Few-body hadronic resonances in lattice QCD

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hadspec

Outline

First half:

- Elastic two-body scattering

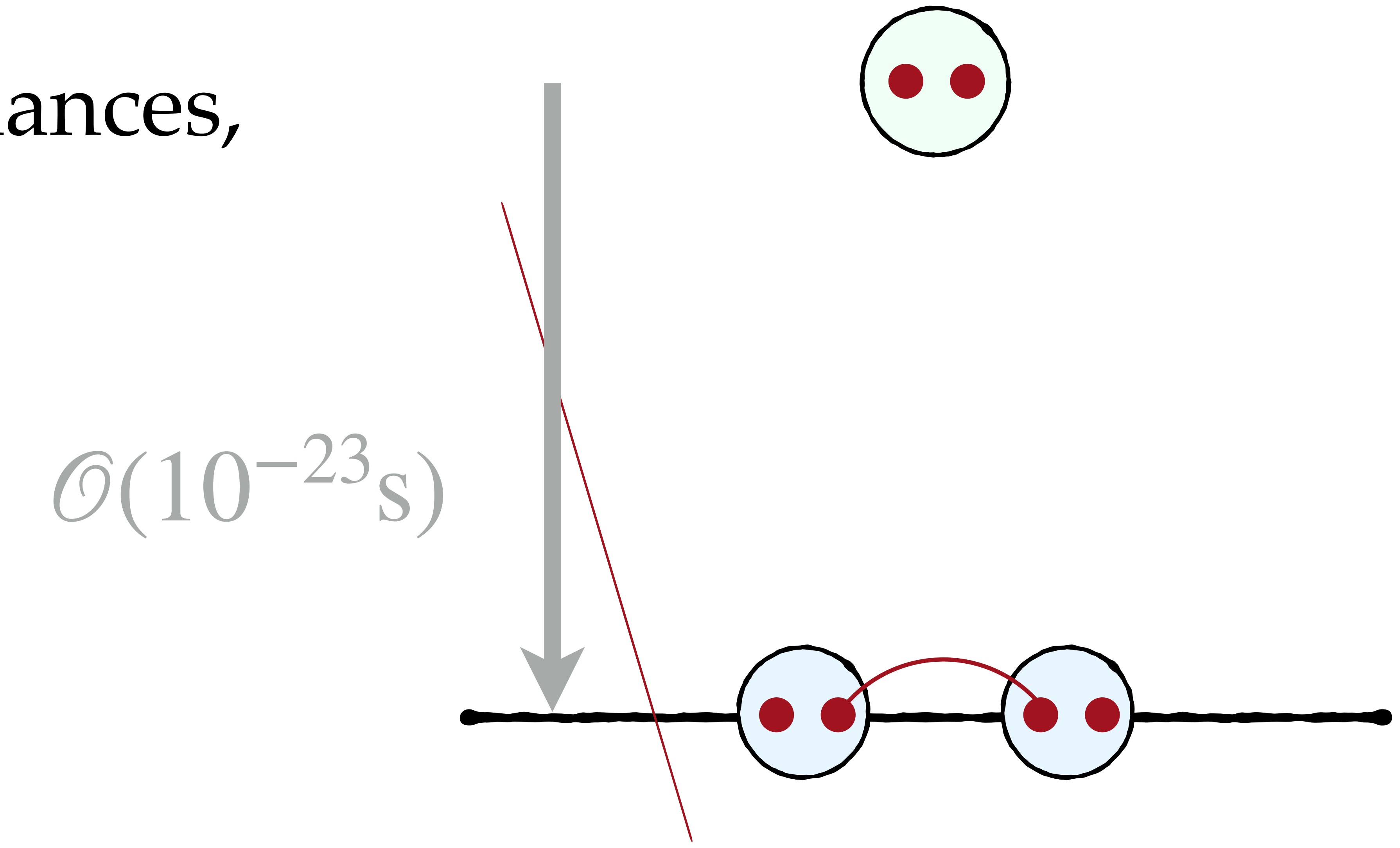
Second half:

- Two-body, coupled-channel scattering
- Three-body scattering

few-body physics in QCD

Big picture:

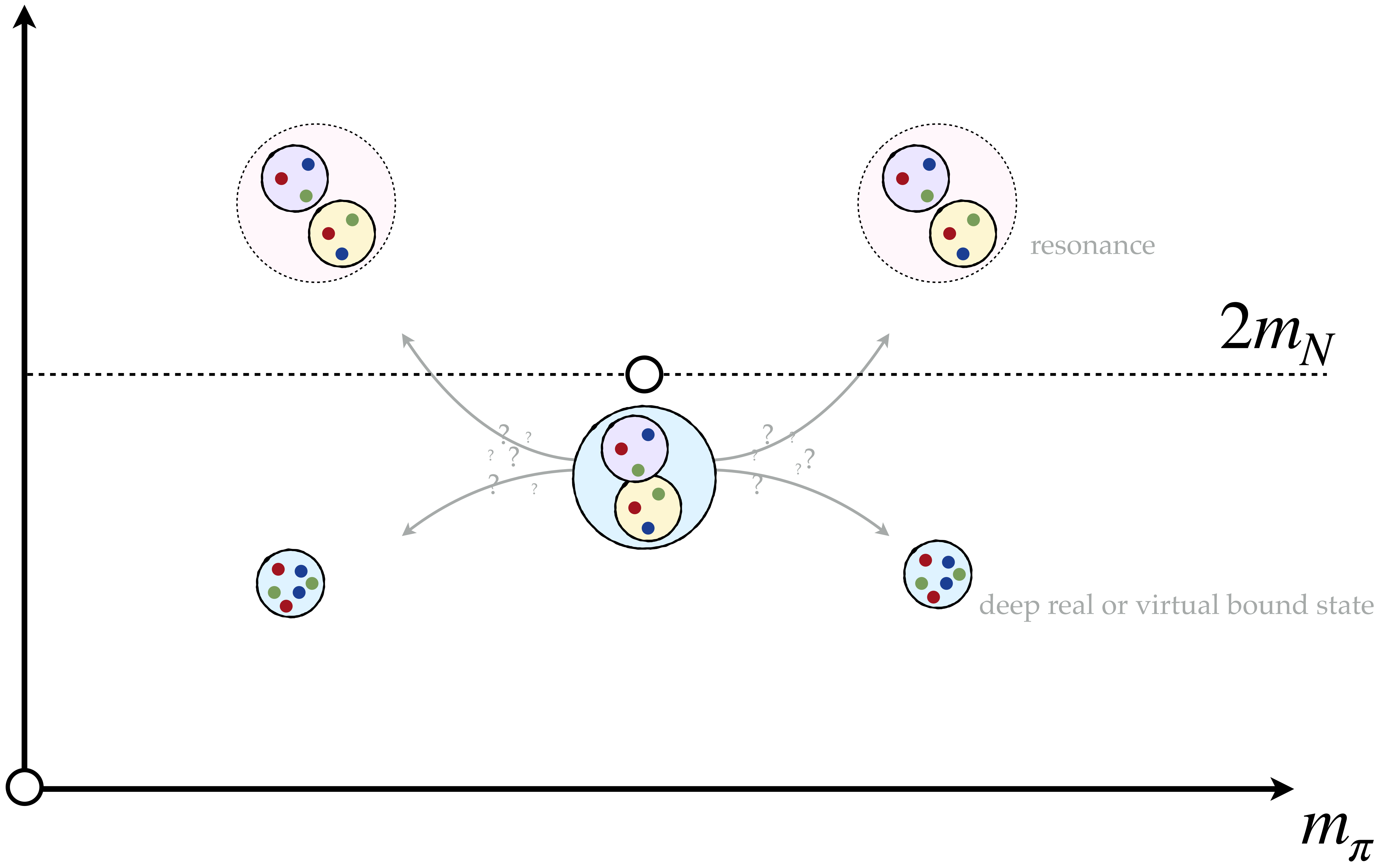
☑ most states in QCD are unstable resonances,



few-body physics in QCD

Big picture:

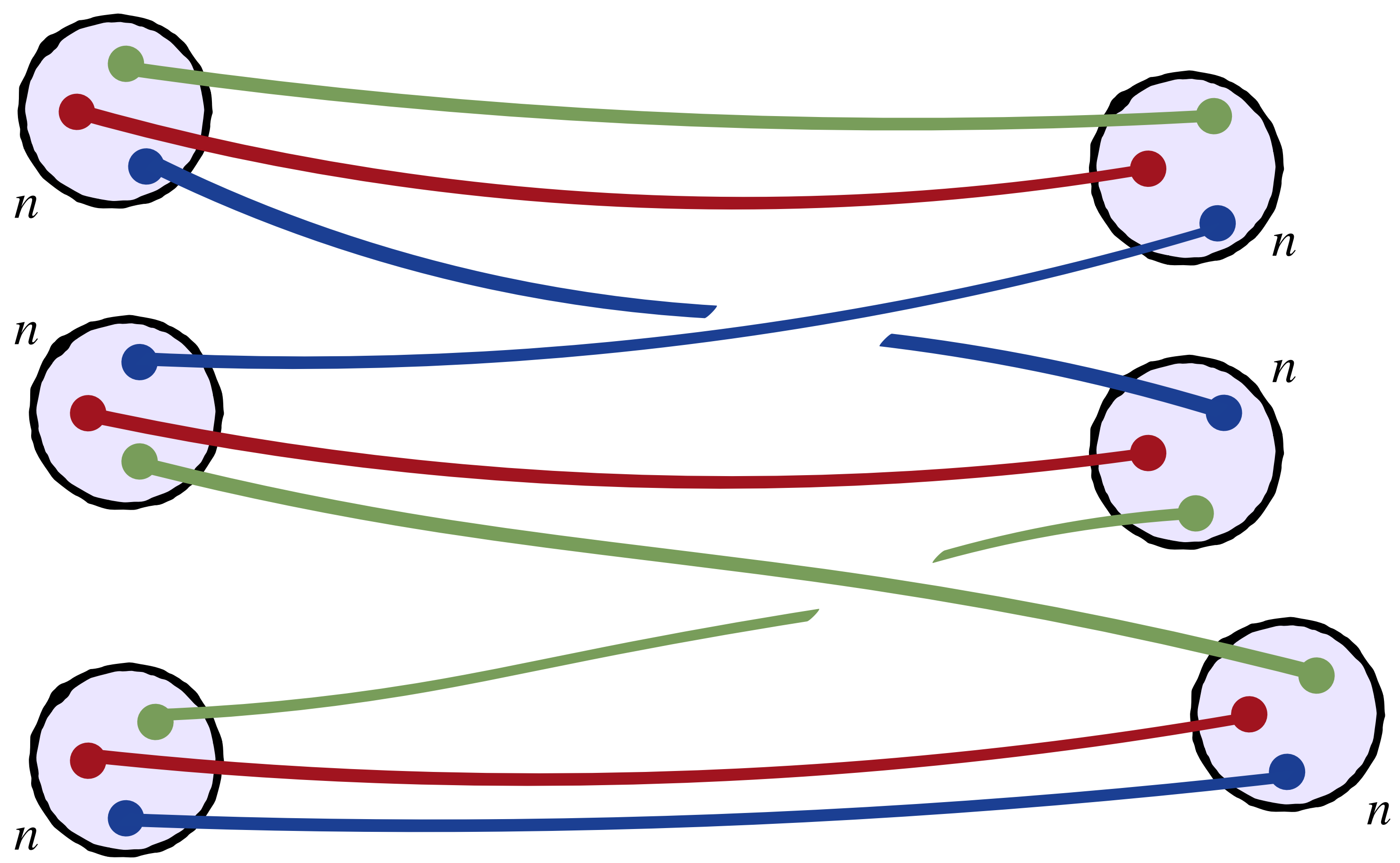
- ☑ most states in QCD are unstable resonances,
- ☑ nuclear physics dependence on nature's fundamental parameters,



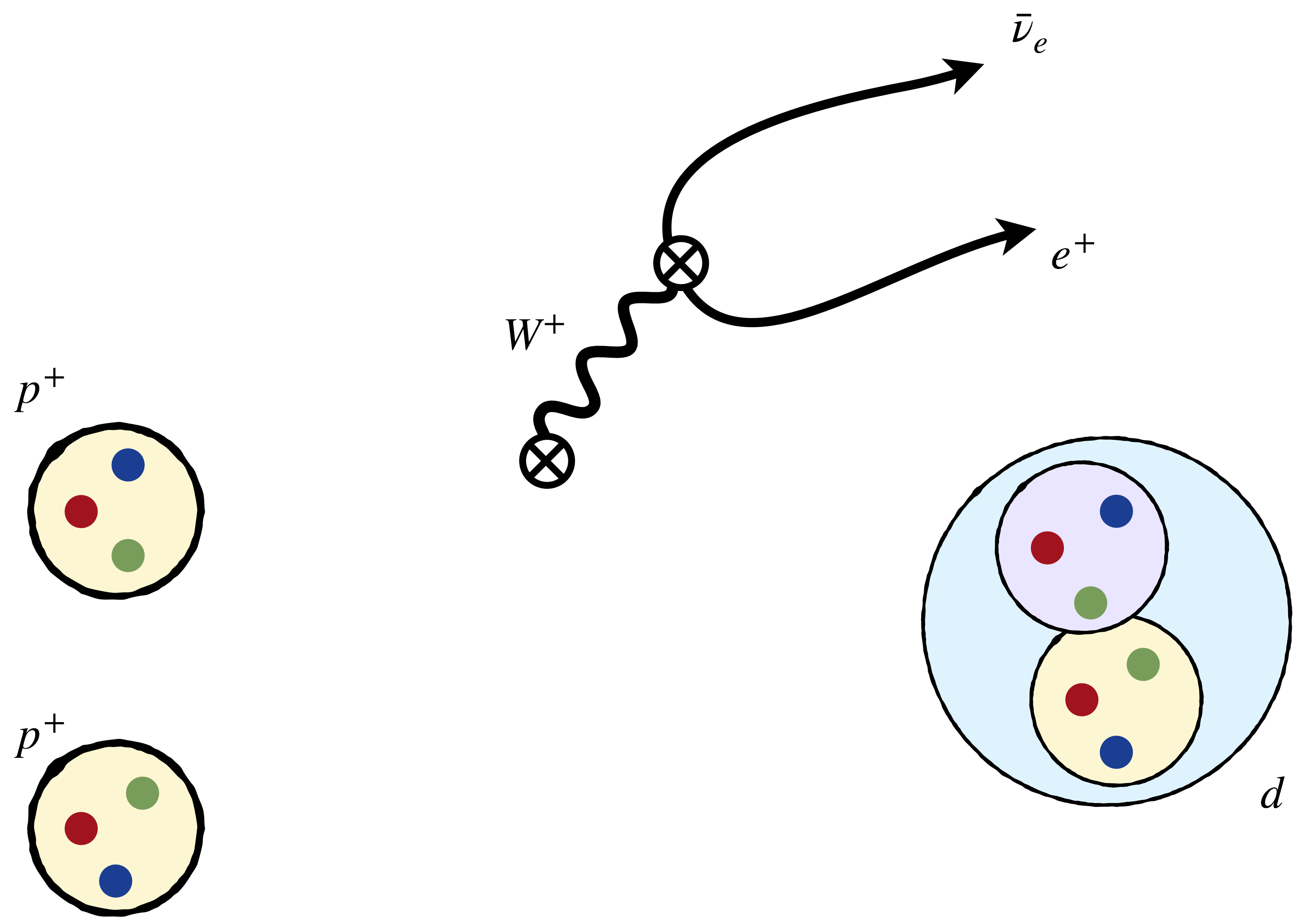
few-body physics in QCD

Big picture:

- ☑ most states in QCD are unstable resonances,
- ☑ nuclear physics dependence on nature's fundamental parameters,
- ☑ determining experimentally inaccessible quantities,



3n scattering

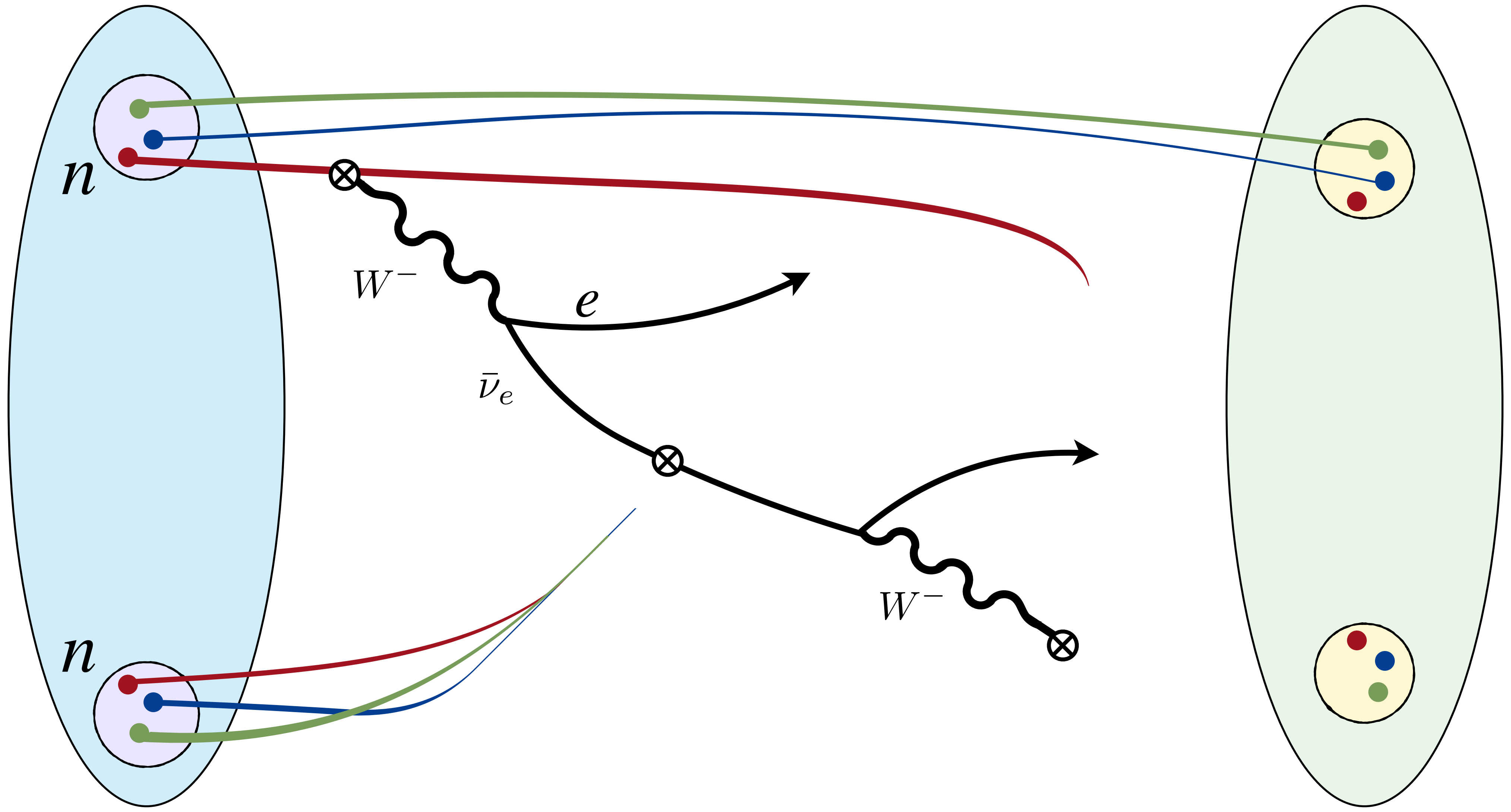


pp fusion

few-body physics in QCD

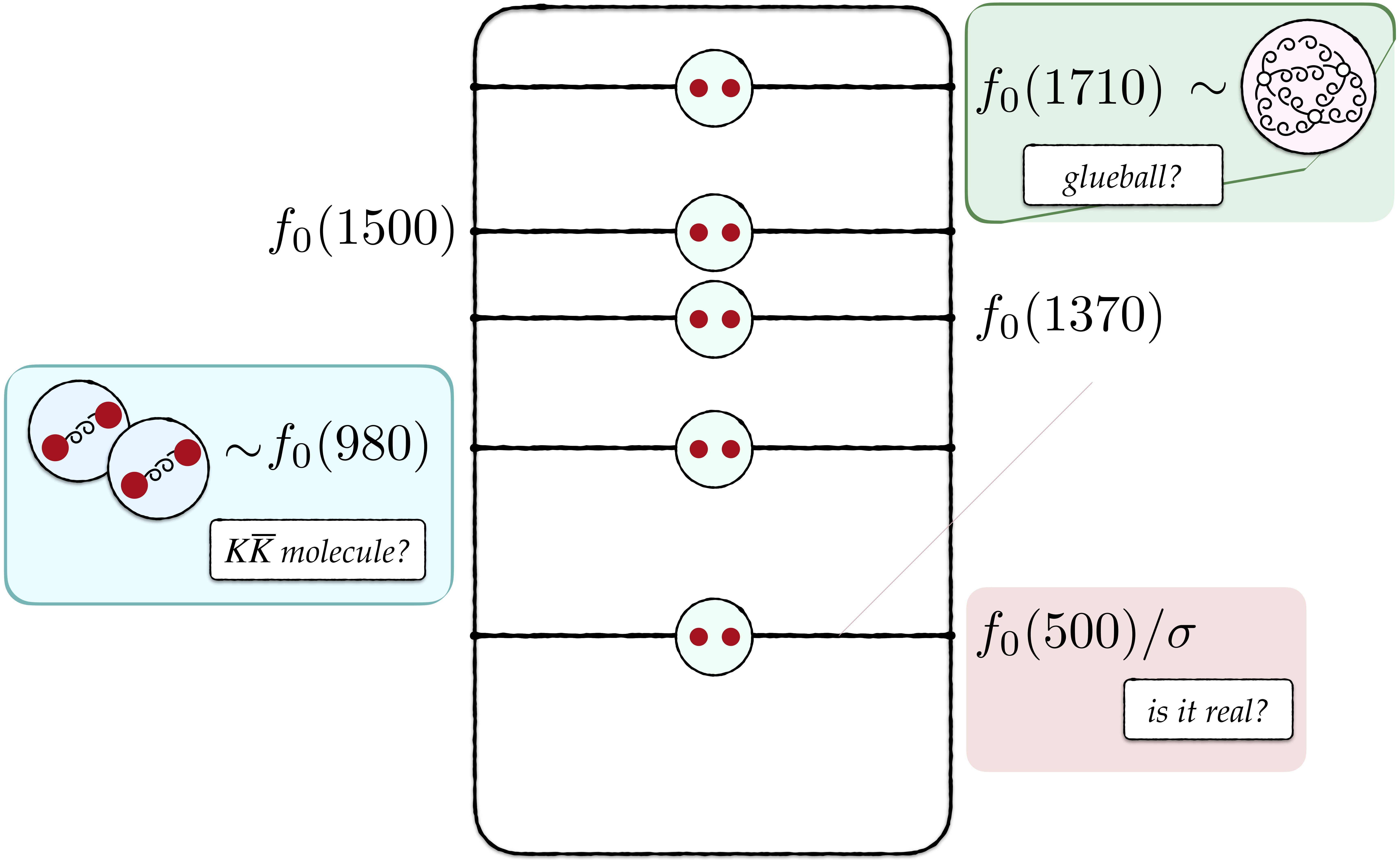
Big picture:

- ☑ most states in QCD are unstable resonances,
- ☑ nuclear physics dependence on nature's fundamental parameters,
- ☑ determining experimentally inaccessible quantities,
- ☑ determining QCD contributions to BSM physics,
- ☑ …



neutrionless double-beta decay

The vacuum channel



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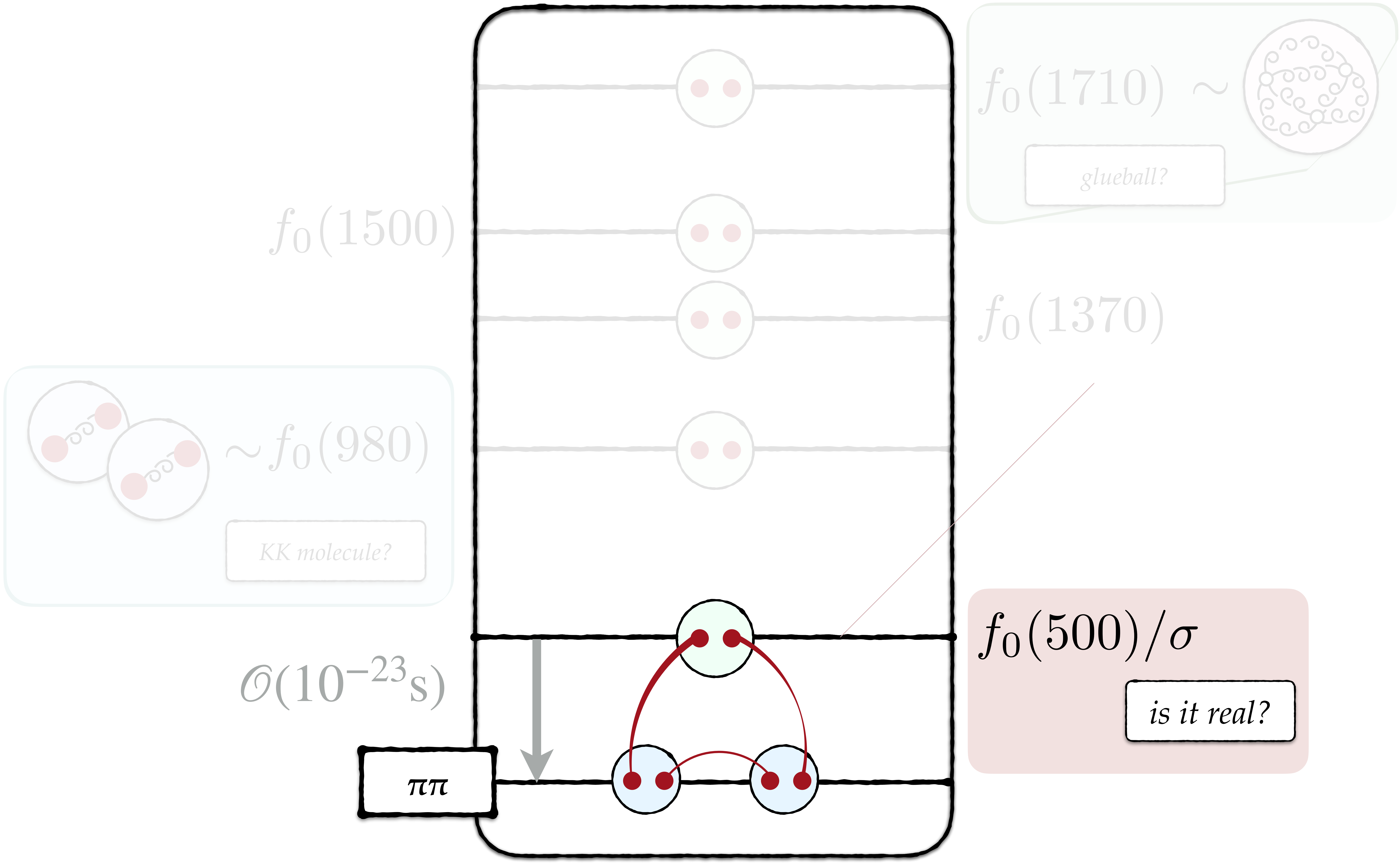

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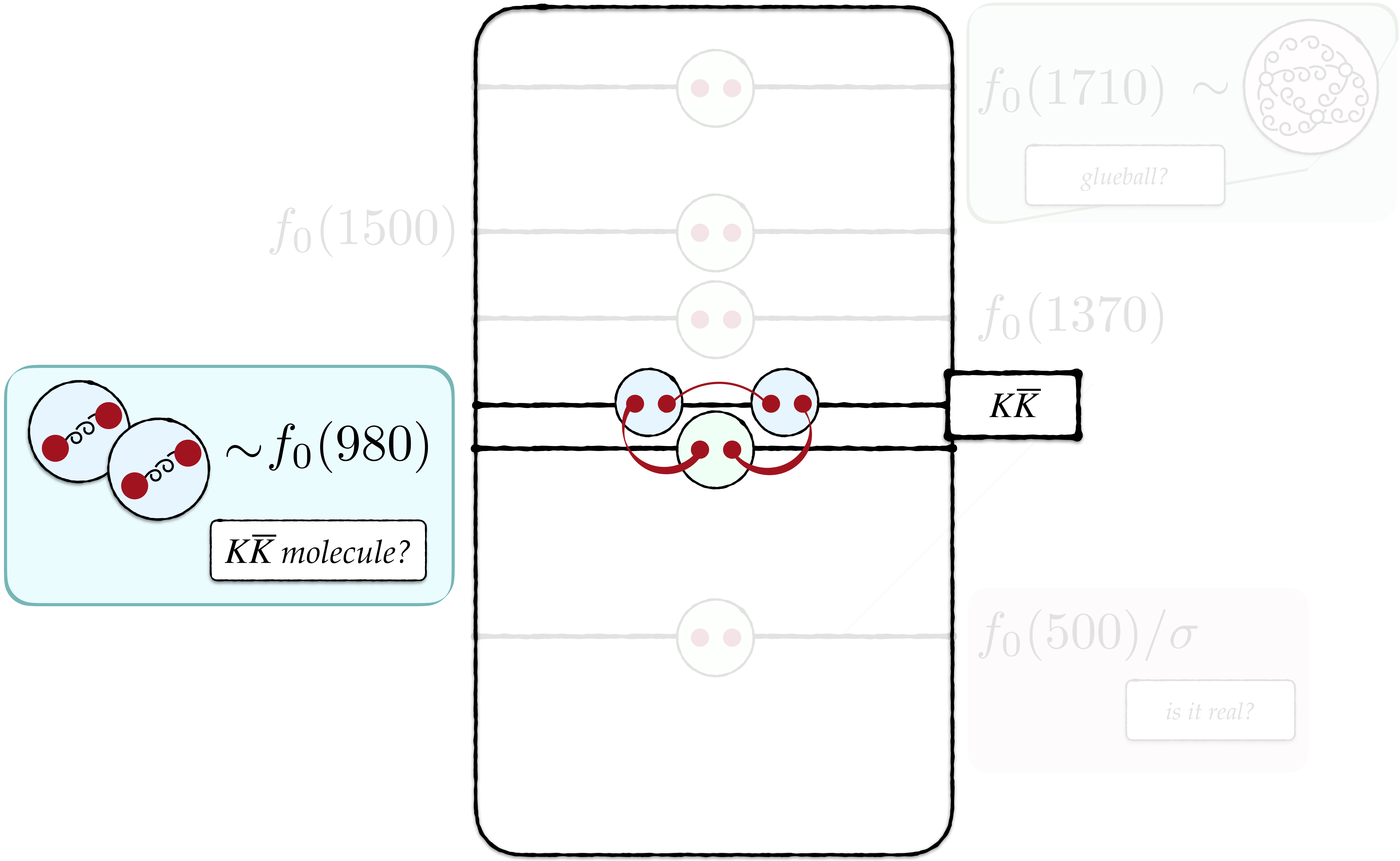

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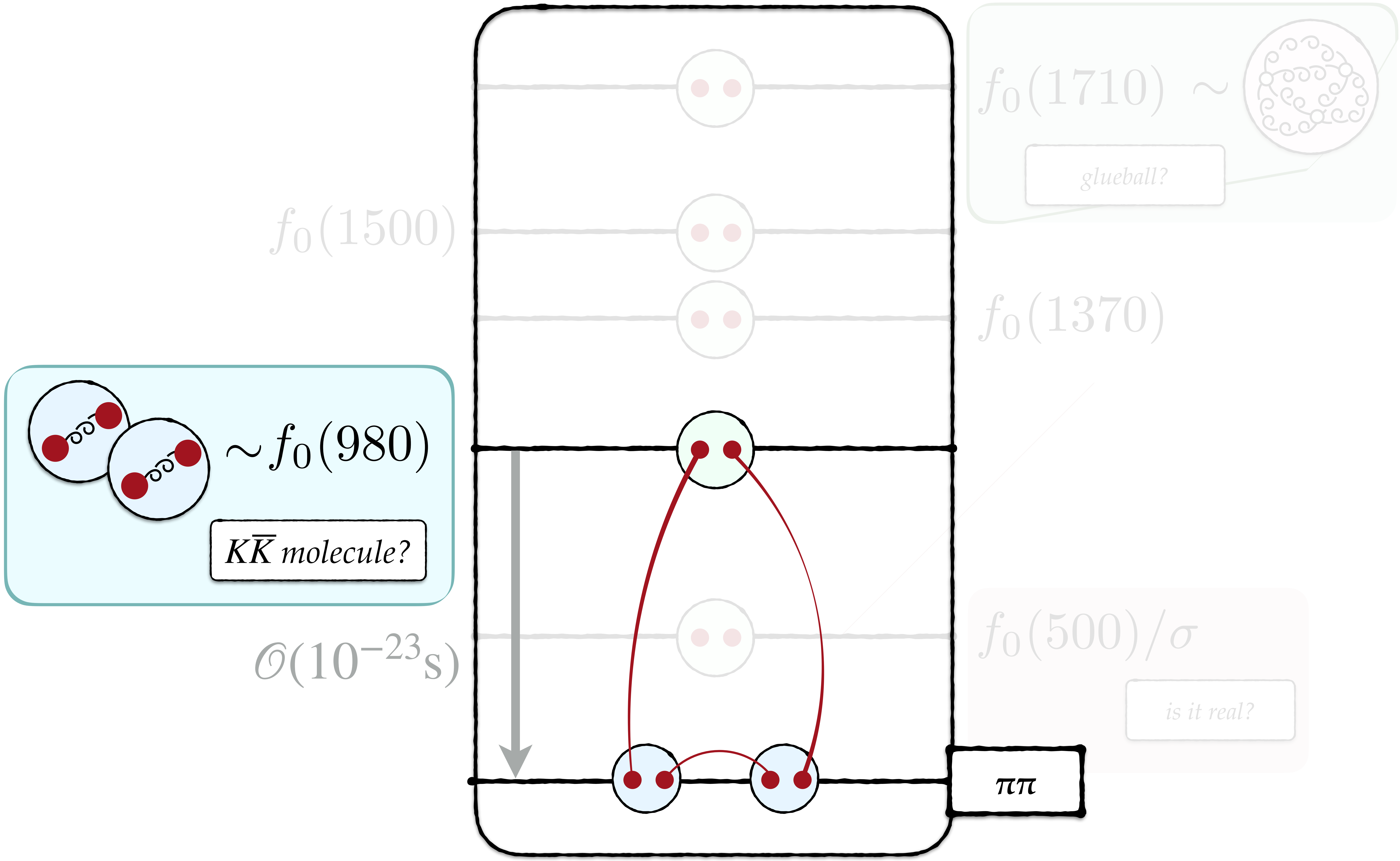
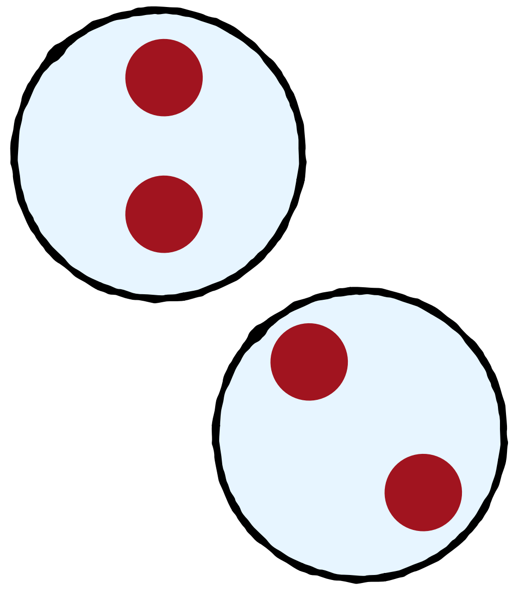


Lattice QCD in a nut shell

Non-perturbative tool to study non-perturbative QCD phenomena:

- ☑ broad and narrow resonances,
- ☑ strongly coupled systems,
- ☑ structure of few-body systems,
- ☑ …

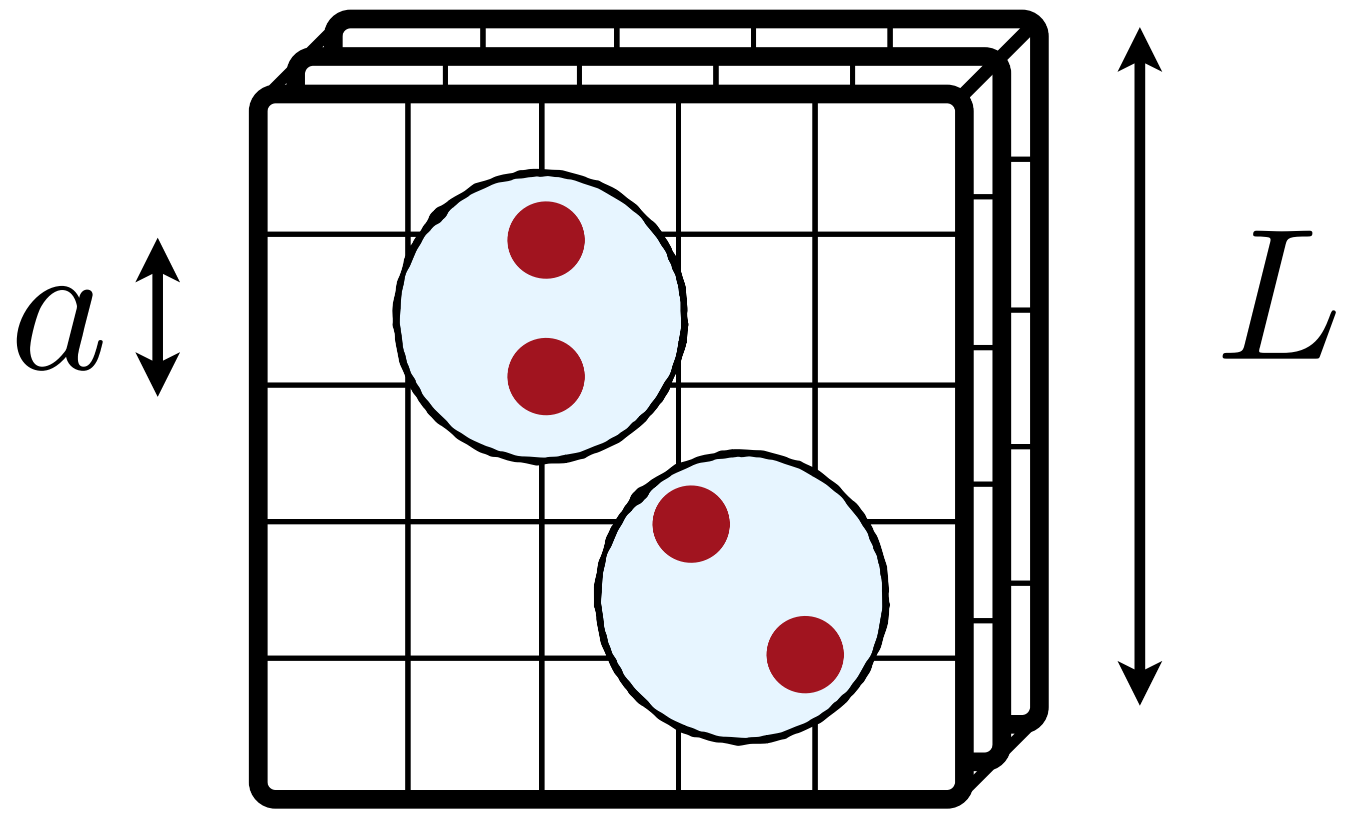
Lattice QCD in a nut shell



let's say you want to study
how two mesons interact…

Lattice QCD in a nut shell

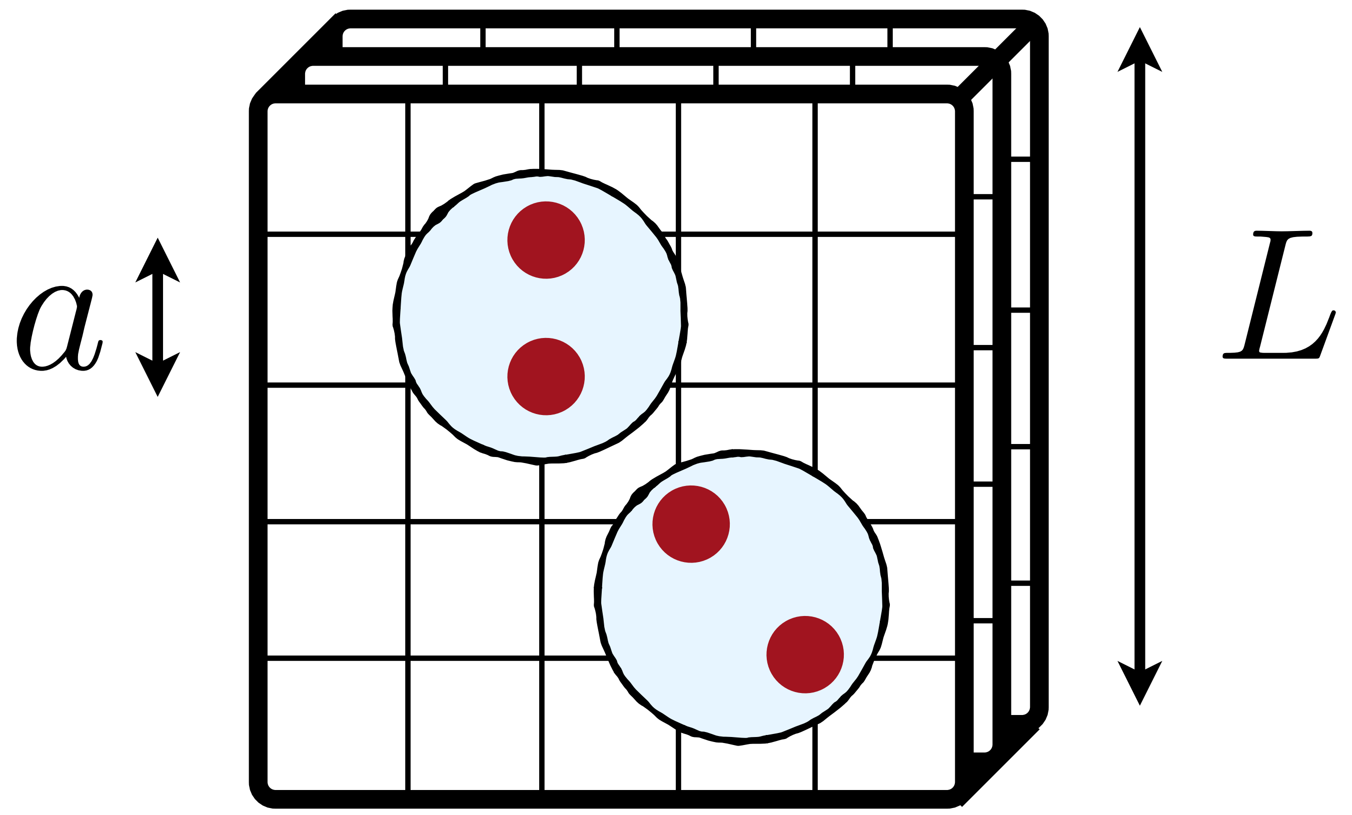
- lattice spacing: $a \sim 0.03 - 0.12$ fm
- finite volume: $L \sim 6 - 12$ fm



$$D_\mu = \Big(\quad \Big) \updownarrow (L/a)^3 \times (T/a)$$

Lattice QCD in a nut shell

- lattice spacing: $a \sim 0.03 - 0.12$ fm
- finite volume: $L \sim 6 - 12$ fm
- quark masses



Advantage over experiment!

Lattice QCD in a nut shell

- lattice spacing: $a \sim 0.03 - 0.12$ fm
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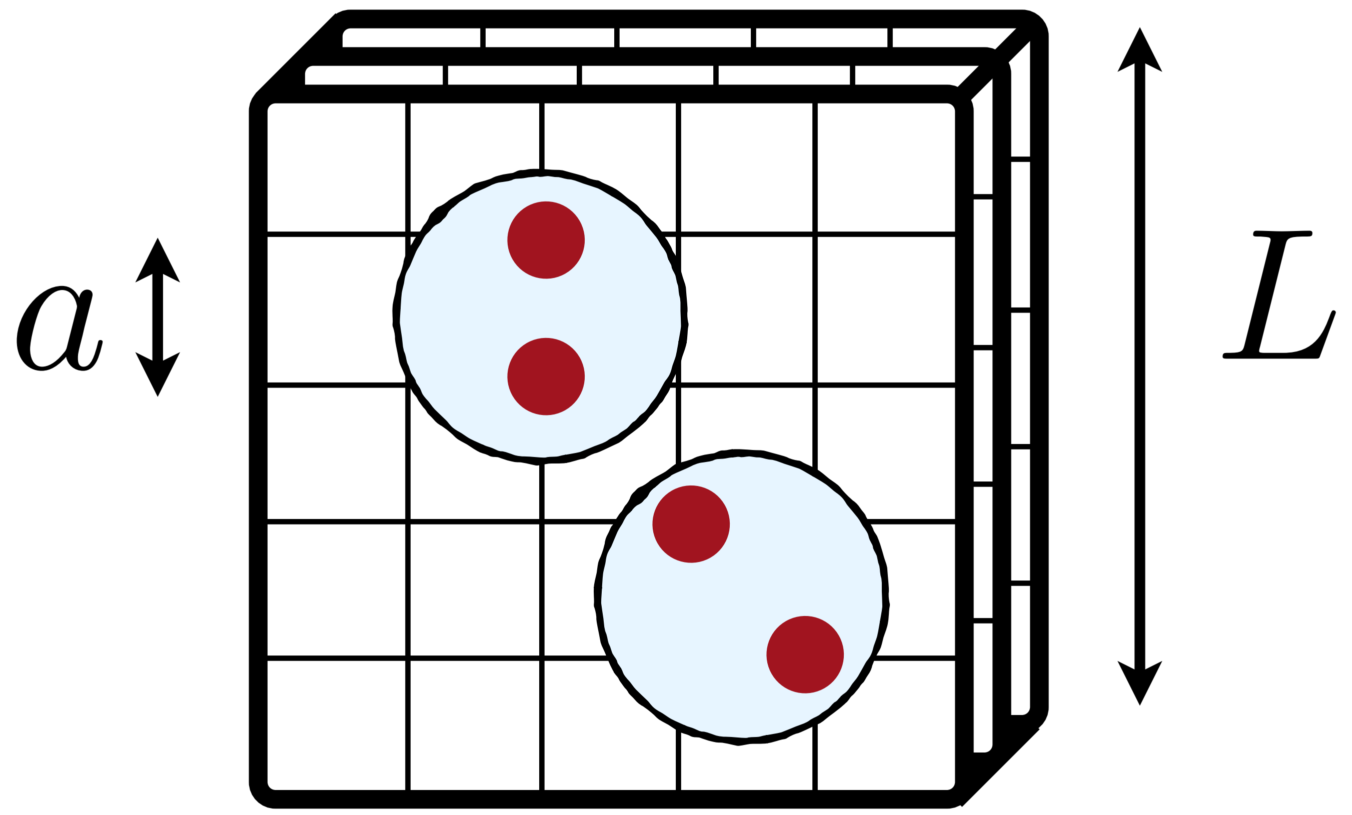
- quark masses
- Euclidean spacetime: $t_M \to -it_E$
- Monte Carlo sampling



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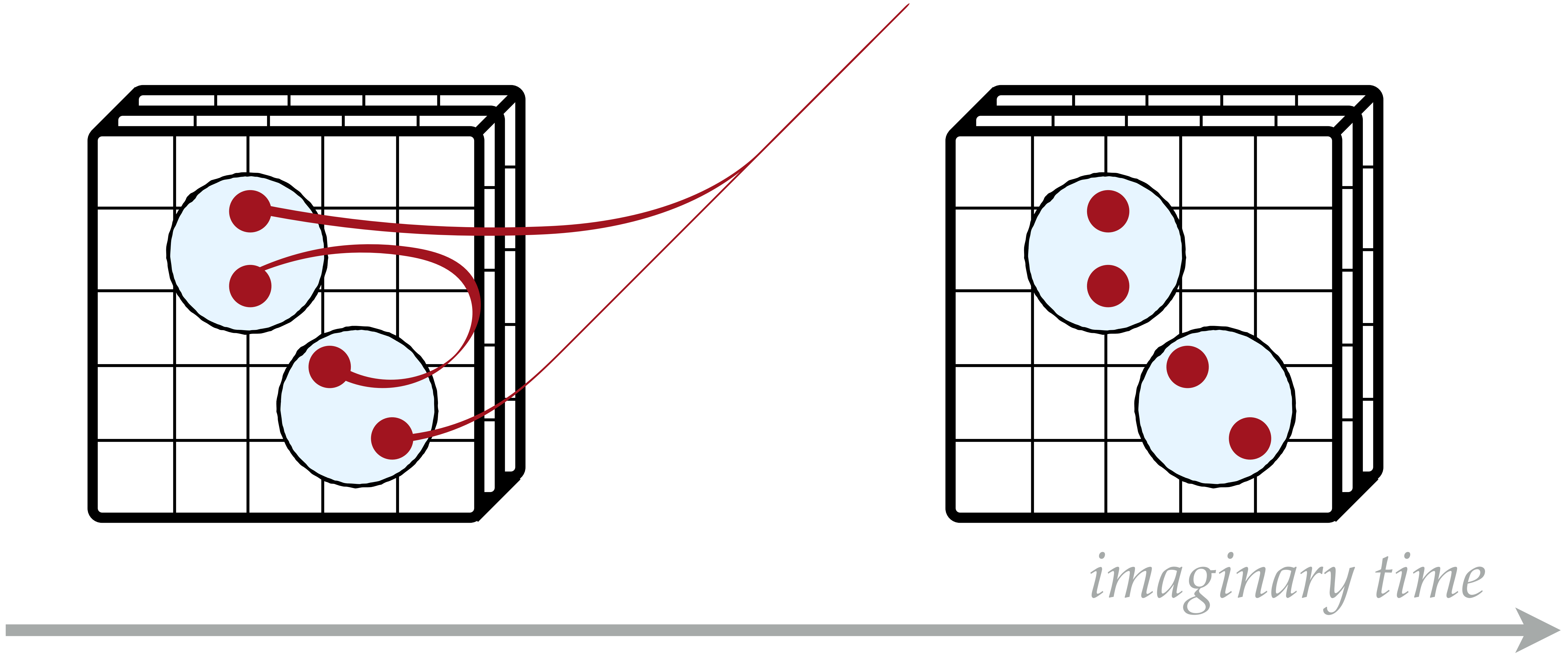
- Monte Carlo sampling
- Correlation functions, …

$$C^{2pt.}(t_E) \equiv \langle 0|\mathcal{O}(t_E)\mathcal{O}^\dagger(0)|0\rangle = \sum_n c_n e^{-E_n t_E}$$



