Multi-fermion systems around universality

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Approaches to theoretical nuclear physics:

Personal motivations

+ Understanding of the **mechanisms** of nuclear properties;

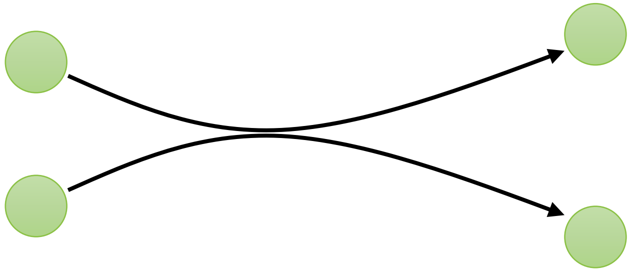
+ Support to **experiments**;

+ **Precision description** of nuclear observables;

+ …

An example of universality – (2-body only)

Unitarity: **The size of a nonrelativistic quantum two-body system** much larger than the **range** of the **interaction** between particles.



S-wave nonrelativistic scattering;
k: center of mass collision momentum;
δ_0: phase shift in $L = 0$ partial wave.

Effective range expansion:

$$k \cot(\delta_0) = -\frac{1}{a_0} + \frac{1}{2} r_0 k^2 + O(k^4)$$

See: Doerte Blume
Blackboard Talk: Few-Body Universality: What Do We Mean by this and Why Do We Care?

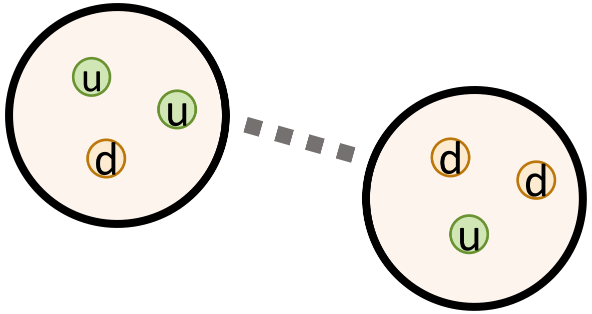
Unitarity / unitary limit: $a_0 \to \infty$, $r_0 \to 0$, (all the other scattering parameters vanish).

An example of universality – unitarity (2-body only)

(the **size of a nonrelativistic quantum two-body system** is much larger than the **interaction range** between particles)

Systems close to the **Unitary limit** can be found in

- **Atomic physics** (Feshbach resonances, $^{6}\mathrm{Li} - {}^{6}\mathrm{Li}$, $^{40}\mathrm{K} - {}^{40}\mathrm{K}$ atoms, …)
- **Nuclear physics** ($n - p$ interaction)
- **Lattice nuclei** (Unphysically large m_π)
- **Hypernuclei** ($\Lambda - n$ interaction)
- **Hadronic physics** ($X(3872)$ Particles)



Atoms (experiments):
C.A. Regal (2003)
M.W. Zwierlein (2003)
M. E. Gehm (2003)
J. T. Stewart (2007)

Nuclei (theory):
U. van Kolck (1999)
S. König (2017)

Hypernuclei (theory):
H.-W. Hammer (2001)
L.C. (2018)

Hadrons (theory):
E. Braaten et al (2003)

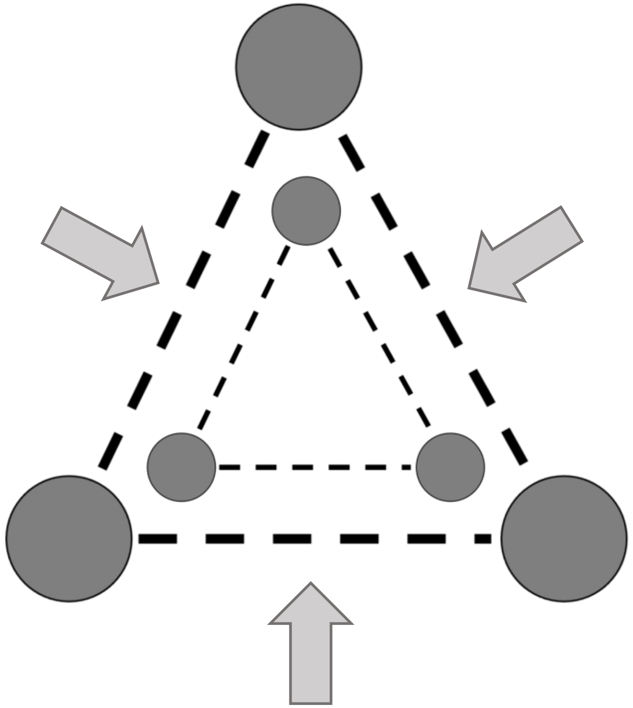
Lattice Nuclei (theory):
N. Barnea et al (2015)
L.C. et al (2017)

Discrete scale invariance: 3+ body

L. H. Thomas (1935)
G. Skorniakov and
K. Ter-Martirosian (1957)

One of the most known and fascinating consequence of unitarity is the

Thomas Collapse / Efimov Effect



In the **unitary limit** a system of **3 bosons/distinguishable particles collapses**

$$r_0 \to 0 \implies E_3 \propto -\left|\frac{1}{r_0^2}\right|$$

A repulsion is needed to **stabilize the system to a finite energy $\boldsymbol{E_3}$**.
$\boldsymbol{E_3}$ **breaks the scale invariance** of the system!
i.e. you have to choose the scale of your system (K, eV, MeV …)

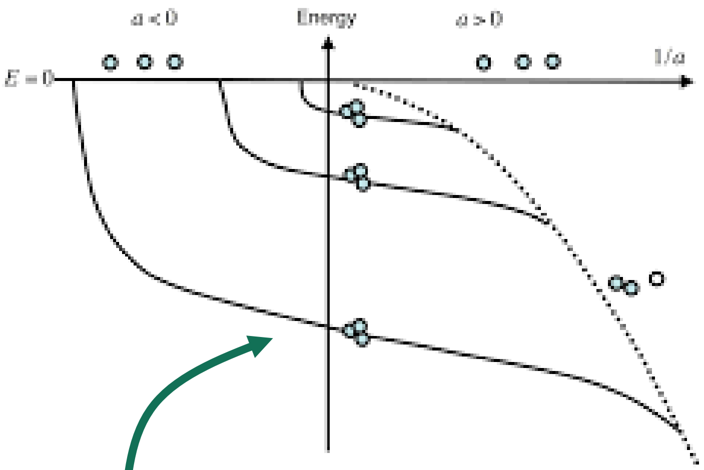
Examples of few-body universality

When a E_3 scale is introduced (maintaining the unitary limit):

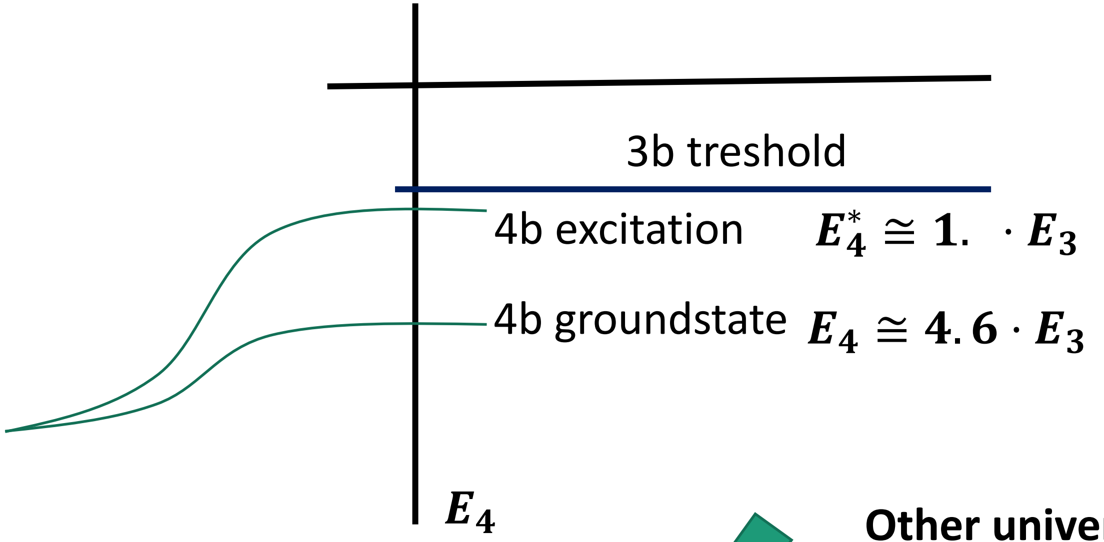
V. Efimov (1970)

- A **tower of states** appears with universal ratios between them
- All the observables are related **to the new scale only**

Efimov states



Groundstate fixed by the 3b force



A. Deltuva (2013)
E. Braaten, H.-W. Hammer (2006)
Rajat K. Bhaduri (2011)

J. Carlson et all. (2017)
P. F.Bedaque et al. (2003)
A.C.Phillips (1968)

Other universality effects>
Tjon line (nuclear physics)
Bosonic drops: $E_N \propto E_3$
Phillips line

Simple and intuitive: Contact theory ($r_0 \to 0$)

- Treat **particles as degrees of freedom** (elementary particles)
- They can interact only **short-range**

  (Short range structure is irrelevant: no quark structure)

  (Long range interactions are negligible: no pion exchange)

- Works for a limited set of energies

- Easy to understand
- Clear limitations
- Expandible
- Minimal inputs required
- Universally transposable

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  (Short range structure is irrelevant: no quark structure)

  (Long range interactions are negligible: no pion exchange)

- Works for a limited set of energies
- Tricky to be properly implemented
- Clear limitations only in the known cases
- Not trivial to be practically expanded beyond 1st order
- Minimal inputs required at the first orders

- Easy to understand
- Clear limitations
- Expandible
- Minimal inputs required
- Universally transposable

A complete theory

$r_{ij} = r_i - r_j$

$$V(r_{ij}) = \delta(r_{ij})$$

Contact theory formally:

$$L = N^\dagger \left(i\partial_0 + \frac{\hbar^2}{2m} \nabla^2 \right) N \; - C_0 \, N^\dagger N^\dagger N N$$

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$$\begin{aligned} L^{N^{>0}LO} = \quad & C_2 \left(N^\dagger \nabla^2 N \, N^\dagger N + h.c. \right) + C_{11} \left(N^\dagger \vec{\nabla} \, N \, N^\dagger \vec{\nabla} N \right) + \\ & C_4 \left(N^\dagger \nabla^4 N \, N^\dagger N + h.c. \right) + \; \ldots \\ & D_0 \left(N^\dagger N^\dagger N^\dagger N N N \right) + E_0 \left(N^\dagger N^\dagger N^\dagger N^\dagger N N N N \right) + \; \ldots \end{aligned}$$

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Including all the derivative/many-body operators one can **express any interaction**

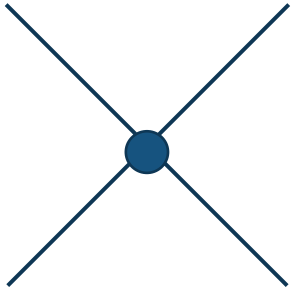
A complete theory

$r_{ij} = r_i - r_j$

Contact theory formally:

$V(r_{ij}) = \delta(r_{ij})$

$$L = N^\dagger \left(i\partial_0 + \frac{\hbar^2}{2m} \nabla^2 \right) N - C_0 \, N^\dagger N^\dagger N N$$



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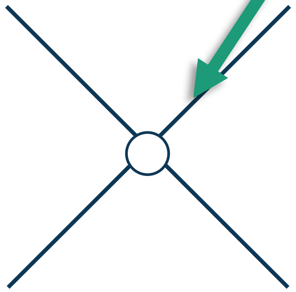

