Few- and Many-Body Systems near Unitarity

Dean Lee
Facility for Rare Isotope Beams
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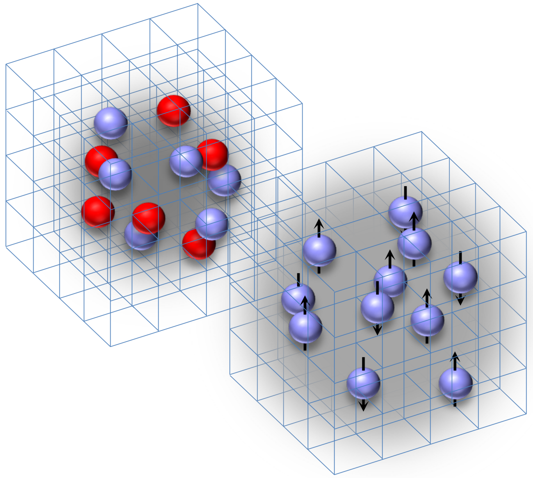

Living Near Unitarity
Kavli Institute for Theoretical Physics
UC Santa Barbara
May 9, 2022

FRIB MICHIGAN STATE UNIVERSITY NUCLEI Nuclear Computational Low-Energy Initiative A SciDAC-4 Project JÜLICH FORSCHUNGSZENTRUM OAK RIDGE National Laboratory LEADERSHIP COMPUTING FACILITY

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Two-component Fermi gases at unitarity

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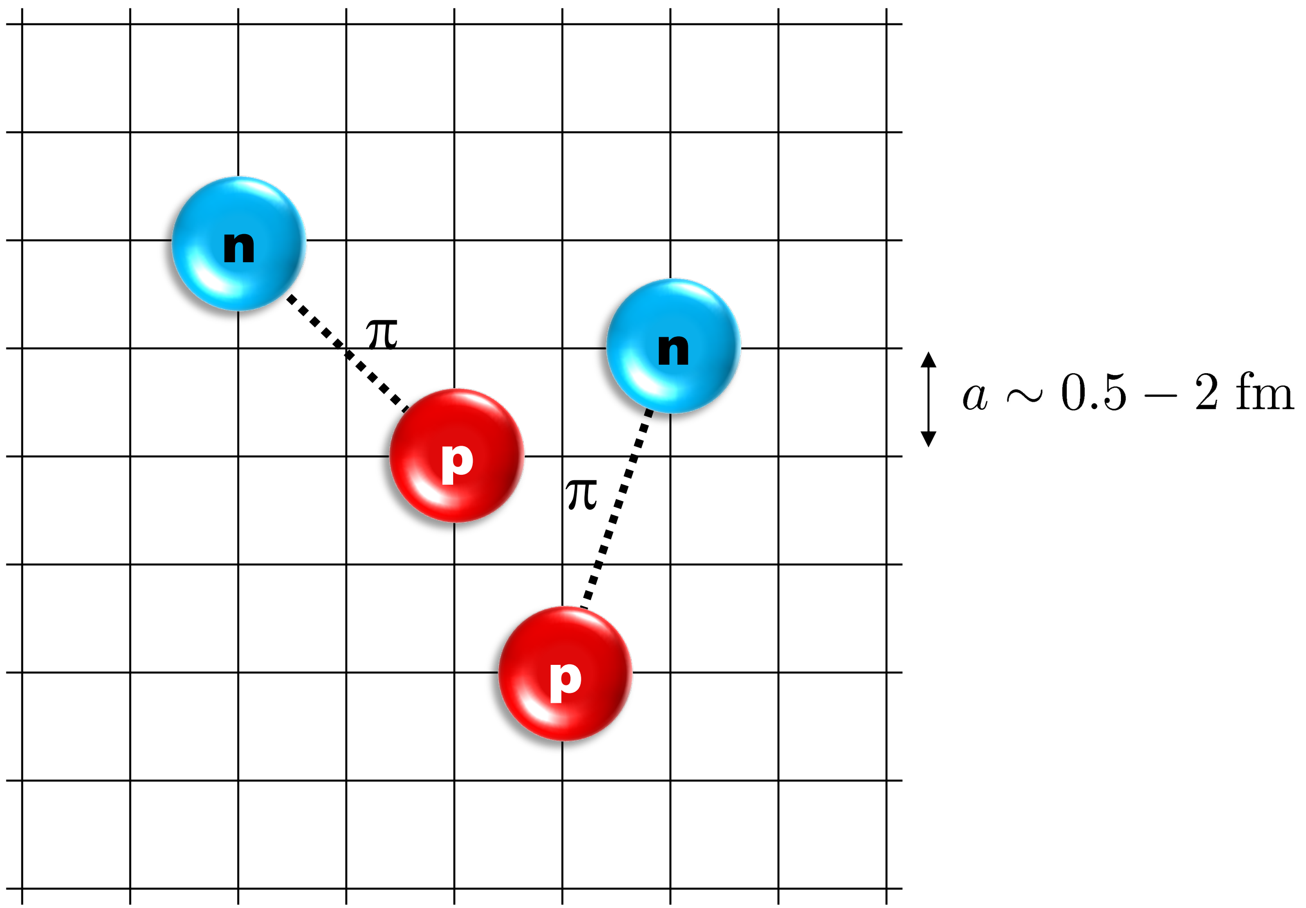
Summary

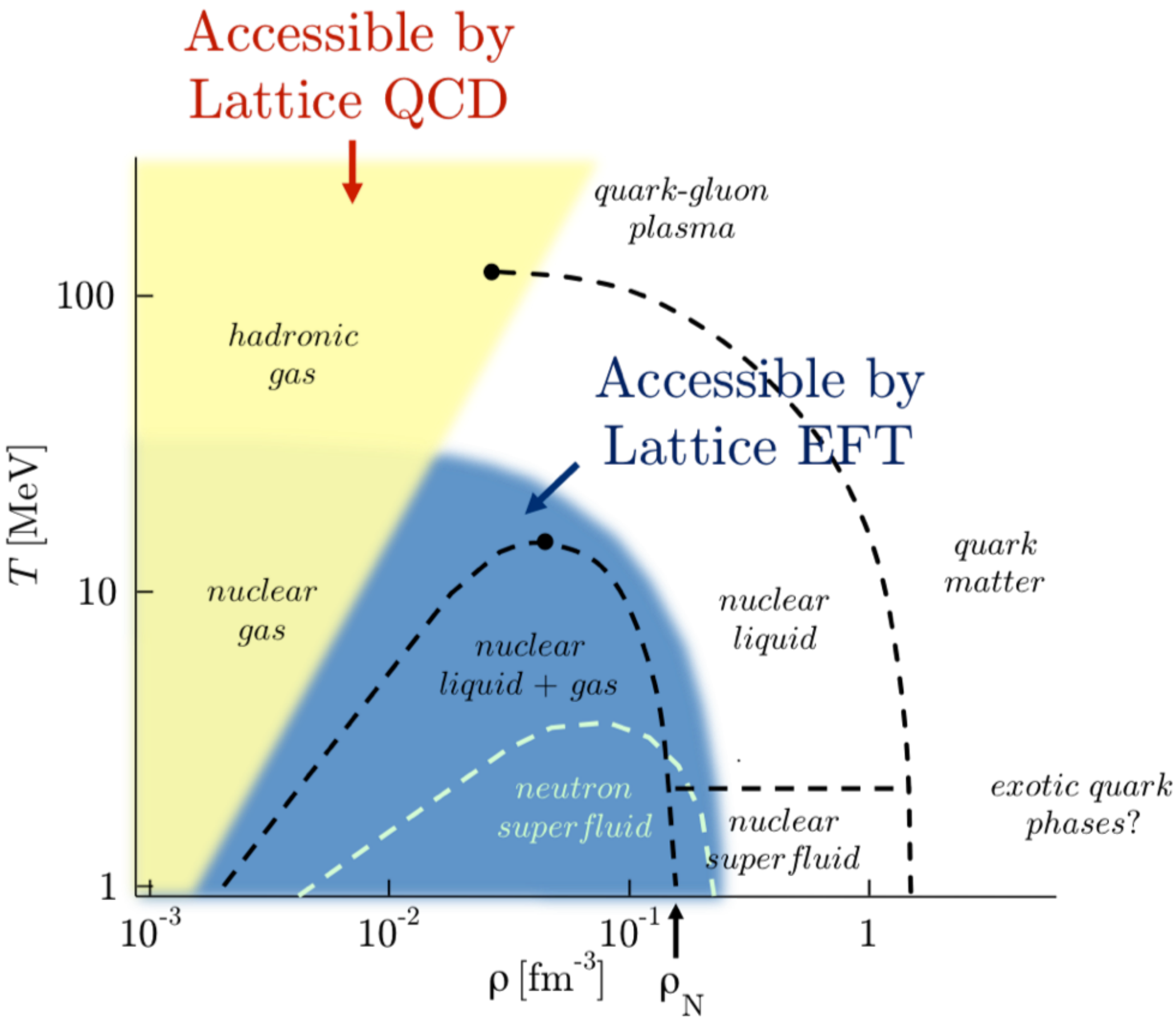
Lattice effective field theory

Lattice effective field theory



D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)

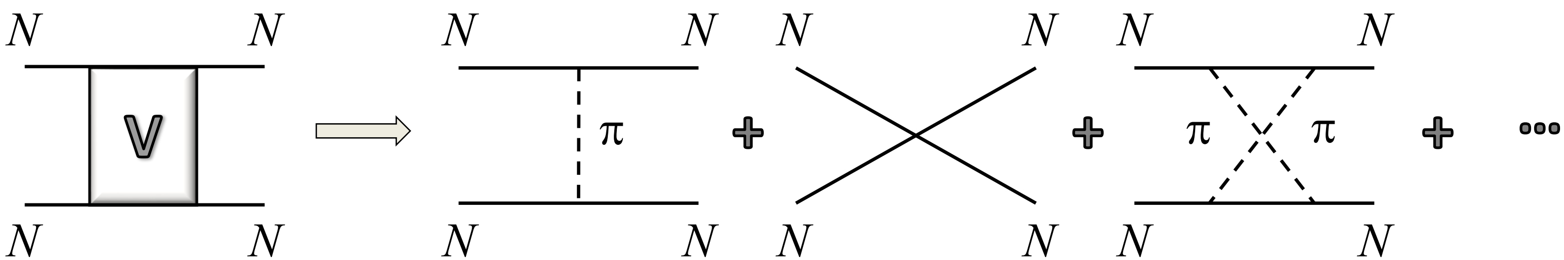
Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer



Accessible by
Lattice QCD
quark-gluon
plasma
100
hadronic
gas
Accessible by
Lattice EFT
T [MeV]
quark
matter
10
nuclear
gas
nuclear
liquid
nuclear
liquid + gas
neutron
superfluid
exotic quark
phases?
nuclear
superfluid
1
10^-3
10^-2
10^-1
1
ρ [fm^-3]
ρ_N

Chiral effective field theory

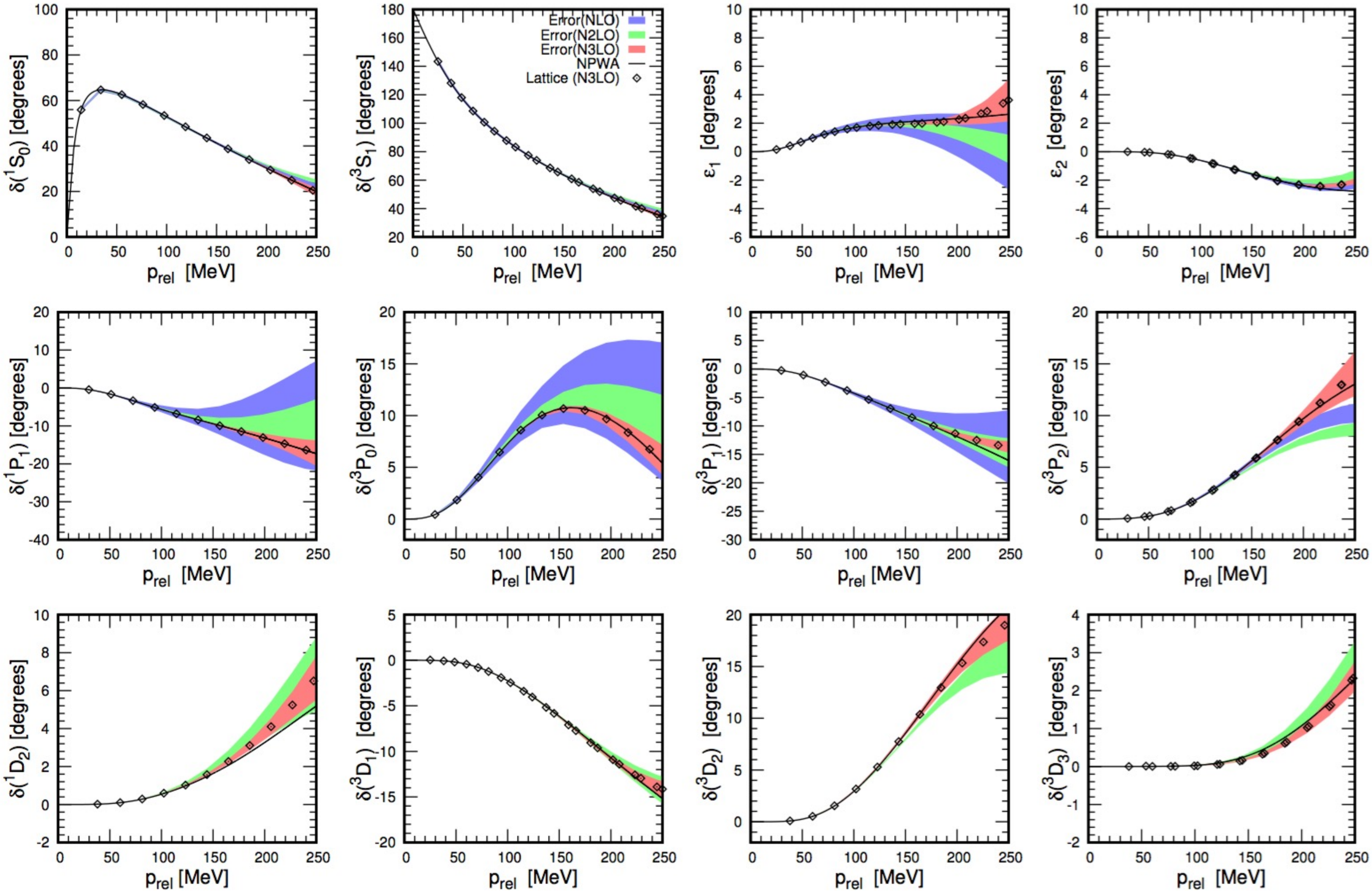
Construct the effective potential order by order