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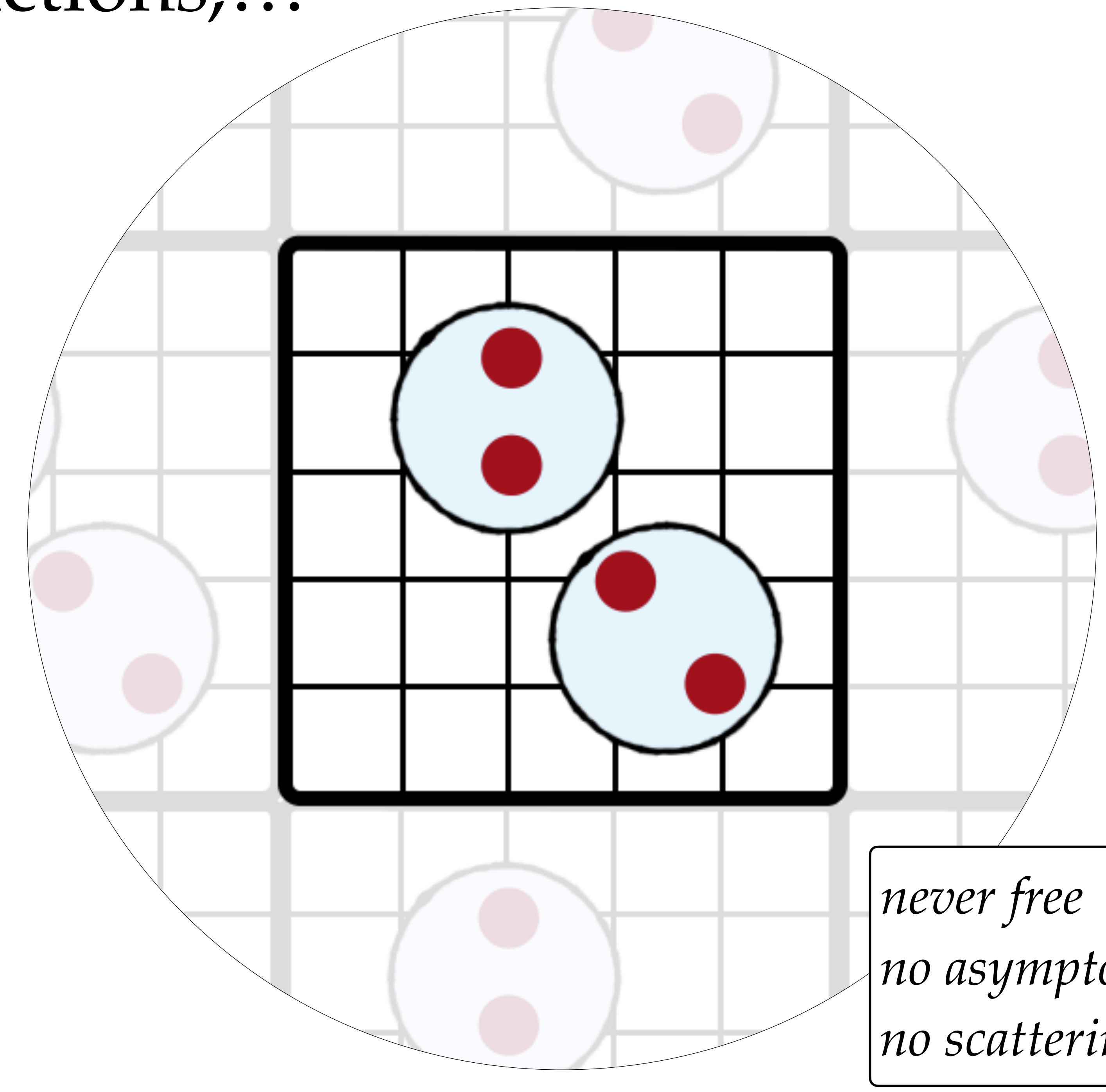
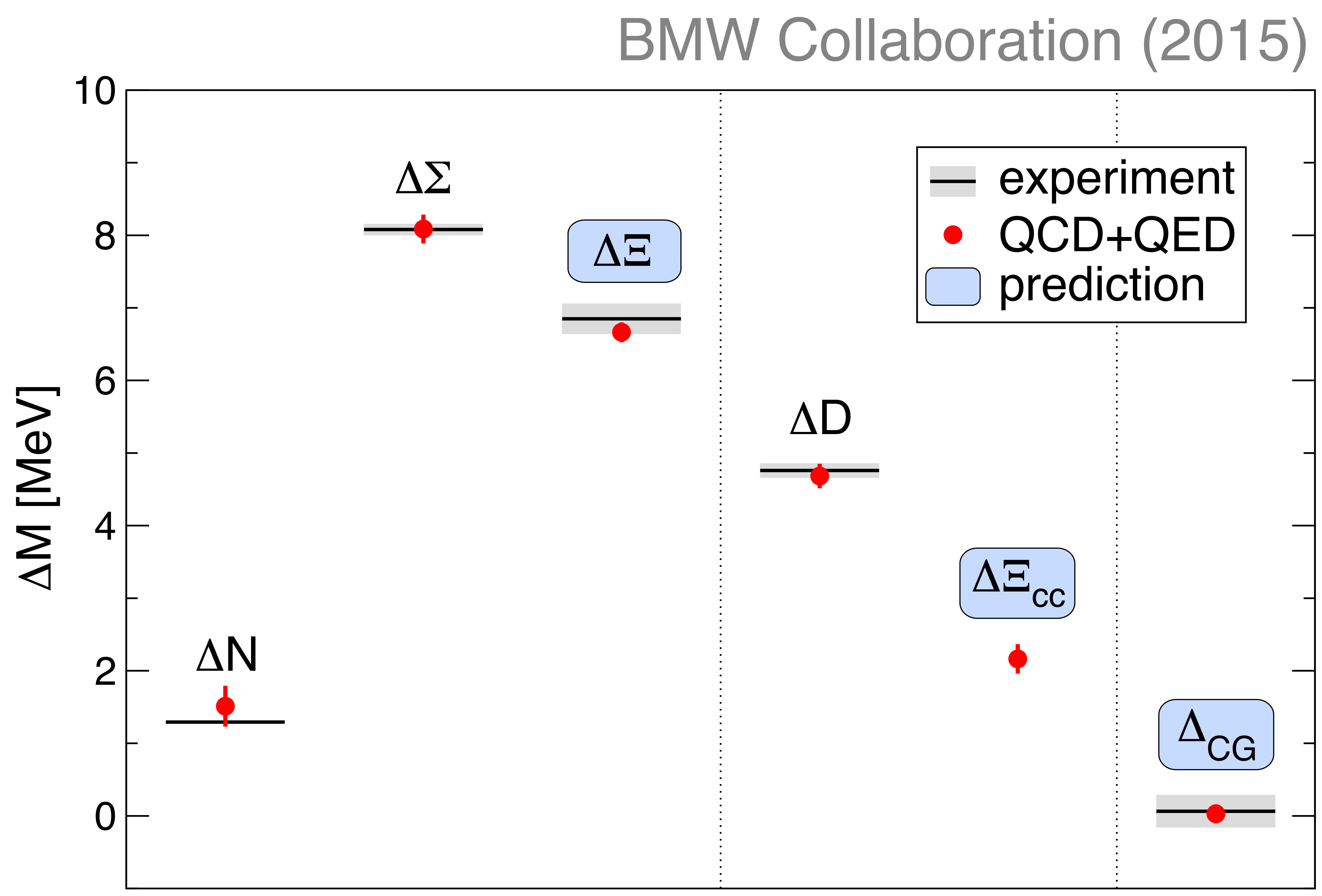
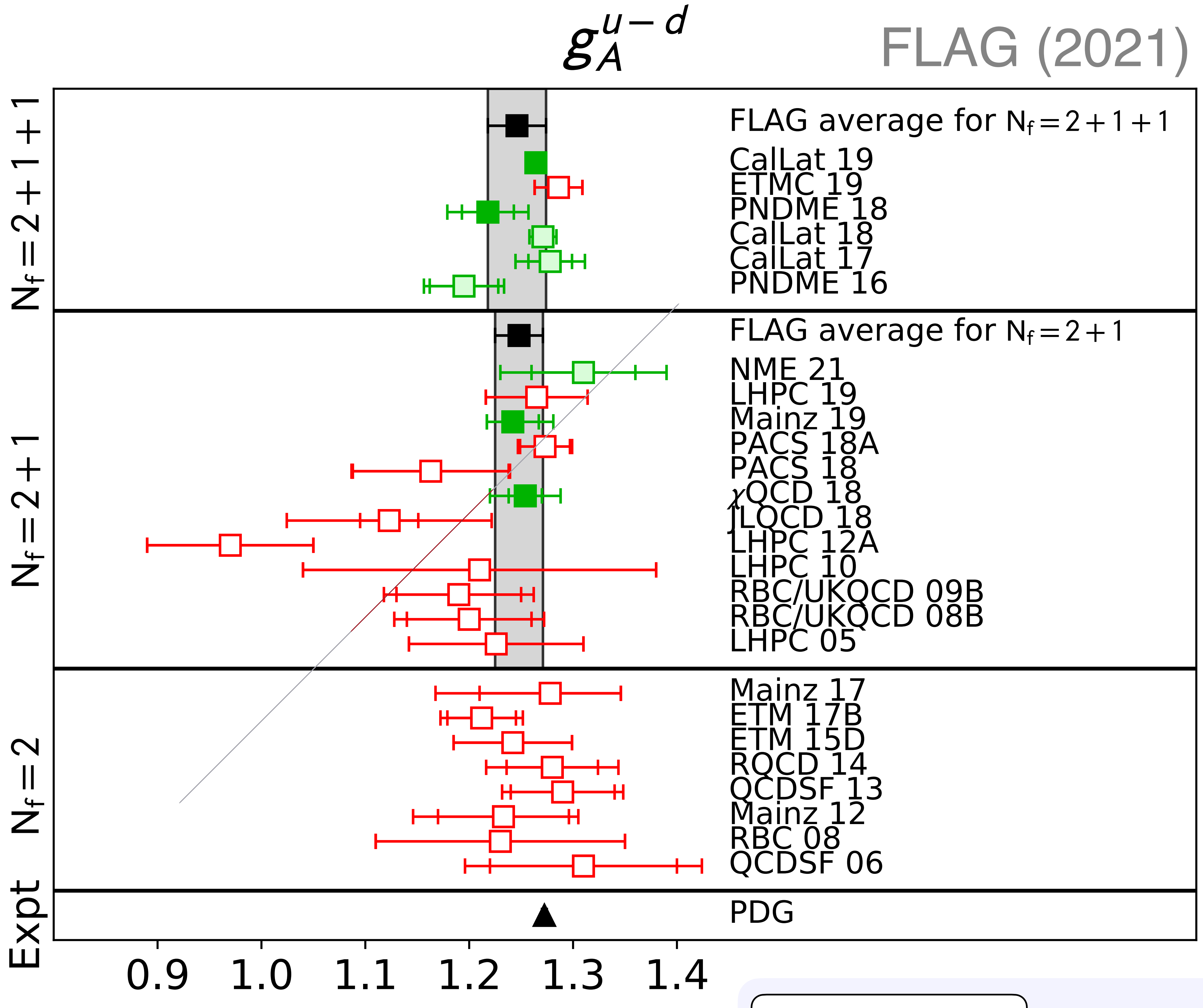
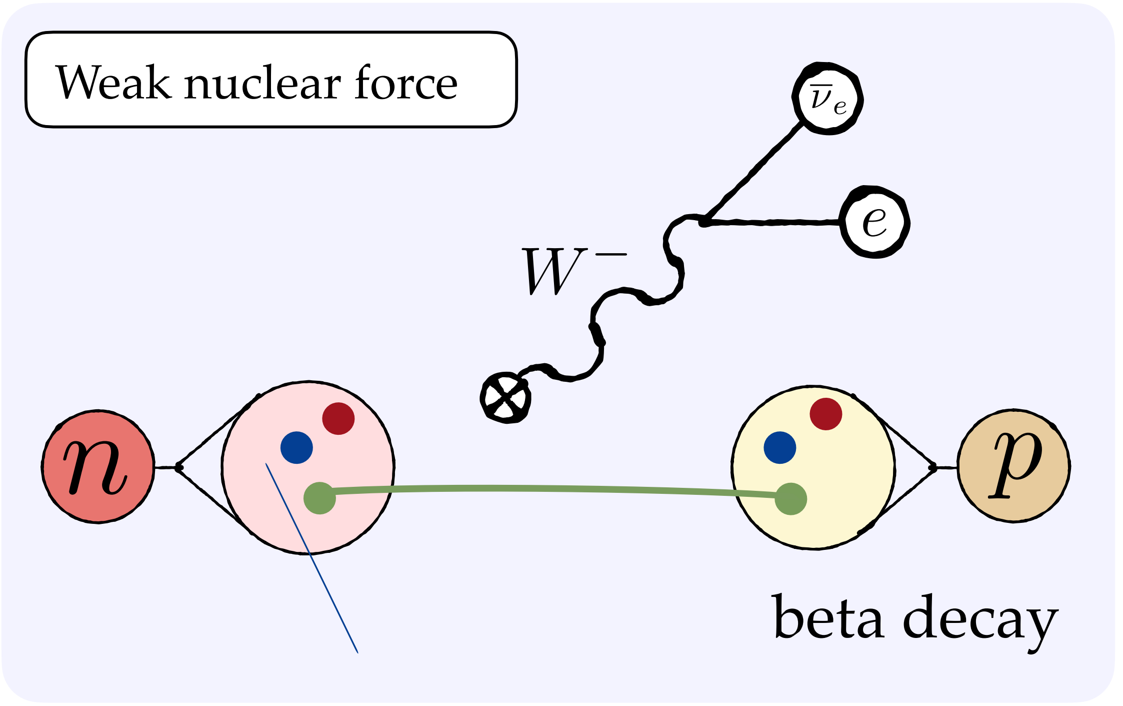


never free
no asymptotic states
no scattering

"Simple quantities" from lattice QCD

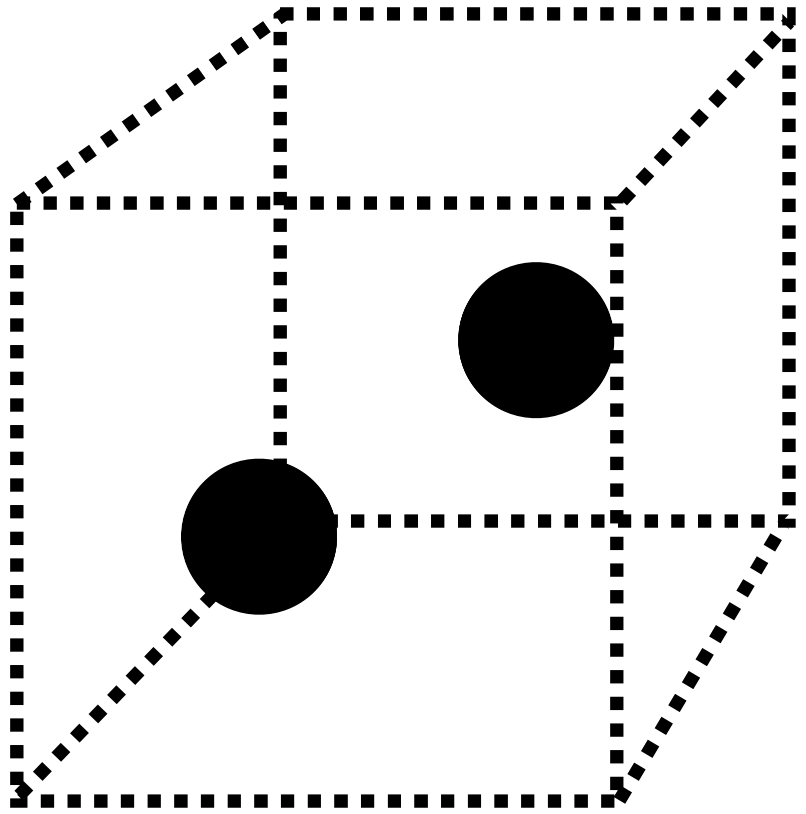






Coleman-Glashow mass difference $\Delta_{CG} = \Delta M_N - \Delta M_\Sigma + \Delta M_\Xi$

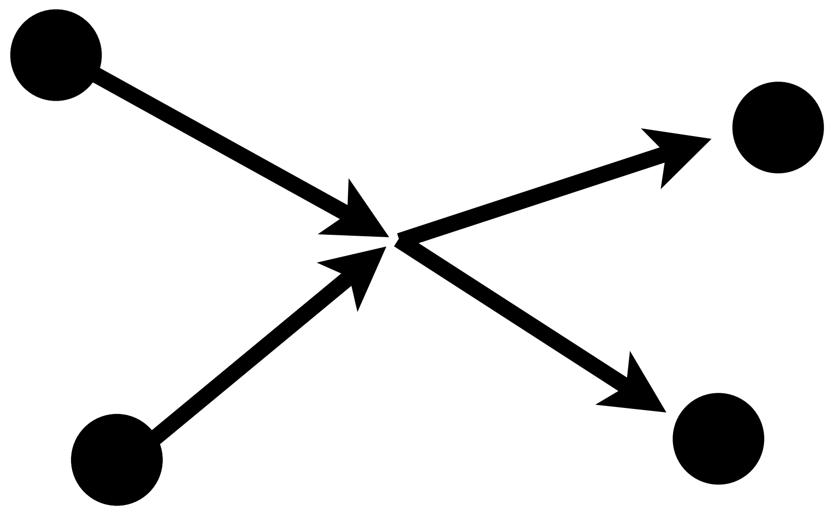
Two-hadron systems



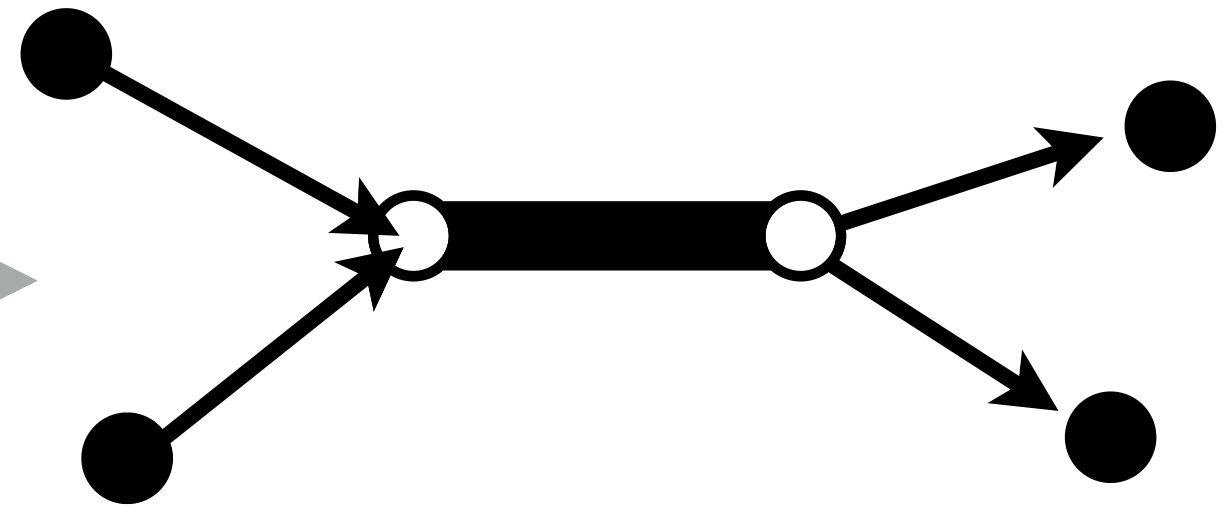
finite-volume
spectroscopy

$$\det\left[F(E_L,L)+\mathcal{M}^{-1}(E_L)\right]=0$$

Lüscher (1986, 1991)
Rummukainen & Gottlieb (1995)
Kim, Sachrajda, & Sharpe (2005)
Christ, Kim & Yamazaki (2005)
Feng, Li, & Liu (2004)
Hansen & Sharpe (2021)
RB & Davoudi (2012)
RB (2014)



infinite-volume
scattering amplitudes



bound state and
resonance poles

2 minimum requirements

Two "musts" for few-body systems:

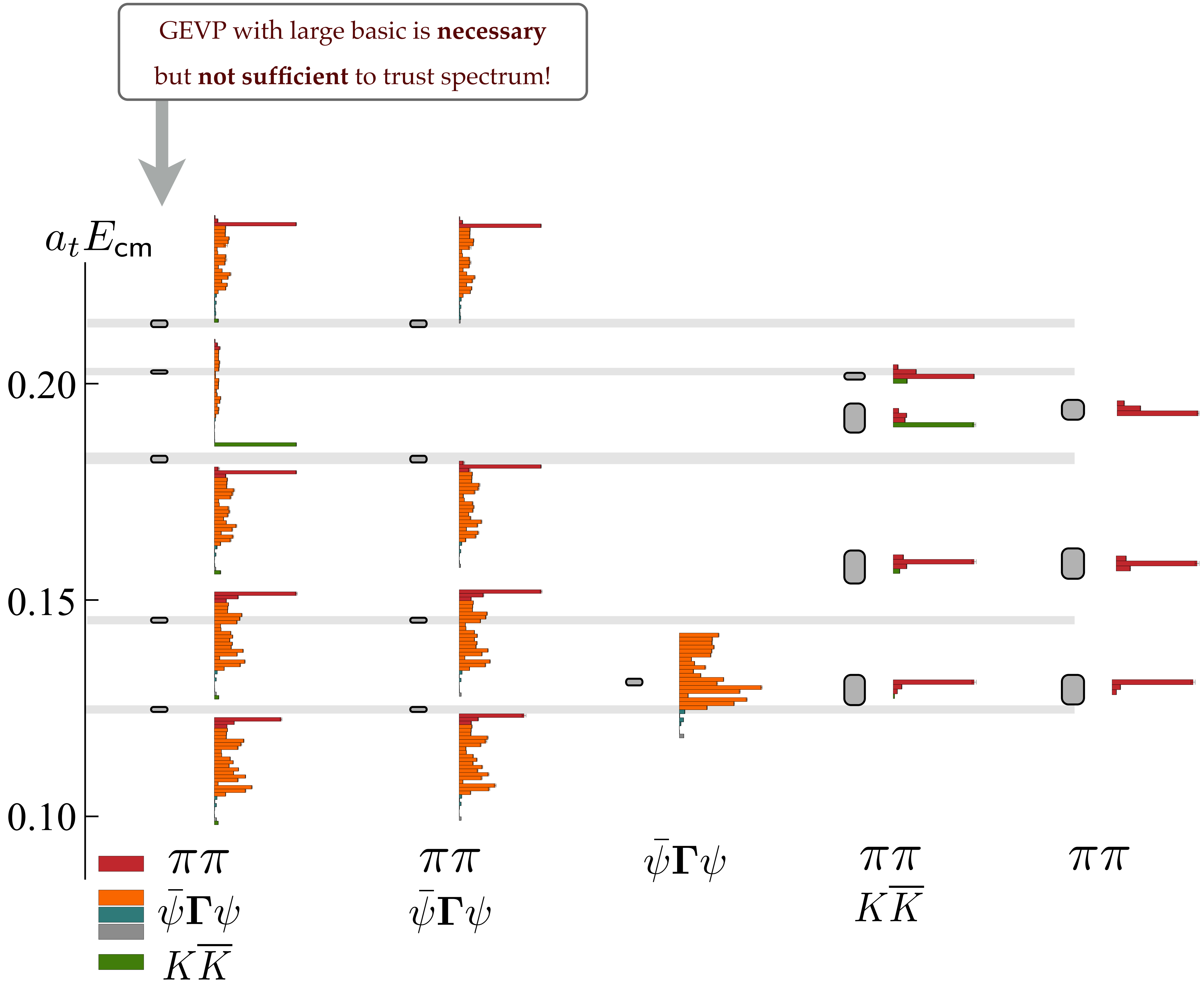
- ☑ Generalized eigenvalue problem (GEVP),
  - ☑ large basis of ops,

    $\mathcal{O}_b \sim \bar{q}\,\Gamma_b\,q, \pi\pi, K\overline{K}, \ldots,$
  - ☑ diagonalization,

    $C^{2pt.}_{ab}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}^\dagger_a(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n}Z^*_{a,n}e^{-E_n t}$

$C(t)\vec{v}^{(n)}(t,t_0) = \lambda_n(t,t_0)C(t_0)\vec{v}^{(n)}(t,t_0)$

- ☑ Finite-volume formalisms.

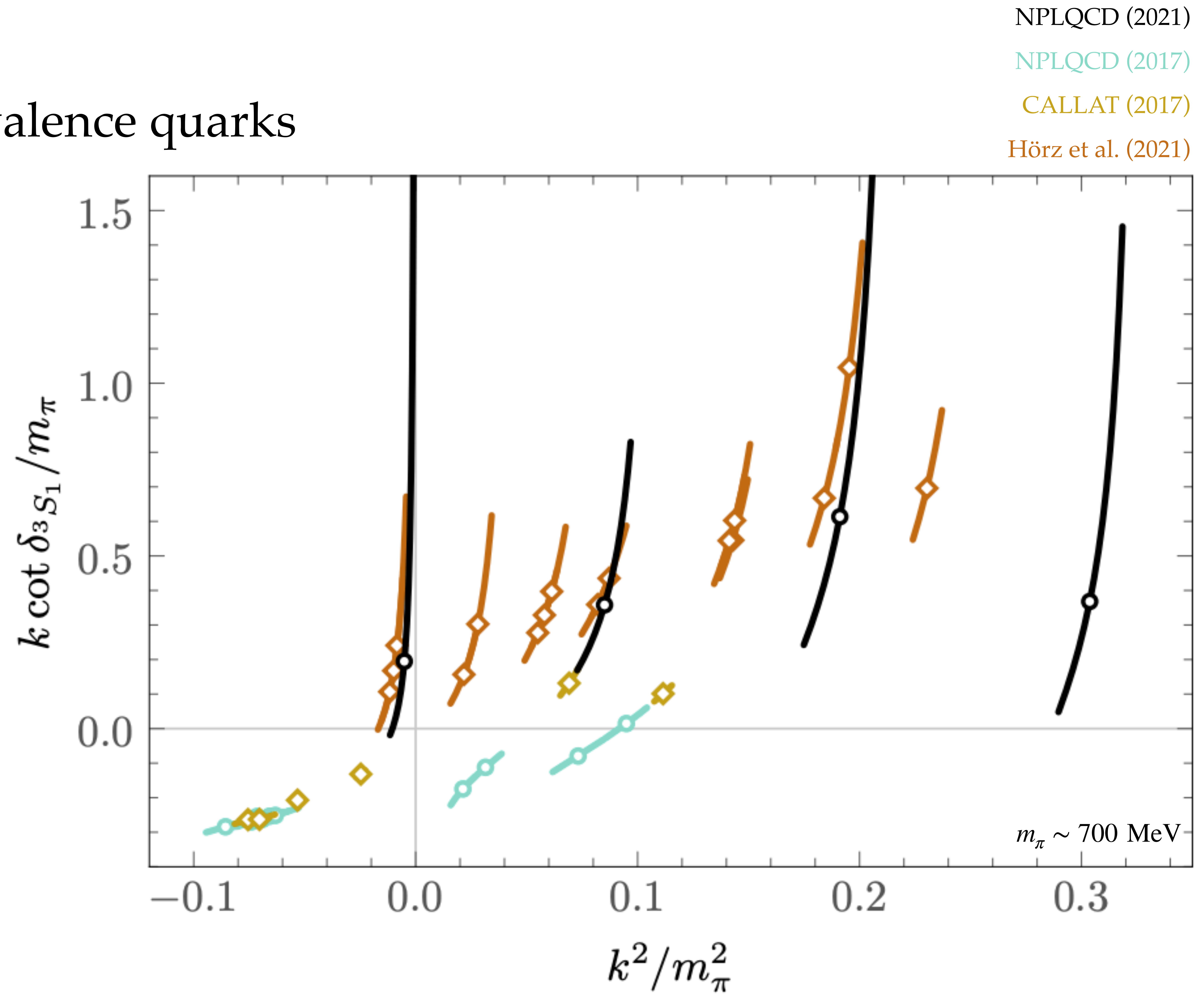


Wilson, RB, Dudek, Edwards, & Thomas (2015)

Why no-nucleons?

Nucleons introduce many challenges

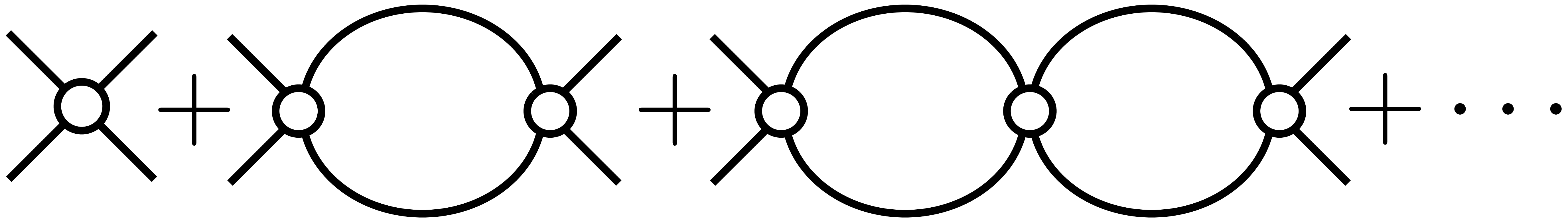
- ☑ More valence quarks
  - ☑ Computational cost grows with number of valence quarks
- ☑ Noise grows with baryon number
  - ☑ more statistics needed
- ☑ Noise grows for lighter quark masses
- ☑ **Hard…not impossible**



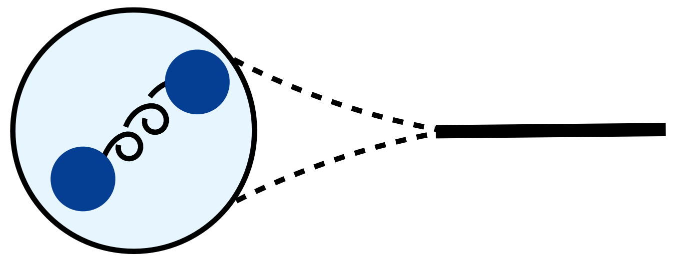
Two-hadron scattering

Unitarity using all orders perturbation theory:

$i\mathcal{M} =$



IR limit of QCD, only interested in hadronic d.o.f.



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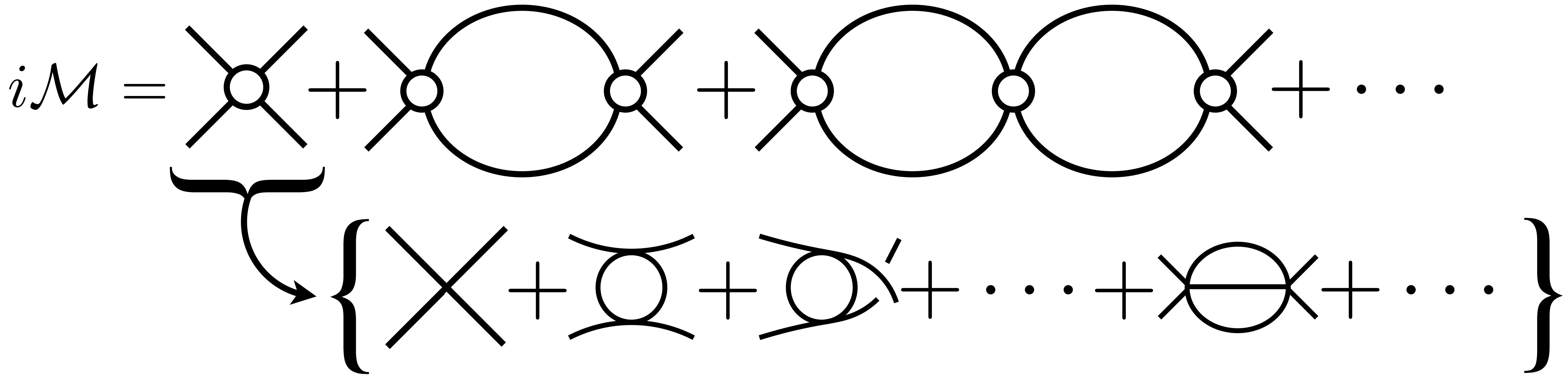

non-perturbative kernel including all diagrams not shown…

"yep, the left hand cut is there"

Two-hadron scattering

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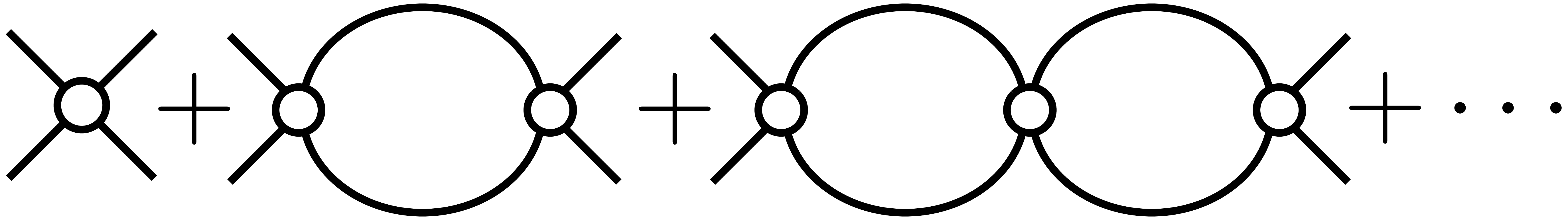
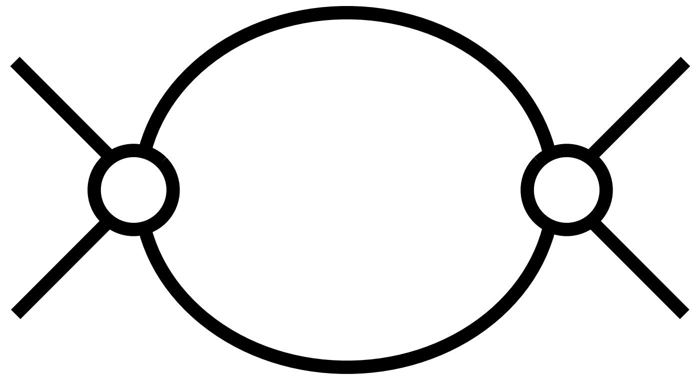


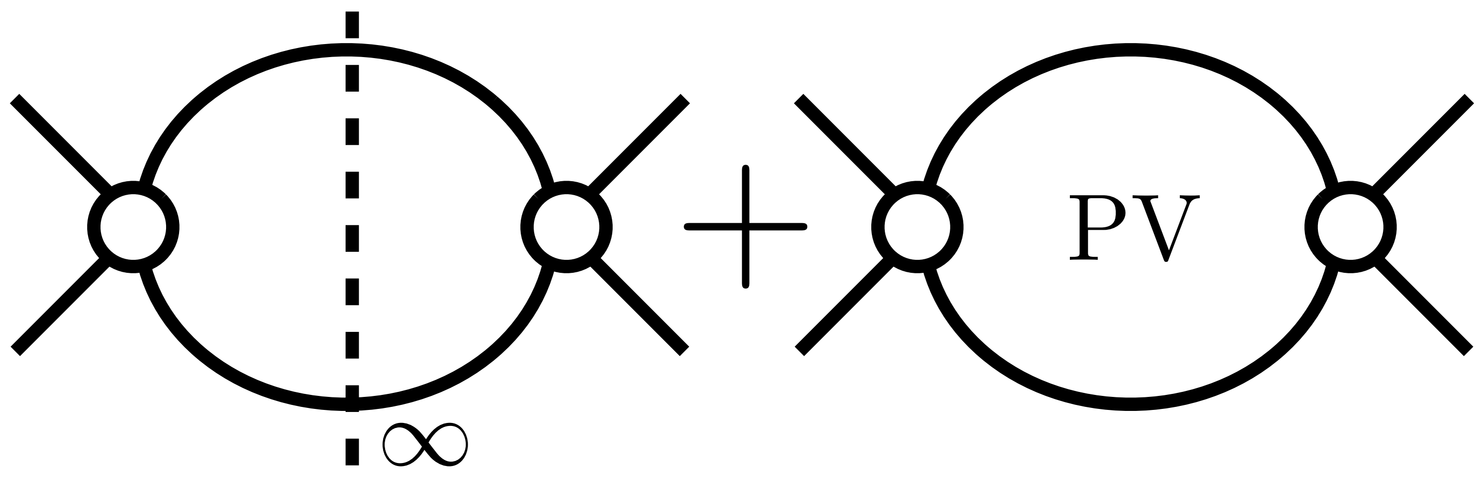


$$= \int \frac{d^4k}{(2\pi)^4}[iB(k,P)]^2 \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(P-k)^2 - m^2 + i\epsilon}$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{[iB(k,P)]^2}{(2\omega_k)^2} \pi\delta(E - 2\omega_k) + \text{“PV integral”}$$

$$= [iB_{on}]\rho\,[iB_{on}] + \text{“PV integral”}$$

$=$



$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity.

Two-hadron scattering

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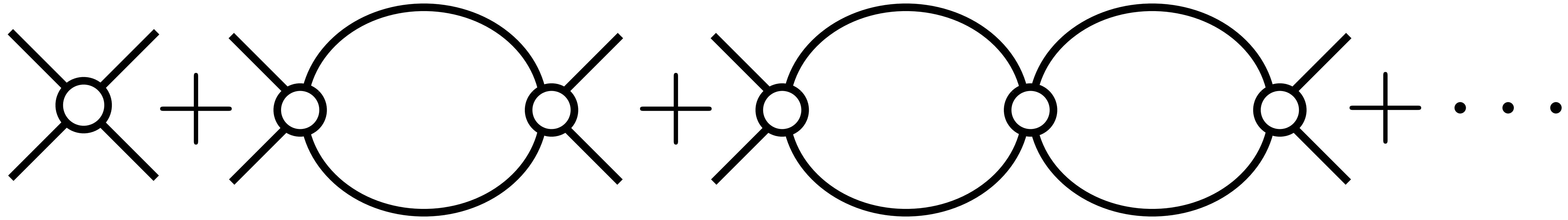
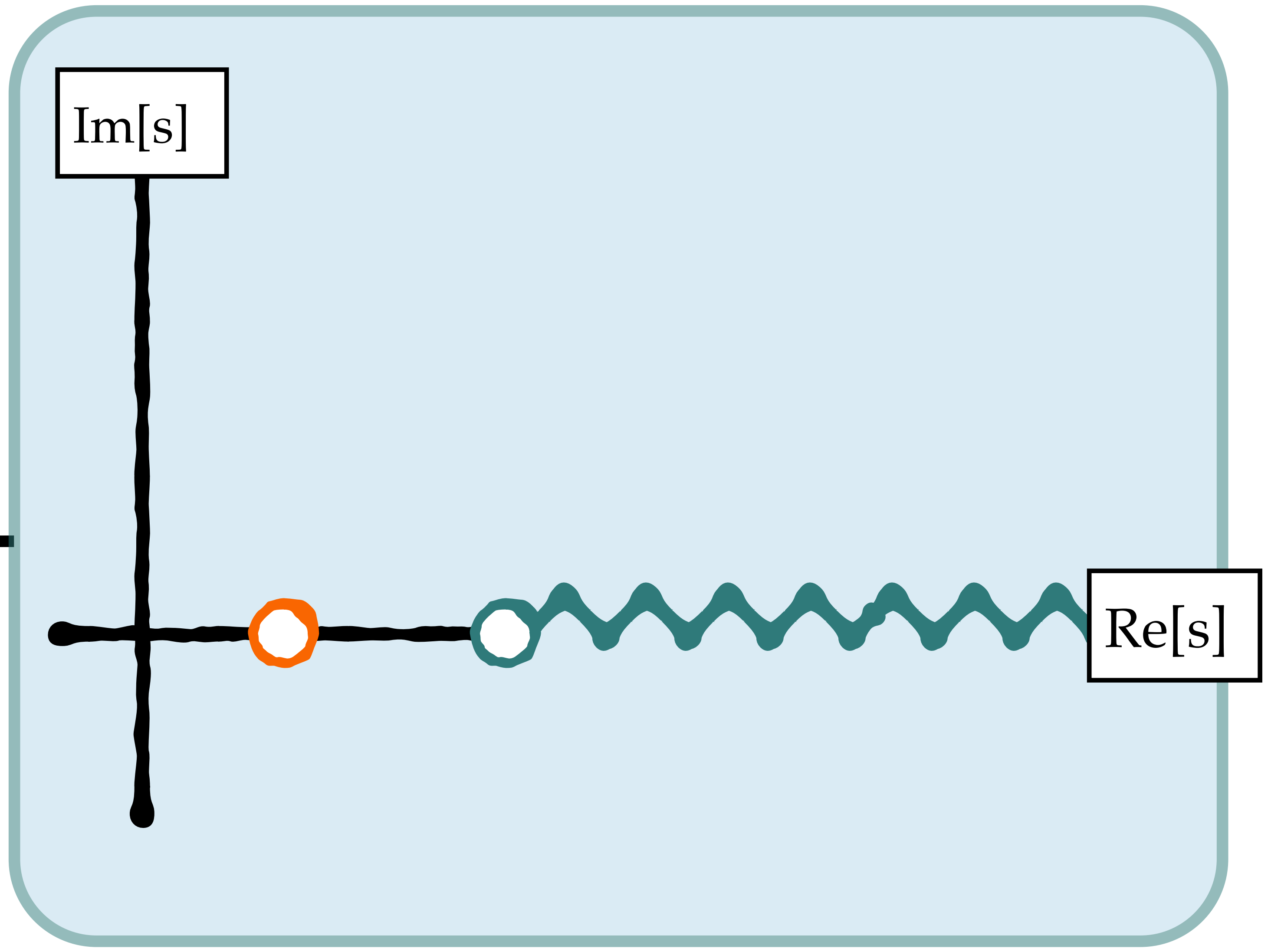
$i\mathcal{M} =$ 



2^N sheets for N open channels

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Two-hadron scattering

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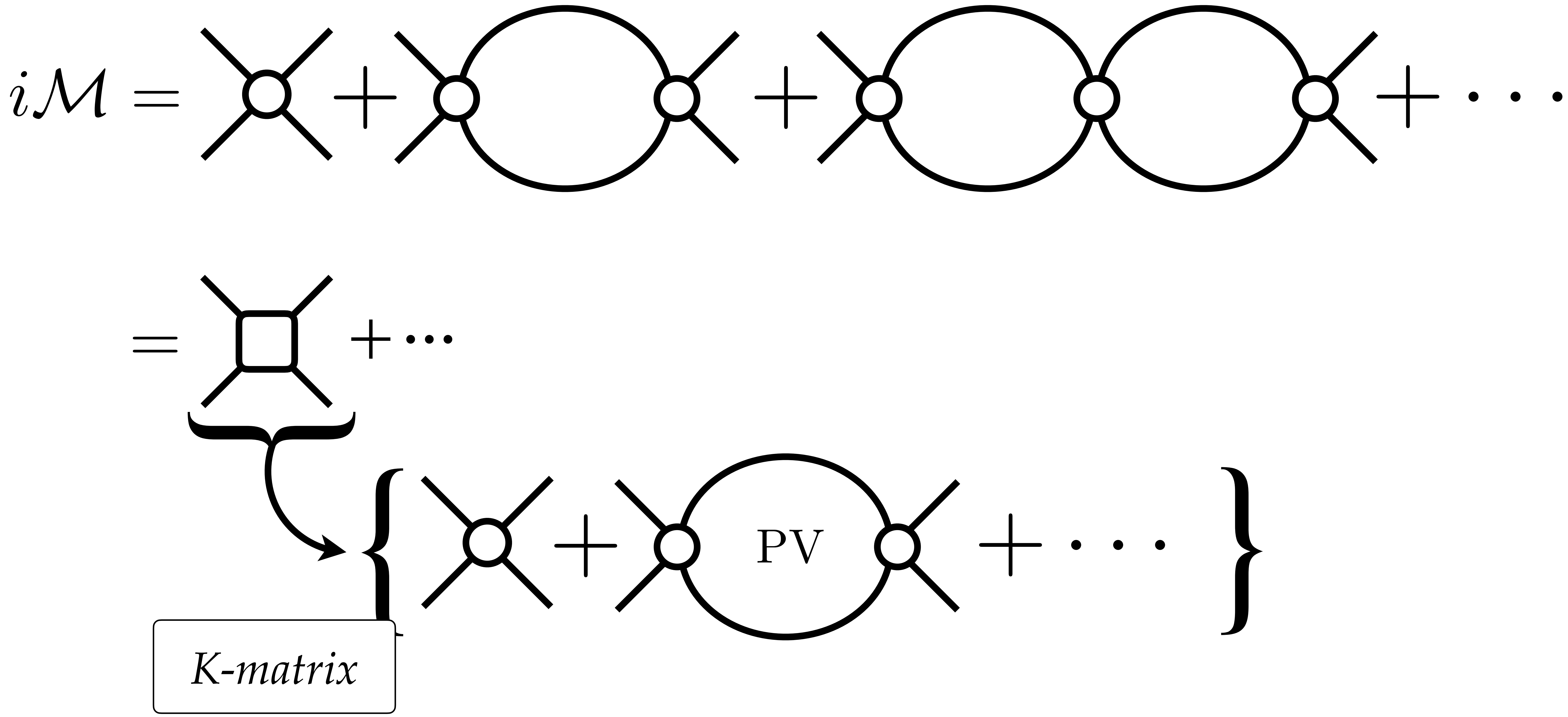

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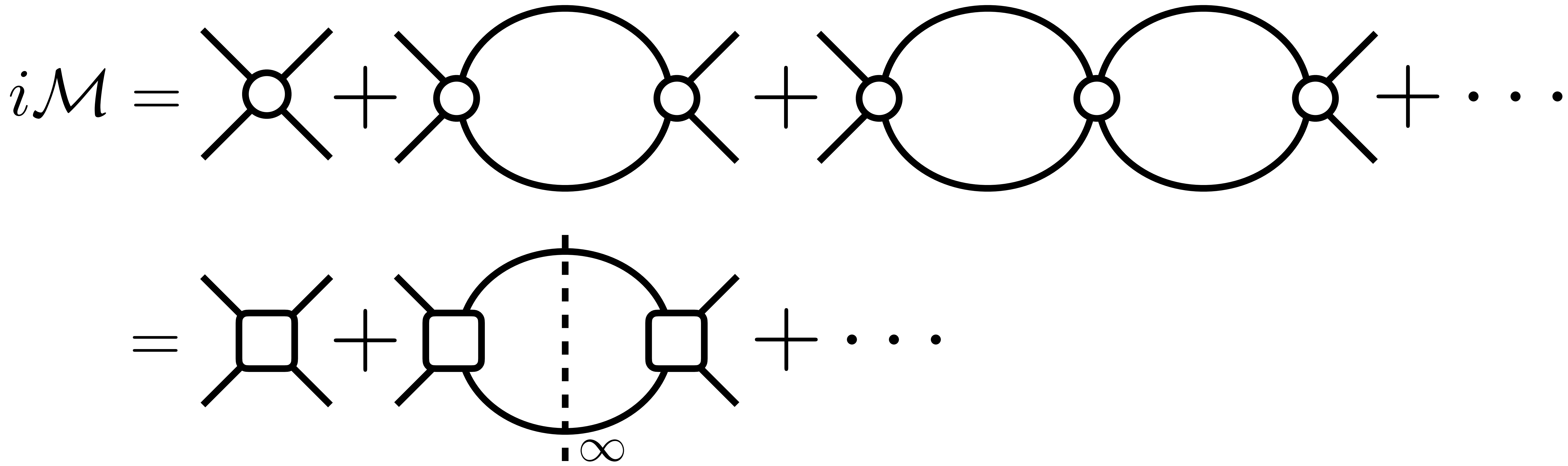

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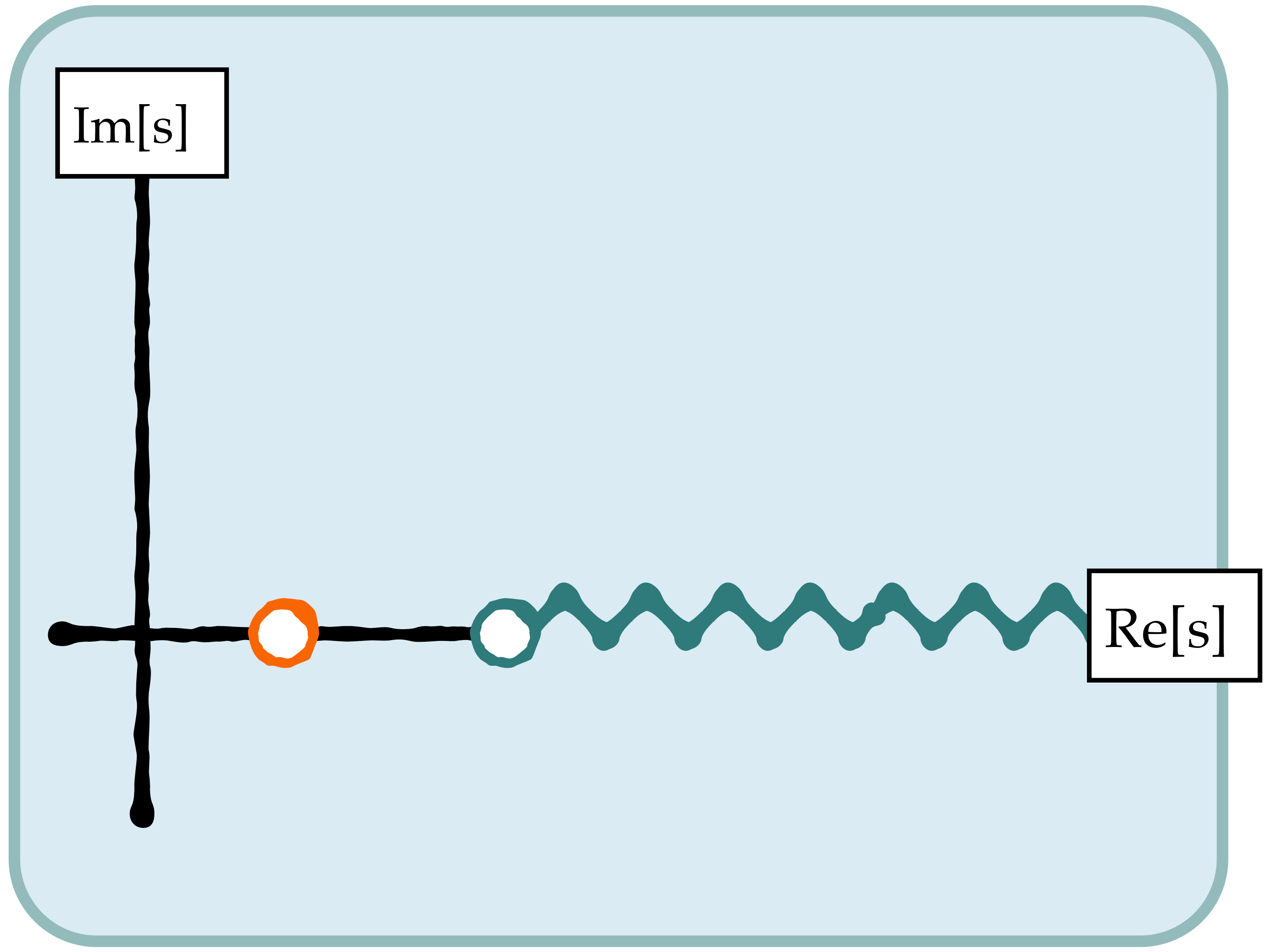

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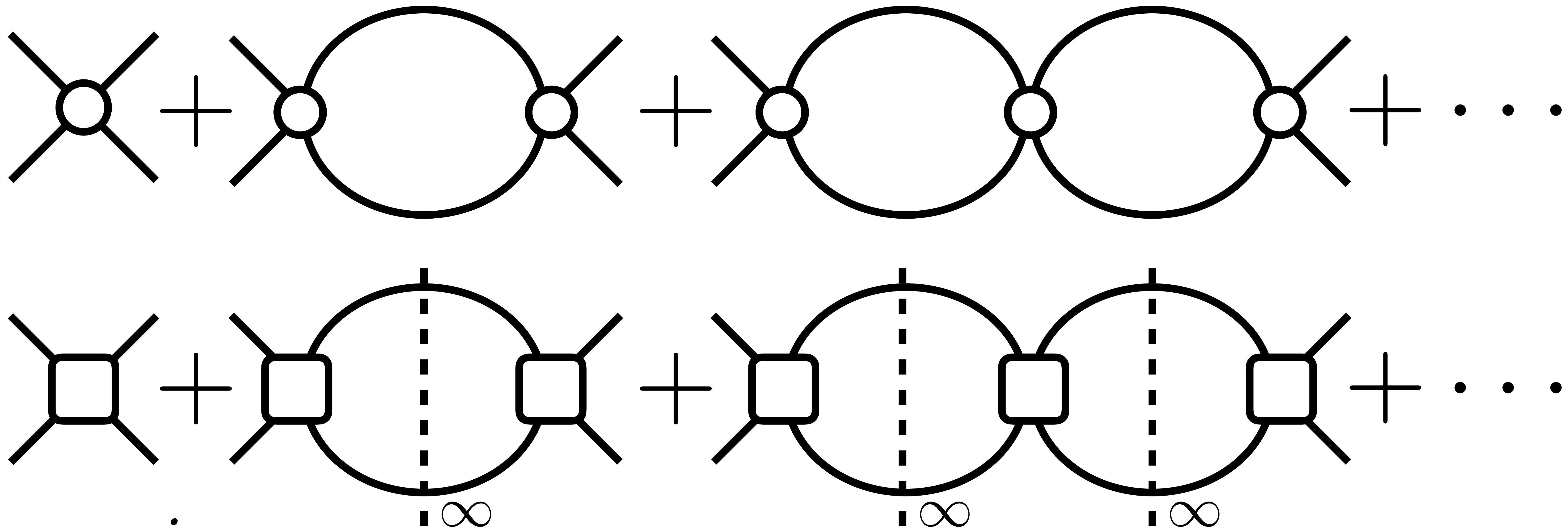
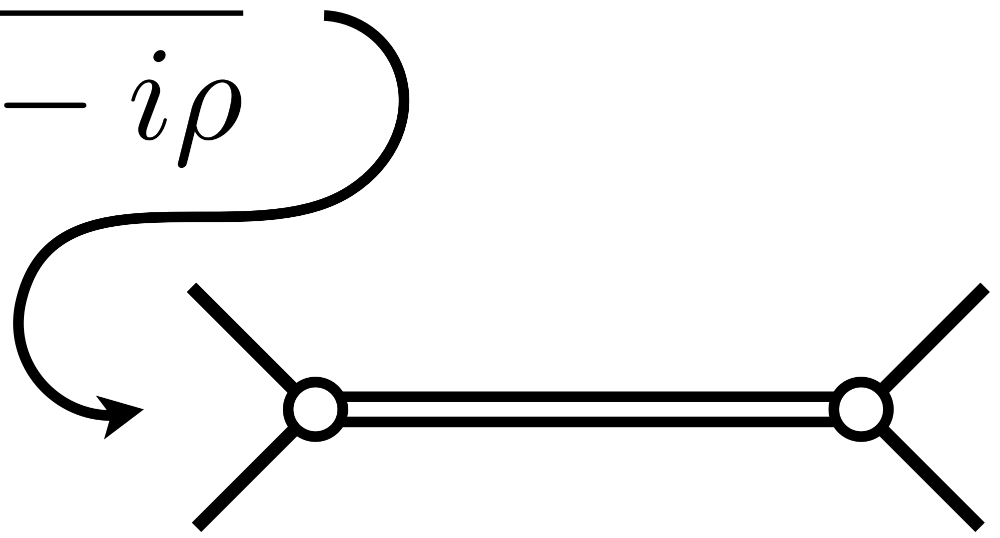
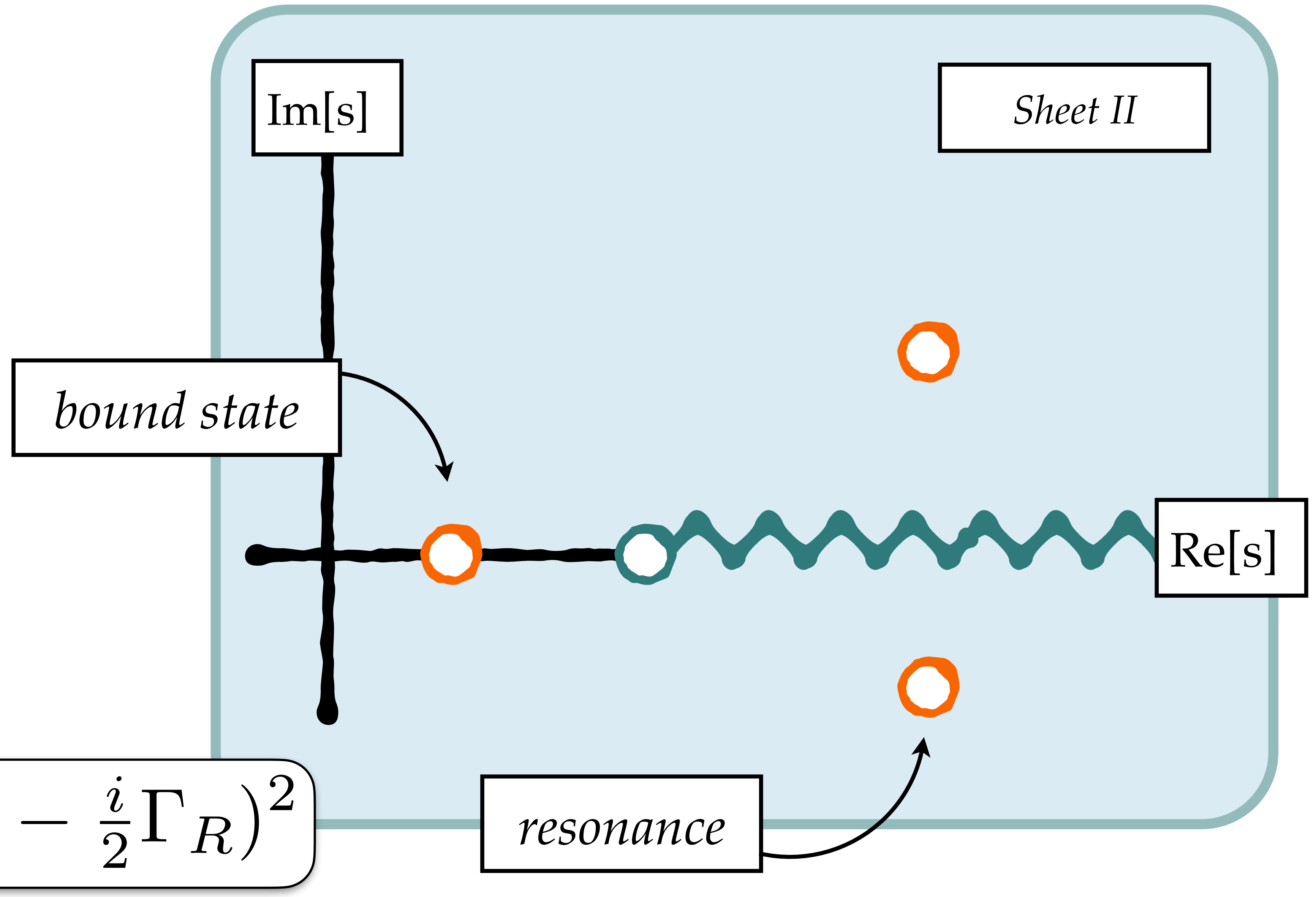


$$= \frac{i}{\mathcal{K}^{-1} - i\rho}$$





$$s_R = \left(E_R - \tfrac{i}{2}\Gamma_R\right)^2$$

Two-hadron scattering

Unitarity using all orders perturbation theory:

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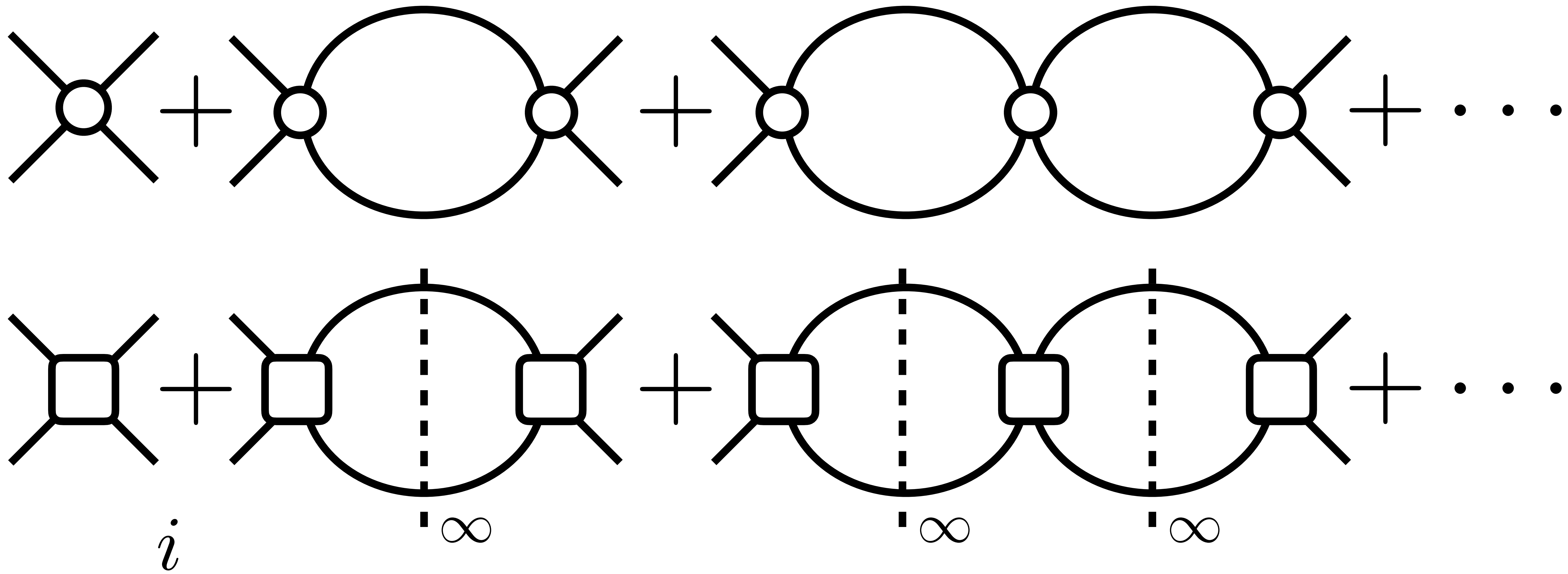


$$= \frac{i}{\mathcal{K}^{-1} - i\rho}$$

Equating this to the elastic S matrix…

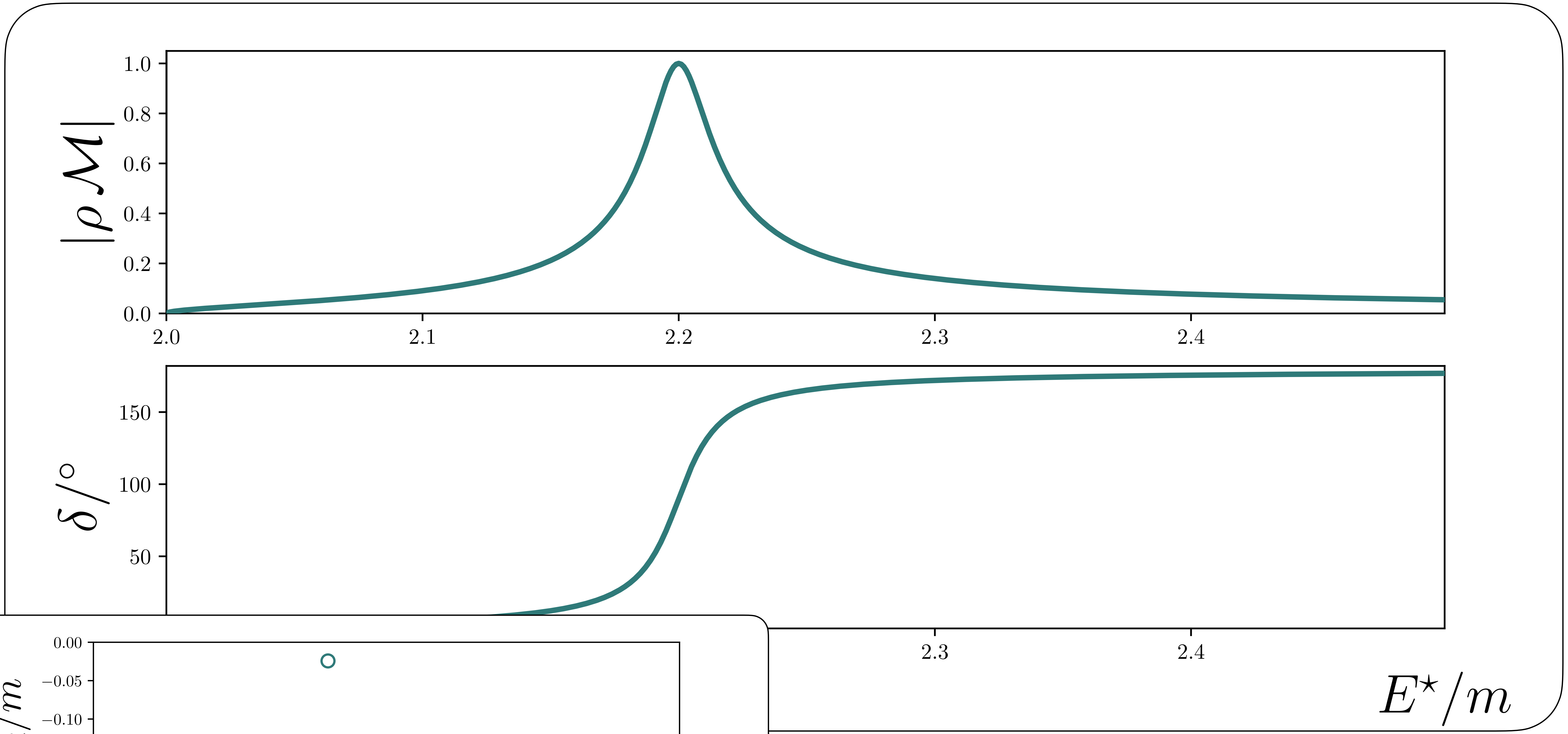
$$S = e^{2i\delta} = 1 + 2i\rho\mathcal{M}$$

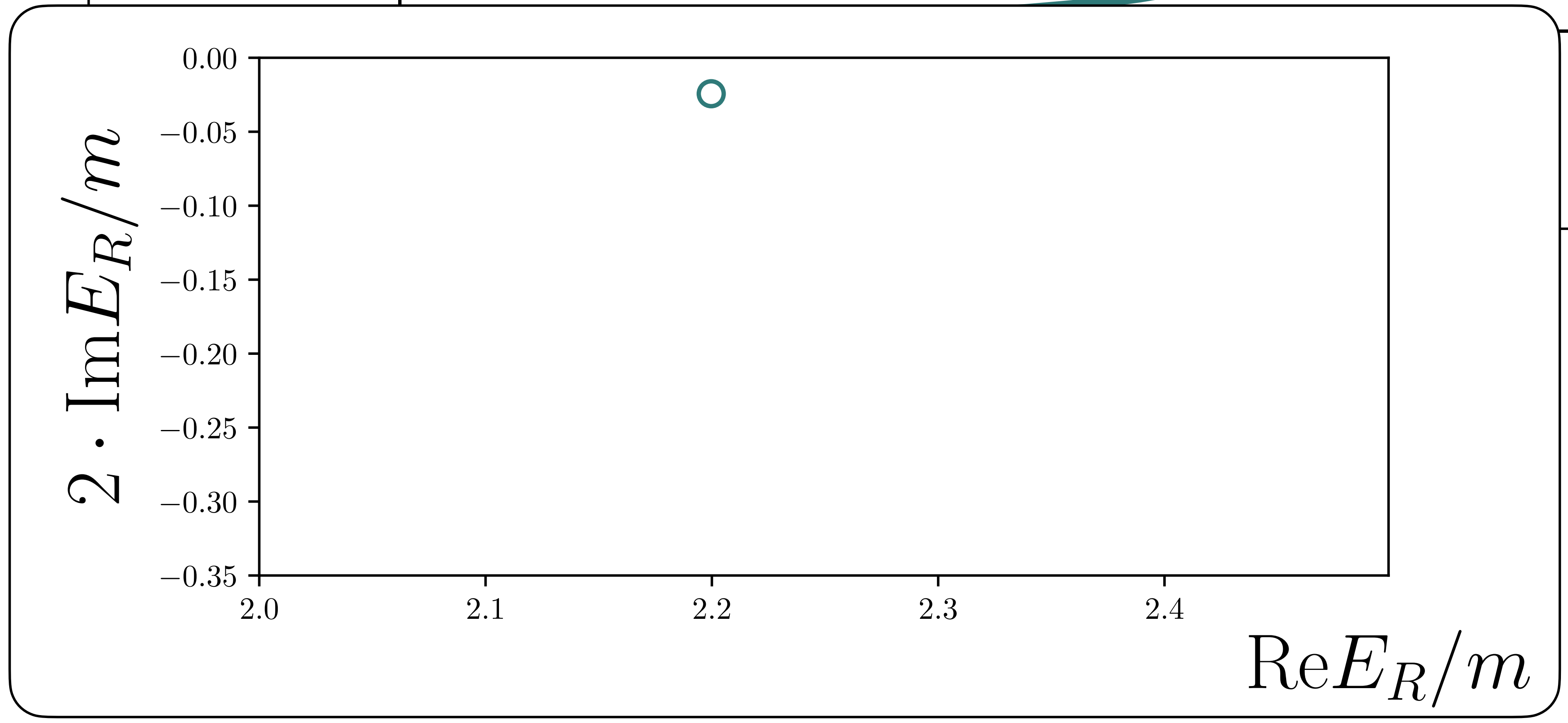
$$\mathcal{K}^{-1} = \rho \cot\delta$$

$$\mathcal{M} = \frac{\sin\delta}{\rho} e^{i\delta}$$

Resonances

To build some intuition: $\mathcal{M} = \dfrac{\sin\delta}{\rho} e^{i\delta}$





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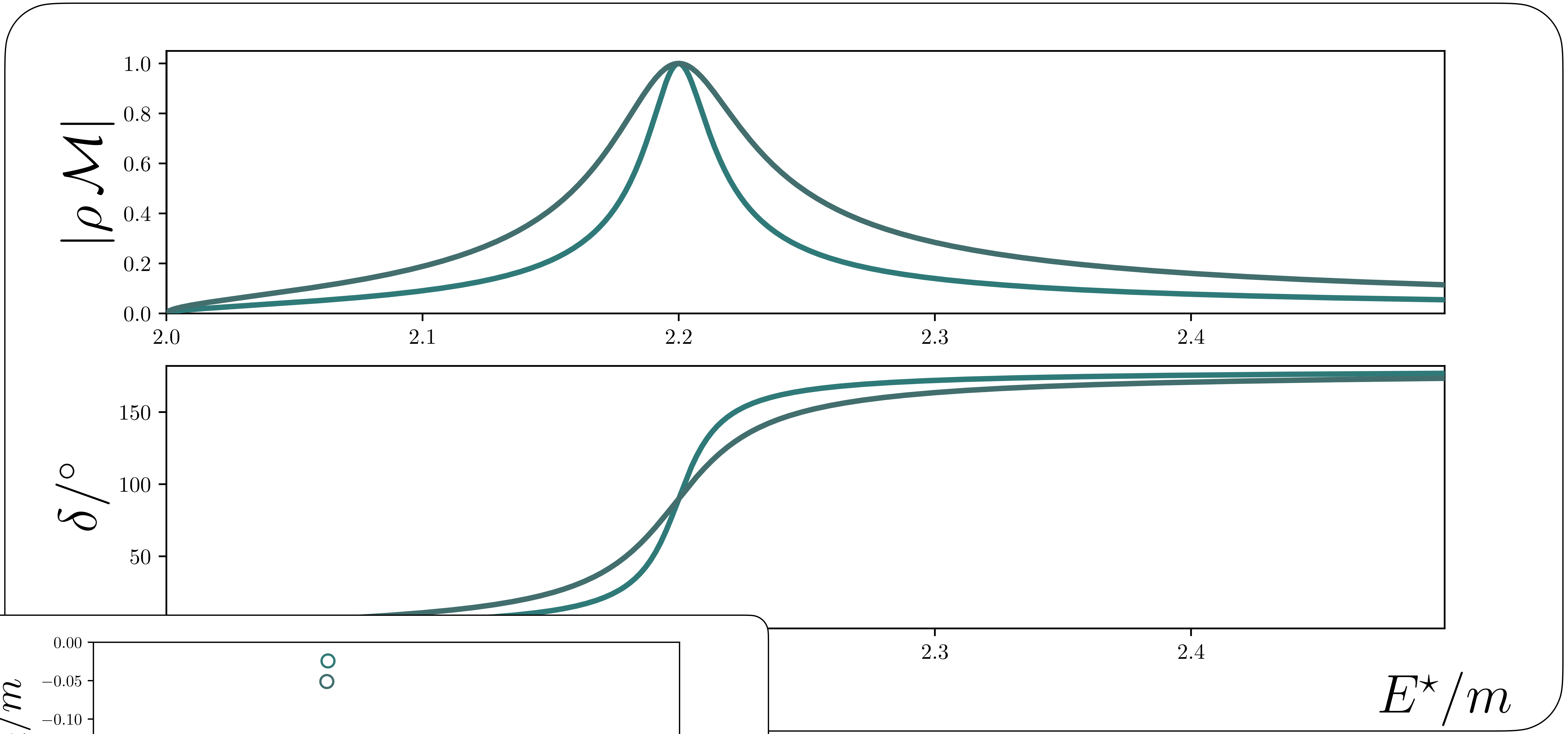

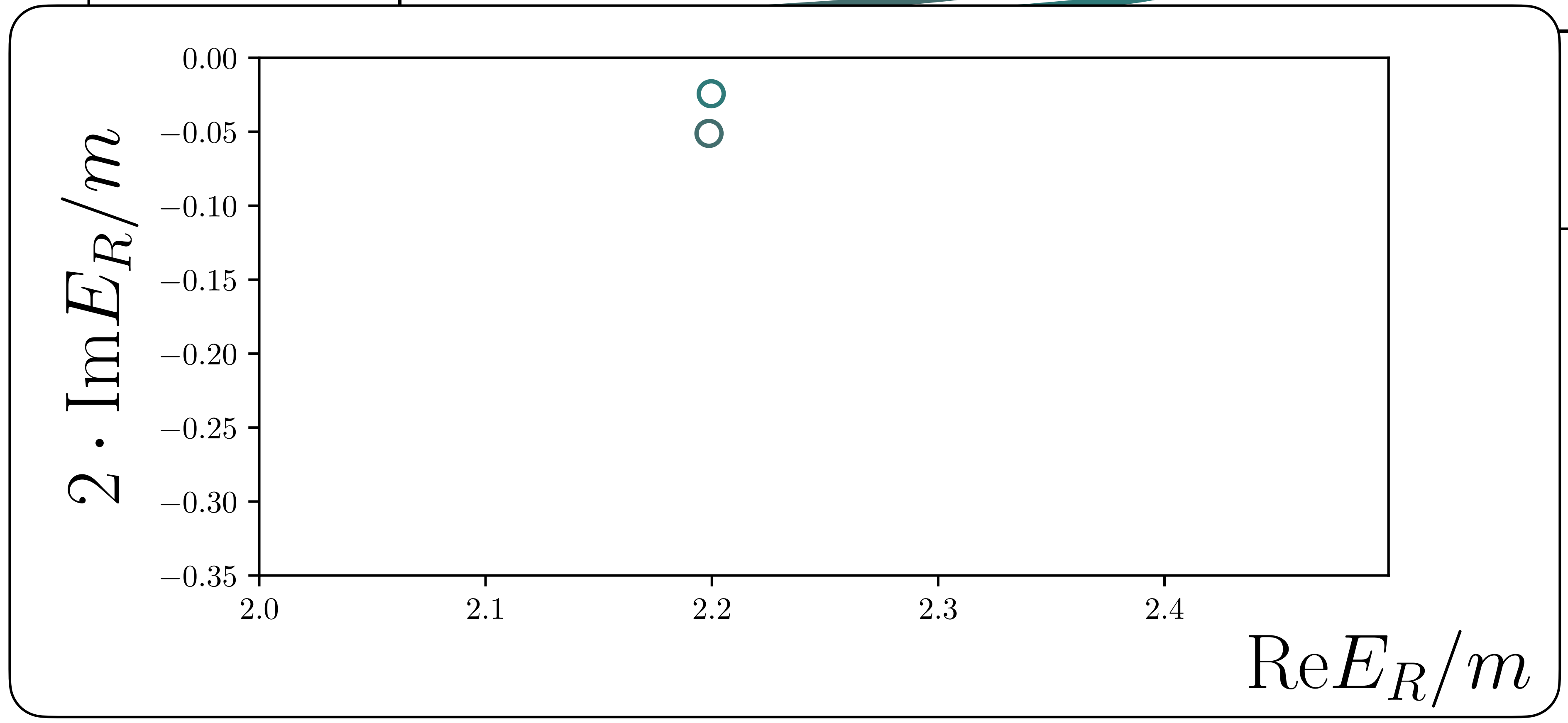

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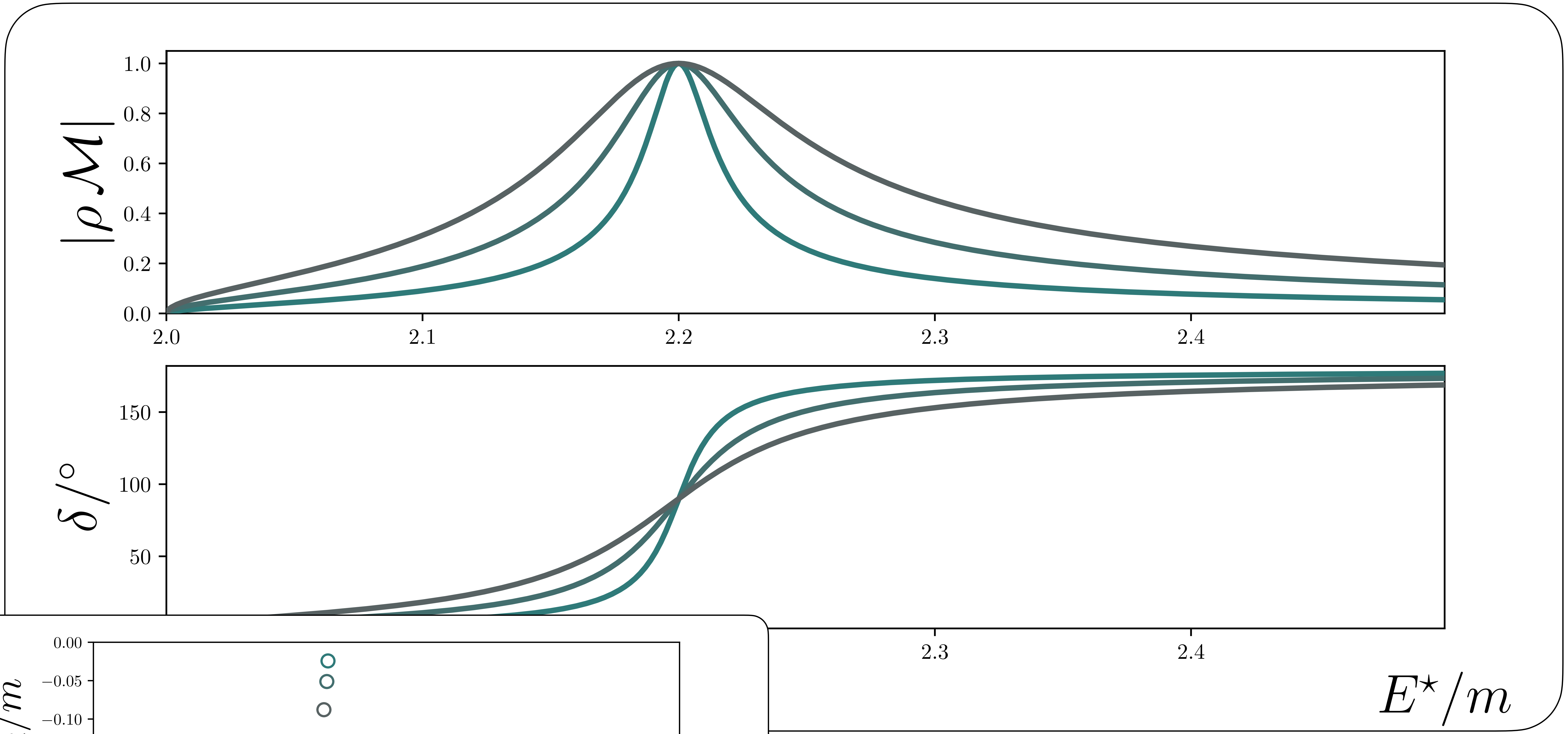

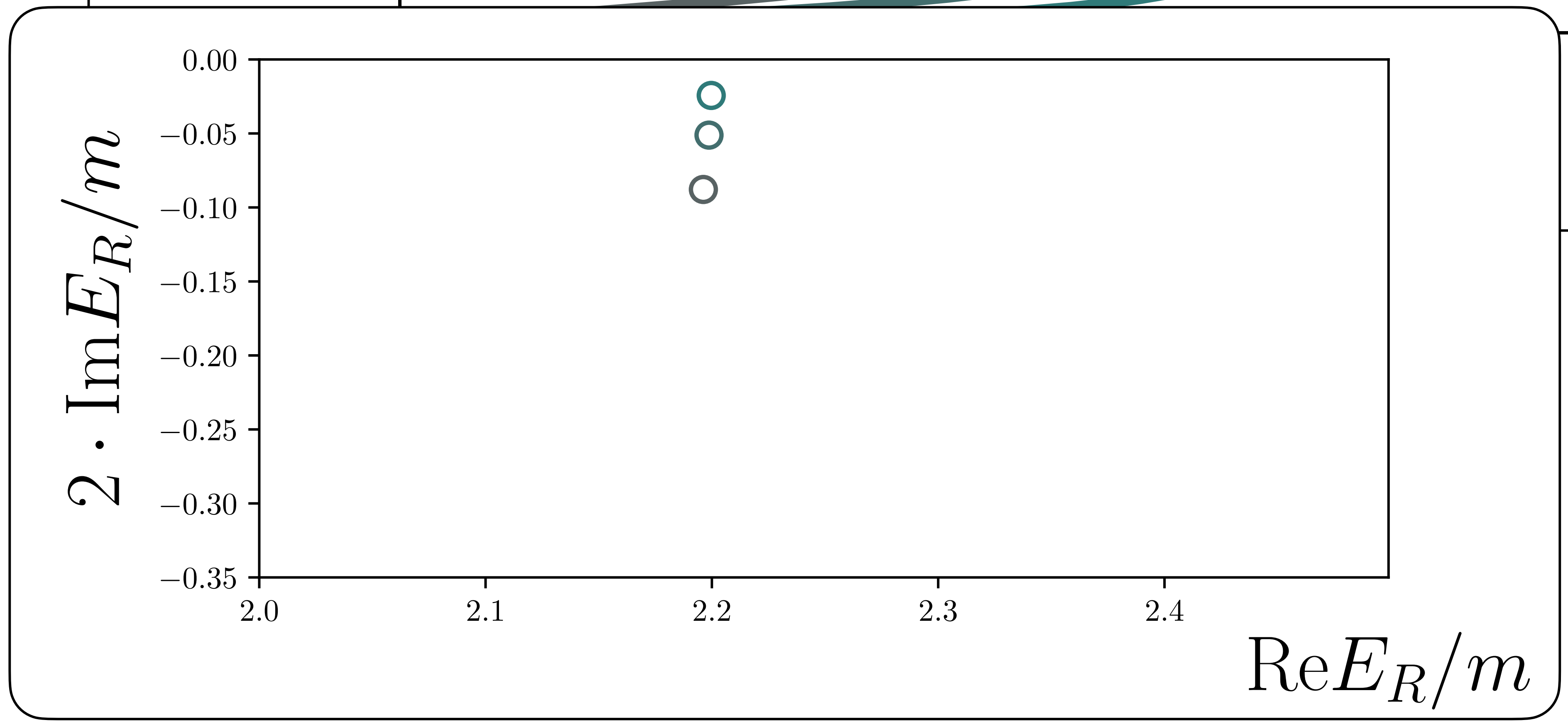

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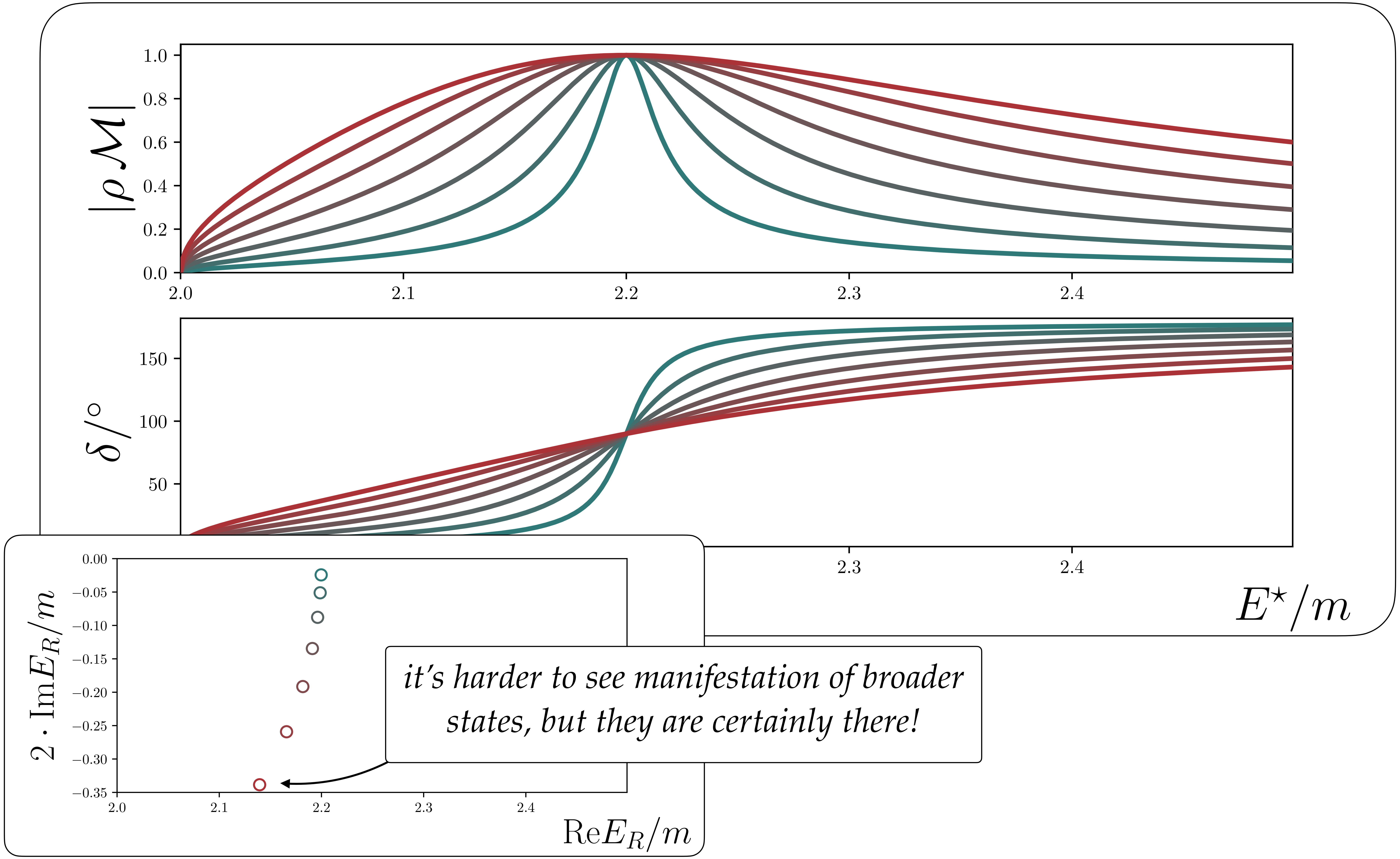
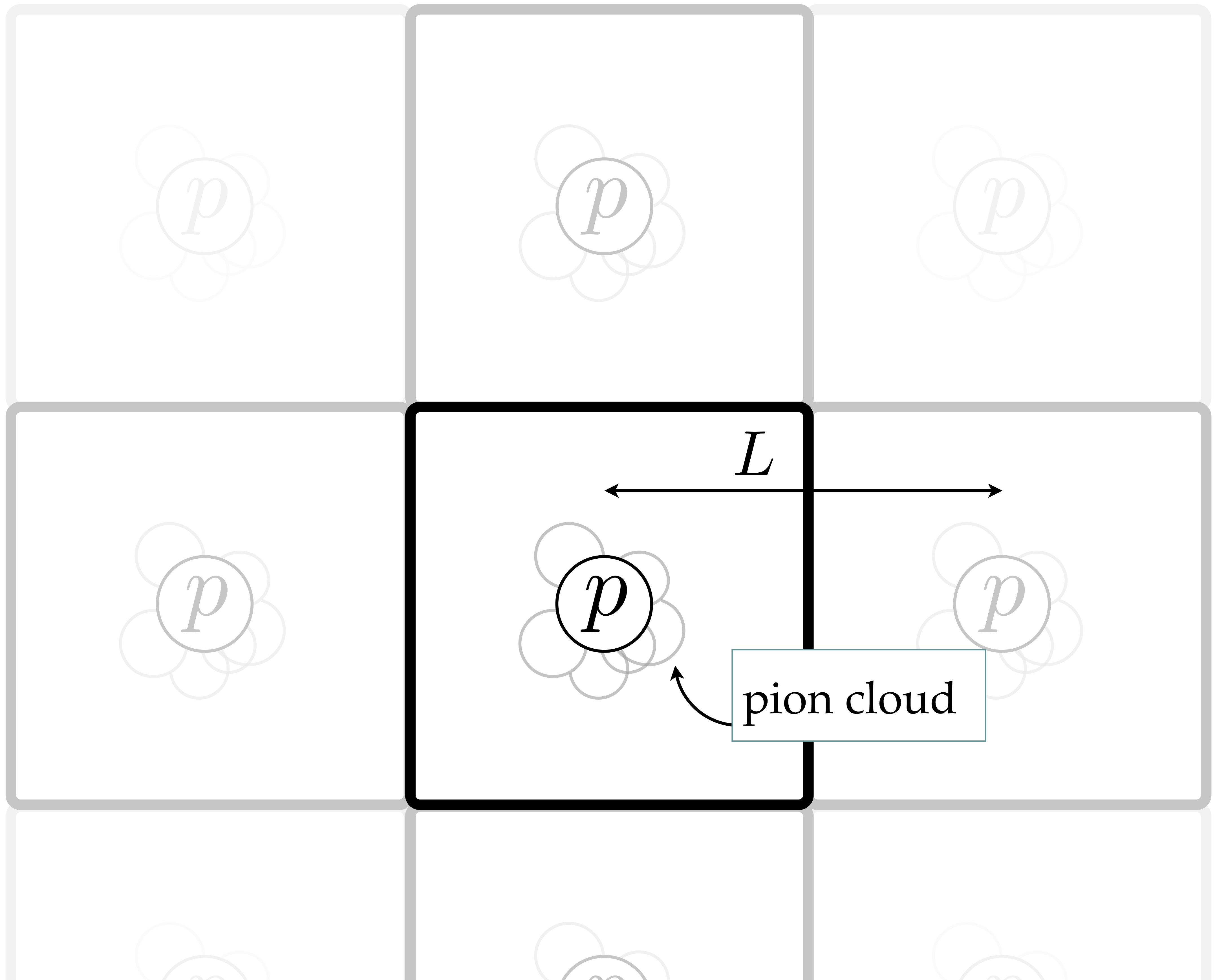


One particle in finite volume

- Stable hadron size ~ $\mathcal{O}(1/m_\pi)$
- If $L \gg m_\pi^{-1}$, finite-volume errors are suppressed

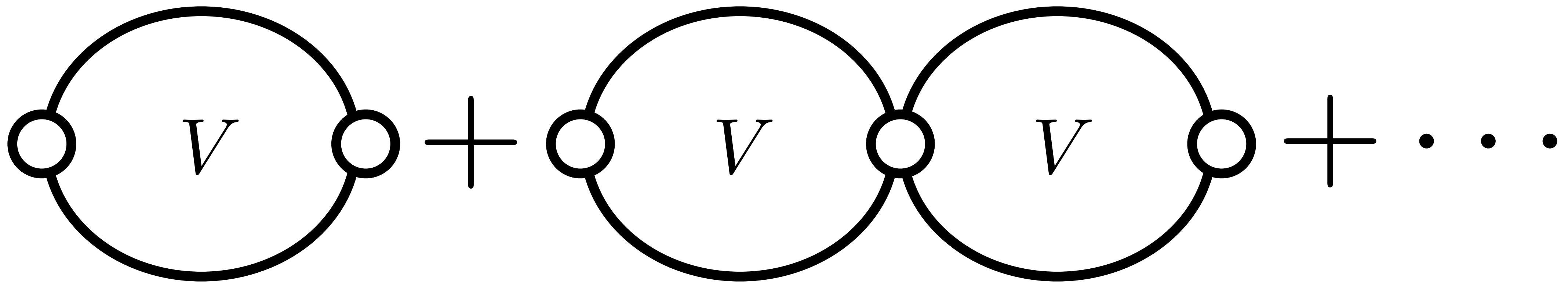


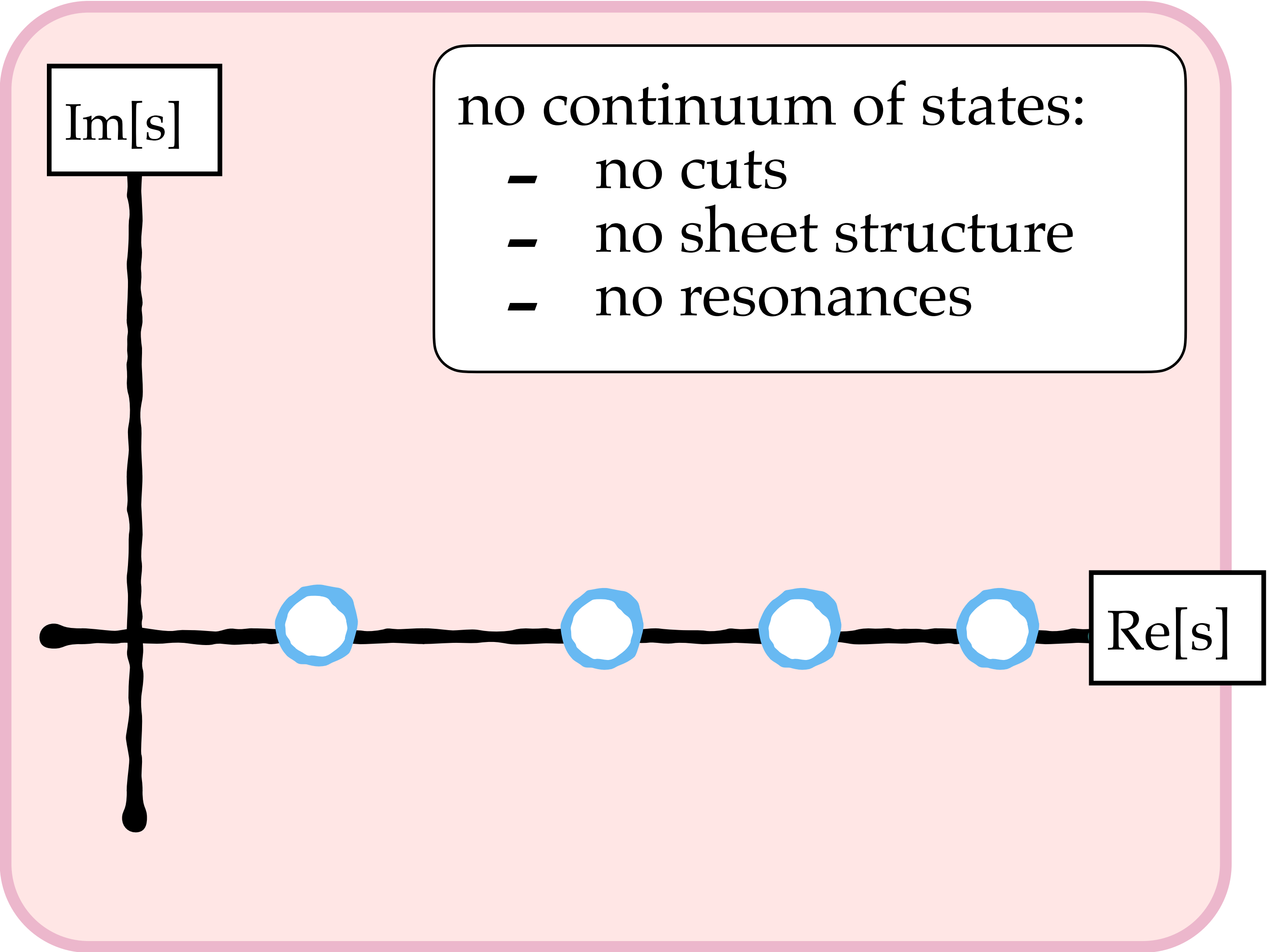
$$m_h(L) = m_h(\infty) + \mathcal{O}(e^{-m_\pi L})$$

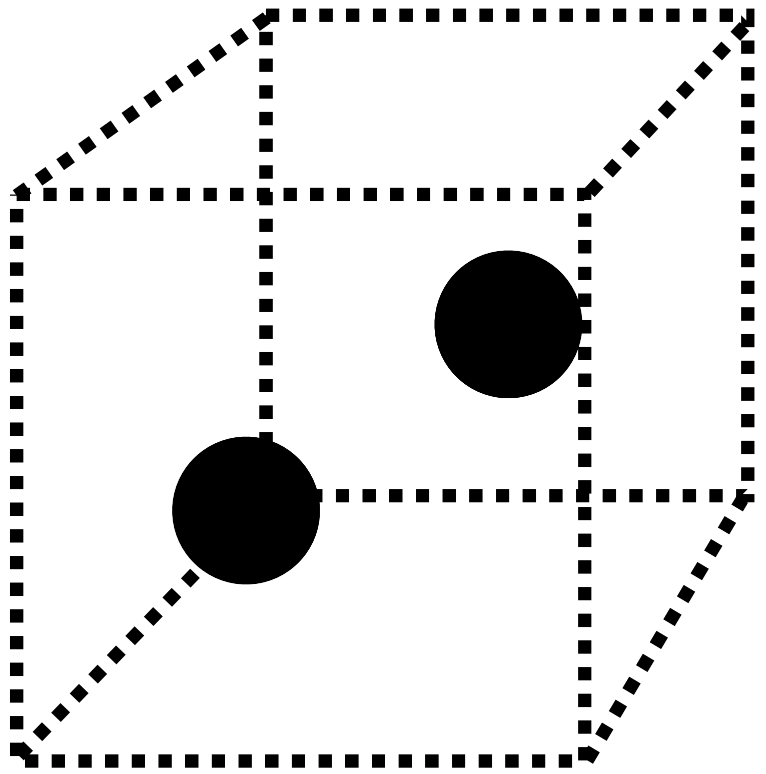
Two particle in finite volume

Consider the finite-volume two-particle correlator (E~$2m$):

$$C_L^{2pt.}(P) =$$



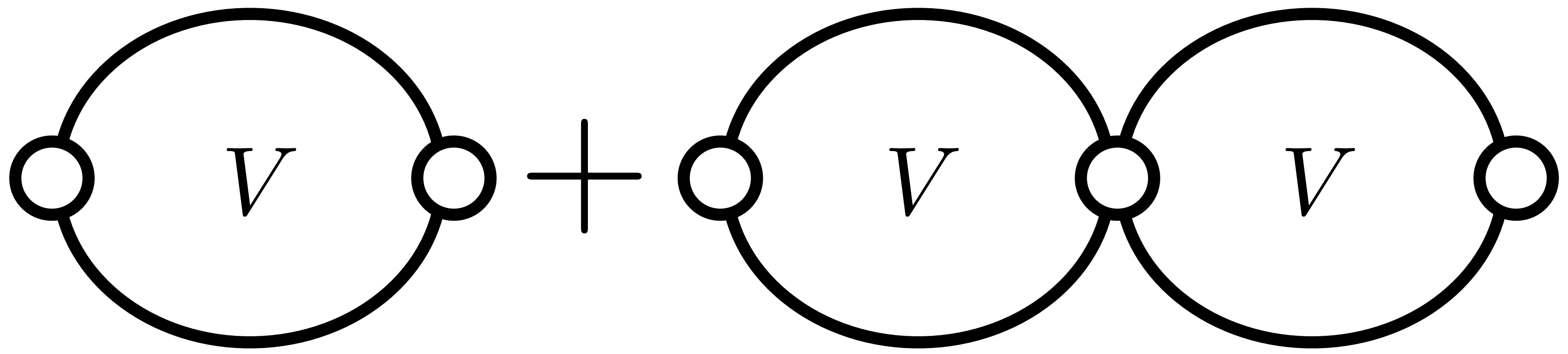




Two particle in finite volume

Consider the finite-volume two-particle correlator (*E~2m):*

$$C_L^{2pt.}(P) =$$



$$+ \cdots$$

$$= C_\infty(P) + \cdots$$

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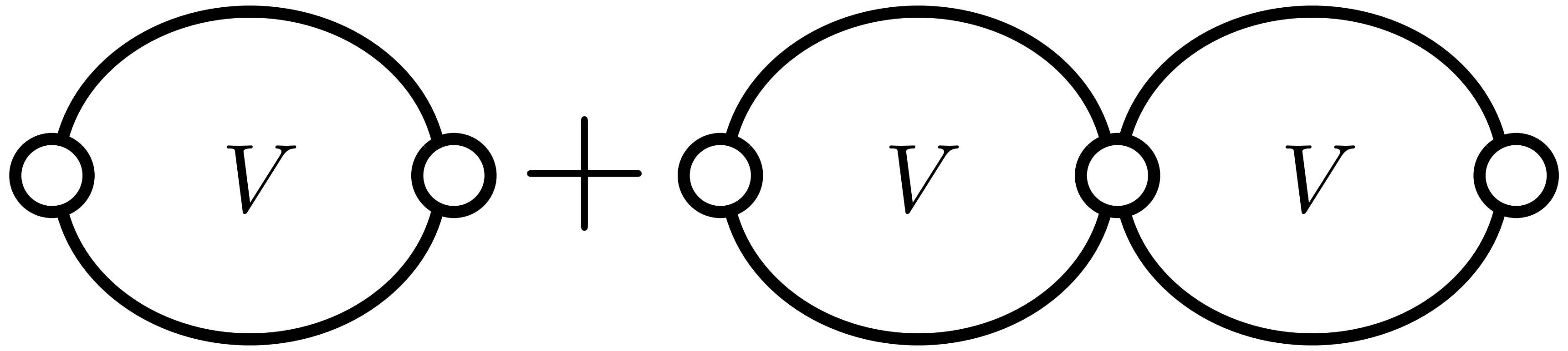
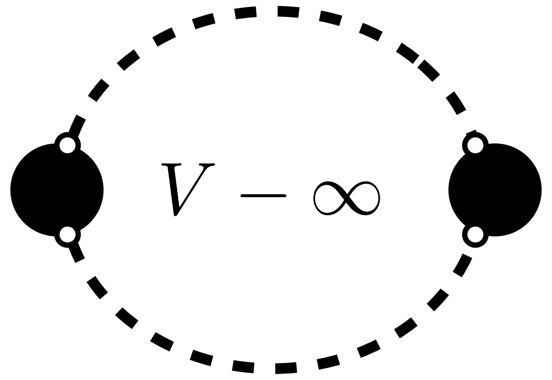
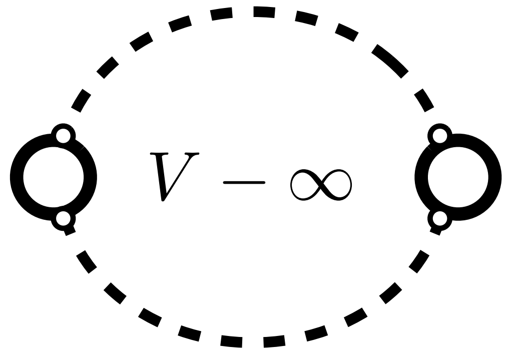
$$C_L^{2pt.}(P) =$$  $$+ \cdots$$

$$= C_\infty(P) +$$  $$+ \cdots$$



$$= (iB)\left(\left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3k}{(2\pi)^3}\right]\frac{1}{(2\omega_k)^2}\frac{i}{E-2\omega_k+i\epsilon}\right)(iB)$$

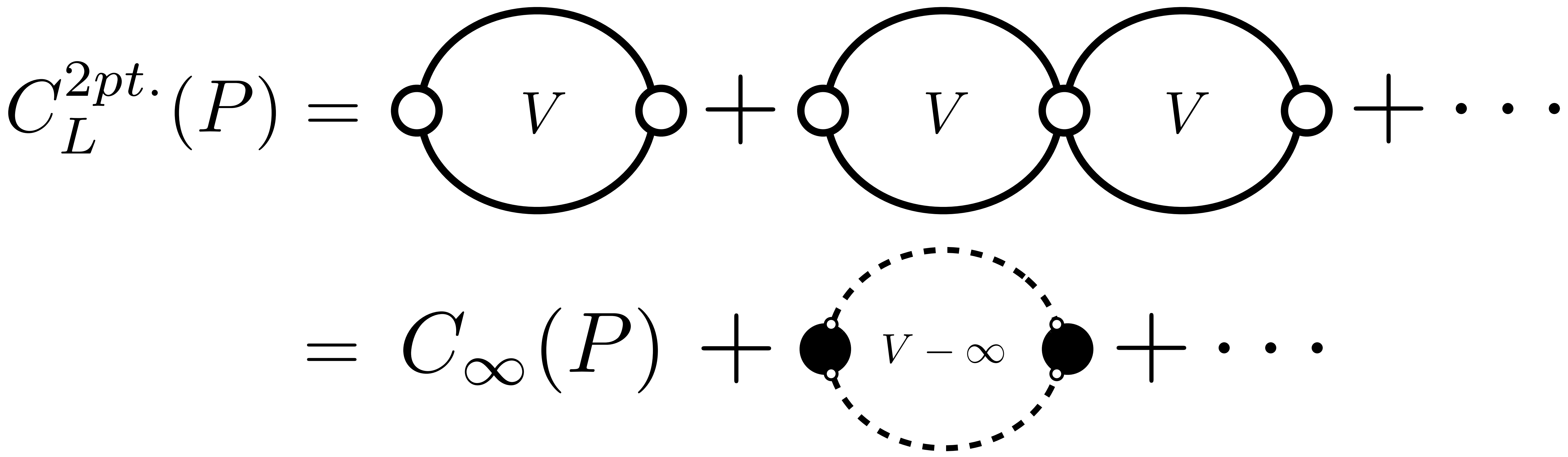
$$\equiv [iB]\, iF\, [iB]$$

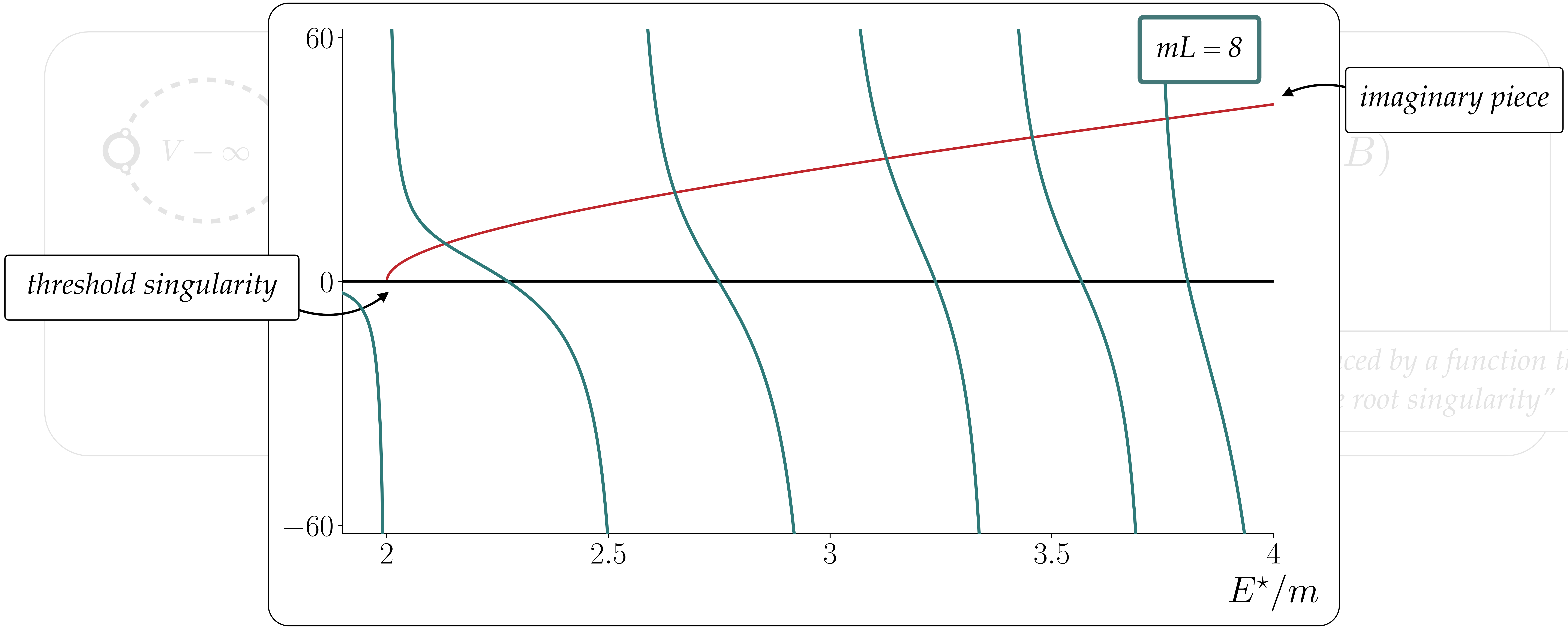
F replaces ρ

a simple square root singularity is replaced by a function that has both simples poles and the square root singularity

Two particle in finite volume

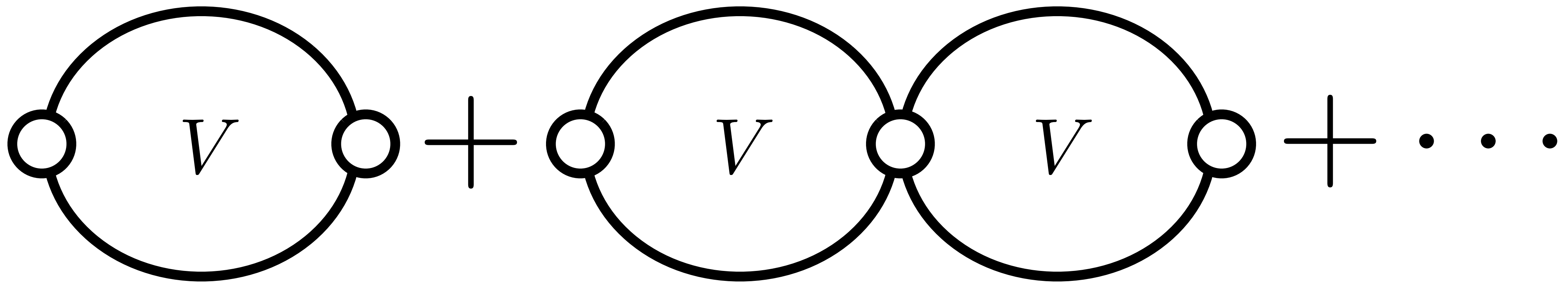
Consider the finite-volume two-particle correlator ($E\sim 2m$):

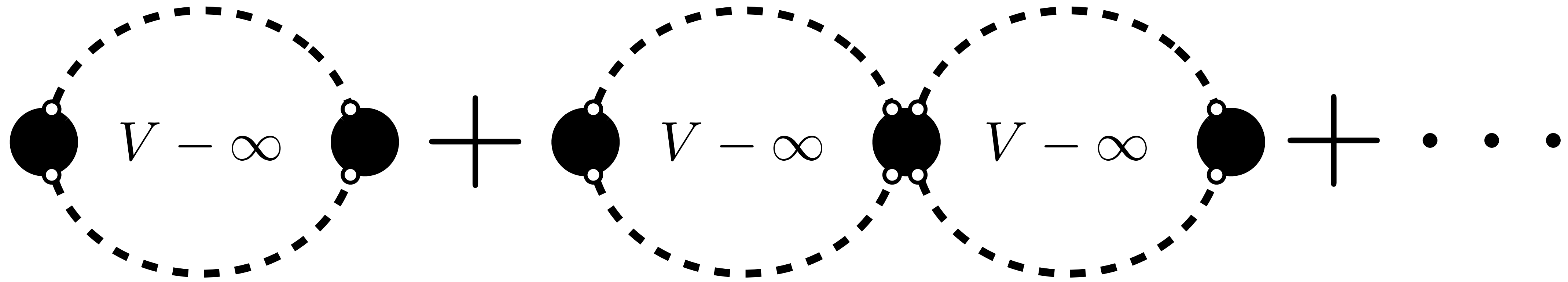




Two particle in finite volume

Consider the finite-volume two-particle correlator (*E*~2*m*):

