# Few-nucleon systems expanded around unitarity

## Sebastian König, NC State University

Linving Near Unitarity @ KITP, Santa Barbara

June 6. 2022

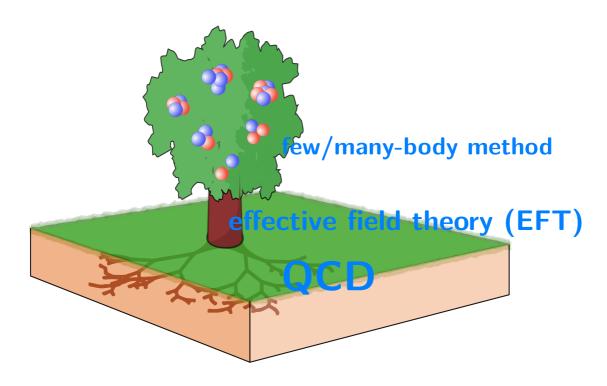
SK, Hammer, Grießhammer, van Kolck, JPG **43** 055106 (2016); PRL **118** 202501 (2017) SK, JPG **44** 064007 (2017); EPJA **56** 4, 113 (2020)



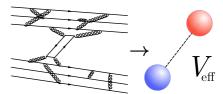




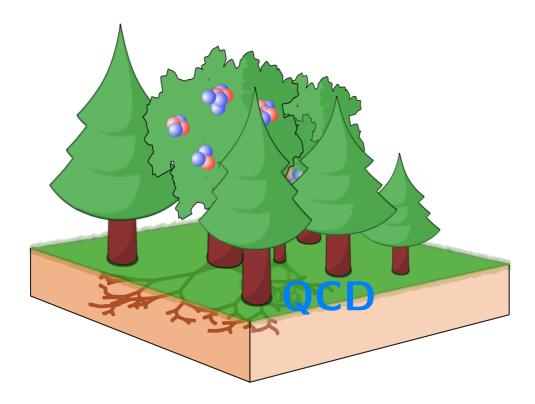
## Nuclear paradise



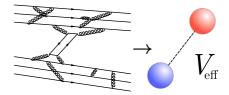
- QCD = underlying theory of strong interaction
- **EFT** = effective description in terms of hadrons
- degrees of freedom depend on resolution scale



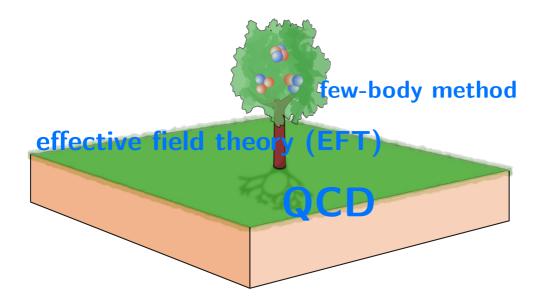
## **Nuclear forest**



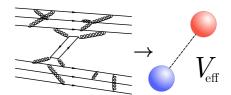
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# Nuclear clearing



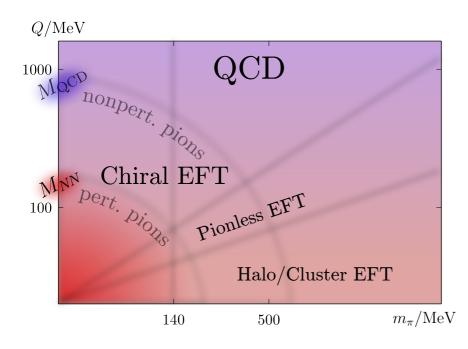
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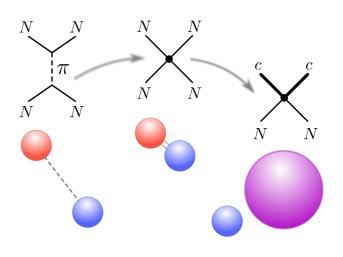


## Nuclear effective field theories

- choose **degrees of freedom** approriate to energy scale
- only restricted by symmetry, ordered by power counting

Hammer, SK, van Kolck, RMP 92 025004 (2020)





- degrees of freedom here: nucleons (and clusters thereof)
- even more effective d.o.f.: rotations, vibrations

Papenbrock, NPA 852 36 (2011); ...

