# A few-body treatment of breakup reactions on halo nuclei

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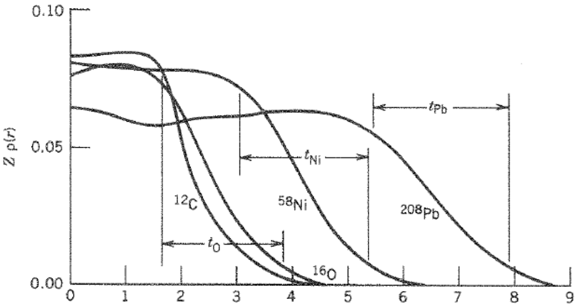
# Stable nuclei

Binding energy well described by Bethe-Weizsäcker

$$E_B = a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z)$$

$\Rightarrow$ stable nuclei $\equiv$ liquid droplet of radius $R \propto A^{1/3}$




[K. S. Krane, *Introductory Nuclear Physics* (Wiley, NY, 1987)]

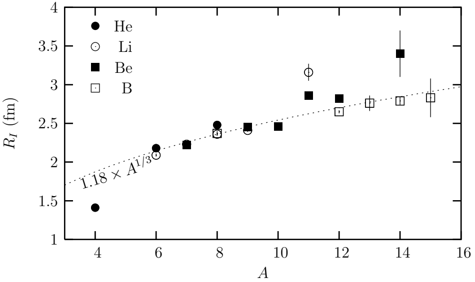
# Unstable nuclei. . .

In the mid-80s, Isao Tanihata used RIBs to measure interaction cross sections of light exotic nuclei.

[I. Tanihata *et al.* PRL 55, 2676 (1985)]

In a simple geometrical model

$$\sigma_I(P,T) = \pi[R_I(P) + R_I(T)]^2$$

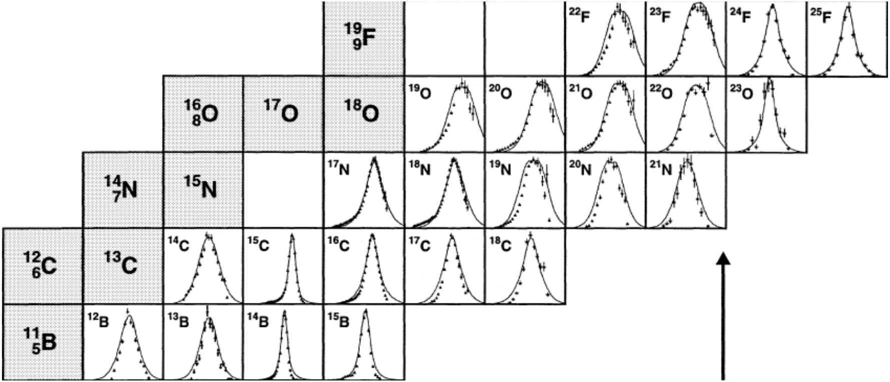



Some nuclei appear larger : $^{6}$He, $^{11}$Be, $^{11}$Li,. . .

⇒ large collective deformation or exotic structure ?

# Parallel-momentum distributions

These nuclei exhibit also a narrow parallel-momentum distribution in one-neutron removal reaction




[E. Sauvan *et al.* PLB, 491, 1 (2000)]

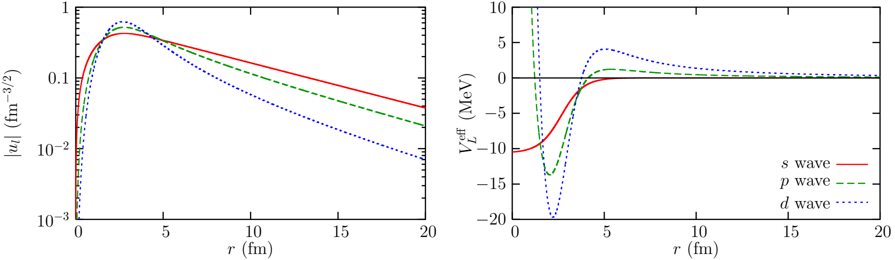
Sign of an extended spacial core-neutron distribution

# Halo structure

Seen as core + one or two neutrons at large distance
[P. G. Hansen and B. Jonson, Europhys. Lett. 4, 409 (1987)]

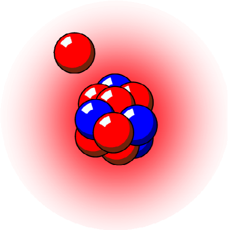
Peculiar structure of nuclei due to small $S_{\rm n}$ or $S_{\rm 2n}$
⇒ neutrons tunnel far from the core to form a halo

Halo only appears for low centrifugal barrier (low $\ell$)



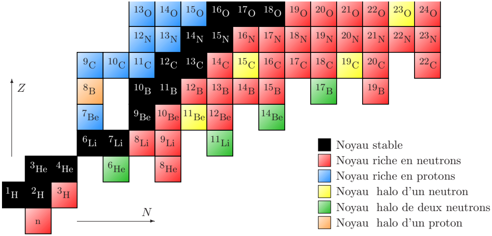
# Halo nuclei

- Light, neutron-rich nuclei
- small $S_{\mathrm{n}}$ or $S_{2\mathrm{n}}$
- low-$\ell$ orbital



One-neutron halo
$^{11}$Be ≡ $^{10}$Be + n
$^{15}$C ≡ $^{14}$C + n

Two-neutron halo
$^{6}$He ≡ $^{4}$He + n + n
$^{11}$Li ≡ $^{9}$Li + n + n




Proton halœs are possible but less probable : $^{8}$B, $^{17}$F

Two-neutron halo nuclei are Borromean…

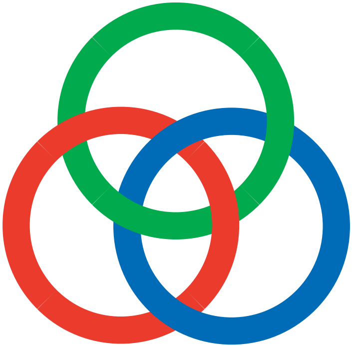
$c$+n+n is bound but not two-body subsystems

e.g. $^{6}$He bound but not $^{5}$He or $^{2}$n

# Borromean nuclei

Named after the Borromean rings. . .