$C_L^{2pt.}(P) =$ 

$= C_\infty(P) +$ 

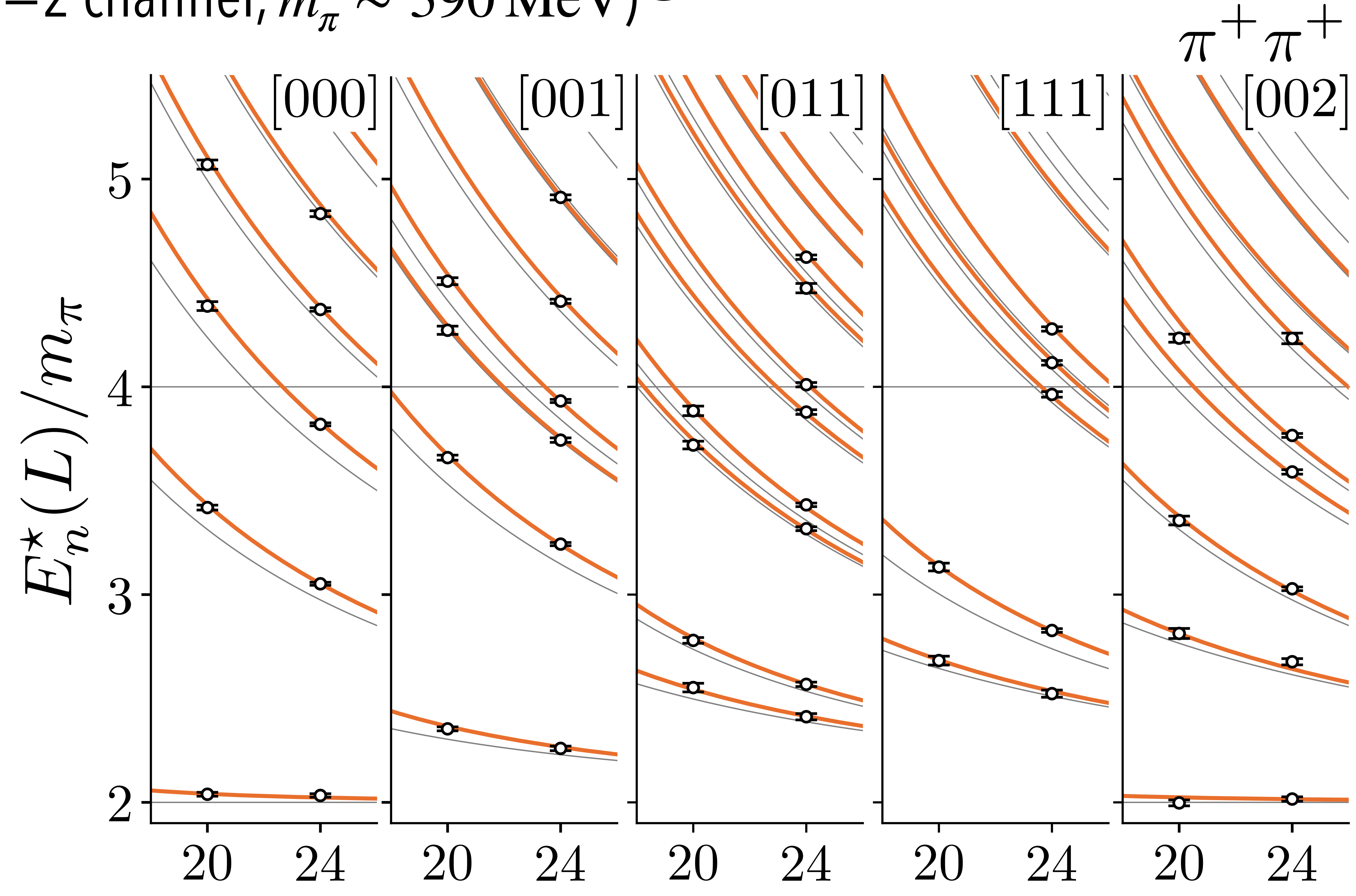
$$= \text{“smooth”} + A\frac{i}{F^{-1} + \mathcal{M}}B^\dagger$$

poles satisfy: $\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$

- Lüscher (1986, 1991)
- Rummukainen & Gottlieb (1995)
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005)
- Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)
- RB (2014)

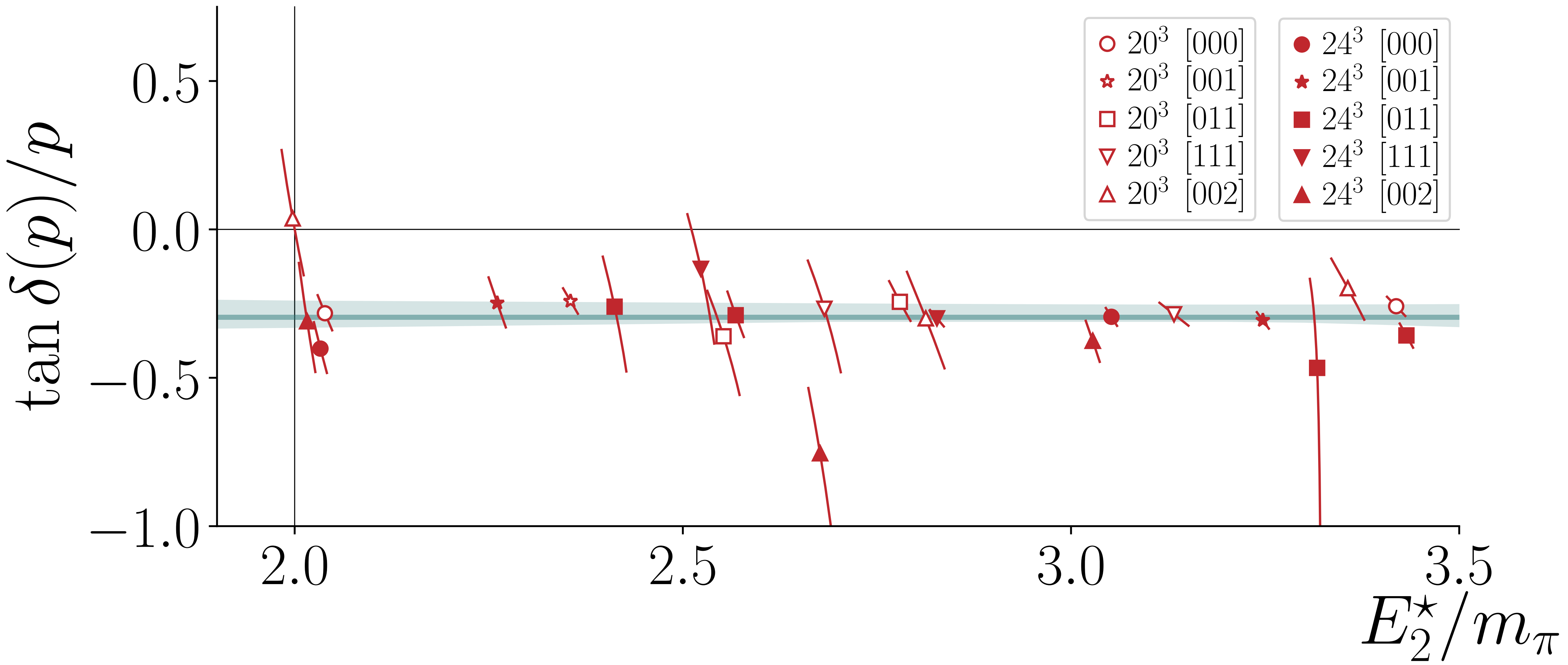
$\pi\pi$ scattering

(I=2 channel, $m_\pi \sim 390\,\mathrm{MeV}$)



$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

$$\mathcal{M} \sim \frac{1}{p\cot\delta - ip}$$



Hansen, RB, Edwards, Thomas, & Wilson (2020)

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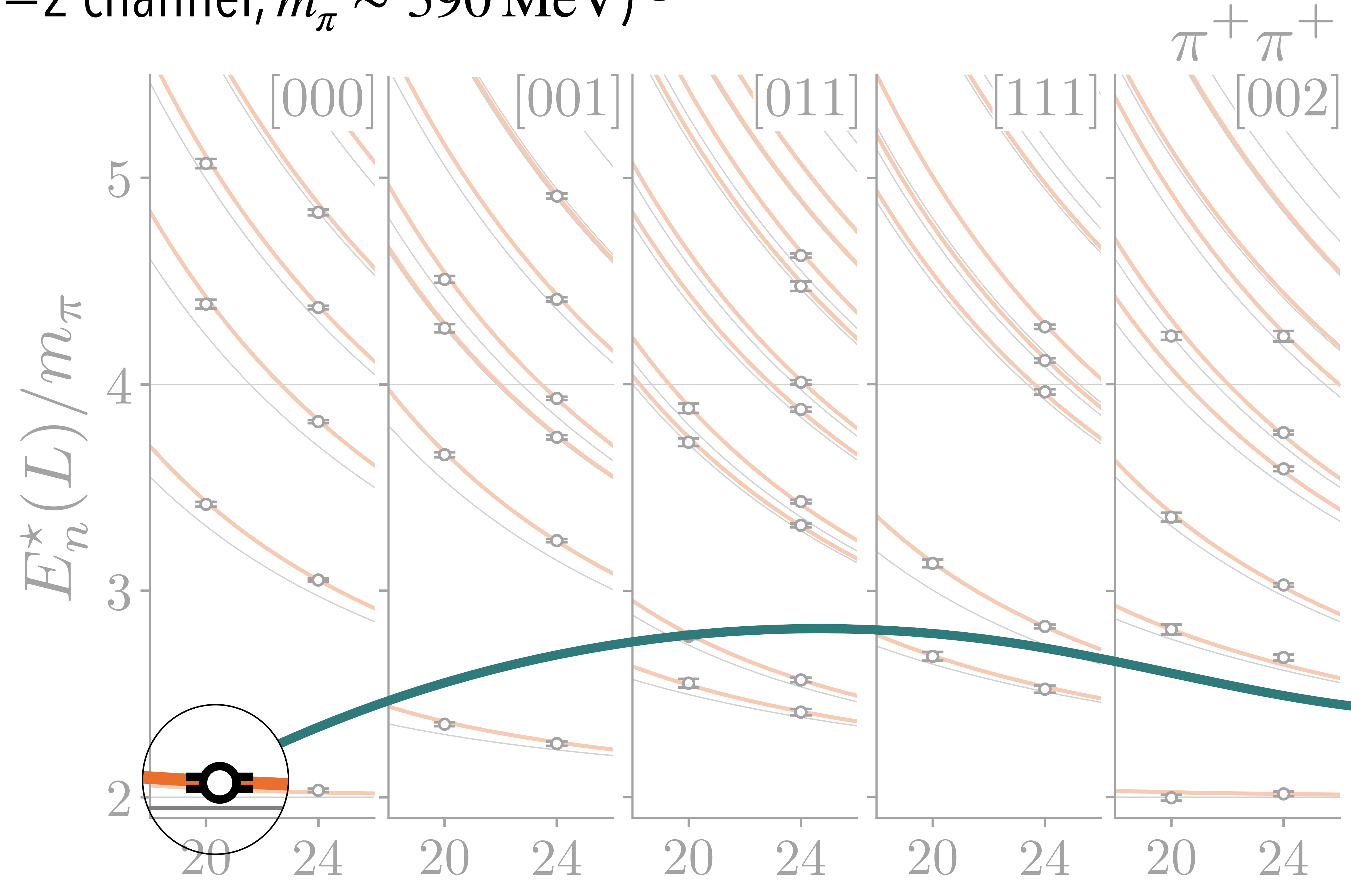
(I=2 channel, $m_\pi \sim 390\,\mathrm{MeV}$)



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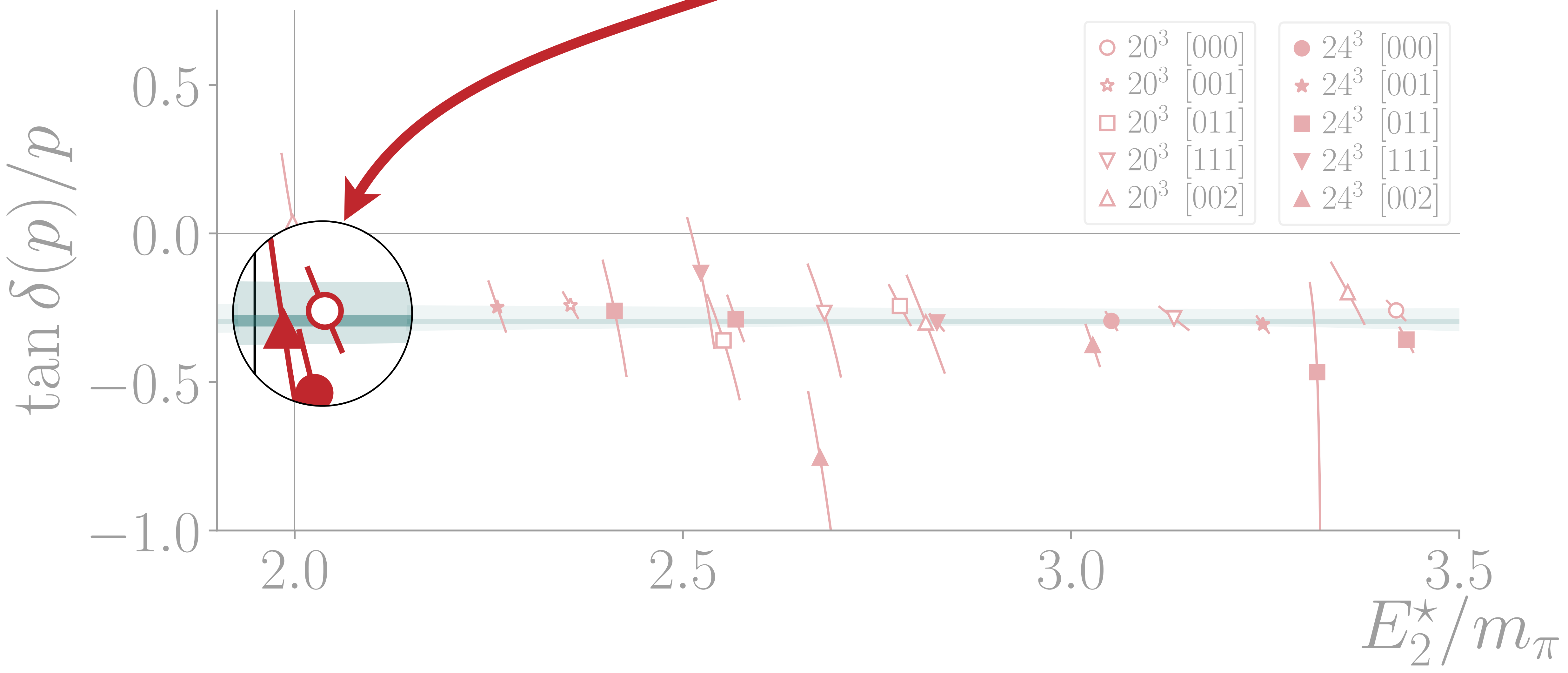
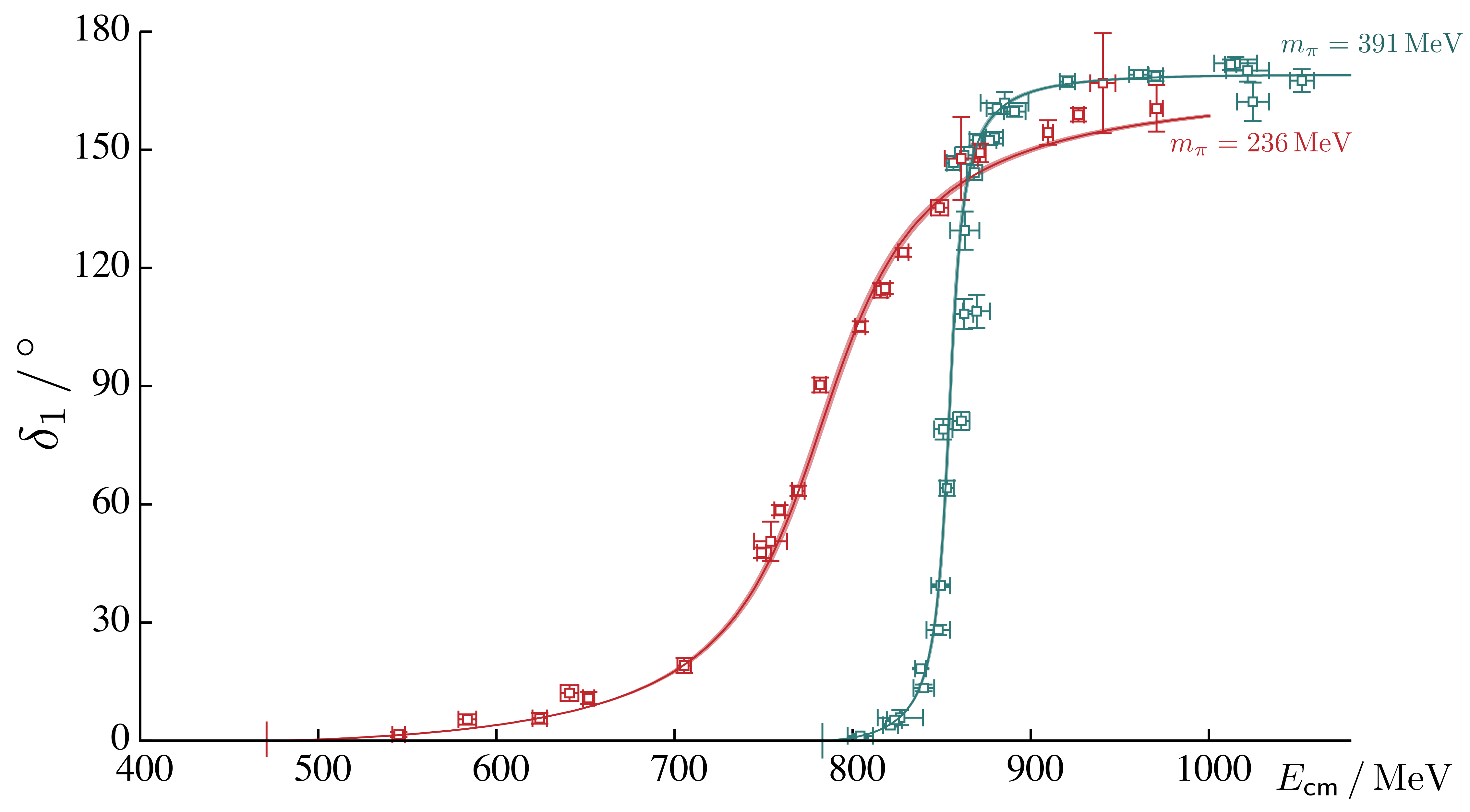
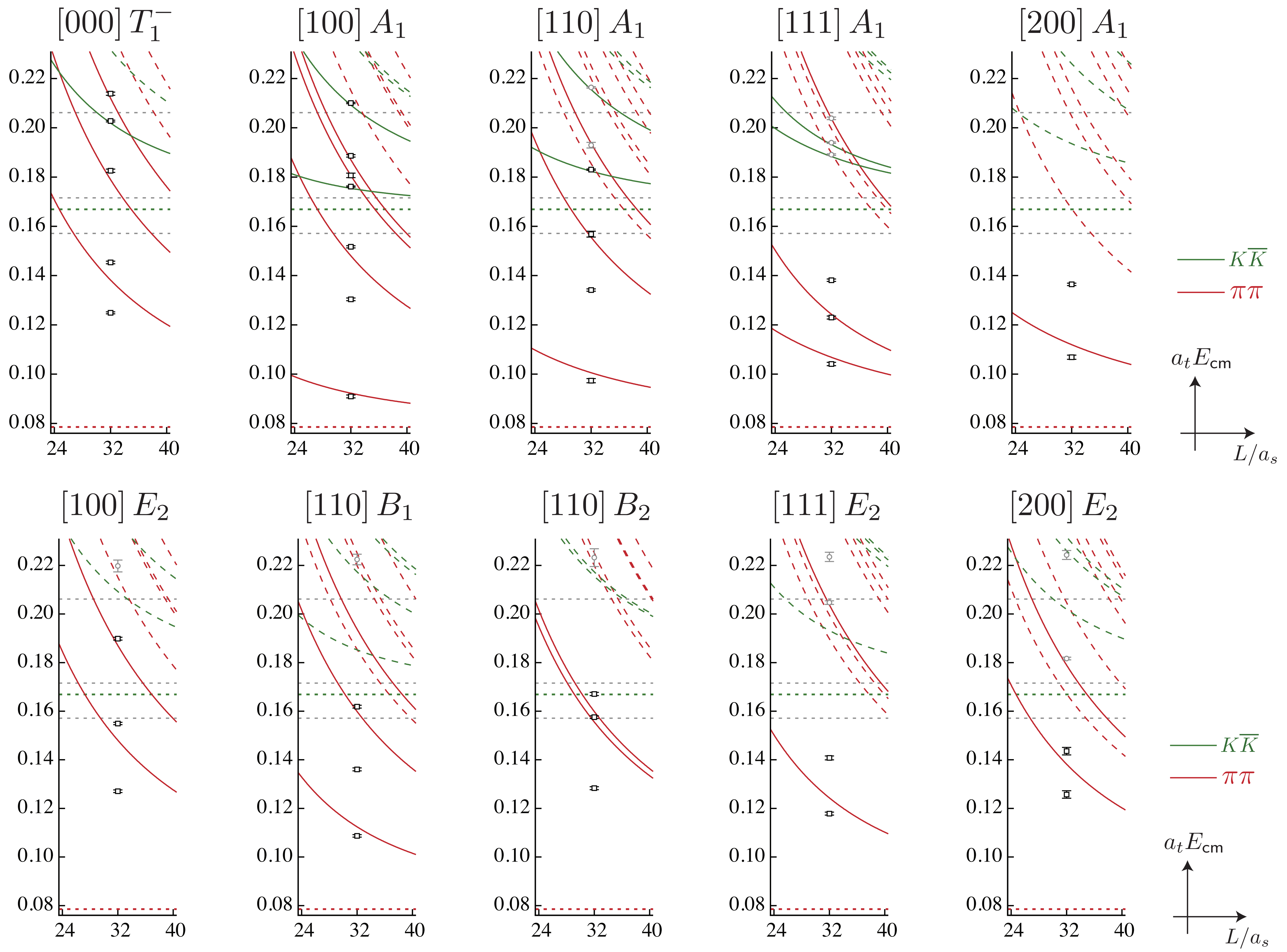


Hansen, RB, Edwards, Thomas, & Wilson (2020)

$\pi\pi$ scattering
(I=1 channel)



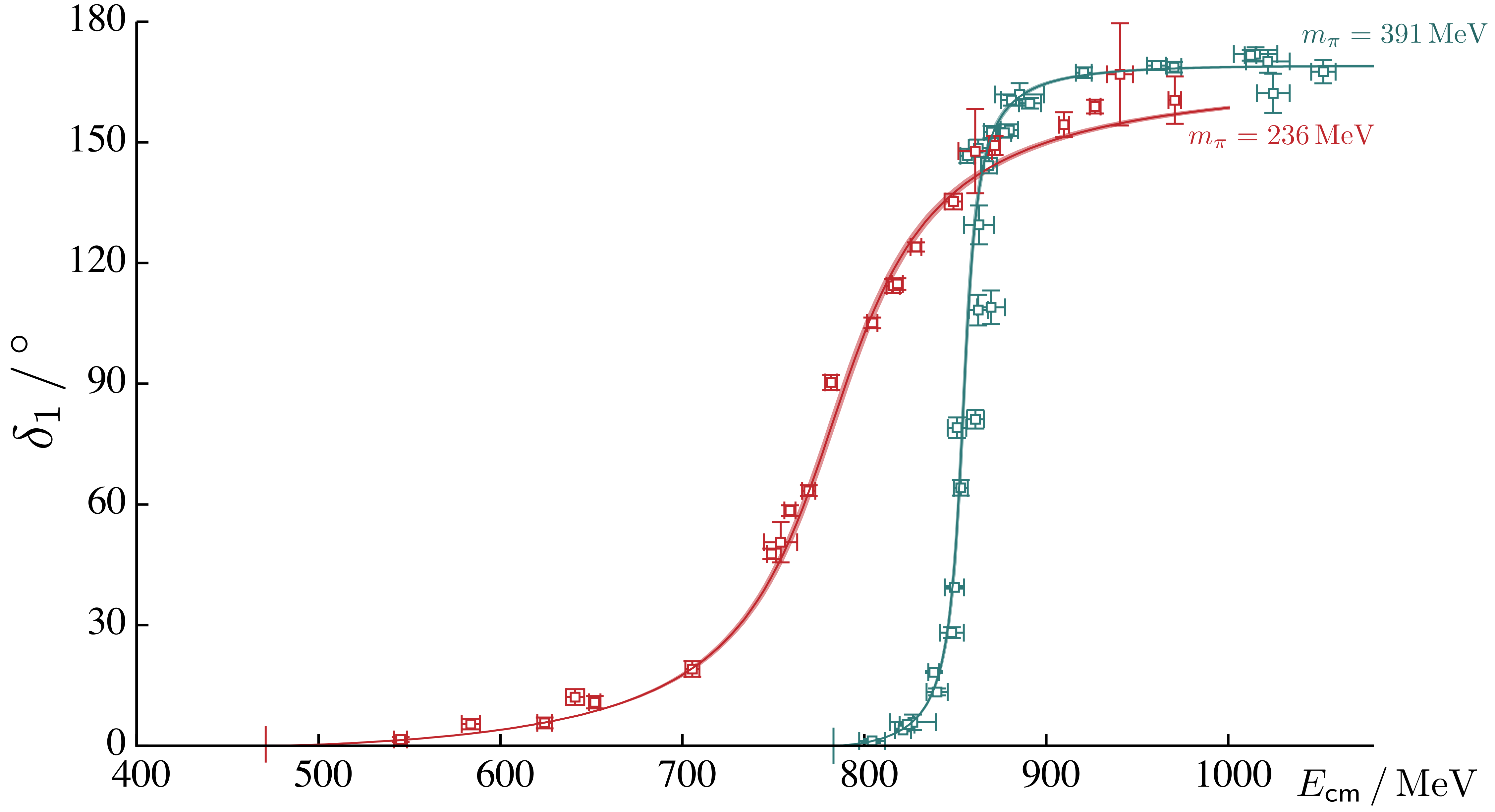


$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$

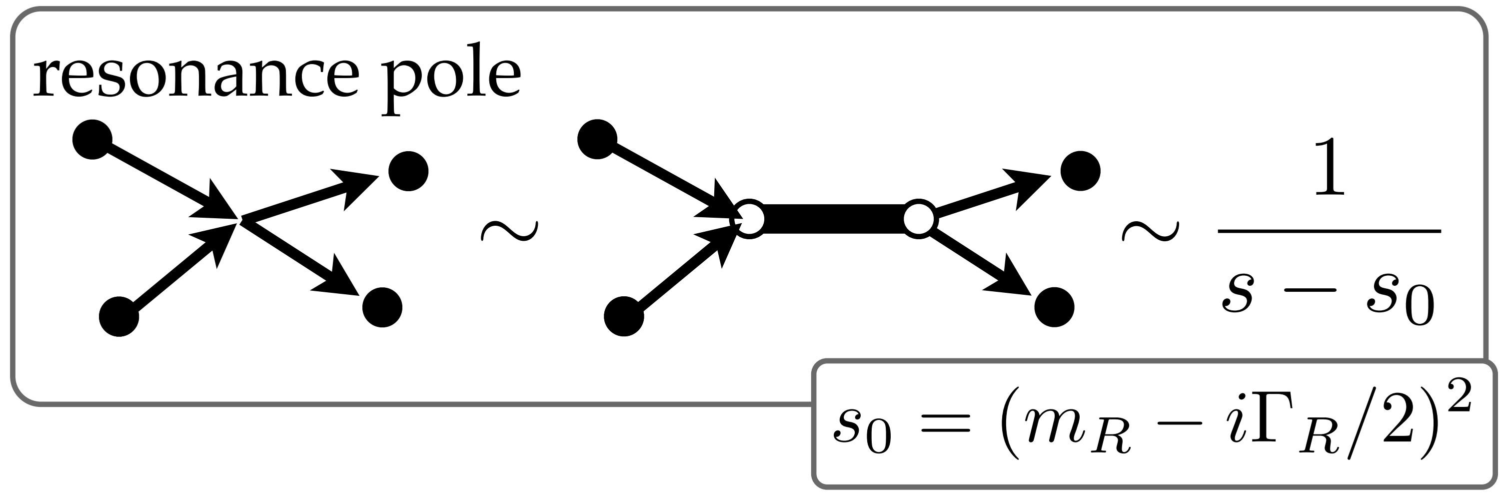
Dudek, Edwards, & Thomas (2012)

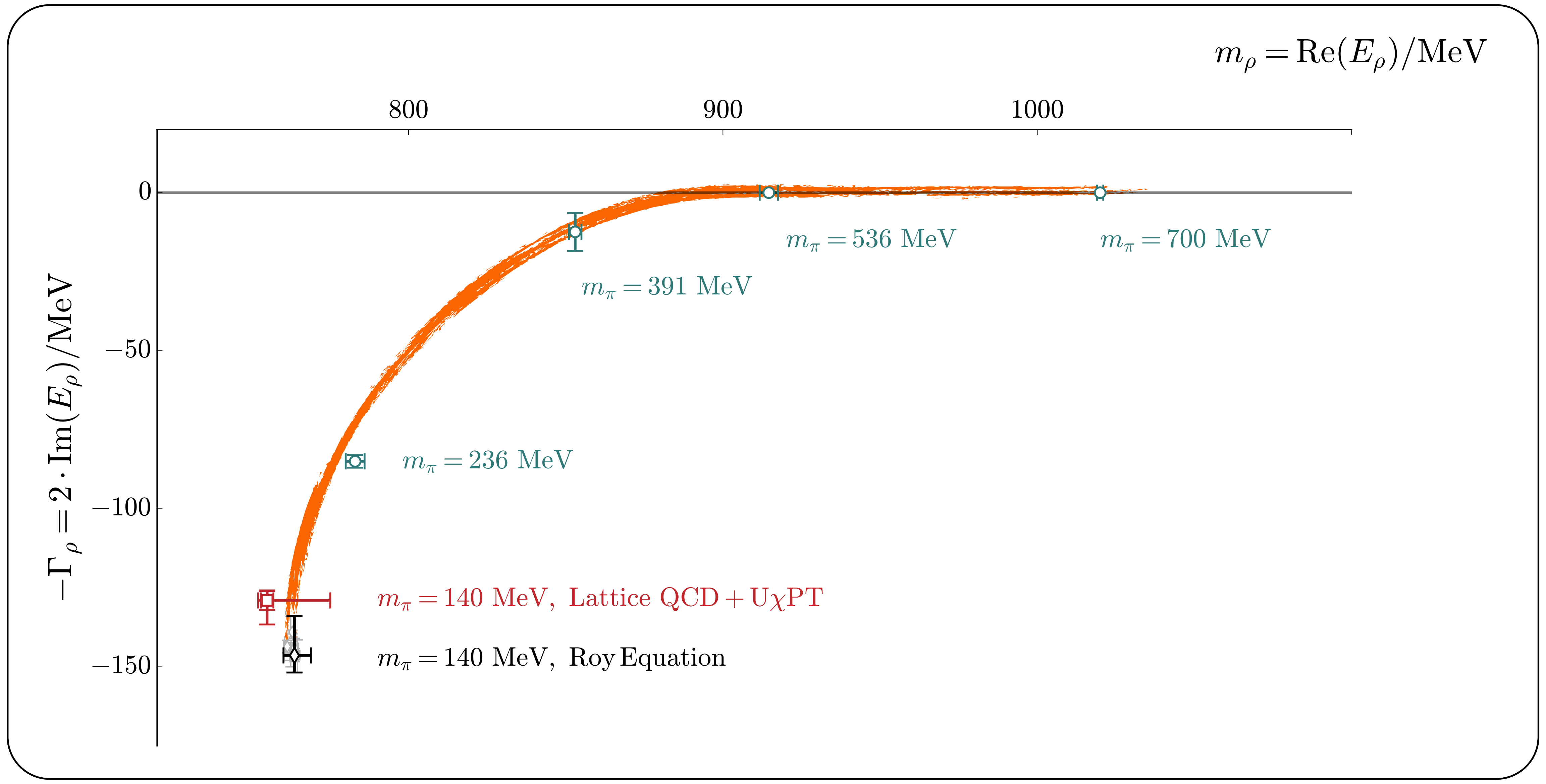
Wilson, RB, Dudek, Edwards, & Thomas (2015)

$\pi\pi$ scattering
(I=1 channel)



$$\mathcal{M} \sim \frac{1}{p\cot\delta - ip}$$



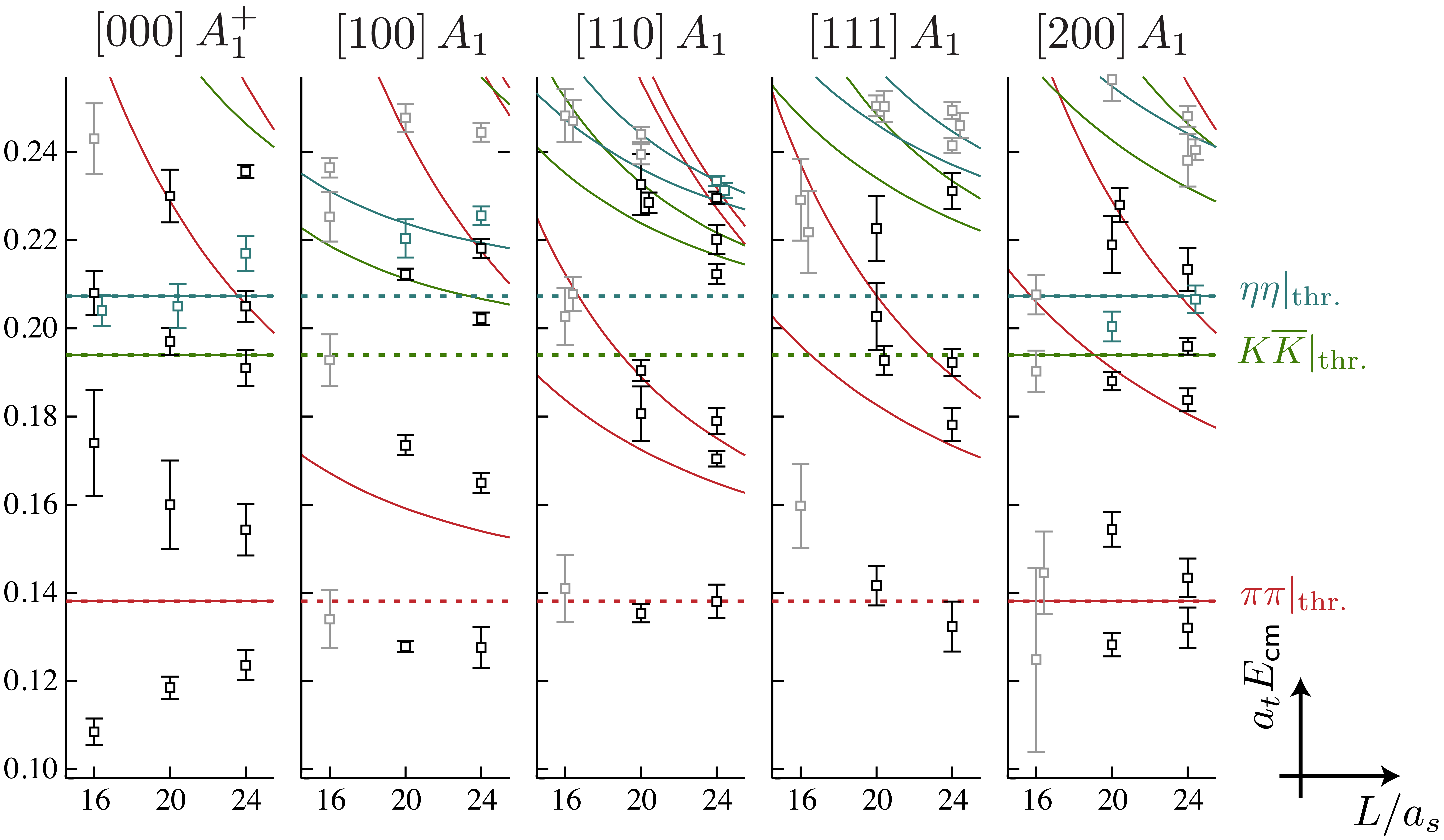


Dudek, Edwards, & Thomas (2012)

Wilson, RB, Dudek, Edwards, & Thomas (2015)

The vacuum channel in a finite volume

(I=0 channel, $m_\pi \sim 390\,\mathrm{MeV}$)



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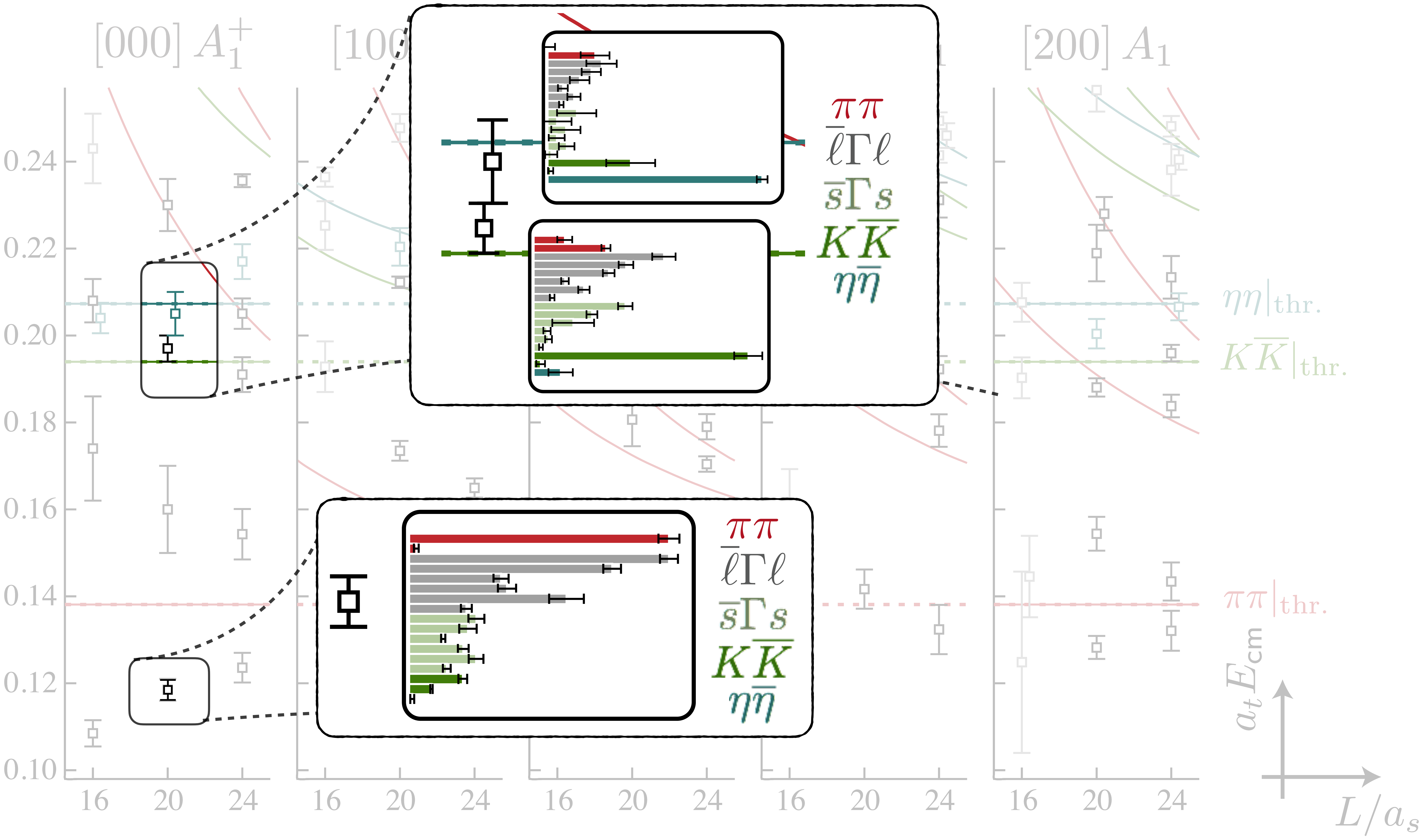

The vacuum channel in a finite volume

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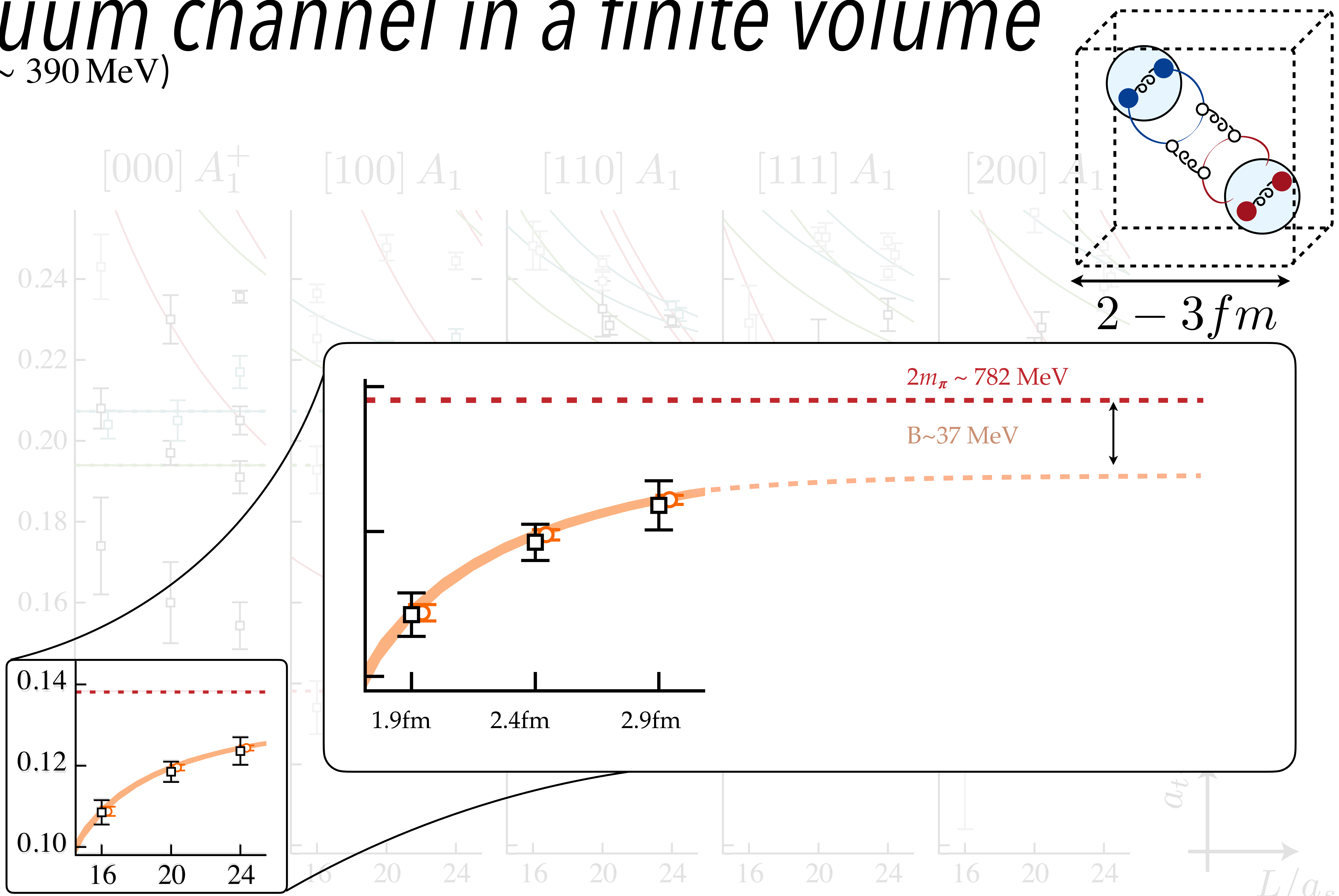

The vacuum channel in a finite volume

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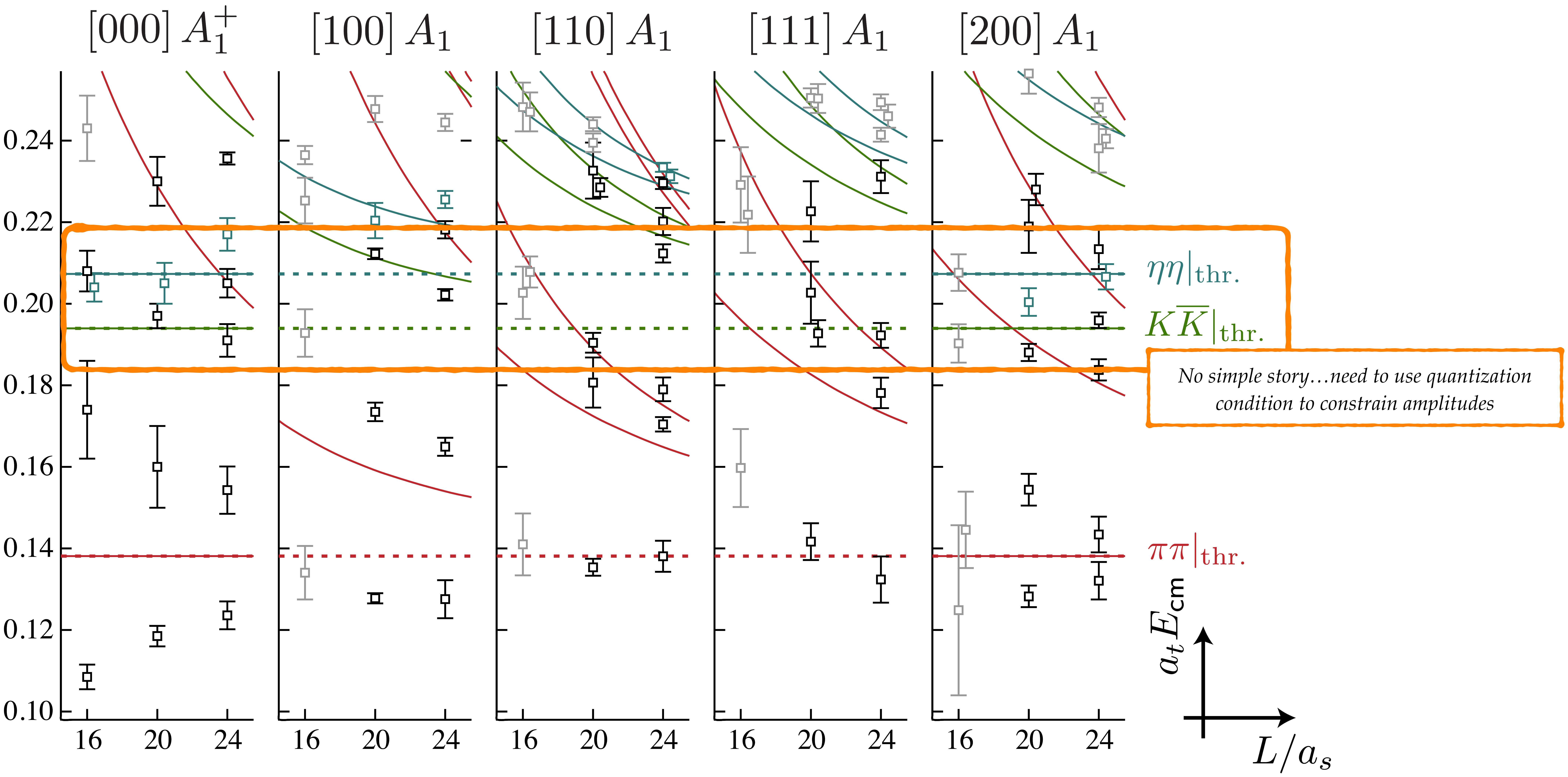
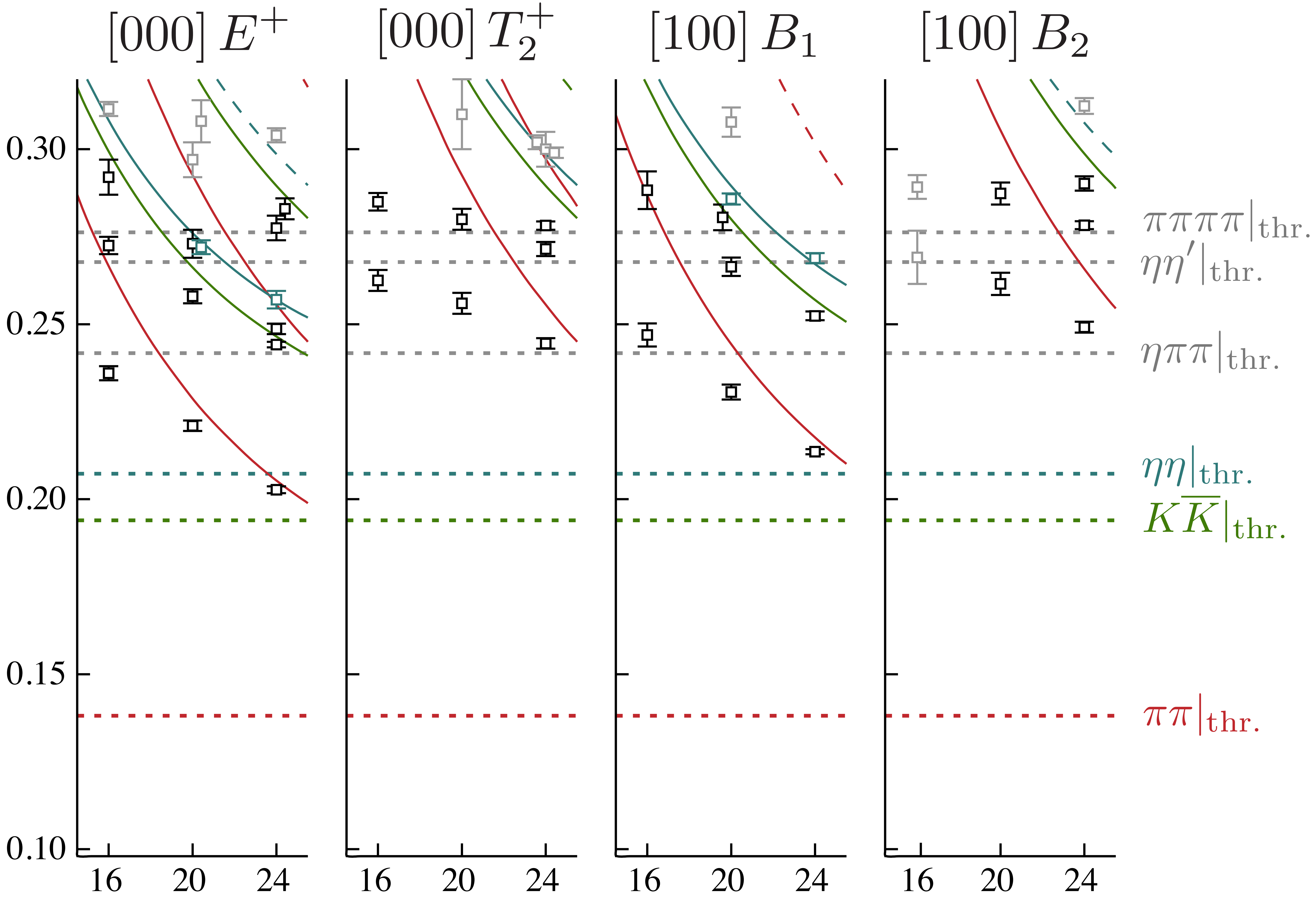


Isoscalar spectra // D-wave dominant

(I=0 channel, $m_\pi \sim 390\,\mathrm{MeV}$)



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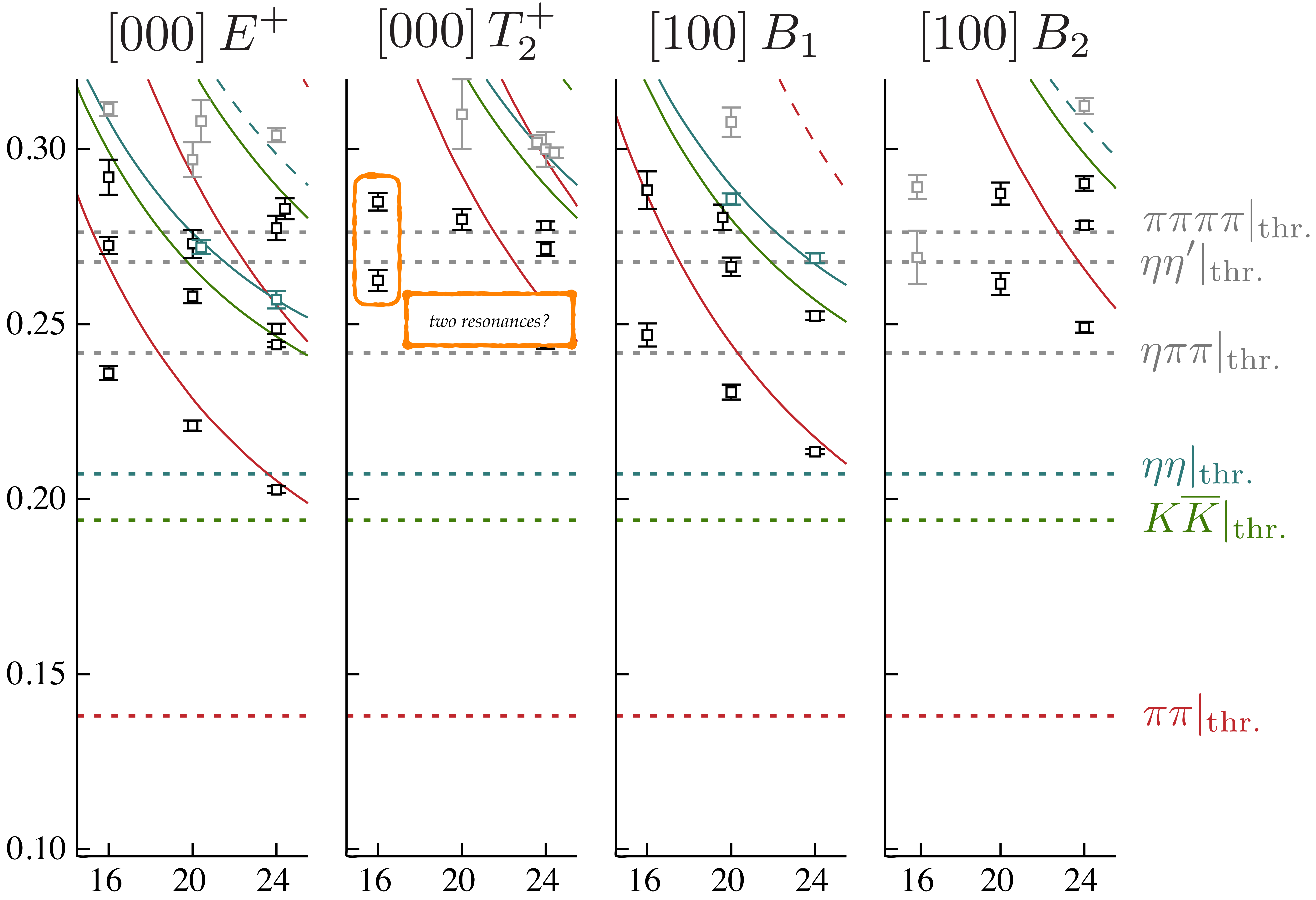