V^{OPEP} Contact interactions V^{TPEP}

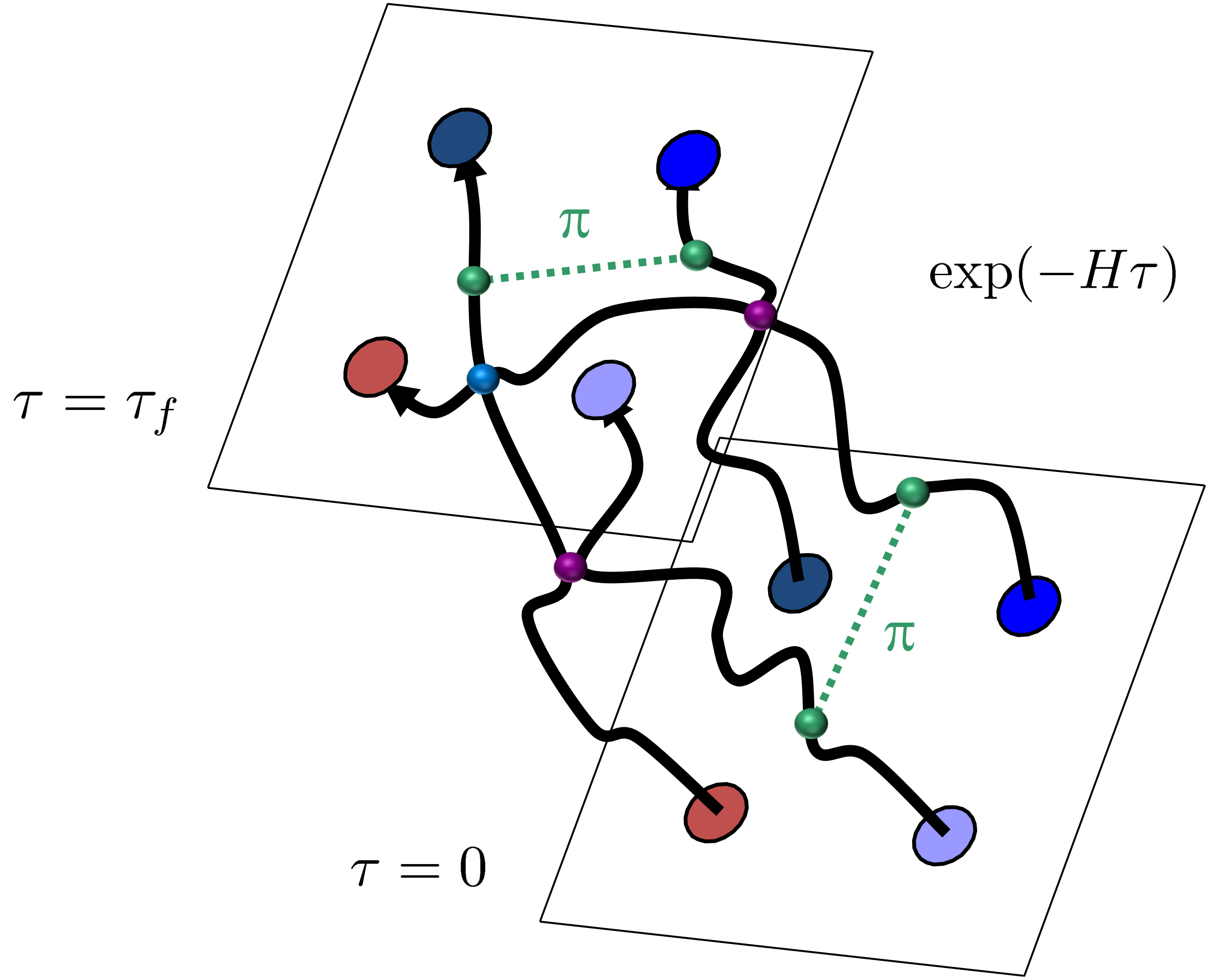
Leading order (LO) Next-to-leading order (NLO)

$a = 0.987\,\mathrm{fm}$



Li, Elhatisari, Epelbaum, D.L., Lu, Meißner, PRC 98, 044002 (2018)

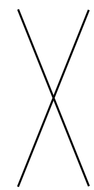
<u>Euclidean time projection</u>



Auxiliary field method

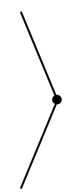
We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^\dagger N)^2\right]$$



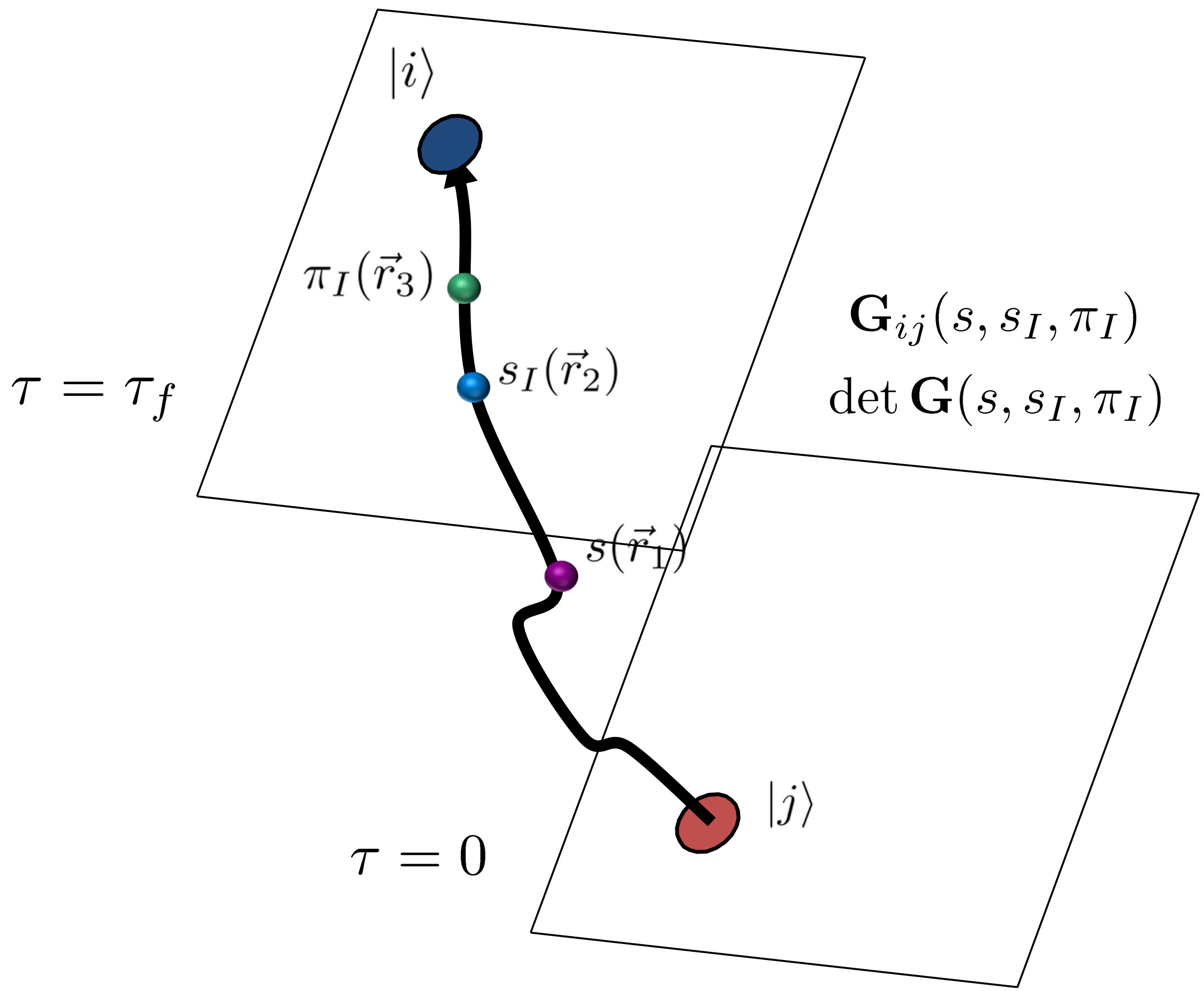
$(N^\dagger N)^2$

$$= \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds \exp\left[-\frac{1}{2}s^2 + \sqrt{-C}\, s(N^\dagger N)\right]$$



$sN^\dagger N$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.


$|i\rangle$
$\pi_I(\vec{r}_3)$
$s_I(\vec{r}_2)$
$\tau = \tau_f$
$\mathbf{G}_{ij}(s, s_I, \pi_I)$
$\det \mathbf{G}(s, s_I, \pi_I)$
$s(\vec{r}_1)$
$|j\rangle$
$\tau = 0$

Two-component Fermi gas at unitarity

Unitarity limit

Consider S-wave two-body scattering in three dimensions at low momenta. The scattering amplitude is

$$f_0(p) \propto \frac{1}{p\cot\delta_0(p) - ip}$$

The effective range expansion: $$p\cot\delta_0(p) = -\frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \cdots$$

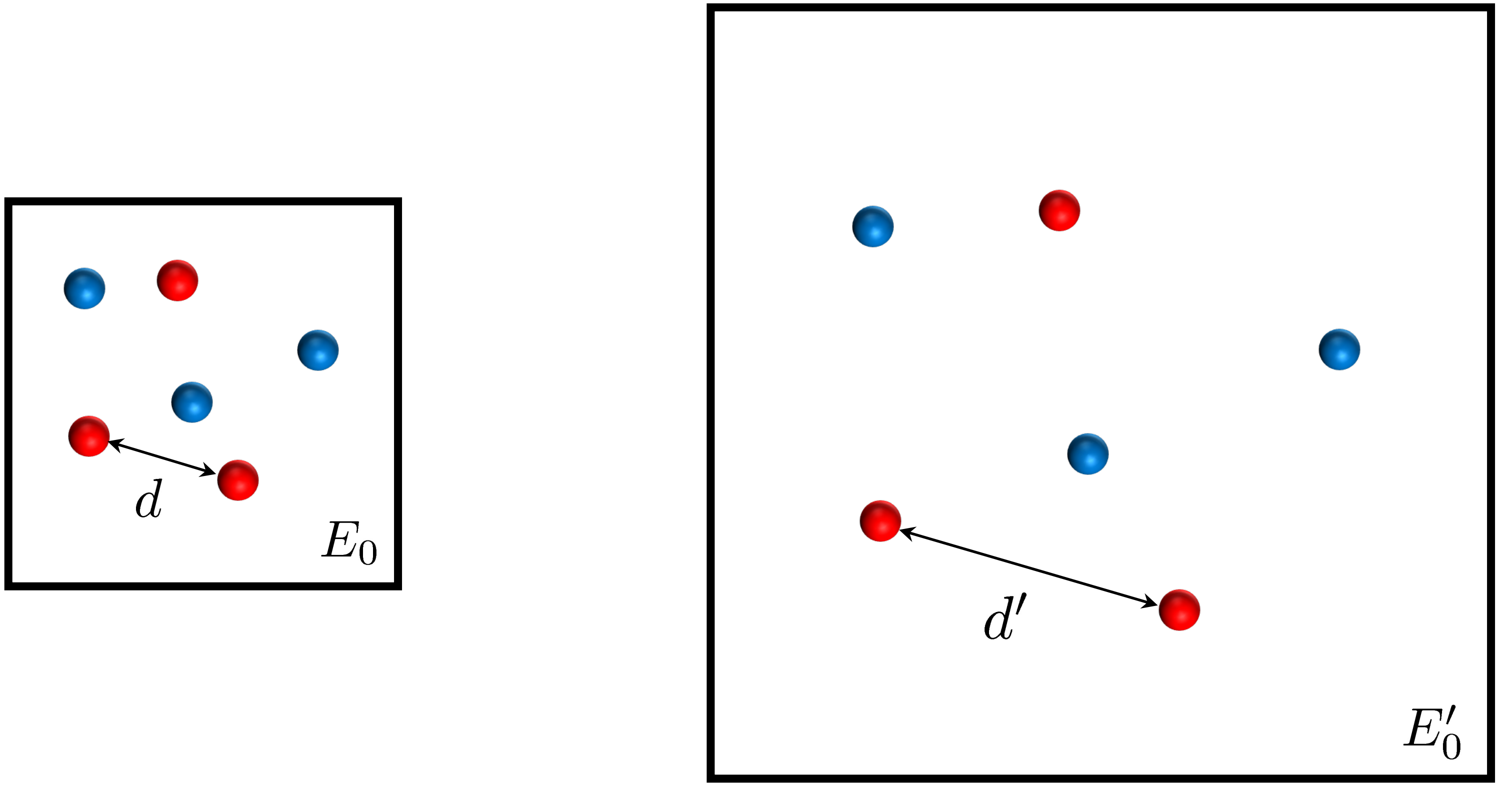
Consider when the scattering length a_0 is infinite. At low momenta, we get

$$p\cot\delta_0(p) \approx 0 \qquad f_0(p) \propto \frac{i}{p} \qquad \text{scale invariant physics}$$

Two-component Fermi gases at unitarity

Consider a non-relativistic two-component Fermi gas at unitarity in the limit that the range of the interactions is zero and the scattering length tuned to infinity.