[M. V. Zhukov *et al.* Phys. Rep. 231, 151 (1993)]



# Breakup reaction

Halo nuclei are fascinating objects

However difficult to study experimentally [$\tau_{1/2}(^{11}\text{Be}) = 13$ s]

How can one probe their structure ?

⇒ require indirect techniques, like reactions

Breakup ≡ dissociation of projectile in constituent clusters
by interaction with target

$$^{11}\text{Be} + {}^{12}\text{C} \rightarrow {}^{10}\text{Be} + \text{n} + {}^{12}\text{C}$$
$$^{8}\text{B} + {}^{208}\text{Pb} \rightarrow {}^{7}\text{Be} + \text{p} + {}^{208}\text{Pb}$$

The target $T$ acts differently on projectile $P$ contituents
⇒ tidal force → breakup

Need a good understanding of the reaction mechanism
i.e. an accurate theoretical description of reaction
coupled to a realistic model of projectile

Elastic breakup ≡ all clusters measured in coincidence
(exclusive measurement)

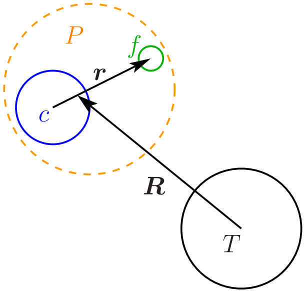
# Framework

Projectile ($P$) modelled as a two-body system :
core ($c$)+loosely bound fragment ($f$) described by

$H_0 = T_r + V_{cf}(\boldsymbol{r})$

$V_{cf}$ adjusted to reproduce $P$ spectrum

Target $T$ seen as structureless particle




$P$-$T$ interaction simulated by optical potentials

⇒ breakup reduces to three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

# Projectile Hamiltonian $H_0$

$$H_0 = -\frac{\hbar^2 \Delta_r}{2\mu_{cf}} + V_{cf}(r)$$

$V_{cf}$ has usually a Woods-Saxon form factor,
but we'll see that Halo-EFT can be efficiently used instead

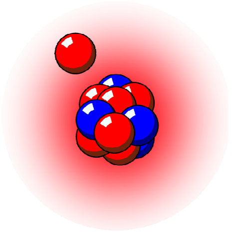
$c$-$f$ relative motion described by $H_0$ eigenstates

- $E_{nl} < 0$ : discrete set of bound states $\quad H_0\, \phi_{nlm}(\boldsymbol{r}) = E_{nl}\, \phi_{nlm}(\boldsymbol{r})$
- $E > 0$ : $c$-$f$ continuum ≡ broken up projectile
  $H_0\, \phi_{klm}(\boldsymbol{r}) = E\, \phi_{klm}(\boldsymbol{r})$ where $E = \hbar^2 k^2 / 2\mu_{cf}$

Breakup ≡ transition from bound state to continuum
through interaction with target (Coulomb and nuclear)

Breakup can take place in one or more steps
will be sensitive to both bound and continuum states

# Example : $^{11}$Be



| | | |
|---|---|---|
| $5/2^+$ | 1.274 | $d5/2$ |
| | $^{10}$Be + n | |
| $1/2^-$ | -0.184 | $0p1/2$ |
| $1/2^+$ | -0.504 | $1s1/2$ |

$^{11}$Be spectrum

$$^{11}\text{Be} \equiv {}^{10}\text{Be}(0^+) + \text{n}$$

$^{10}$Be cluster assumed in $0^+$ ground state (extreme shell model)

$\Rightarrow$ spin and parity of $^{11}$Be states fixed by angular momenta $l$ and $j$ of n :

- $1/2^+$ ground state in $s1/2$
- $1/2^-$ excited state in $p1/2$
- $5/2^+$ resonance in $d5/2$

$\Rightarrow$ fit $V_0$ in $s1/2$, $p1/2$ and $d5/2$ waves (but not in $p3/2\ldots$)

# Projectile-target interaction : $V_{cT}$ and $V_{fT}$

The breakup channel is explicitly included in the collision description

However other channels not included :

- absorption of the fragment by the target
- excitation of the core
- …

$c$-$T$ and $f$-$T$ interactions described by optical potentials $V_{cT}$ and $V_{fT}$
Their imaginary parts simulate the absorption to other channels

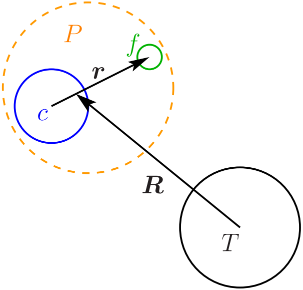
Usually chosen in the literature

- $V_{cT}$ : problematic if $c$-$T$ scattering not measured
  $\Rightarrow$ extrapolate what exists or use folding technique
- $V_{fT}$ : many $N$-$T$ global potentials exist
  [Becchetti and Greenlees, Phys. Rev. 182, 1190 (1969)]
  [Koning and Delaroche NPA 713, 231 (2003) ]

# Three-body Scattering Problem

Within this framework breakup reduces to three-body problem

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$




with the initial condition

$$\Psi(\boldsymbol{r},\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ+\cdots}\phi_{n_0l_0m_0}(\boldsymbol{r})$$

⇔ $P$ in its ground state $\phi_{n_0l_0m_0}$ impinging on $T$

Various methods developed to solve that equation

[Recent review : Baye, P.C., Lecture Notes in Physics 848, 121 (2012)]

- Coupled-channel method with discretised continuum (CDCC)
- Time-dependent approach (TD) (semiclassical)
- Eikonal approximation
- . . .