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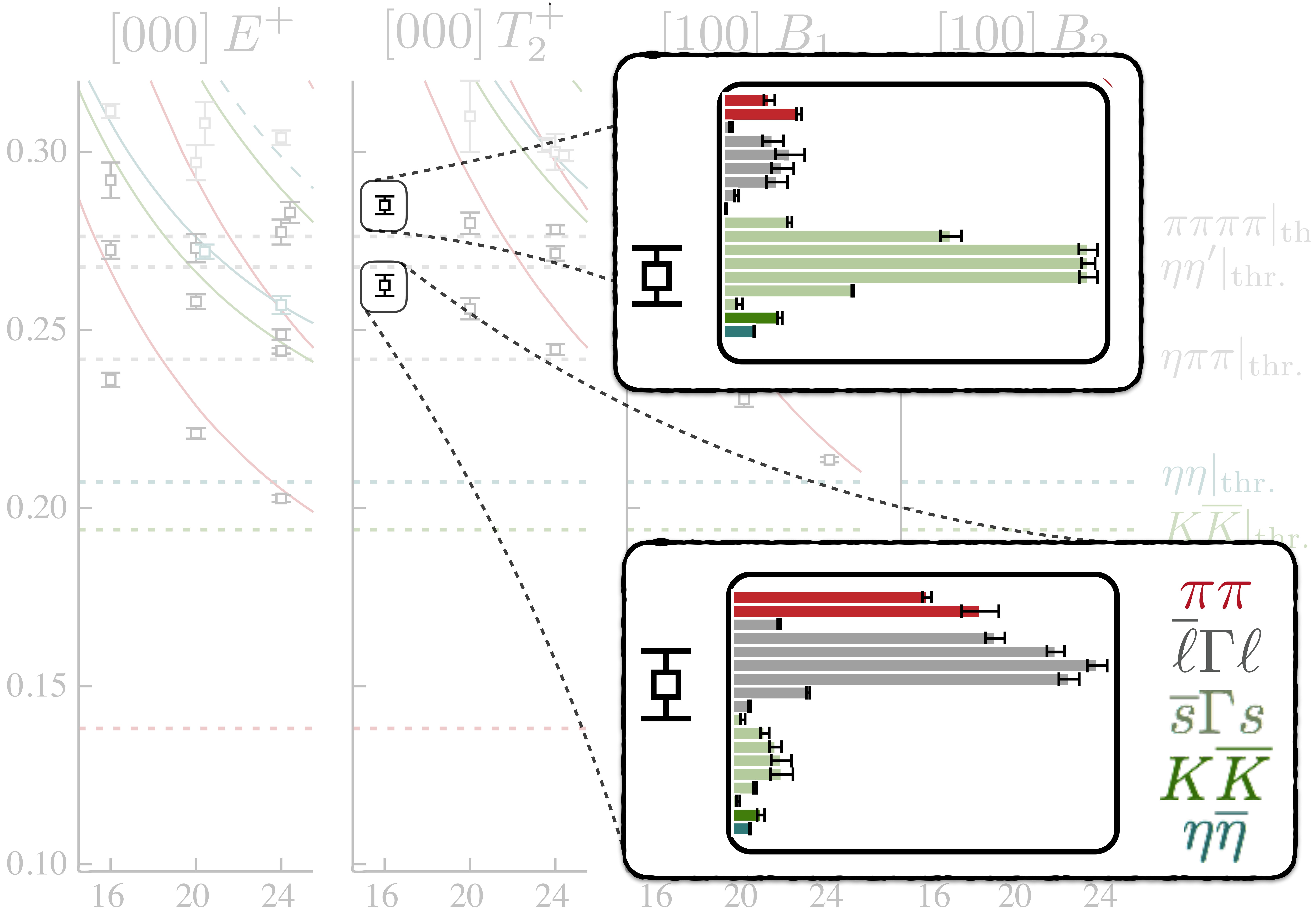
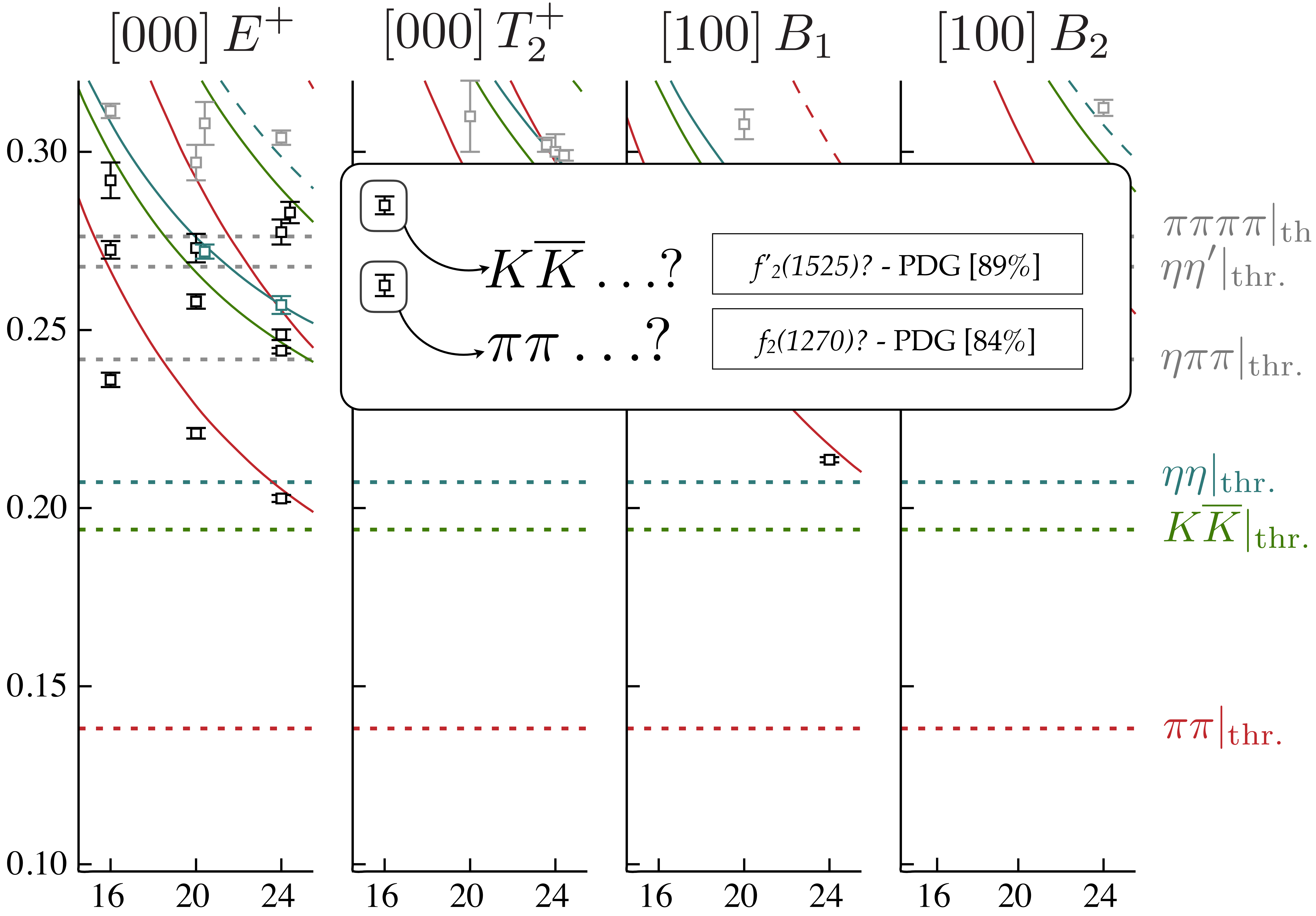
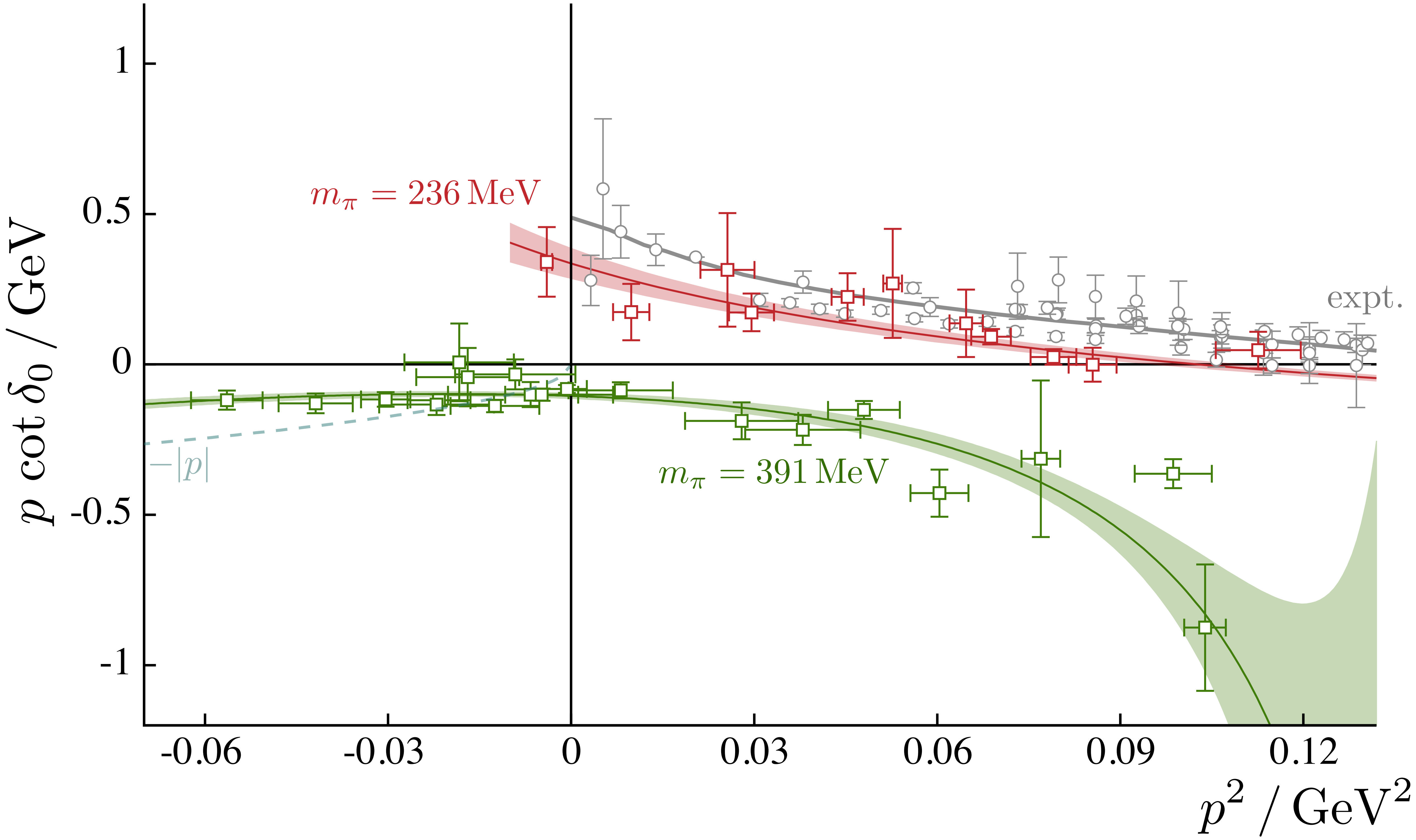


Isoscalar spectra // D-wave dominant

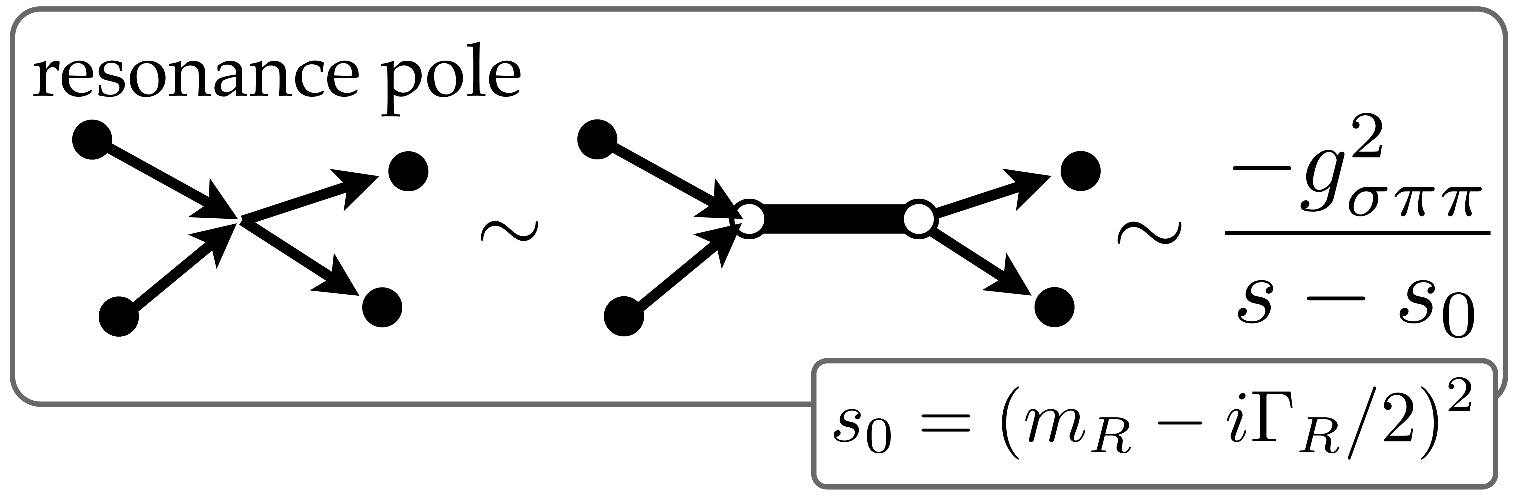
(I=0 channel, $m_\pi \sim 390\,\mathrm{MeV}$)

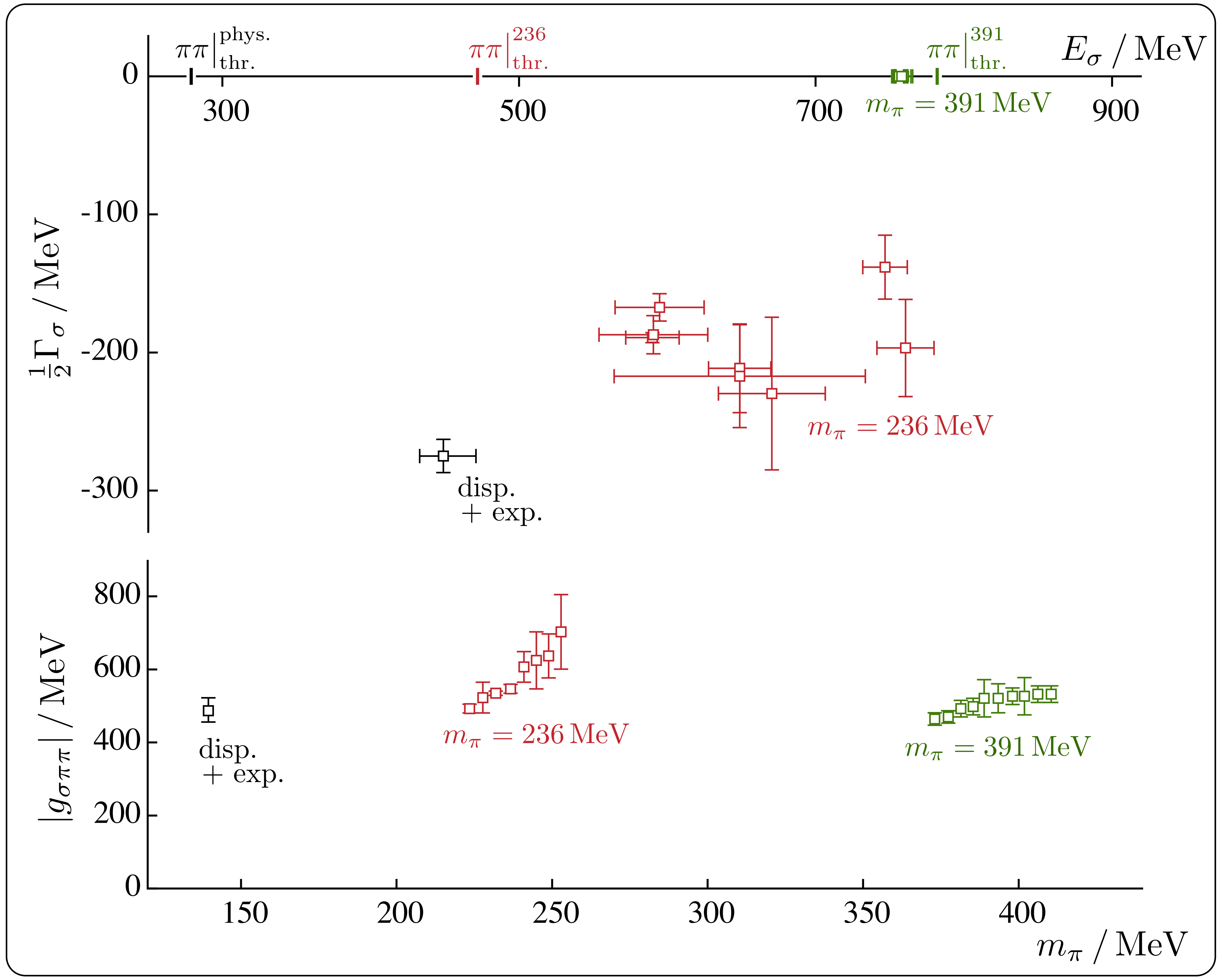


$\pi\pi$ scattering
(I=0 // S-wave channel)



$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$





Dudek, Edwards, & Thomas (2012)
Wilson, RB, Dudek, Edwards, & Thomas (2015)

Outline

First half:

- [x] Elastic two-body scattering

Second half:

- [ ] Two-body, coupled-channel scattering
- [ ] Three-body scattering

Coupled-channels analysis

- Above $2m_K$, there is not a one-to-one correspondence

$$\det\begin{bmatrix} F^{-1}_{\pi\pi} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F^{-1}_{K\overline{K}} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$

- In general, must constrain $(1/2)\,[N^2 + N]$ functions of energy
- Need that many energy levels at the same energy
- Alternatively, parametrize scattering amplitude and do a global fit

Hansen & Sharpe / RB & Davoudi (2012)

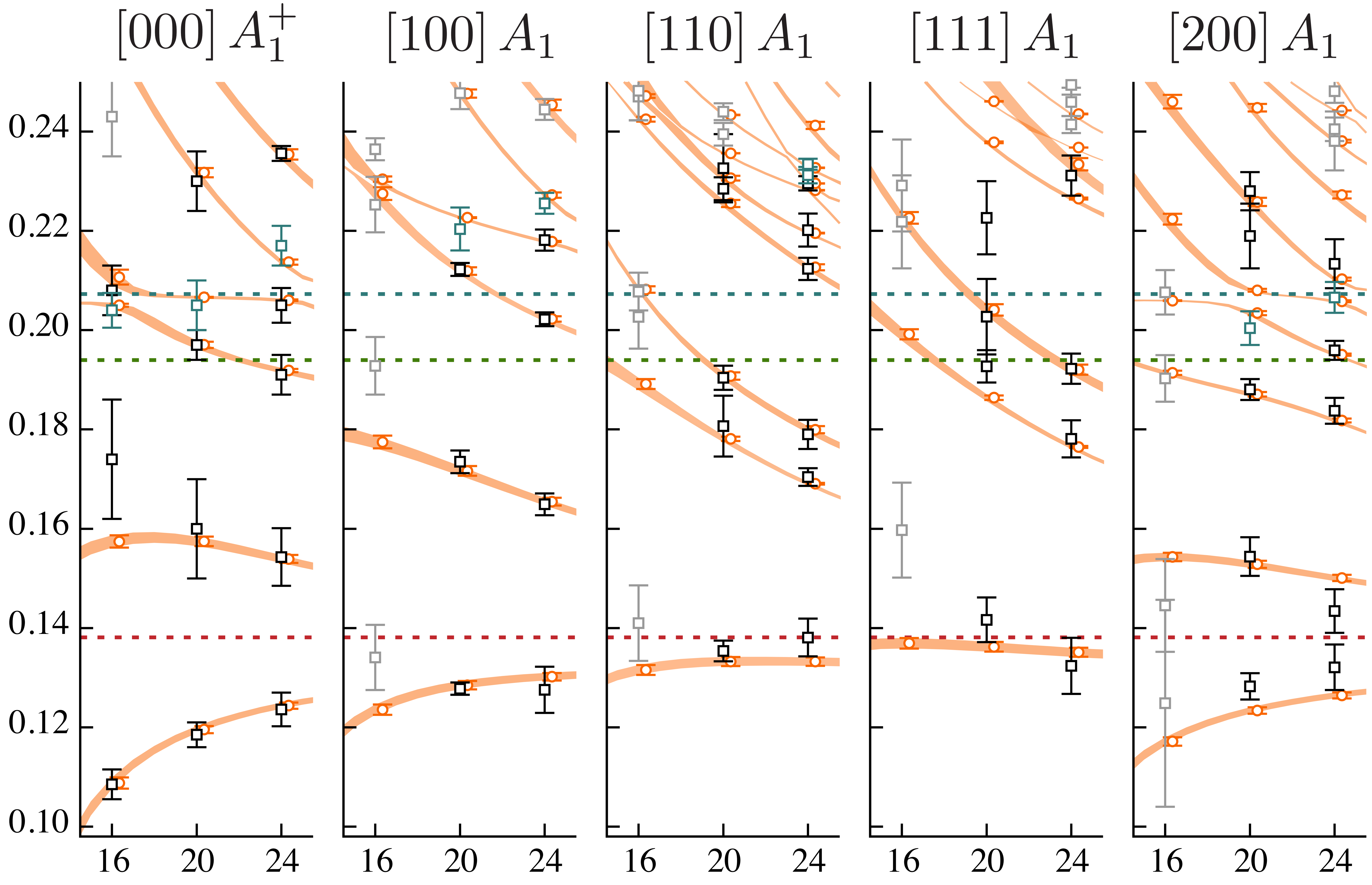
Coupled-channels analysis

- S-wave above $2m_\pi$, $2m_K$, and $2m_\eta$

- Ansatz $\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels



Coupled-channels analysis

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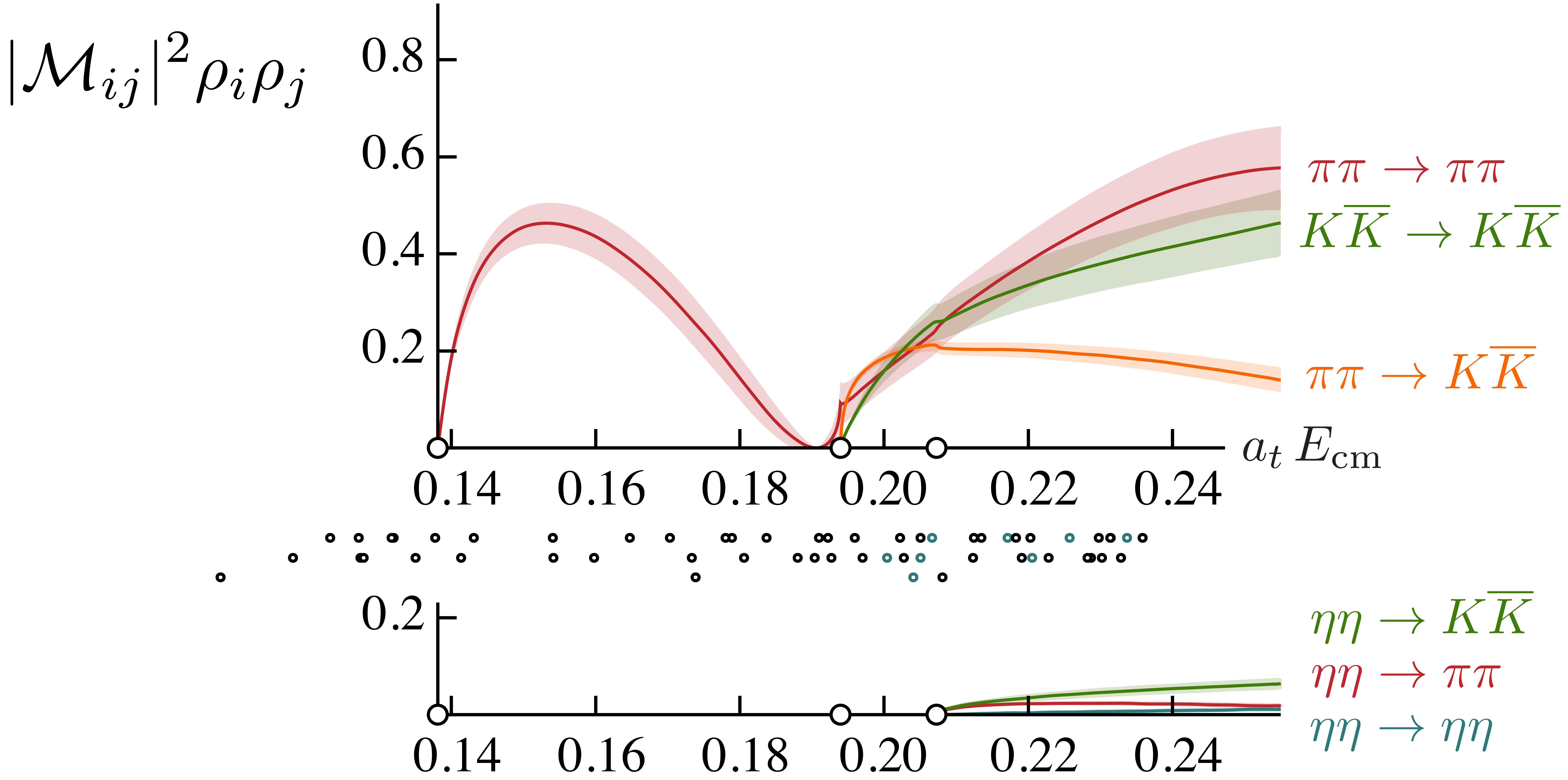


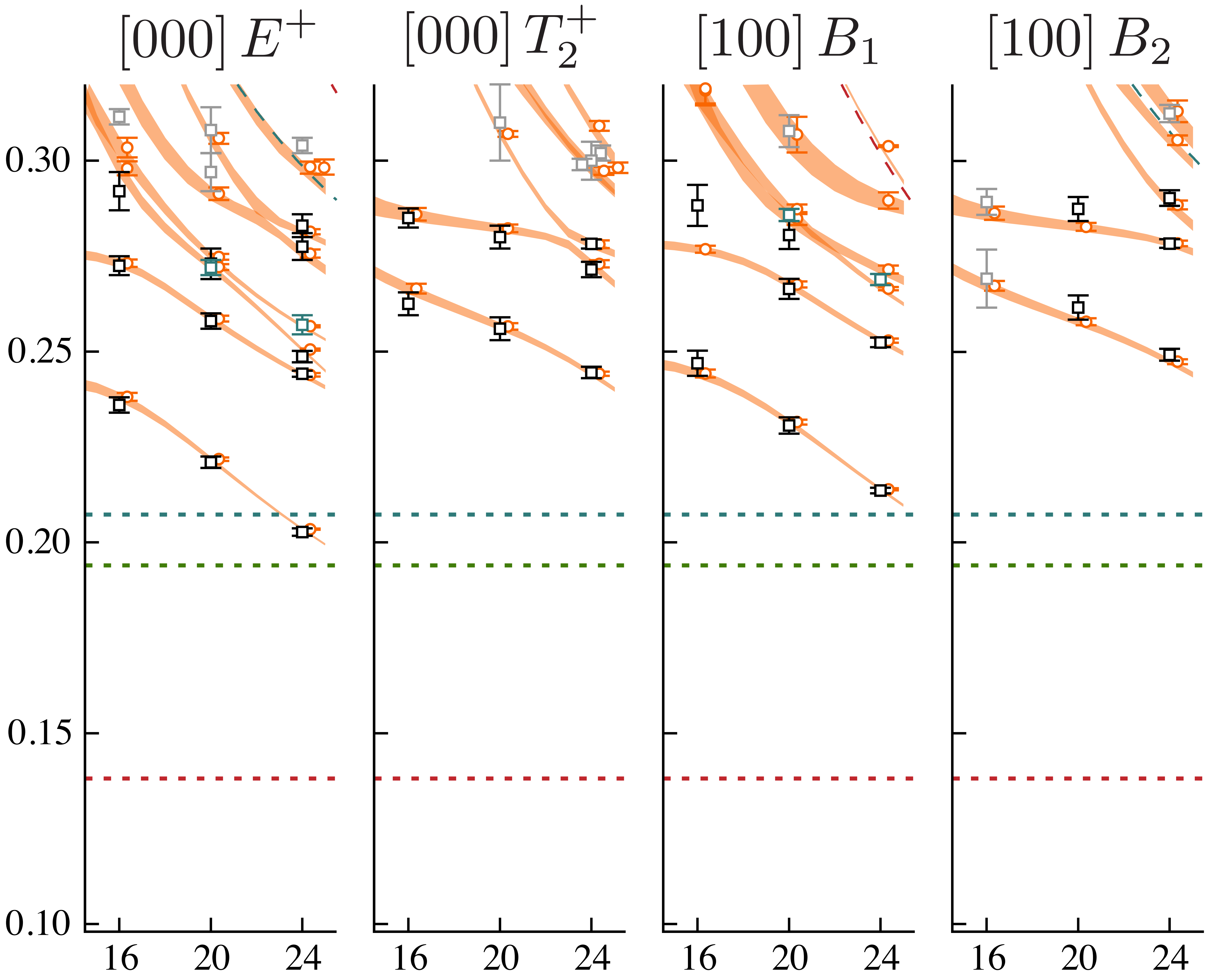
Coupled-channels analysis

- D-wave above $2m_\pi$, $2m_K$, and $2m_\eta$

- Ansatz $$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$ $\gamma_{\eta\eta} \neq 0$, $\gamma_{ij} = 0$ otherwise

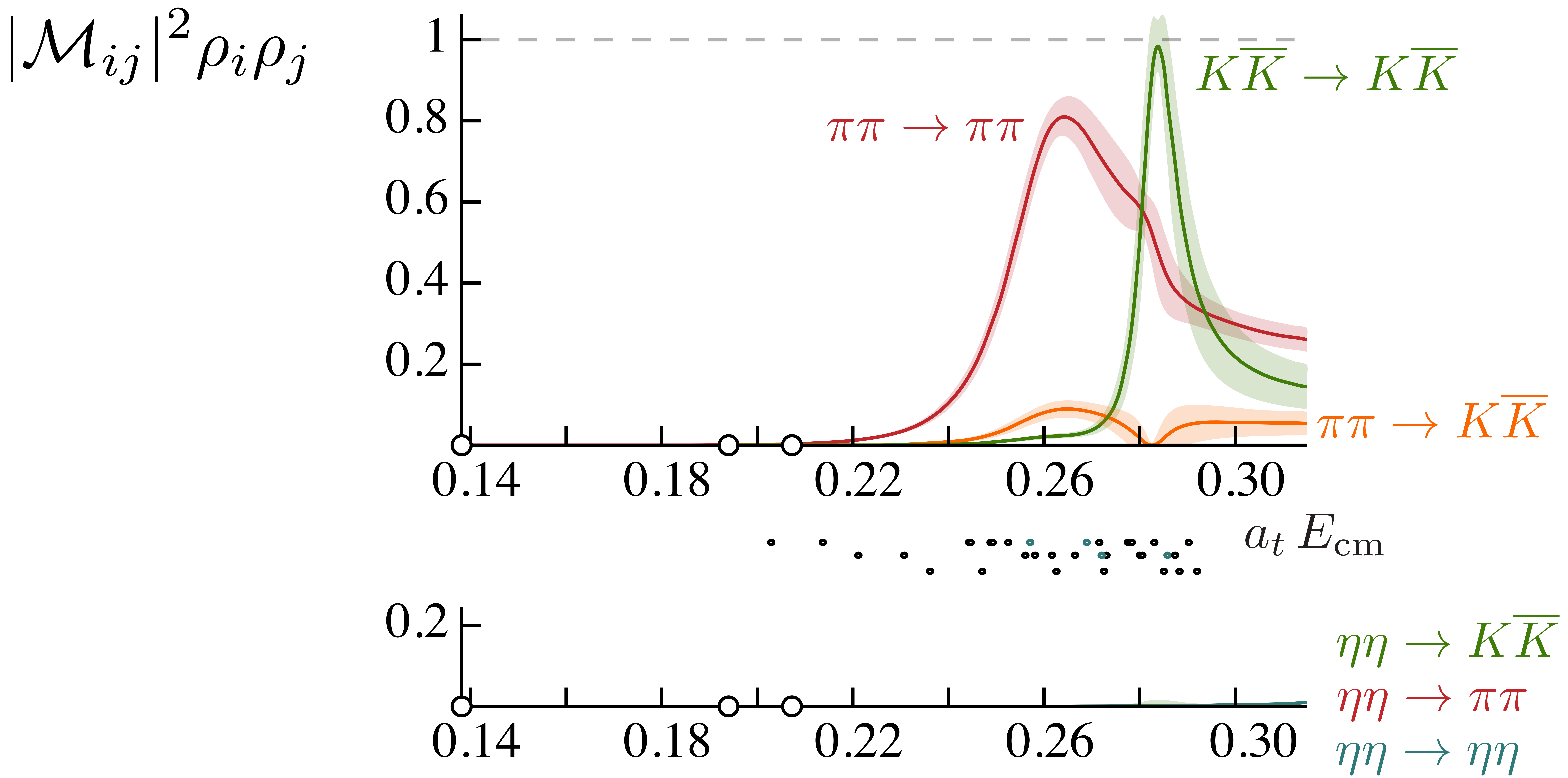
$$\chi^2/N_{\rm dof} = \frac{28.9}{34-9} = 1.15$$

34 energy levels

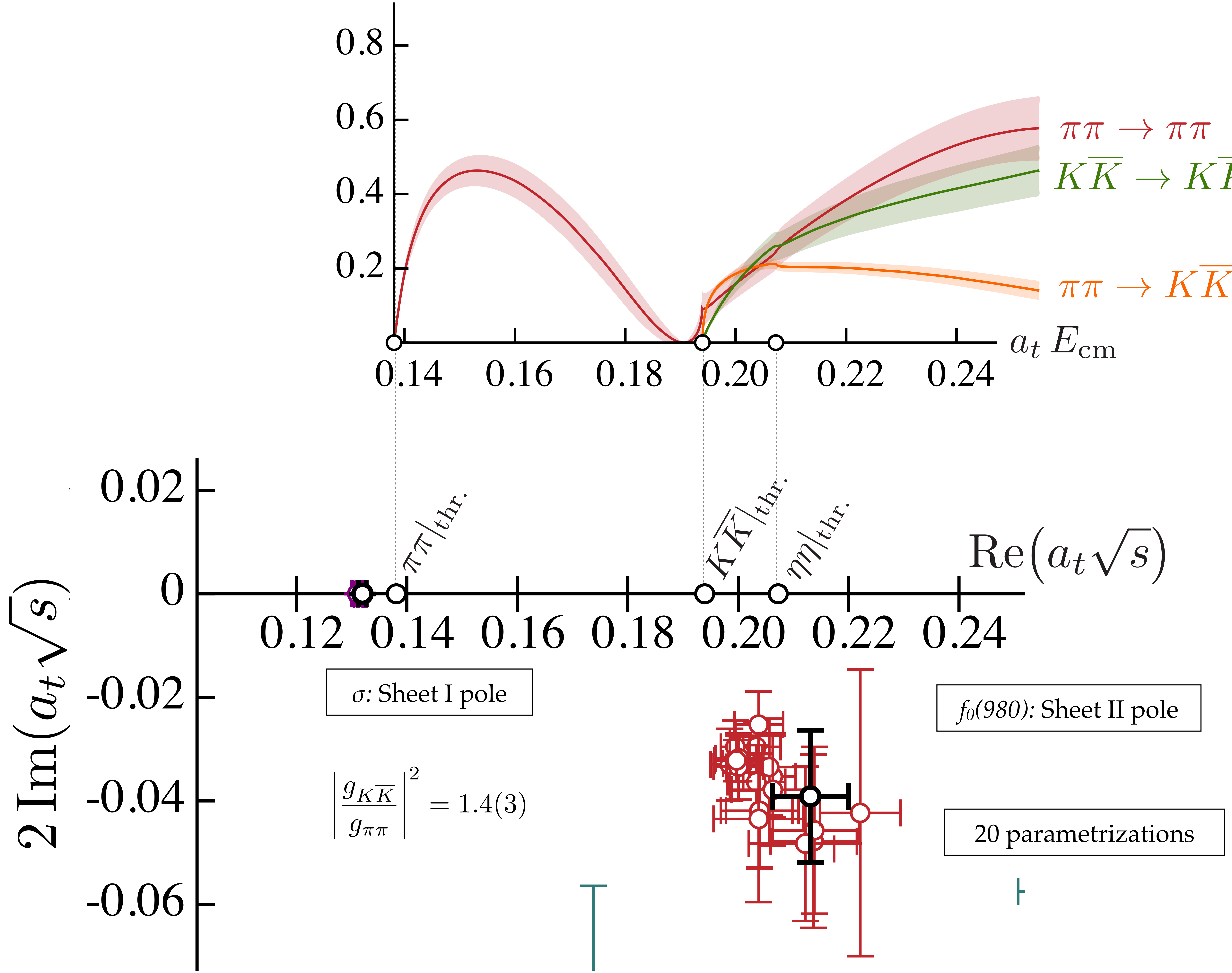


Coupled-channels analysis

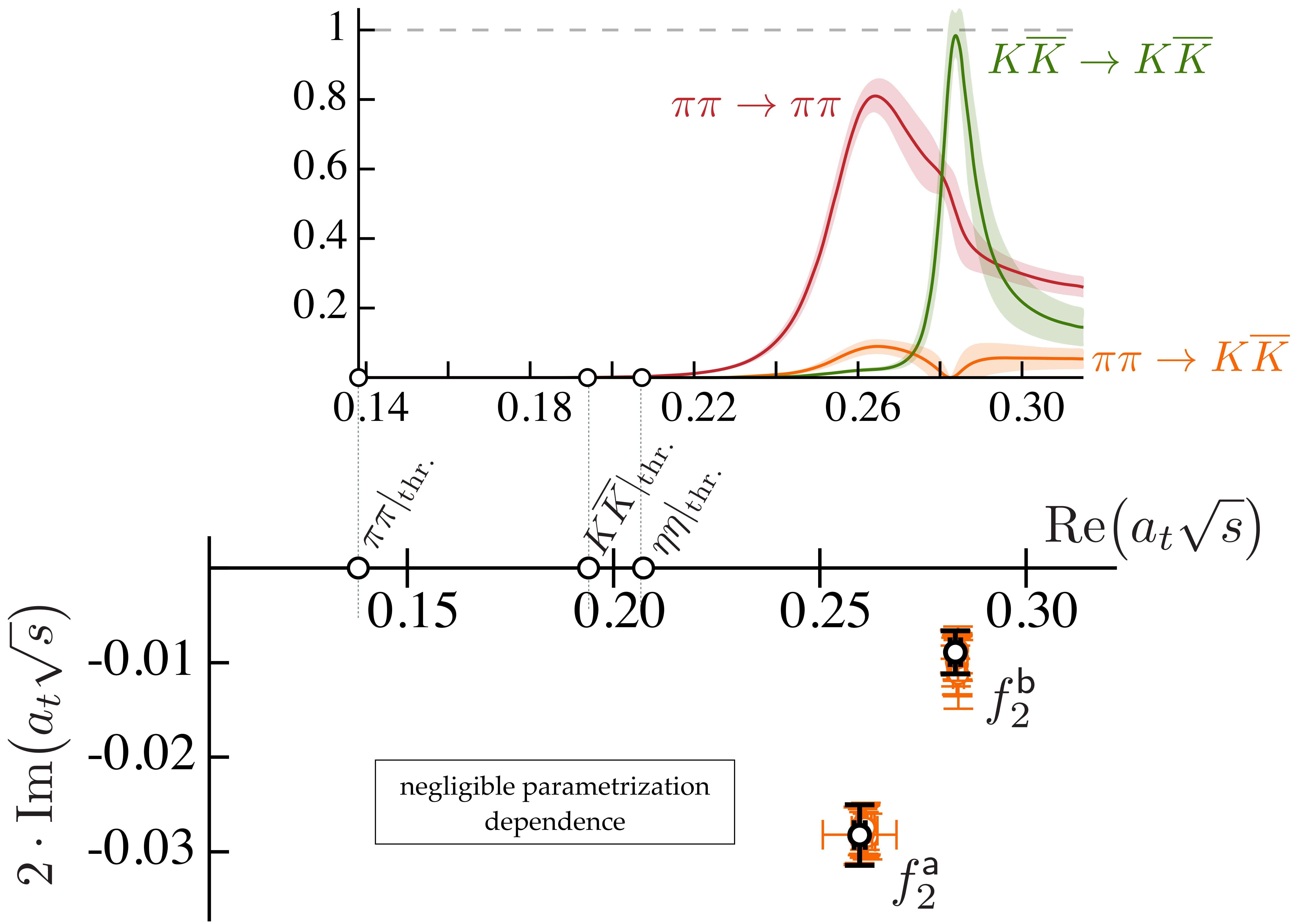
- D-wave above $2m_\pi$, $2m_K$, and $2m_\eta$
- Ansatz $K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$ $\gamma_{\eta\eta} \neq 0$, $\gamma_{ij} = 0$ otherwise



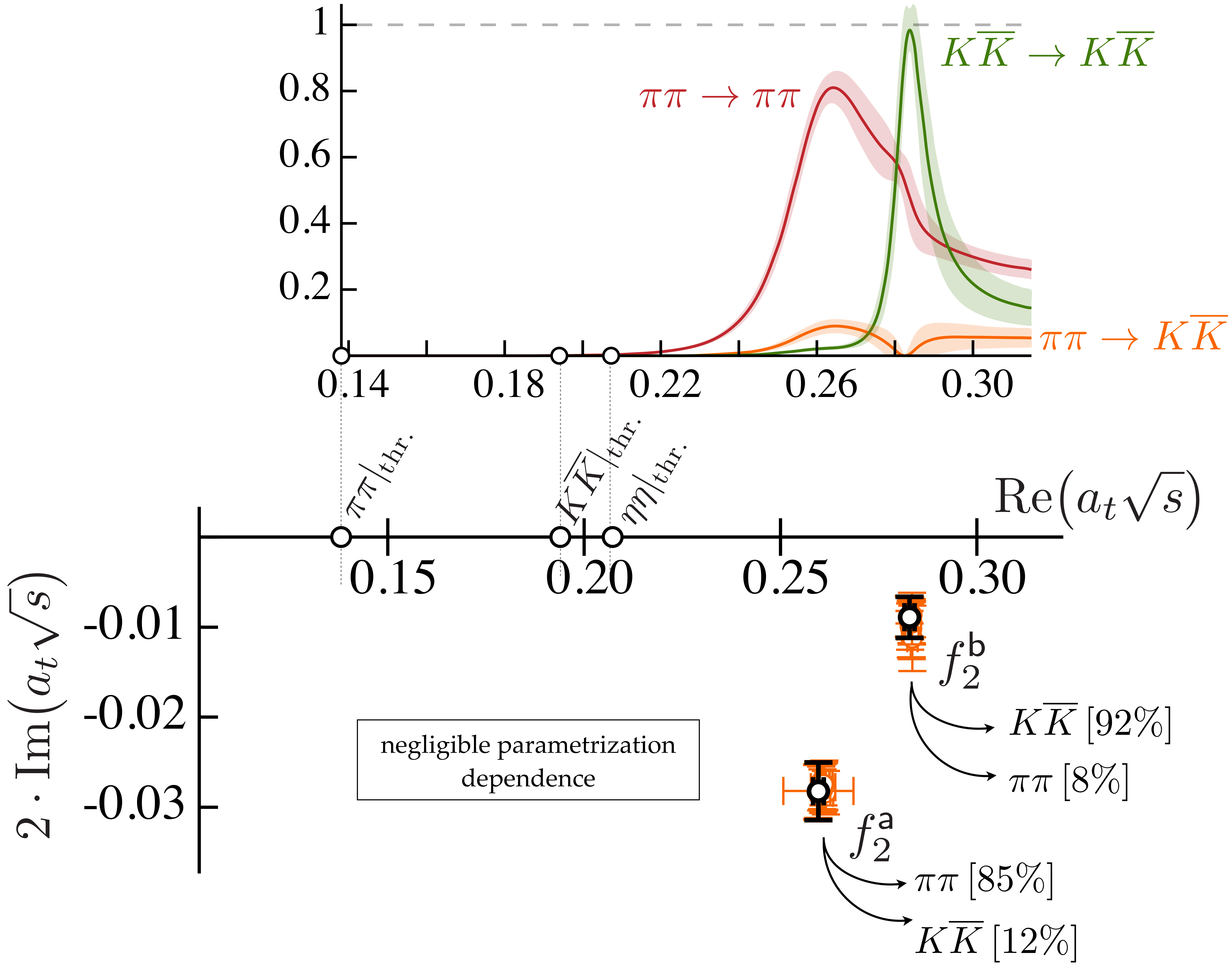
Scalar poles: σ and $f_0(980)$



Tensor poles: the f_2's

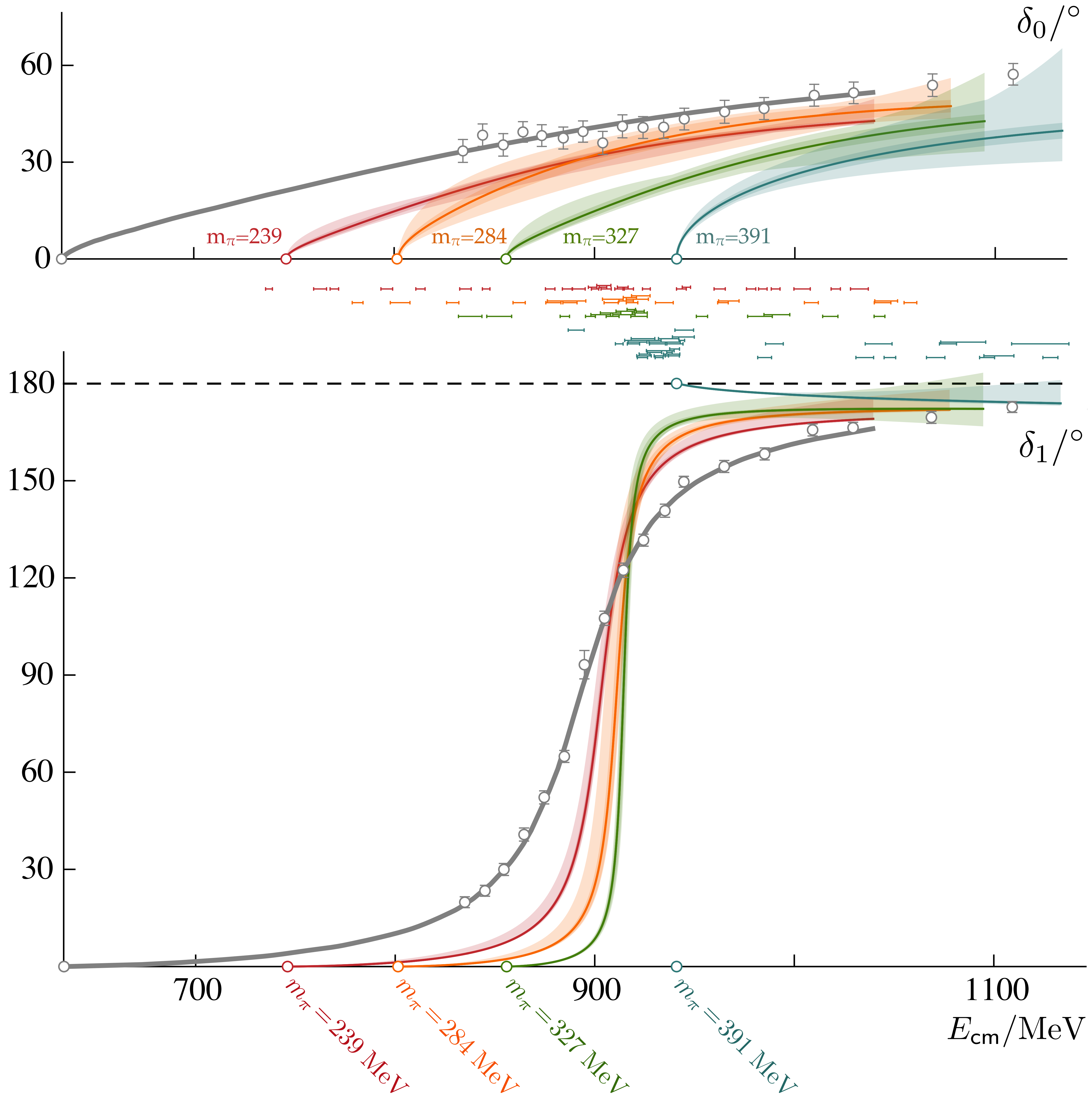


Tensor poles: the f_2's



πK scattering
(I=1/2 channel)

$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$



Wilson, RB, Dudek, Edwards, & Thomas (2019)

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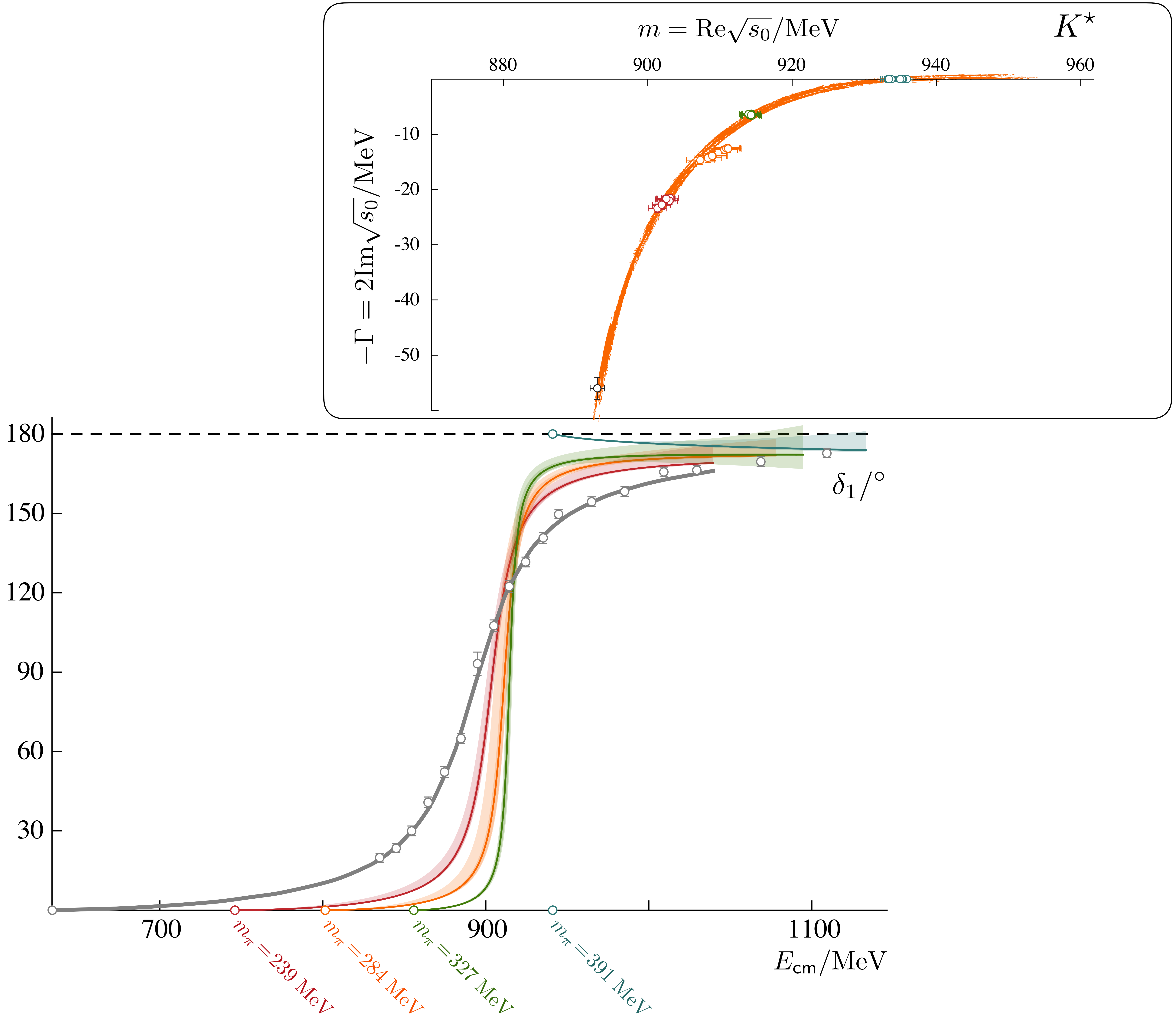
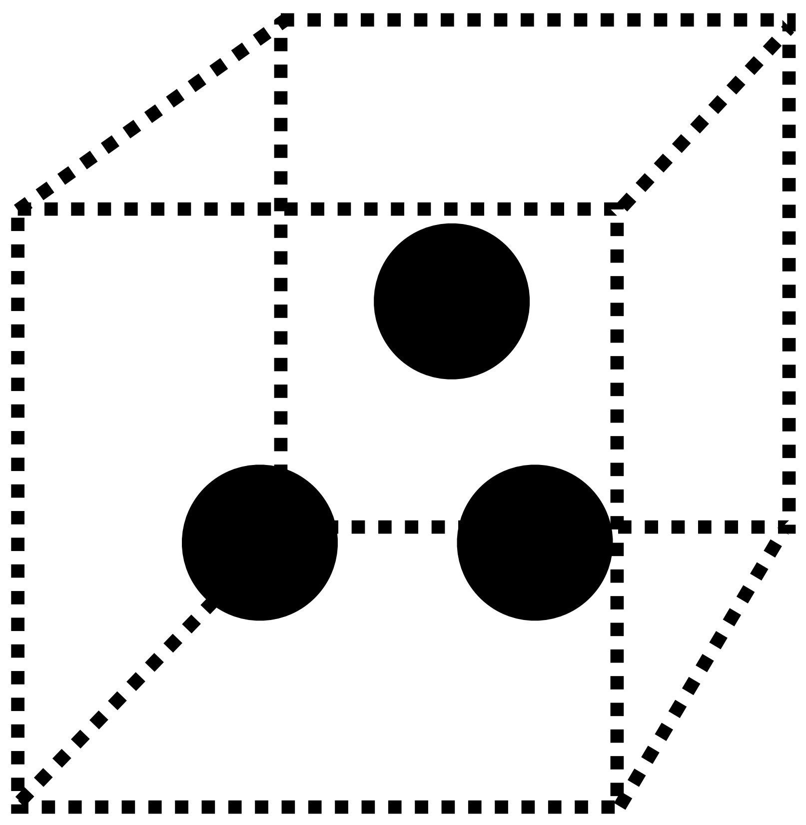


Wilson, RB, Dudek, Edwards, & Thomas (2019)

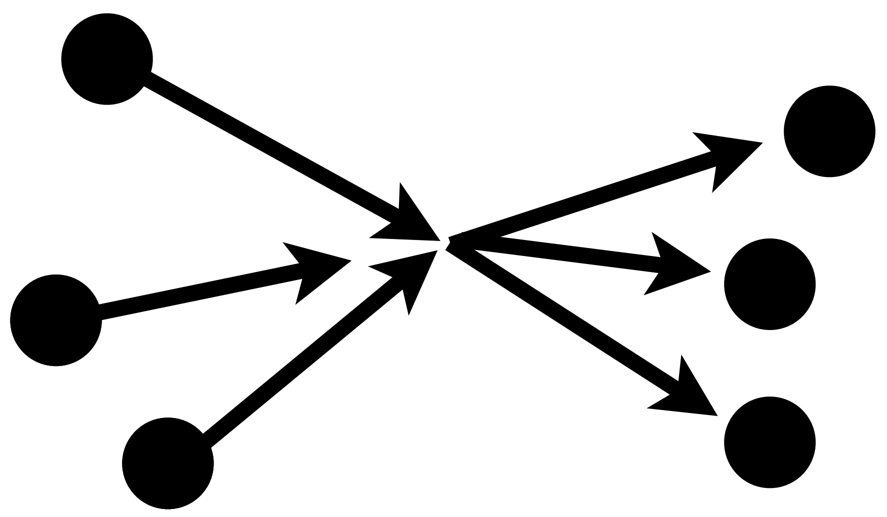
Three-hadron systems



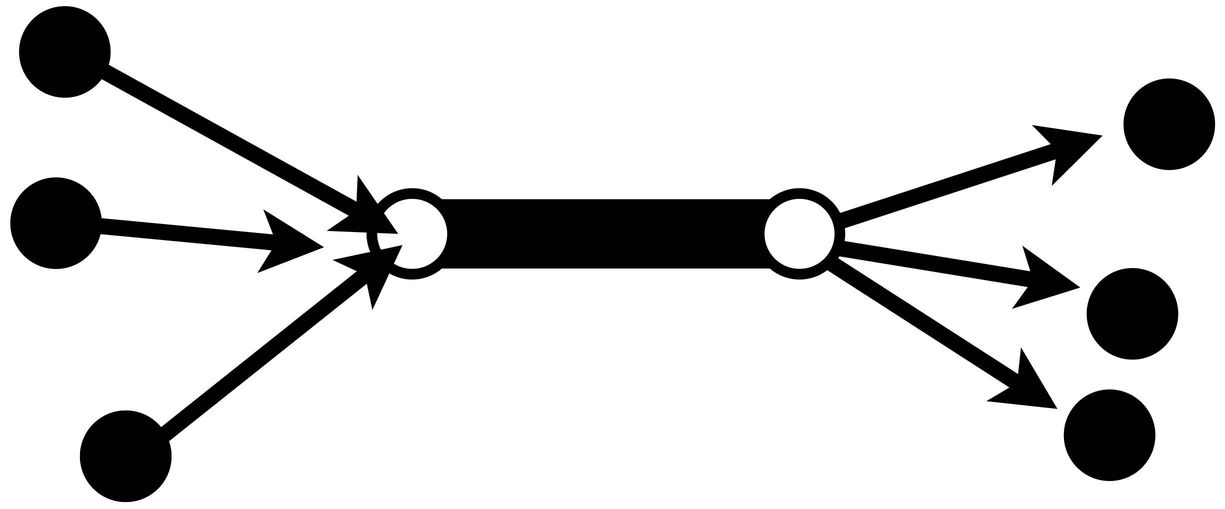
finite-volume
spectroscopy

$$\det\left[F_3(E_L,L)+\mathcal{K}_{\mathrm{df},3}^{-1}(E_L)\right]=0$$

Hansen & Sharpe ('14, '15)
Mai & Döring ('17)
RB, Hansen & Sharpe ('18)
Hansen, Romero-Lopez & Sharpe ('20)
Blanton & Sharpe ('20)
Jackura et al. ('20)



infinite-volume
scattering amplitudes



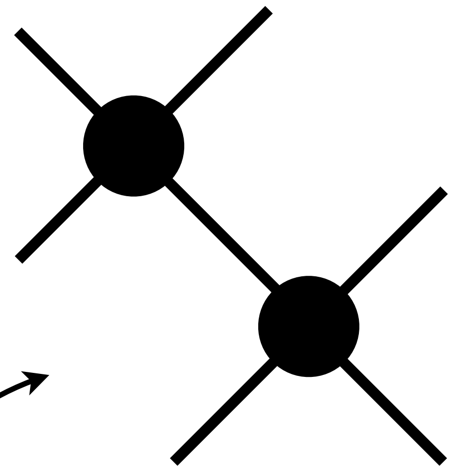
bound state and
resonance poles

Three hadrons in an infinite volume

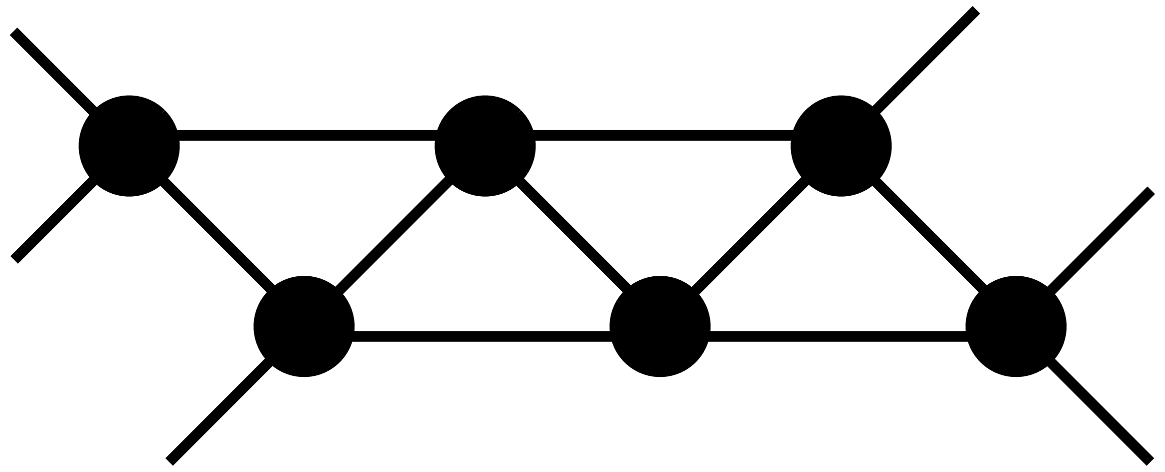
The three-body scattering amplitude using all orders perturbation theory.

