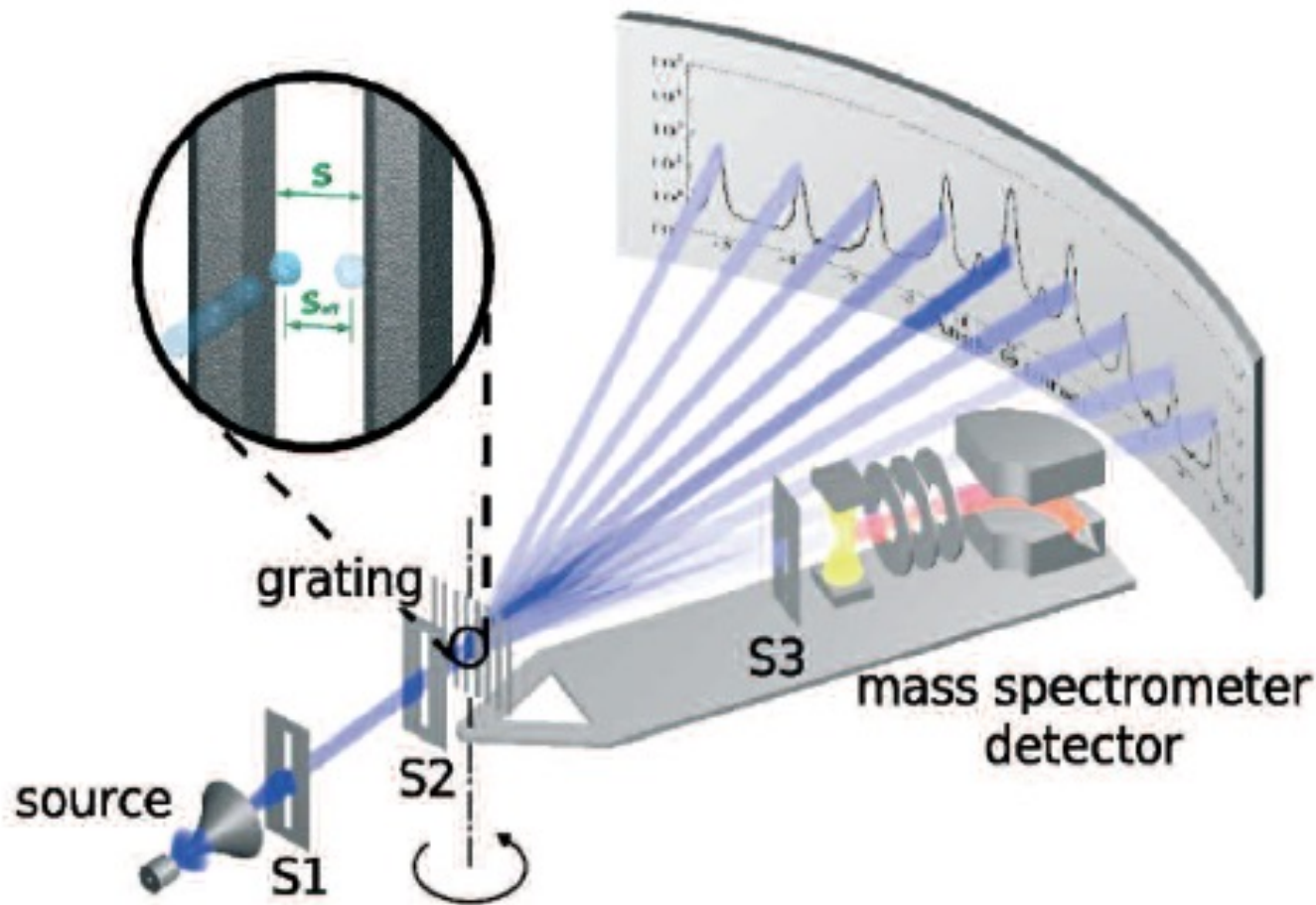


$N > 20$ energies are well described by liquid drop model with volume and surface terms (no Coulomb, asymmetry, or pairing terms).

Rich interplay between many-body nuclear physics and quantum droplet community [e.g., Pandharipande et al., PRL 50, 1676 (1983); Stringari et al., JCP 87, 5021 (1987); Sindzingre et al., PRL 63, 1601 (1989)].

Helium Droplets: Matter Wave Diffraction Experiment

Kornilov, Toennies, 10.1051/epr:2007003



De Broglie wave length λ :

$$\lambda = h/(Mv)$$

Diffraction angle θ :

$$\sin \theta = n \frac{\lambda}{d} = n \frac{h}{\underline{N \cdot m \cdot v \cdot d}}$$

v: velocity

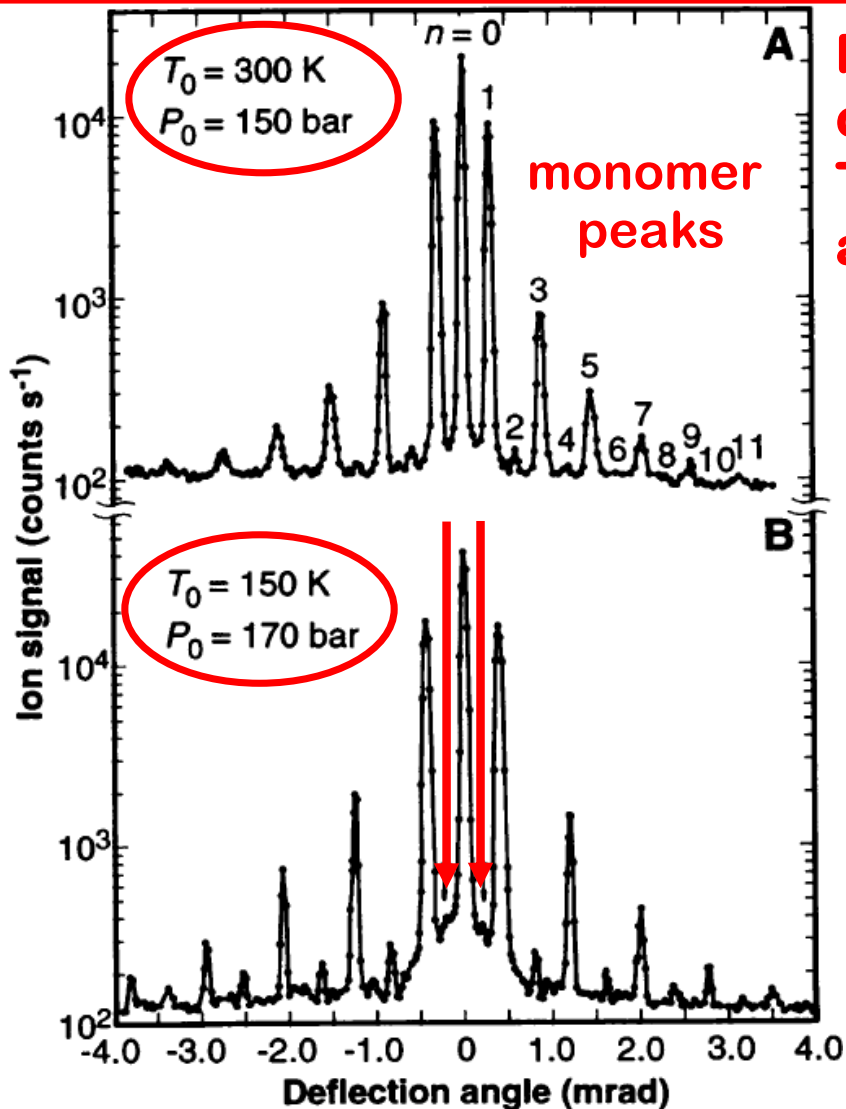
n: diffraction order

m: mass of helium atom

N: number of helium Atoms

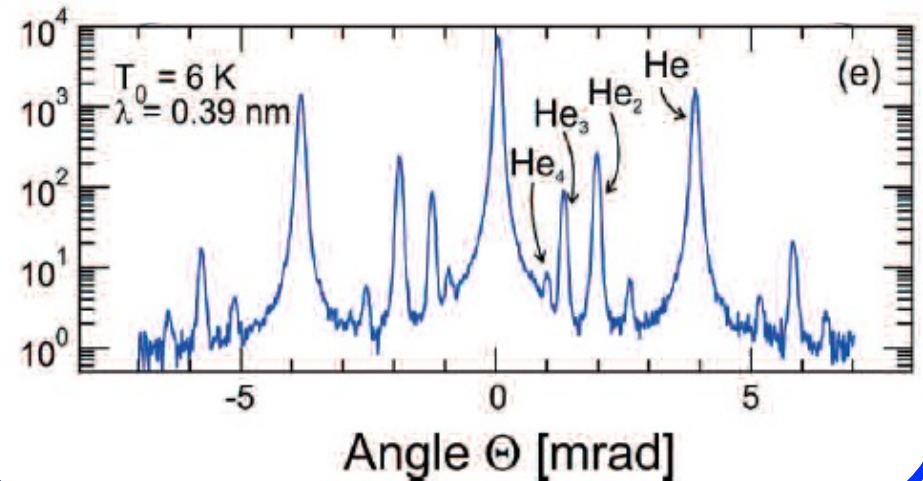
Nm: mass of cluster

Observation of Bosonic Helium Dimer: $^4\text{He}_2$



Fragile helium dimer forms in beam and can be isolated. Schoellkopf and Toennies, *Science* 266, 1345 (1994); see also Luo et al., *JCP* 98, 3564 (1993).

Nozzle temperature and pressure can be adjusted. Kornilov, Toennies, [10.1051/epl:2007003](https://doi.org/10.1051/epl/2007003)



Debate on Helium Dimer

creasing with each passing year. When a recent PhD in a physical science said that helium formed diatomic molecules, I knew we were in trouble!

**R. Bruce Doak,
Physics Today
65(8), 10 (2012)**

...that would be, let's see, 16 years ago now. Schöllkopf and Toennies diffracted helium atoms and dimers from a manmade transmission diffraction grating to show, beyond the shadow of a doubt, that the neutral helium dimer exists as a stable diatomic species (albeit extremely weakly bound). Given the highly quantum mechanical nature of this extraordinary dimer and the fact that it has perhaps the most weakly bound ground state of any dimer, it is of considerable fundamental interest.

Helium Dimer and Trimers

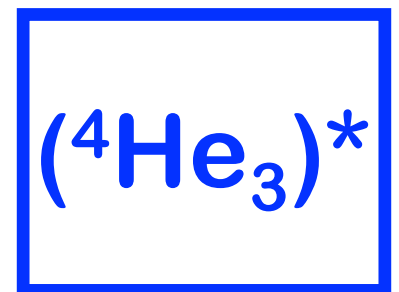
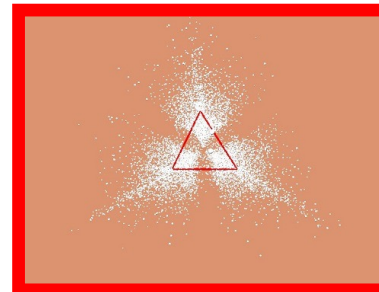
$$1 \text{ K} = 8.6 \times 10^{-5} \text{ eV}$$

- Dimer:

- ^4He - ^4He bound state energy $E_{\text{dimer}} = -1.625\text{mK}$. No $J > 0$ bound states. ^4He - ^3He does not support bound state.
- Two-body s-wave scattering length $a_s = 170.86a_0$.
- Two-body effective range $r_{\text{eff}} = 15.2a_0$
(alternatively, two-body van der Waals length $r_{\text{vdW}} = 5.1a_0$).

- Trimer:

- Two $J = 0$ bound states with $E_{\text{trimer}} = -131.8\text{mK}$ and -2.65mK .
- No $J > 0$ bound states.



Discussed later
in this talk.

Helium Dimer and Trimers

$$1 \text{ K} = 8.6 \times 10^{-5} \text{ eV}$$

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(alternatively, two-body van der Waals length $r_{\text{vdW}} = 5.1a_0$).

- Trimer:

- Two $J = 0$ bound states with $E_{\text{trimer}} = -1.3$
 -2.65mK .
- No $J > 0$ bound states.

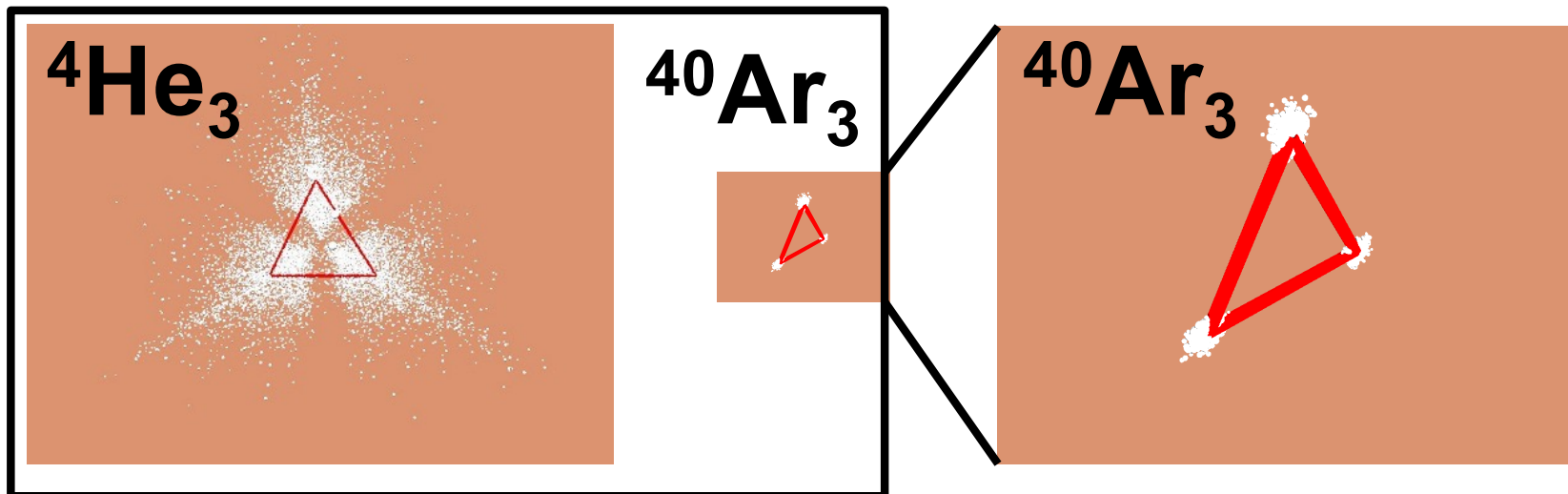
Close to hard wall at small internuclear distances is a challenge for some numerical approaches: "Less soft" than "typical" nuclear potentials.

- Nuclear physics: Deuteron and triton.

in this talk.

Comparison With Other Neutral Rare Gas Clusters

- ^4He , ^{10}Ne , ^{20}Ar : composite bosons (energy scales are such that these atoms can be considered as point particles; consider only nuclear degrees of freedom).
- Dimer (potential minimum at $5-10a_0$):
 - ^4He - ^4He binding energy: $E_{\text{dimer}} = -1.3\text{mK}$.
 - ^{10}Ne - ^{10}Ne binding energy: $E_{\text{dimer}} = -20.1\text{K}$.
 - ^{20}Ar - ^{20}Ar binding energy: $E_{\text{dimer}} = -101\text{K}$.



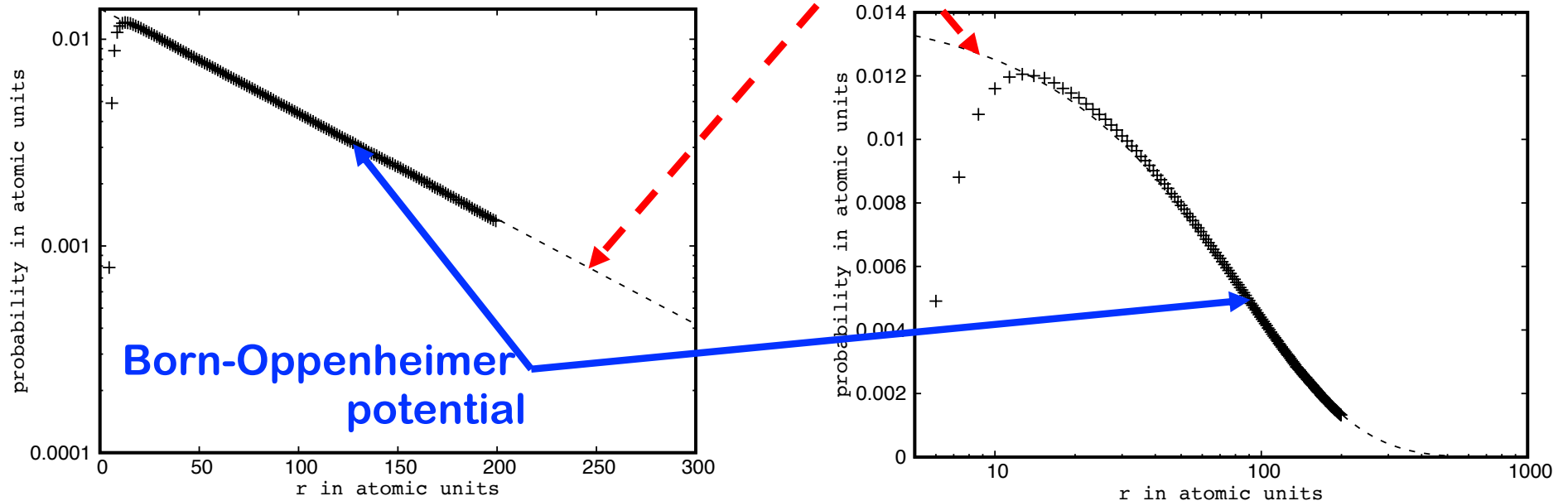
Helium Dimer

$$1 \text{ K} = 8.6 \times 10^{-5} \text{ eV}$$

- Using modern Born-Oppenheimer potential:
 - ^4He - ^4He bound state energy $E_{dimer} = -1.625 \text{ mK}$.
 - Two-body s-wave scattering length $a_s = 170.86 a_0$.
- **Question: Is the ^4He dimer universal?**
 - $E_{dimer} = -5.147 \cdot 10^{-9} \text{ a. u.}$
 - Zero-range theory: $E_{dimer} = -\frac{\hbar^2}{m a_s^2} = -4.69 \cdot 10^{-9} \text{ a. u.} (\approx 91\%)$
 - Including effective range correction ($r_{eff} = 15.2 a_0$):
$$E_{dimer} = -\frac{\hbar^2}{m r_{eff}^2} \left(1 - \sqrt{1 - \frac{2r_{eff}}{a_s}} \right)^2 = -5.17 \cdot 10^{-9} \text{ a. u.} (\approx 100\%)$$