# Coupled-Channel method

The eigenstates of $H_0$ $\{|\phi_i\rangle\}$ are a basis in $\boldsymbol{r}$ : $\quad H_0|\phi_i\rangle = E_i|\phi_i\rangle$

Idea : expand $\Psi$ on that basis : $\quad \Psi(\boldsymbol{r},\boldsymbol{R}) = \sum_i \chi_i(\boldsymbol{R})\langle\boldsymbol{r}|\phi_i\rangle$

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

$$\Leftrightarrow \sum_i T_R\chi_i(\boldsymbol{R})|\phi_i\rangle + \chi_i(\boldsymbol{R})H_0|\phi_i\rangle + (V_{cT} + V_{fT})\chi_i(\boldsymbol{R})|\phi_i\rangle$$

$$= \sum_i E_T\chi_i(\boldsymbol{R})|\phi_i\rangle$$

$$\langle\phi_j| \quad \Downarrow$$

$$T_R\chi_j(\boldsymbol{R}) + E_j\chi_j(\boldsymbol{R}) + \sum_i \langle\phi_j|V_{cT} + V_{fT}|\phi_i\rangle\chi_i(\boldsymbol{R}) = E_T\,\chi_j(\boldsymbol{R})$$

This is a set of coupled equations in $\chi_i(\boldsymbol{R})$
where the coupling terms are $\langle\phi_j|V_{cT} + V_{fT}|\phi_i\rangle$
i.e. connect the various projectile states through the $P$-$T$ interaction
Problem : continuum states $\phi_{klm}$ are not discrete. . .

# Discretising the Continuum

Model of breakup requires description of continuum
must be tractable in computations, i.e. discrete

[Rawitscher, PRC 9, 2210 (1974)]

$$\phi_{klm} \quad \text{with } k \in \mathbb{R}^+ \rightarrow \phi_{ilm} \quad \text{with } i \in \mathbb{N}$$

Various methods exist :

- mid-point : divide continuum in bins $[E_i - \Delta E_i/2, E_i + \Delta E_i/2]$ and choose $\phi_{ilm}(\boldsymbol{r}) = \phi_{k_i lm}(\boldsymbol{r})$ to describe bin $i$
- average the wave function over the bin

$$\phi_{ilm}(\boldsymbol{r}) = \frac{1}{W_i} \int_{E_i-\frac{\Delta E_i}{2}}^{E_i+\frac{\Delta E_i}{2}} f_i(E)\, \phi_{klm}(\boldsymbol{r})\, dE \quad \text{with } W_i = \int_{E_i-\frac{\Delta E_i}{2}}^{E_i+\frac{\Delta E_i}{2}} f_i(E) dE$$

  $\Rightarrow$ square-integrable wave functions $\phi_{ilm}$

- pseudo-states : solve $H_0\, \phi_{ilm} = E\, \phi_{ilm}$ on a finite basis
  $\Rightarrow$ square-integrable wave functions $\phi_{ilm}$ but $E_i$ not chosen

# CDCC

Continuum Discretised Coupled-Channel : CDCC
[Austern *et al.* , Phys. Rep. 154, 125 (1987)]
[Tostevin, Nunes, Thompson, PRC 63, 024617 (2001)]
Recent review : [Yahiro *et al.* , PTEP 2012 01A206 (2012)]

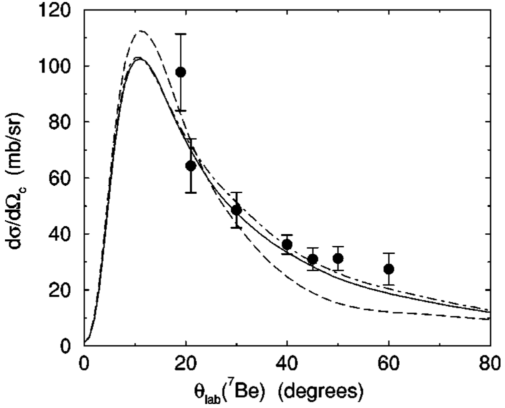
Fully quantal approximation
No approximation on $P$-$T$ motion, nor restriction on energy
But expensive computationally (at high energies)

Various codes have been written to solve these coupled equations
FRESCO written by Ian Thompson is free on `www.fresco.org.uk`
[Thompson, Comput. Phys. Rep. 7, 167 (1988)]

# Example : $^8$B breakup

$$^{8}\mathrm{B} + {}^{58}\mathrm{Ni} \rightarrow {}^{7}\mathrm{Be} + \mathrm{p} + {}^{58}\mathrm{Ni} \quad @26\mathrm{MeV}$$

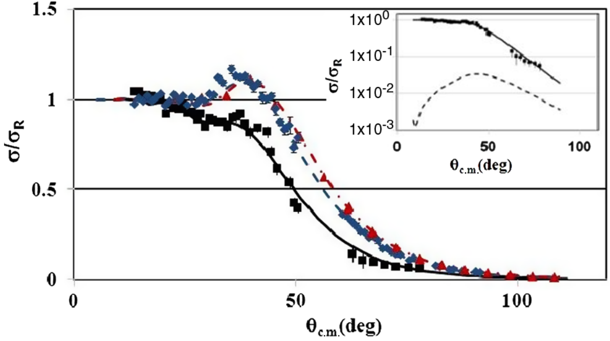
Exp. :[V. Guimarães *et al.* PRL 84, 1862 (2000)]




Th. :[Tostevin *et al.* PRC 63, 024617 (2001)]