Sum over all connected 3-to-3 diagrams…

$$i\mathcal{M}_{3,\mathrm{con.}} =$$



$$+ \cdots +$$



$$+ \cdots$$

$$G \sim \frac{1}{(P-p-k)^2 - m^2}$$

satisfies an integral equations

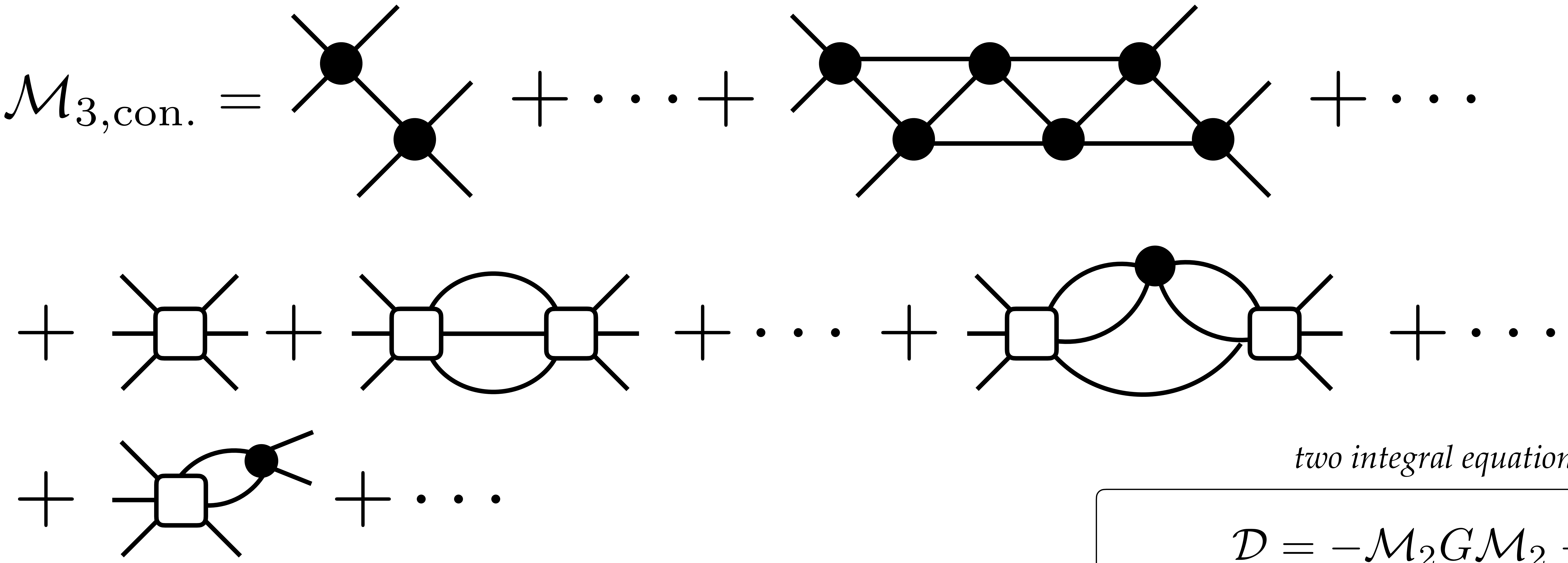
$$i\mathcal{D} = i\mathcal{M}_2 iG i\mathcal{M}_2 + \int i\mathcal{M}_2 iG\, i\mathcal{D}$$

Three hadrons in an infinite volume

The three-body scattering amplitude using all orders perturbation theory.

Sum over all connected 3-to-3 diagrams…

$$i\mathcal{M}_{3,\text{con.}} =$$



two integral equations and you're done!

$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \int \mathcal{M}_2 G\, \mathcal{D}$$

$$\mathcal{L} = \frac{1}{3} + \mathcal{M}_2 \rho - \mathcal{D}\rho$$

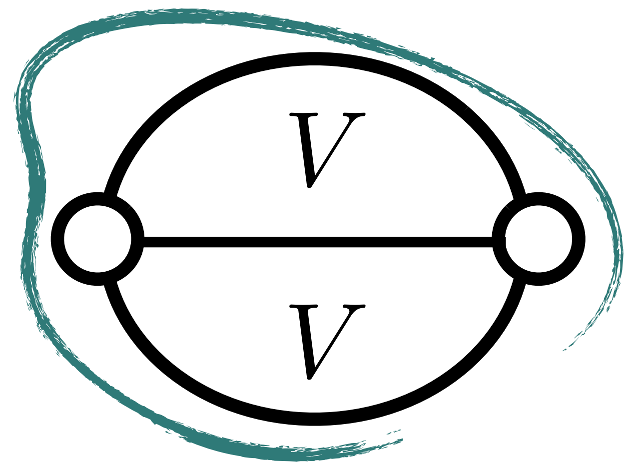
$$\mathcal{T} = \mathcal{K}_{\text{df},3} - \int \mathcal{K}_{\text{df},3}\, \rho\, \mathcal{L}\, \mathcal{T}$$

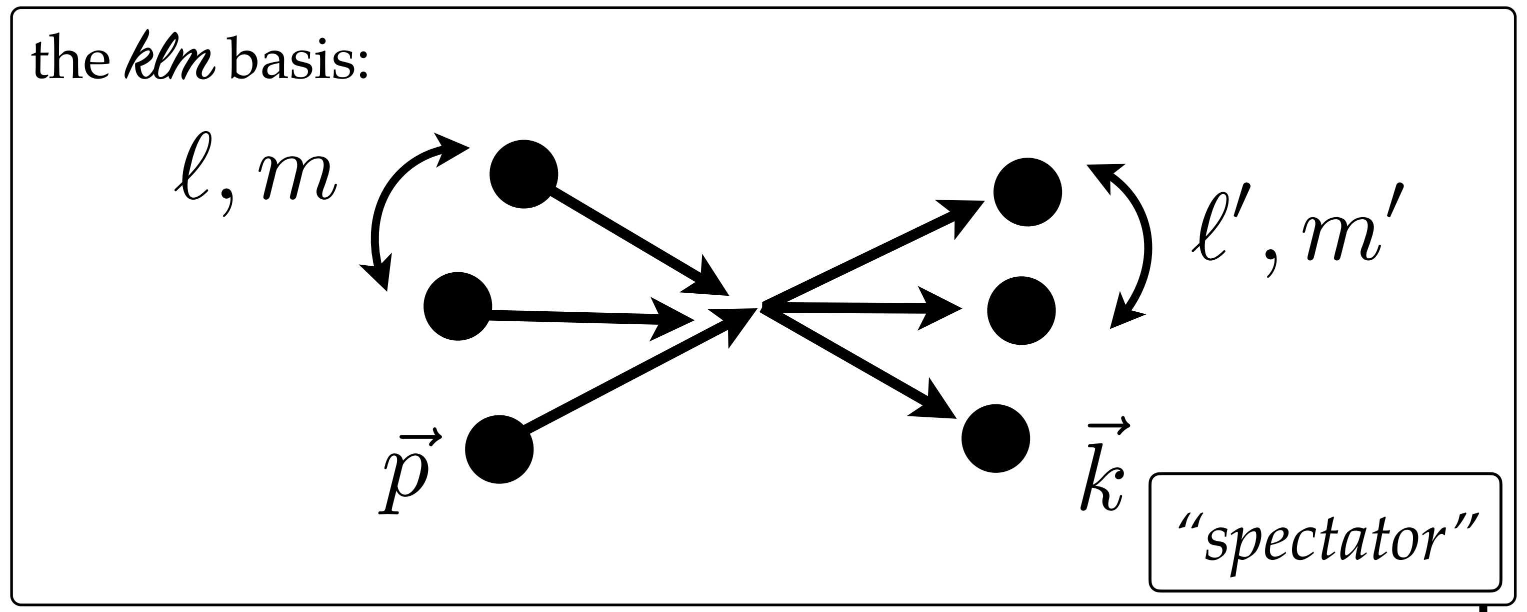
$$\mathcal{M}_{3,\text{con.}} = \mathcal{S}\left\{ \mathcal{D} + \mathcal{L}\, \mathcal{T}\, \mathcal{L}^T \right\}$$

Three hadrons in a finite volume

Finite-volume correlator…

$$C_L^{2pt.}(P) = \bigcirc\!\!\!-\!\!\!\bigcirc + \cdots$$



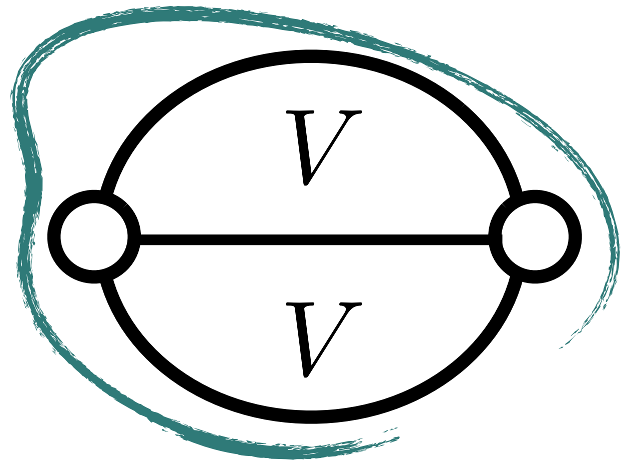


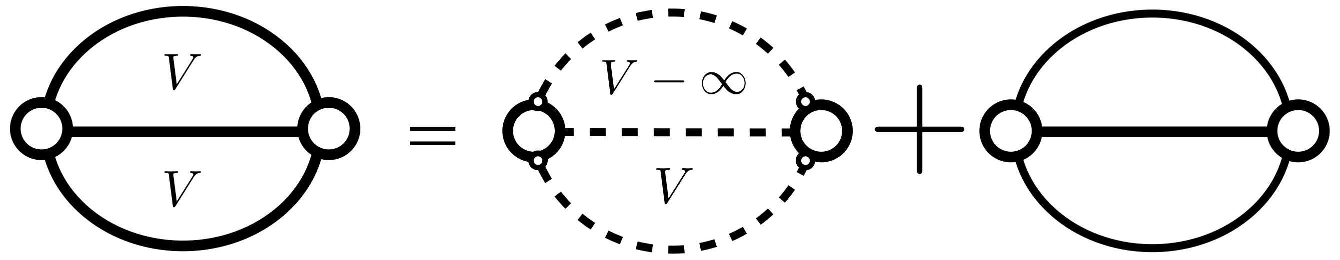
not a natural basis in a infinite volume…that's ok!

More degrees of freedom.
In infinite volume = (6 particles with 3 unfixed momenta) - (10 Poincare generators) = 8 d.o.f.
The *klm* basis has 1 + 4 + 6 = 11 d.o.f. …also ok!

Three hadrons in a finite volume

Finite-volume correlator…

$C_L^{2pt.}(P) =$  $+ \cdots$



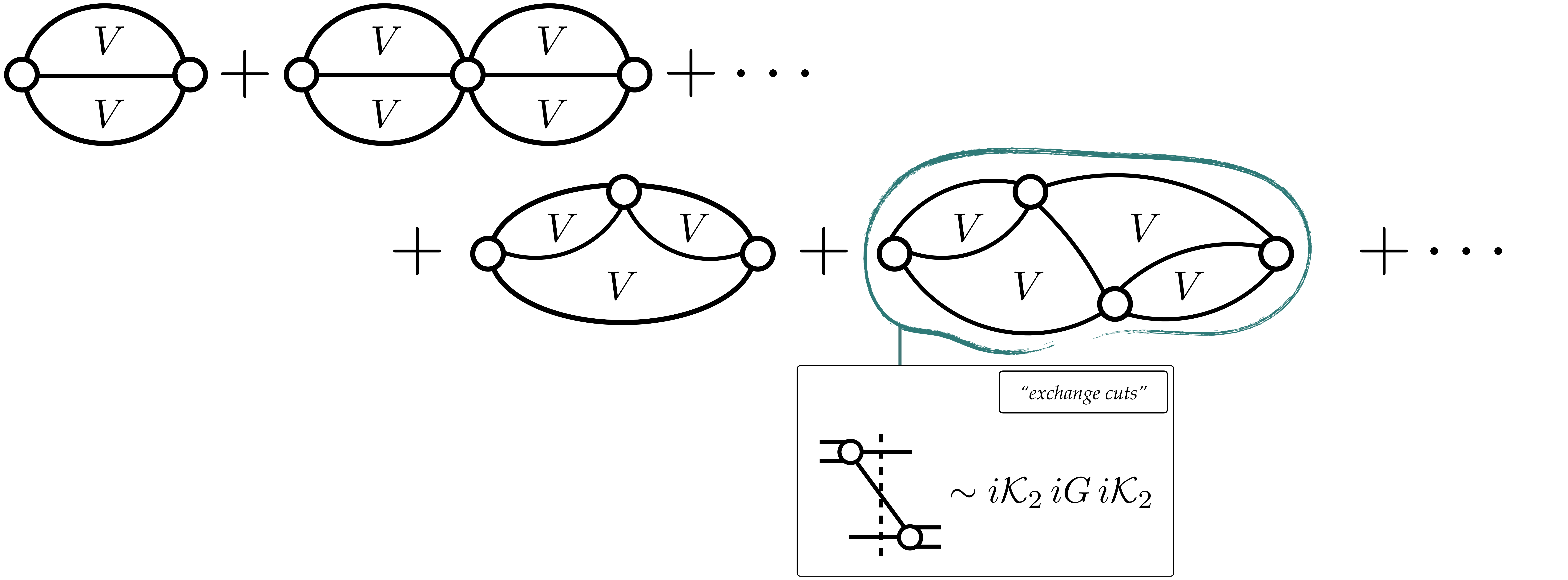
cut-off

$$F_{k\ell m;p\ell' m'} = \delta_{kp} \frac{H(\vec{k})}{4\omega_k} \left[\frac{1}{L^3} \sum_{\vec{a}}^{\text{UV}} - \int_a^{\text{UV}} \right] \frac{4\pi\, Y_{\ell m}(\hat{a})\, Y^*_{\ell' m'}(\hat{a})}{4\omega_a \omega_{ka}(E - \omega_k - \omega_a - \omega_{ka})}$$

Three hadrons in a finite volume

Finite-volume correlator…

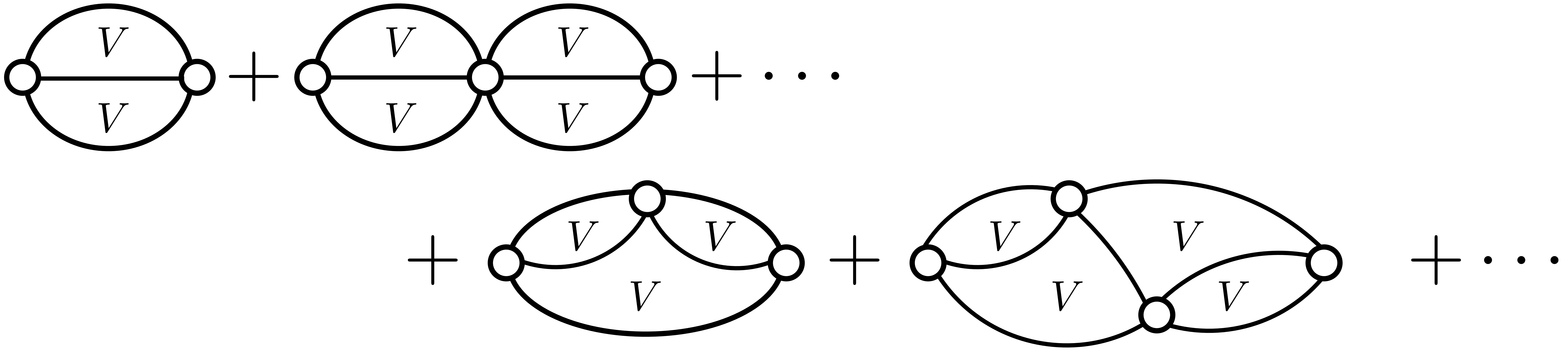
$$C_L^{2pt.}(P) =$$



Three hadrons in a finite volume

Finite-volume correlator…

$$C_L^{2pt.}(P) =$$



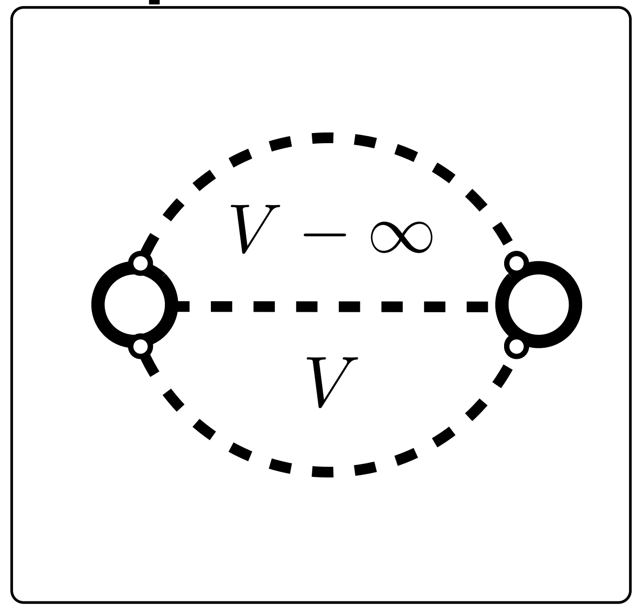
after a lot of massaging…

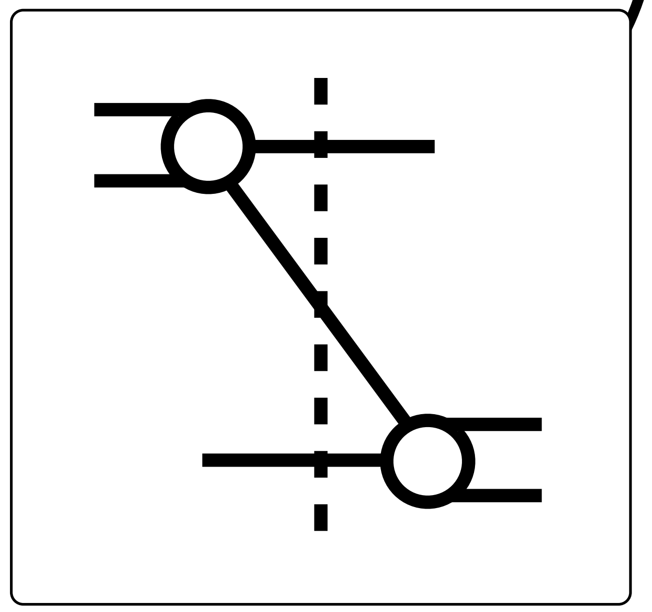
$$\det\left[F_3(E_L, L) + \mathcal{K}_{\mathrm{df},3}^{-1}(E_L)\right] = 0$$

Isotropic approximation

Consider the case where two-body system is an S-wave

$$[F_3^s]_{kp} = \frac{1}{L^3}\left[\frac{F^s}{3} - F^s \frac{1}{1/(2\omega\mathcal{K}_2^s) + F^s + G^s} F^s\right]_{kp}$$





Furthermore, consider the case where the K-matrix does not depend on the spectator momentum. Then the quantization condition reduces to:

$$F_3^{\mathrm{iso}}(E,\vec{P},L) = -1/\mathcal{K}_{\mathrm{df},3}^{\mathrm{iso}}(E^*)$$

Where:

$$F_3^{\mathrm{iso}}(E,L) = \sum_{k,p}[F_3^s]_{kp}$$

Solving integral equations

Assuming $\mathscr{K}_{\text{df}} = 0$ and partial wave-projecting onto $\ell = 0$ sector,

$$\mathcal{D}_{\mathsf{s}}^{(u,u)}(p,k) = -\mathcal{M}_2(E^{\star}_{2,p})G_{\mathsf{s}}(p,k,\epsilon)\mathcal{M}_2(E^{\star}_{2,k}) - \mathcal{M}_2(E^{\star}_{2,p})\int_0^{k_{\mathsf{max}}} \frac{k'^2\,dk'}{(2\pi)^2\omega_{k'}}\,G_{\mathsf{s}}(p,k',\epsilon)\mathcal{D}_{\mathsf{s}}^{(u,u)}(k',k),$$

- Discretized momenta: meshes, splines,…
- Soften or isolate possible singularities: non-zero epsilon, integrate out poles analytically, …
- Write as a matrix form:

$$\boldsymbol{D}(N,\epsilon) = -\boldsymbol{\mathcal{M}}\cdot\boldsymbol{G}(\epsilon)\cdot\boldsymbol{\mathcal{M}} - \boldsymbol{\mathcal{M}}\cdot\boldsymbol{G}(\epsilon)\cdot\boldsymbol{P}\cdot\boldsymbol{D}(N,\epsilon)$$

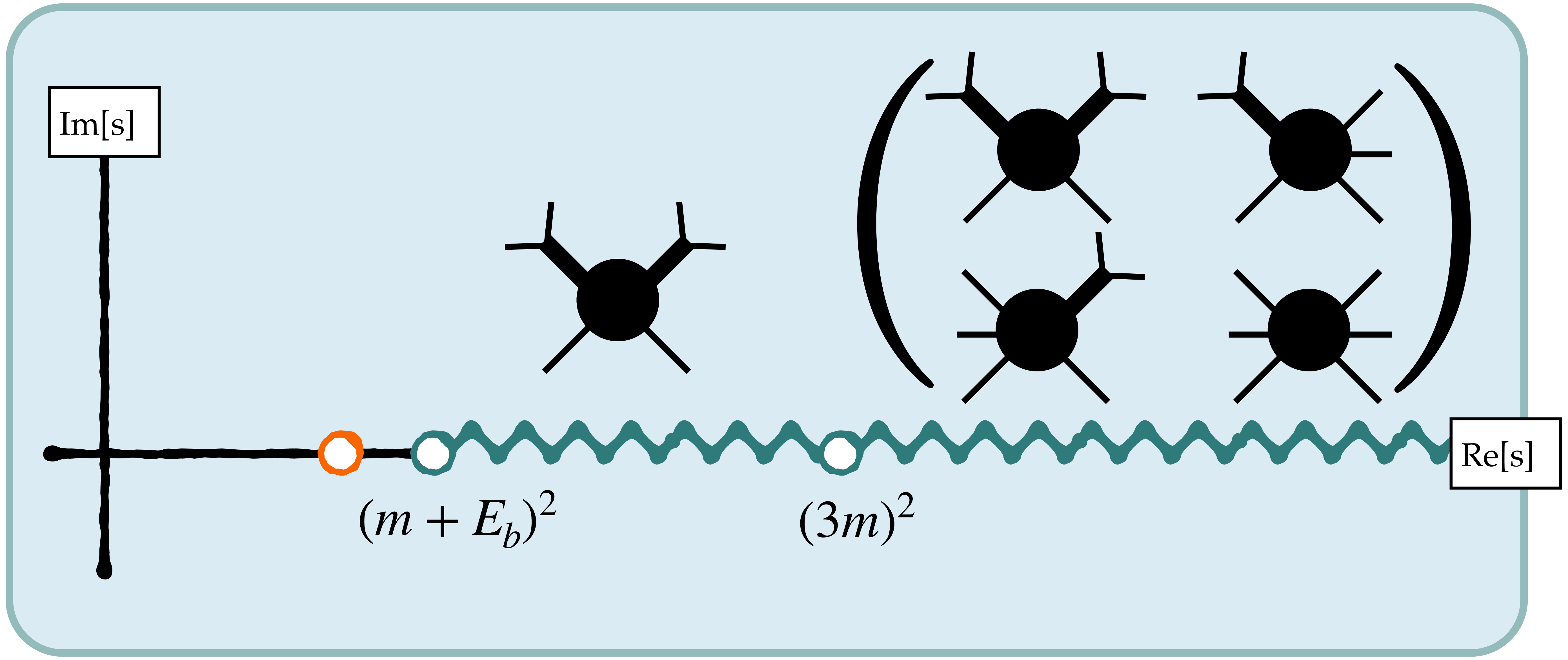
- Recover the exact results when taking $\epsilon = \eta/N \to 0$.

Jackura, RB, Dawid, Islam, & McCarty (2020)

Consistency checks for toy model

Consider a toy theory with a two-body bound state

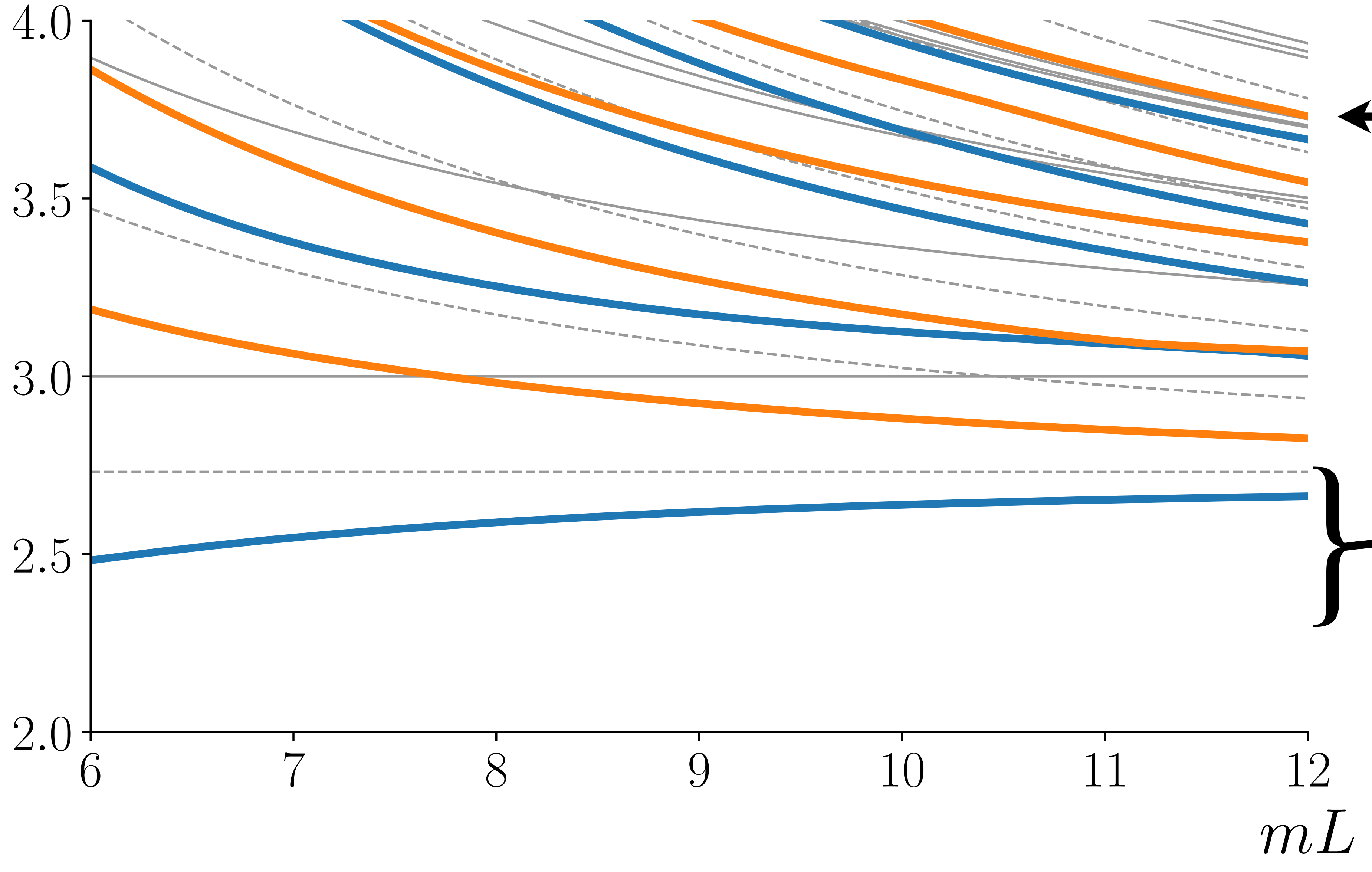
- Simple case $q \cot \delta = -\dfrac{1}{a}$
- if $a > 0$, there is a bound state with binding energy $E_b = 2\sqrt{m^2 - \dfrac{1}{a^2}}$



Consistency checks for toy model

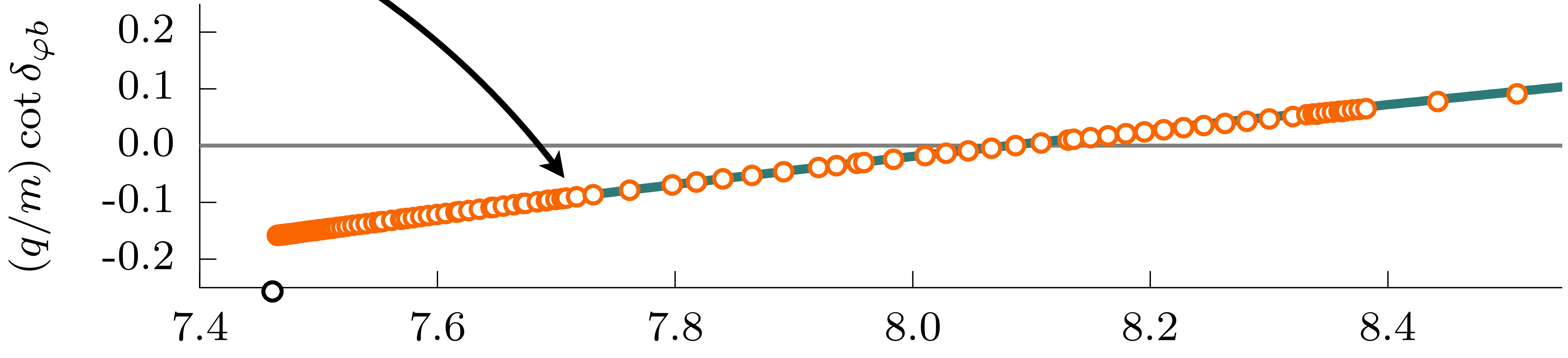
Alternatively to solving the integral equations, one can instead:

- Fix $ma = 2$ and $\mathscr{K}_{\text{df}} = 0$
- obtain E_L using 3-body quantization by Hansen & Sharpe (2014),
- obtain $q \cot \delta_{\varphi b}$ using the two-body Lüscher formalism.



$$\det\left[F_3(E_L, L) + \mathcal{K}_{\text{df},3}^{-1}(E_L)\right] = 0$$

$$\det\left[F(E_L, L) + \mathcal{M}^{-1}(E_L)\right] = 0$$

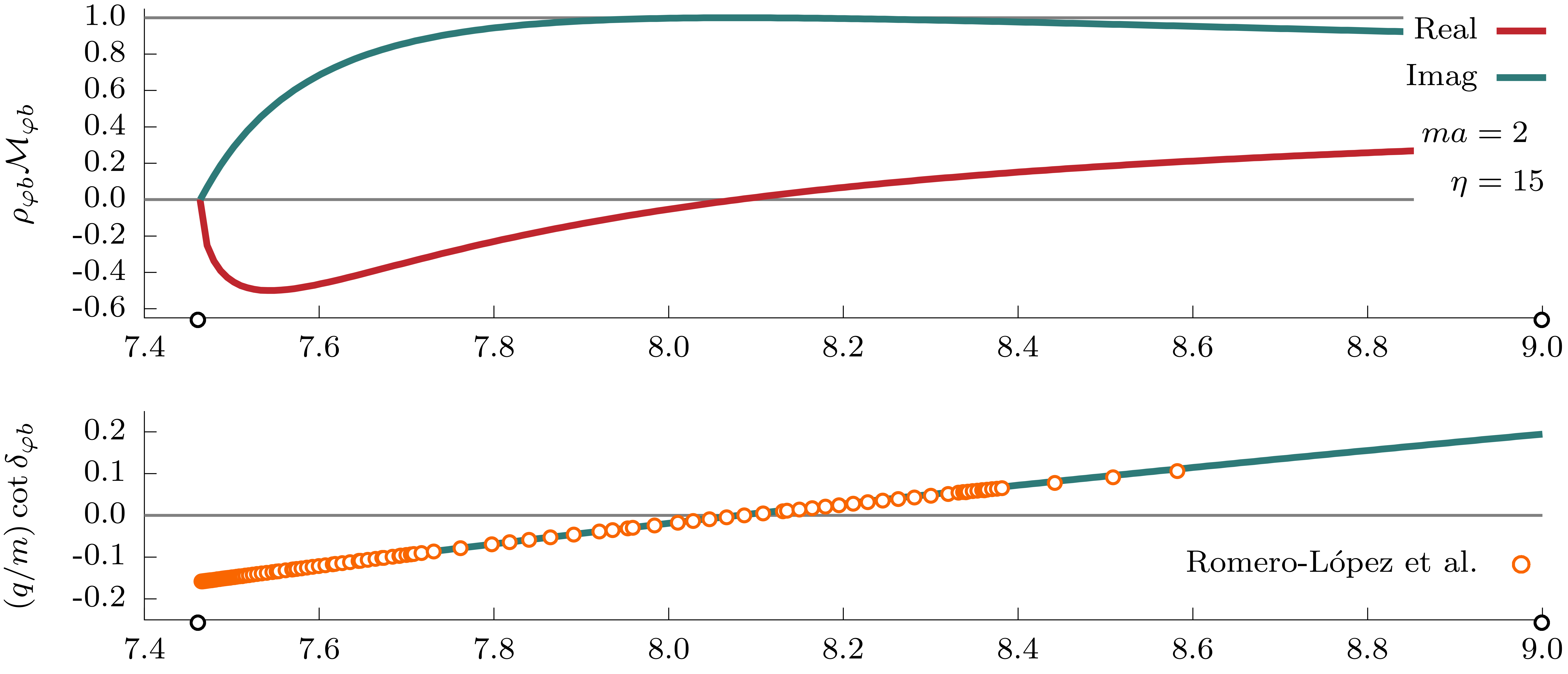


Romero-López, Sharpe, Blanton, RB, & Hansen (2019)

Consistency checks for toy model

One can also solve the integral equations to find agreement!

Two independent methods reproducing same exact amplitudes!

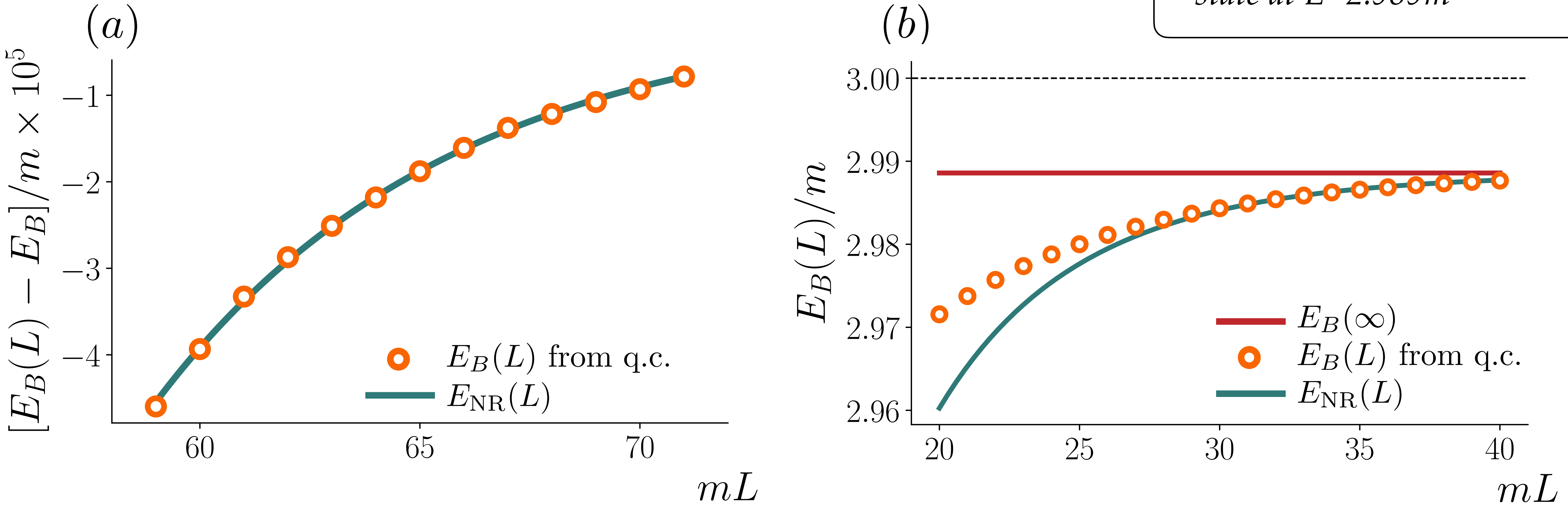


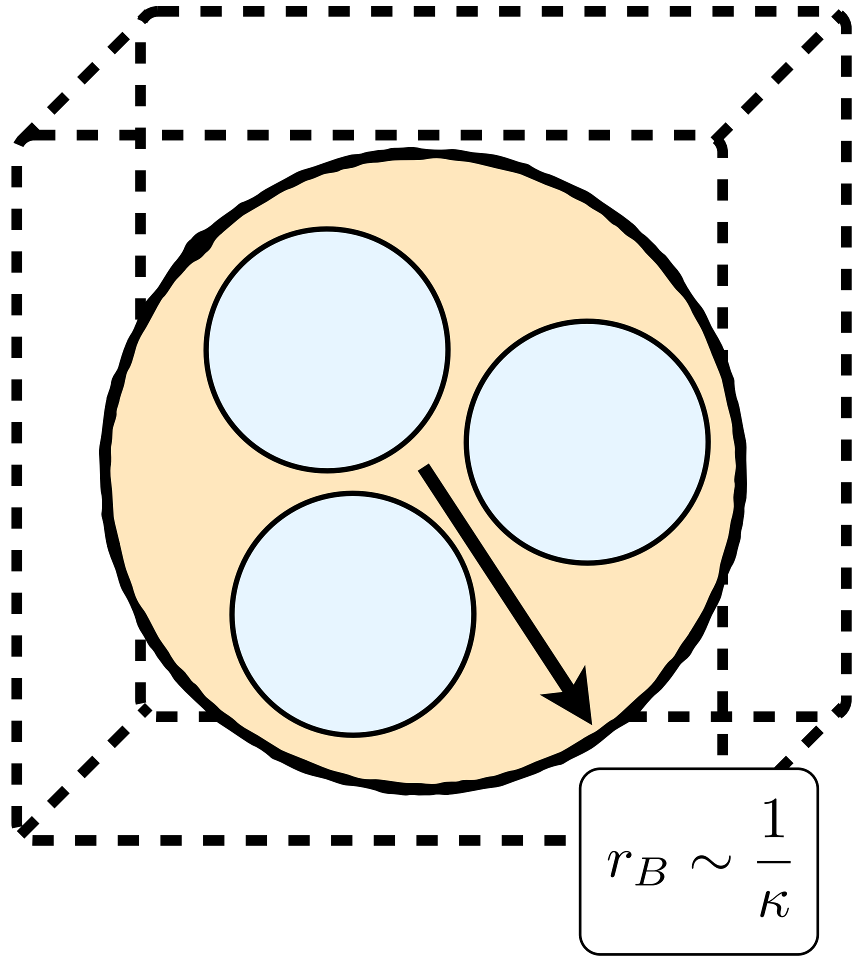
Jackura, RB, Dawid, Islam, & McCarty (2020)

Efimov-like physics

$$p \cot \delta_2 = -\frac{1}{a}$$

- ma=-10^4
- K_{df} is tuned to have a bound state at E=2.989m





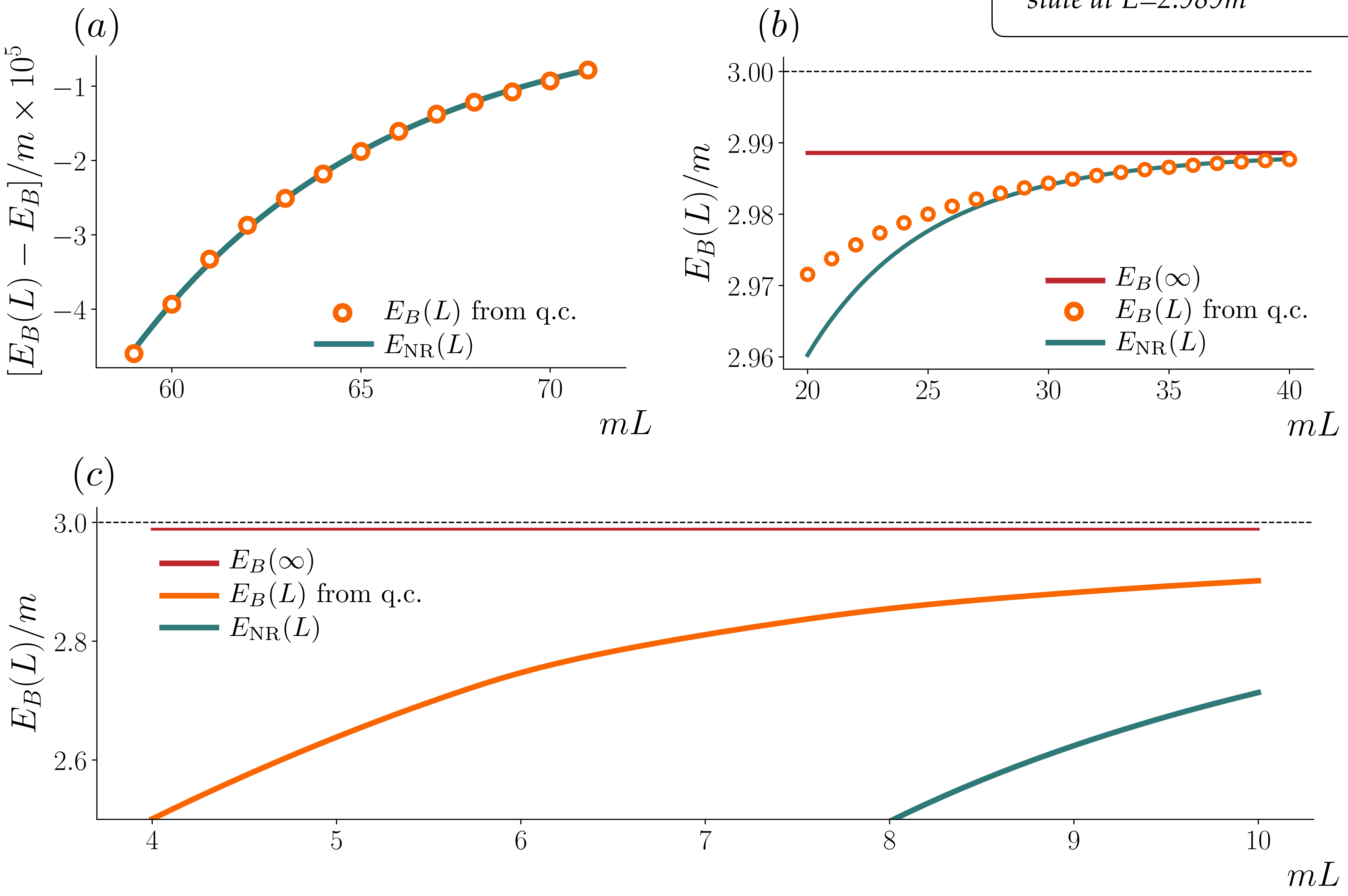
$$E_B(L) = 3 - \kappa^2 - 96.35|A|^2\kappa^2 \frac{e^{-2\kappa L/\sqrt{3}}}{(\kappa L)^{3/2}}$$

Meißner, Ríos, & Rusetsky (2015)