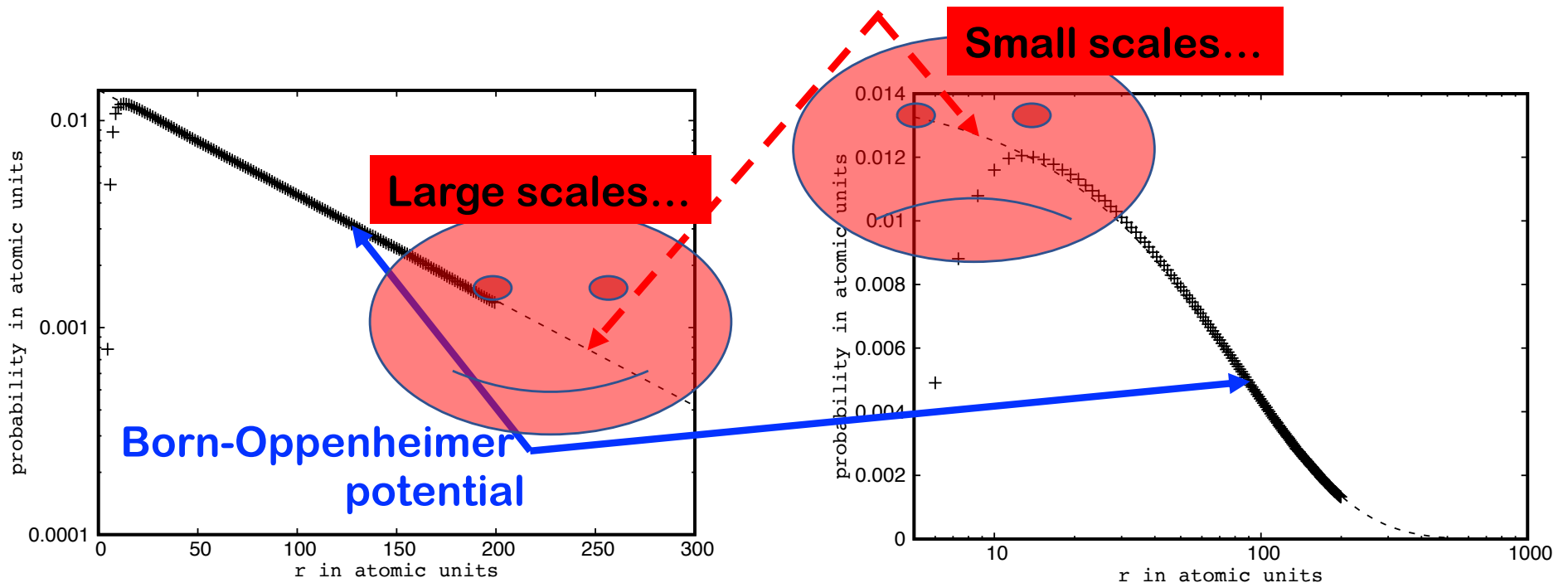
Visualizing the Difference

- Using modern Born-Oppenheimer potential:
 - ^4He - ^4He bound state energy $E_{dimer} = -1.625$ mK.
 - Two-body s-wave scattering length $a_s = 170.86 a_0$.
 - Zero-range theory: **probability** $= \frac{2}{a_s} \exp\left(-2\frac{r}{a_s}\right)$.

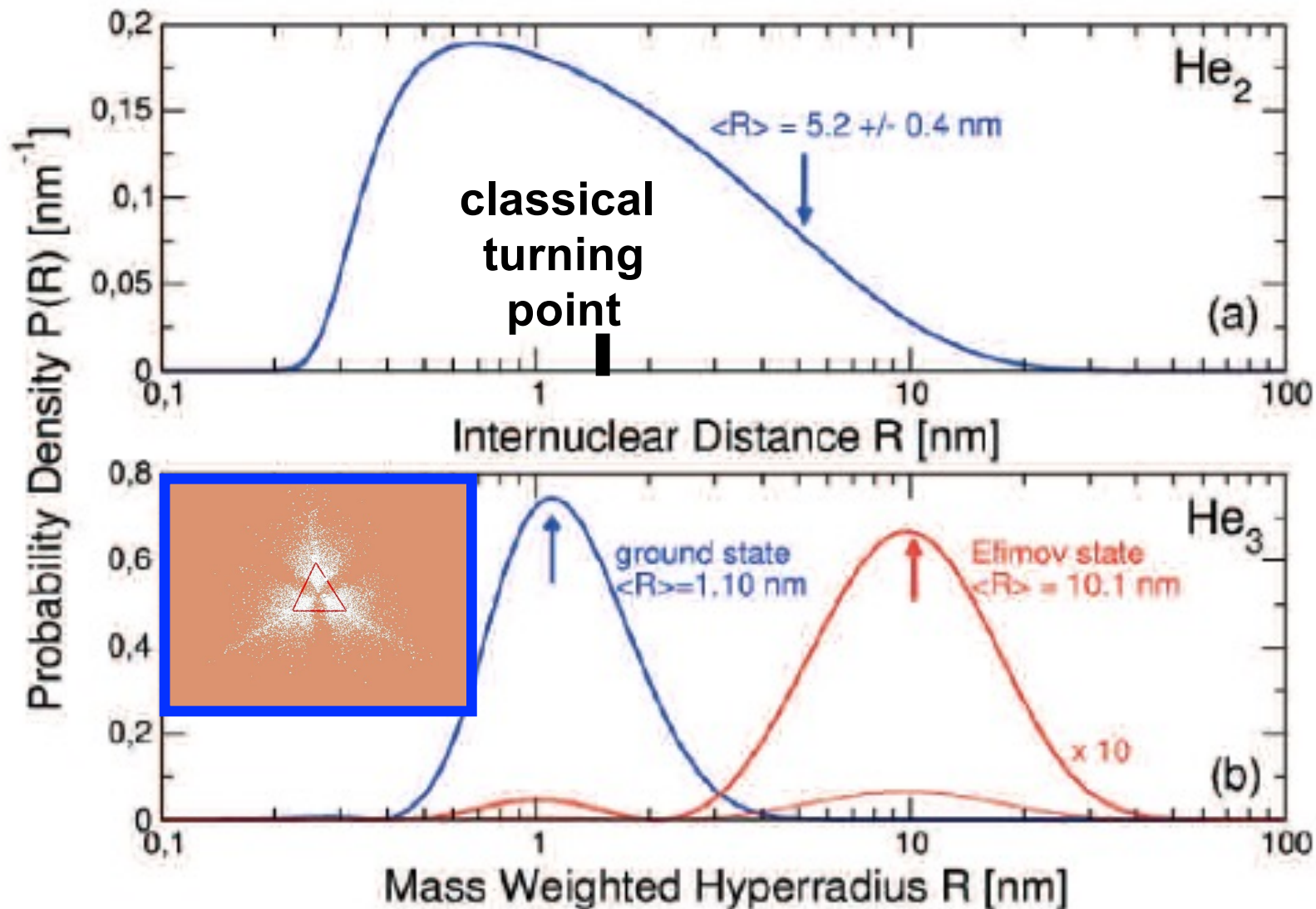


Visualizing The Difference

- Using modern Born-Oppenheimer potential:
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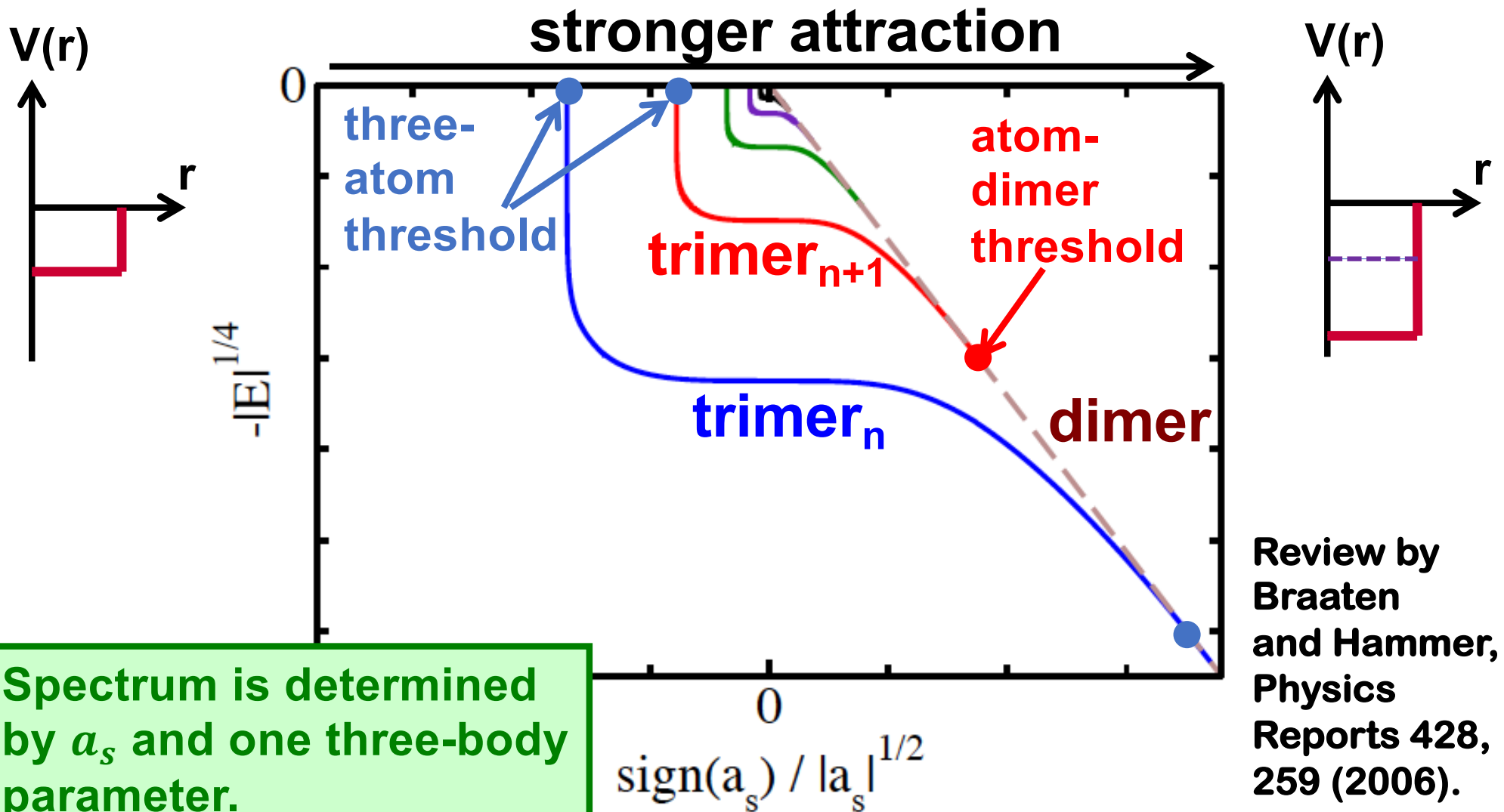


Comparison: Helium Dimer and Trimers



Kornilov,
Toennies,
10.1051/epr:
2007003

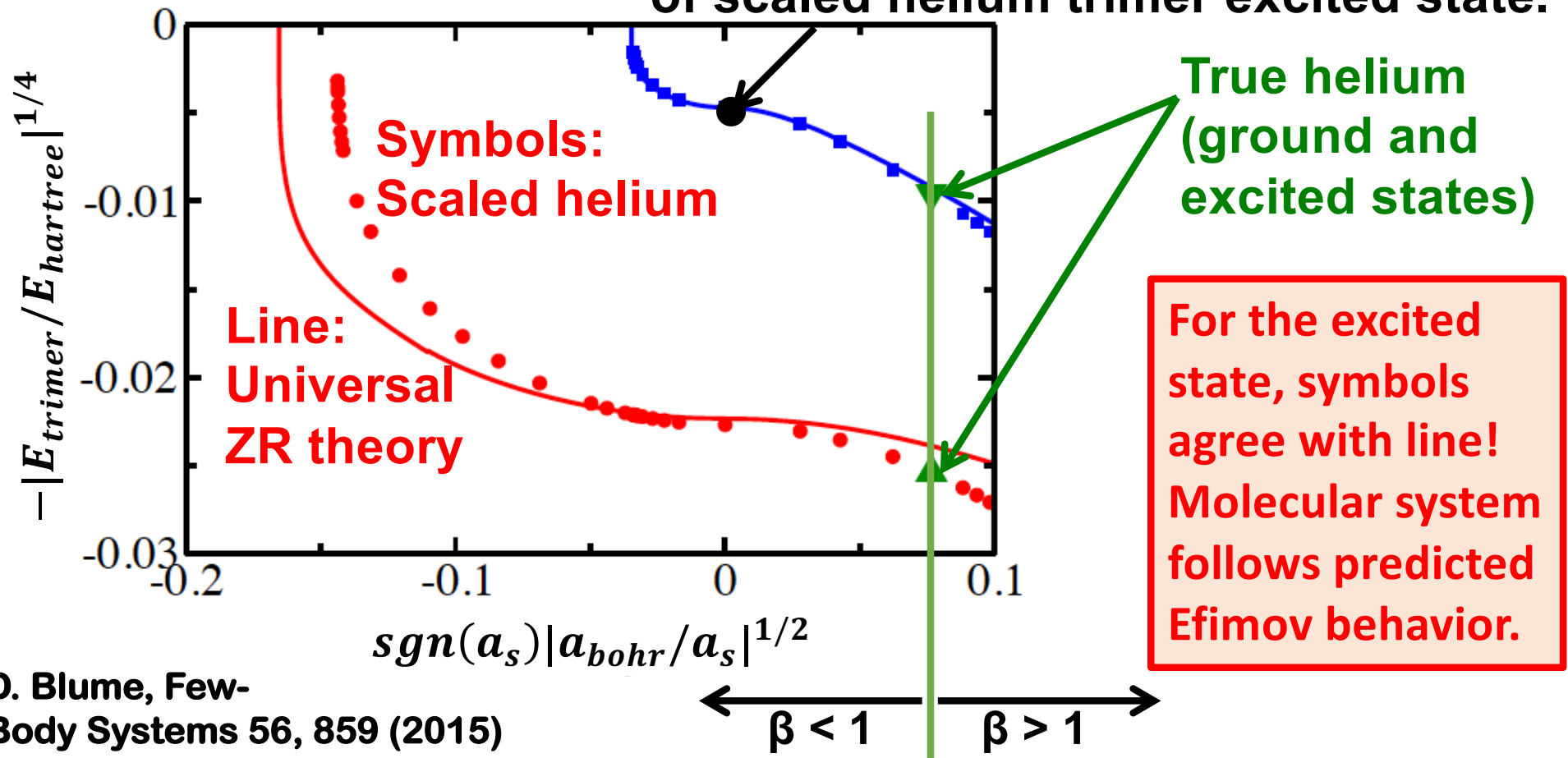
Finite s-wave Scattering Length: Universally Linked States



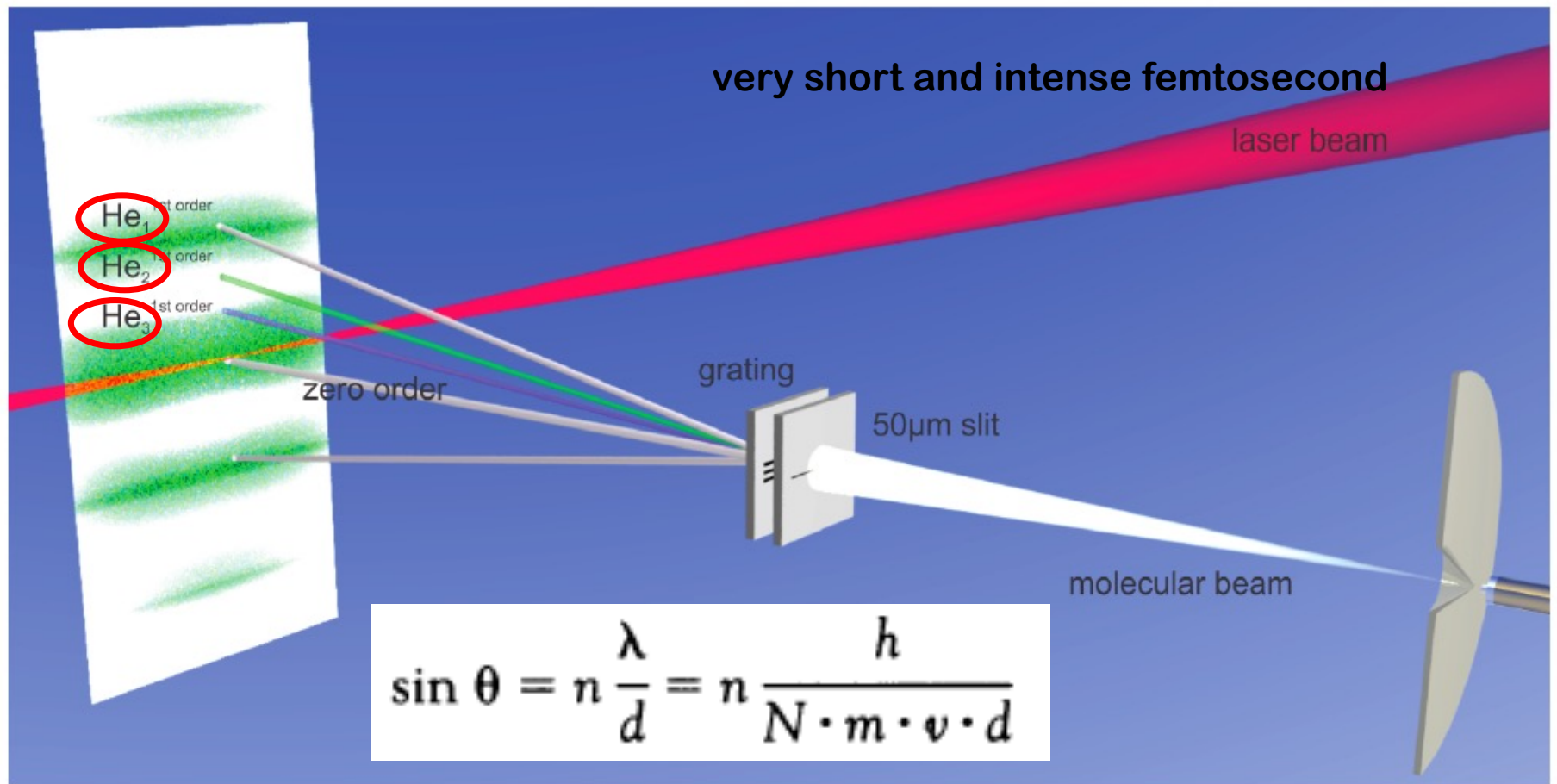
Helium Trimer Excited State is an Efimov State

$$\beta V_{\text{He-He}}(r_{12}) + \beta V_{\text{He-He}}(r_{23}) + \beta V_{\text{He-He}}(r_{31}).$$