Since the unitarity limit has no intrinsic length scales, all many-body observables must equal some dimensionless number times the appropriate power of the Fermi momentum, k_F. For energies and temperatures, there is an overall factor of the inverse particle mass.



$$E_0 d^2 = E'_0 d'^2$$

For example, the energy per particle must be proportional the Fermi energy, E_F. This similar to the case for a free non-interacting Fermi gas. For the free Fermi gas, the ground state energy per particle in the thermodynamic limit is

$$\frac{E_0^{\text{free}}}{A} = \frac{3}{5} E_F$$

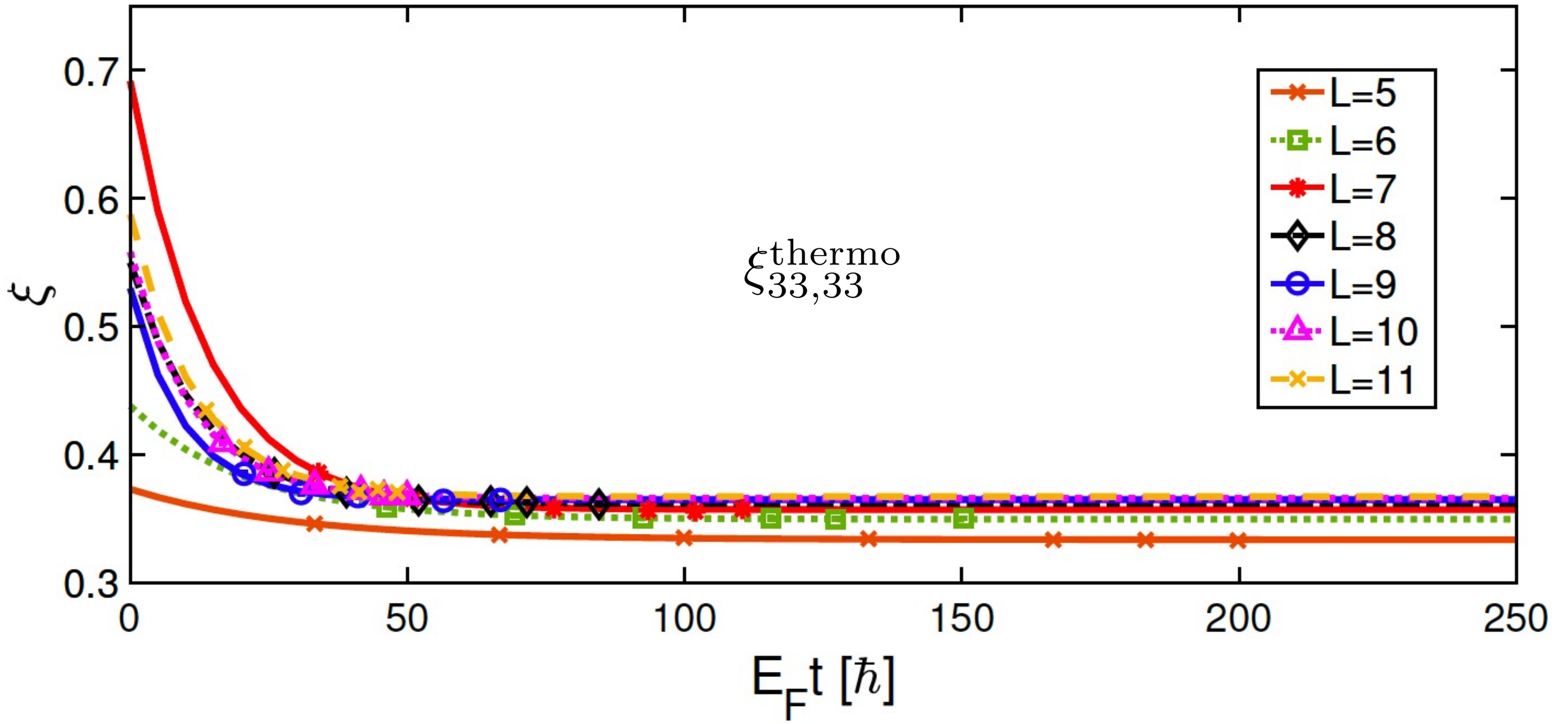
It is conventional to define the Bertsch parameter ξ for the ratio between the ground state energy per particle in the unitary limit and the ground state energy per particle for the free gas.

$$\frac{E_0}{A} = \xi \frac{E_0^{\text{free}}}{A} = \frac{3}{5} \xi E_F$$

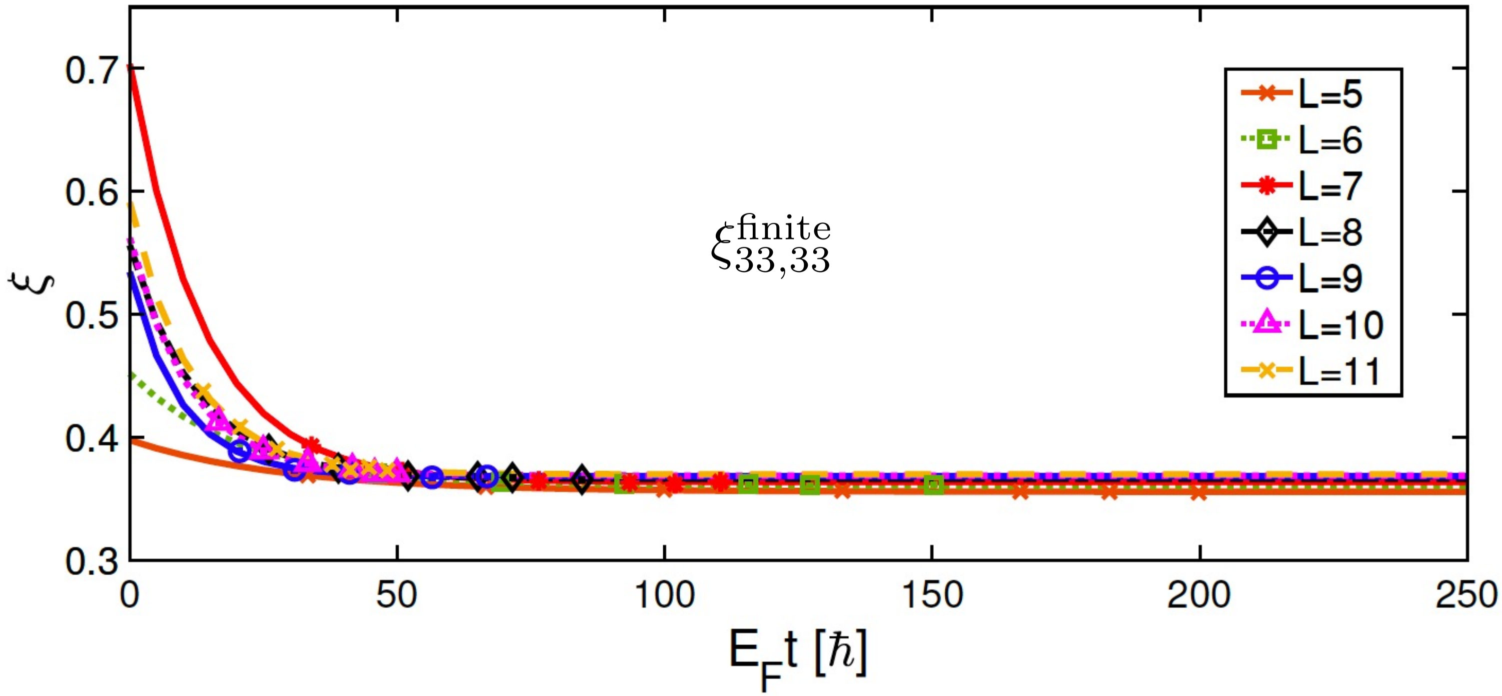
Lattice simulation results

$$N_\uparrow = N_\downarrow = 33$$

$$\xi_{33,33}^{\text{finite}} = \frac{E_{33,33}}{E_{33,33}^{\text{free}}} \qquad \xi_{33,33}^{\text{thermo}} = \frac{E_{33,33}}{3(N_\uparrow + N_\downarrow)k_F^2/(10m)}$$



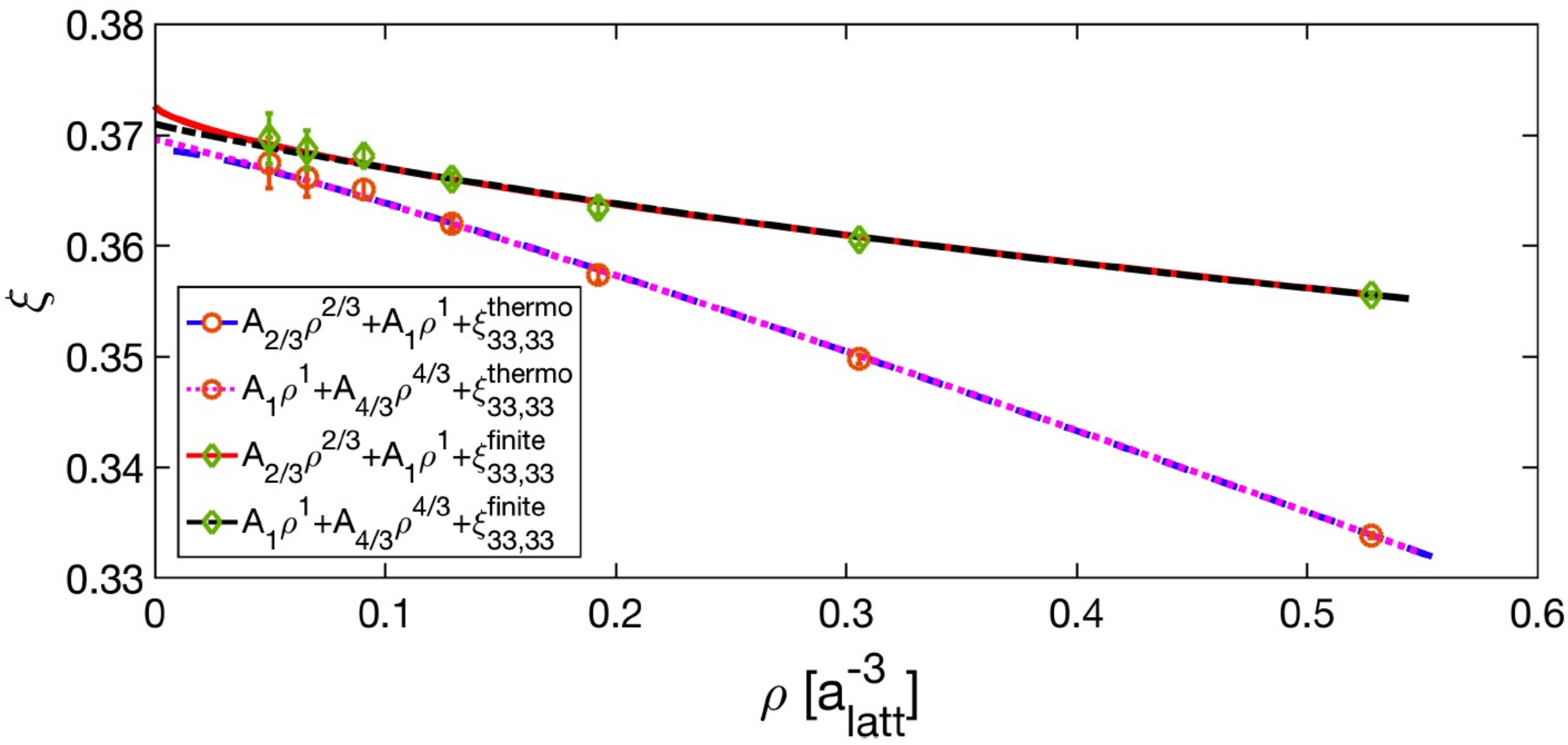
He, Li, Lu, D.L., PRA 101, 063615 (2020)



He, Li, Lu, D.L., PRA 101, 063615 (2020)

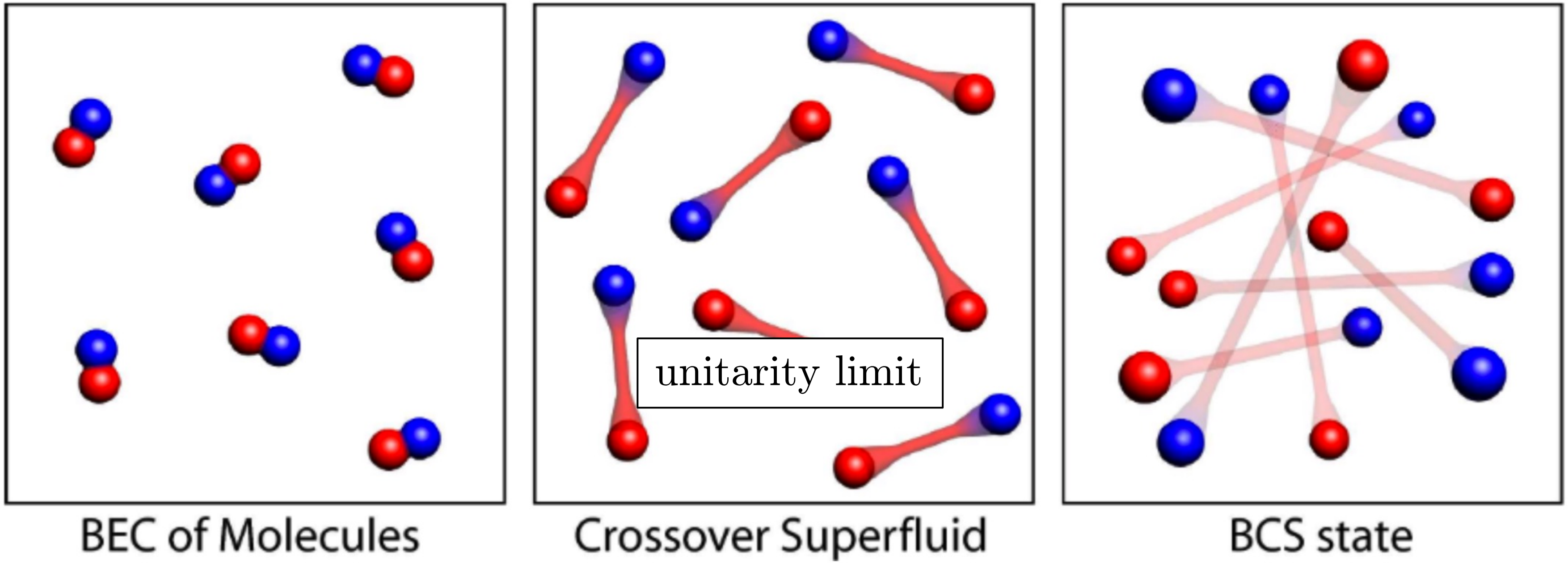
$$\xi_{33,33}^{\text{thermo}} = 0.372(2) \qquad \xi_{33,33}^{\text{finite}} = 0.369(2)$$

He, Li, Lu, D.L., PRA 101, 063615 (2020)