# Influence of breakup on the elastic channel

$^{9,10,11}$Be + Zn @ 24.5MeV

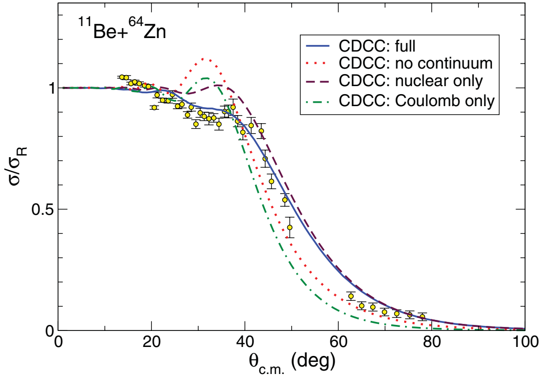



Exp. : [A. Di Pietro *et al.* PRL 105, 022701 (2010)]

- $^{9,10}$Be elastic scattering reproduced with usual optical potentials
- $^{11}$Be elastic scattering strongly affected by breakup channel

# Influence of breakup on the elastic channel

$^{11}$Be + Zn @ 24.5MeV

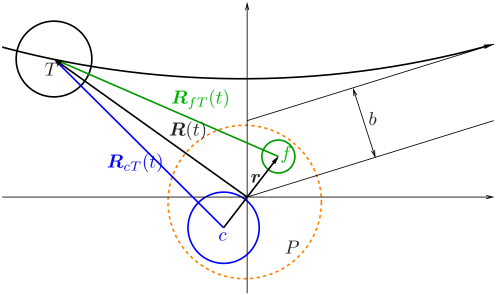



[A. Di Pietro *et al.* PRC 85, 054607 (2012)]

- $^{9,10}$Be elastic scattering reproduced with usual optical potentials
- $^{11}$Be elastic scattering strongly affected by breakup channel
  Confirmed by CDCC calculations

# Time-dependent model

$P$-$T$ motion described by classical trajectory $\boldsymbol{R}(t)$ defined by $V_{\text{traj}}(\boldsymbol{R})$




$P$ structure described quantum-mechanically by $H_0$
Time-dependent potentials simulate $P$-$T$ interaction

⇒ time-dependent (TD) Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\Psi(\boldsymbol{r}, \boldsymbol{b}, t) = [H_0 + V_{cT}(t) + V_{fT}(t) - V_{\text{traj}}(t)]\Psi(\boldsymbol{r}, \boldsymbol{b}, t)$$

Solved for each $b$ with initial condition $\Psi^{(m_0)} \underset{t\to-\infty}{\longrightarrow} \phi_{n_0 l_0 m_0}$

# Numerical resolution of the TD Schrödinger equation

Time-step evolution approximating the evolution operator

$$\Psi^{(m_0)}(\boldsymbol{r}, b, t + \Delta t) \quad = \quad U(t + \Delta t, t)\,\Psi^{(m_0)}(\boldsymbol{r}, b, t)$$

with $U(t', t) = \exp[\frac{i}{\hbar} \int_t^{t'} H(\tau) d\tau]$ and $\Psi^{(m_0)}(\boldsymbol{r}, b, t \to -\infty) = \phi_{n_0 l_0 m_0}(\boldsymbol{r})$

- Faster computation compared to CDCC
  because each trajectory treated separately
- Lacks quantum interferences between trajectories

Many codes developed to solve TD

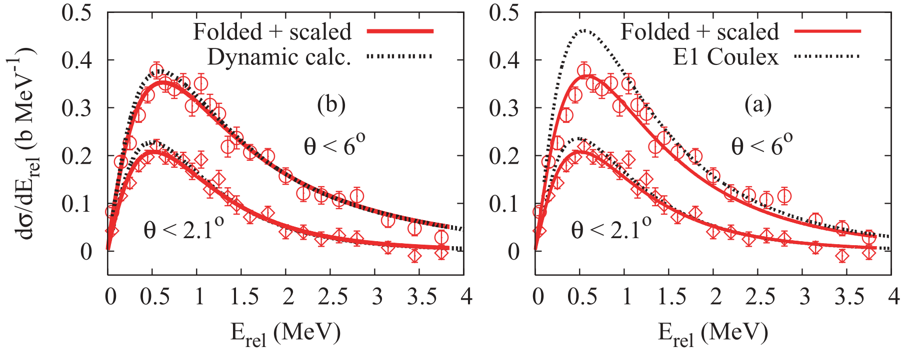
- Partial-wave expansion of $\Psi$ :
  [Kido, Yabana, and Suzuki, PRC 50, R1276 (1994)]
  [Esbensen, Bertsch and Bertulani, NPA 581, 107 (1995)]
  [Typel and Wolter, Z. Naturforsch.A 54, 63 (1999)]
- Expansion on a 3D spherical mesh :
  [P. C., Melezhik and Baye, PRC 68, 014612 (2003)]
- Expansion on 3D cubic lattice : [Fallot *et al.* NPA700, 70 (2002)]

# Example : $^{15}$C Coulomb breakup

$^{15}\mathrm{C} \equiv {}^{14}\mathrm{C}(0^+) + \mathrm{n}$

$$^{15}\mathrm{C} + {}^{208}\mathrm{Pb} \rightarrow {}^{14}\mathrm{C} + \mathrm{n} + {}^{208}\mathrm{Pb} \quad @68A\mathrm{MeV}$$

Exp. :[Nakamura *et al.* PRC 79, 035805 (2009)]




Th. :[Esbensen, PRC 80, 024608 (2009)]

Higher-order effects play a significant role

Perturbation treatment of the reaction can be treacherous

# Eikonal approximation

Three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with condition $\Psi^{(m_0)} \underset{Z\to-\infty}{\longrightarrow} e^{iKZ}\phi_{n_0 l_0 m_0}$

Eikonal approximation : factorise $\Psi = e^{iKZ}\widehat{\Psi}$

$$T_R\Psi = e^{iKZ}[T_R + vP_Z + \frac{\mu_{PT}}{2}v^2]\widehat{\Psi}$$

Neglecting $T_R$ vs $P_Z$ and using $E_T = \frac{1}{2}\mu_{PT}v^2 + E_{n_0 l_0}$

$$i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - E_{n_0 l_0} + V_{cT} + V_{fT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

solved for each $\boldsymbol{b}$ with condition $\widehat{\Psi}^{(m_0)} \underset{Z\to-\infty}{\longrightarrow} \phi_{n_0 l_0 m_0}(\boldsymbol{r})$