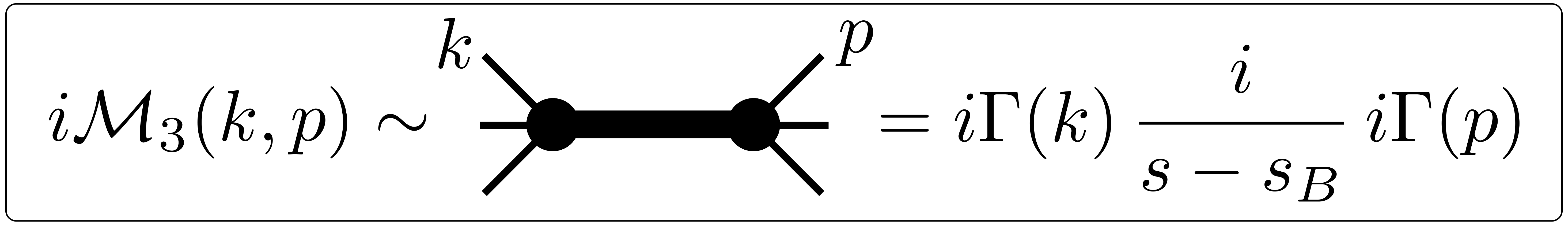
Efimov-like physics

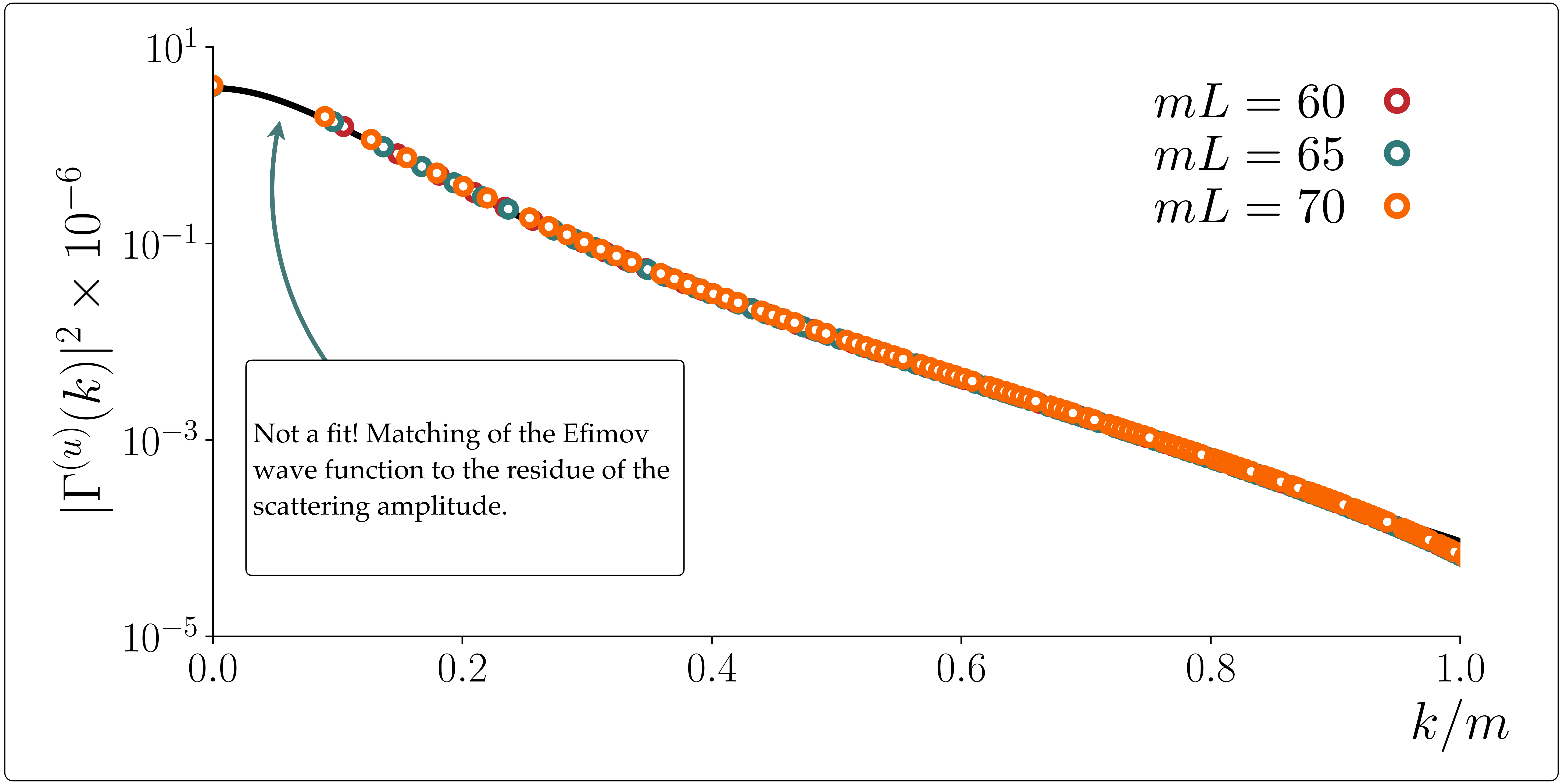
$$p \cot \delta_2 = -\frac{1}{a}$$

- ma=-10^4
- K_{df} is tuned to have a bound state at E=2.989m



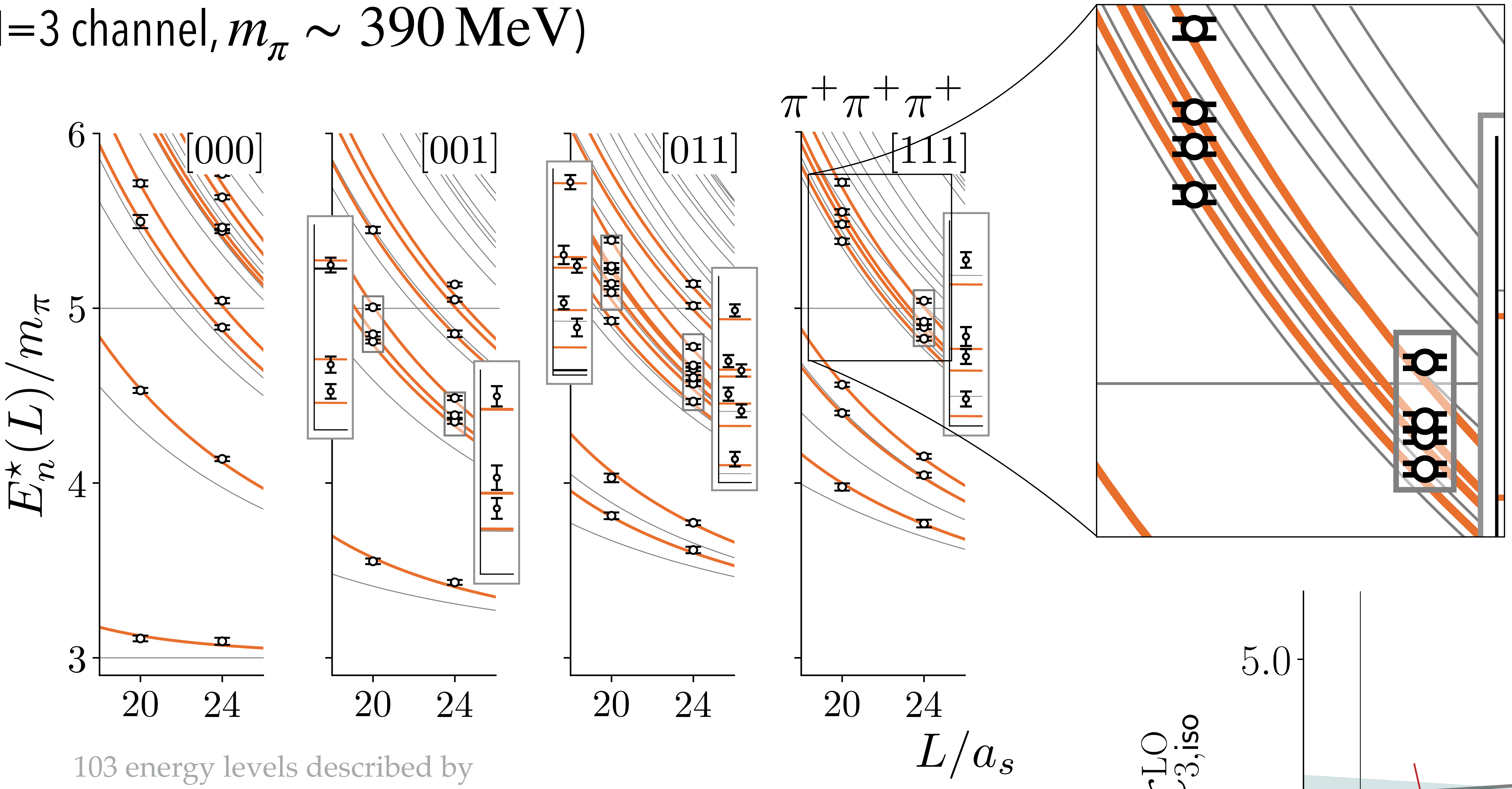
Efimov-like physics



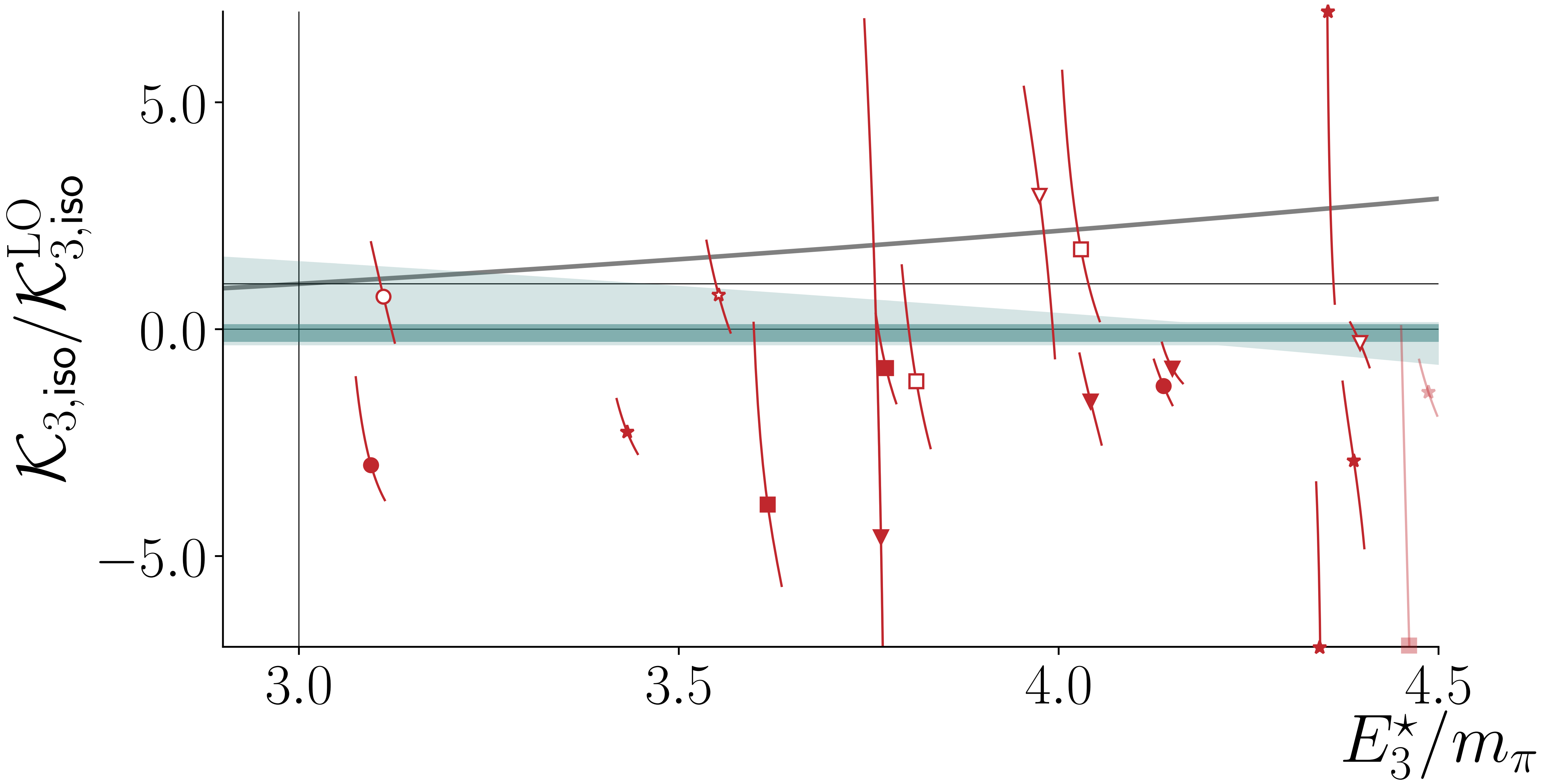


$\pi\pi\pi$

(I=3 channel, $m_\pi \sim 390\,\text{MeV}$)



103 energy levels described by three numbers: m_π, $a_{\pi\pi}$, $\mathcal{K}_{3,\text{iso}}$



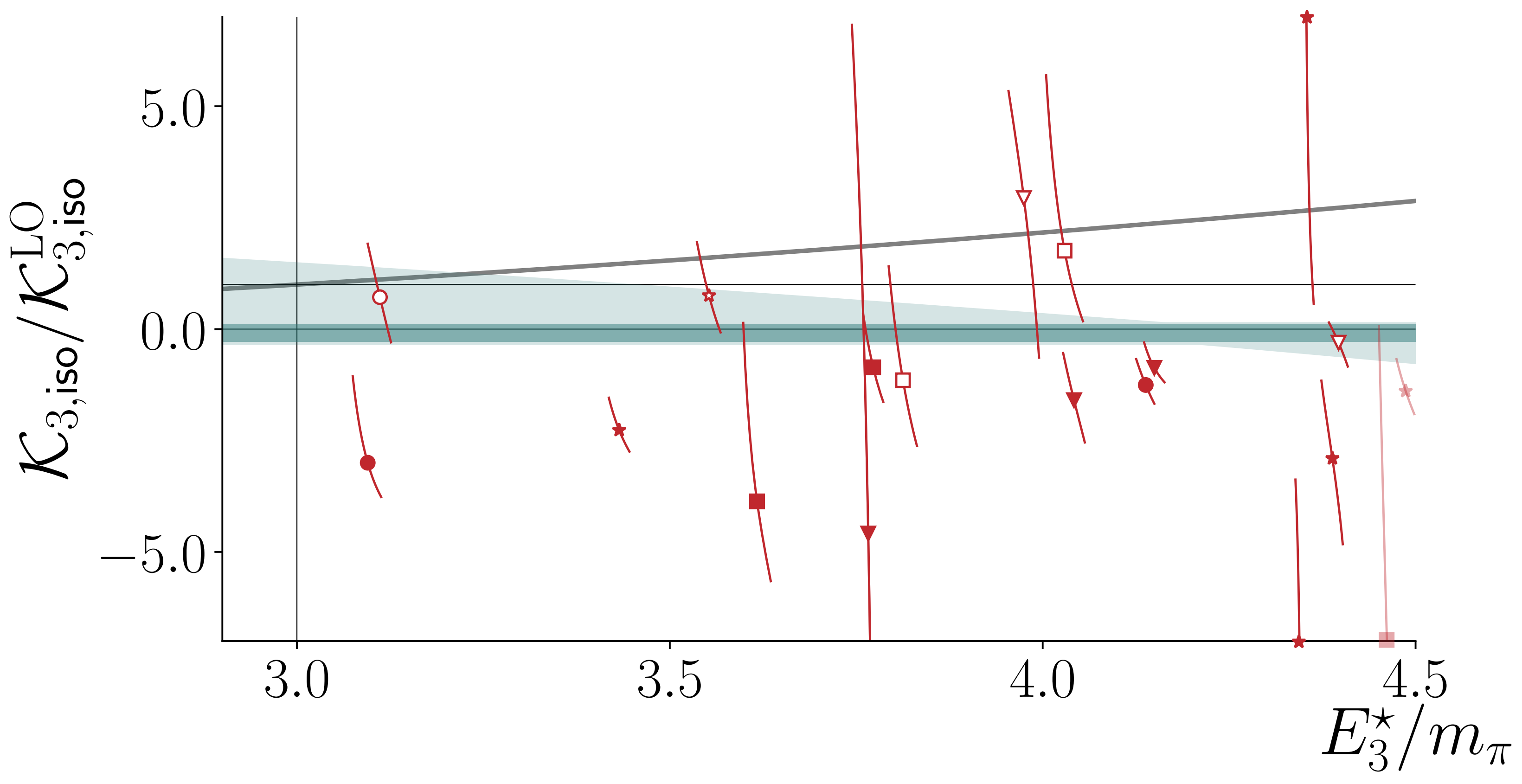
$m_\pi \sim 390\,\text{MeV}$

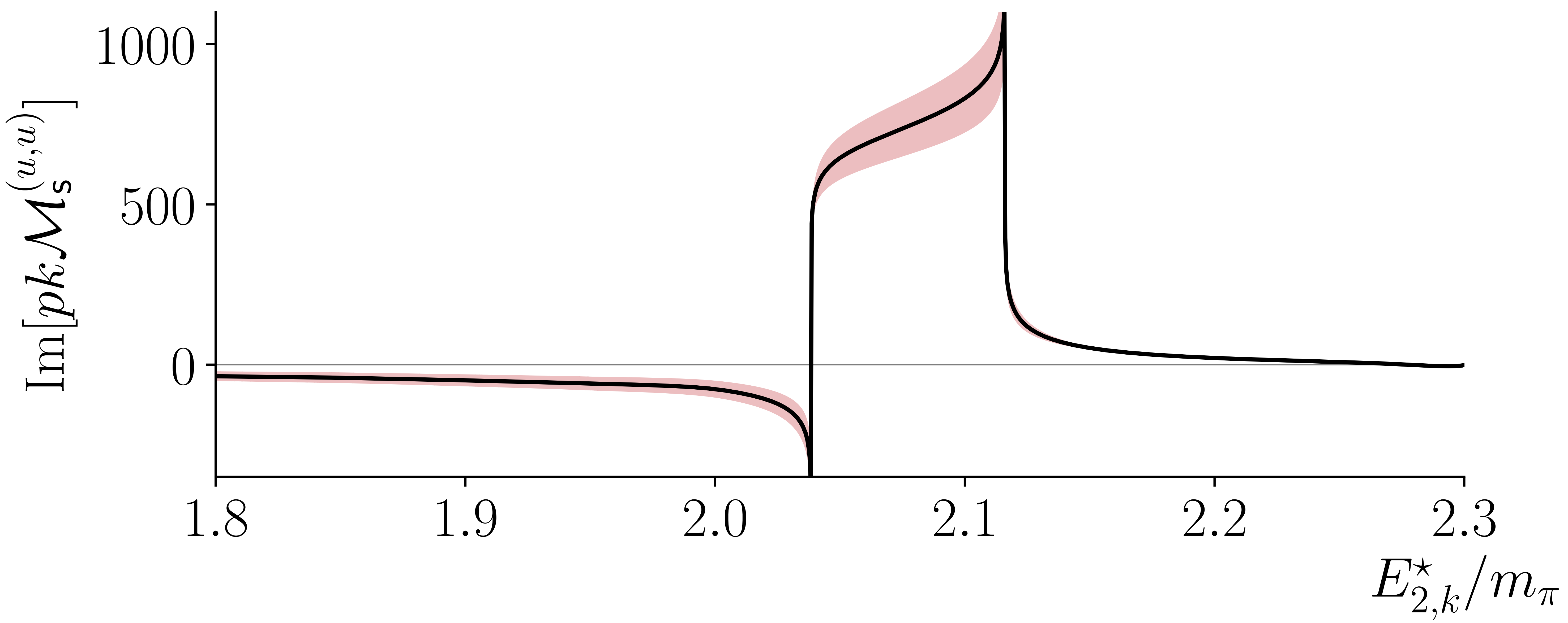
Hansen, RB, Edwards, Thomas, & Wilson (2020)

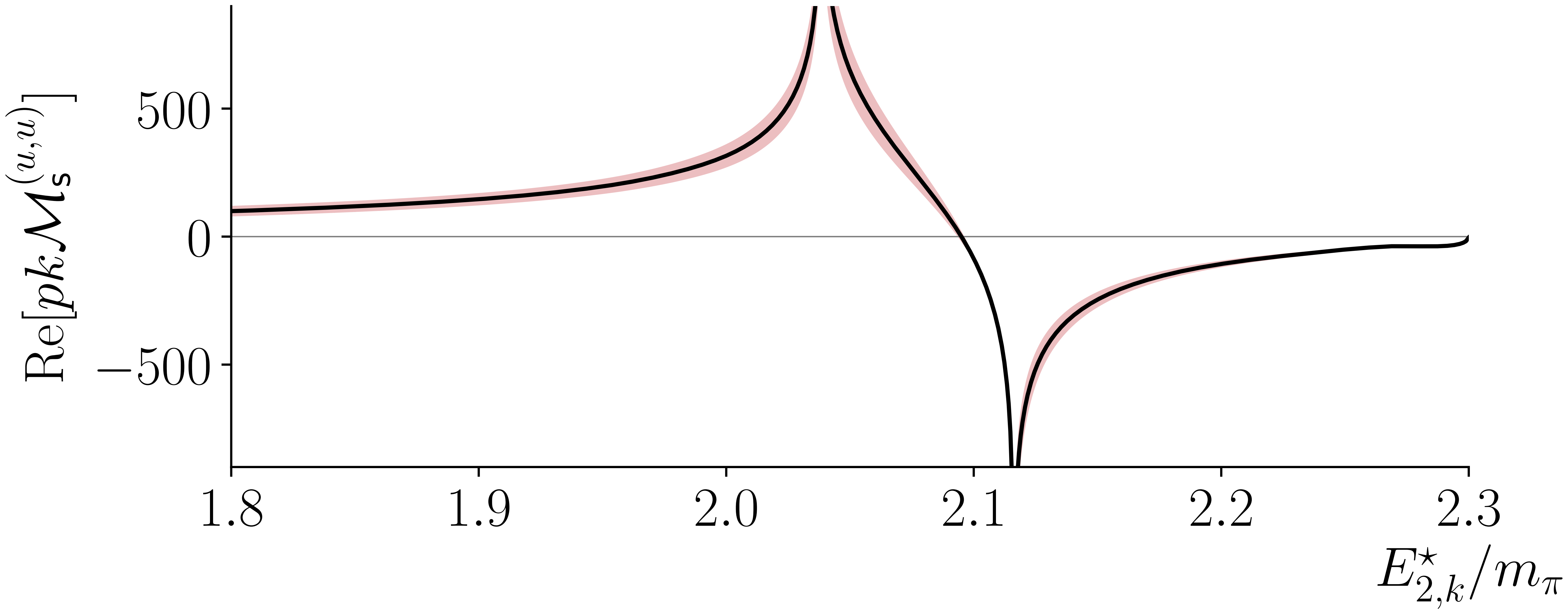
$\pi\pi\pi$ scattering

(I=3 channel, $m_\pi \sim 390\,\mathrm{MeV}$)

first 3body scattering amplitude from the lattice QCD!







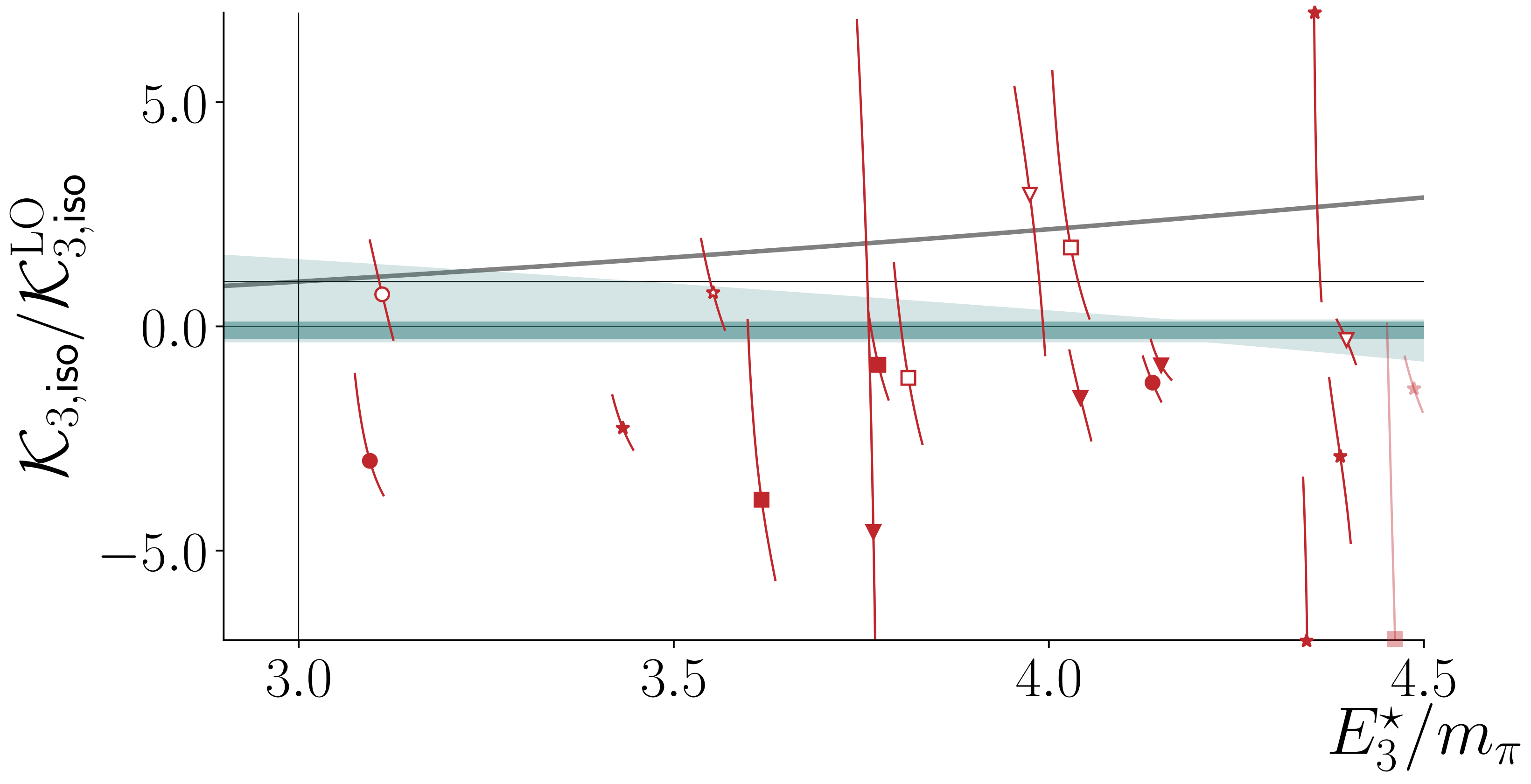
$m_\pi \sim 390\,\mathrm{MeV}$

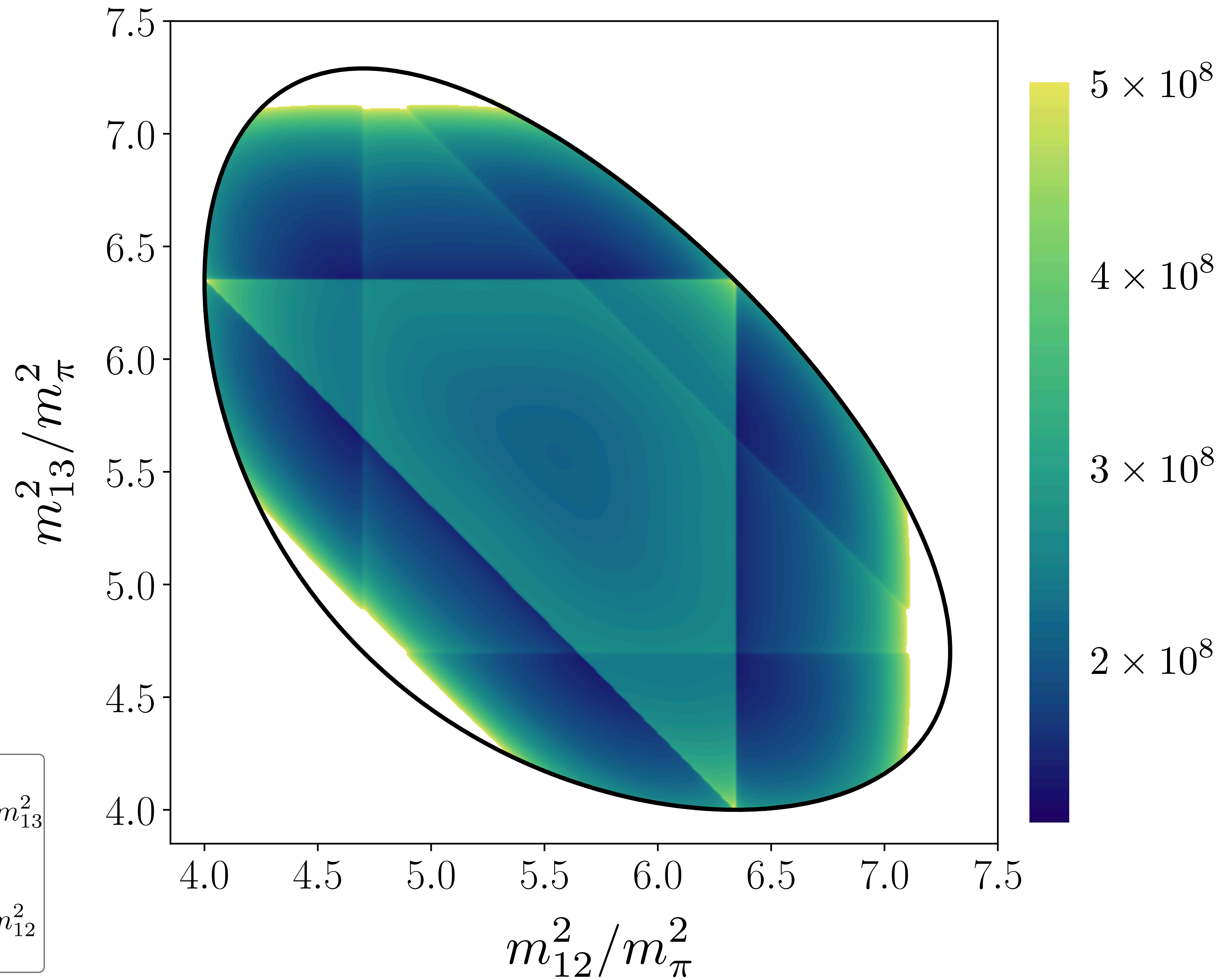
Hansen, RB, Edwards, Thomas, & Wilson (2020)

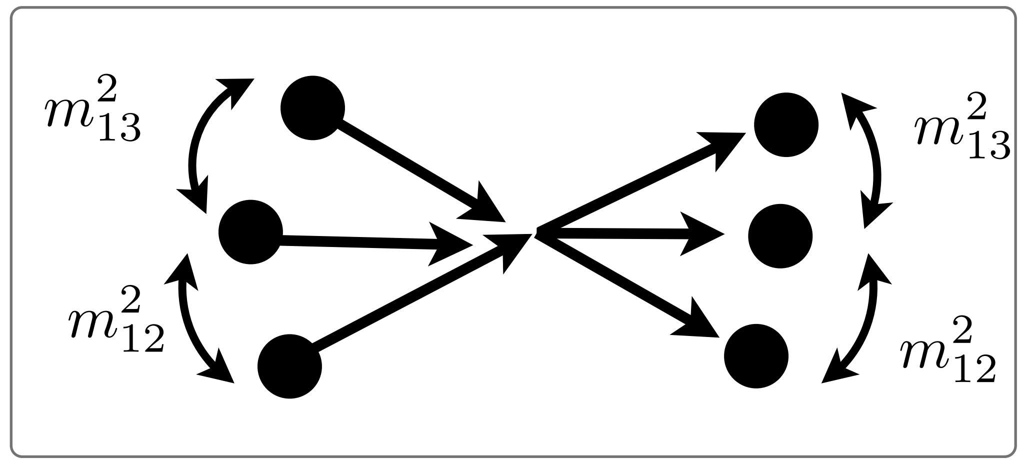
$$\mathcal{D}_s^{(u,u)}(p,k) = -\mathcal{M}_2(E_{2,p}^\star)G_s(p,k,\epsilon)\mathcal{M}_2(E_{2,k}^\star) - \mathcal{M}_2(E_{2,p}^\star)\int_0^{k_{\max}} \frac{k'^2\,dk'}{(2\pi)^2\omega_{k'}}\,G_s(p,k',\epsilon)\mathcal{D}_s^{(u,u)}(k',k),$$

$\pi\pi\pi$ scattering
(I=3 channel, $m_\pi \sim 390\,\mathrm{MeV}$)

first 3body scattering amplitude from the lattice QCD!



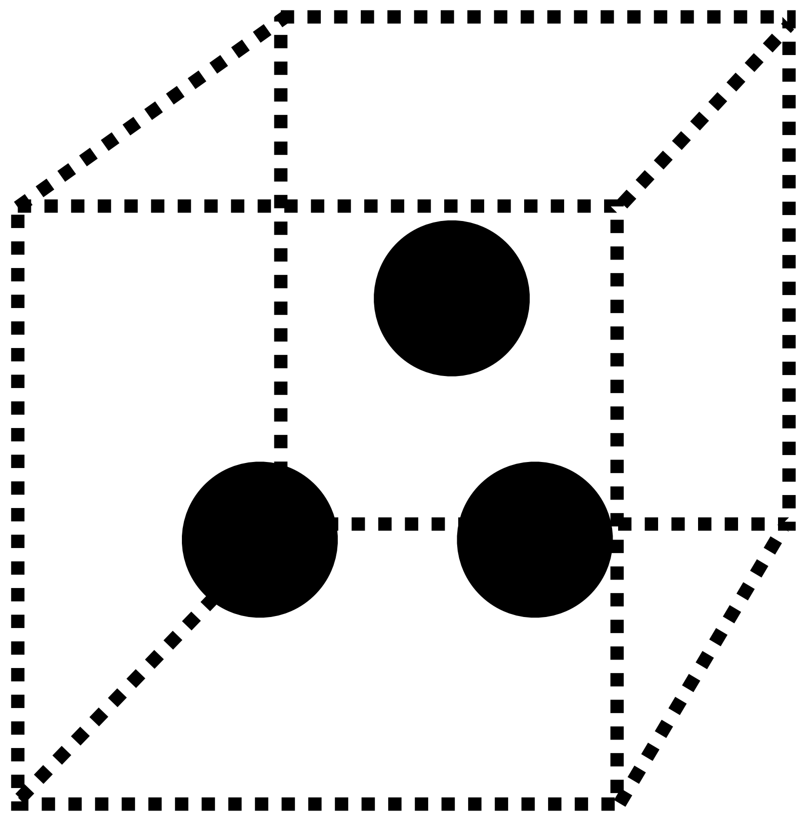


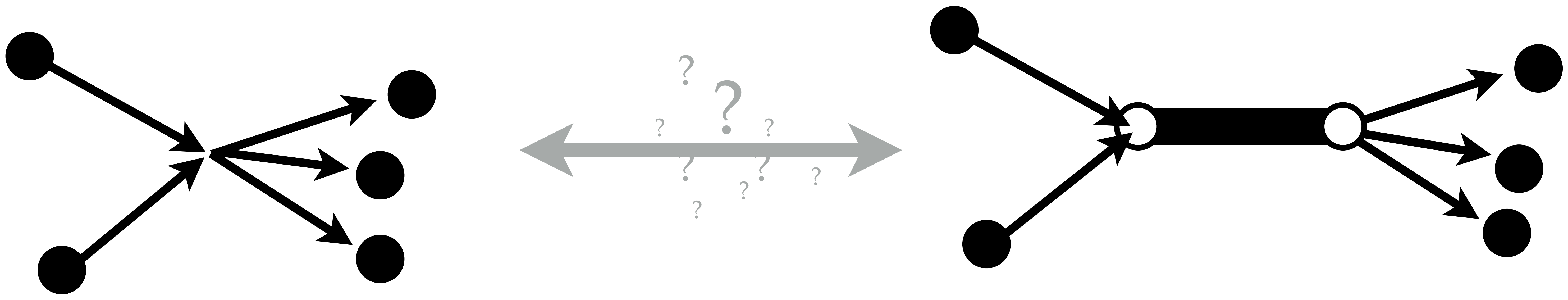


$m_\pi \sim 390\,\mathrm{MeV}$

Hansen, RB, Edwards, Thomas, & Wilson (2020)

We are just getting started!





Hansen & Sharpe ('14, '15)

Mai & Döring ('17)

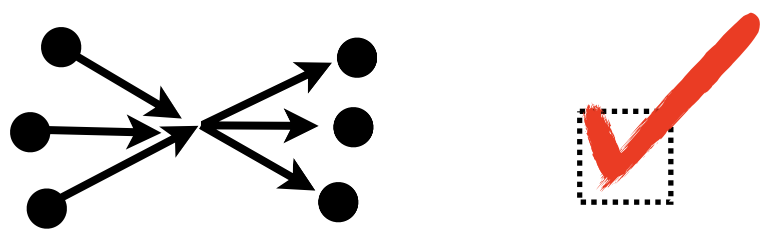
RB, Hansen & Sharpe ('18)

Hansen, Romero-Lopez & Sharpe ('20)

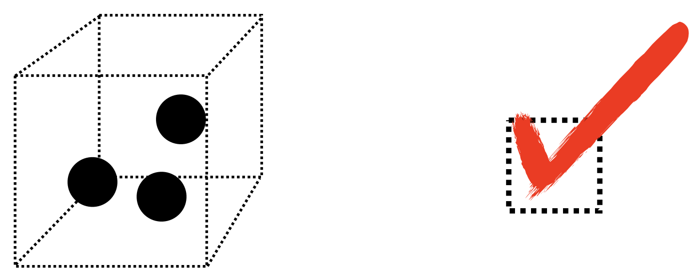
Blanton & Sharpe ('20)

Jackura et al. ('20)

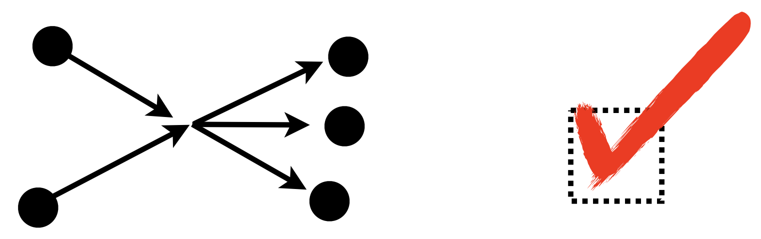
Unitarity in 3Body systems



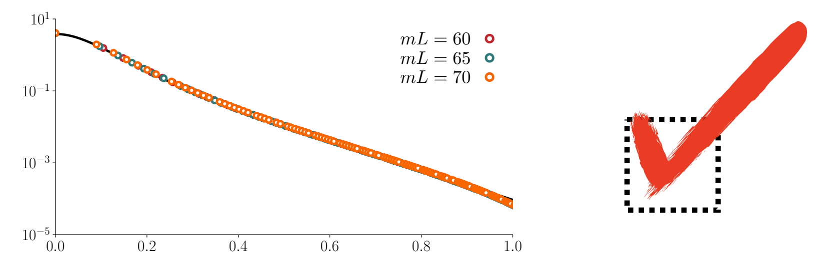
FV formalism for spinless states



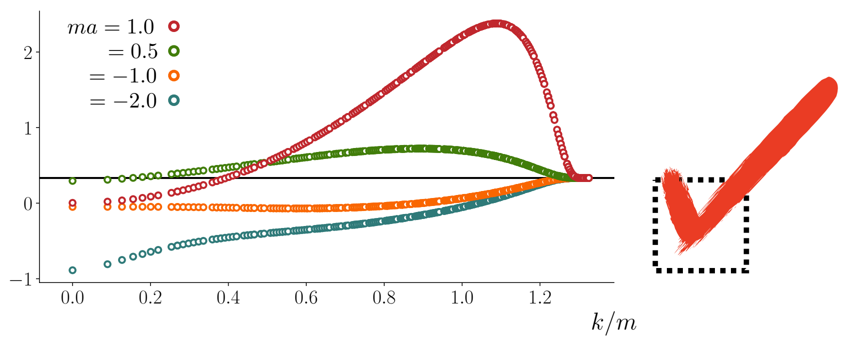
coupled 2/3B systems



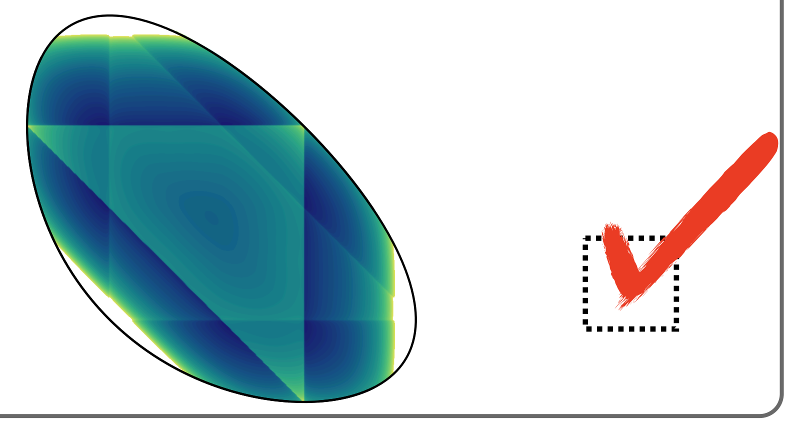
analytic and numerical checks



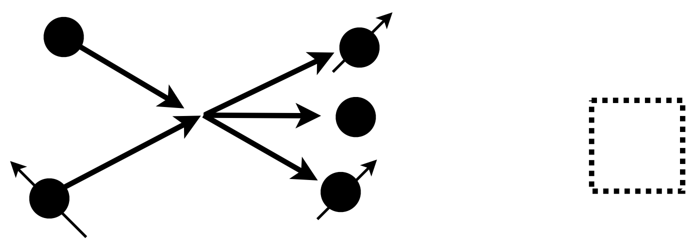
integral equations



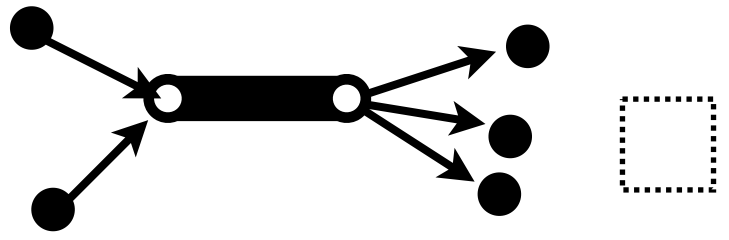
Actual calculations



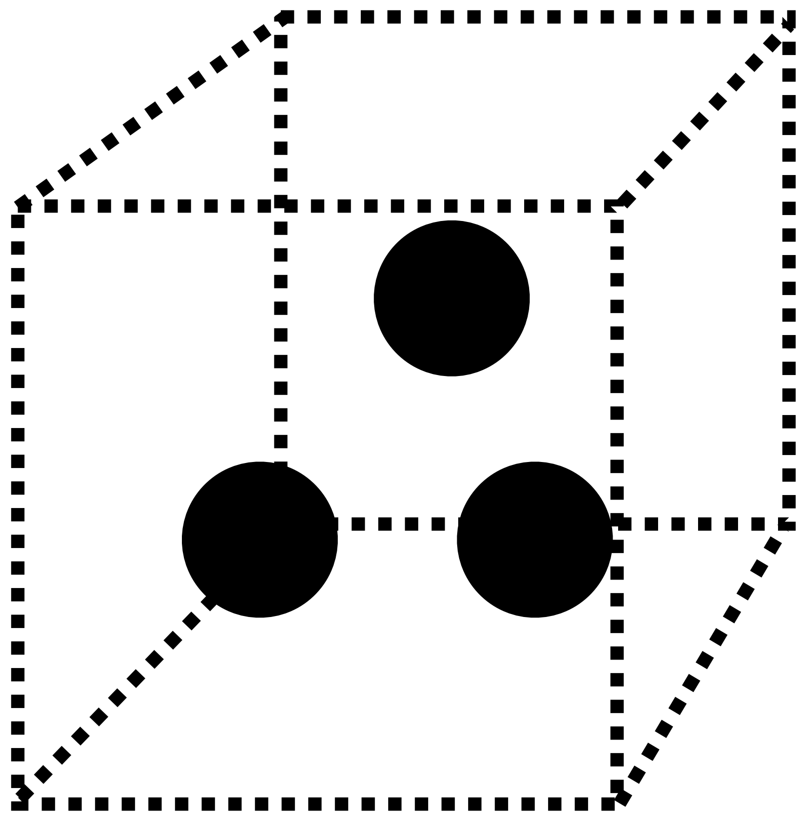
Spin, multichannels



Analytic continuation



We are just getting started!



Hansen & Sharpe ('14, '15)

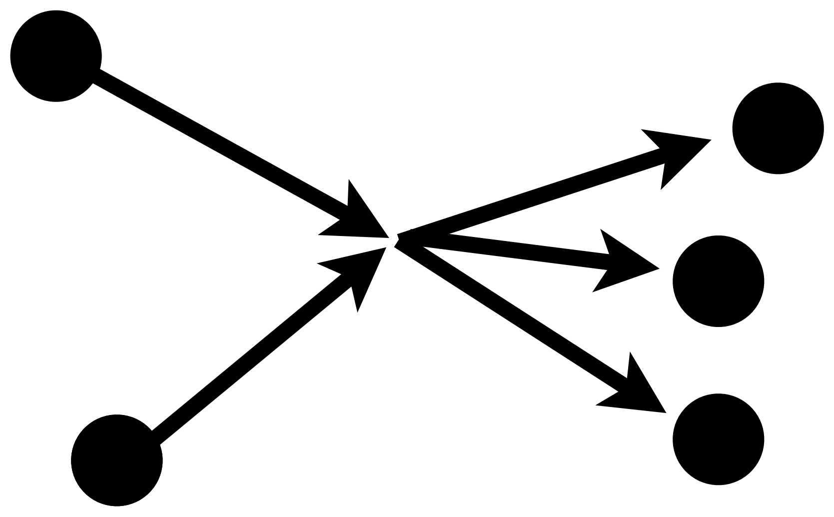
Mai & Döring ('17)

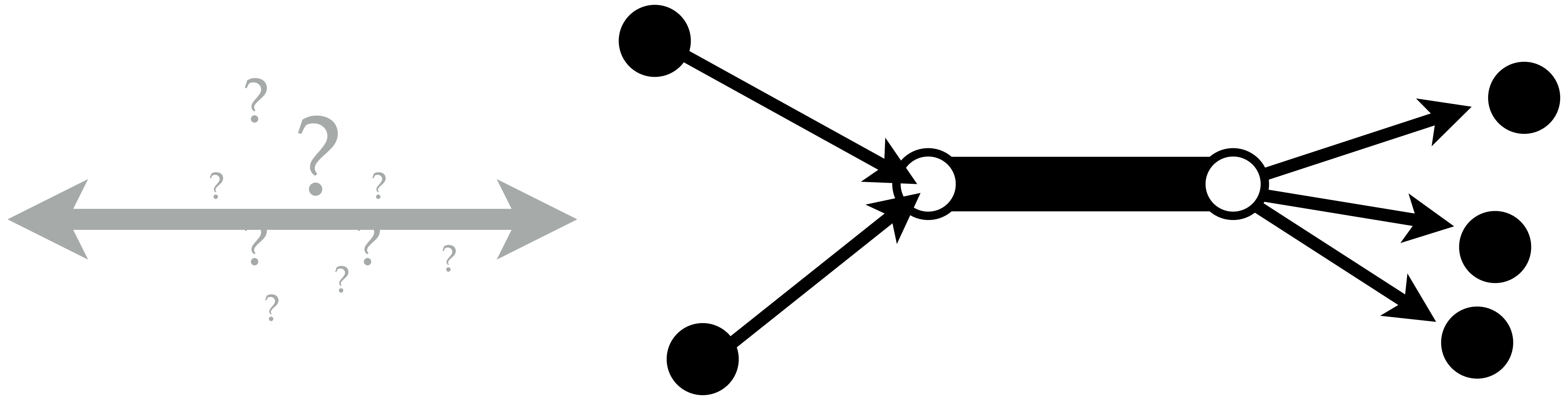
RB, Hansen & Sharpe ('18)

Hansen, Romero-Lopez & Sharpe ('20)

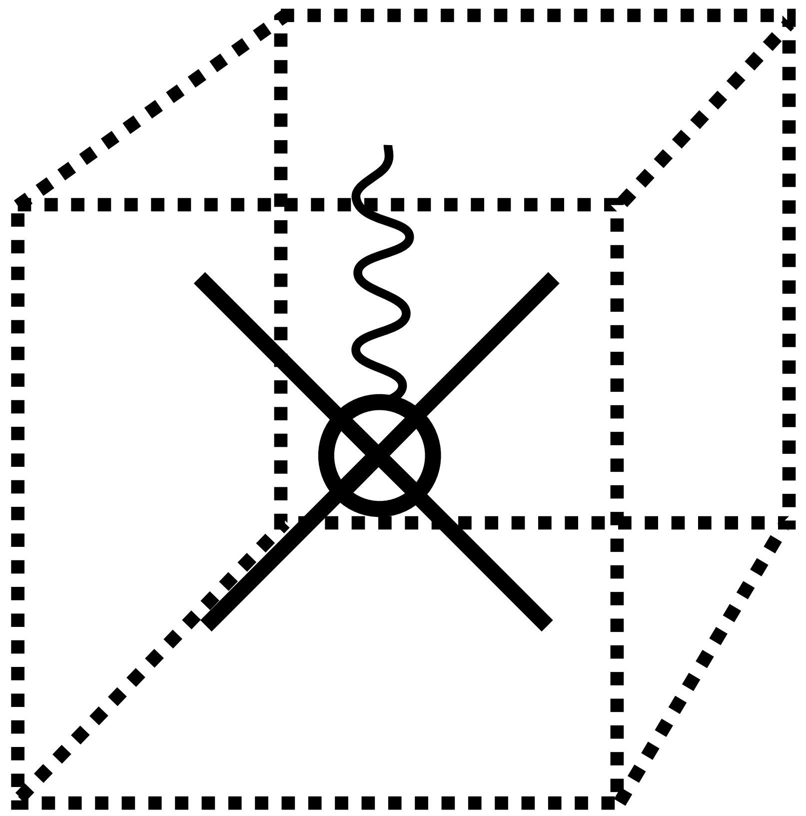
Blanton & Sharpe ('20)

Jackura et al. ('20)



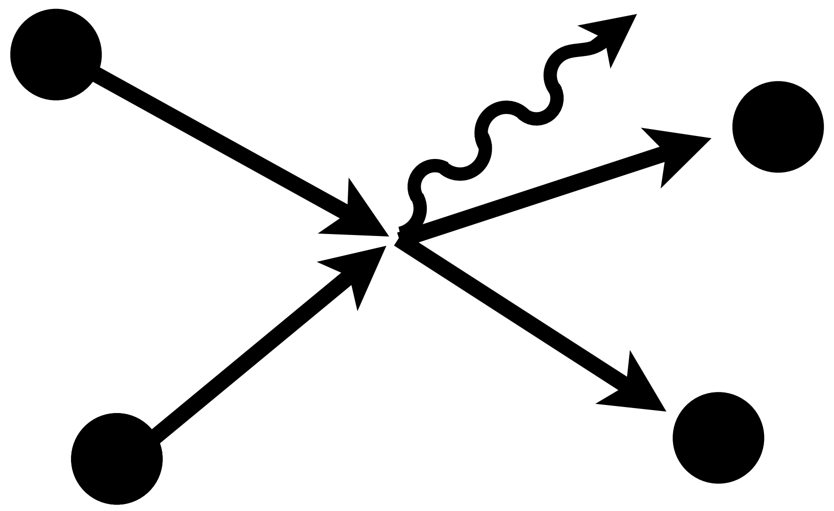


Structural information

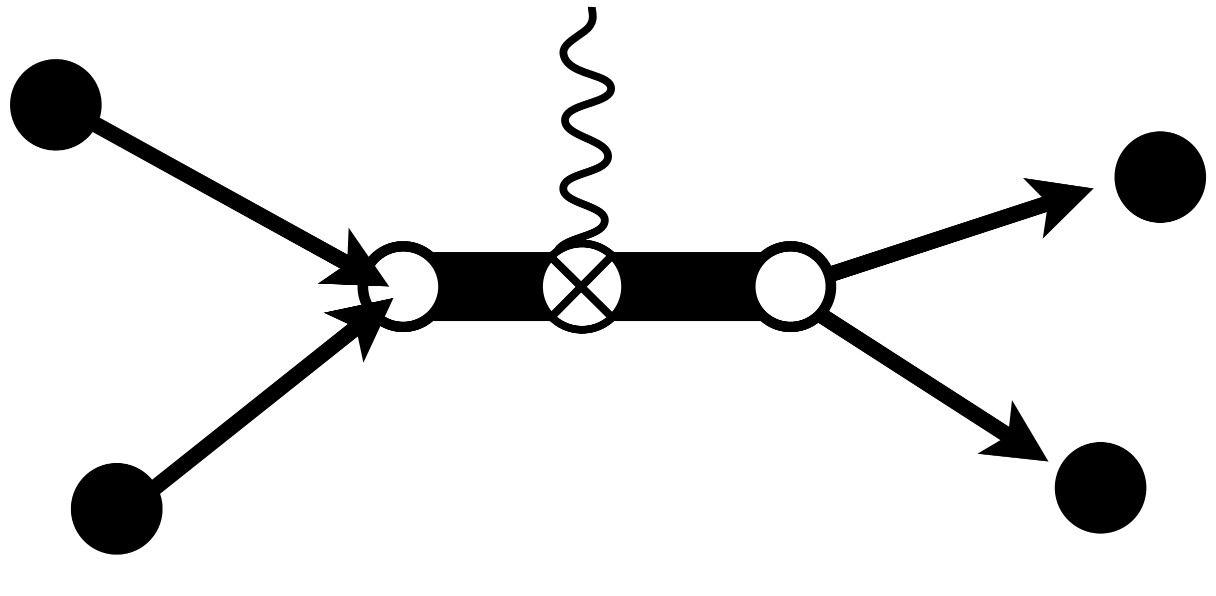


RB & Hansen ('15)

Baroni, RB, Jackura, Hansen & Ortega-Gama ('18)



RB, Jackura, Ortega-Gama, Sherman ('21)



A review/introduction

slightly out of date

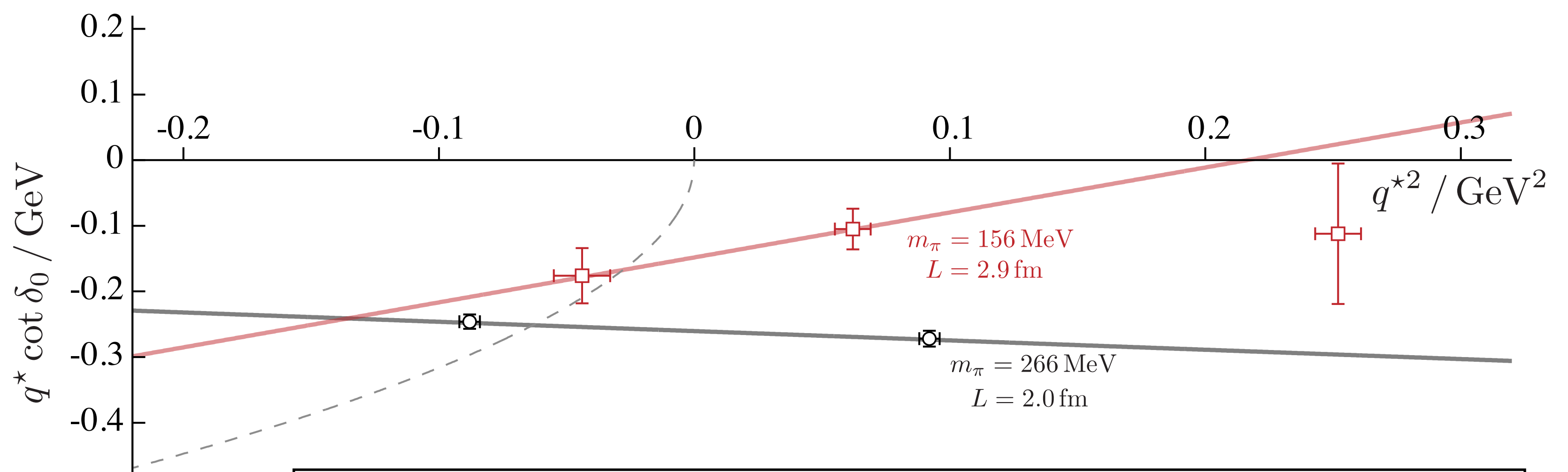
Scattering processes and resonances from lattice QCD

Raúl A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] and Ross D. Young[3,‡]

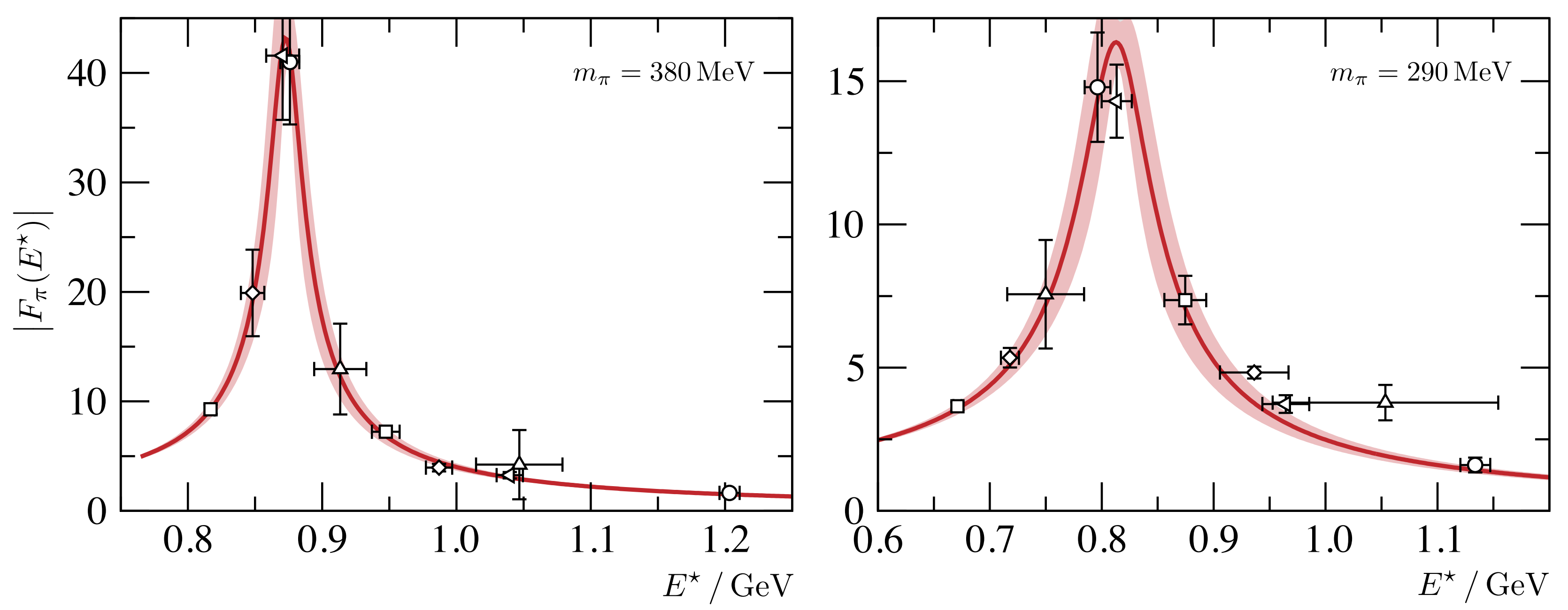
[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