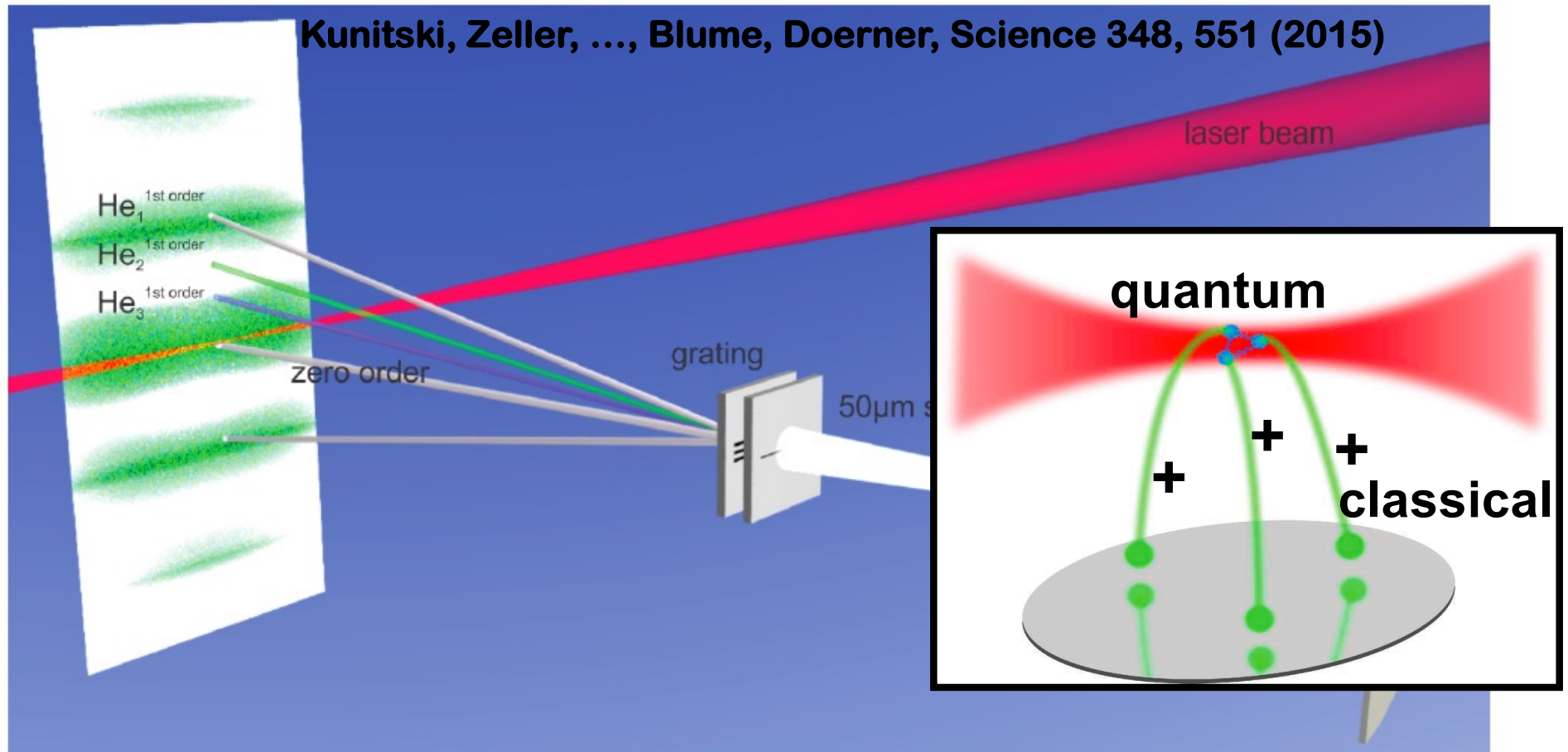
Three-body parameter is chosen such that ZR energy agrees with energy of scaled helium trimer excited state.



Probing Helium Trimer Excited Efimov State

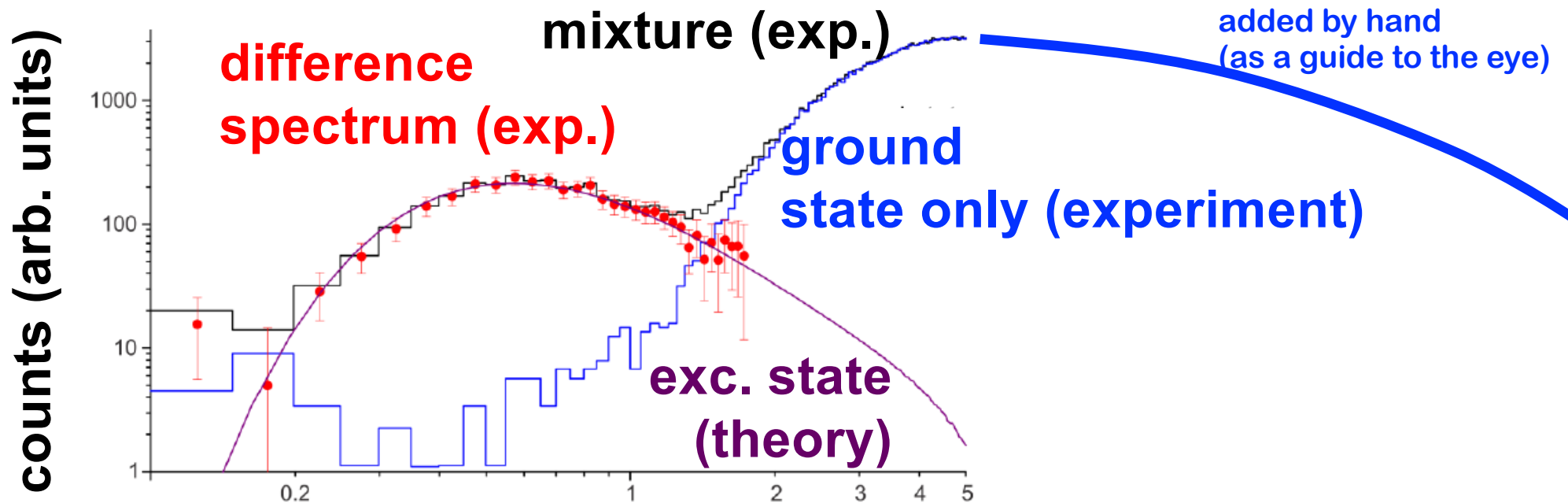


Probing Helium Trimer Excited Efimov State



**He₃ signal contains ground state trimer *and* excited state trimer.
Laser beam ionizes trimer: Coulomb explosion of ⁴He₃ (3 ions).**

Kinetic Energy Release Measurement: Observing $(^4\text{He}_3)^*$

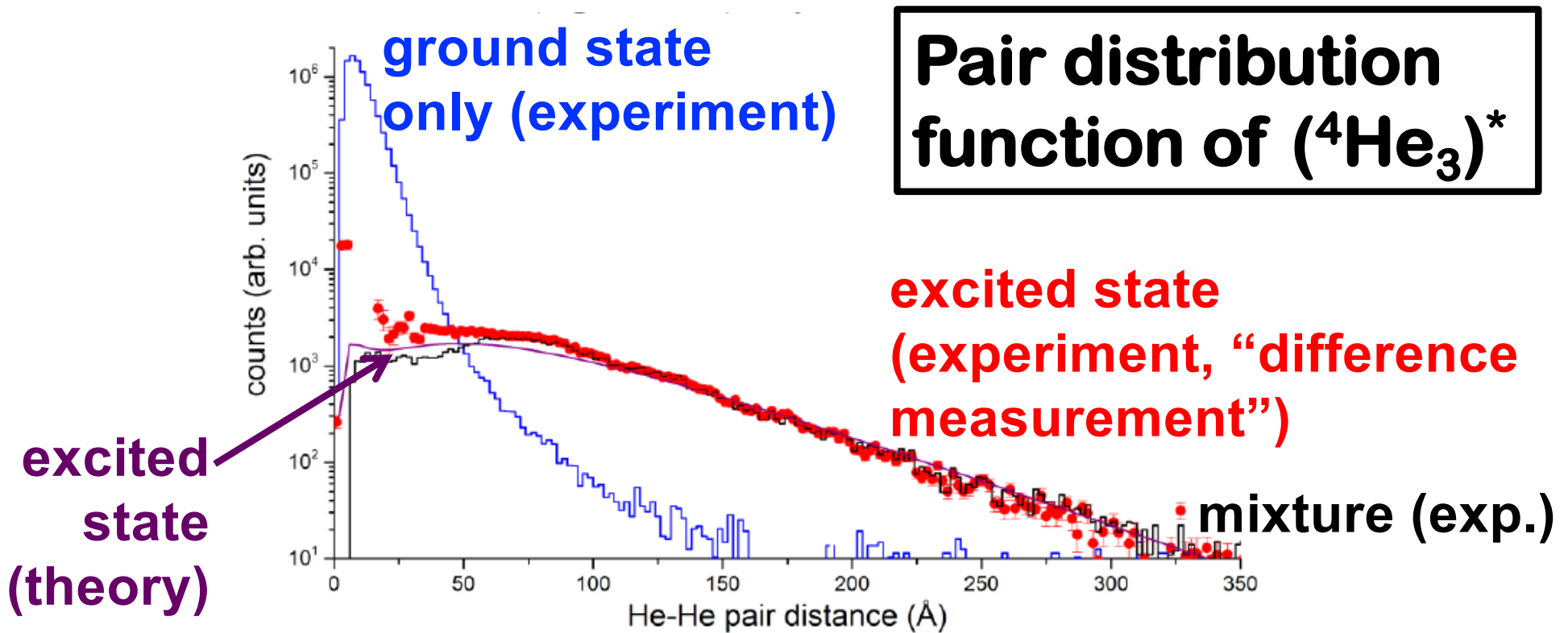


kinetic energy release (KER) in eV (log scale)

The ionization is instantaneous and the He-ions are distributed according to the quantum mechanical eigen states of the ground and excited helium trimers.

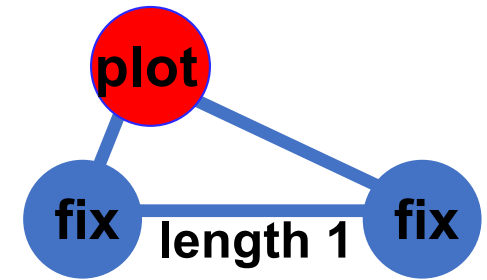
Large r_{12} , r_{23} and r_{31} correspond to small $\text{KER} = 1/r_{12} + 1/r_{23} + 1/r_{31}$.

Reconstructing Real Space Properties

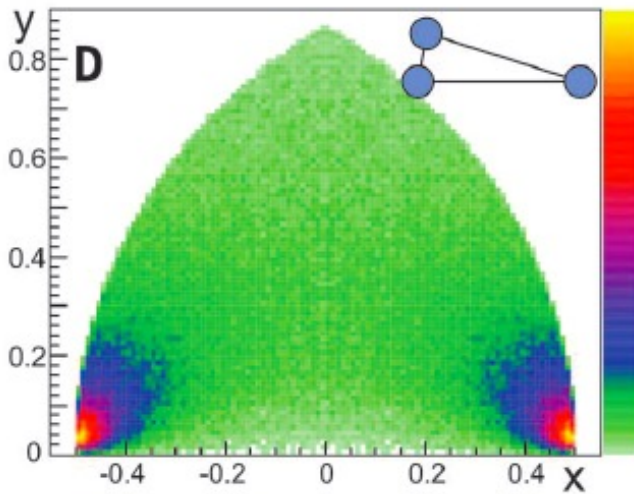


The excited state is eight times larger than the ground state. Assuming an “atom-dimer geometry”, the tail can be fit to extract the binding energy of the excited helium trimer. Fit to experimental data yields 2.6(2)mK. Theory 2.65mK [Hiyama et al., PRA 85, 062505 (2012)].

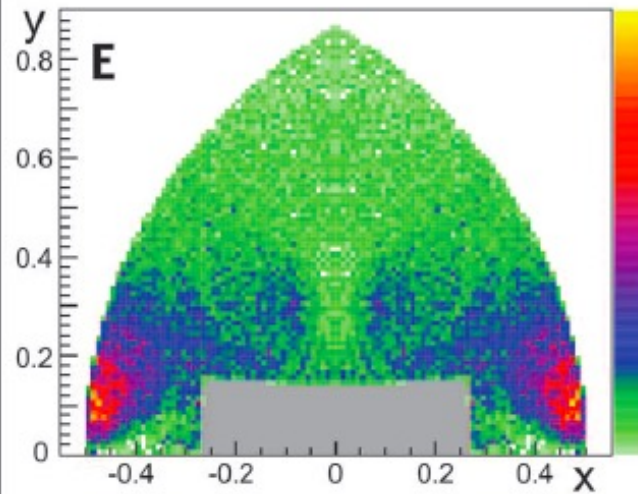
Normalized Structural Properties of ${}^4\text{He}_3$



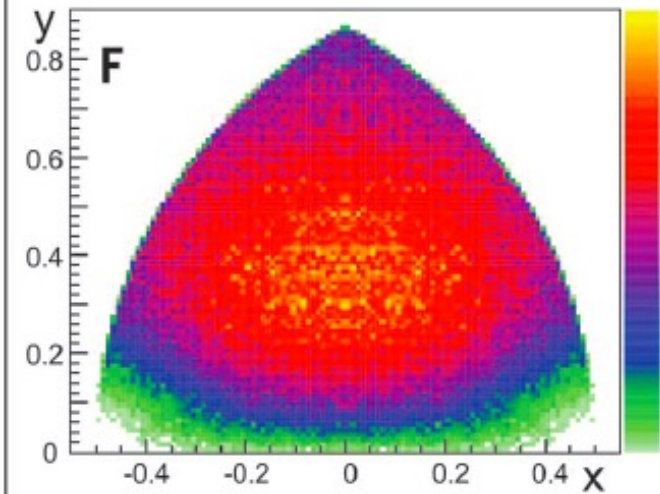
excited state:
theory



excited state:
experiment



ground state:
theory



Divide all three interparticle distances by largest r_{ij} and plot k^{th} atom (positive y): Corresponds to placing atoms i and j at $(-1/2, 0)$ and $(1/2, 0)$.

Ground state and excited states have distinct characteristics!!!
Message: Reconstruction of quantum mechanical trimer density.

Summary of Introductory Part

- ${}^4\text{He}_N$ droplets can be realized experimentally in size-selective manner.
- Access to structural properties (accessing real-space structures beyond $N=3$ is non-trivial due to reconstruction algorithm).
- ${}^4\text{He}$ - ${}^4\text{He}$ well described by effective range theory (two-parameter theory).
- ${}^4\text{He}_3$: Ground state has Efimov characteristics and excited state is Efimov trimer (s-wave scattering length and three-body parameter).
- If we want more “info,” what can be done?
 - Spectroscopy (need more than one bound energy level).
 - Isotope substitution (${}^4\text{He} \rightarrow {}^3\text{He}$).
 - Tuning of interactions (this talk: ground state droplets).
 - Dynamics (this talk: dimer).

Structural Properties: Short- and Long-Range Characteristics

Emphasized during introductory part:

- Universal long-range behavior ($r \gg r_{eff}$).
- Large probability to find particles at large separations.
- Low-energy theory starts with s-wave scattering length and then systematically adds corrections.

Atomic systems:

- Van der Waals universality.
- Highly repulsive short-range potential.

Which long-range behaviors “collapse”?

Which short-range behaviors “collapse”?

Strategy: Consider different two-body potentials (realistic and effective models), both at the physical point and at unitarity.

Different (Helium-Helium) Interaction Potentials

Strategy: Consider different two-body potentials [realistic (**Model I**) and effective models (**Model II**)], both at the physical point and at unitarity.

Model I: Realistic potential with hard inner wall.

$$V_{tot} = \sum_{j=1}^{N-1} \sum_{k>j}^N V_{realistic}(r_{jk})$$

Unitarity realized by applying overall scaling factor.

HFD-HE2.
CPKMJS.
TTY.
LM2M2.

Model II: Effective low-energy potential model.

$$V_{tot} = \sum_{j=1}^{N-1} \sum_{k>j}^N V_{2,gauss}(r_{jk}) + \sum_{j=1}^{N-2} \sum_{k>j}^{N-1} \sum_{l>k}^N V_{3,gauss}(R_{jkl})$$

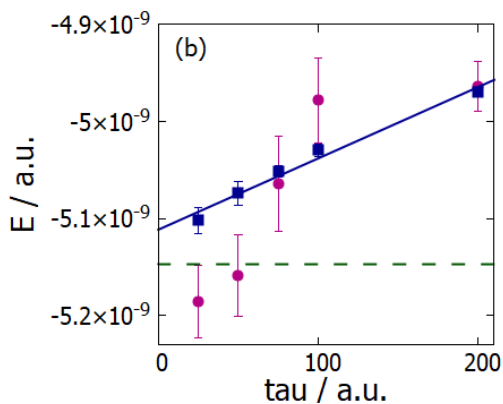
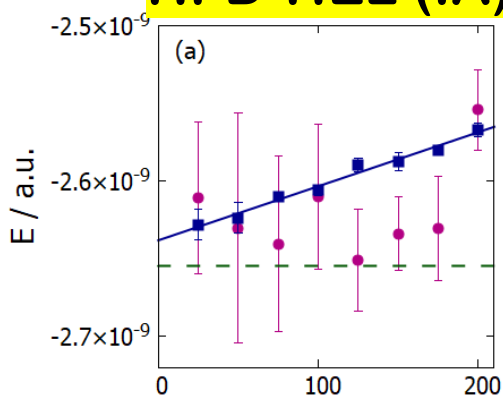
matched to
HFD-HE2 and
scaled-HFD-HE2.

See Kievsky et al., PRA 96, 040501 (2017); PRA 102, 063320 (2020):
2-body range and depth → s-wave scattering length and effective range.
3-body range and depth → three- and four-body energy.