This is the dynamical eikonal approximation (DEA)

[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

Same equation as TD with straight line trajectories

# Eikonal cross section

After some mathematical developments…

[Goldstein, Baye, P.C. PRC 73, 024602 (2006)]

$$\frac{d\sigma_{\rm bu}}{dEd\Omega} \propto \frac{1}{2l_0+1} \sum_{m_0} \sum_{lm} \left| \int_0^\infty J_{|m_0-m|}(qb)\, S^{(m_0)}_{klm}(b)\; bdb \right|^2 ,$$

$S^{(m_0)}_{klm}(b) = \langle \phi_{klm} | \widehat{\Psi}^{(m_0)}(Z \to \infty) \rangle$ are breakup amplitudes

$$\frac{d\sigma_{\rm bu}}{dEd\Omega} \xrightarrow{\int d\Omega} \frac{d\sigma_{\rm bu}}{dE}$$

⇒ Dynamical eikonal extends TD

takes into account interferences between *trajectories*

(sum of breakup amplitudes)

# Usual Eikonal

$$i\hbar v \frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - E_{n_0 l_0} + V_{cT} + V_{fT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

The usual eikonal uses adiabatic approx. $H_0 - E_{n_0 l_0} \sim 0$

$$\widehat{\Psi}^{(m_0)}_{\text{eik}}(\boldsymbol{r},\boldsymbol{b},Z) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{Z} dZ' \left[V_{cT}(\boldsymbol{r},\boldsymbol{b},Z') + V_{fT}(\boldsymbol{r},\boldsymbol{b},Z')\right]\right\}\phi_{n_0 l_0 m_0}(\boldsymbol{r})$$

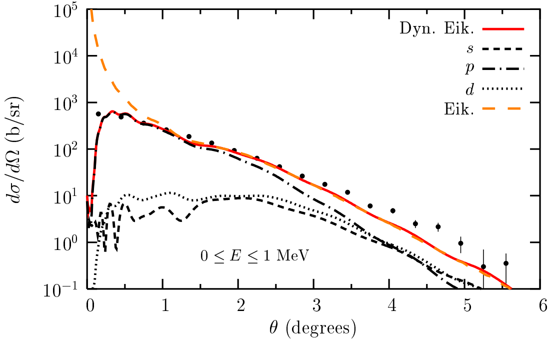
- Easy to interpret and implement
- Neglects internal dynamics of projectile

⇒ dynamical eikonal generalises eikonal

# Example : $^{11}$Be Coulomb breakup

$$^{11}\mathrm{Be} + {}^{208}\mathrm{Pb} \rightarrow {}^{10}\mathrm{Be} + \mathrm{n} + {}^{208}\mathrm{Pb} \quad @69A\mathrm{MeV}$$

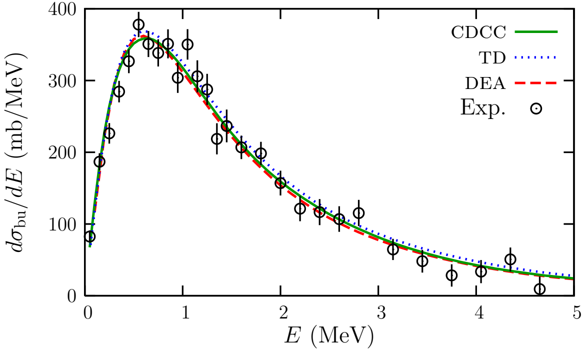
Exp. :[Fukuda *et al.* PRC 68, 054606 (2004)]




Th. :[Goldstein, Baye, P.C. PRC 73, 024602 (2006)]

- DEA exhibits interferences (oscillations)
- Usual eikonal diverges at forward angles (adiabatic approx.)

# $^{15}$C+Pb @ $68A$MeV : energy distribution

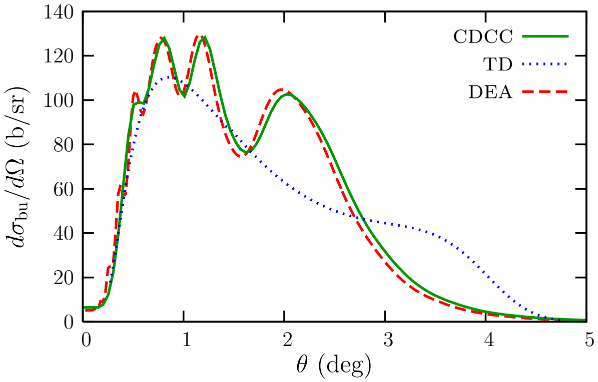



- Excellent agreement between all three models
  [P.C., Esbensen and Nunes, PRC 85, 044604 (2012)]
- Excellent agreement with experiment
  [Nakamura *et al.* PRC 79, 035805 (2009)]

⇒ Confirms the validity of the approximations

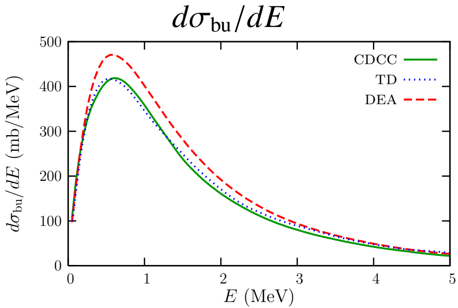
. . . and the two-body structure of $^{15}$C