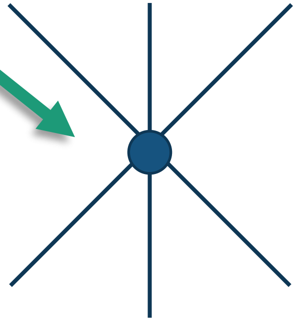

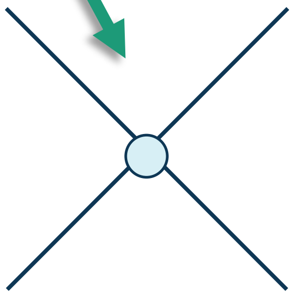

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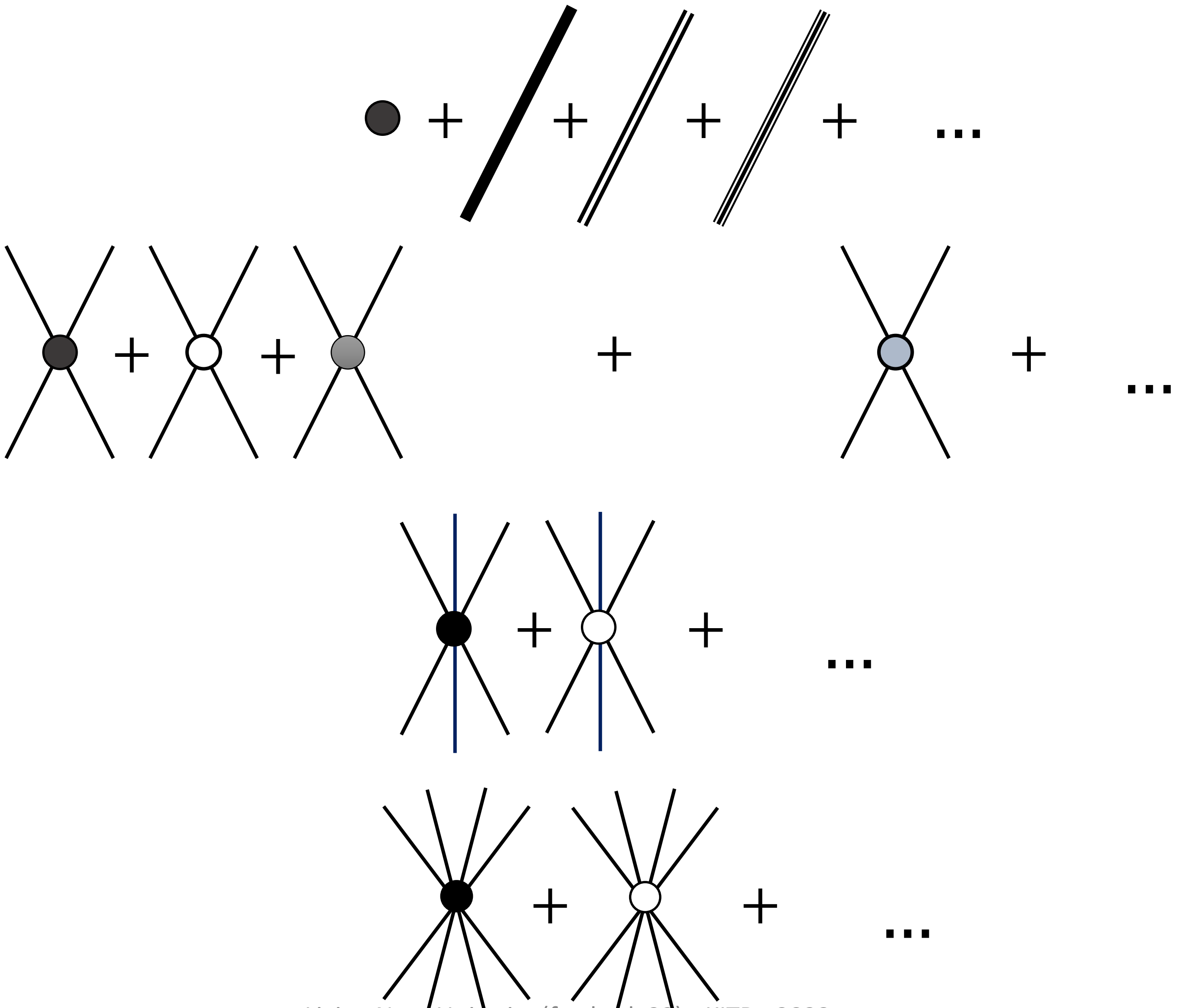
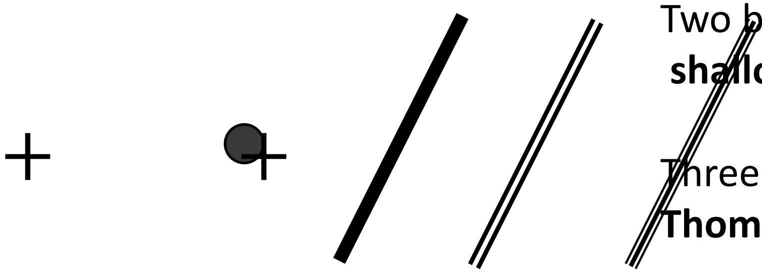
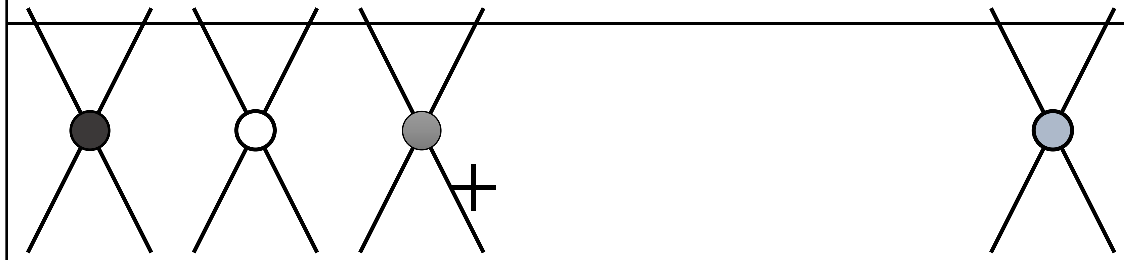
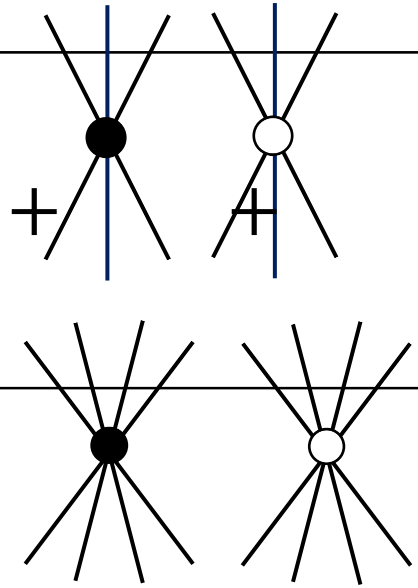
Pionless EFT powercounting



U. Van Kolck - Progress in Particle and Nuclear Physics 43, 1999, 337-418

Pionless EFT powercounting

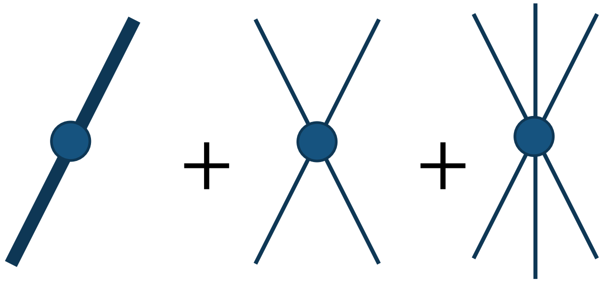
LO	 Two body is needed to provide **shallow states**. Three-body needed to avoid **Thomas collapse**.
NLO	
N^2LO	 $\underset{\substack{\wr \\ M_{hi}}}{C_2}\overleftarrow{\nabla}\cdot\overrightarrow{\nabla} \sim c_2 \frac{Q^2}{M_{hi}}$

Pionless EFT powercounting

In the nuclear case: $\Gamma_{\mathrm{NN}} = \frac{\mathrm{Q}}{\mathrm{m}_\pi} = 0.5 \sim 0.8$

Momentumless 2-3 body

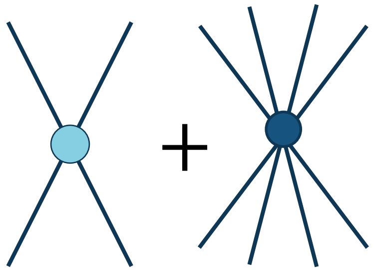
LO



1

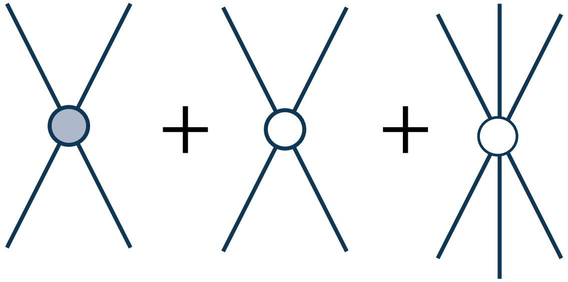
Momentum dependent / 4-body

NLO



$O(\Gamma)$

$\mathrm{N}^2\mathrm{LO}$



$O(\Gamma^2)$

$O(\Gamma^{\geq 3})$

G.P. Lepage, How to renormalize the Schrödinger equation (1997)
U. van Kolck, Nucl.Phys. A645 273-302 (1999)
J.-W. Chen, et al. Nucl.Phys. A653 (1999)
S. König, H. W. Grießhammer, H. W. Hammer, and U. van Kolck, J. Phys. G43, 055106 (2016)
B. Bazak, PRL 122.143001 (2019)

Contact Renormalizability

The **Lagrangian** can be transformed into a **Hamiltonian** that may be used in **many-body calculations**

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0\, \delta\left(\vec{r_i}, \vec{r_j}\right) + \sum_{ijk} D_0\, \delta\left(\vec{r_i}, \vec{r_j}, \vec{r_k}\right)$$

Regularize the interaction to smear the contact interactions

*May have 2 two-body channels
(E.g. in nuclear physics)

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C^\lambda\, e^{-\frac{\lambda^2 r_{ij}^2}{4}} + \sum_{ijk} D^\lambda \sum_{cyc}\left[e^{-\frac{\lambda^2\left(r_{ij}^2 + r_{ik}^2\right)}{4}}\right]$$

Renormalization fixes the dependence of C_λ and D_λ to observables
C^λ and D^λ fitted on **two- and three-body observables**.

Any $\lambda \to \infty$ **any** observable becomes λ independent

Duality

universality

Unitary limit: $a_0 = \infty$

$r_0 = 0$

Finite three-body scale: $0 > E_3 > -\infty$

(contact) EFT

(nonrelativistic)

<u>LO</u>

$$\mathcal{L} = N^\dagger\left(\partial_0 + \frac{\nabla^2}{2m}\right)N + $$
$$+ C_0 N^\dagger N^\dagger N N + D_0 N^\dagger N^\dagger N^\dagger N N N$$

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$$\mathcal{L} = N^\dagger \left(\partial_0 + \frac{\nabla^2}{2m} \right) N + \\ + C_0 N^\dagger N^\dagger N N + D_0 N^\dagger N^\dagger N^\dagger N N N$$

However, no physical system is perfectly in the unitary limit

S. König (2016)

N^nLO

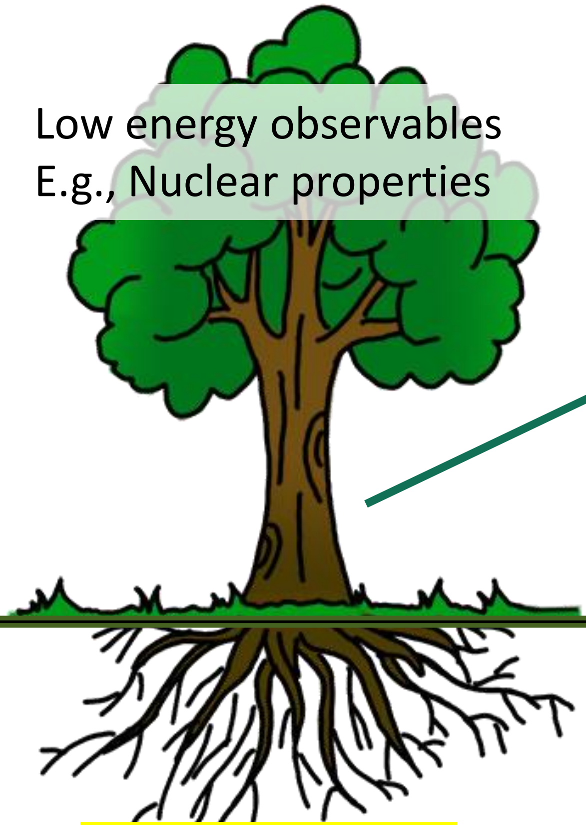
Physical systems can be close to the limit:
e.g. $|a_{n-n}| = (|-23.|\,\mathrm{fm}) \gg (\mathrm{r}_0 \sim 2.7\ \mathrm{fm})$

Deviation from the universal limit
are needed to predict physical phenomena.