Diffusion Monte Carlo: Unbiased Energies

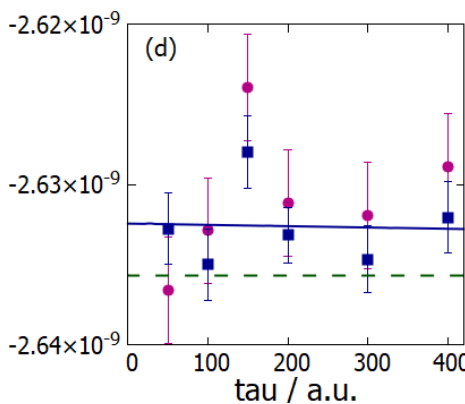
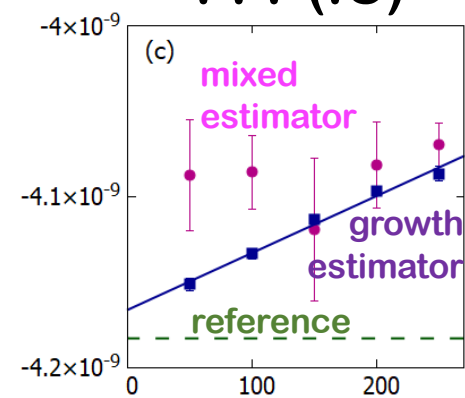
dimer (physical point)

HFD-HE2 (IA)



CPKMJS (IB)

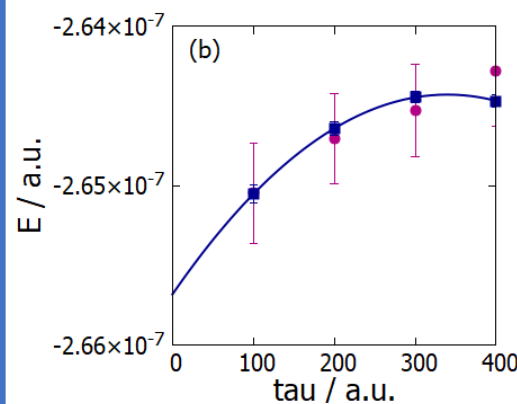
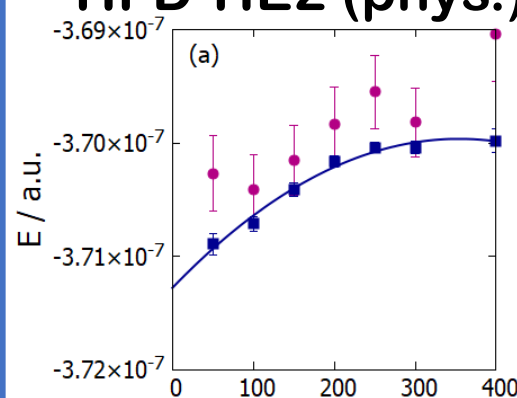
TTY (IC)



Gauss (II)

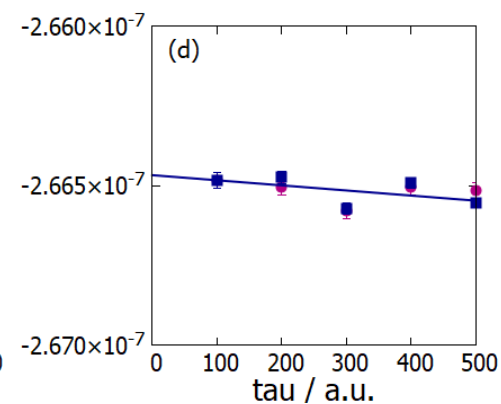
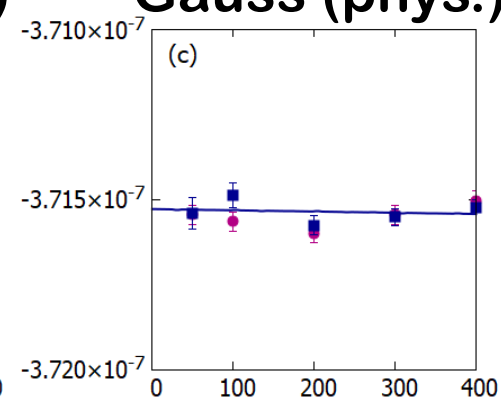
trimer (phys. & unit.)

HFD-HE2 (phys.)



HFD-HE2 (unit.)

Gauss (phys.)



Gauss (unit.)

Diffusion Monte Carlo: Unbiased Energies

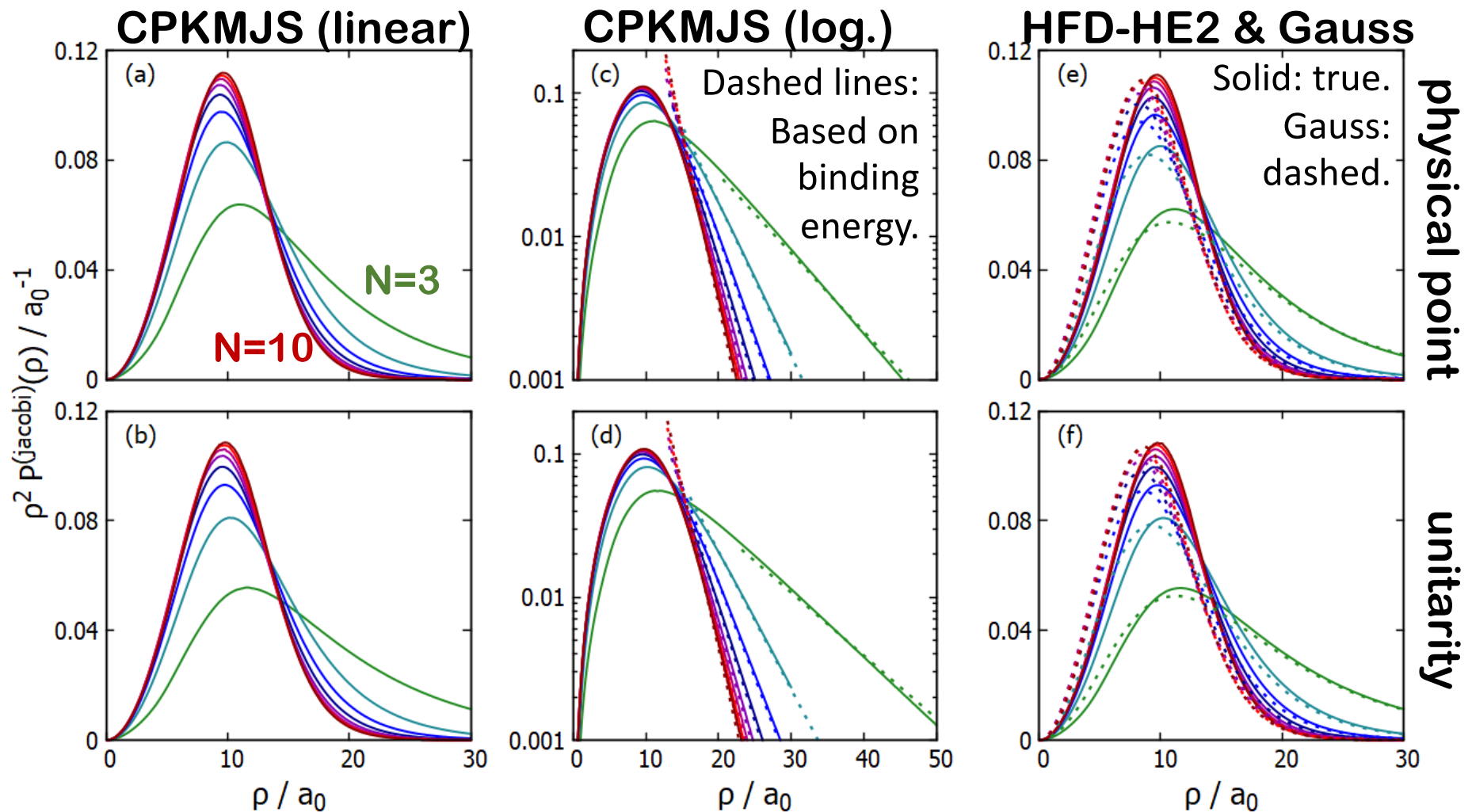
Physical point:

N	$E_{\text{HFD-HE2}}^{(d)}$ (Model IA)	E_{CPKMJS} (Model IB)	E_{TTY} (Model IC)	E_{GAUSS} (Model II)	$E_{\text{CPKMJS}}/E_{\text{HFD-HE2}}$ (in percent)	$E_{\text{GAUSS}}/E_{\text{HFD-HE2}}$ (in percent)
2 ^(a)	-2.645×10^{-9}	-5.147×10^{-9}	-4.183×10^{-9}	-2.6357×10^{-9}	195	100
3 ^(b)	$-3.713(3) \times 10^{-7}$	$-4.174(5) \times 10^{-7}$	$-4.006(3) \times 10^{-7}$	$-3.715(1) \times 10^{-7}$	112	100
4 ^(c)	$-1.688(1) \times 10^{-6}$	$-1.815(1) \times 10^{-6}$	$-1.768(1) \times 10^{-6}$	$-1.6984(1) \times 10^{-6}$	108	101
5 ^(c)	$-3.966(1) \times 10^{-6}$	$-4.201(1) \times 10^{-6}$	$-4.112(1) \times 10^{-6}$	$-3.9622(3) \times 10^{-6}$	106	100
6 ^(c)	$-7.102(2) \times 10^{-6}$	$-7.467(2) \times 10^{-6}$	$-7.325(1) \times 10^{-6}$	$-7.0166(4) \times 10^{-6}$	105	99
7 ^(c)	$-1.0986(5) \times 10^{-5}$	$-1.150(1) \times 10^{-5}$	$-1.130(1) \times 10^{-5}$	$-1.0737(1) \times 10^{-5}$	106	98
8 ^(c)	$-1.5531(6) \times 10^{-5}$	$-1.621(1) \times 10^{-5}$	$-1.594(1) \times 10^{-5}$	$-1.5030(1) \times 10^{-5}$	104	97
9 ^(c)	$-2.066(1) \times 10^{-5}$	$-2.152(1) \times 10^{-5}$	$-2.1176(8) \times 10^{-5}$	$-1.9830(2) \times 10^{-5}$	104	96
10 ^(c)	$-2.631(1) \times 10^{-5}$	$-2.736(1) \times 10^{-5}$	$-2.694(1) \times 10^{-5}$	$-2.5083(2) \times 10^{-5}$	104	95

Unitarity:

N	$E_{\text{HFD-HE2}}^{(d)}$ (Model IA)	E_{CPKMJS} (Model IB)	E_{GAUSS} (Model II)	$E_{\text{CPKMJS}}/E_{\text{HFD-HE2}}$ (in percent)	$E_{\text{GAUSS}}/E_{\text{HFD-HE2}}$ (in percent)
3 ^(b)	$-2.656(6) \times 10^{-7}$	$-2.65(1) \times 10^{-7}$	$-2.665(1) \times 10^{-7}$	100	100
4 ^(c)	$-1.391(1) \times 10^{-6}$	$-1.395(5) \times 10^{-6}$	$-1.4028(1) \times 10^{-6}$	100	101
5 ^(c)	$-3.411(3) \times 10^{-6}$	$-3.418(4) \times 10^{-6}$	$-3.4130(2) \times 10^{-6}$	100	100
6 ^(c)	$-6.235(6) \times 10^{-6}$	$-6.241(4) \times 10^{-6}$	$-6.1642(4) \times 10^{-6}$	100	99
7 ^(c)	$-9.764(9) \times 10^{-6}$	$-9.773(4) \times 10^{-6}$	$-9.5379(6) \times 10^{-6}$	100	98
8 ^(c)	$-1.391(1) \times 10^{-5}$	$-1.392(8) \times 10^{-5}$	$-1.3446(1) \times 10^{-5}$	100	97
9 ^(c)	$-1.861(1) \times 10^{-5}$	$-1.863(10) \times 10^{-5}$	$-1.7824(1) \times 10^{-5}$	100	96
10 ^(c)	$-2.379(2) \times 10^{-5}$	$-2.382(12) \times 10^{-5}$	$-2.2626(2) \times 10^{-5}$	100	95

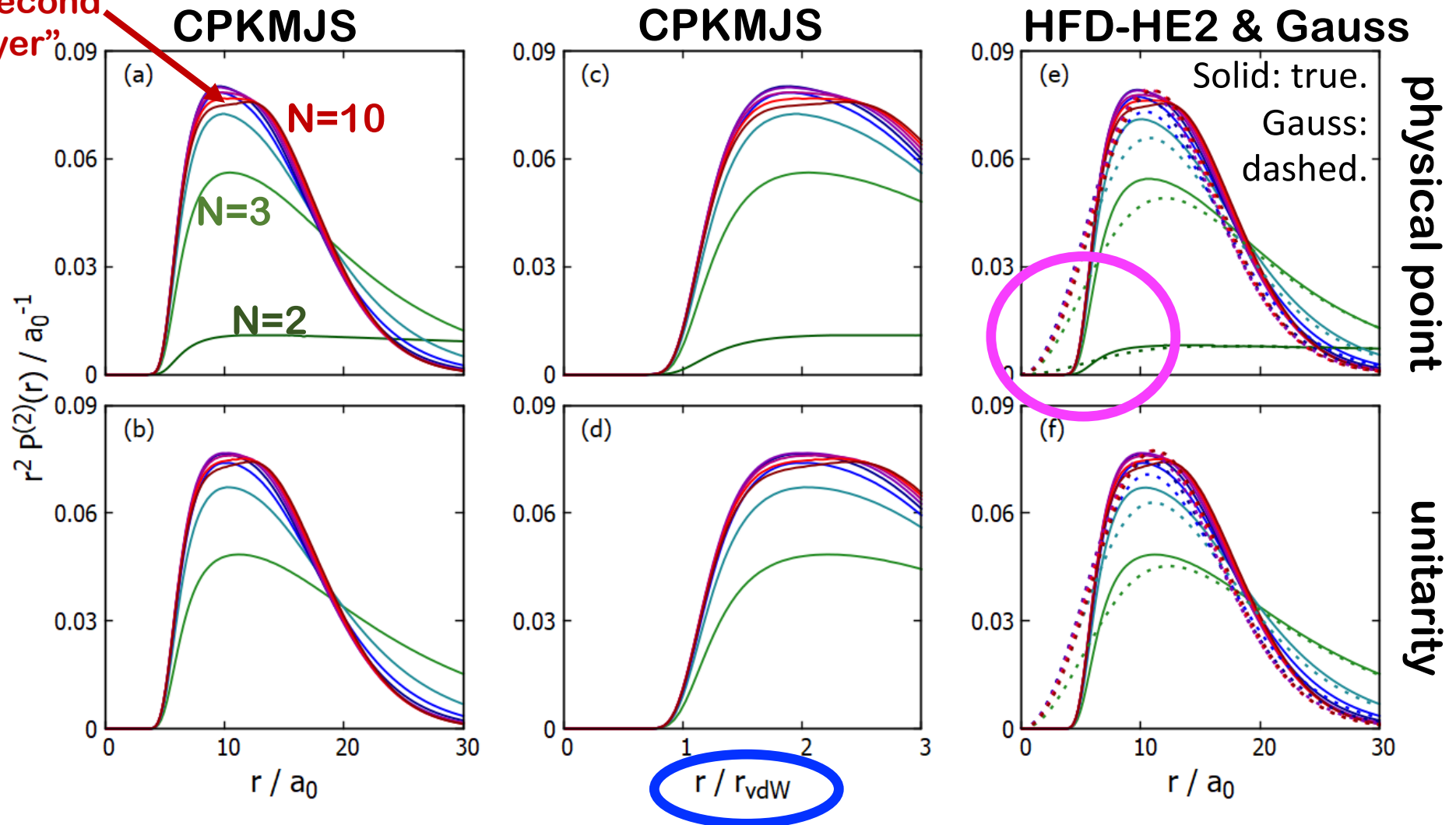
${}^4\text{He}_N$: N^{th} Atom Relative to Center-of-Mass of $N-1$ Atoms



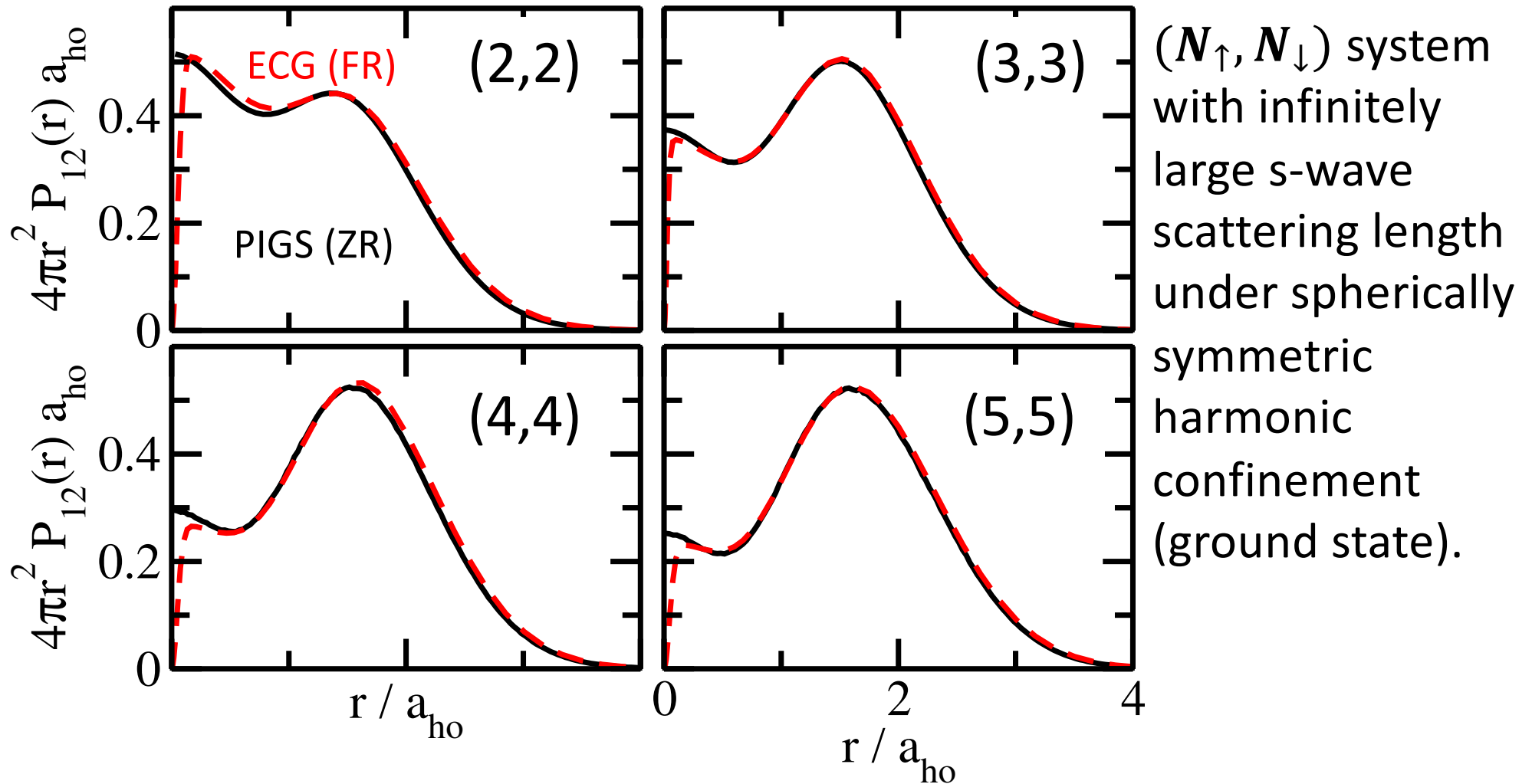
Results obtained using forward walking: Reynolds et al., J. Stat. Phys. 43, 1017 (1986).