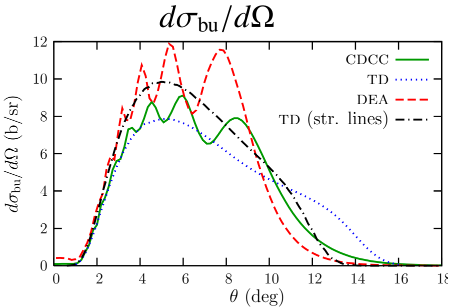
# $^{15}$C+Pb @ $68A$MeV : angular distribution




- TD lacks quantum interferences
  but reproduces the general trend at small $\theta$
- DEA exhibits quantum interferences
  though much less time consuming than CDCC

# $^{15}$C + Pb @ $20A$MeV




TD ≡ CDCC
DEA too high

TD gives trend of CDCC
(lacks oscillations)
DEA peaks too early

DEA≠CDCC due to Coulomb deflection
Eikonal is a high-energy approximation
A semiclassical correction can account for the Coulomb deflection
[Fukui, Ogata, P.C. PRC 90, 034617 (2014)]

# And now what ?

We now have models of breakup of halo nuclei
They reliably describe the reaction mechanism

- What do we learn from breakup on the structure of halo nuclei ?
- $^{11}$Be calculated *ab initio* by [Calci *et al.* PRL 117, 242501 (2016)]
  How can we test their predictions ?

Studies on the sensitivity of breakup calculations to projectile description :

[P.C. & Nunes PRC73, 014615 (2006), *ibid.* 75, 054609 (2007)]

We address both questions within Halo-EFT

[C. Bertulani, H.-W. Hammer, U. Van Kolck, NPA 712, 37 (2002)]

[H.-W. Hammer, C. Ji, D. R. Phillips JPG 44, 103002 (2017)]

# Ab initio description of $^{11}$Be

NCSMC calculation of $^{11}$Be [Calci *et al.* PRL 117, 242501 (2016)]

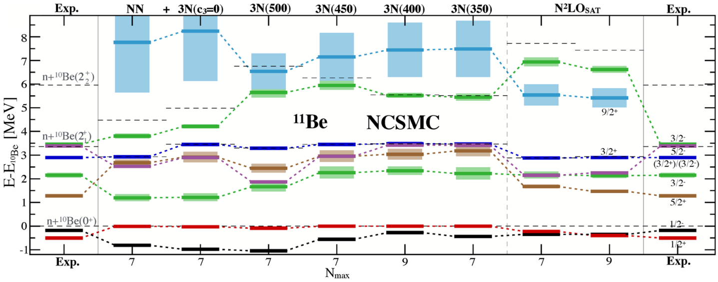



FIG. 2. NCSMC spectrum of $^{11}$Be with respect to the $n + {}^{10}$Be threshold. Dashed black lines indicate the energies of the $^{10}$Be states. Light boxes indicate resonance widths. Experimental energies are taken from Refs. [1,51].

- $\frac{1}{2}^+$ ground state :
  $\epsilon_{\frac{1}{2}^+} = -0.500$ MeV
  $C_{\frac{1}{2}^+} = 0.786$ fm$^{-1/2}$
  $S_{1s\frac{1}{2}} = 0.90$
- $\frac{1}{2}^-$ bound excited state :
  $\epsilon_{\frac{1}{2}^-} = -0.184$ MeV
  $C_{\frac{1}{2}^-} = 0.129$ fm$^{-1/2}$
  $S_{0p\frac{1}{2}} = 0.85$

Calci *et al.* also predict the $^{10}$Be-n phaseshift

# $^{10}$Be-n Halo-EFT potential

Replace $^{10}$Be-n interaction by effective potential in each partial wave

Use Halo EFT : clear separation of scales (in energy or in distance)
⇒ provides an expansion parameter (small scale / large scale)
along which the low-energy behaviour is expanded

[C. Bertulani, H.-W. Hammer, U. Van Kolck, NPA 712, 37 (2002)]

[H.-W. Hammer, C. Ji, D. R. Phillips JPG 44, 103002 (2017)]

Use narrow Gaussian potentials @ LO

$$V_{lj}(r) = V_0^{lj}\ e^{-\frac{r^2}{2\sigma^2}}$$

- In $s\frac{1}{2}$ fit $V_0^{lj}$ to reproduce
  - $\epsilon_{nlj}$ (known experimentally)
- For $l > 0$ : $V_{lj} = 0$

$\sigma$ = 1.2, 1.5 or 2 fm used to evaluate the sensitivity of calculations to short-range physics

# $^{10}$Be-n Halo-EFT potential

Replace $^{10}$Be-n interaction by effective potential in each partial wave

Use Halo EFT : clear separation of scales (in energy or in distance)
⇒ provides an expansion parameter (small scale / large scale)
along which the low-energy behaviour is expanded

[C. Bertulani, H.-W. Hammer, U. Van Kolck, NPA 712, 37 (2002)]

[H.-W. Hammer, C. Ji, D. R. Phillips JPG 44, 103002 (2017)]

Use narrow Gaussian potentials @ NLO

$$V_{lj}(r) = V_0^{lj}\, e^{-\frac{r^2}{2\sigma^2}} + V_2^{lj}\, r^2 e^{-\frac{r^2}{2\sigma^2}}$$

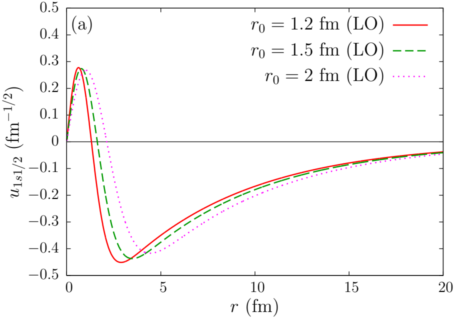
- In $s\frac{1}{2}$ and $p\frac{1}{2}$ : fit $V_0^{lj}$ and $V_2^{lj}$ to reproduce
  - $\epsilon_{nlj}$ (known experimentally)
  - $C_{nlj}$ (predicted *ab initio*) [Calci *et al.* PRL 117, 242501 (2016)]
- $V_{p3/2} = 0$ to reproduce *ab initio* $\delta_{3/2^-} \sim 0$
- For $l > 1$ : $V_{lj} = 0$