## **Outline**

Prelude ✓

Introduction

**Bound states and scattering** 

Form factors and charge radii

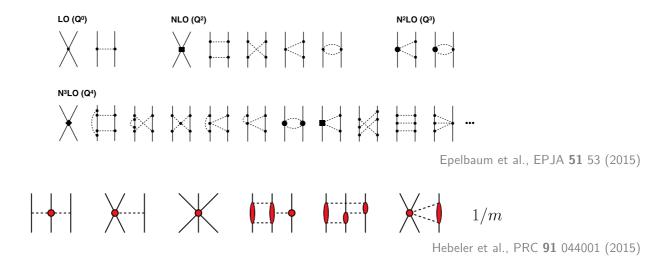
Something else

## **Nuclear complexity**

## Many remarkable results based on chiral E(F)T

• expand in  $(Q \sim M_\pi)/M_{\rm OCD}$ , derive potential (2N, 3N, ...)

Weinberg (90); Rho (91); Ordoñez + van Kolck (92); van Kolck (93); Epelbaum et al. (98); Entem + Machleidt (03); ...



#### However...

- still lots of detail in the interaction
- rather large number of parameters

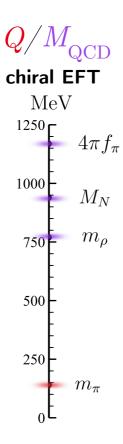
# Why so complicated?

Do we need all those details?

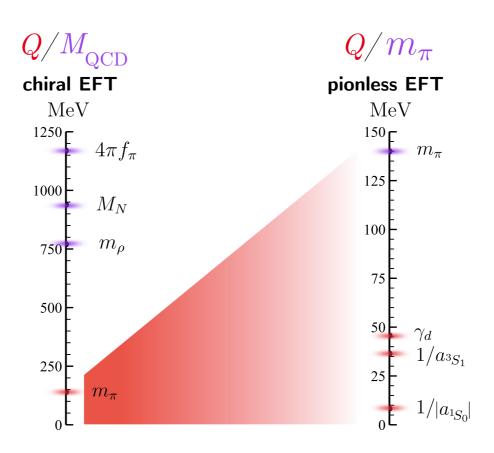
Can't we have something simpler?

What's in a nucleus?

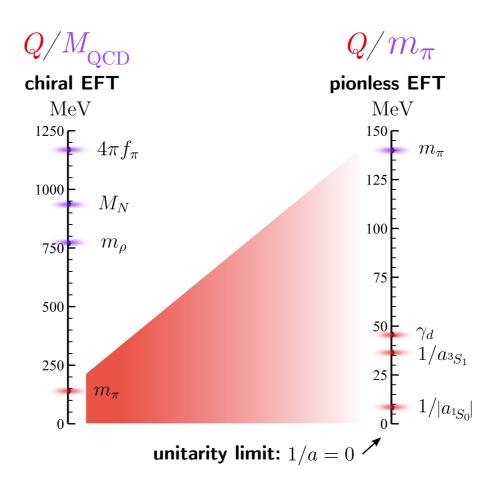
# **Nuclear scales**



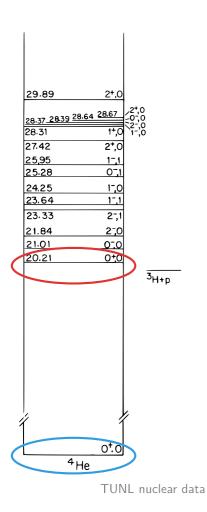
# Nuclear scales



## Nuclear scales

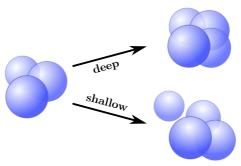


## Efimov trimers and tetramers



- <sup>3</sup>H as Efimov state Efimov, PLB **33** 563 (1970); Bedaque et al.(2000)
- two associated tetramers for each Efimov state

Hammer+Platter, EPJA 32 13 (2007); von Stecher, JPB 43 101002 (2010); ...