Pair Distribution Function $P^{(2)}(r)$ for $N = 2 - 10$

“second layer”



Contrasting with Two-Component Fermions at Unitarity

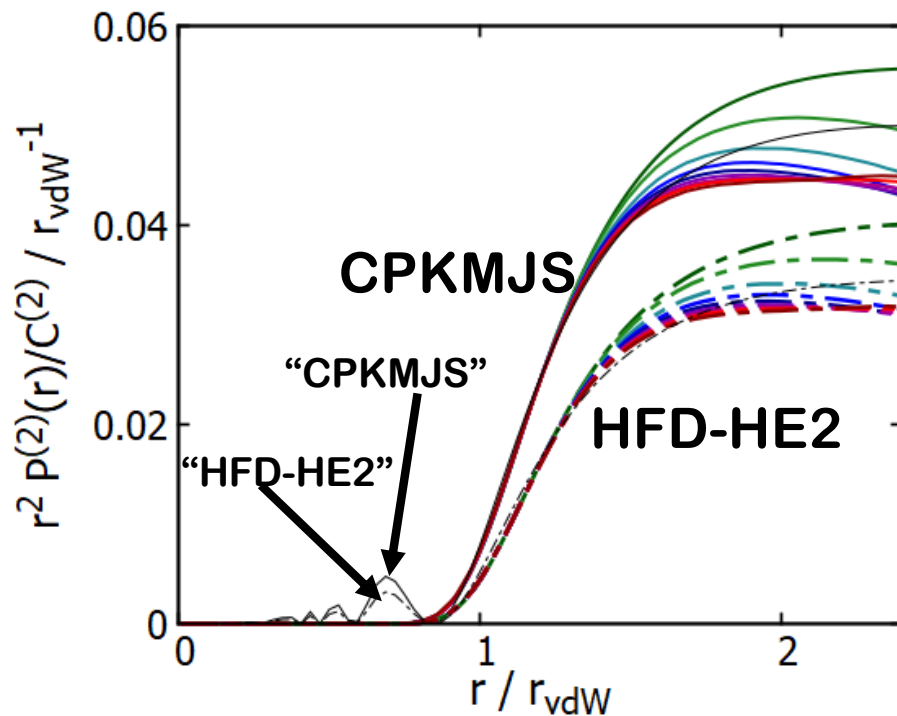


$P^{(2)}(r)$, Two-Body Contact $C_N^{(2)}$, van der Waals Universality

$$\hat{P}_N^{(2)}(r) = \frac{2}{N(N-1)} \sum_{j=1}^{N-1} \sum_{k>j}^N \frac{\delta(r_{j,k} - r)}{r^2}$$

$$\int_0^\infty P_N^{(2)}(r) r^2 dr = 1$$

$$P_N^{(2)}(r) \xrightarrow{\text{small } r} C_N^{(2)} P_2^{(2)}(r)$$



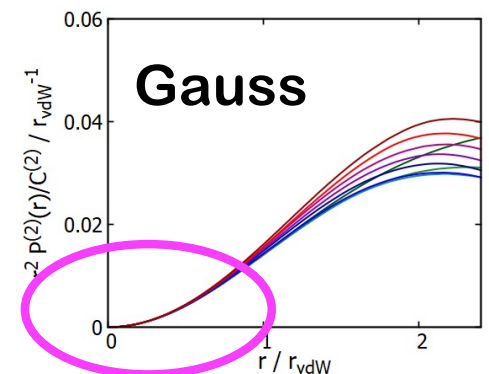
$$\frac{a_s}{r_{vdW}} = 46.95 \text{ for HFD-HE2.}$$

$$\frac{a_s}{r_{vdW}} = 33.63 \text{ for CPKMJS.}$$

See Naidon et al., PRL 112, 105301 (2014); PRA 90, 022106 (2014) for vdW universality of trimer at unitarity.

$$\varphi_{vdW}(r) = B \left[\Gamma(5/4) x^{1/2} J_{1/4}(2x^{-2}) - \frac{r_{vdW}}{a_s} \Gamma(3/4) x^{1/2} J_{-1/4}(2x^{-2}) \right]$$

Flambaum et al., PRA 59, 1998 (1999).
Gao, PRA 58, 4222 (1998).



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$$\int_0^\infty P_N^{(2)}(r) r^2 dr = 1$$

$$P_N^{(2)}(r) \xrightarrow{\text{small } r} C_N^{(2)} P_2^{(2)}(r)$$

N	$C_N^{(2)} N(N-1)/2$ HFD-HE2	$C_N^{(2)} N(N-1)/2$ CPKMJS	$C_N^{(2)} N(N-1)/2$ TTY	$C_N^{(2)} N(N-1)/2$ LM2M2 [34]	HFD-HE2/CPKMJS (in percent)	TTY/CPKMJS (in percent)	LM2M2/CPKMJS (in percent)
2	1	1	1	1	100	100	100
3	22.3	16.9	18.4	17.8(0.03)	132	109	106
4	62.4	46.2	50.5	48.8(1)	135	109	106
5	117	86.1	94.2	91.1(3)	136	109	106
6	184	134	147	143(0.6)	137	110	106
7	259	189	207	201(0.8)	137	110	106
8	343	250	274	267(1)	138	110	107
9	434	315	346	338(2)	138	110	107
10	533	386	424	415(2)	138	110	108

Assuming that the pair distribution functions for different potential models agree for larger N , we find that the ratio of the two-body contacts for larger N is given, to leading order, by the ratio between a_s/r_{vdW} for the two interaction potentials.

1.40

1.10

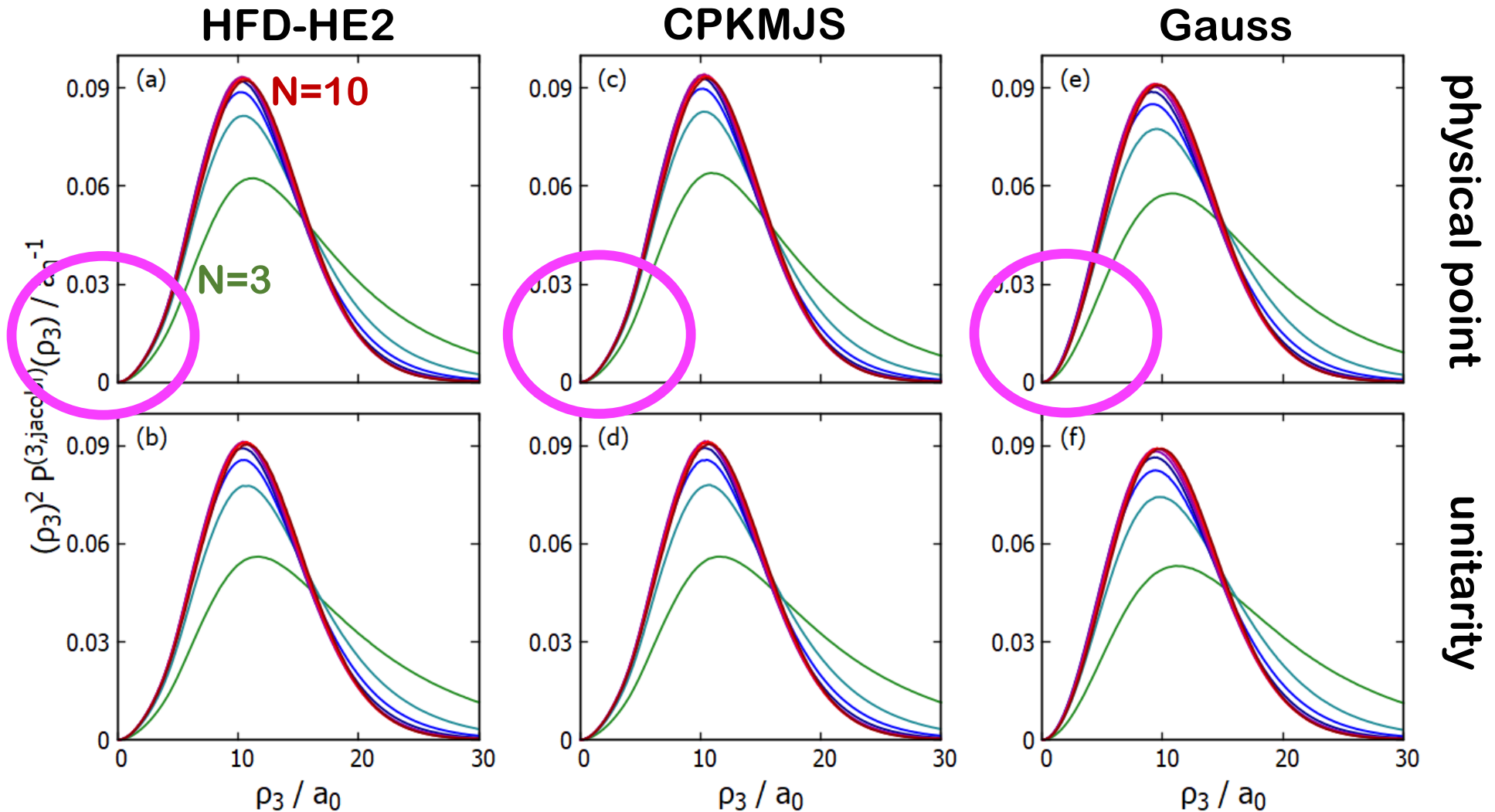
1.13

Bazak et al., PRA 101, 010501 (2020)

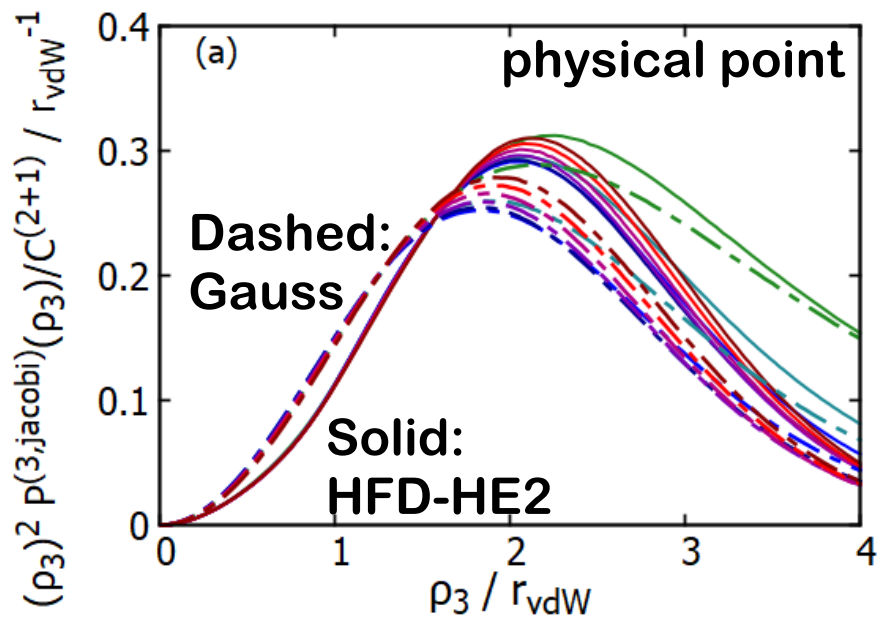
Yates, Blume, PRA 105, 022824 (2022)

See Kim et al., Annu. Rev. Nucl. Sci. 24, 96 (1974): “asymptotic normalization constant”

Three-Body (Sub-Cluster) Correlations for $N = 3 - 10$

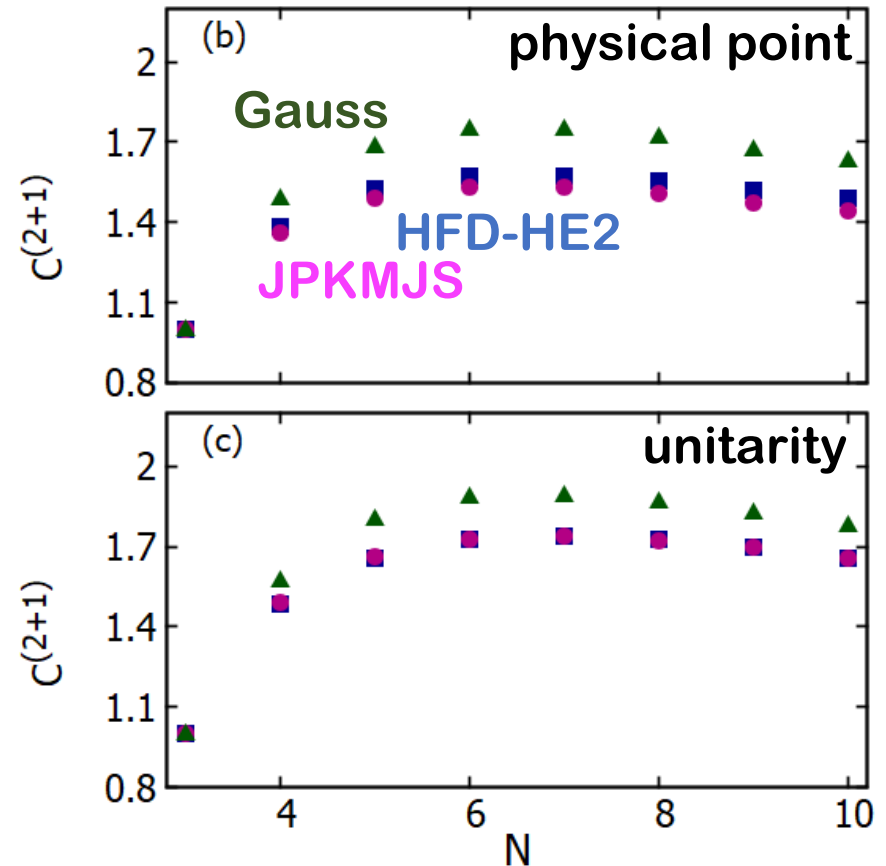


Short-Range “2+1 Contact”



$$P_N^{(3,jacobi)}(\rho_3) \xrightarrow{\text{small } \rho_3} C_N^{(2+1)} P_3^{(3,jacobi)}(\rho_3)$$

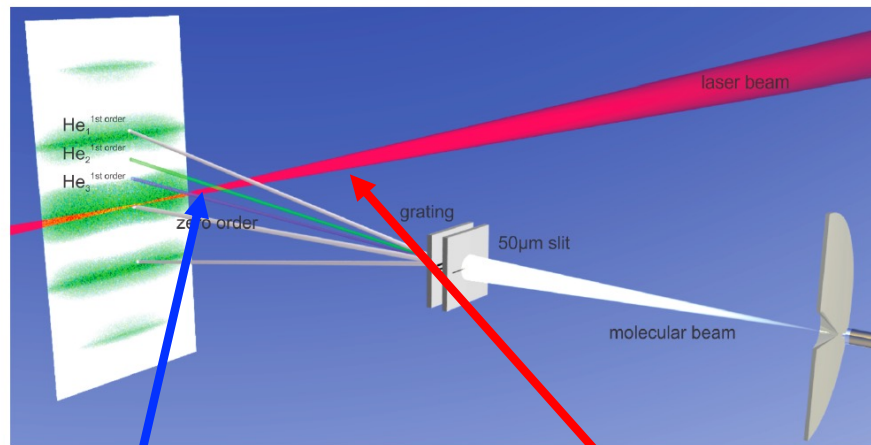
$$\Psi(\vec{r}_1, \dots, \vec{r}_N) \xrightarrow{\text{small } \rho_{jk,l}} \Phi(\vec{\rho}_{jk,l}) B_N^{(2+1)}(\vec{r}_{j,k}, \vec{R}_{j,k,l}, \{\vec{r}_{n;n \neq j,k,l}\})$$



Summary

- ${}^4\text{He}_N$ droplets can be realized experimentally in size-selective manner.
- Access to structural properties (accessing real-space structures beyond $N=3$ is non-trivial due to reconstruction algorithm).
- ${}^4\text{He}$ - ${}^4\text{He}$ well described by effective range theory (two-parameter theory).
- ${}^4\text{He}_3$: Ground state has Efimov characteristics and excited state is Efimov trimer (s-wave scattering length and three-body parameter).
- If we want more “info,” what can be done?
 - Spectroscopy (need more than one bound energy level).
 - Isotope substitution (${}^4\text{He} \rightarrow {}^3\text{He}$).
 - **Tuning of interactions (this talk: ground state droplets).**
 - **Dynamics (this talk: dimer).**

Basic Concept

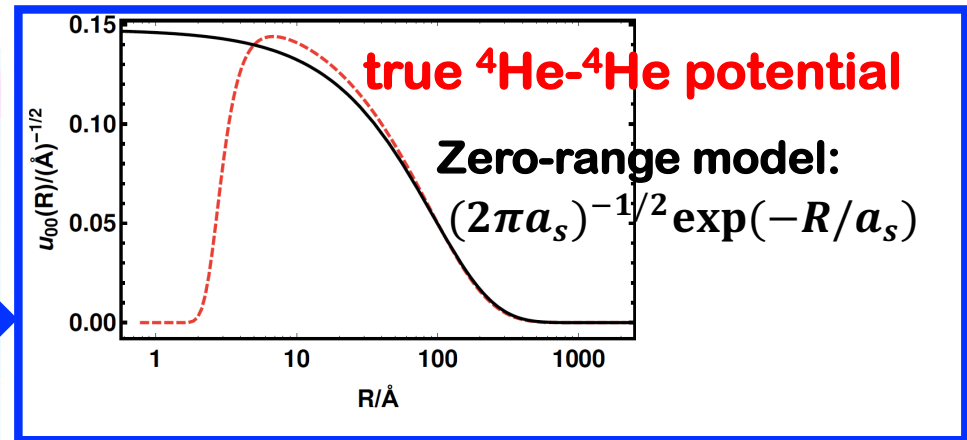
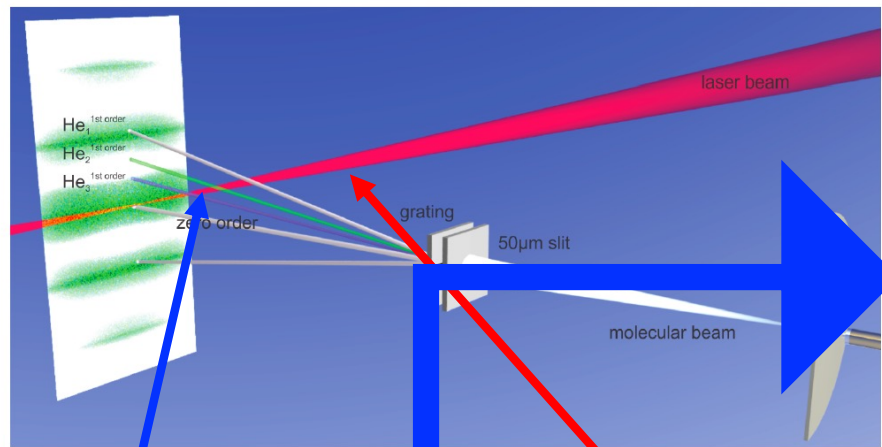


Prepare universal initial state (i.e., state that is dominated by s-wave scattering length).