$\sigma =$ 1.2, 1.5 or 2 fm used to evaluate the sensitivity of calculations to short-range physics

# $s\frac{1}{2}$ : @ LO potentials fitted to $\epsilon_{\frac{1}{2}^+}$

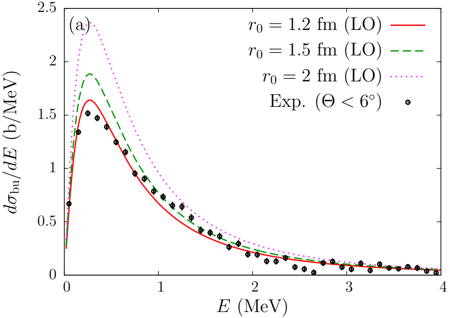
Potentials fitted to $\epsilon_{1s\frac{1}{2}} = -0.503$ MeV

Ground-state wave function




$^{11}$Be+Pb→$^{10}$Be+n+Pb @69$A$MeV

Exp : [Fukuda *et al.* PRC 70, 054606 (2004)]



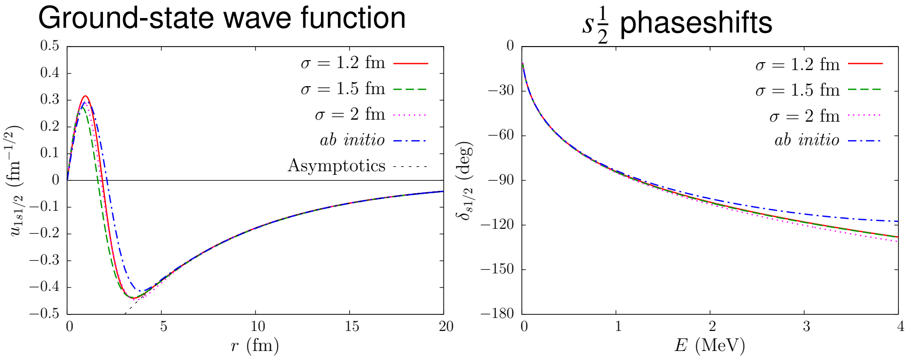

- Wave functions : same exponential decay but different $C_{s\frac{1}{2}}$
- Breakup cross sections scales as $\left|C_{s\frac{1}{2}}\right|^2$

  ⇒ reaction is peripheral

[P.C. & Nunes PRC73, 014615 (2006), *ibid.* 75, 054609 (2007)]

# $s\frac{1}{2}$ : @ NLO potentials fitted to $\epsilon_{\frac{1}{2}^+}$ and $C_{\frac{1}{2}^+}$

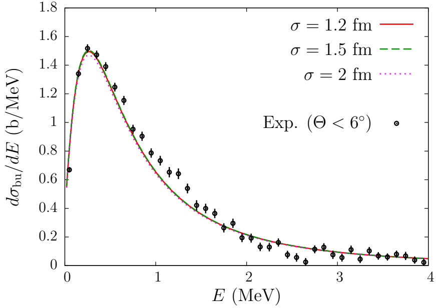
Potentials fitted to $\epsilon_{1s\frac{1}{2}} = -0.503$ MeV and $C_{1s\frac{1}{2}} = 0.786$ fm$^{-1/2}$




- Wave functions : same asymptotics but different interior
- $\delta_{s\frac{1}{2}}$ : all effective potentials are in good agreement with *ab initio* up to 1.5 MeV (same effective-range expansion)
- Similar results obtained for $p\frac{1}{2}$ (excited bound state)

# Coulomb breakup : $^{11}$Be+Pb→$^{10}$Be+n+Pb

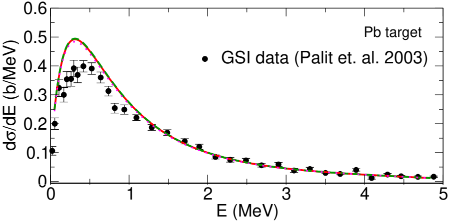
RIKEN : $69A$ MeV




Exp : [Fukuda *et al.* PRC 70, 054606 (2004)]

Th. : [P.C., Phillips & Hammer, PRC 98, 034610]

GSI : $520A$ MeV



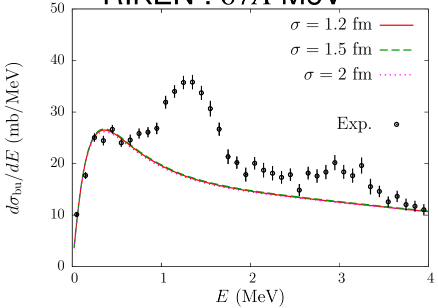

Exp : [Palit *et al.* PRC 68, 034318 (2003)]

Th. : [Moschini & P.C. PLB 790 367 (2019)]

- All calculations provide very similar results for all $\sigma$ despite the difference in the internal part of the wave function
  ⇒ reaction is peripheral [P.C. & Nunes PRC75, 054609 (2007)]
- Excellent agreement with data (no fitting parameter)
  ⇒ confirms *ab initio* ANC and phaseshift

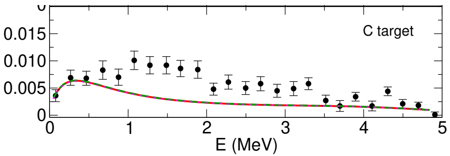
# Nuclear breakup : $^{11}$Be+C→$^{10}$Be+n+C

RIKEN : $67A$ MeV




Exp : [Nakamura *et al.* PRC 70, 054606 (2004)]
Th. : [P.C., Phillips & Hammer, PRC 98, 034610]