Other lattice calculations: 0.372(5) Carlson, Gandolfi, Schmidt, Zhang PRA 84 061602(R) (2011)
Experiment: 0.376(4) Ku, Sommer, Cheuk, Zwierlein, Science 335 (2012) 563

Superfluidity pairing and condensate



Ketterle, Zwierlein

Superfluidity and pairing correlations

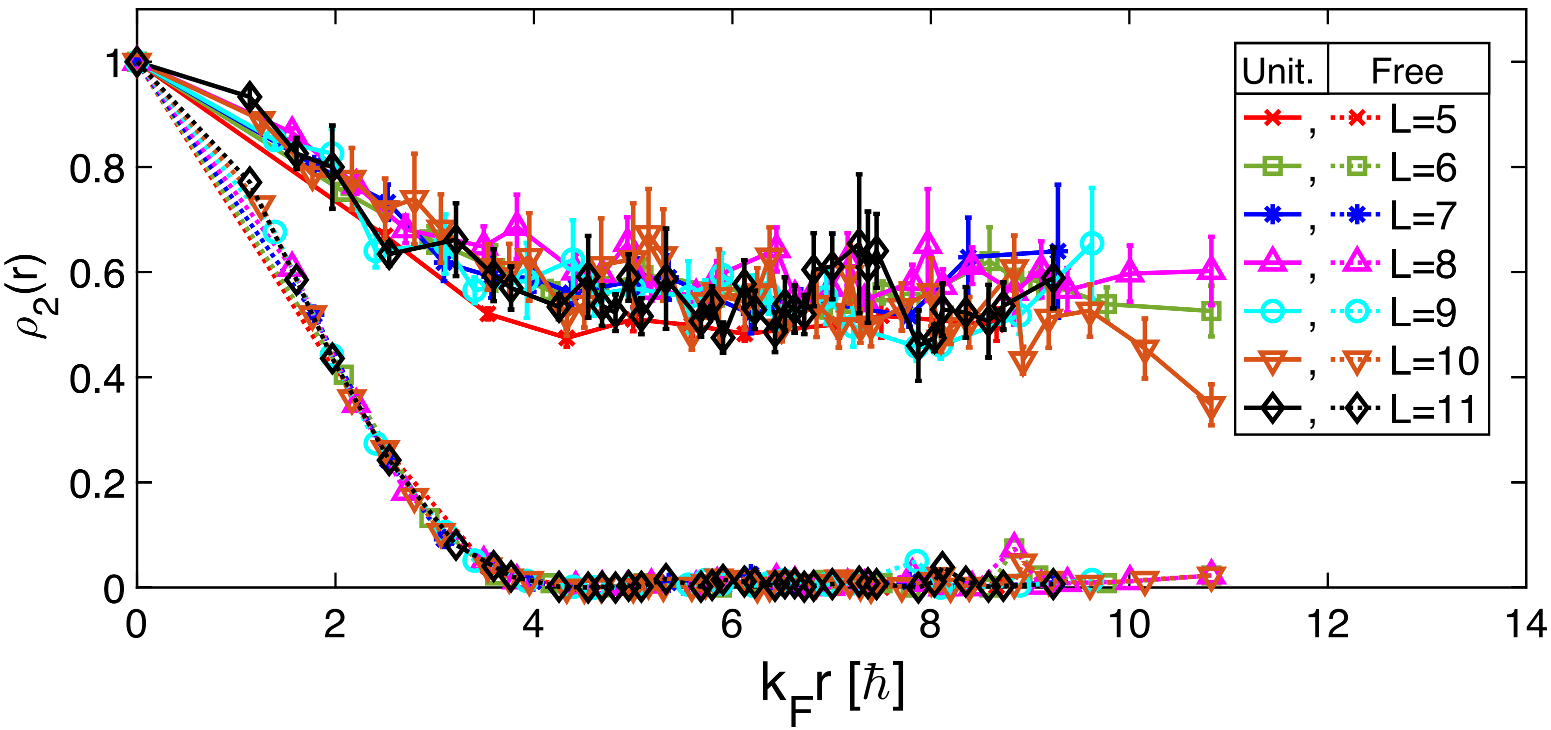
The two-body density matrix is defined as

$$\rho_2(\vec{r}\,'_1, \vec{r}\,'_2, \vec{r}_1, \vec{r}_2) = \left\langle \psi^\dagger_\downarrow(\vec{r}\,'_2)\psi^\dagger_\uparrow(\vec{r}\,'_1)\psi_\uparrow(\vec{r}_1)\psi_\downarrow(\vec{r}_2) \right\rangle$$

Long-range correlations in the two-body density matrix is a signature for pair superfluidity:

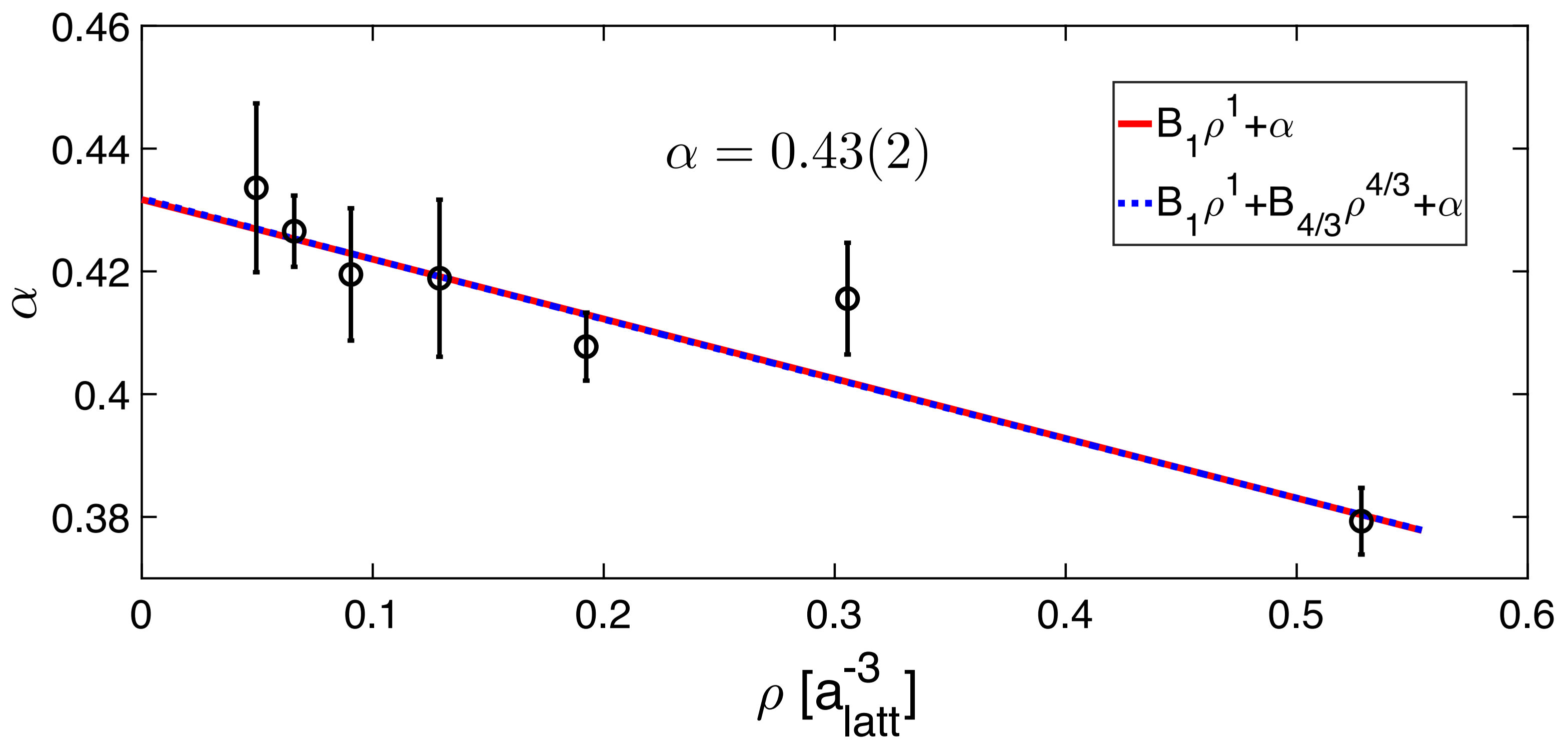
$$\rho_2(\vec{r}\,'_1, \vec{r}\,'_2, \vec{r}_1, \vec{r}_2) \to \alpha N/2 \cdot \phi^*(|\vec{r}\,'_1 - \vec{r}\,'_2|)\phi(|\vec{r}_1 - \vec{r}_2|)$$
$$|\vec{r}_1 - \vec{r}\,'_1|, |\vec{r}_2 - \vec{r}\,'_2| \to \infty$$

Yang, RMP 34, 694 (1962)



He, Li, Lu, D.L., PRA 101, 063615 (2020)

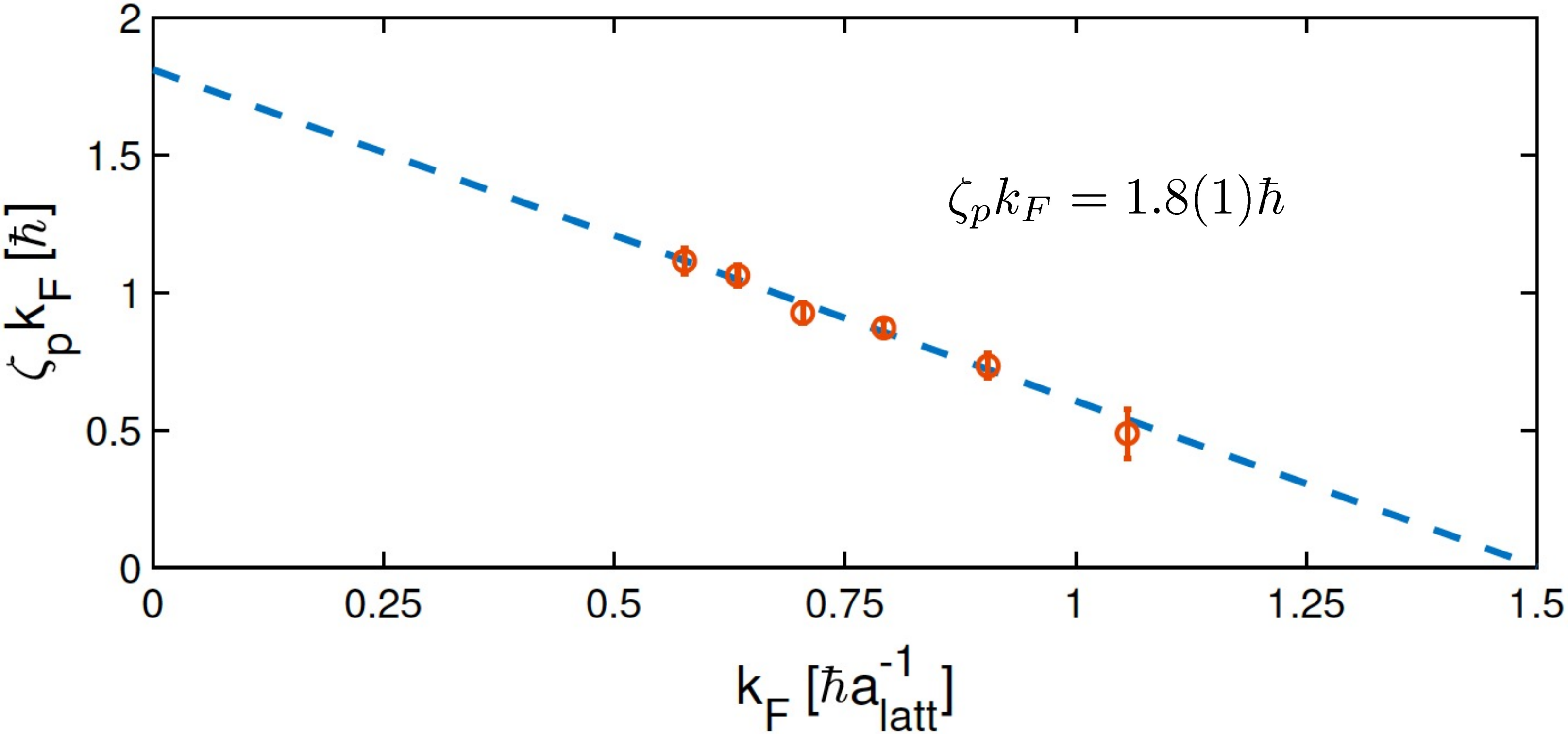
Superfluid condensate fraction



Experiment: 0.46(7) Zwierlein, Stan, Schunck, Raupach, Kerman, Ketterle, PRL 92 (2004) 120403

Pair wave function

$$\Phi(r) \propto \frac{e^{-r/\zeta_p}}{r}$$



He, Li, Lu, D.L., PRA 101, 063615 (2020)