Effective field theory **powercounting**

i.e. subleading perturbative corrections
define the specific physical system.

From the fundamental theory to the low energy physics



Low energy observables
E.g., Nuclear properties

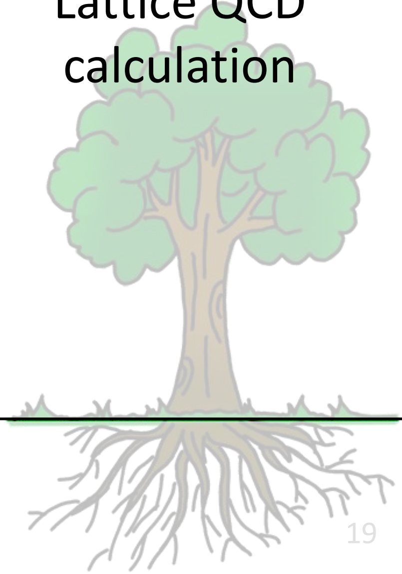
Pionless / Contact EFT

QCD

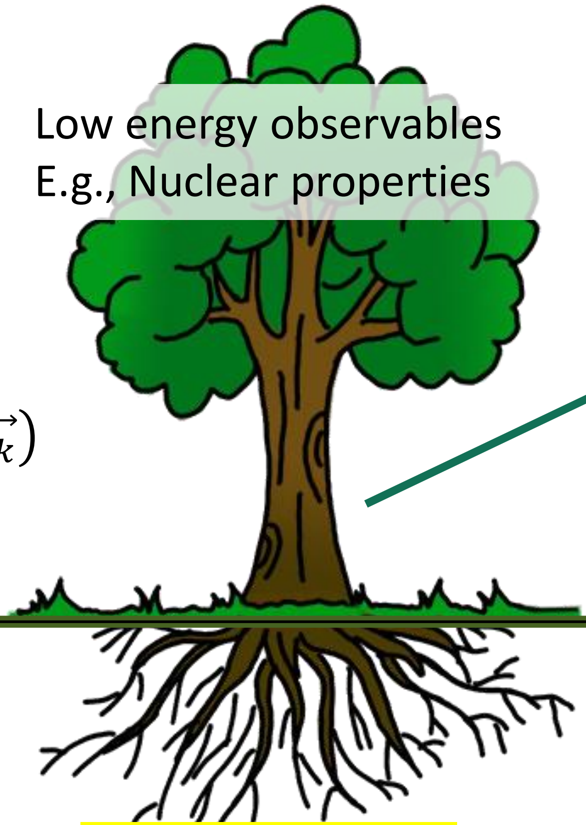
Nuclear data from experiments:

- Experimental spin singlet and triplet scattering length,
- deuterium binding energy
- Energy of few-body systems (e.g., $B(^3\mathrm{H})$)
- …

Lattice QCD calculation



From the fundamental theory to the low energy physics



Low energy observables
E.g., Nuclear properties

Pionless / Contact EFT

$$\sum_{ij} C_0^{\{0,1\}}\, \delta\left(\vec{r_i},\vec{r_j}\right) + \sum_{ijk} D_0\, \delta\left(\vec{r_i},\vec{r_j},\vec{r_k}\right)$$

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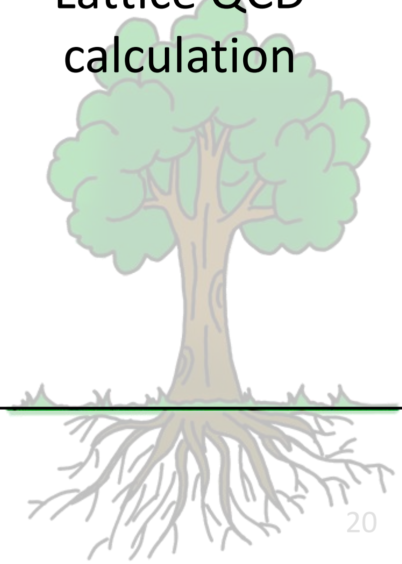
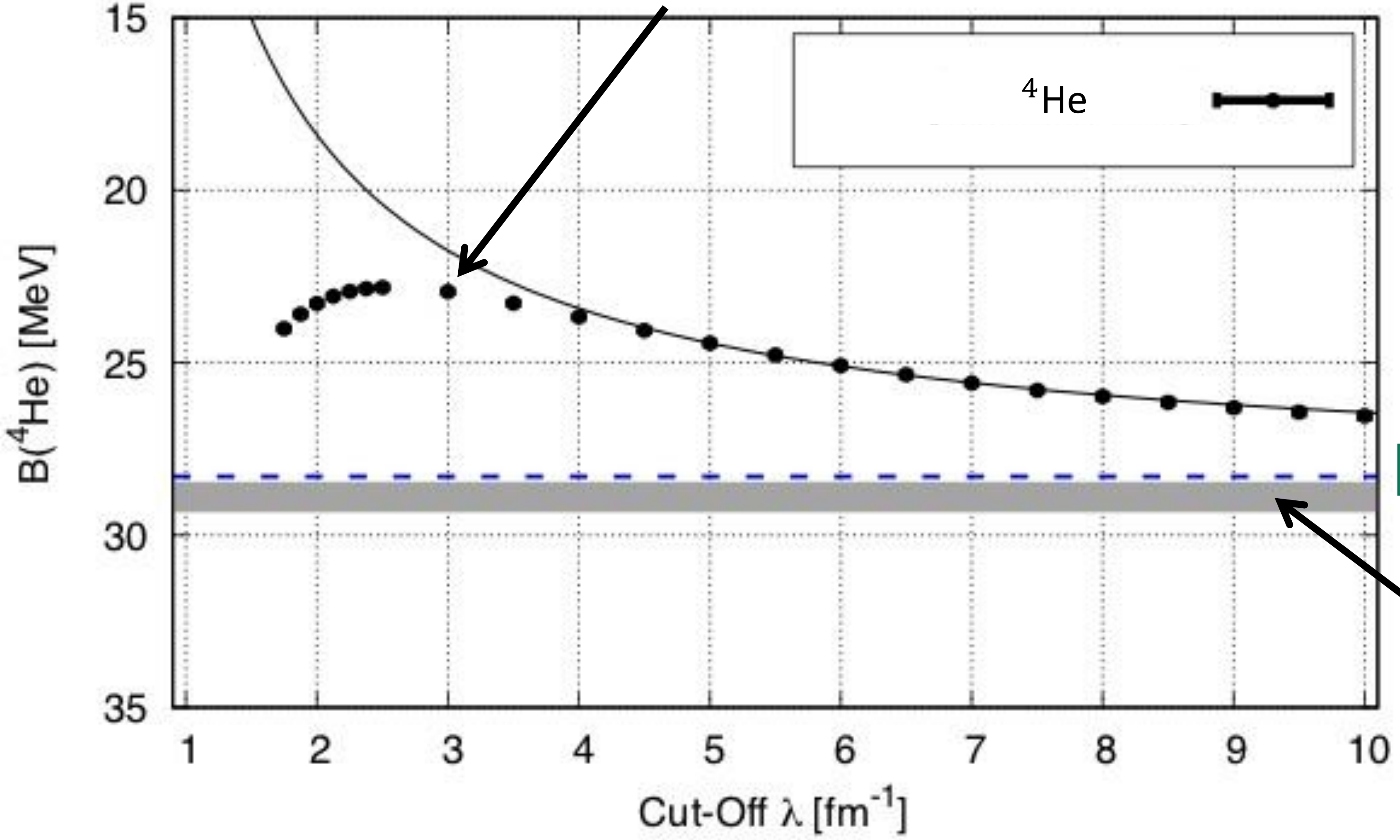


^{4}He

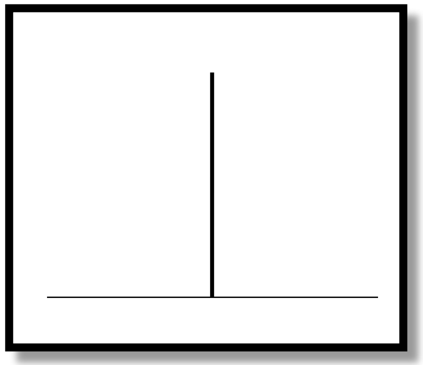
SVM calculations with uncertainty



Everything works great with Pionless EFT up to 4-nucleons.

$$E_\lambda \propto \frac{1}{\lambda}$$

Contact limit



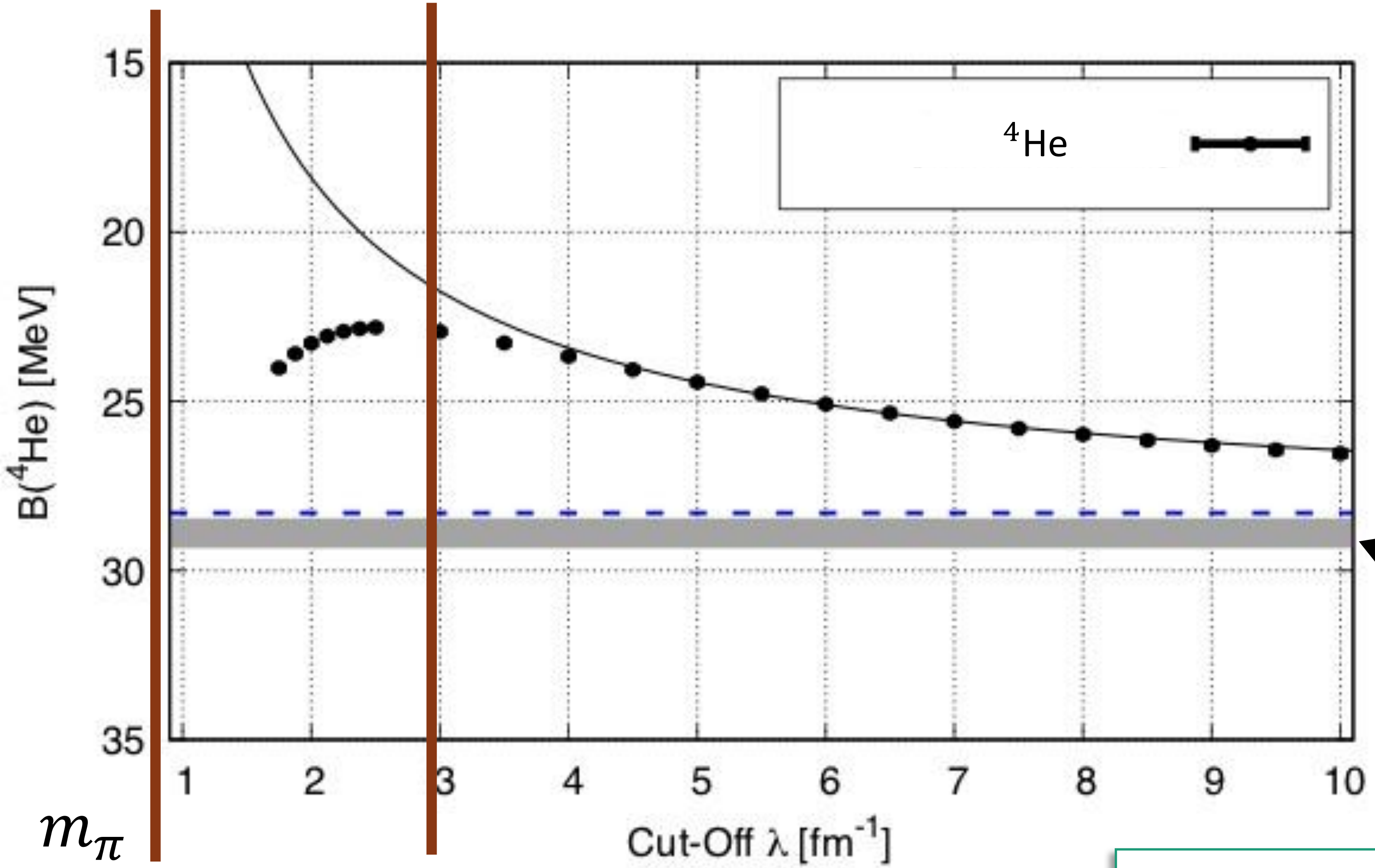
- - - - Experimental data

▬ Extrapolation

Calculations done with **few-body stochastic variational diagonalization method (SVM)**: Y. Suzuki, K. Varga (2003)

L. C., N. Barnea, and A. Gal Phys. Rev. Lett. **121**, 102502

^{4}He



LO pionless EFT theory fitted on **two- and three-body** observables **predicts well ^{4}He energy**!