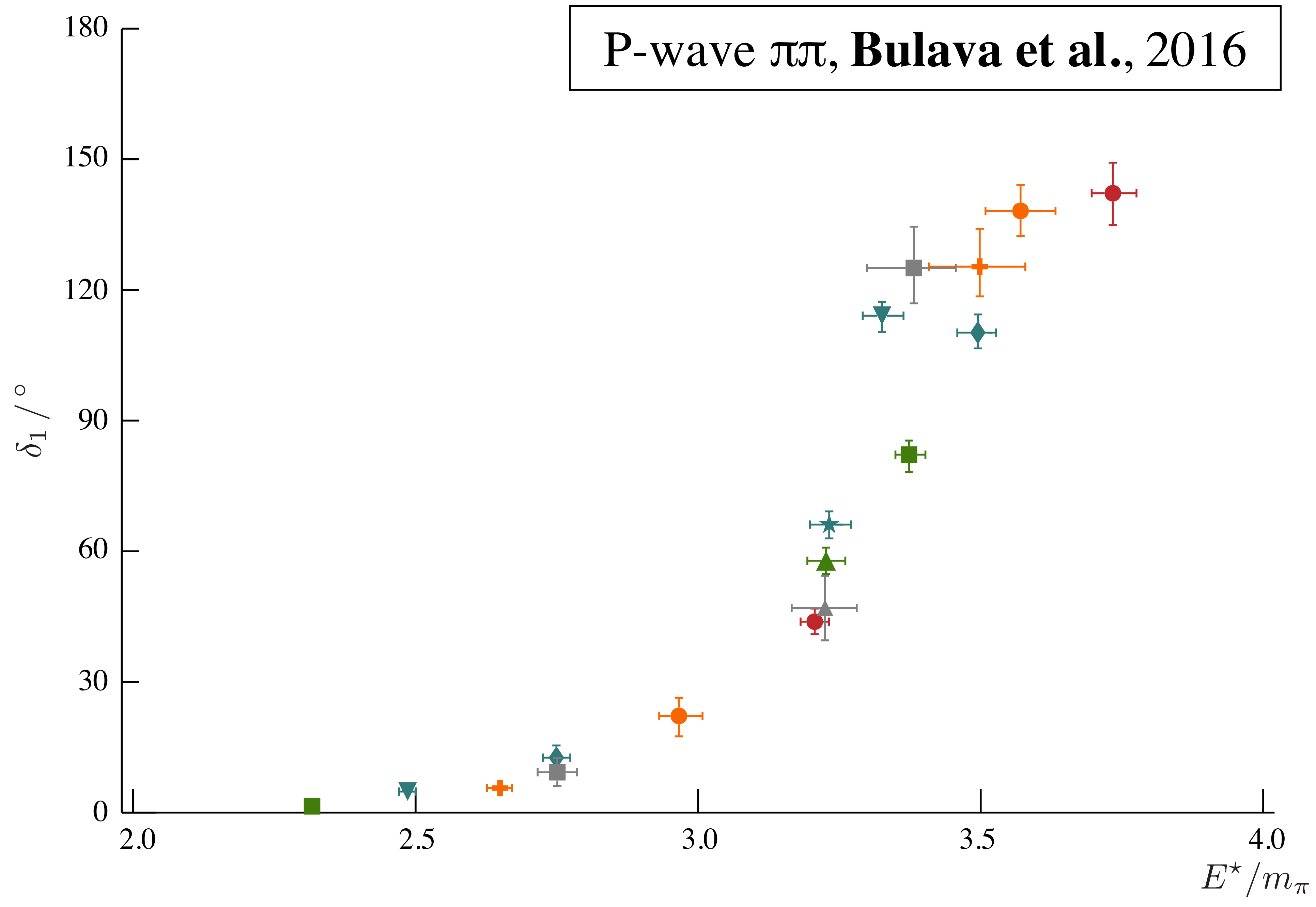
[3] *Special Research Center for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide 5005, Australia*



S-wave DK and the $D^\star_{s0}$, **Lang et al.**, 2014



γ*-to-ππ, **Feng et al.**, 2015



P-wave ππ, **Bulava et al.**, 2016

HadSpec+

Jefferson Lab

Edwards

Old Dominion University / Jefferson Lab

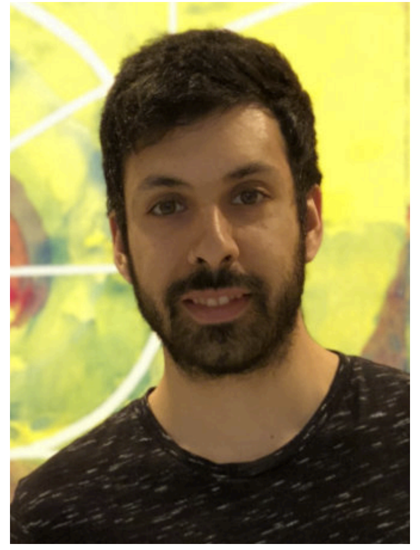
Jackura

DAMTP, University of Cambridge

Wilson

Thomas

MIT

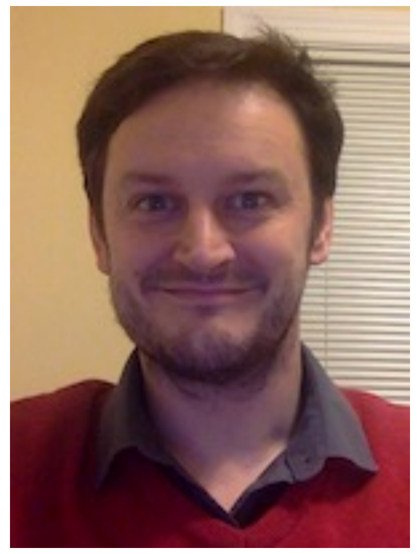


Romero-Lopez

Old Dominion University

Islam

William and Mary / Jefferson Lab



Dudek

Edinburgh

Hansen

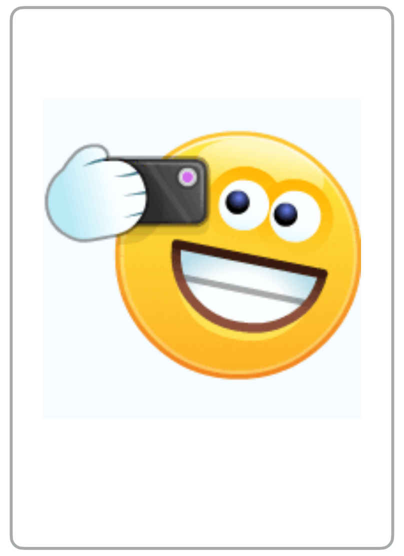
University of Washington

Sharpe

Indiana University

Dawid

University of Maryland



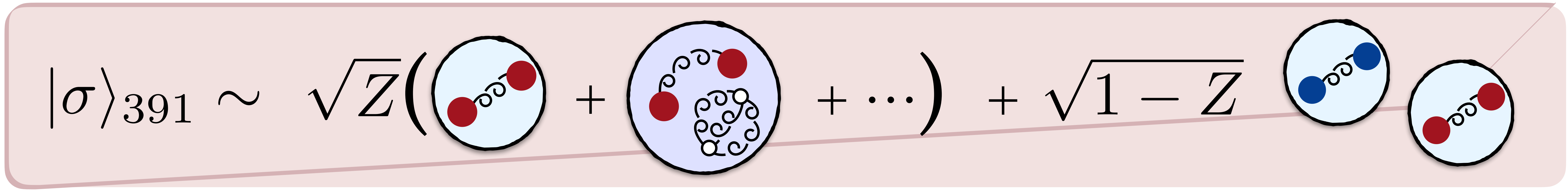
Blanton

hadspec

back-up slides

Weinberg compositeness criterion for the σ

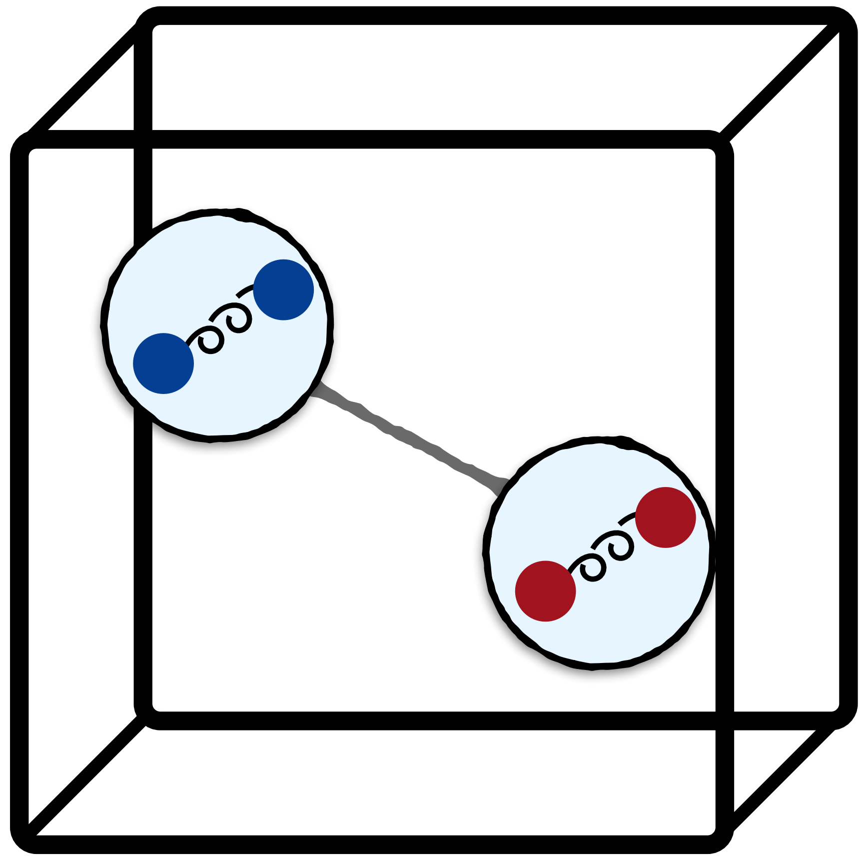
- The σ is a bound state, so we can apply Weinberg's criterion

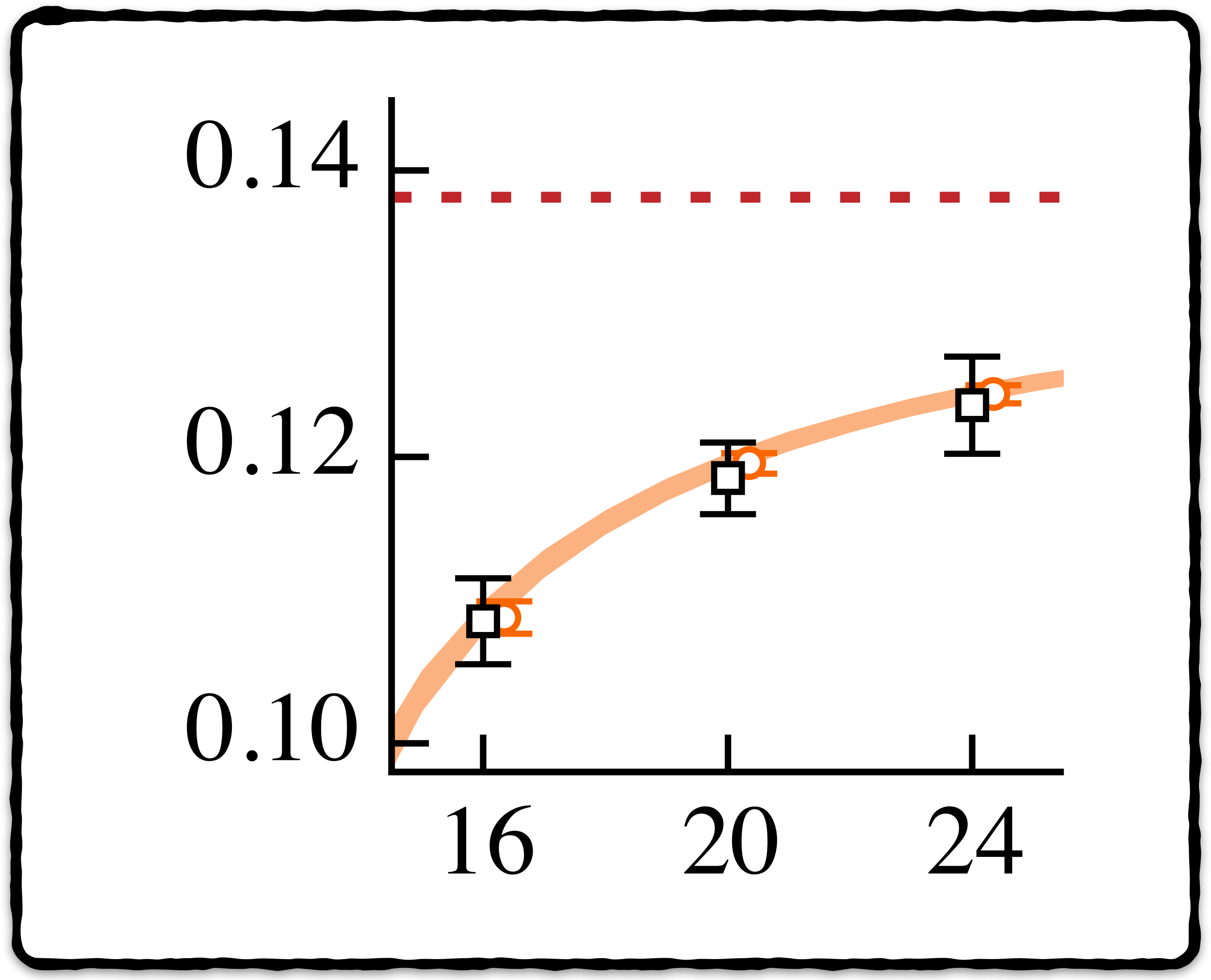


- Can relate Z to scattering information

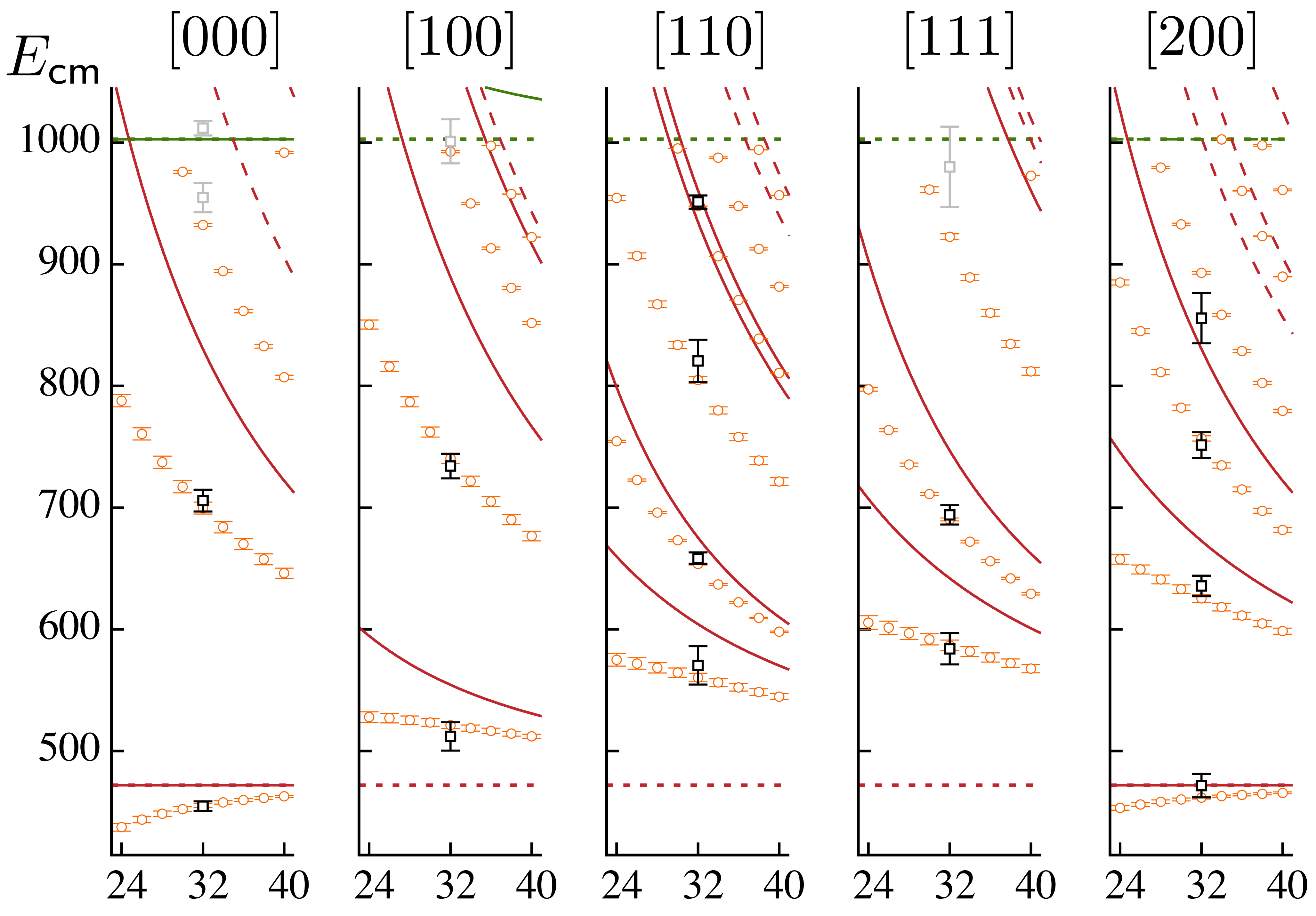
$$a = -2\frac{1-Z}{2-Z}\frac{1}{\sqrt{m_\pi B_\sigma}}, \qquad r = -\frac{Z}{1-Z}\frac{1}{\sqrt{m_\pi B_\sigma}}$$

- To obtain: $Z \sim 0.3(1)$
- Consistent with the large FV effects

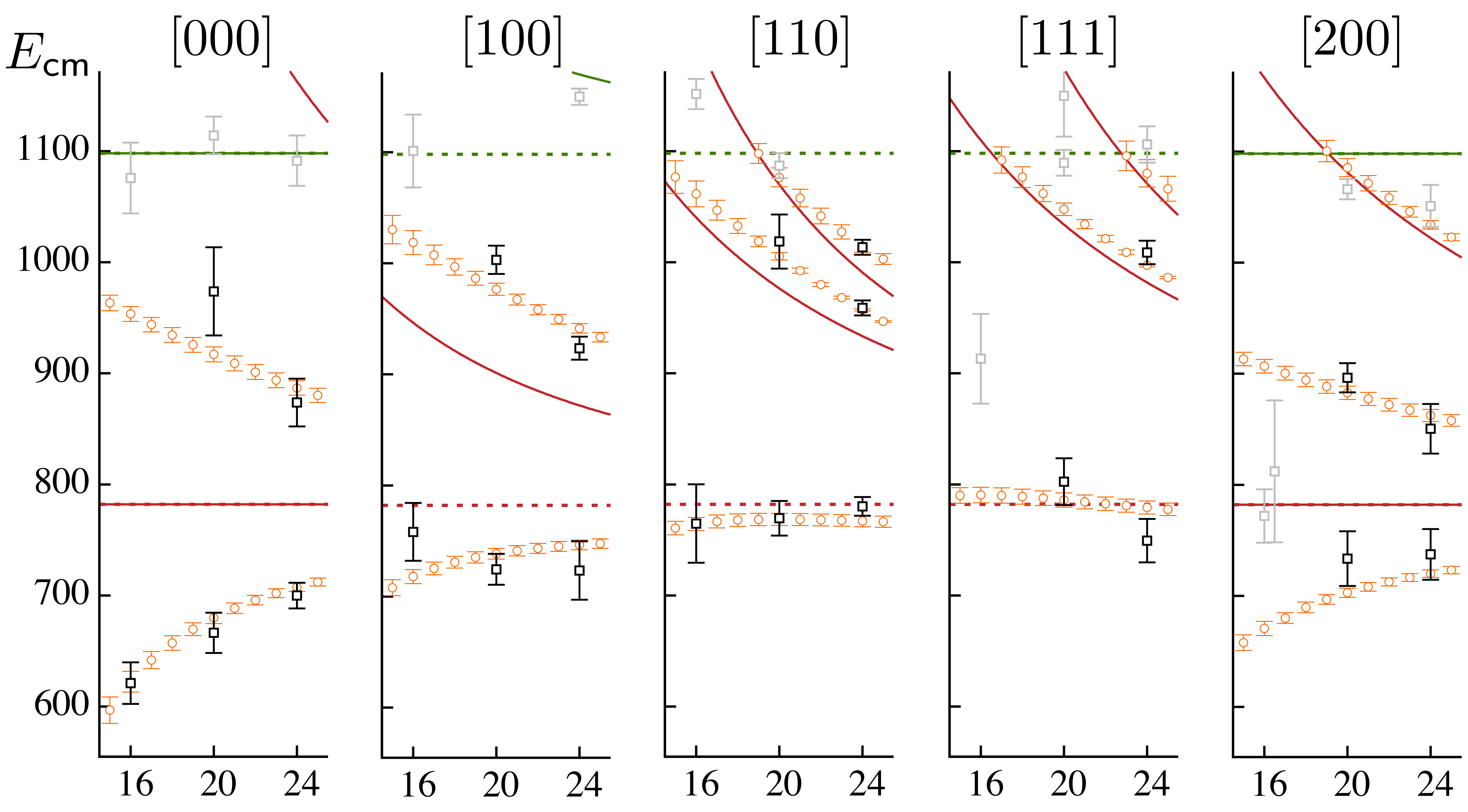




Isoscalar, elastic spectrum fits

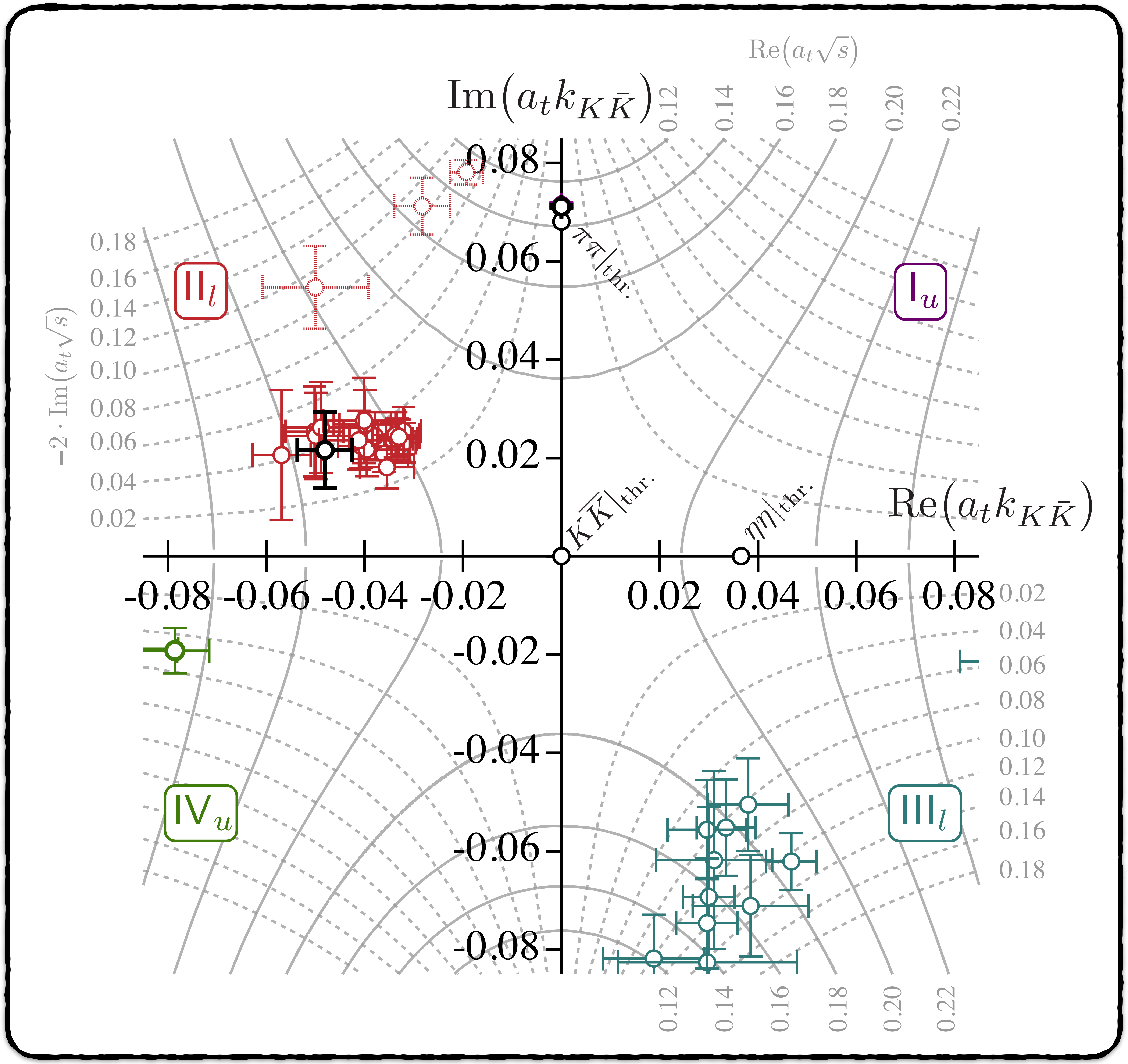


$m_\pi \sim 260\,\mathrm{MeV}$

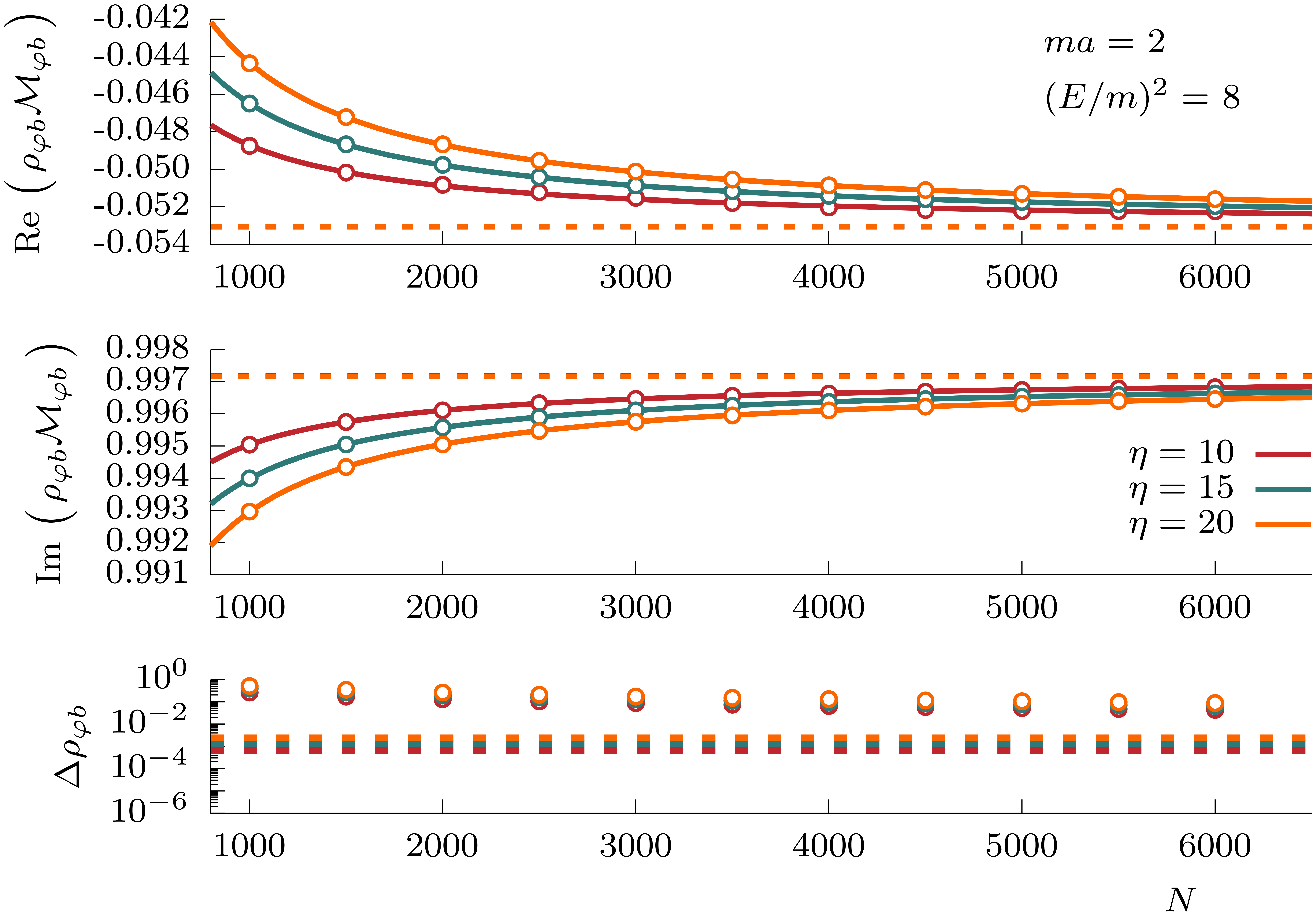


$m_\pi \sim 390\,\mathrm{MeV}$

Complex momentum plane

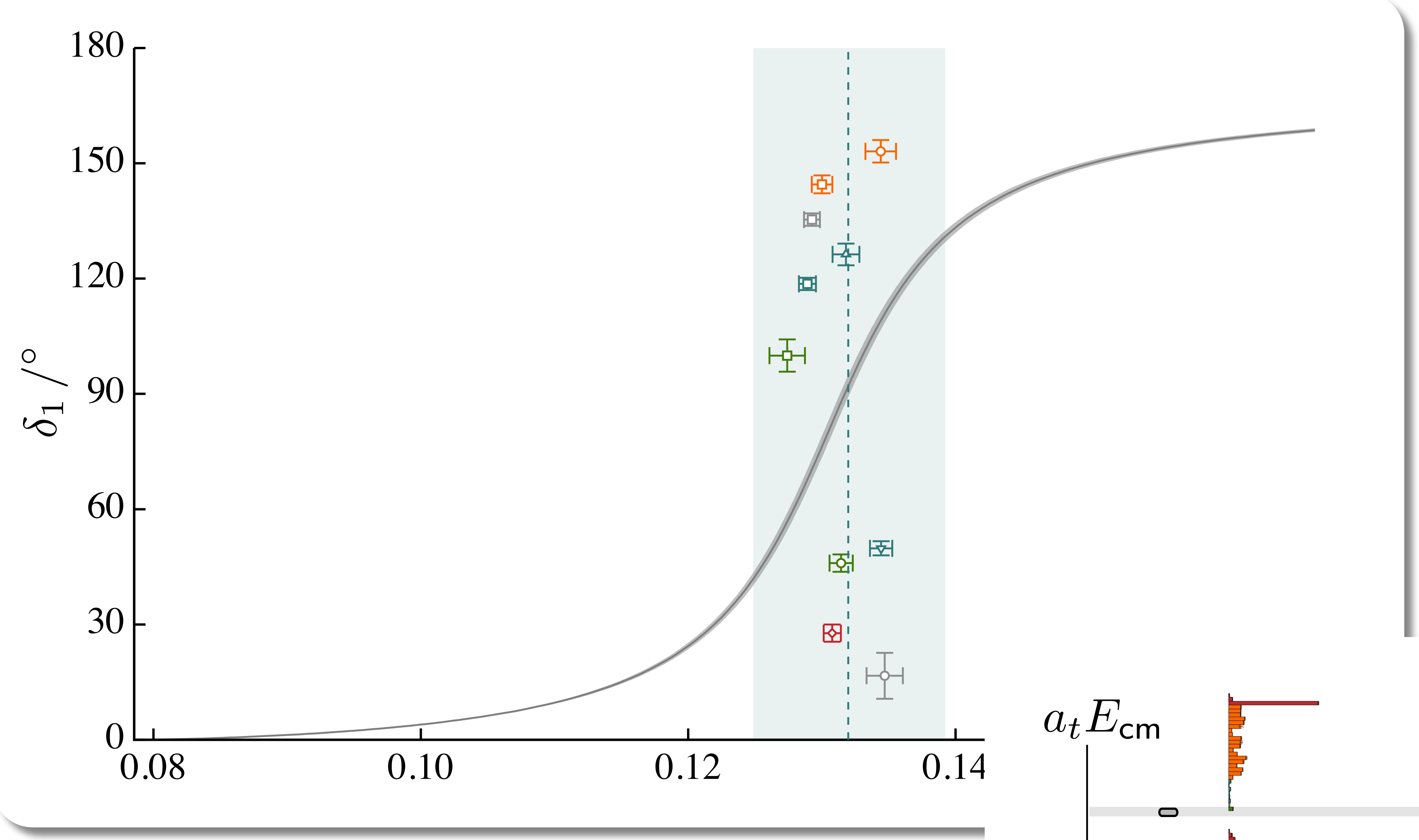


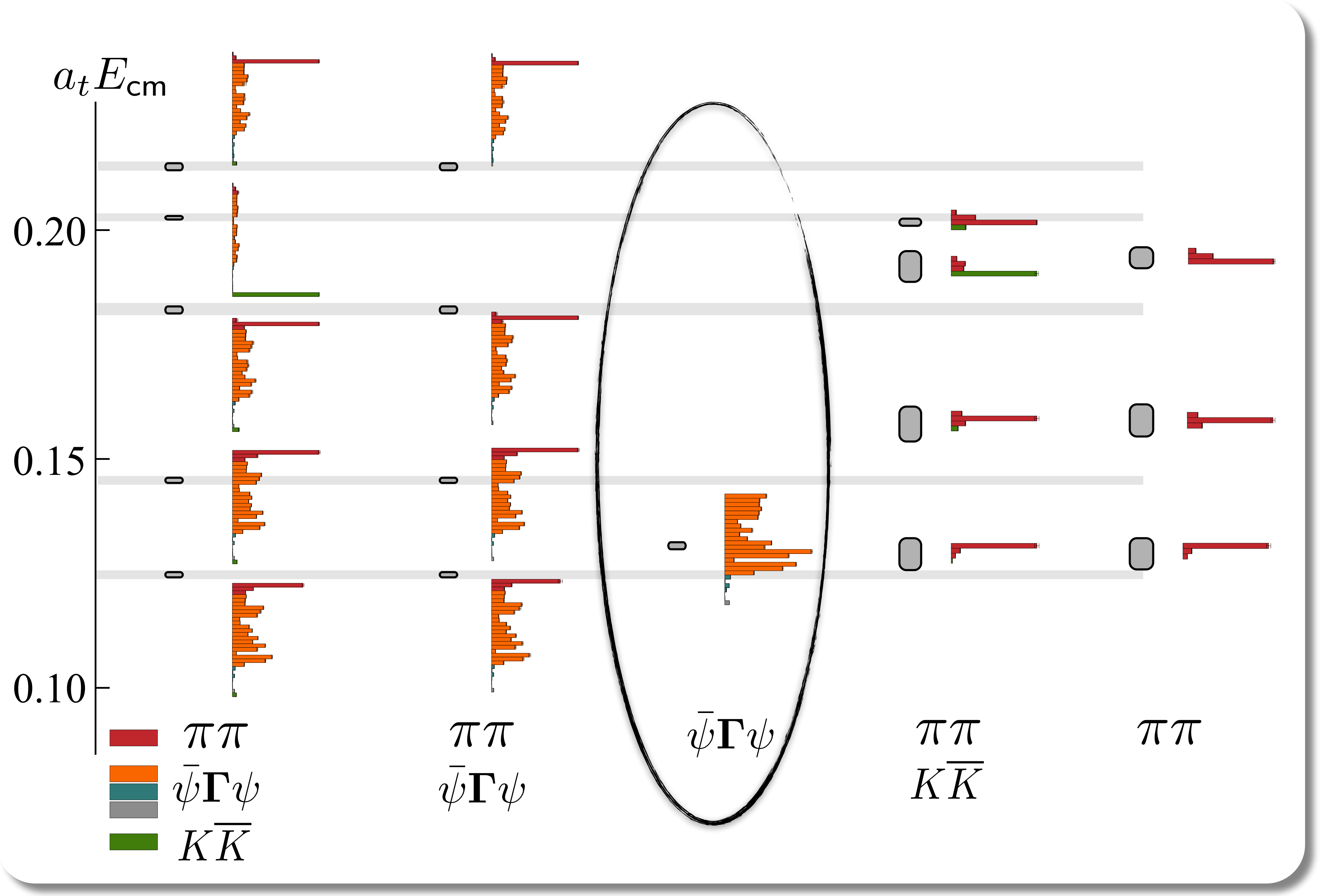
Convergence tests



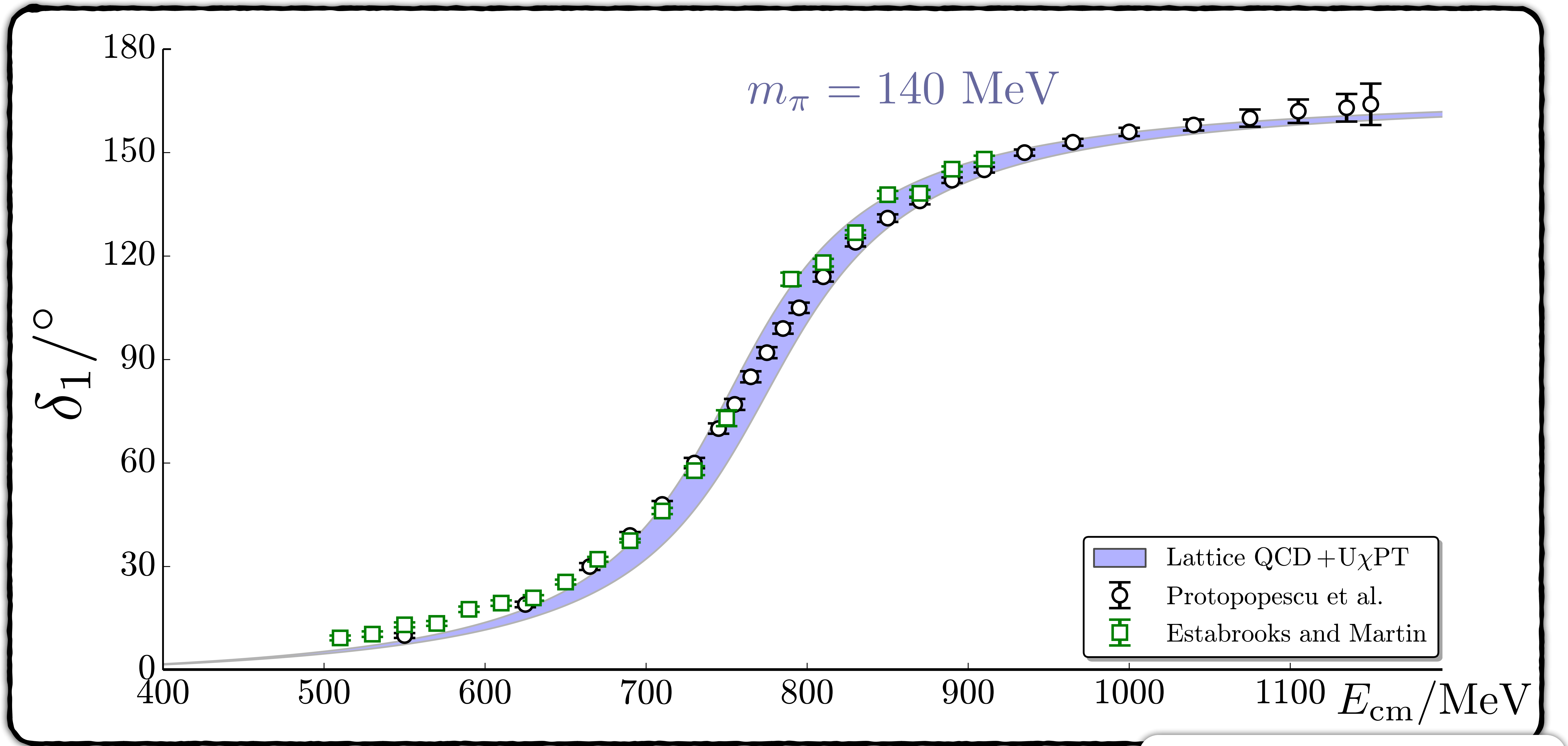
Jackura, RB, Dawid, Islam, & McCarty (2020)

The incorrect answer





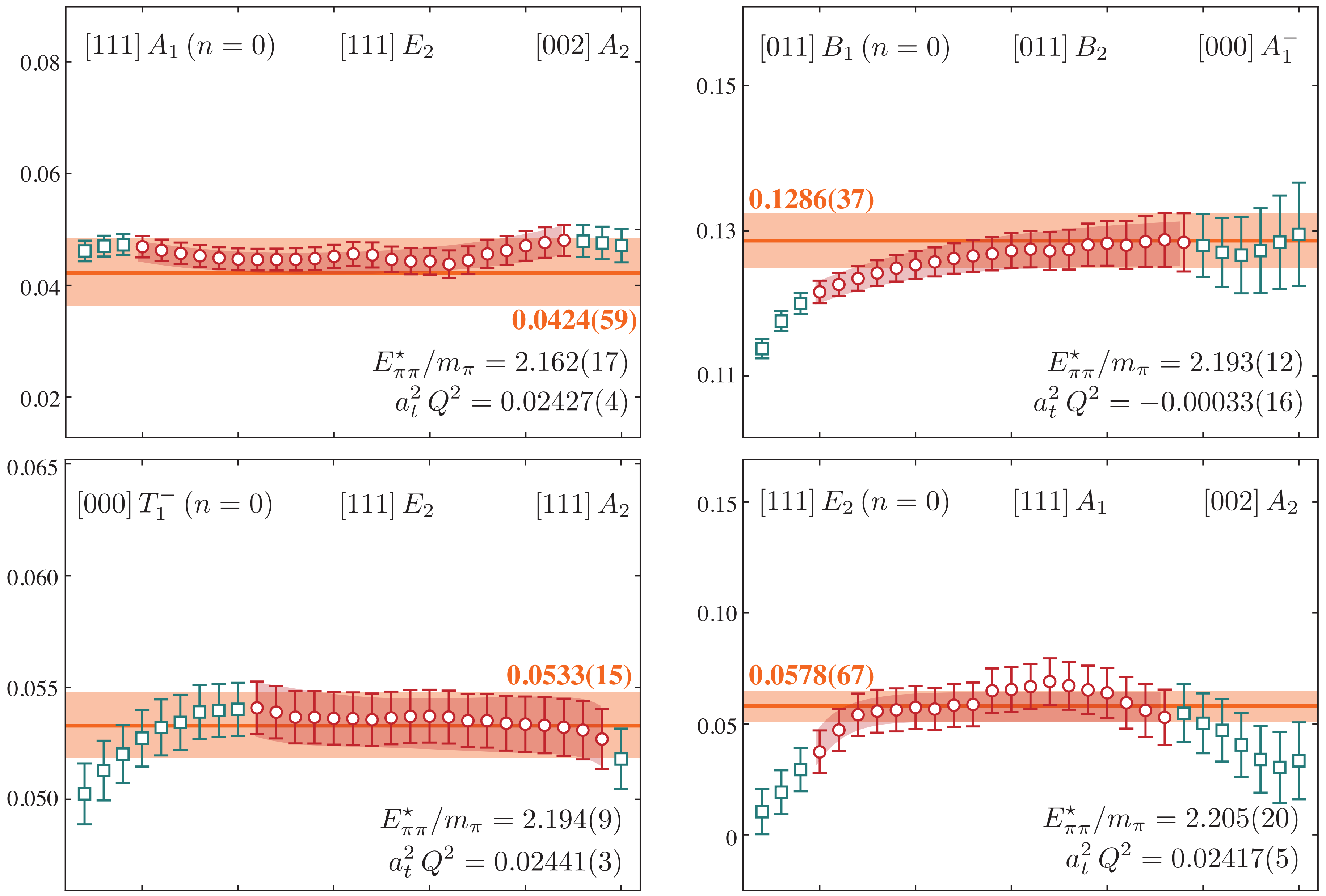
Comparison with experiment

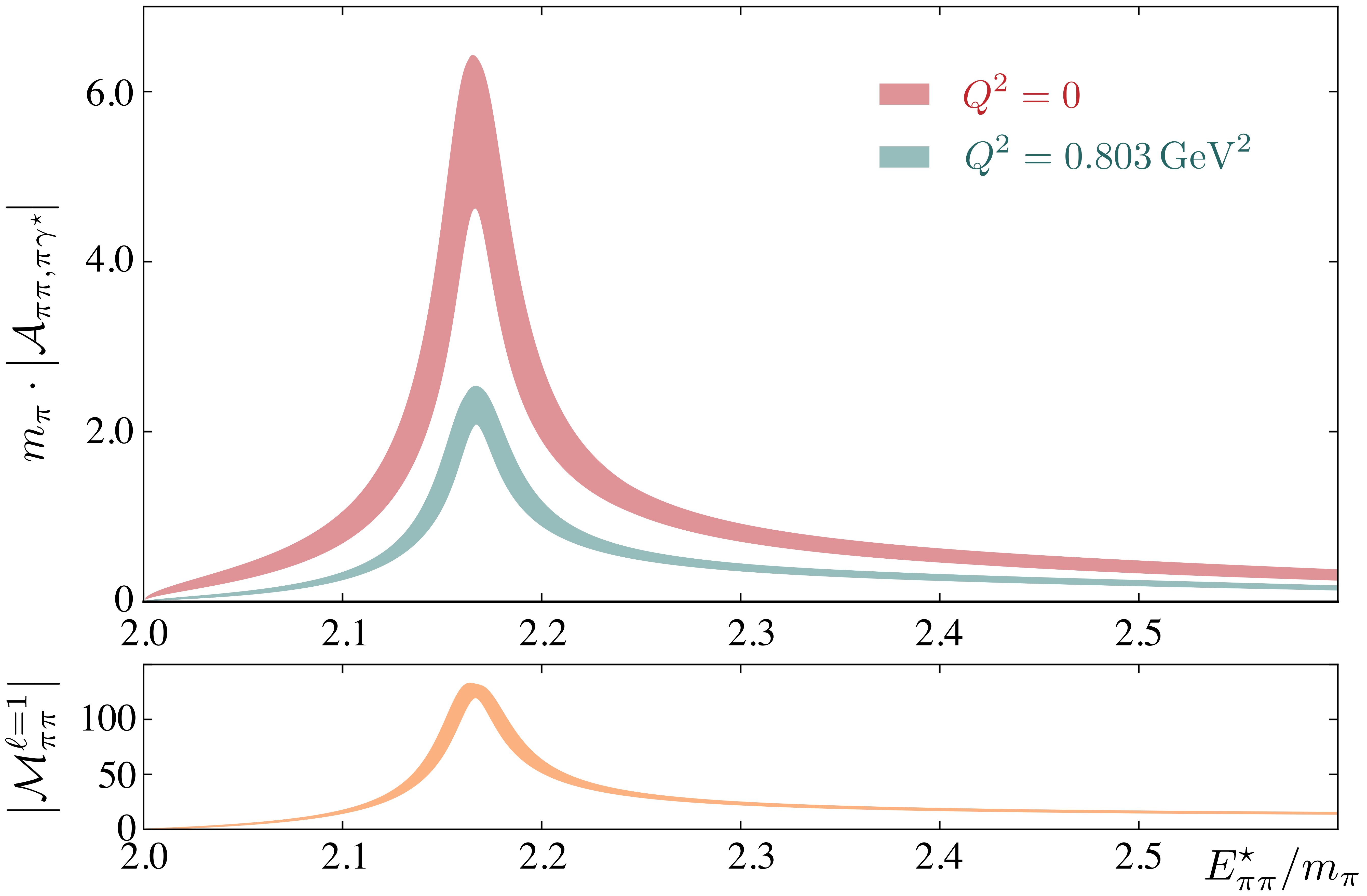


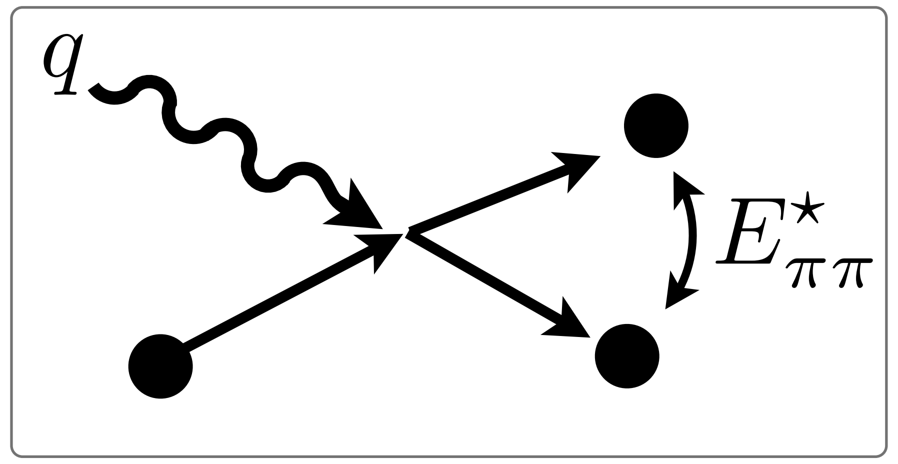
Bolton, RB & Wilson (2016)

- *UχPT* in a nut-shell:
  - enforce unitarity exactly: $\mathcal{M}^{-1}_{U\chi PT} \equiv \text{Re}\left(\mathcal{M}^{-1}_{\chi PT}\right) - i\rho$
  - *treat dynamics perturbatively:* $\text{Re}\left(\mathcal{M}^{-1}_{\chi PT}\right) = \text{Re}\left(\frac{1}{\mathcal{M}_2 + \mathcal{M}_4 + \cdots}\right) \approx \frac{1}{\mathcal{M}_2} - \frac{\text{Re}(\mathcal{M}_4)}{\mathcal{M}_2^2}$

$\pi\gamma \Rightarrow \pi\pi$



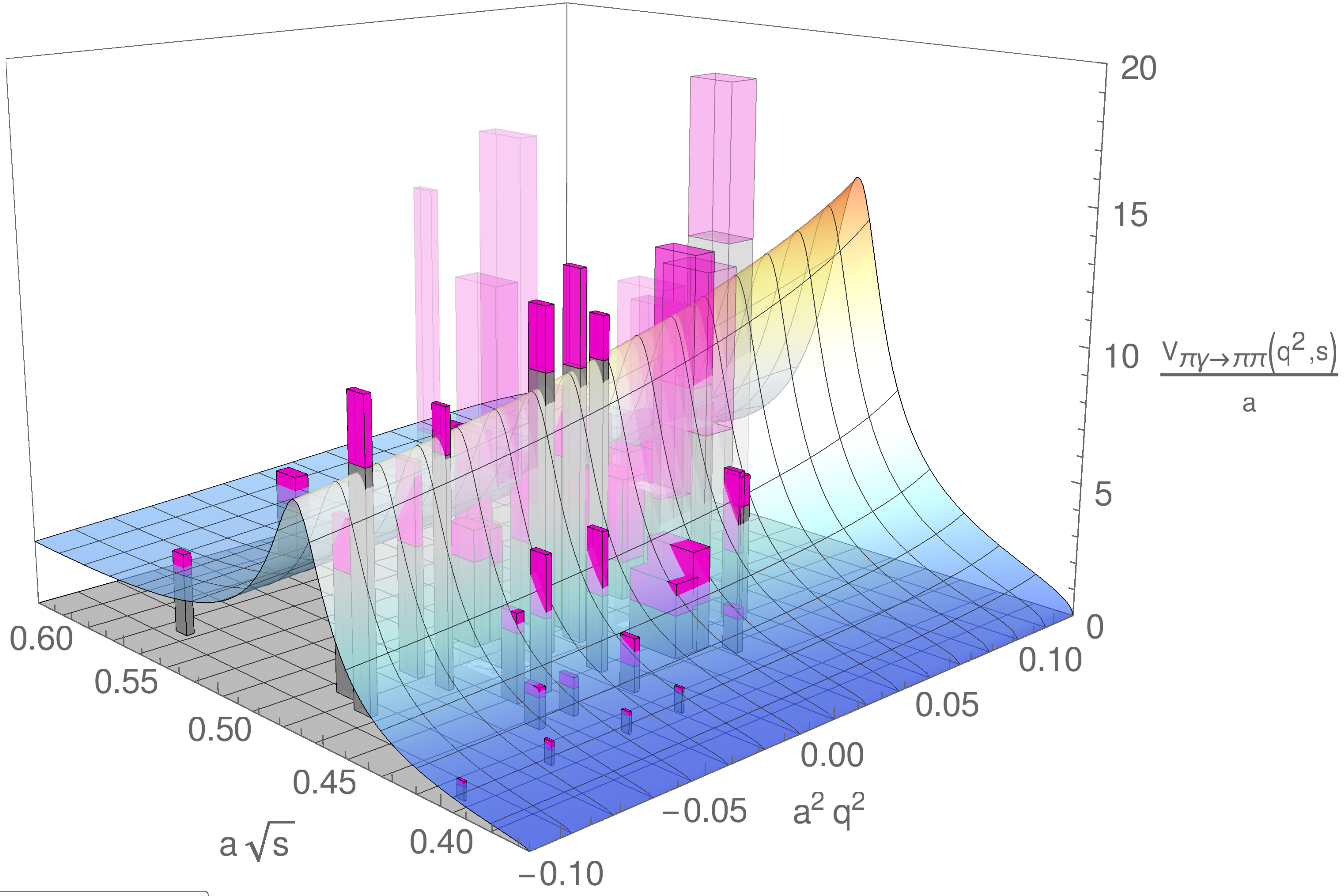


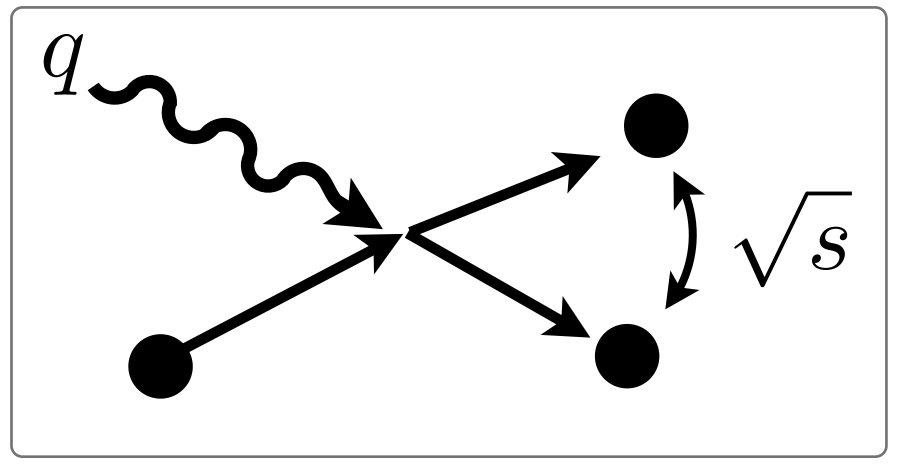


$m_\pi \sim 390\,\mathrm{MeV}$

RB, Dudek, Edwards, Shultz, & Thomas (2015)

$\pi\gamma \Rightarrow \pi\pi$





$m_\pi \sim 320\,\mathrm{MeV}$

Alexandrou, Leskovec, Meinel, Negele,
Paul, Petschlies, Pochinsky, Rendon, Syritsyn(2018)