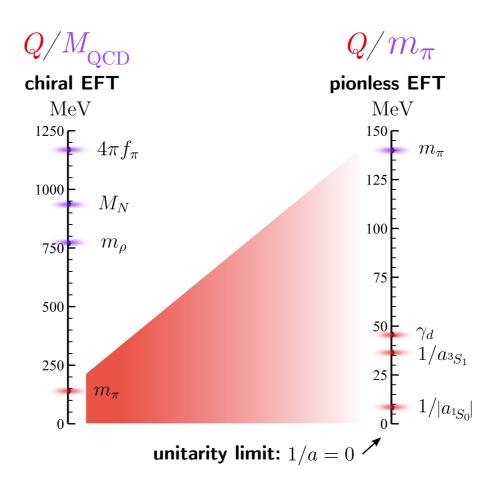


- at unitarity
  - ►  $B_4/B_3 \simeq$  4.611,  $B_{4*}/B_3 \simeq$  1.002

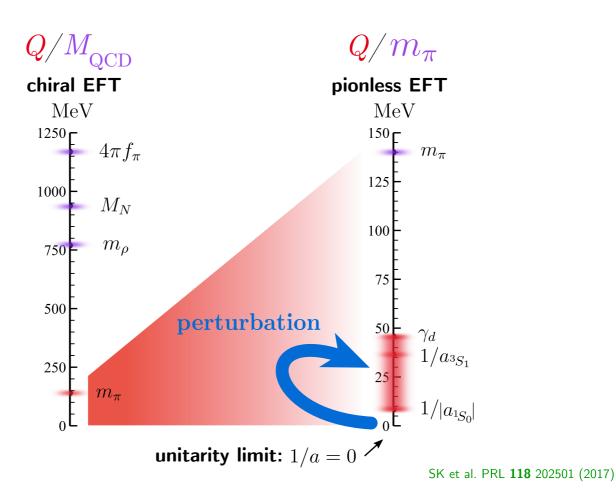
Deltuva, PRA 82 040701 (2010)

- in <sup>4</sup>He
  - ground state at  $B_{lpha}/B_{H}\simeq 3.66$
  - resonance at  $B_{\alpha*}/B_H \simeq 1.05$  (where  $B_H = 7.72$ )

## Nuclear scales revisited

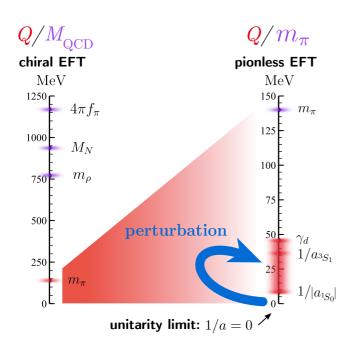


## Nuclear scales revisited



## New nuclear paradise

Capture gross features at leading order, build up the rest as perturbative "fine structure!"



- take unitarity limit as leading order
  - ► infinite S-wave scattering lengths
  - ► deuteron at zero energy
- shift focus away from two-body details
- physics in universality regime

## New nuclear paradise

Capture gross features at leading order, build up the rest as perturbative "fine structure!"