Going beyond two components

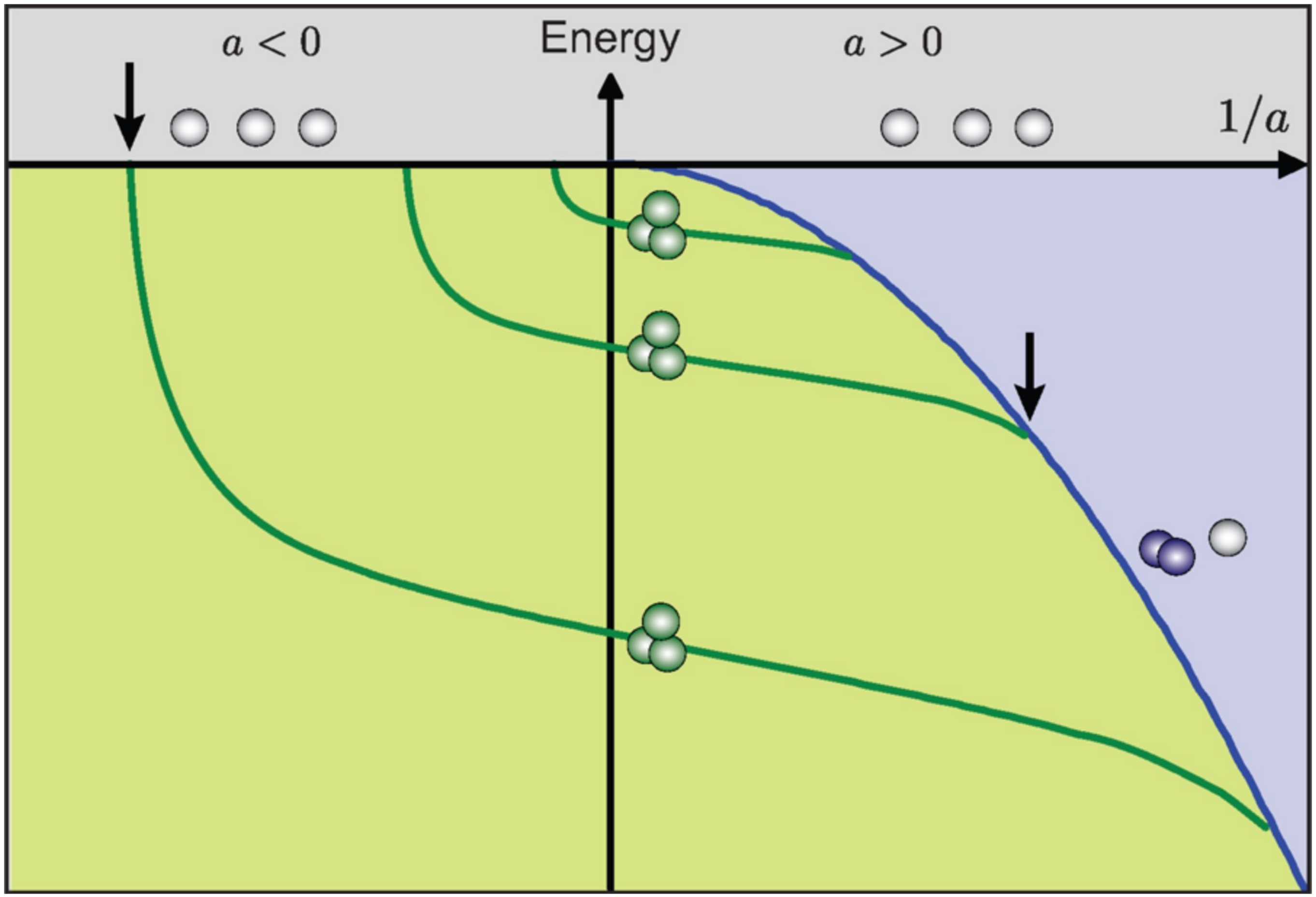
In quantum mechanics and quantum field theory, scale invariance can be spoiled by quantum scale anomalies. This happens when there are bound states, which necessarily correspond to discrete energy levels.

Nevertheless, it may happen that a discrete subgroup of the scale symmetry is preserved for the dynamics of certain sectors of the Hilbert space.

This phenomenon was first noted by Efimov for bound states of three bosons when the two-body interactions are tuned to the unitarity limit.

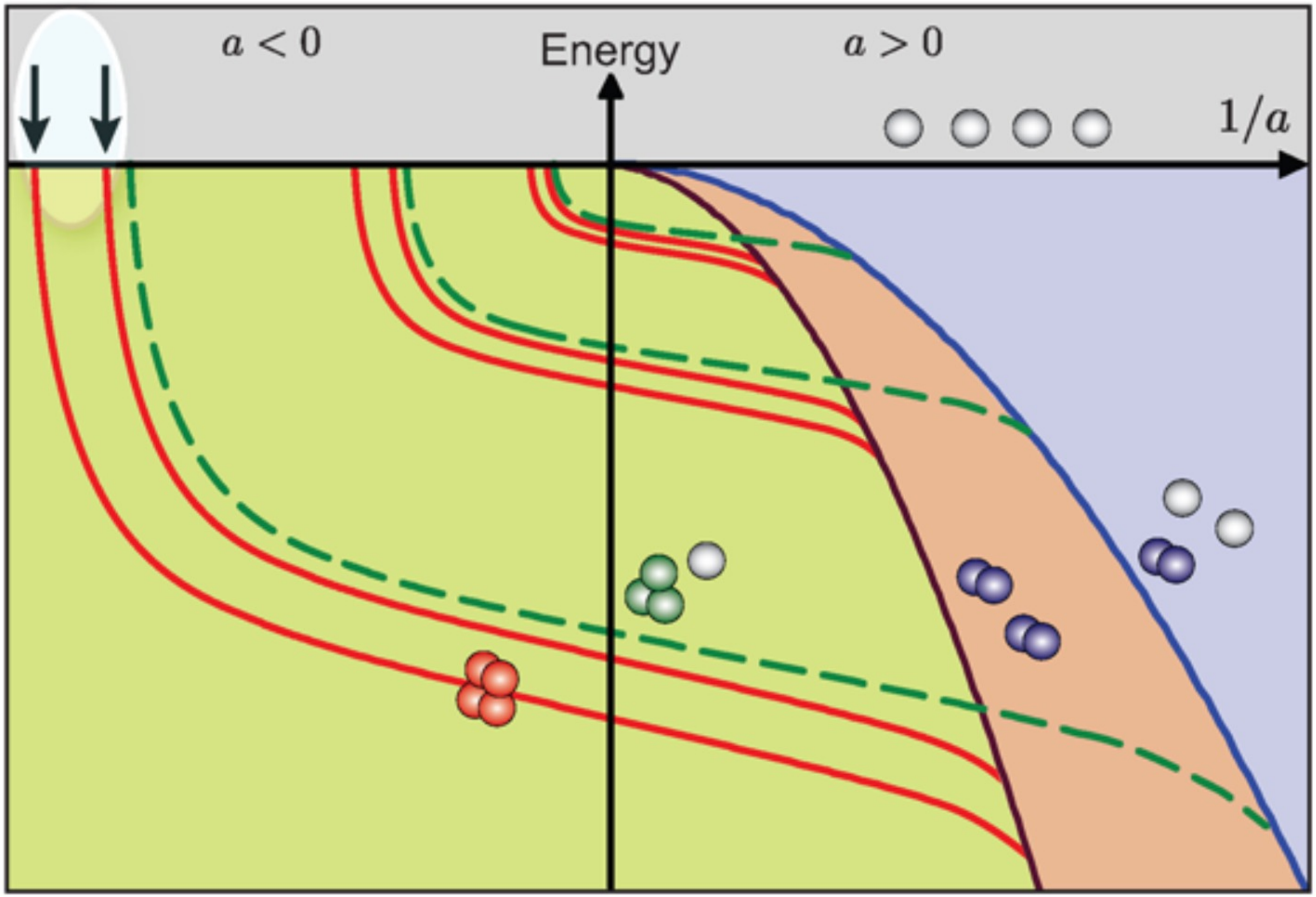
Efimov, Sov. J. Nucl. Phys. 12, 589 (1971); Efimov, Phys. Rev. C47 1876 (1993)
Bedaque, Hammer, van Kolck, Phys. Rev. Lett. 82 463 (1999)

Efimov trimers



Ferlaino, Grimm, Physics 3, 9 (2010)

Efimov tetramers



Platter, Hammer, Meißner, PRA 70, 052101 (2004)
Ferlaino, Koop, Berninger, Harm, D'Incao, Nägerl, Grimm, PRL 102, 140401 (2009)
Ferlaino, Grimm, Physics 3, 9 (2010)

Is nuclear physics near unitarity?

Nuclear Physics Around the Unitarity Limit

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(Received 18 July 2016; revised manuscript received 10 February 2017; published 15 May 2017)

We argue that many features of the structure of nuclei emerge from a strictly perturbative expansion around the unitarity limit, where the two-nucleon *S* waves have bound states at zero energy. In this limit, the gross features of states in the nuclear chart are correlated to only one dimensionful parameter, which is related to the breaking of scale invariance to a discrete scaling symmetry and set by the triton binding energy. Observables are moved to their physical values by small *perturbative* corrections, much like in descriptions of the fine structure of atomic spectra. We provide evidence in favor of the conjecture that light, and possibly heavier, nuclei are bound weakly enough to be insensitive to the details of the interactions but strongly enough to be insensitive to the exact size of the two-nucleon system.

DOI: 10.1103/PhysRevLett.118.202501

König, Grießhammer, Hammer, van Kolck, PRL 118, 202501 (2017)

Clustering of Four-Component Unitary Fermions

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[1]Department of Physics, University of Guelph, Guelph, Ontario N1G 2W1, Canada
[2]Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
[3]Institut de Physique Nucléaire, CNRS-IN2P3, Université Paris-Sud, Université Paris-Saclay, 91406 Orsay, France
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(Received 12 August 2019; accepted 13 March 2020; published 9 April 2020)

Ab initio nuclear physics tackles the problem of strongly interacting four-component fermions. The same setting could foreseeably be probed experimentally in ultracold atomic systems, where two- and three-component experiments have led to major breakthroughs in recent years. Both due to the problem's inherent interest and as a pathway to nuclear physics, in this Letter we study four-component fermions at unitarity via the use of quantum Monte Carlo methods. We explore novel forms of the trial wave function and find one which leads to a ground state of the eight-particle system whose energy is almost equal to that of two four-particle systems. We investigate the clustering properties involved and also extrapolate to the zero-range limit. In addition to being experimentally testable, our results impact the prospects of developing nuclear physics as a perturbation around the unitary limit.

DOI: 10.1103/PhysRevLett.124.143402

Dawkins, Carlson, van Kolck, Gezerlis, PRL 124, 143402 (2020)

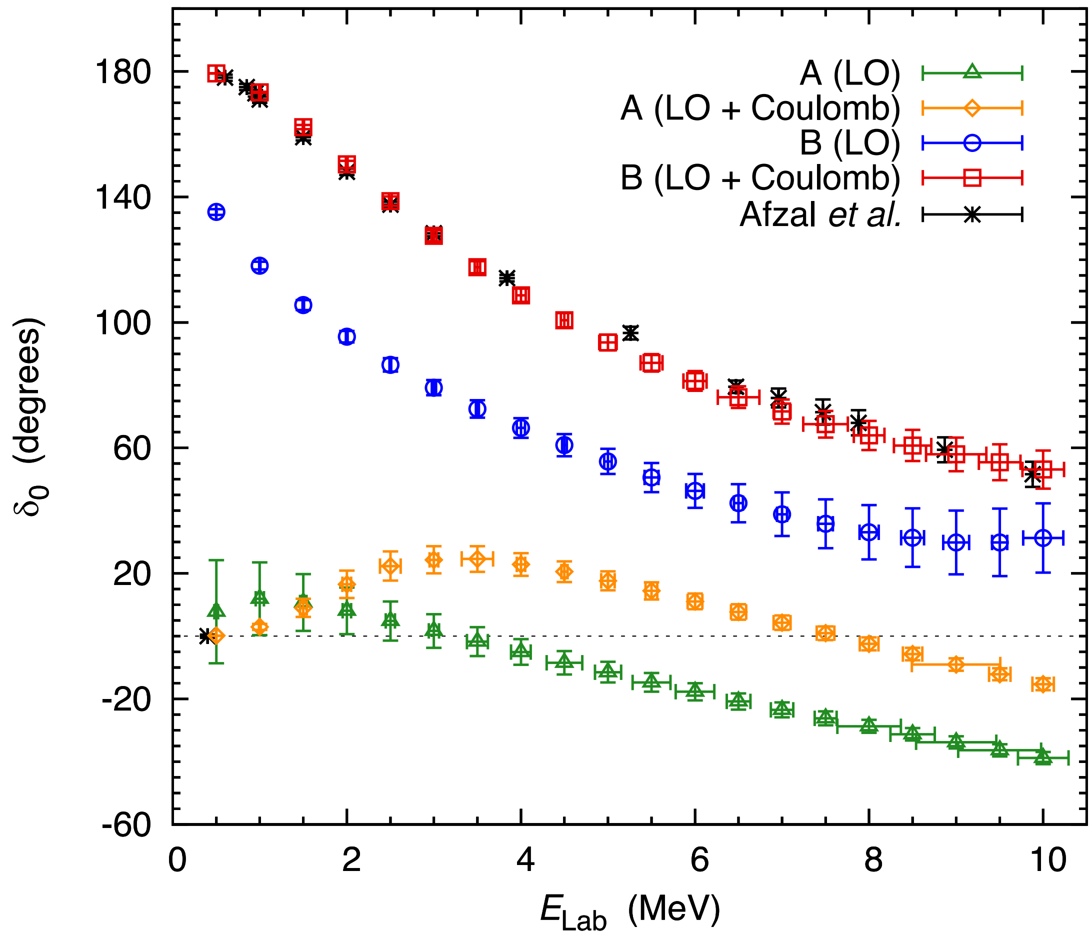
A tale of two interactions

We consider two different interactions, A and B, at leading order (LO) in chiral effective field theory. They both have the same one-pion exchange potential and Coulomb potential. The difference between A and B resides with their short-range interactions.