Interrogate the initial state: fast and intense pump laser that takes the system out of equilibrium.

Wait for a variable time (delay) and apply even shorter and more intense probe laser that allows us to look at time-evolved system.

Basic Concept

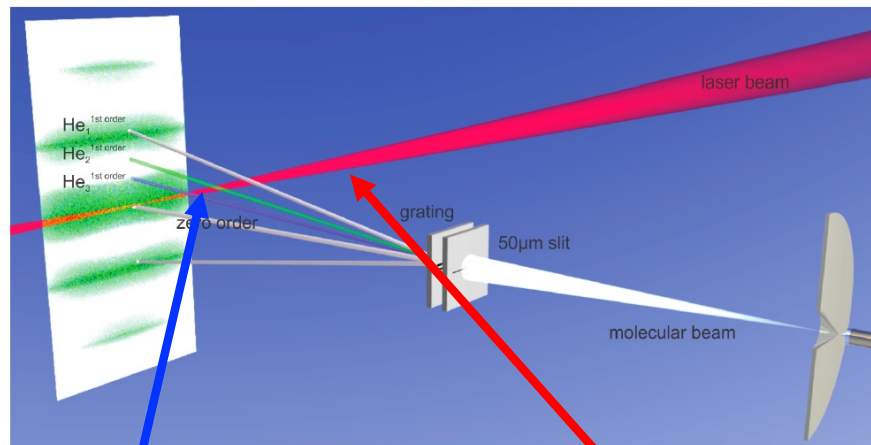


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Basic Concept



Sort of like...



From vectorstock.com

Prepare universal initial state (i.e., state that is dominated by s-wave scattering length).

Interrogate the initial state: fast and intense pump laser that takes the system out of equilibrium.

Wait for a variable time (delay) and apply even shorter and more intense probe laser that allows us to look at time-evolved system.

Two Exciting Fields

**(ultra)cold atoms:
universal physics**

**fast intense
lasers**

**(ultra)cold atoms: fast intense
universal physics lasers**

Selected Works in This Direction

PHYSICAL REVIEW LETTERS 124, 253201 (2020)

ARTICLE

DOI: 10.1038/s41467-019-04556-3

OPEN

Quantum simulation of ultrafast trapped ultracold atoms

Ruwan Senaratne¹, Shankari V. Rajagopal¹, Toshihiko Shimasaki¹, Daniel E. Dotti¹, Kurt M. Fujiwara¹, Kevin ...¹, Zachary A. Geiger¹ & David M. Weld¹

Found Phys (2014) 44:813–818
DOI 10.1007/s10701-014-9773-5

Optically Engineered Quantum States in Ultrafast and Ultracold Systems

Kenji Ohmori

PRL 103, 260401 (2009)

Ultrafast Creation of Overlapping Rydberg Electrons in an Atomic BEC and Mott-Insulator Lattice

M. Mizoguchi,^{1,2} Y. Zhang,^{1,3} M. Kunimi,¹ A. Tanaka,¹ S. Takeda,^{1,2,†} N. Takei[Ⓞ],^{1,2,‡} V. Bharti[Ⓞ],¹ K. Koyasu,^{1,2} T. Kishimoto[Ⓞ],⁴ D. Jaksch[Ⓞ],^{5,6} A. Glaetzle,^{5,6} M. Kiffner[Ⓞ],^{5,6} G. Masella[Ⓞ],⁷ G. Pupillo,⁷ M. Weidemüller[Ⓞ],^{8,9} and K. Ohmori^{1,2,*}

PHYSICAL REVIEW A 95, 011403(R) (2017)

Ultracold-atom quantum simulator for attosecond science

Simon Sala, Johann Förster, and Alejandro Saenz

AG Moderne Optik, Institut für Physik, Humboldt-Universität zu Berlin, Newtonstraße 15, 12489 Berlin, Germany

(Received 23 November 2016; published 25 January 2017)

PHYSICAL REVIEW LETTERS

week ending
31 DECEMBER 2009

Pump-Probe Spectroscopy of Two-Body Correlations in Ultracold Gases

Christiane P. Koch^{1,*} and Ronnie Kosloff²

(ultra)cold atoms: fast intense universal physics lasers

RAPID COMMUNICATIONS

Laser spectroscopic characterization of the nuclear clock isomer ^{229m}Th

Johannes Thielking¹, Maxim V. Okhapkin¹, Przemysław Głowacki^{1,†},
David M. Meier¹, Lars von der Wense², Benedict Seiferle²,
Christoph E. Düllmann^{3,4,5}, Peter G. Thirolf², Ekkehard Peik¹

¹ Physikalisch-Technische Bundesanstalt, 38116 Braunschweig, Germany.

² Ludwig-Maximilians-Universität München, 85748 Garching, Germany.

³ GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany.

⁴ Helmholtz-Institut Mainz, 55099 Mainz, Germany.

⁵ Johannes Gutenberg-Universität, 55099 Mainz, Germany.

The isotope ^{229}Th is the only nucleus in the energy range of a few electron volts, valence shell of atoms, but about four

(ultra
univ

Found Phys (2014) 44:813–818
DOI 10.1007/s10701-014-9773-5

Optically Engineered Quantum States in Ultracold and Ultracold Systems

Kenji Ohmori

PRL 103, 260401 (2009)

Pump-Pro

Laser Probing of Neutron-Rich Nuclei in Light Atoms

Z.-T. Lu

Physics Division, Argonne National Laboratory, Lemont, Illinois 60439, USA and
Department of Physics and Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637, USA

P. Mueller

Physics Division, Argonne National Laboratory, Lemont, Illinois 60439, USA

G. W. F. Drake

Department of Physics, University of Windsor, Windsor, Ontario N9B 3P4 Canada

W. Nörtershäuser

Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany

Steven C. Pieper

Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

Z.-C. Yan

State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, and Center for Cold Atom Physics, Chinese Academy of Sciences, Wuhan 430071, China and
Department of Physics, University of New Brunswick, Fredericton, New Brunswick E3B 5A3 Canada

(Dated: June 5 2013)

The neutron-rich ^6He and ^8He isotopes exhibit an exotic nuclear structure that consists of a tightly bound ^4He -like core with additional neutrons orbiting at a relatively large distance, forming a halo. Recent experimental efforts have succeeded in laser trapping and cooling these short-lived, rare

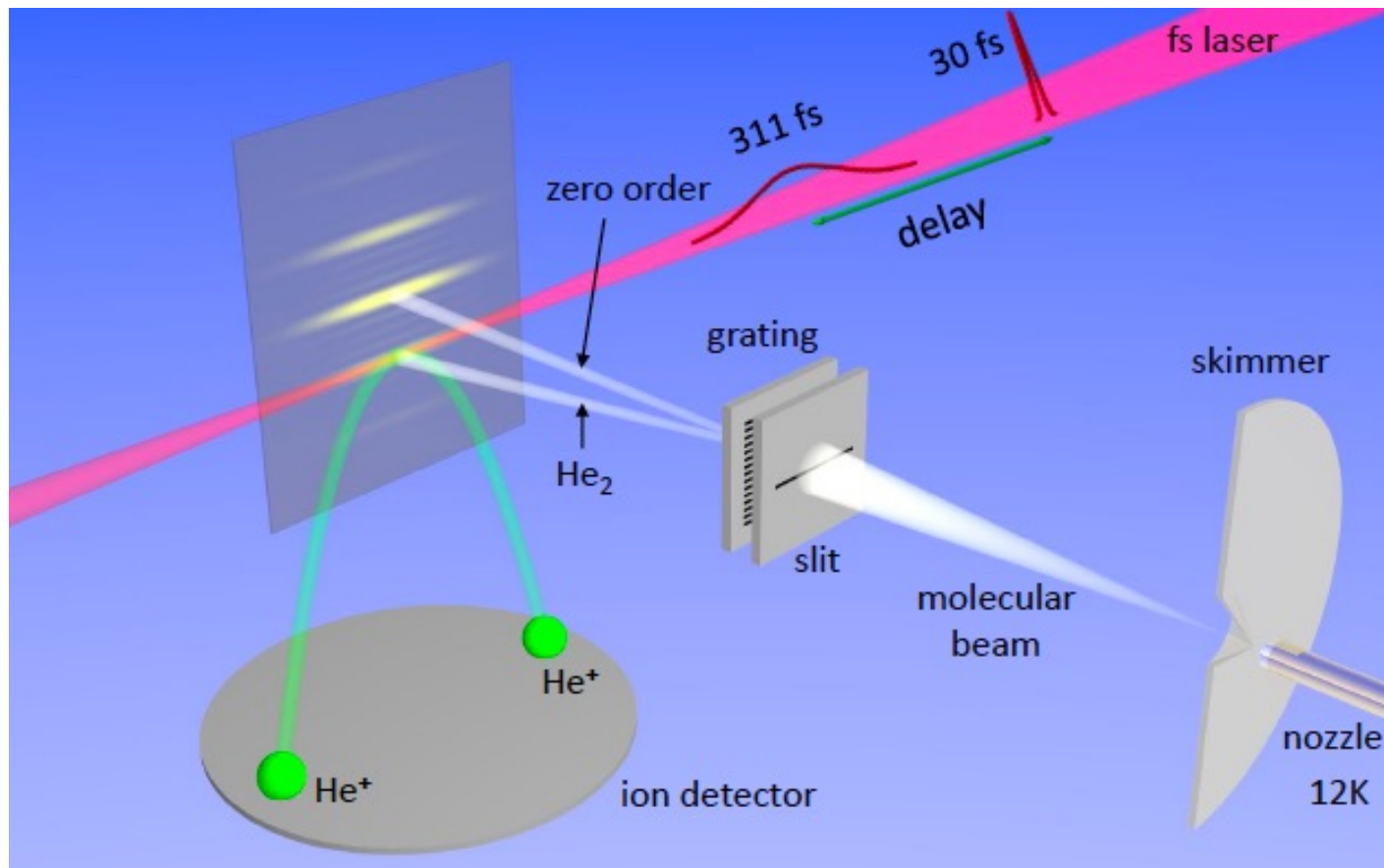
Christiane P. Koch^{1,*} and Ronnie Kosloff^{6,2}

his

PHYSICAL REVIEW LETTERS 124, 253201 (2020)

Using Rydberg Electrons in an

Pump-Probe Spectroscopy of Isolated Helium Dimers



Pump pulse: pulse length of 311 fs and intensity of $1.3 \times 10^{14} \text{ W/cm}^2$.
Probe pulse rips off two electrons (Coulomb explosion).

What do we expect to happen as a function of the delay time???

What Do The Numbers Mean?

Pump pulse: pulse length of 311 fs and intensity of $1.3 \times 10^{14} \text{ W/cm}^2$.

Probe pulse: rips off two electrons (Coulomb explosion).

What do we expect to happen as a function of the delay time???

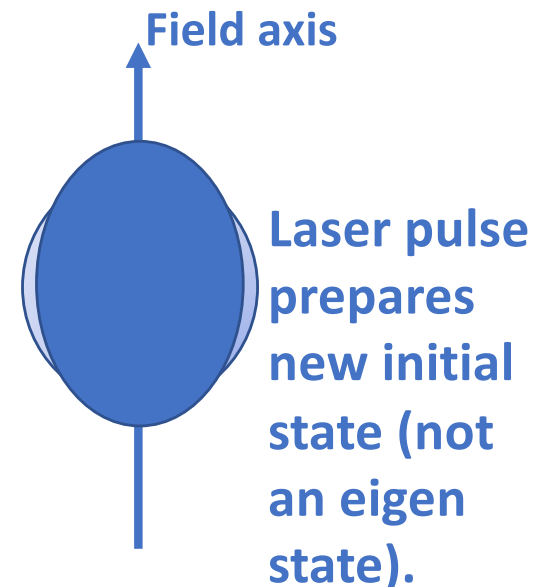
Binding energy of 1mK corresponds to $50 \text{ ns} = 5 \cdot 10^7 \text{ fs}$. The 311 fs pump laser is extremely short compared to the natural time scale of the helium dimer: laser pulse acts as a “kick.”

Solar: $\frac{10^3 \text{ W}}{\text{m}^2}$.

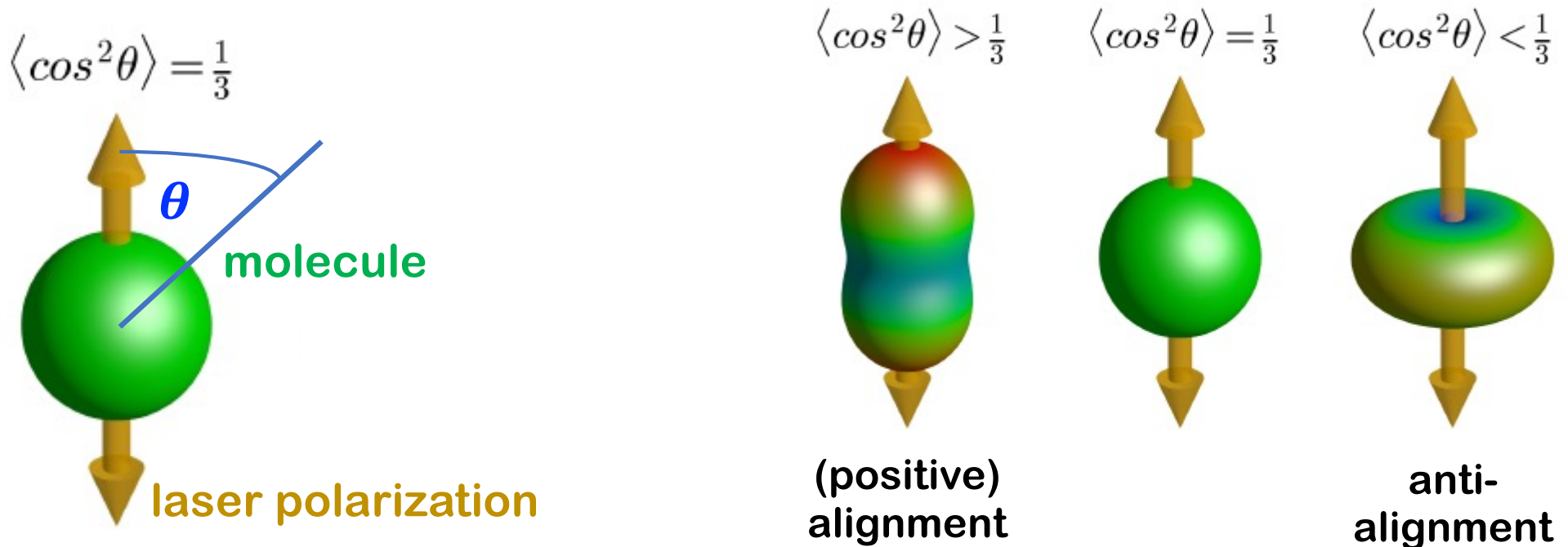
Laser pointer: $\frac{10^6 \text{ W}}{\text{m}^2}$.

Pump pulse: $1.3 \cdot \frac{10^{13} \text{ W}}{\text{cm}^2} = 1.3 \cdot \frac{10^{17} \text{ W}}{\text{m}^2}$.

Roughly, we need to worry about electronic degrees of freedom at intensities $> \frac{10^{15} \text{ W}}{\text{cm}^2}$ (probe pulse).



What Does The Pulse Do To Helium Dimer?



Without the laser, the molecule is spherically symmetric (no alignment): The helium dimer has vanishing relative orbital angular momentum.

Will show: Helium dimer can be aligned. However, since the $J > 0$ partial wave components are not bound, they will “run away” (dissociative wave packet). Heavier non-universal dimers behave very differently.

Pump-Probe Experiments: Field Induced Alignment

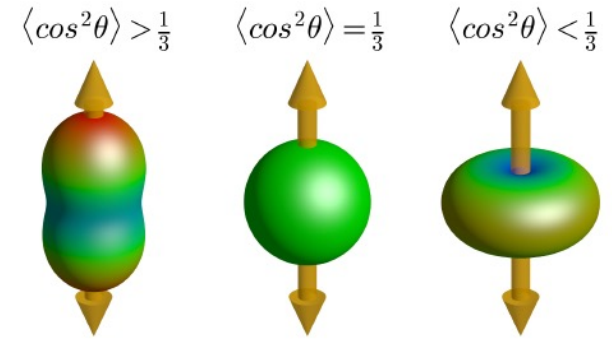
Long history of electric-field induced alignment of molecules:
Unique **rotational** dynamics for molecules such as I_2 , N_2 ,...

E.g., “Colloquium: Aligning molecules with strong laser pulses”,
RMP 75, 543 (2003) by Stapelfeldt and Seideman, >1000
citations:

“We review the theoretical and experimental status of intense laser alignment—a field at the interface between intense laser physics and **chemical dynamics** with potential applications ranging from high harmonic generation and nanoscale processing to stereodynamics and control of chemical reactions.”