### **Nuclear sweet spot**

$$ullet$$
  $1/a$   $< Q_A < 1/R \sim m_\pi$ 

$$ullet \ Q_A = \sqrt{2 M_N B_A/A}$$

SK et al. PRL 118 202501 (2017)

Α	2	3	4	• • •	56
$\mathbf{Q_AR}$	0.3	0.5	8.0	• • •	0.9

van Kolck (2018)

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van Kolck (2018)

- discrete scale invariance as guiding principle (Efimov effect!)
  - ▶ near equivalence to bosonic clusters, exact SU(4) symmetry

Wigner, Phys. Rev. **51** 106 (1937); Mehen et al., PRL **83** 931 (1999); Bedaque et al., NPA **676** 357 (2000) Vanasse+Phillips, FB Syst. **58** 26 (2017)

cf. also Kievsky+Gattobigio, EPJ Web Conf. 113 03001 (2016), ...

## Unitarity prescription

SK et al., PRL 118 202501 (2017)

## (1) describe strong force with contact interaction

$$C_0 = \underbrace{C_0^{(0)}}_{ ext{leading order (LO)}} + C_0^{(1)} + \cdots$$

- ullet momentum cutoff  $\Lambda$  gives "smearing"
- fit  $C_0^{(0)}$  to get  $a=\infty$  in both NN S-wave channels

## (2) fix Efimov spectrum to physical triton energy

- pionless LO three-body force
- triton as "anchor" at each order

Bedaque et al., NPA 676 357 (2000)



## (3) include in perturbation theory

- finite a, Coulomb
- range corrections
- all further higher-order corrections

# Leading order has a single parameter, all the rest is a perturbation!

# Implementation

## Unified (2-, 3-, 4-body) numerical framework

## Two-nucleon system

- ullet separable regulator for contact interactions:  $V=C_0|g
  angle\langle g|$
- can be solved analytically to get scattering amplitudes



## Three-nucleon system

- ullet Faddeev equations:  $|\psi
  angle = G_0 t P |\psi
  angle + G_0 t |\psi_3
  angle$  ,  $|\psi_3
  angle = \cdots$
- used to fit three-body force



## Four-nucleon system

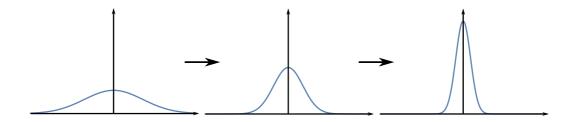
- ullet Faddeev-Yakubowsky equations: two components  $|\psi_{A,B}
  angle$
- need full wavefunction for perturbation theory:

$$ullet \ket{\Psi} = (1{-}P_{34}{-}PP_{34})(1+P)\ket{\psi_A} + (1{+}P)(1{+}P)\ket{\psi_B}$$



## The cutoff

- ullet increasing the momentum cutoff  $\Lambda$  decreases interaction range
- ullet RG invariance: fix  $C=C(\Lambda)$  to keep input observables invariant



- ullet predicted observables should converge as  $\Lambda$  increases...
- ...but individual contributions generally do not, e.g.:

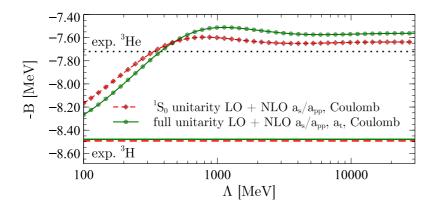
$\Lambda/\mathrm{MeV}$	800	1000	1200	1400
$E_{ m kin}/{ m MeV}$	+113.67	+140.58	+168.44	+197.09
$E_{ m pot}/{ m MeV}$	-139.77	-167.47	-195.76	-224.62

## Trinucleon energy difference

- at LO <sup>3</sup>H and <sup>3</sup>He are degenerate (exact isospin symmetry)
- ullet Coulomb correction enters together with  $1/a_{s,pp}$  at NLO

 $\Delta E_3$ : -

predict binding energy difference



	LO	NLO	exp.
<sup>3</sup> H	8.48	8.48	8.48
<sup>3</sup> He	8.48	7.6(2)	7.72
			(red = input)

- range corrections cancel at NLO
  - ► leading order is isospin symmetric
  - ullet small isospin breaking  $r_{pp} 
    eq r_{np}$  (5%) relegated to next higher order