Interaction A	**Interaction B**
Nonlocal short-range interaction	Nonlocal short-range interaction + Local short-range interaction

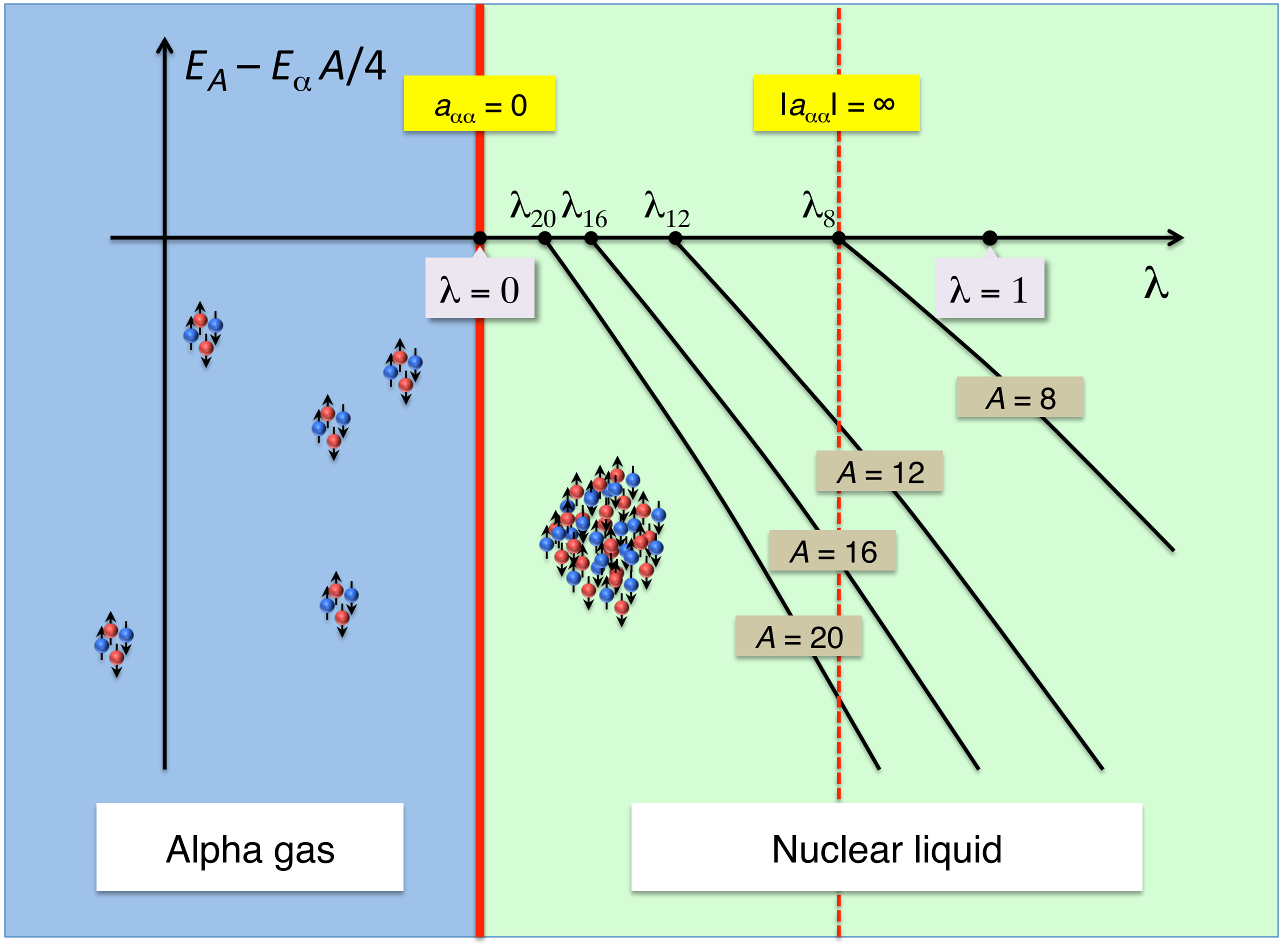
Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, D.L., Rupak, PRL 117, 132501 (2016)

alpha-alpha S-wave scattering



Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, D.L., Rupak, PRL 117, 132501 (2016)

Nuclear physics near a quantum phase transition



Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, D.L., Rupak, PRL 117, 132501 (2016)

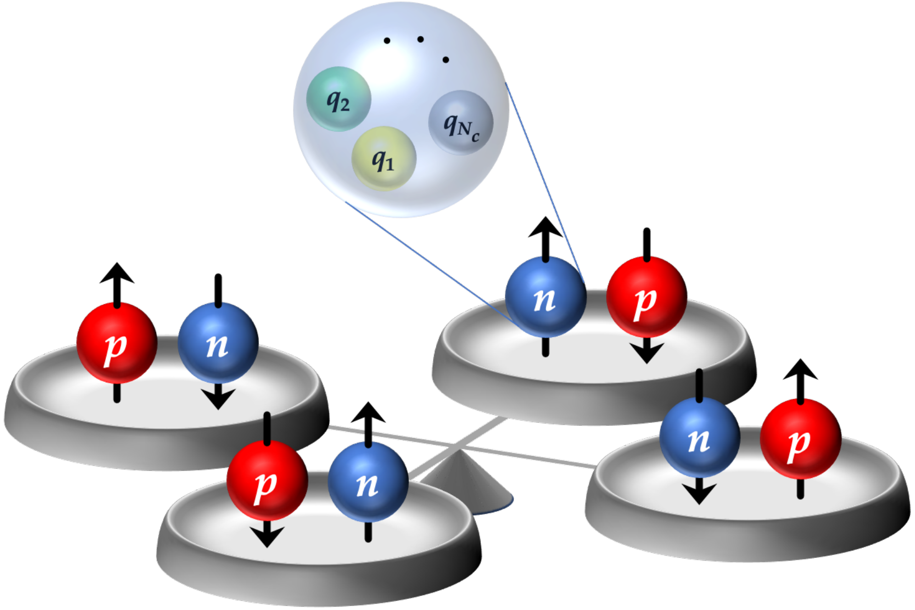
Hidden spin-isospin exchange symmetry

Kaplan, Savage, PLB 365, 244 (1996)

Kaplan, Manohar, PRC 56, 76 (1997)

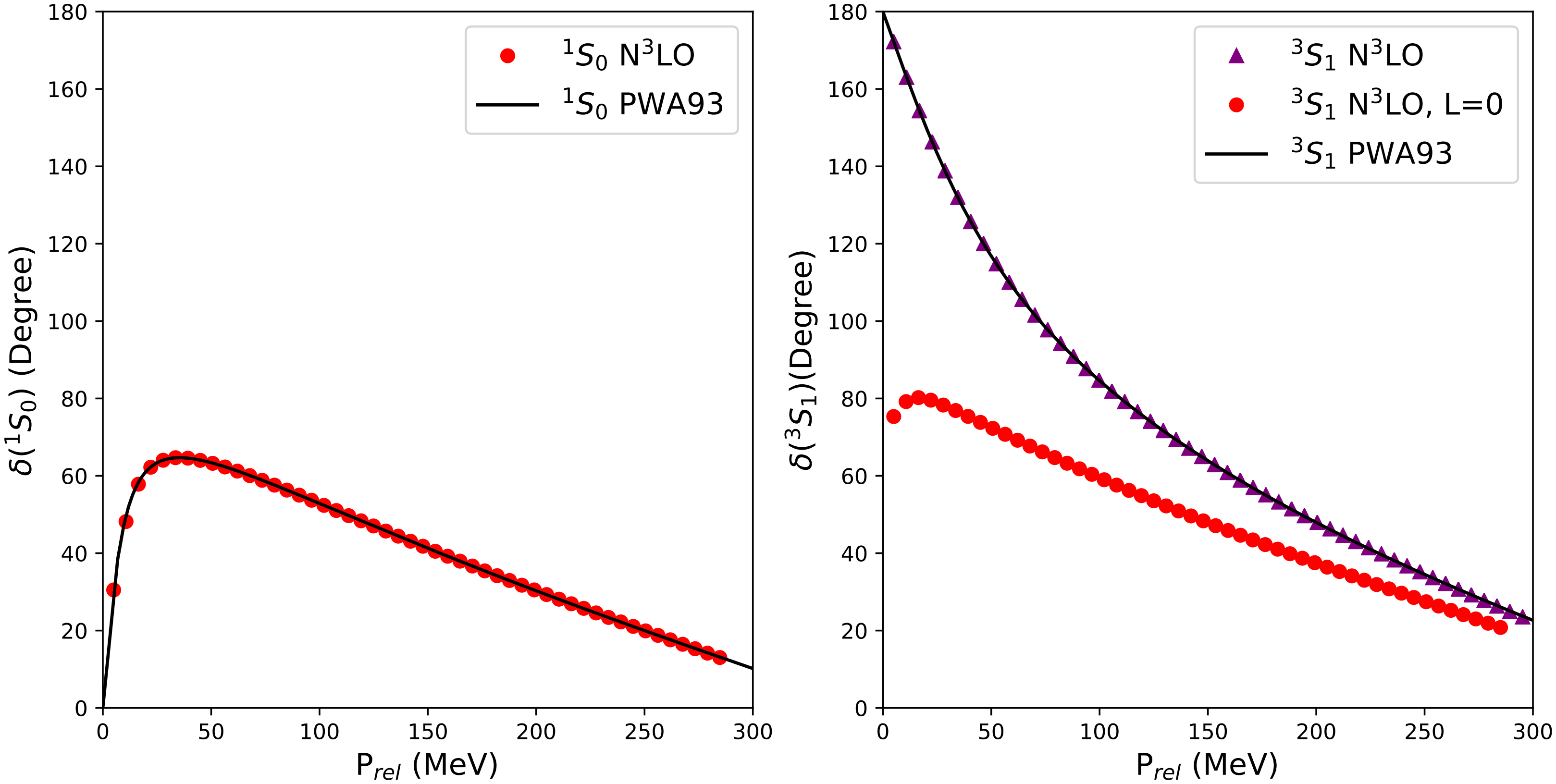
Calle Gordon, Arriola, PRC 80, 014002 (2009)

$$V^{2N}_{\text{large}-N_c} = V_C + \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2 W_S + (3\hat{r} \cdot \vec{\sigma}_1 \hat{r} \cdot \vec{\sigma}_2 - \vec{\sigma}_1 \cdot \vec{\sigma}_2) \vec{\tau}_1 \cdot \vec{\tau}_2 W_T$$



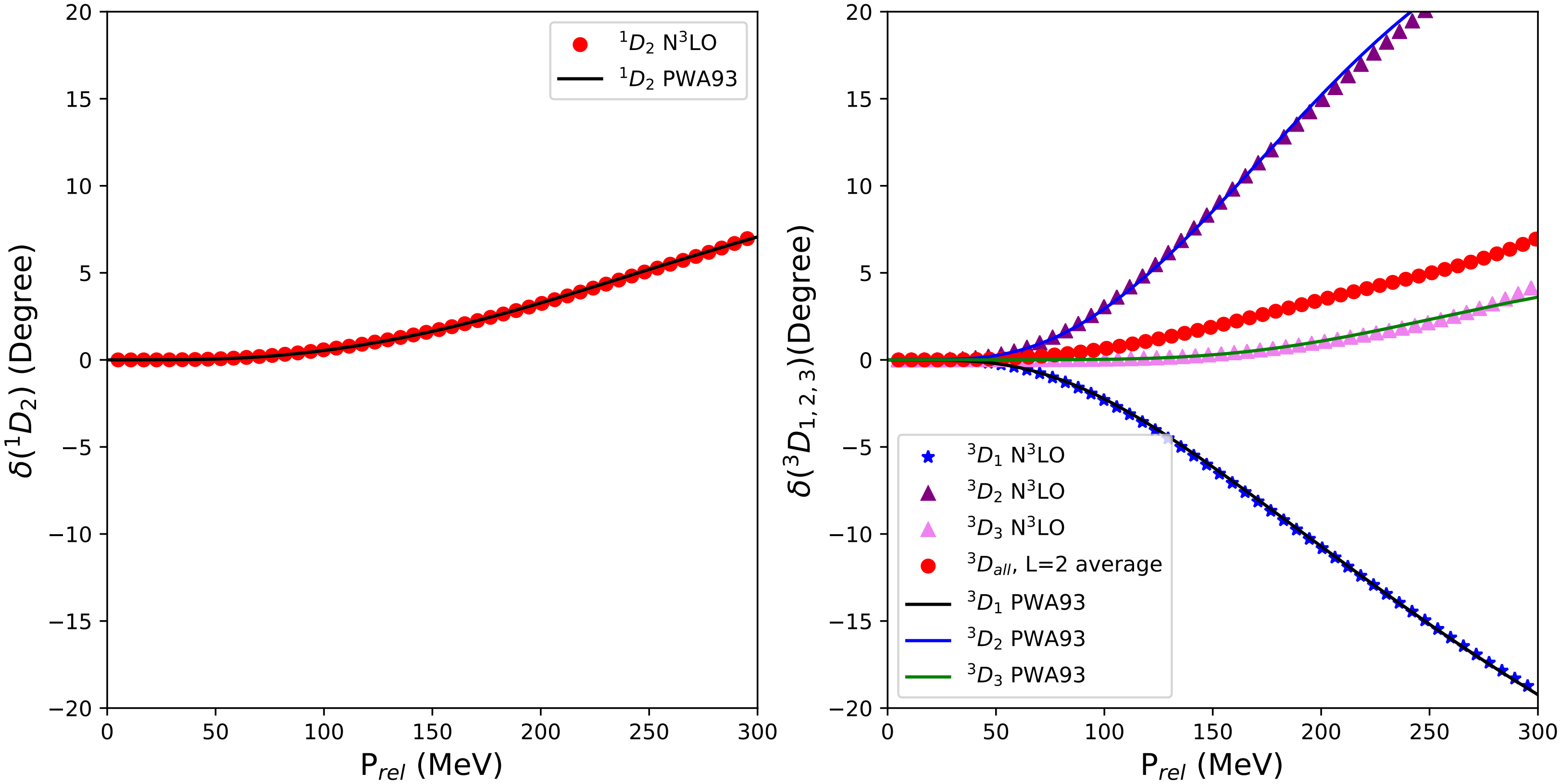
D.L., Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, Meißner, PRL 127, 062501 (2021)

$$\Lambda_{\mathrm{large}-N_c} \sim 500 \text{ MeV}$$



D.L., Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, Meißner, PRL 127, 062501 (2021)

$$\Lambda_{\mathrm{large}-N_c} \sim 500 \text{ MeV}$$



D.L., Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, Meißner, PRL 127, 062501 (2021)