Fitted on: $a_{n-n} = -18.63$ fm
$B_d = -2.22$ MeV
$B_t = -8.48$ MeV

Experimental data: 28.3 MeV
Extrapolation: 29.2±0.5 MeV

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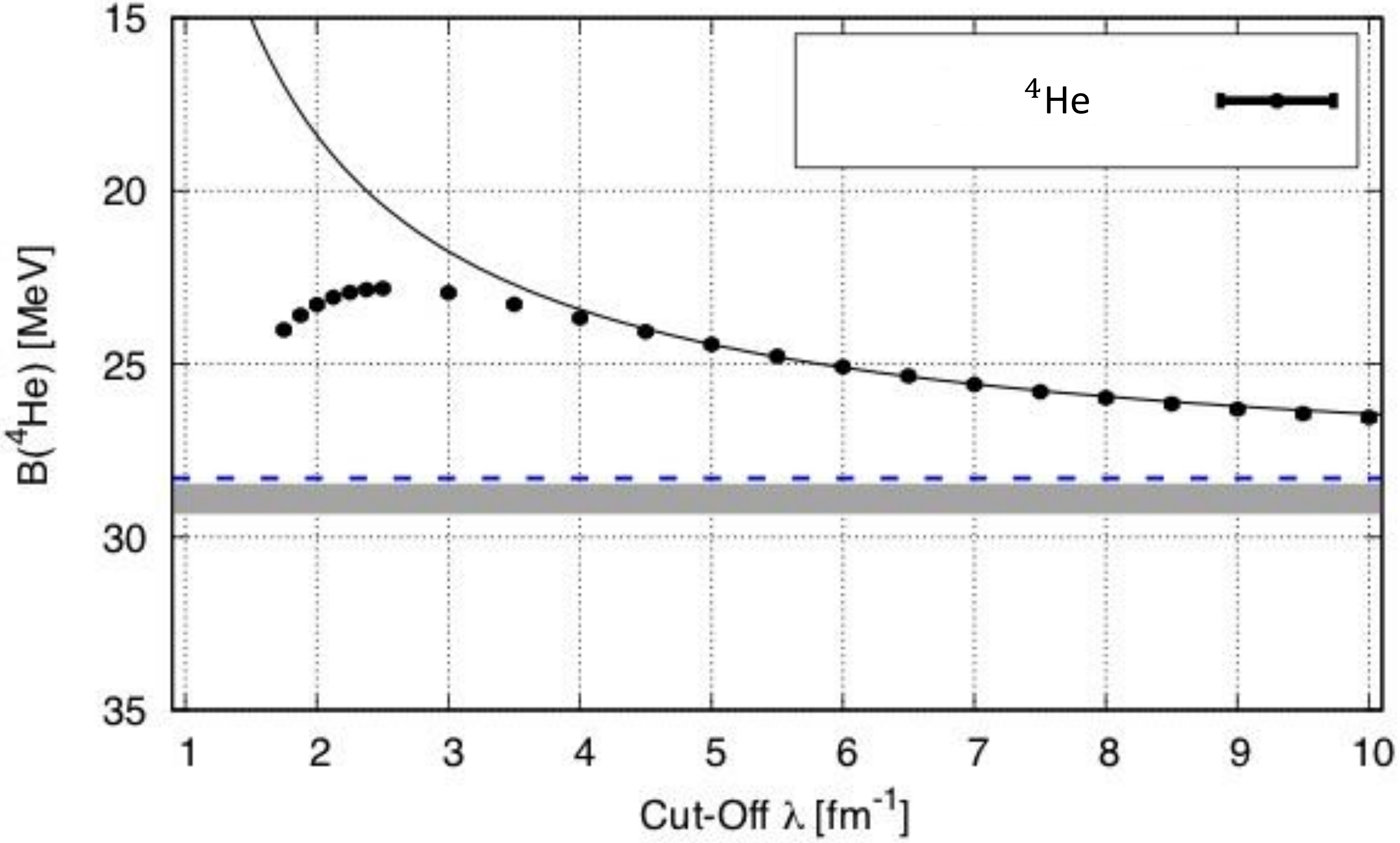
^{4}He



Everything works fine with Pionless EFT up to 4-nucleons.

^{16}O - Monte Carlo calculation

Phys.Lett.B 772 (2017) 839-848

S-wave system

Λ $[\mathrm{fm}^{-1}]$	$^{4}\mathrm{He}$ Energy [MeV]
2	-23.17(2)
4	-23.63(3)
6	-24.06(2)
8	-26.04(5)
∞	$-30^{0.3(sys)}_{2.0(stat)}$
Exp	-28.296

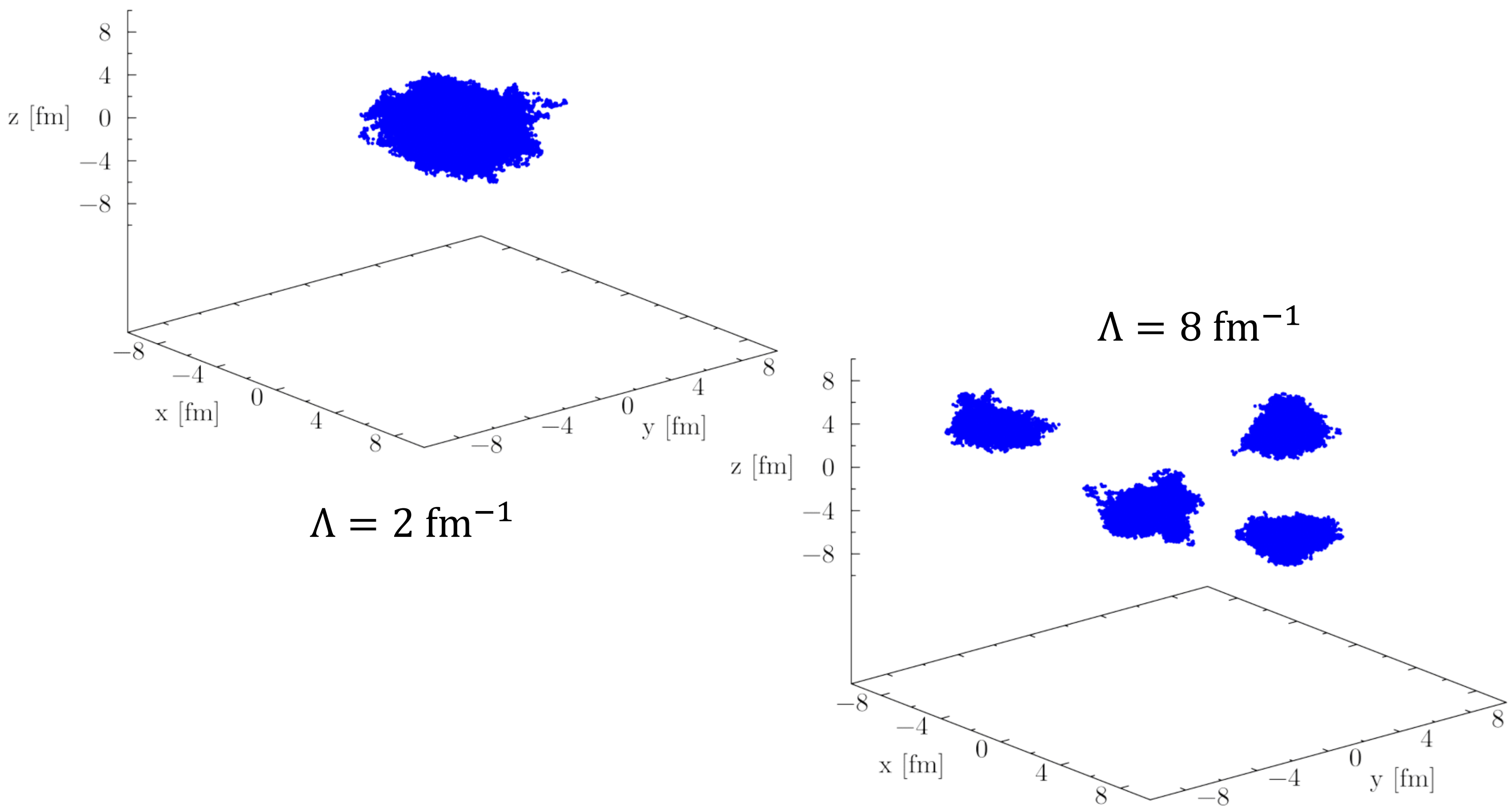
P-wave system

Λ $[\mathrm{fm}^{-1}]$	$^{16}\mathrm{O}$ Energy [MeV]	4α treshold [MeV]
2	-97.19(6)	-92.68(8)
4	-92.23(14)	-94.52(9)
6	-97.51(14)	-100.24(8)
8	-100.97(20)	-104.2(2)
∞	$-115^{1(sys)}_{8(stat)}$	$-120^{1(sys)}_{8(stat)}$

- All the errors shown are statistical errors from Monte Carlo method.

Be(^{16}O) ~ 127 MeV
Be(4α) ~ 113 MeV
It is only 10% difference!