SK et al. JPG 43 055106 (2017)

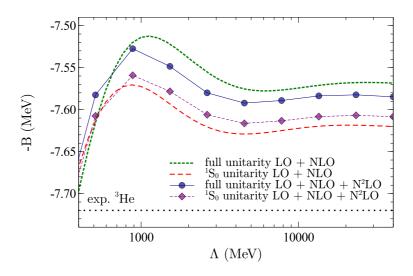
## Unitarity expansion at second order

various contributions at next-to-next-to-leading order

SK et al., JPG **43** 055106 (2016)

- quadratic scattering-length corrections, two-photon exchange
- ullet quadratic range corrections, isospin-breaking:  $r_{pp} 
  eq r_{np}$
- mixed Coulomb and range corrections
- use efficient method to calculate T-matrix in pert. theory

Vanasse, PRC 88 044001 (2013)



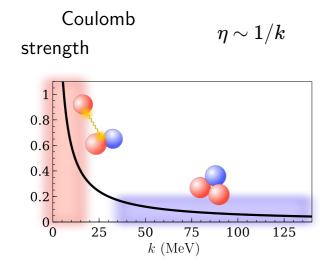
SK, JPG 44 064007 (2017)

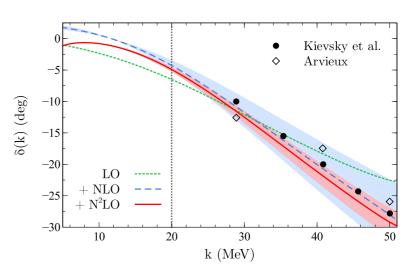
- good convergence of 1/a + Coulomb expansion up to  $N^2LO$
- need isospin breaking 3NF if range corrections are included

## Proton-deuteron scattering

- strength of Coulomb interaction depends on momentum scale
- perturbative effect in bound states and scattering beyond very small energies
- possible to study p-d scattering in  ${}^1S_0$  unitarity expansion

SK et al., JPG 43 055106 (2016)



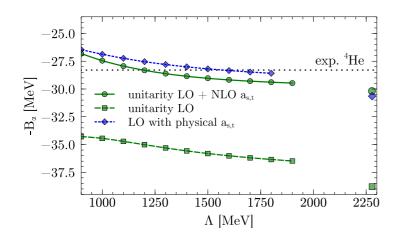


- Coulomb-subtracted phase shifts calculated in perturbation theory
- uncertainty bands estimated based on EFT expansion parameter
  - ► 30% leading-order band not shown

## Four nucleons

- unitarity expansion converges well in three-nucleon sector ✓
- ullet further test:  $^4$ He with  $Q_4\sim 115\,$  MeV
- good standard pionless LO description established previously

Platter et al., PLB 607 254 (2005)



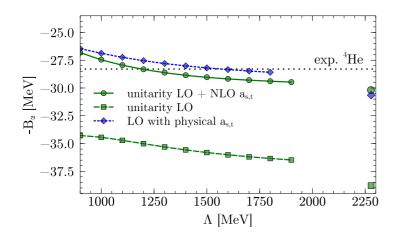
#### **NLO** incomplete: 1/a corr. only

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		(1	red = input

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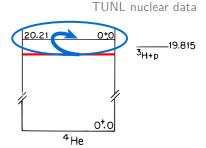
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- <sup>4</sup>He resonance state 0.3 MeV above  ${}^3H + p$  threshold
- just below threshold at unitarity LO
- simplified calculations give a shift of 0.2 0.5 MeV



## Summary so far

## Pionless EFT expansion of light nuclei around unitarity

- leading order at unitarity limit (infinite scattering length)
- everything else as perturbative fine structure

SK et al., PRL 118 202501 (2017); SK, JPG 44 064007 (2017)