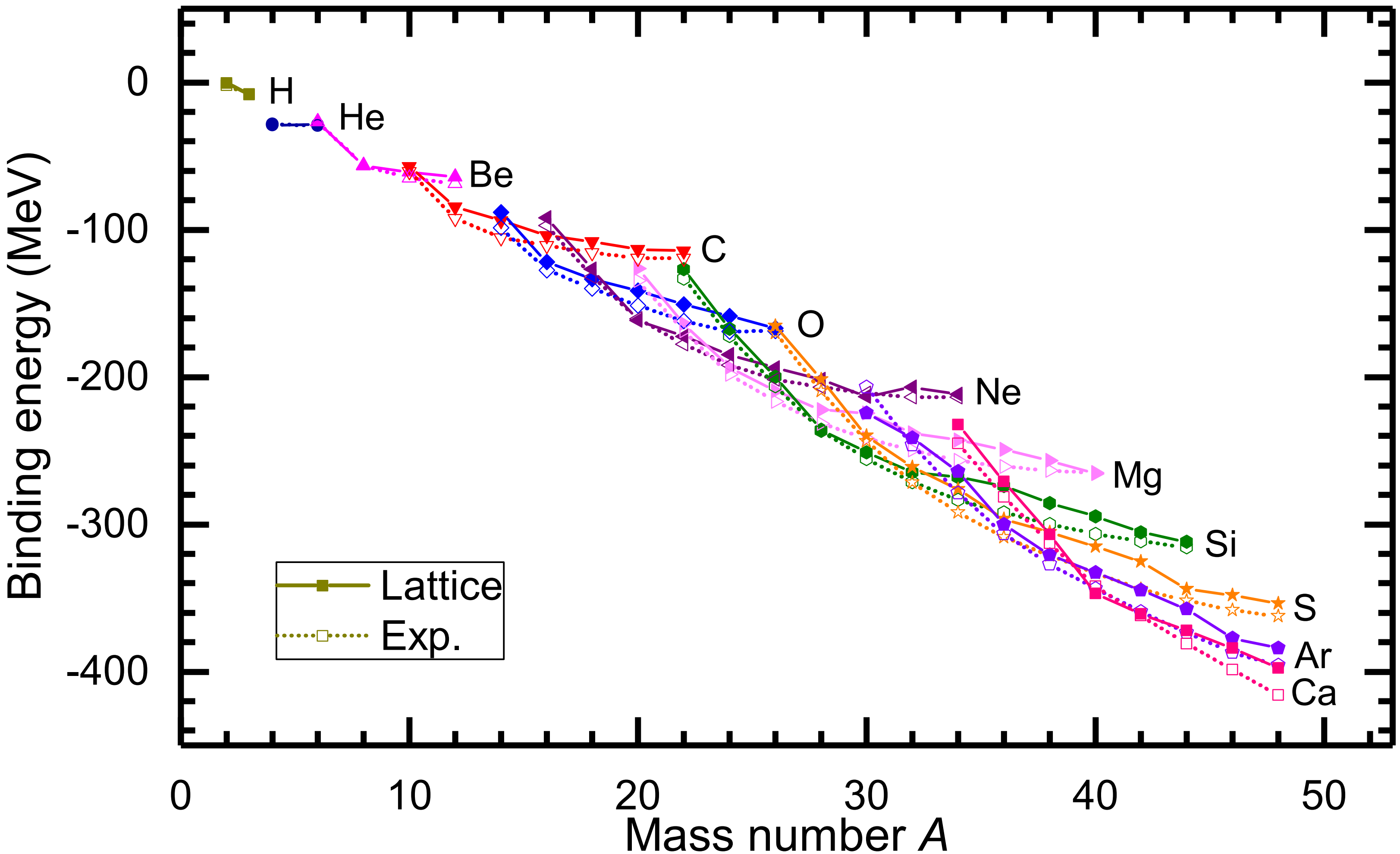
Essential elements for nuclear binding

What is the minimal nuclear interaction that can reproduce the ground state properties of light nuclei, medium-mass nuclei, and neutron matter simultaneously with no more than a few percent error in the energies and charge radii?

We construct an interaction with only four parameters.

1. Strength of the two-nucleon S-wave interaction
2. Range of the two-nucleon S-wave interaction
3. Strength of three-nucleon contact interaction
4. Range of the local part of the two-nucleon interaction

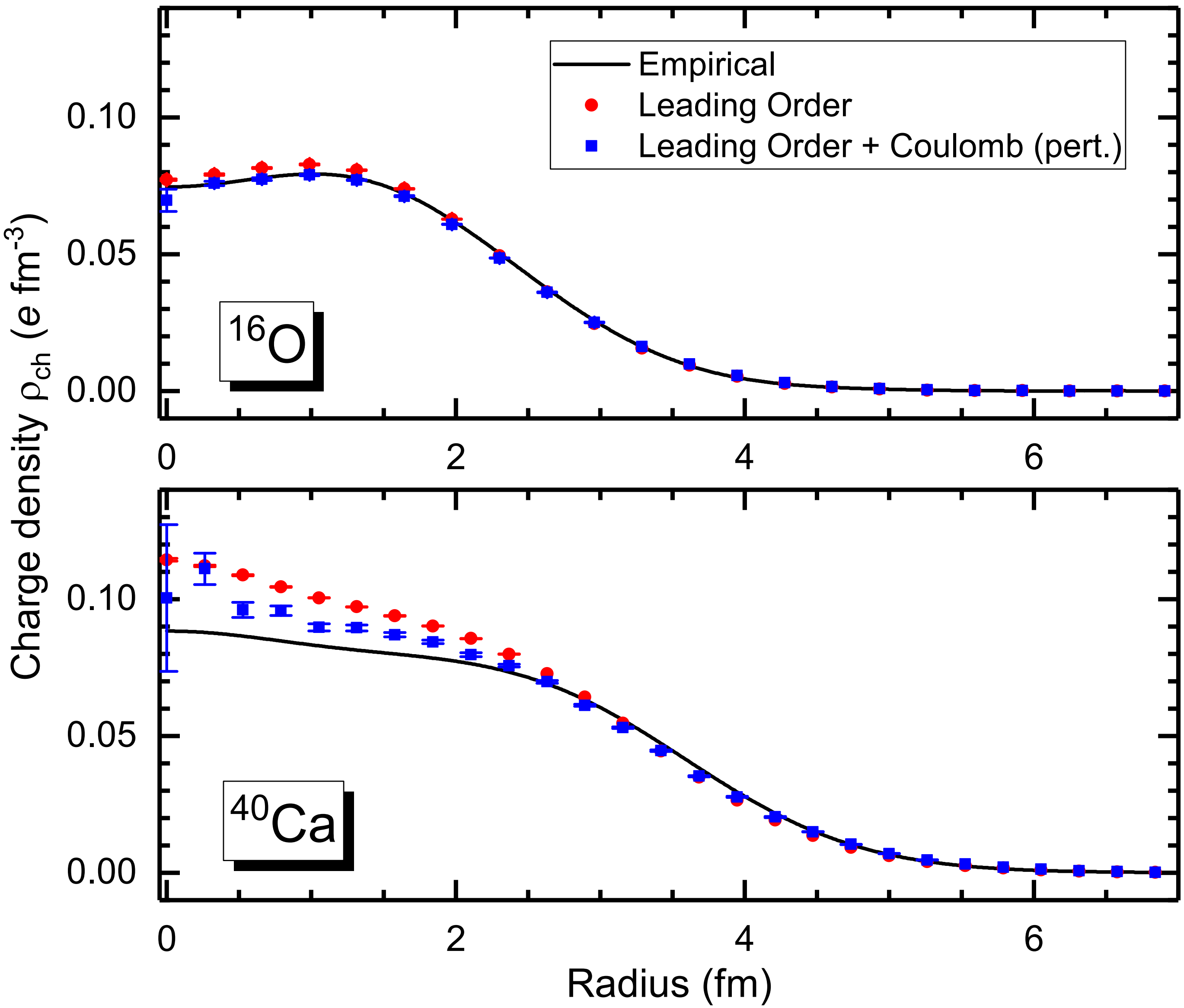
Except for the Coulomb potential, the interaction is invariant under Wigner's SU(4) symmetry.



Lu, Li, Elhatisari, D.L., Epelbaum, Meißner, PLB 797, 134863 (2019)

	B	Exp.	$R_{\rm ch}$	Exp.
^{3}H	8.48(2)(0)	8.48	1.90(1)(1)	1.76
^{3}He	7.75(2)(0)	7.72	1.99(1)(1)	1.97
^{4}He	28.89(1)(1)	28.3	1.72(1)(3)	1.68
^{16}O	121.9(1)(3)	127.6	2.74(1)(1)	2.70
^{20}Ne	161.6(1)(1)	160.6	2.95(1)(1)	3.01
^{24}Mg	193.5(02)(17)	198.3	3.13(1)(2)	3.06
^{28}Si	235.8(04)(17)	236.5	3.26(1)(1)	3.12
^{40}Ca	346.8(6)(5)	342.1	3.42(1)(3)	3.48

Lu, Li, Elhatisari, D.L., Epelbaum, Meißner, PLB 797, 134863 (2019)

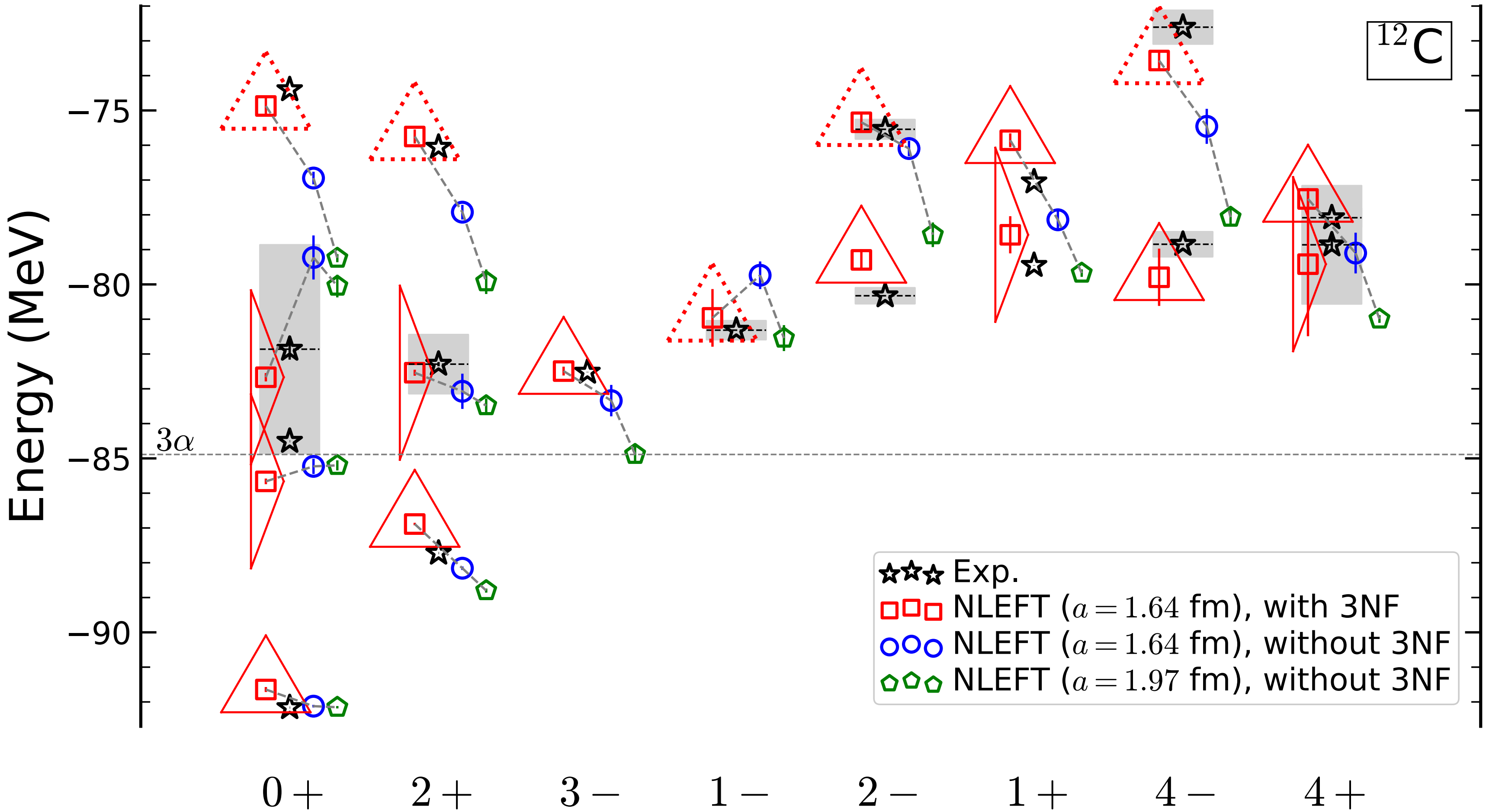


Lu, Li, Elhatisari, D.L., Epelbaum, Meißner, PLB 797, 134863 (2019)

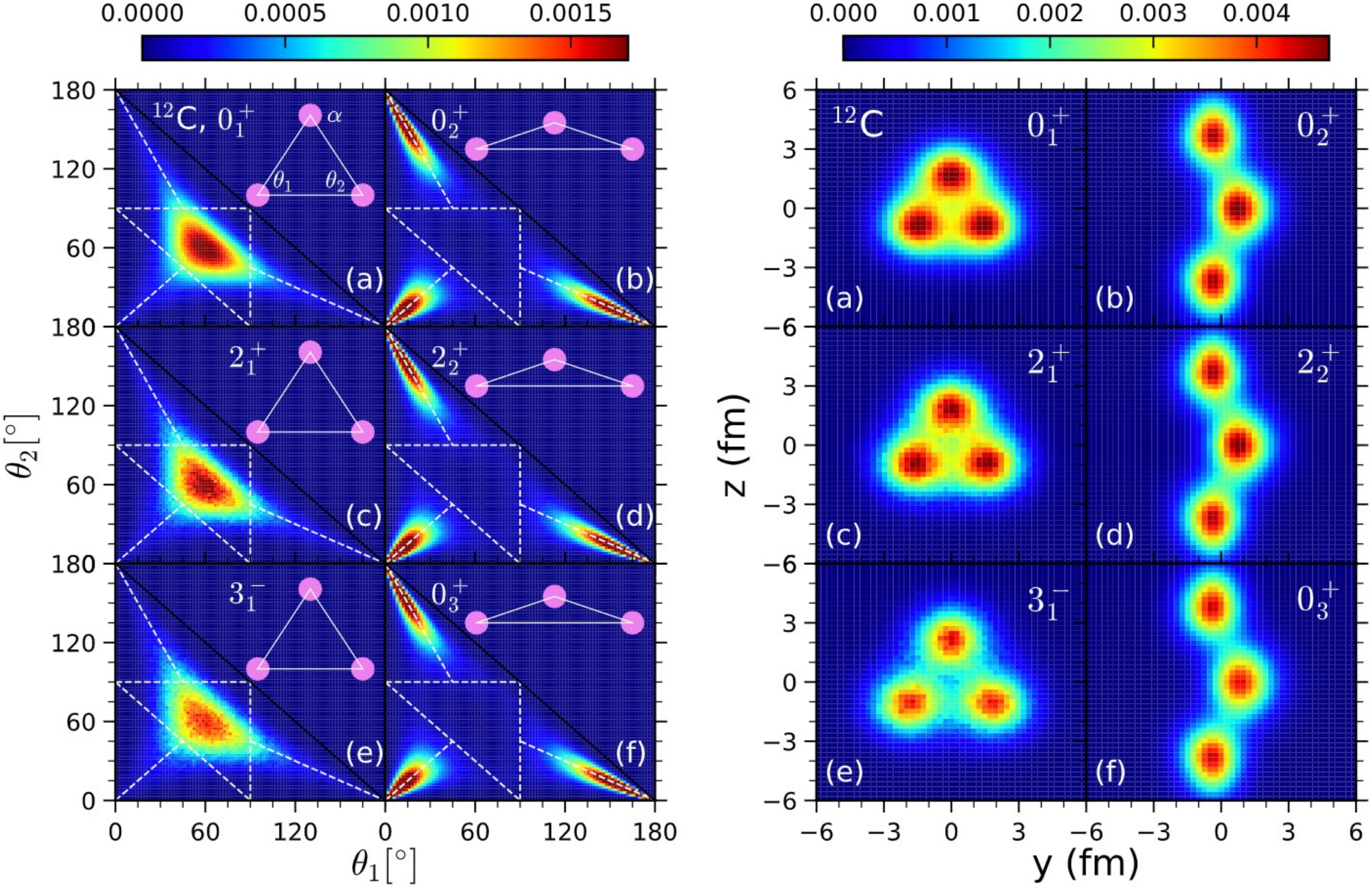
Structure and spectrum of ^{12}C

Shen, Lähde, D.L. Meißner, EPJA 57, 276 (2021)