GSI : $520A$ MeV




Exp : [Palit *et al.* PRC 68, 034318 (2003)]
Th. : [Moschini & P.C. PLB 790 367 (2019)]

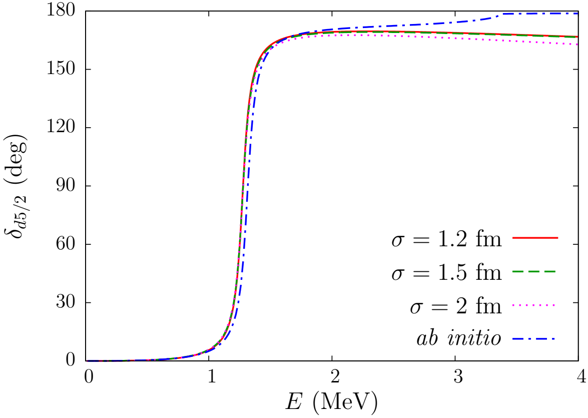
- All potentials produce very similar breakup cross sections
  ⇒ still peripheral (even if nuclear dominated)

[P.C. & Nunes PRC 75, 054609 (2007)]

- Order of magnitude of experiment well reproduced
- Breakup strength missing at the $5/2^+$ and $3/2^+$ resonances

⇒ for this observable, the continuum must be better described

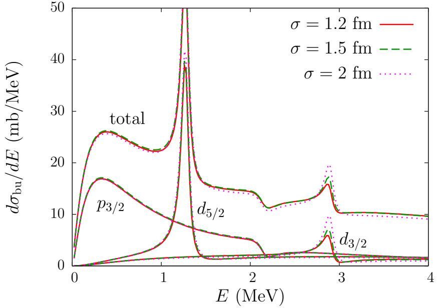
# $d\frac{5}{2}$ : potentials fitted to $\epsilon^{\text{res}}_{\frac{5}{2}^+}$ and $\Gamma_{\frac{5}{2}^+}$



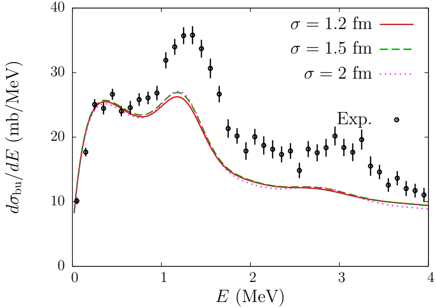

- Identical $\delta_{d\frac{5}{2}}$ up to 1.5 MeV
  up to 5 MeV for the narrow potentials ($\sigma$ = 1.2 or 1.5 fm)
- Excellent agreement with ab initio results up to 2 MeV

# $^{11}$Be+C→$^{10}$Be+n+C @ 67$A$MeV (beyond NLO)

Total breakup cross section and dominant contributions




Folded with energy resolution [Fukuda *et al.* PRC 70, 054606 (2004)]



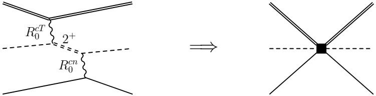

- All potentials produce similar breakup cross sections
- In nuclear breakup, resonances play significant role

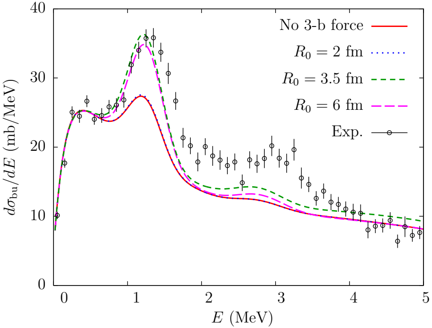
  [P.C., Goldstein & Baye PRC 70, 064605 (2004)]
- Still, resonant breakup not correctly described
  degrees of freedom [$^{10}$Be$(2^+)$] missing in the effective model

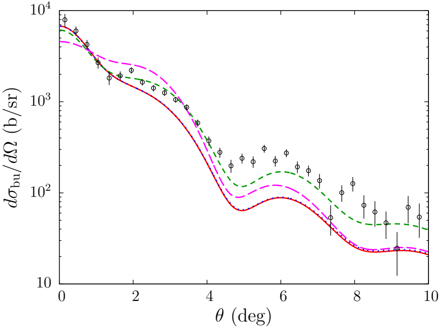
  [Moro & Lay PRL 109, 232502 (2012)]

# Simulating core excitation with 3-b force

Virtual excitation of $^{10}\text{Be}(2^+)$
can be simulated by 3 body force :




- 3-b force can efficiently simulate $^{10}$Be excitation

  [P.C., Phillips & Hammer PLB 825, 136847 (2022)]
- The range in the $c$-$T$ distance should equal that of $V_{cT}$ (3.5 fm)
  - too small ($R_0 = 2$ fm) : no effect
  - too large ($R_0 = 6$ fm) : erroneous angular distribution

# Summary and prospect

- Halo nuclei studied mostly through reactions
- Mechanism of breakup with halo nuclei understood
  To what are these reactions sensitive ?
  How can we relate *ab initio* calculations to reaction observables ?
  Halo EFT : [P.C., Phillips, Hammer, PRC 98, 034610 (2018)]
  Efficient way to include the significant degrees of freedom
- Using one Halo-EFT description of $^{11}$Be, we reproduce Coulomb and nuclear breakup
  - $70A$ MeV : [P.C., Phillips, Hammer, PRC 98, 034610 (2018)]
  - $520A$ MeV : [Moschini & P.C. PLB 790 367 (2019)]
- These reactions are peripheral ⇒ sensitive to ANC and phaseshifts
- Agreement with data validates the *ab initio* predictions
- Same results on $^{15}$C : [Moschini, Yang & P.C., PRC 100, 044615 (2019)]
- Future :
  - Extend to other nuclei (e.g., $^{31}$Ne)
  - Include core excitation in Halo EFT

# Thanks to my collaborators

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