$E_B$	LO	NLO*	N <sup>2</sup> LO	exp.
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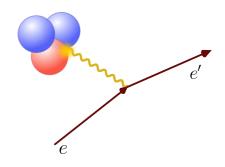
• "half unitarity" ( ${}^1S_0$  only) expansion for perturbative Coulomb in p-d scattering

# What about observables beyond binding energies?

# Charge form factors and radii

$$ullet \ F_C(q) = \langle \Psi | 
ho(q) | \Psi 
angle \ 
ightharpoons \langle r^2 
angle = -rac{1}{6} rac{d^2}{dq^2} F_C$$
 ,  $\, q o 0$ 

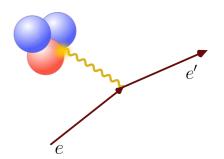
- ullet  $|\Psi
  angle=$  full few-body wavefunction, ho= e.m. current
- ullet point charge radii: subtract effects from  $r_p$  and  $r_n$



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#### Perturbative corrections

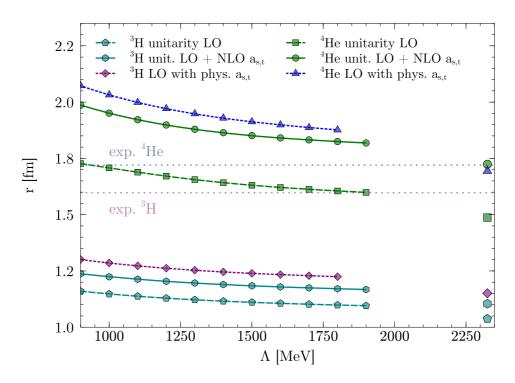
- ullet need corrections to wavefunctions:  $|\Psi
  angle=|\Psi_0
  angle+|\Psi_1
  angle+\cdots$ 
  - ullet corresponding to underlying EFT expansion  $V=V_0+V_1+\cdots$
- can be obtained from inhomogeneous Faddeev/Faddeev-Yakubowsky equations

$$ullet \left[ 1 - G_0 t_0 P 
ight] |\psi_1
angle = B_1 (G_0 + G_0 t_0 G_0) |\psi_0
angle + G_0 t_1 P |\psi_0
angle$$

- ▶ inclusion of three-body forces somewhat tedious but straightforward
- ▶ note: LO kernel is singular at  $E = -B_0$
- ullet calculate  $|\Psi_1
  angle=(1+P)|\psi_1
  angle$  , enforce orthogonality to  $|\Psi_0
  angle$
- ullet  $F_{C,0}(q)=\langle\Psi_0|
  ho(q)|\Psi_0
  angle$  ,  $F_{C,1}(q)=2\langle\Psi_1|
  ho(q)|\Psi_0
  angle$  ,  $\dots$
- ullet analogous expansion for  $\langle r^2 
  angle$

SK, EPJA 56 4, 113 (2020)

# Charge radii results



- triton result in excellent agreement with previous pionless calculations
  - ► range corrections are large Vanasse, PRC 95 024002 (2017); Vanasse+Phillips, FB Syst. 58 26 (2017)
- ullet universal relation satisfied well at unitarity:  $M_N B_{^3H} \langle r_0^2 
  angle_{^3H} = (1+s_0^2)/9 pprox 0.224$
- <sup>4</sup>He result remarkably close to experimental value!

## Summary and outlook

## Pionless EFT expansion of light nuclei around unitarity

- leading order at unitarity limit (infinite scattering length)
- everything else as perturbative fine structure

SK et al., PRL 118 202501 (2017); SK, JPG 44 064007 (2017)

$E_B$	LO	NLO*	N <sup>2</sup> LO	exp.
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$\sqrt{\langle r^2  angle}$	LO	NLO*	exp.	
<sup>3</sup> H	1.04(31)	1.10(33)	1.59	
<sup>4</sup> He	1.49(45)	1.73(52)	1.72	
(operates in MoV radii in fm)				

(energies in MeV, radii in fm)

- ullet "half unitarity" ( $^1S_0$  only) expansion allows perturbative Coulomb in p-d scattering
- good convergence of 3N and 4N radii in pure unitarity expansion