Work on helium dimer adds “physical dynamics” to the list!

Alignment $\langle \cos^2 \theta \rangle$ for N_2

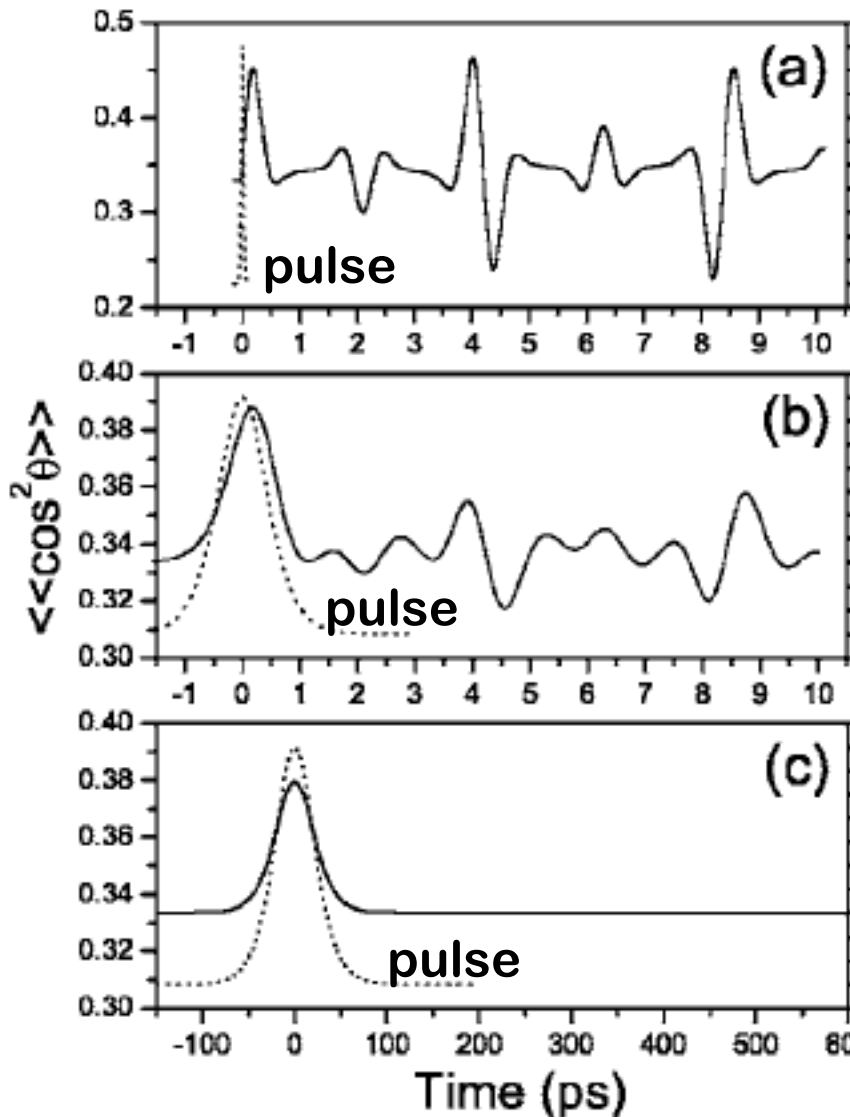


Pulse length 50 fs.

Intensity $2.5 \times 10^{13} \frac{W}{cm^2}$.

Impulse regime.

Torres et al.,
PRA 72,
023420 (2005)



**Alignment signal of $1/3 =$
spherically symmetric.**

**“Rotational revivals” require
particular phase relation:**

$$E_J = B_0 J(J + 1) - D_0 J^2 (J + 1)^2.$$

Pulse length 50 ps.

Intensity $2.5 \times 10^{12} \frac{W}{cm^2}$.

Adiabatic regime.

“Kicking” the $^4\text{He}_2$: Pump-Probe Experiments

Entirely new regime:

Recall: $^4\text{He}_2$ dimer supports exactly one (extremely weakly-bound) state. State is largely universal.

What happens when one applies short ($\sim 310\text{fs}$), intense ($\sim 10^{14}\text{W/cm}^2$) kick?

Separation of time scales (binding energy of 1mK corresponds to 50ns):

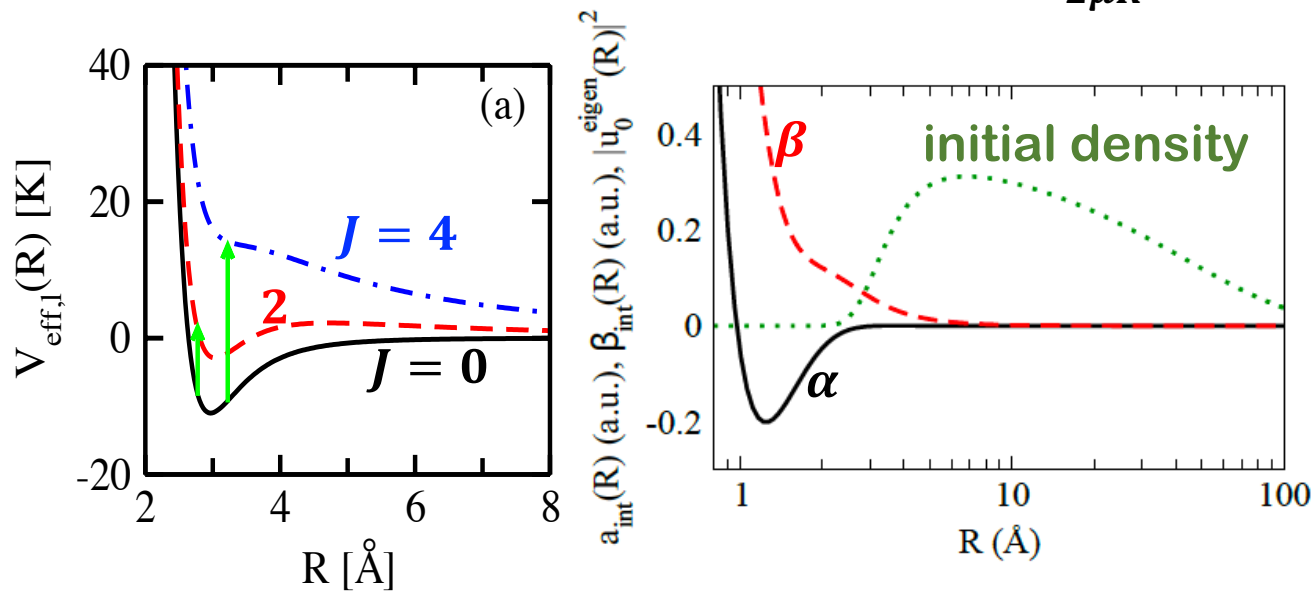
Kick is non-adiabatic (quench); in fact, we can simulate it by a delta-function pulse.

Variety of theory predictions:

Friedrich et al., Collect. Czech. Chem. Commun. 63, 1089 (1998); Nielsen et al., PRL 82, 2844 (1999); Bruch, JCP 112, 9773 (2000).

Theoretical Treatment

Dimer potential $V_{eff,J}(R) = V_{He-He}(R) + \frac{\hbar^2 J(J+1)}{2\mu R^2}$



Laser-molecule interaction:

$$V_{lm} = -\frac{1}{2} \epsilon^2(t) [\alpha(R) Y_{00}(\hat{R}) + \beta(R) Y_{20}(\hat{R})]$$



Gaussian profile

Solve time-dependent Schroedinger equation using spherical coordinates:

$$\Psi(R, \theta, t) = \sum_{J=0,2,\dots} \frac{u_J(R, t)}{R} Y_{J0}(\hat{R})$$

Pulse couples different partial waves.

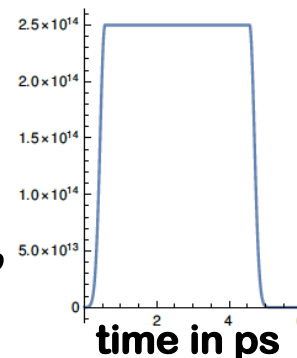
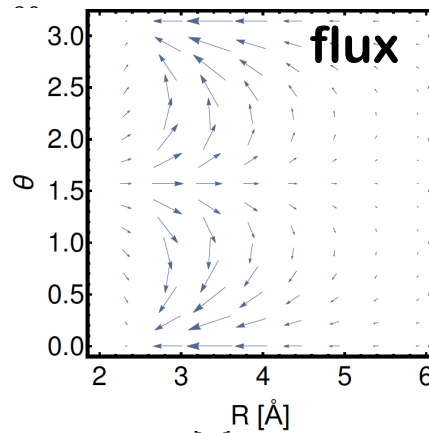
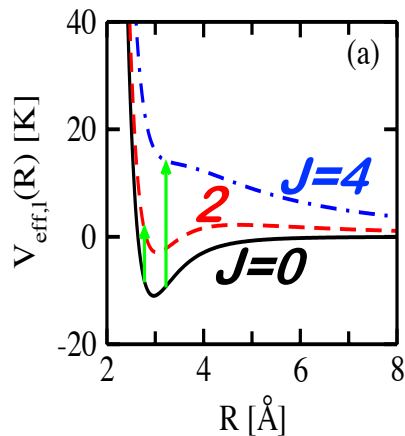
When pulse is off, the channels are decoupled.

$^4\text{He}-^4\text{He}$ In Time-Dependent Electric Field

In what follows, the initial state will be the $J = 0$ eigenstate of the zero-field Hamiltonian of $^4\text{He}-^4\text{He}$ system.

Scenario 1 (non-adiabatic laser kick):

$$\varepsilon(t) = \varepsilon_0 \exp\left(-2 \ln 2 \left(\frac{t-t_{\text{ref}}}{\tau}\right)^2\right); \tau \approx 300\text{fs}.$$



Scenario 2 (“slow”): Gaussian turn-on, hold for several ps, Gaussian turn-off.

Solve time-dependent Schroedinger equation using spherical coordinates:

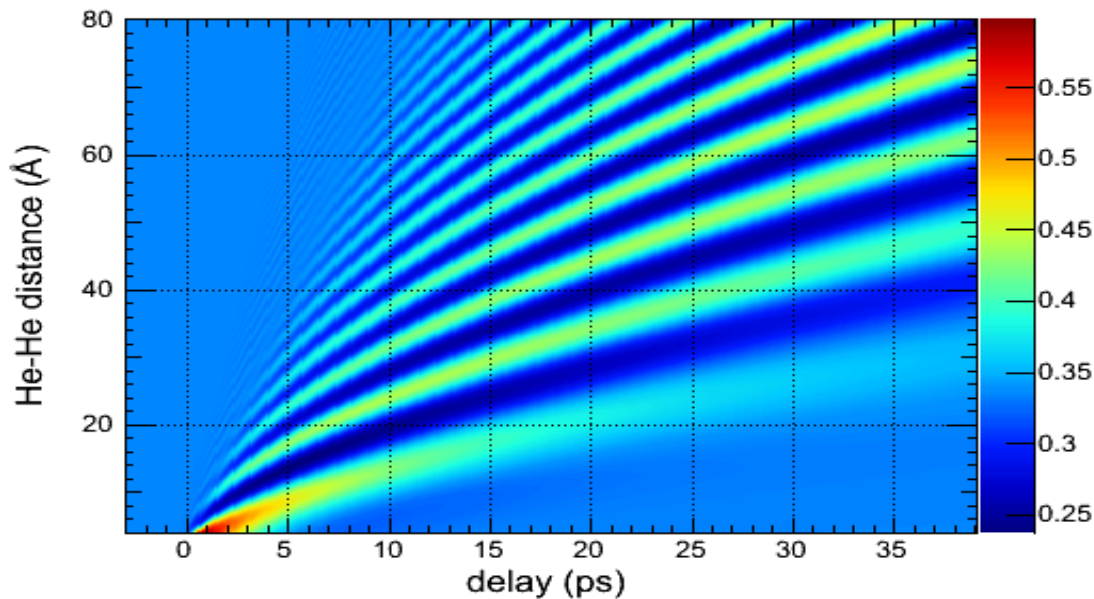
$$\Psi(R, \theta, t) = \sum_{J=0,2,\dots} \frac{u_J(R, t)}{R} Y_{J0}(\hat{R})$$

Laser couples different partial waves.

When laser is off, the channels are decoupled.

Scenario 1: Theory Result

$$C_2(R, t) = \frac{\int_0^\pi \Psi^*(R, \theta, t) \cos^2 \theta \Psi(R, \theta, t) \sin \theta d\theta}{\int_0^\pi |\Psi(R, \theta, t)|^2 \sin \theta d\theta}$$



Interference between J=0 and J=2 partial waves. J=2 portion “travels” on structureless background.

Solve time-dependent Schroedinger equation using spherical coordinates:

$$\Psi(R, \theta, t) = \sum_{J=0,2,\dots} \frac{u_J(R, t)}{R} Y_{J0}(\hat{R})$$

Pulse couples different partial waves.

When pulse is off, the channels are decoupled.

Origin Of The Interference Pattern?

Expand: $\Psi(R, \theta, t) = \sum_{J=0,2,4,\dots} R^{-1} u_J(R, t) Y_{J0}(\theta)$

$$u_J(R, t) = \exp(i\gamma_J(R, t)) |u_J(R, t)| \quad \& \quad \tan(\gamma_J(R, t)) = \frac{\text{Im}(u_J(R, t))}{\text{Re}(u_J(R, t))}$$

Plug in:
$$C_2(R, t) = \frac{\int_0^\pi \Psi^*(R, \theta, t) \cos^2 \theta \Psi(R, \theta, t) \sin \theta d\theta}{\int_0^\pi |\Psi(R, \theta, t)|^2 \sin \theta d\theta}$$

$$C_2(R, t) = \frac{1}{3} + \frac{4}{3\sqrt{5}} \text{Re} \left(\frac{u_2(R, t)}{u_0(R, t)} \right) + \dots$$

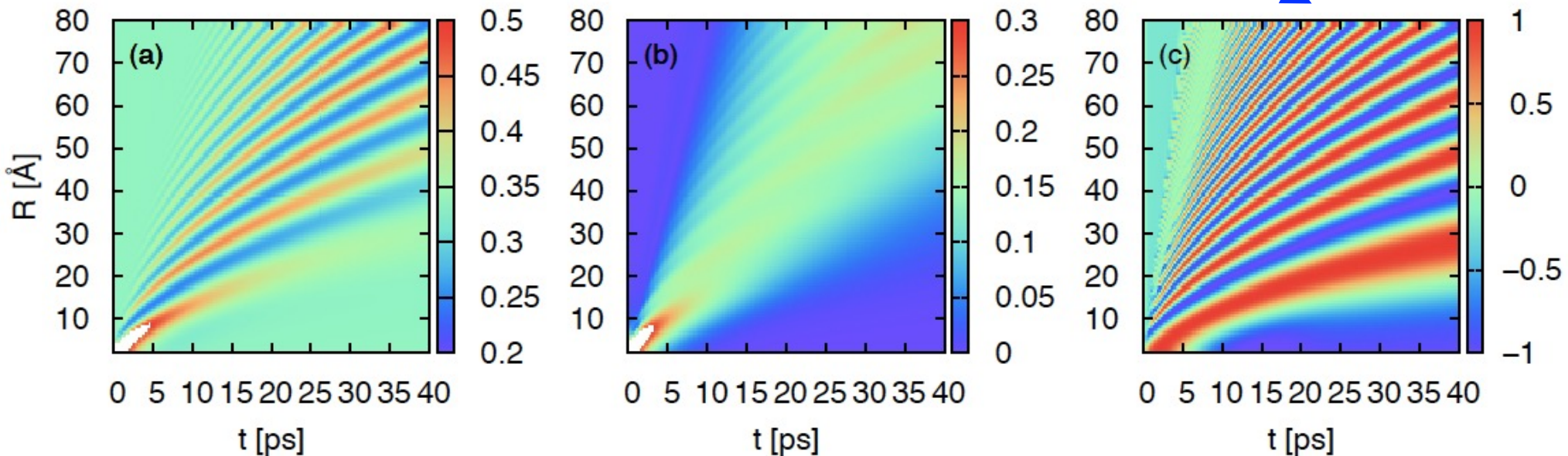
$$C_2(R, t) = \frac{1}{3} + \frac{4}{3\sqrt{5}} \left| \frac{u_2(R, t)}{u_0(R, t)} \right| \cos(\gamma_2(R, t) - \gamma_0(R, t)) + \dots$$

Interference Pattern Due To $J = 0$ and $J = 2$ Phases

$$C_2(R, t) = \frac{1}{3} + \frac{4}{3\sqrt{5}} \left| \frac{u_2(R, t)}{u_0(R, t)} \right| \cos(\gamma_2(R, t) - \gamma_0(R, t)) + \dots$$

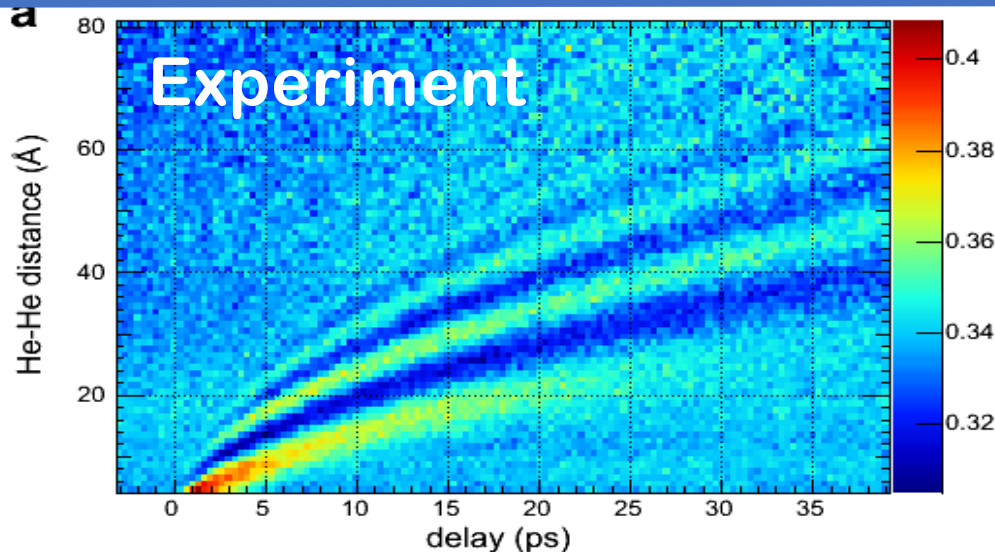
Only plotting
 $J = 0$ and 2

$\gamma_0 \approx \text{const}$

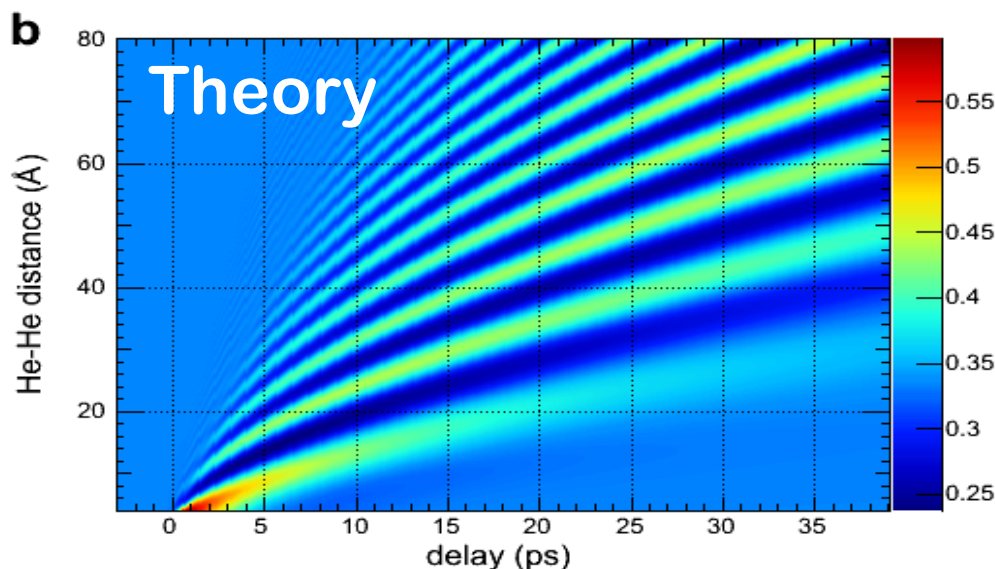


Alignment signal $\cos^2\theta$ can be interpreted as measuring $\gamma_2(R, t)$.