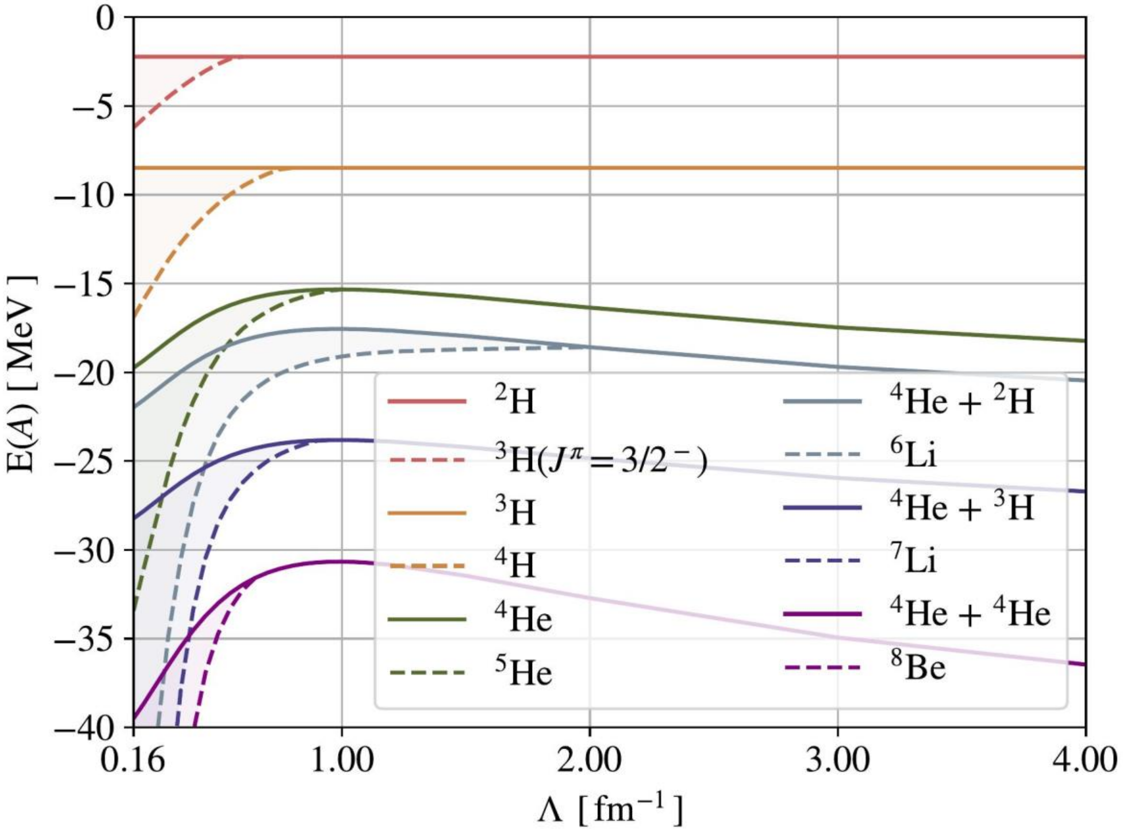
Oxygen density ($m_\pi = 140$ MeV)



L.C. *et al., Phys.Lett.B* 772 (2017) 839-848

Many-fermion systems

Input data: SU(4) theory fitted on B(n-p) and B(^{3}H)



We know that with finite range stabilization is possible, e.g.:
S. König, et al. (2017)
A. Bansal (2018)
…

Multi-fermion systems with contact theories, PLB 816 (2021)
Instability is also observed in Pionless:
I. Stetcu et al. (2006) and W. G. Dawkins et al. (2020)
And in a different framework in M. Gattobigio, et al. (2019)
For the idea of how to enhance stability: Section 6.2.2 of Pascal Naidon and Shimpei Endo Rept.Prog.Phys. 80 (2017) 5, 056001

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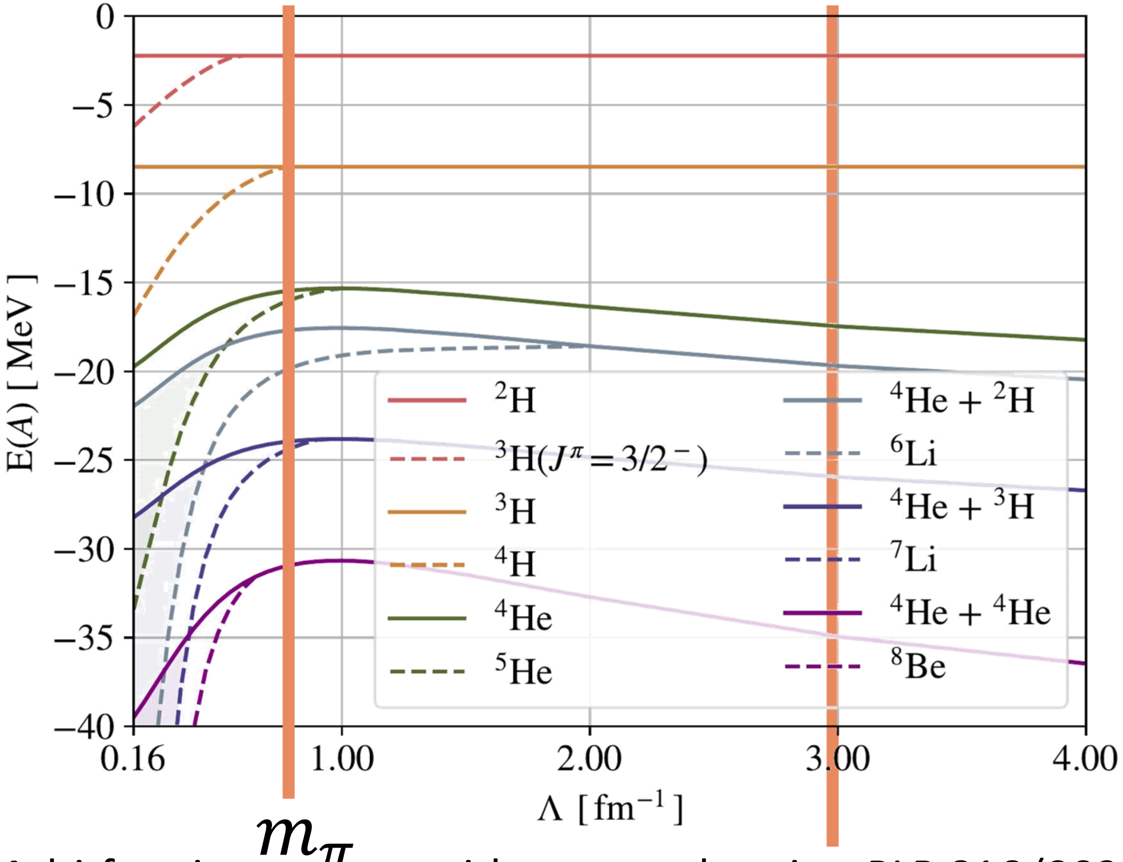
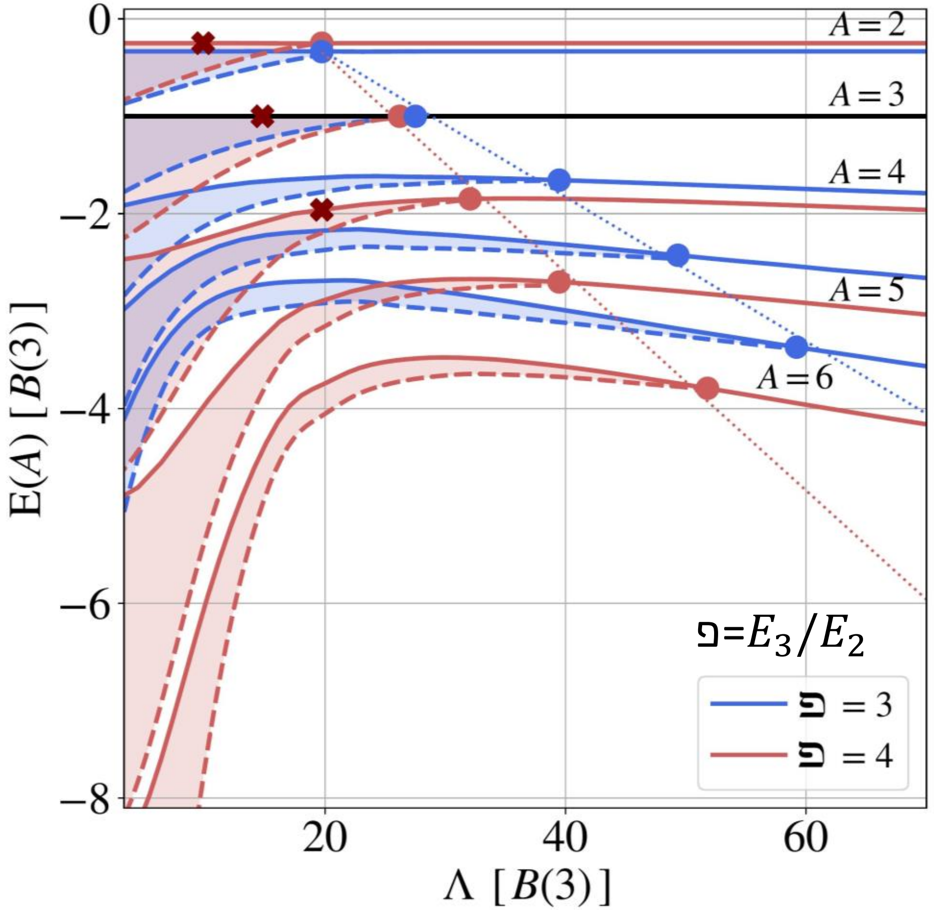


Toy system:



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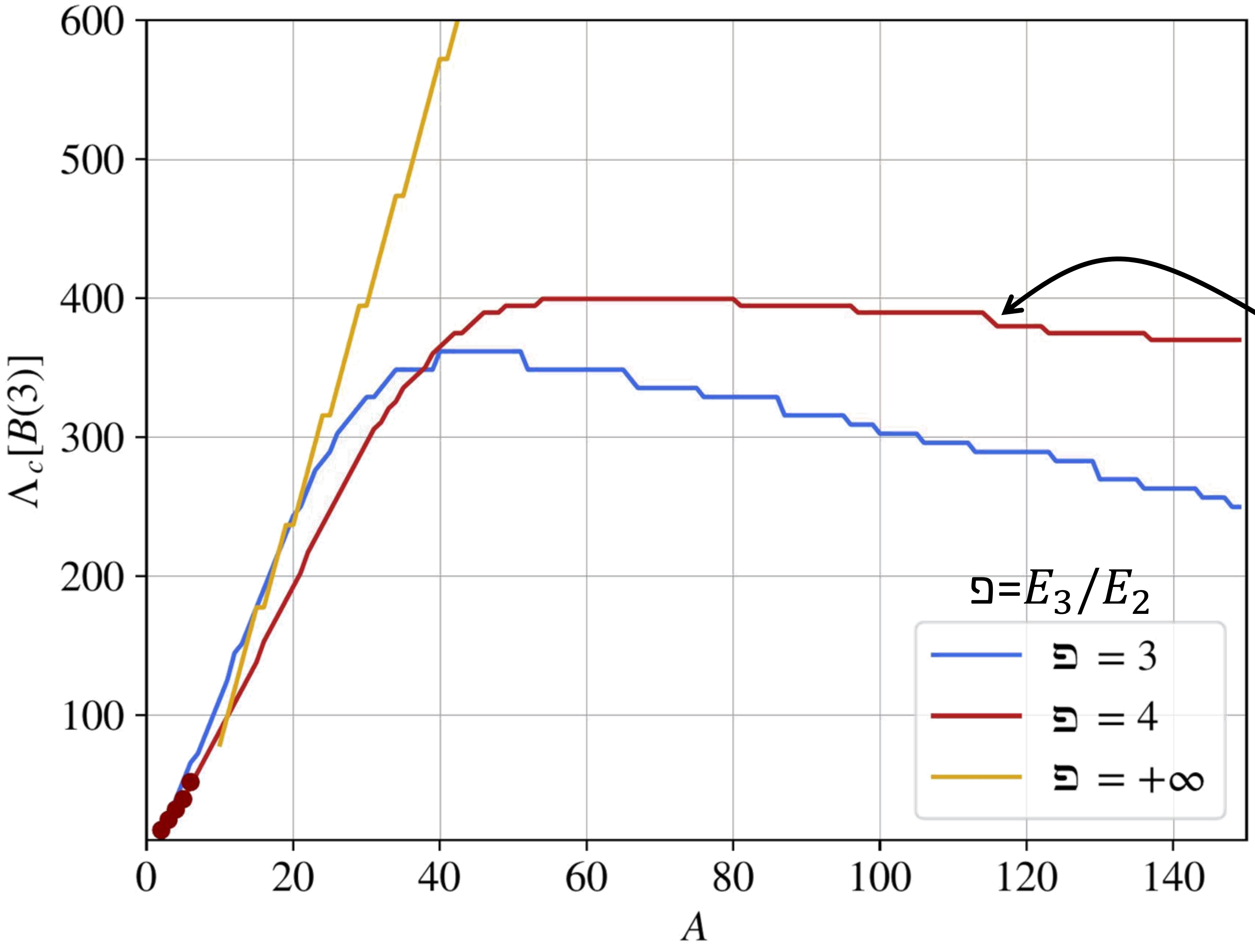
Best case scenario: a heavy bosonic core of A particles + One particle in P-wave
e.g. **(A-1) identical bosons** and **two identical fermions** , all with the same mass.