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- ullet "half unitarity" ( $^1S_0$  only) expansion allows perturbative Coulomb in p-d scattering
- good convergence of 3N and 4N radii in pure unitarity expansion
- inclusion of range correction requires a four-body force at NLO

Bazak, Kirscher, SK et al., PRL 122 143001 (2019)

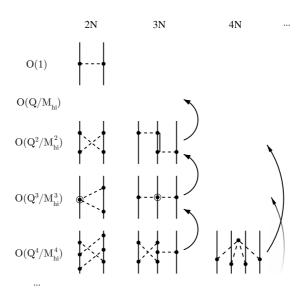
- ► still leaves <sup>4</sup>He radius and other oberservables as predictions
- ► full 4N NLO calculation (including range + Coulomb) still to be done

# And now for something completely different...

# And now for something completely different...?

## **Chiral EFT**

- ullet expansion around **chiral limit**  $(m_\pi=0)$ , assumes  $Q\sim m_\pi$
- pion exchange determines nuclear interaction at large and intermediate range
- further details enter as contact interactions and delta excitations



# Partly perturbative pions

• consider the effective total potential:

$$V(r) = V_{
m OPE}(r) + rac{L(L+1)}{r^2}$$

ullet for L>0, this "shields" the nucleons from the singular attraction of  $V_{
m OPE}(r)$ 

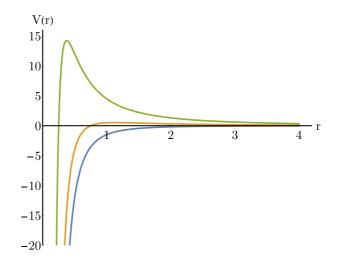
Nogga, Timmermans, van Kolck, NPA 2005

 critical momentum characterizes perturbativeness

Birse, PRC 2006

possibly complicated by contact terms

Wu+Long, PRC 2019



### **Bottom line**

- ullet pion exchange becomes perturbative for L sufficiently large
- however, where to draw the line is not necessarily clear

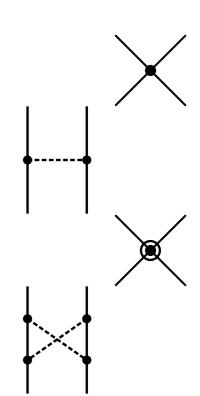
# Minimally non-perturbative pion scheme

## A priori, we adopt the following power counting:

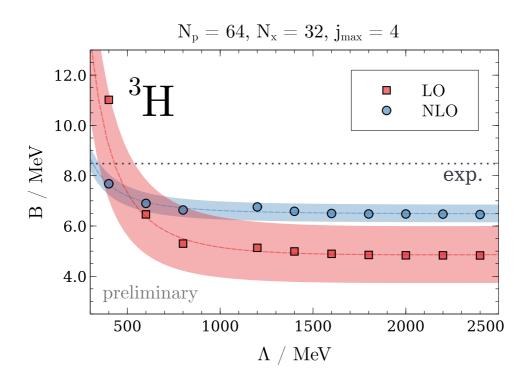
- Leading order (LO):  $\mathcal{O}(0)$ 
  - OPE for  ${}^{3}S_{1}$ - ${}^{3}D_{1}$ ,  ${}^{1}S_{0}$ ,  ${}^{3}P_{0}$
  - contact terms for each of these
  - ► only for <sup>3</sup>P<sub>0</sub> this is a promotion compared to naive counting
    Wu+Long, PRC 2019
- Next-to-leading order (NLO):  $\mathcal{O}(\mathbf{Q}/\mathbf{M_{hi}})$ 
  - ► OPE for other partial waves:  ${}^{1}P_{1}$ ,  ${}^{3}P_{1}$ ,  ${}^{3}P_{2}$ - ${}^{3}F_{2}$ , ...
  - ► correction to <sup>1</sup>S<sub>0</sub> contact term

Long+Yang, PRC 2012

- ullet Next-to-leading order (N2LO):  $\mathcal{O}(\mathbf{Q^2/M_{hi}^2})$ 
  - ▶ two-pion exchange (TPE)
  - ▶ three-nucleon forces
  - ▶ ...