Comparison With Experiment



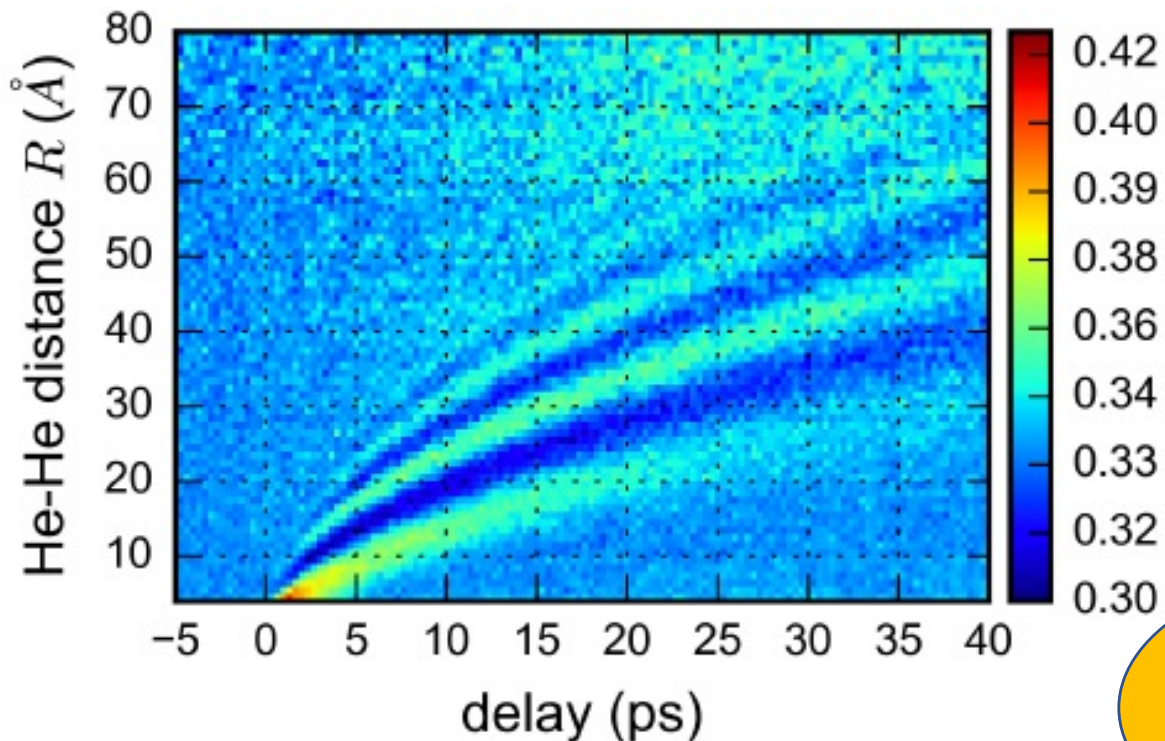
Experimental data by
Maksim Kunitski,
Reinhard
Doerner et al.
(Frankfurt University)



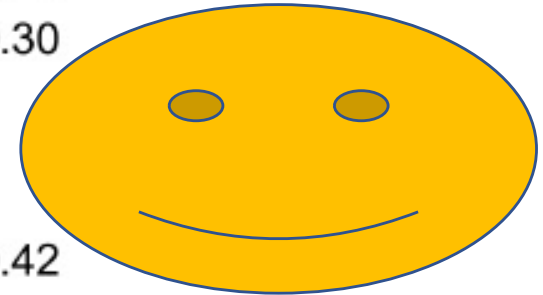
Agreement is
qualitative but not
quantitative.

Need to account for
finite experimental
resolution.

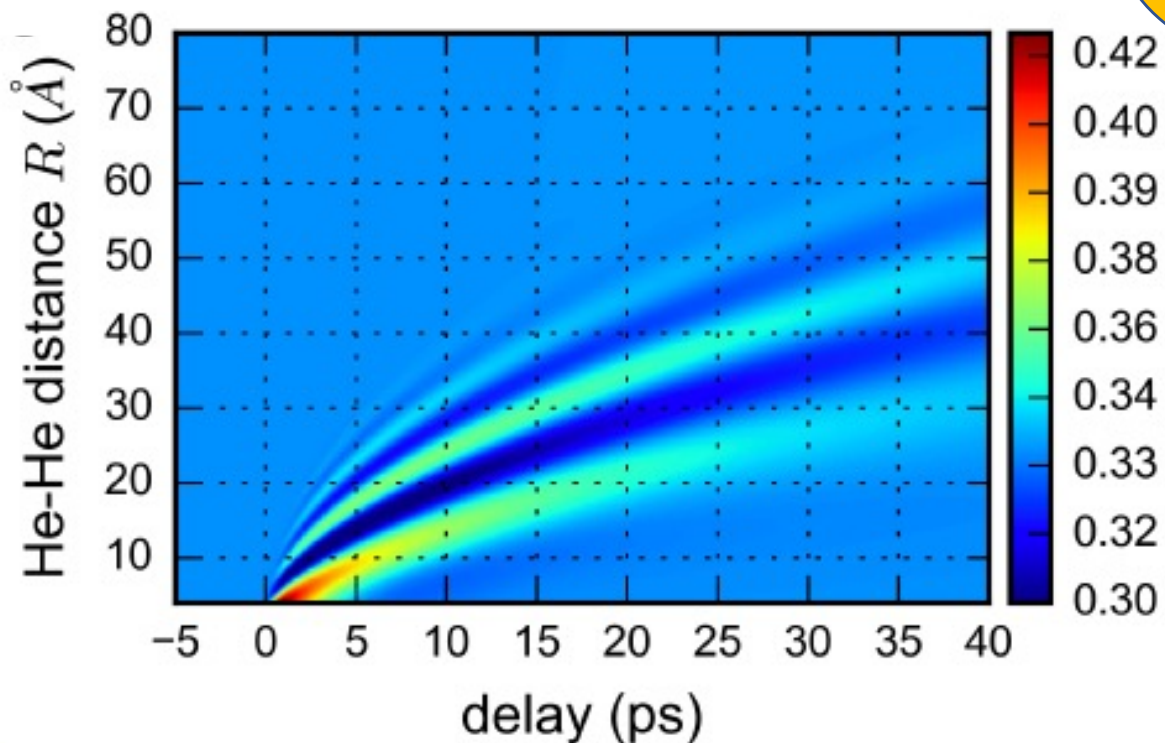
Experiment



$\langle \cos^2 \theta \rangle$



Parameter-free theory (using measured pulse length, intensity, spatial imaging resolution)



$\langle \cos^2 \theta \rangle$

**Kunitski,
Guan,...,Blume,
Doerner,
Nature Physics
(2021).**

Kicking the ^4He Dimer

For the first time: Intense laser used to probe dynamics at single-atom level using universal, scattering length dominated initial state.

“Rotationless” ^4He dimer can be aligned! Note, it’s the continuum portion of the wave packet...

**Pattern due to interference between $J=0$ and $J=2$ channels:
Measurement of spatially and time dependent relative phase between these two partial wave channels. State tomography!**

Many outstanding challenges:

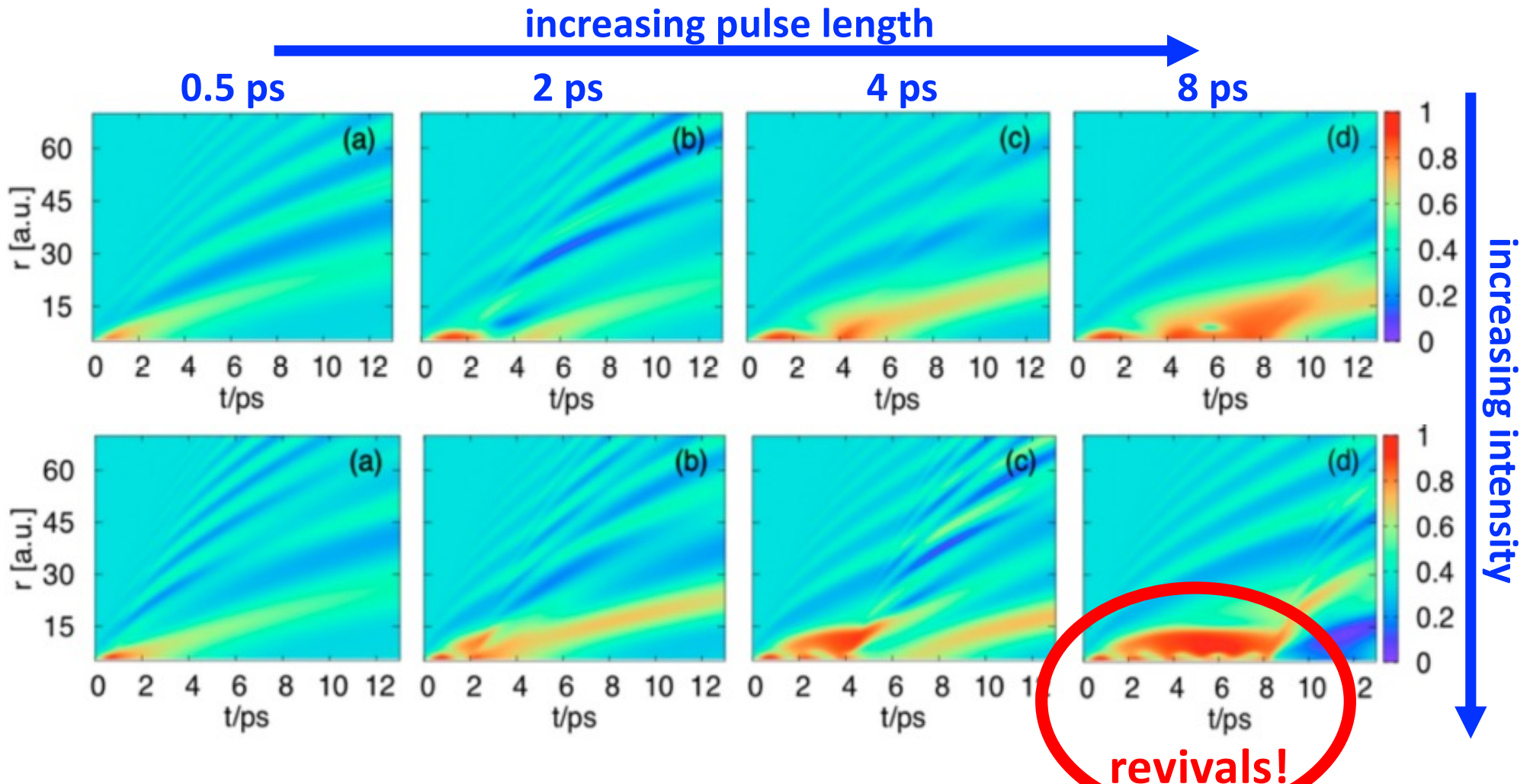
Resonances as in ultracold atoms? Need longer pulses...

Time-dependent modulation of interaction strength?

Dynamics of (Efimov) trimers? Need to populate it first...

Larger clusters.

Scenario 2: Longer Pulses



Guan and Blume, PRA 99, 033416 (2019). Awaiting experimental realization...

Signature Of Field-Induced $^4\text{He}_2$ Bound States?

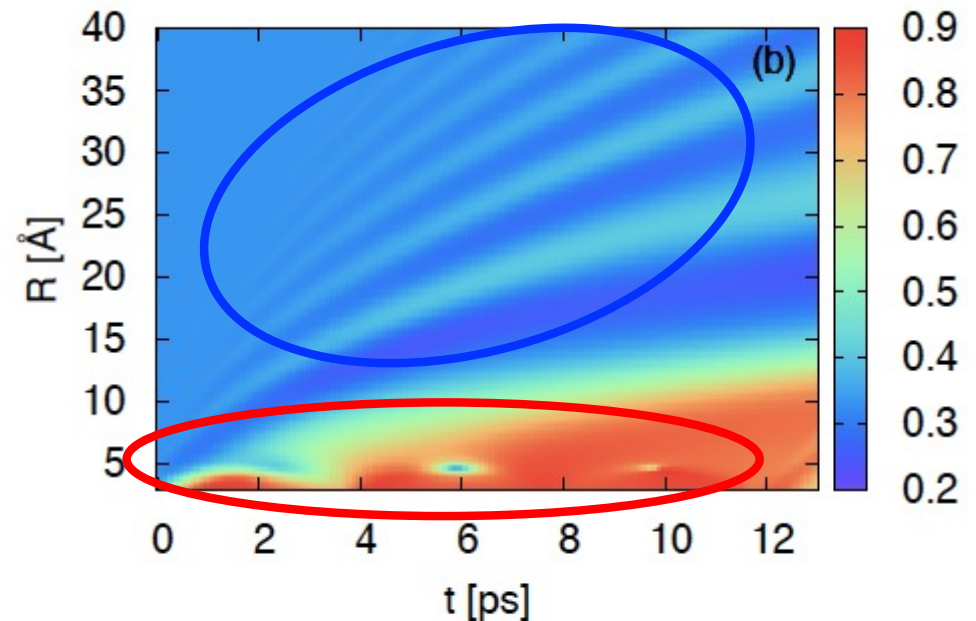
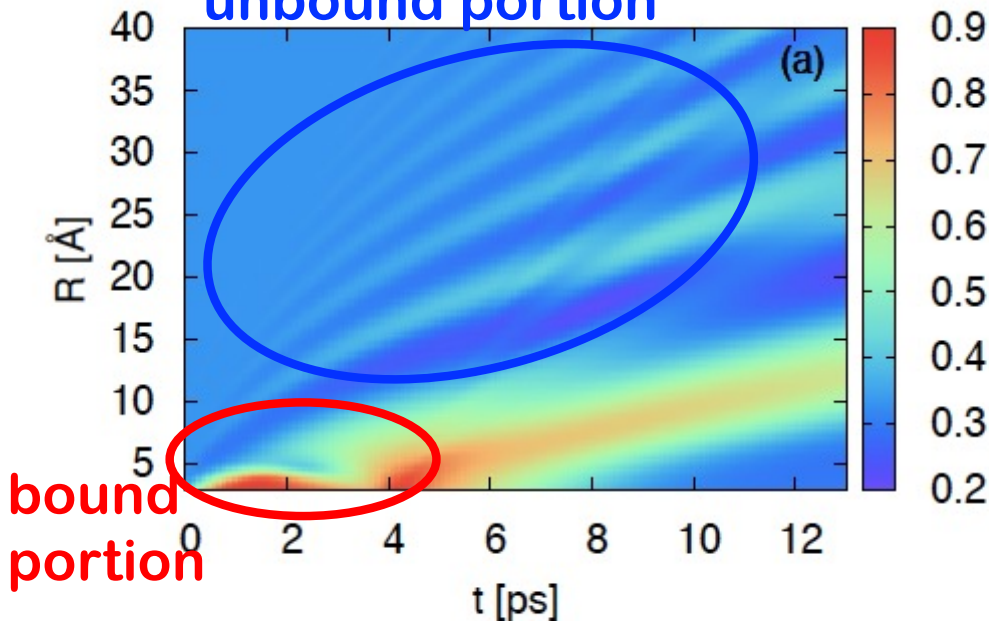
$$C_2(R, t) = \frac{\int_0^\pi \Psi^*(R, \theta, t) \cos^2 \theta \Psi(R, \theta, t) \sin \theta d\theta}{\int_0^\pi |\Psi(R, \theta, t)|^2 \sin \theta d\theta}$$

max. intensity
 $2.5 \cdot 10^{14} \text{W/cm}^2$

4ps hold time

12ps hold time

unbound portion



Fingerprint of revivals in time-dependent response of system: Dimer oscillates between deeply-bound state and weakly-bound state.

Summary

- ${}^4\text{He}_N$ droplets can be realized experimentally in size-selective manner.
- Access to structural properties (accessing real-space structures beyond $N=3$ is non-trivial due to reconstruction algorithm).
- ${}^4\text{He}$ - ${}^4\text{He}$ well described by effective range theory (two-parameter theory).
- ${}^4\text{He}_3$: Ground state has Efimov characteristics and excited state is Efimov trimer (s-wave scattering length and three-body parameter).
- If we want more “info,” what can be done?
 - Spectroscopy (need more than one bound energy level).
 - Isotope substitution (${}^4\text{He} \rightarrow {}^3\text{He}$).
 - Tuning of interactions (this talk: ground state droplets).
 - Dynamics (this talk: dimer).

Thank You!