A -> infinity

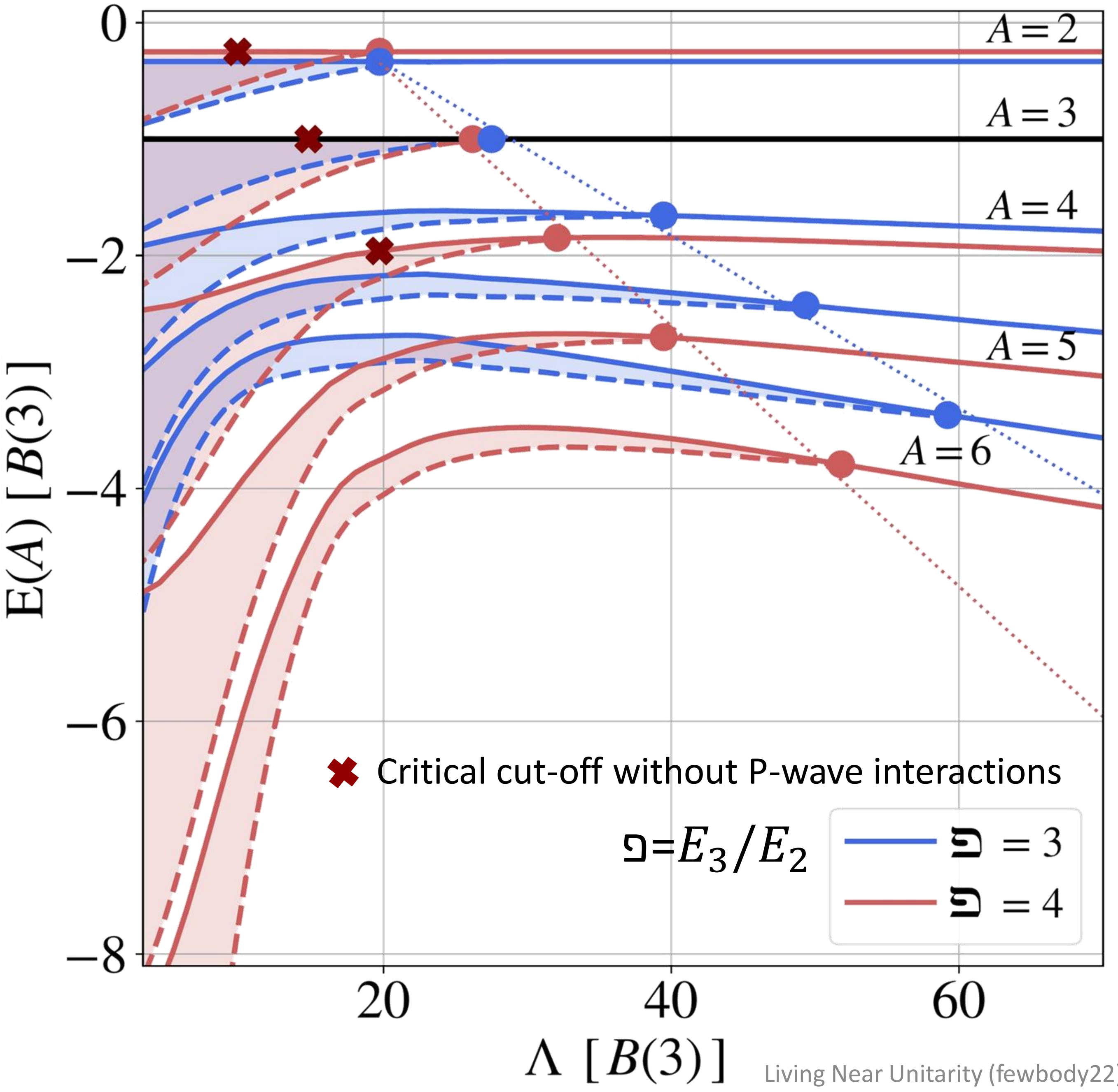


Resonating group method (with a folded interaction) in the **local approximation**.

The discretization is due to the method used to find Λ_c.

No finite number of particle for which $\Lambda_c \to \infty$:
no stable P-wave systems in sight.

Why we see a maximum in Λ_c is still puzzling.



What is the **secret ingredient** we are missing?

- Effective range corrections?
- P-wave interaction?
- ~~Shape parameters?~~
- ~~Many body forces?~~
- …

Gaussians with large width include
a lot of contributions.

Projecting the interaction in S-wave
(to remove P-waves and to keep the range)
we notice that **the critical cut-off halves**.

Final conclusion:
We don't know if we need range or p-waves

5He 6He

J. Kirscher, H. W. Grießhammer, D. Shukla, H. M. Hofmann: arXiv:0909.5606

Breaks in $\alpha + n$ and $\alpha + n + n$

7Li 8Be

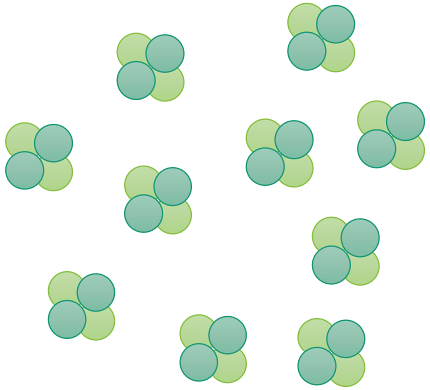
Our calculations in SU(4) symmetry

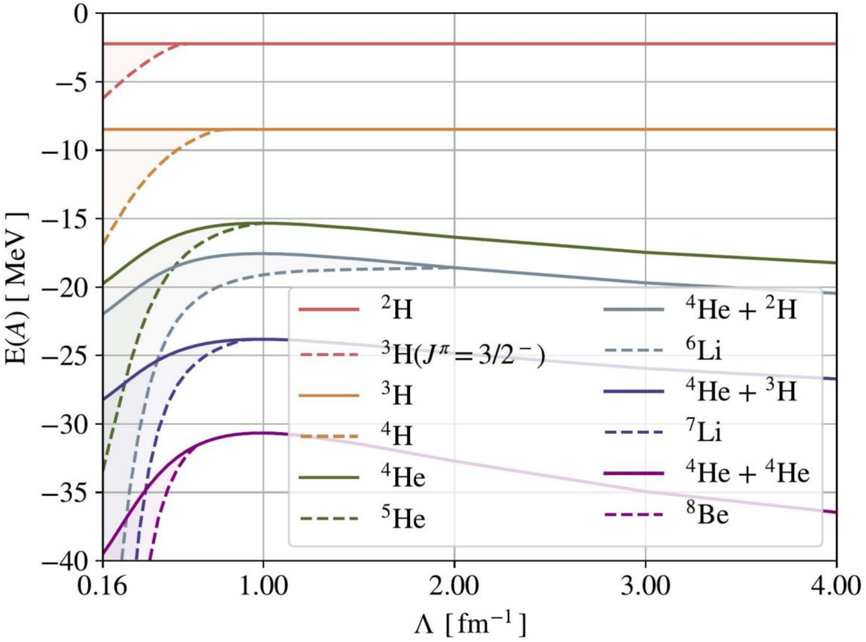
Breaks in $\alpha + d$ and $\alpha + \alpha$

40Ca

QMC calculation suggests the breaking in:

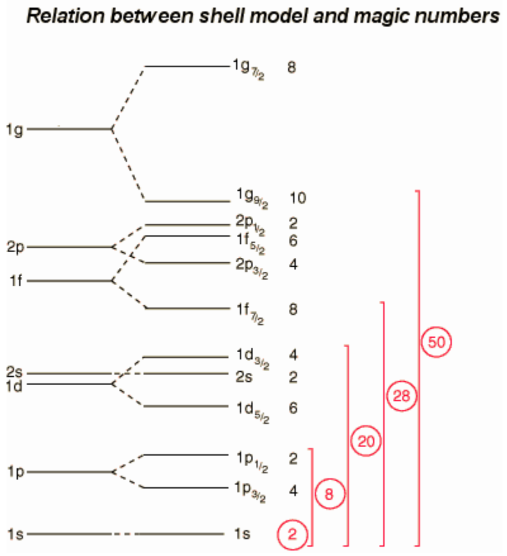
Breaks in $\alpha + \alpha + \alpha +$ …





P-wave systems

In a shell model representation



Will they ever bind?

What to do now?

5He 6He

J. Kirscher, H. W
H. M. Hofmann:

Breaks in α

40Ca

QMC calc

Break

Is this the end?

Do we need to **change the powercounting** of the theory?
(this would be complicated for renormalization reasons)
Should we abandon renormalizability?

- - - - -

Maybe there is not need to be dramatic.

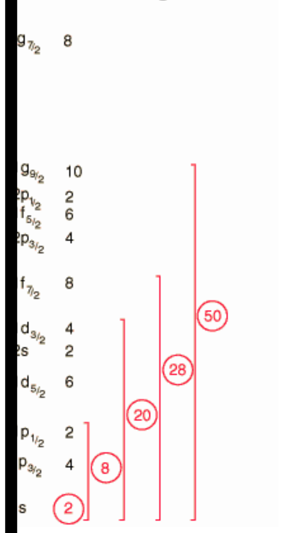
$B(^{16}O) \sim 127$ MeV

$B(4\alpha) \sim 113$ MeV

It is only **10% of difference** and the LO error is larger than that!
$\sim 30\%$ in the most optimistic case.

Λ [fm^{-1}]

P-wave systems
l representation

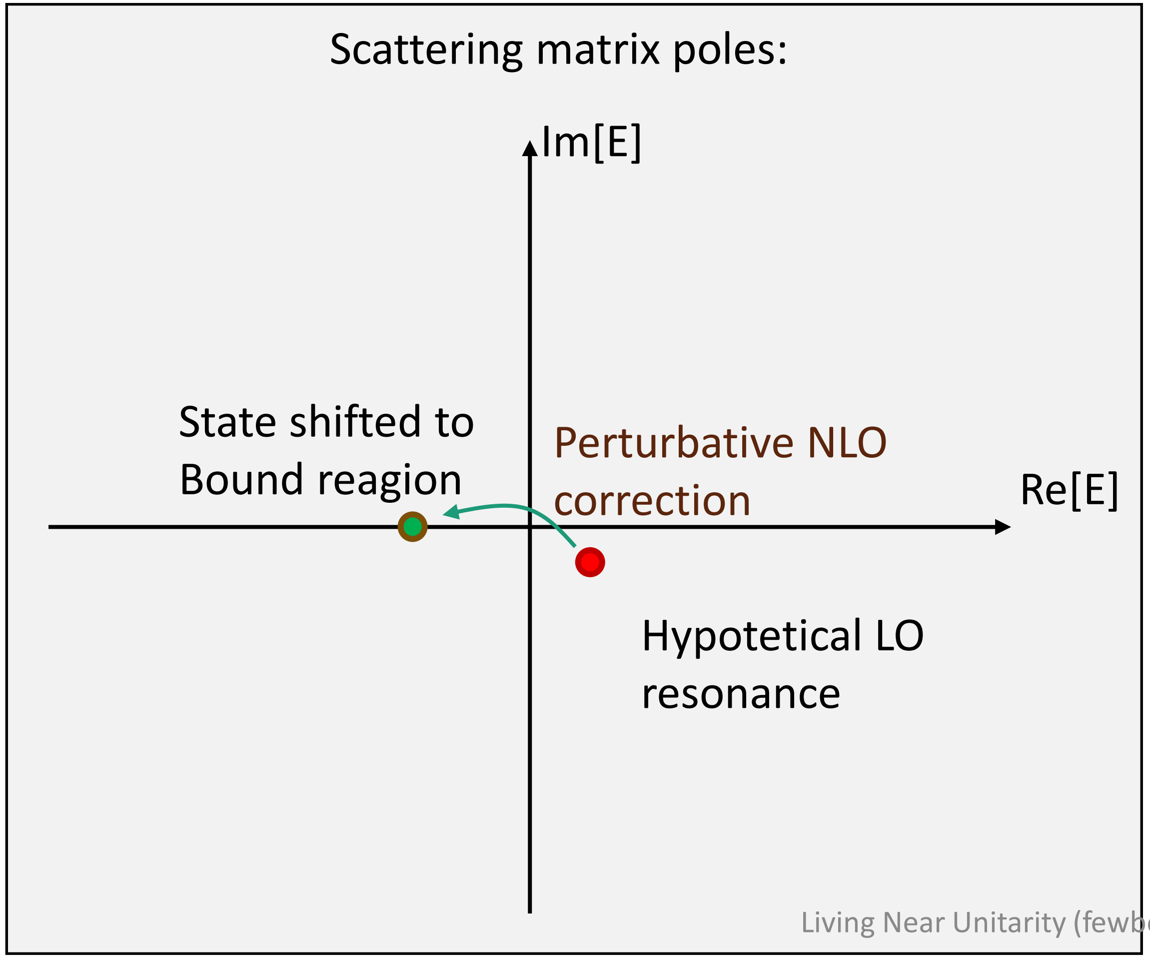


ever bind?