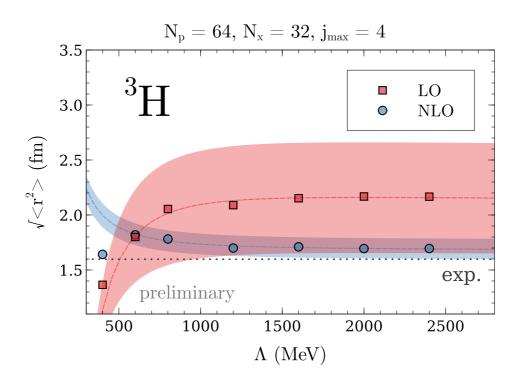


# <sup>3</sup>H binding energy



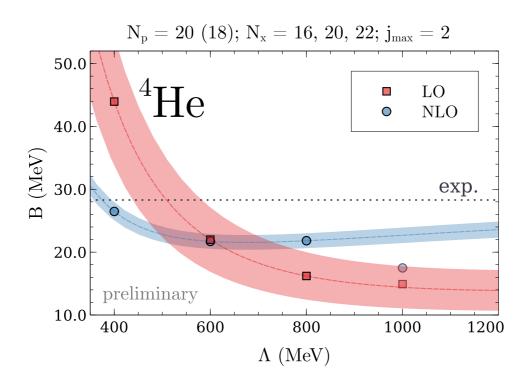
- leading order underbound by about 40%
- reduced cutoff dependence and underbinding at NLO

# <sup>3</sup>H charge radius



- established RG invariance for radius calculation
- NLO surprisingly close to experiment

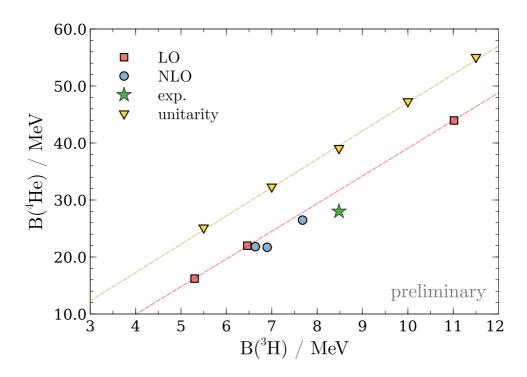
# <sup>4</sup>He binding energy



- underbinding comparable to <sup>3</sup>H result
- NLO trend also similar, but needs further investigation
- no Coulomb correction included, needs to be derived carefully

SK et al., JPG 2016

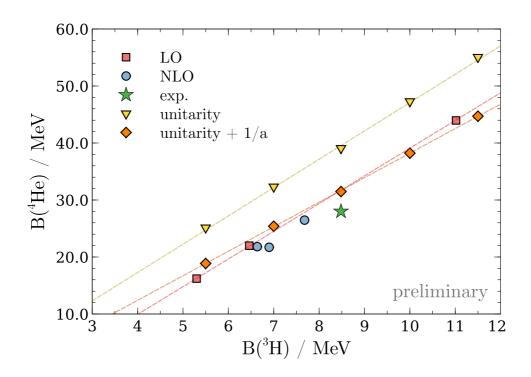
# Tjon line



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## Thanks...

### ...to my collaborators...

- H.-W. Hammer (TU Darmstadt)
- H. Grießhammer (George Washington U.)
- U. van Kolck (U. Arizona and IJCLab Orsay)
- B. Long, R. Peng, S. Liu (Sichuan U.)

## ...for support, funding, and computing time...







• Jülich Supercomputing Center

## Thanks...

## ...to my collaborators...

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