Resonance

One little step further is necessary:

If a resonance is close to the **threshold**, it might be possible to move it with a **subleading correction** (there is no proof this is possible, nor proof this is not possible)



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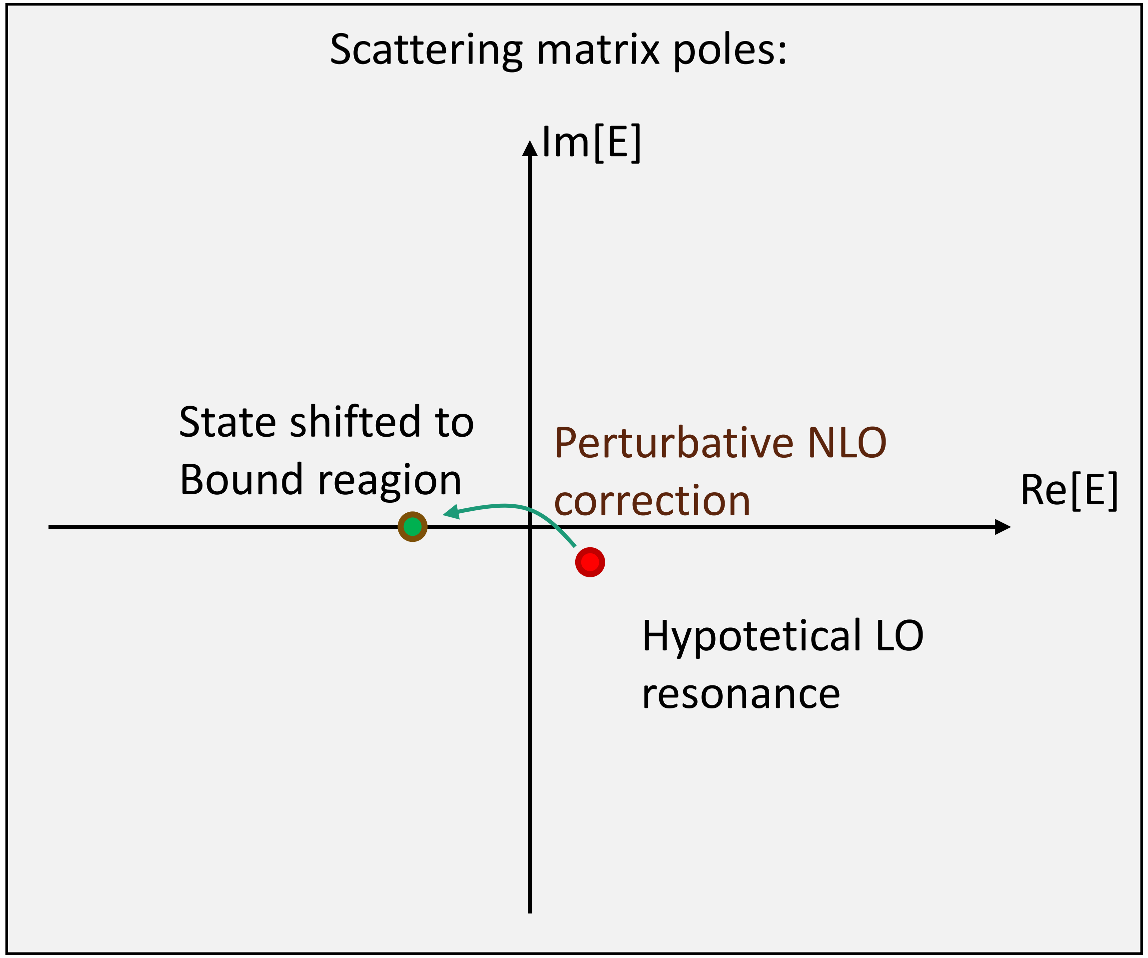
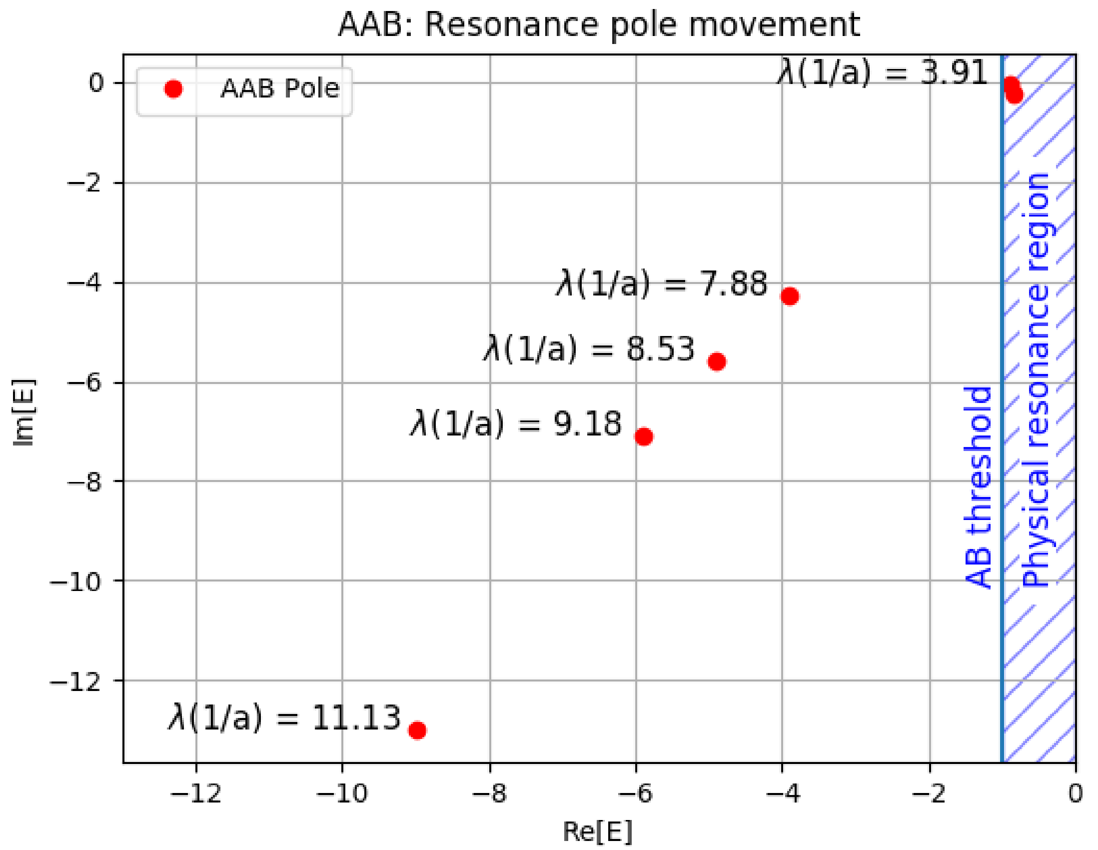


Easiest case: *dimer − fermion*

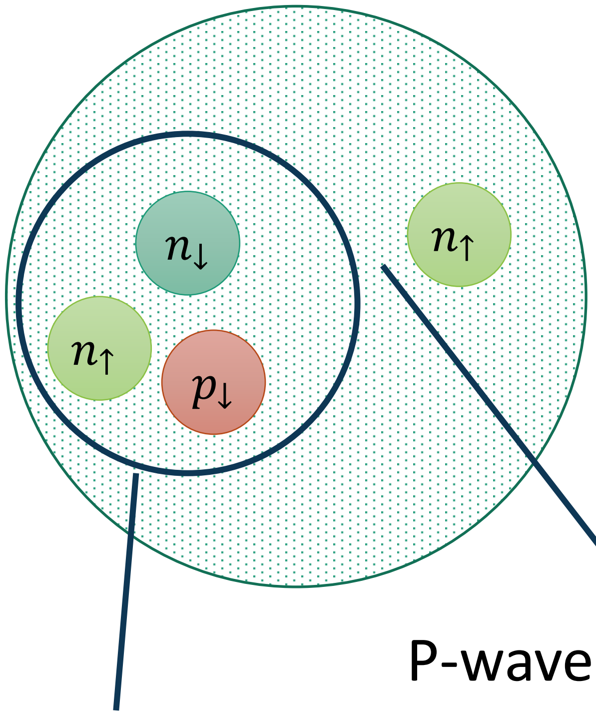
Known **three-fermion** case:
No physical resonance is found.

No scale invariance breaking,
Three-body force might change picture.



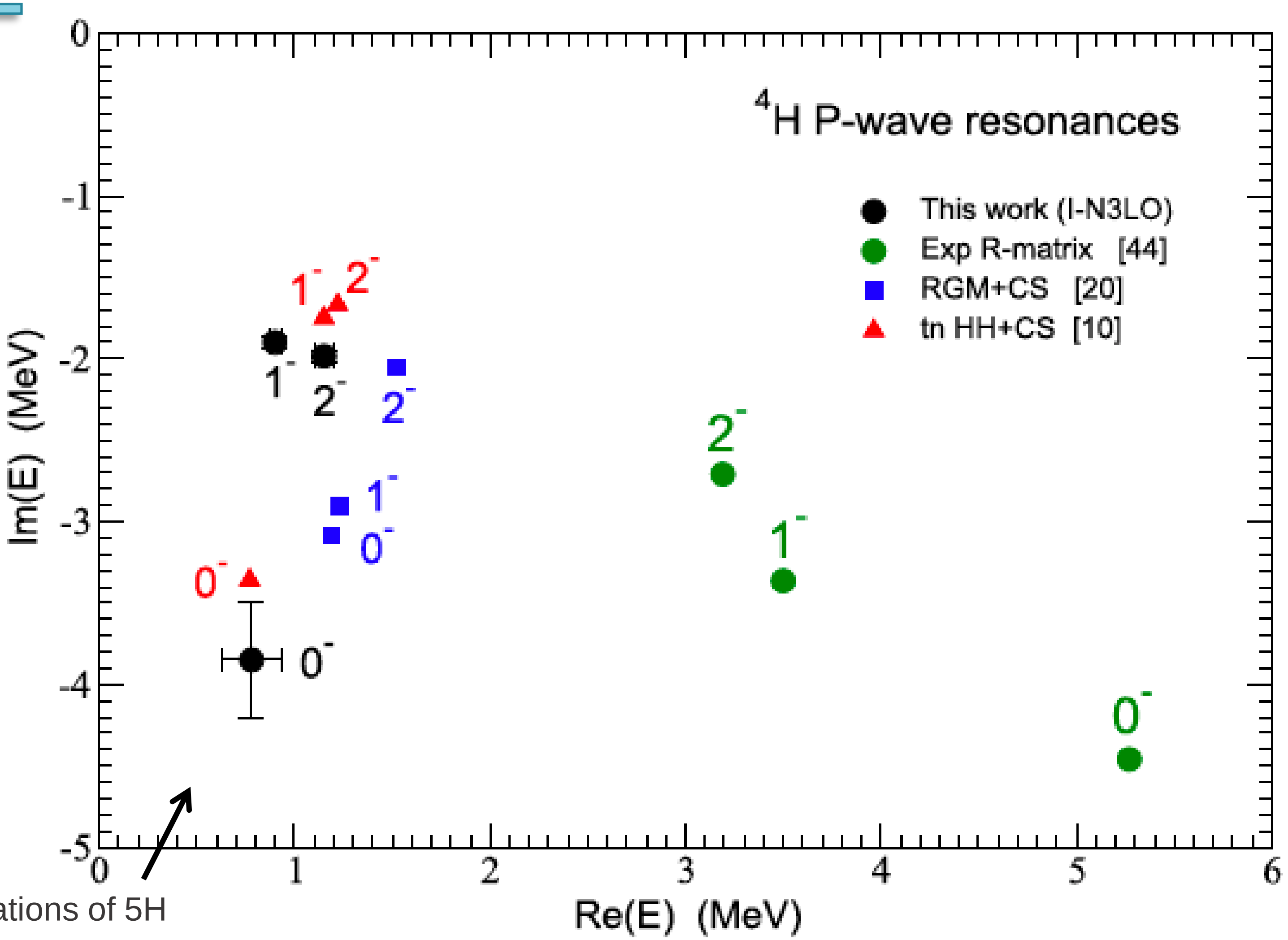
See. S. Dietz et al. https://arxiv.org/abs/2109.11356 (2022)
J. Balal Habashi et al. (2020)

4H resonance:
the minimal nuclear system with an Efimovian component



P-wave interaction

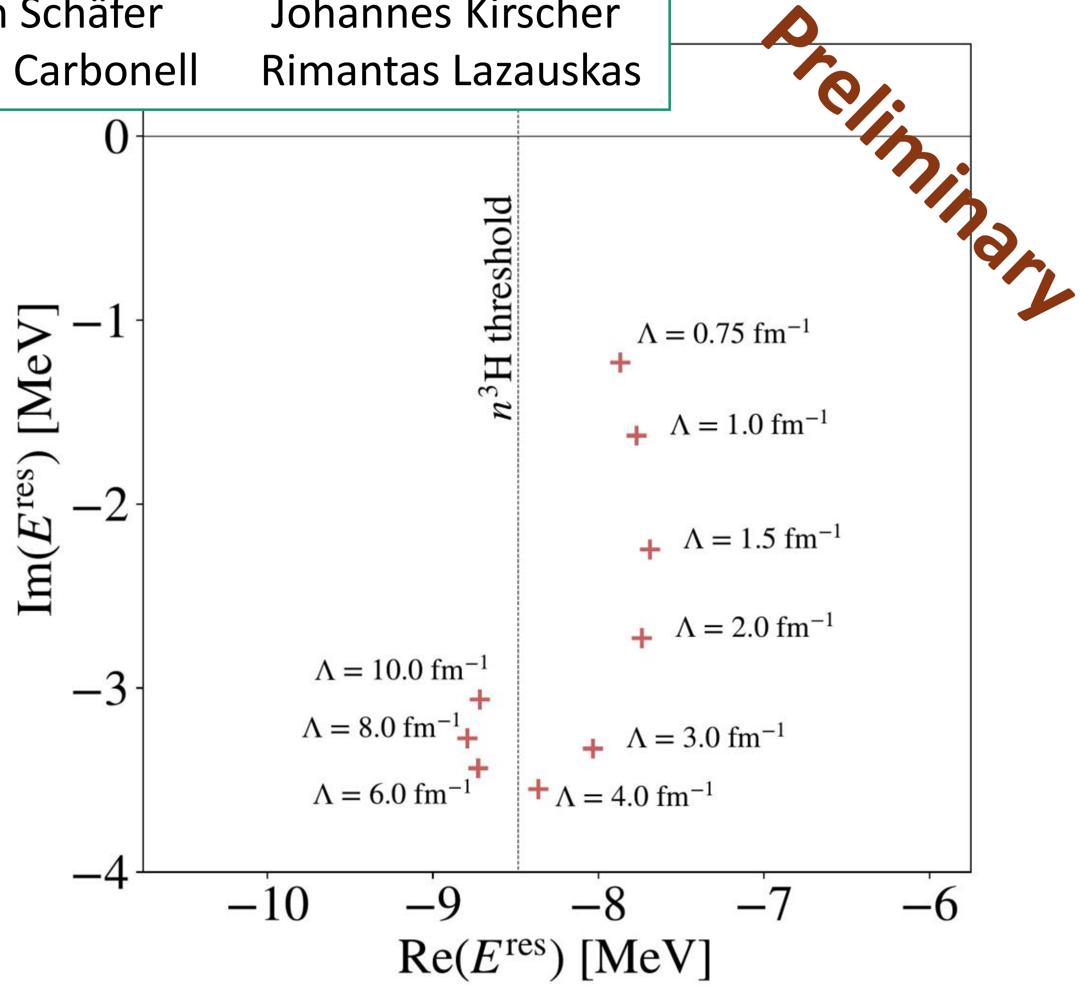
Efimovian system

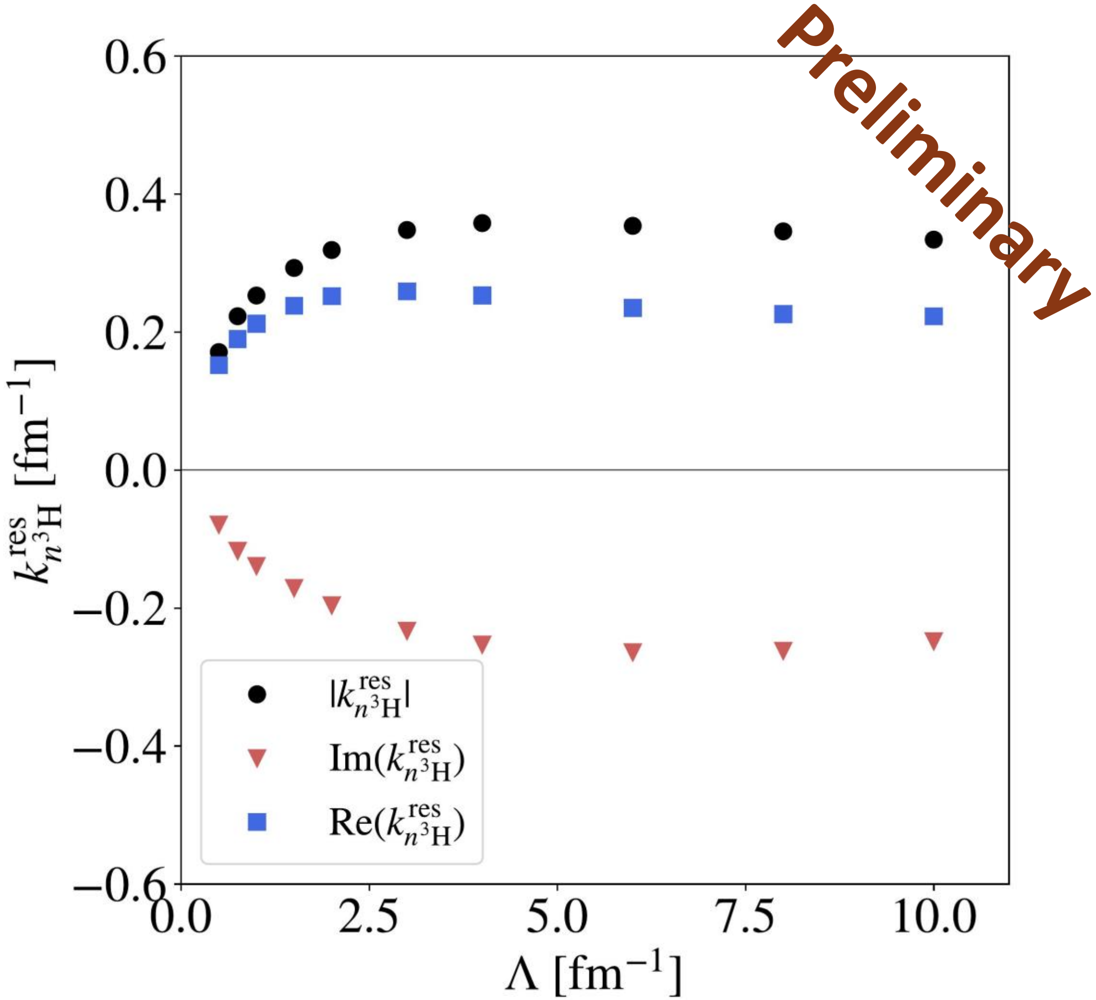


R. Lazauskas, E. Hiyama, and J. Carbonell, “Ab initio calculations of 5H resonant states,”Phys. Lett. B, vol. 791, pp. 335–341, 2019.

Contact EFT: a sub-threshold 4-body resonance is present

In preparation with:
Martin Schäfer Johannes Kirscher
Jaume Carbonell Rimantas Lazauskas





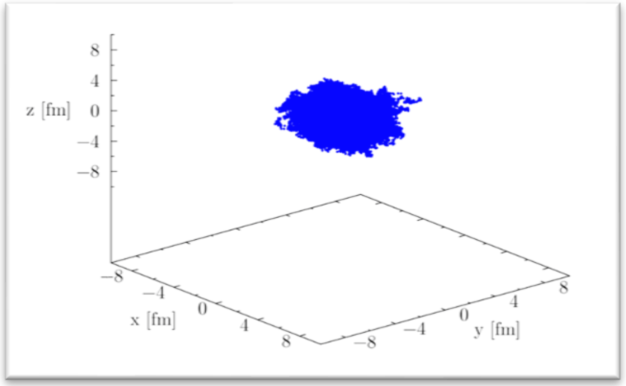
In Pionless SU(4) theory fitted on B(n-p) and B(^{3}H) a stable pole appears inside the convergence radius of the theory

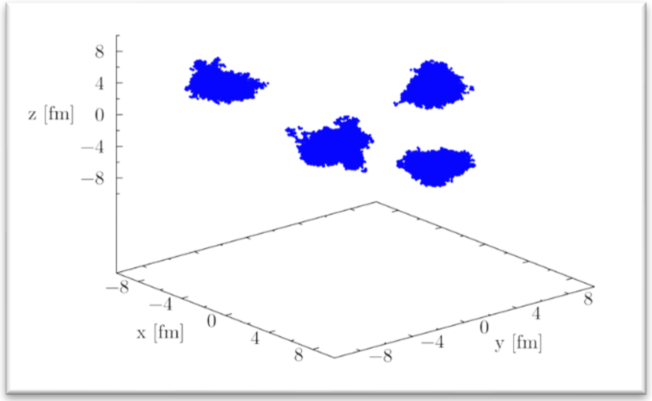
Contact EFT: a sub-threshold resonance is present

Contact theory → everything fine in **S-wave**
→ **no P-wave stable states**

A resonance is found in ^{4}H → many-body **P-shell poles can be created**

Can the resonant pole be moved to the bounded region with a **perturbative** NLO insertion?
Can we restore oxygen stability?





If yes, the answer will take us a step closer to the understanding of the nuclear interaction

Summary

Many physical system **live around unitarity**
They share a **common description,** but each slightly **deviate** from universality

Effective field theory grasp universality at LO
and gives a **consistent framework** to handle deviations

- ✓ Contact EFT works extremely well for **boson-like** systems.
- × **No stable P-shell systems in sight**
  (even if the theory should be convergent).

P-wave states exist in the theory (resonances)
may they be moved using perturbation theory?