State	$a = 1.97$ fm	$a = 1.64$ fm	Experiment
0_1^+	$-92.15(3)$	$-92.12(4)$	-92.162
2_1^+	$-88.87(4)$	$-88.19(17)$	-87.722
0_2^+	$-85.20(15)$	$-85.23(22)$	-84.508
3_1^-	$-84.9(2)$	$-83.3(5)$	$-82.521(5)$
2_2^+	$-83.5(2)$	$-83.1(5)$	$-82.29(6)$
0_3^+	$-80.0(3)$	$-79.2(6)$	$-81.9(3)$
1_1^-	$-81.5(4)$	$-79.7(4)$	$-81.315(4)$
2_1^-	$-78.6(2)$	$-76.1(2)$	$-80.326(4)$
1_1^+	$-79.67(11)$	$-78.14(24)$	$-79.452(6)$
4_1^-	$-78.1(2)$	$-75.5(5)$	$-78.846(20)$
4_1^+	$-80.99(11)$	$-79.1(6)$	$-78.083(5)$
2_3^+	$-79.9(4)$	$-77.9(2)$	-76.056
0_4^+	$-79.25(11)$	$-76.94(18)$	-74.402



Shen, Lähde, D.L. Meißner, arXiv:2202.13596



Shen, Lähde, D.L. Meißner, arXiv:2202.13596

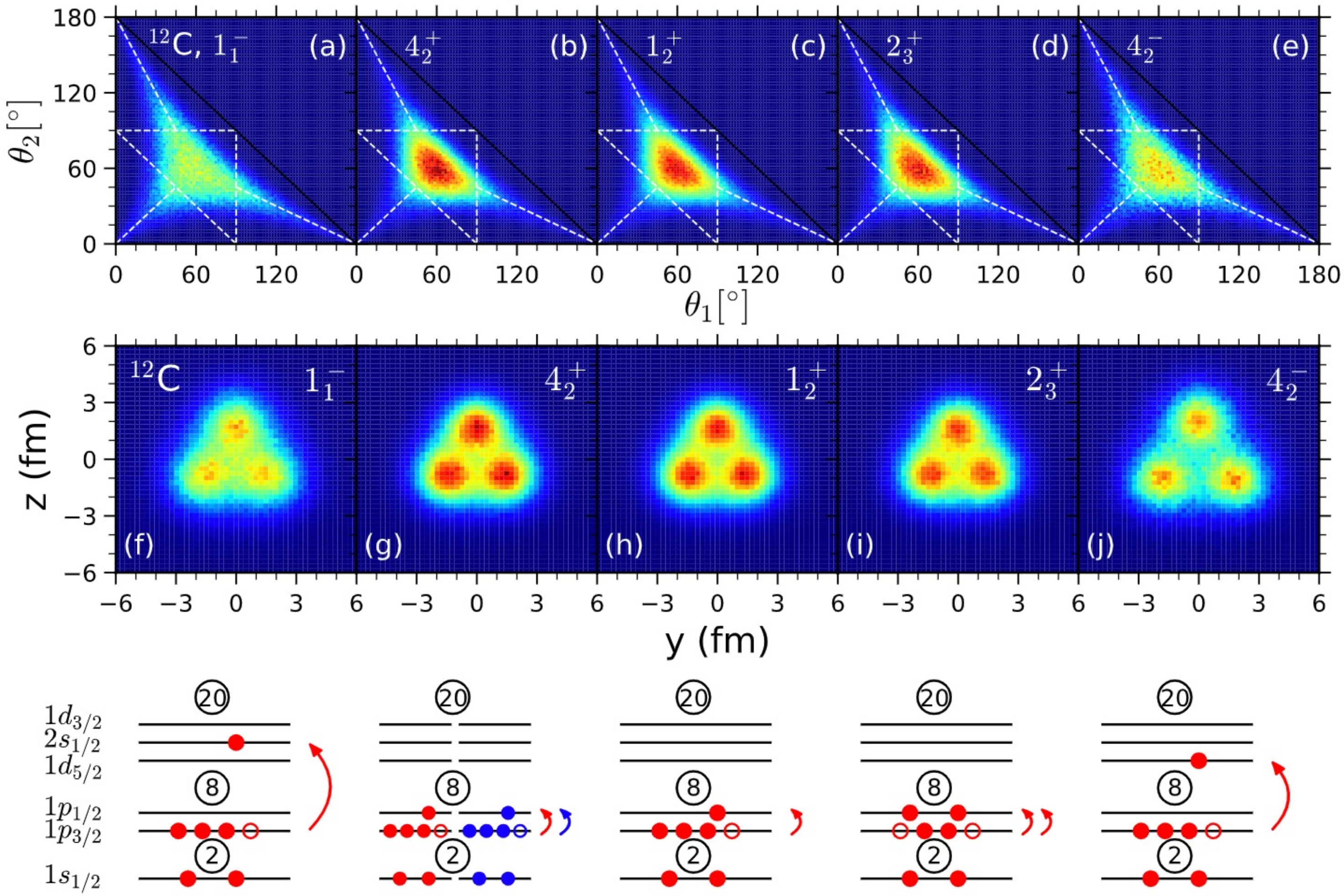


Figure S3: **Top Panel:** Density distribution for the two inner angles of the triangle formed by the three alpha clusters. **Middle Panel:** Tomographic projection of the nuclear density. **Lower Panel:** Sketch of the orbitals for the shell model initial states used in each of these calculations.

Shen, Lähde, D.L. Meißner, arXiv:2202.13596

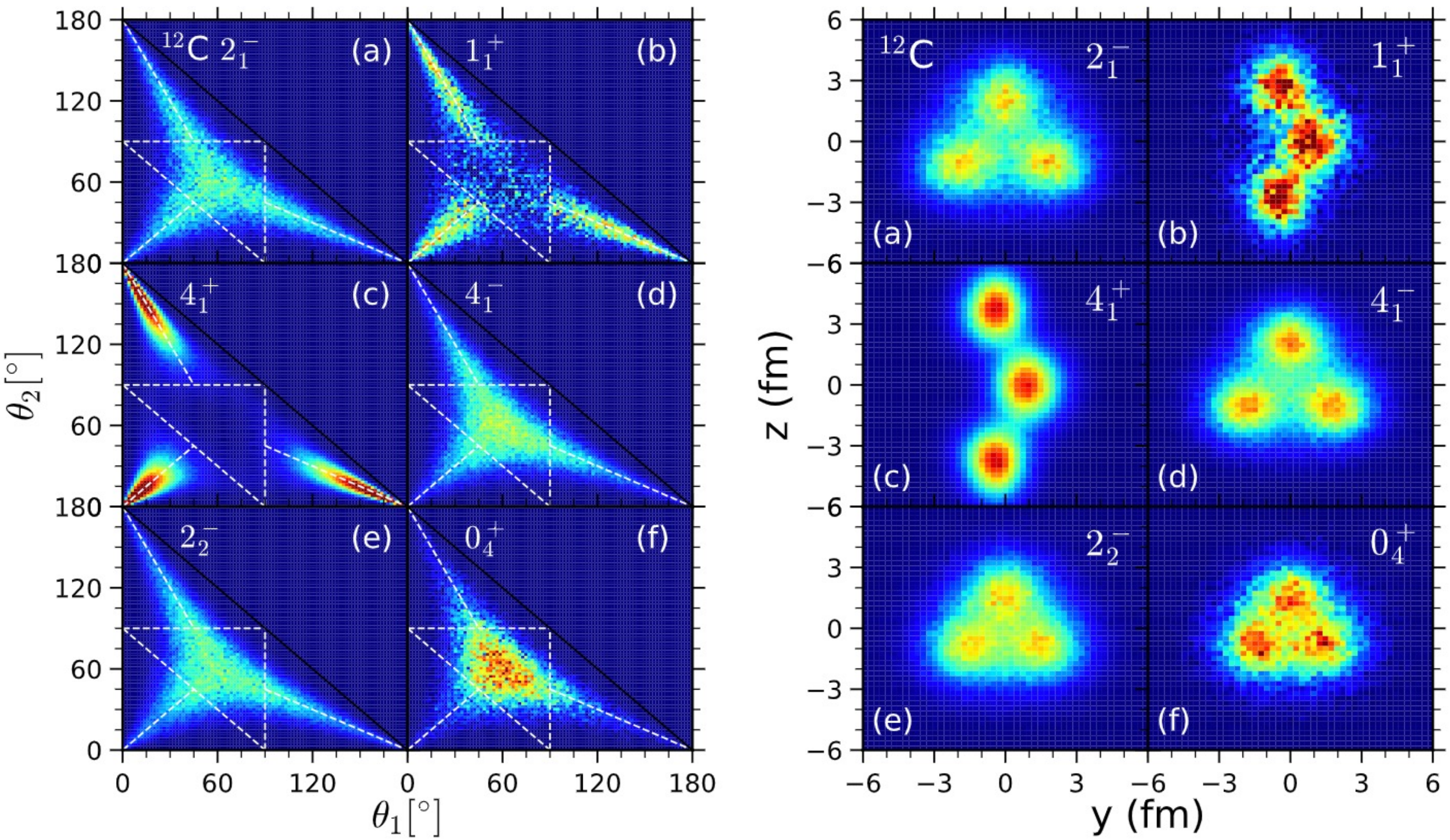
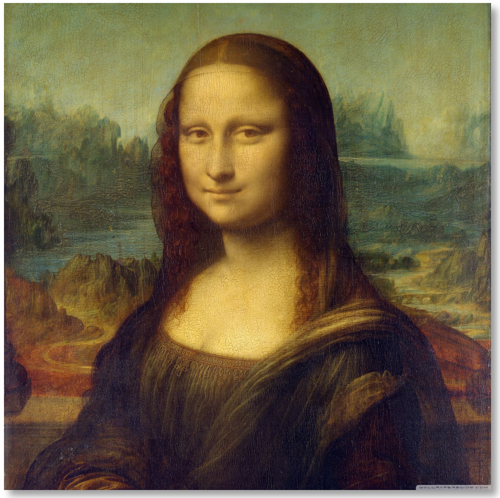


Figure S4: **Left Panel:** Density distribution for the two inner angles of the triangle formed by the three alpha clusters. **Right Panel:** Tomographic projection of the nuclear density. From (a) to (f), the selected states are ordered by their energies from low to high.

Shen, Lähde, D.L. Meißner, arXiv:2202.13596

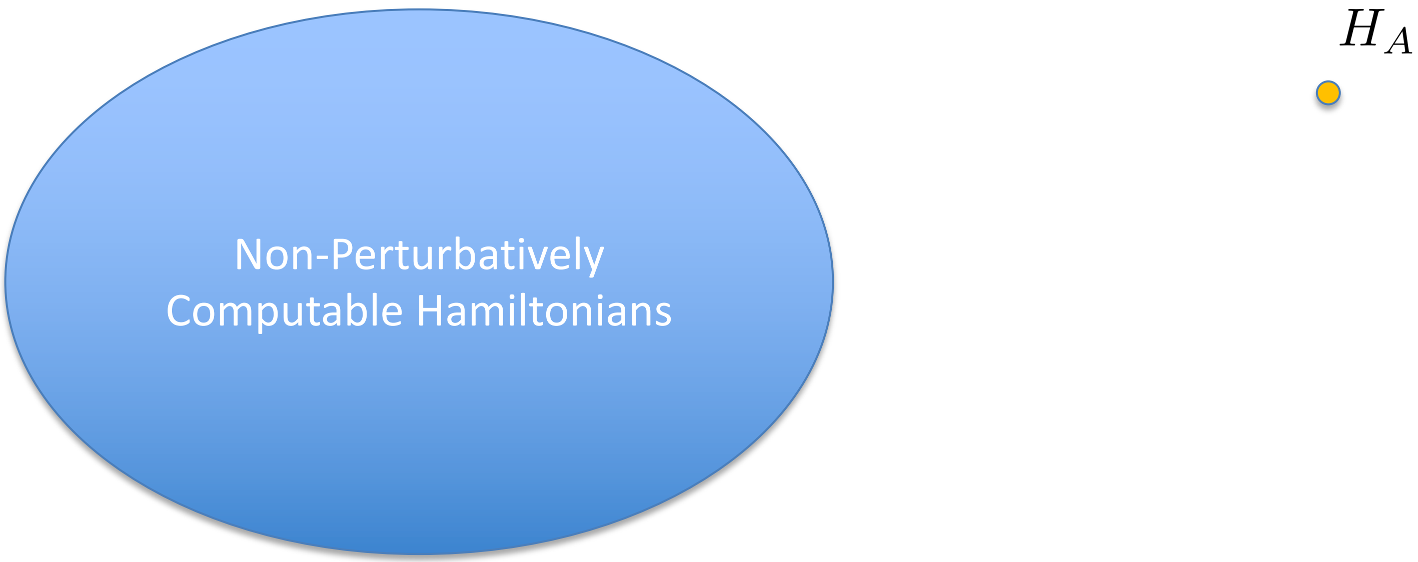
Wave function matching

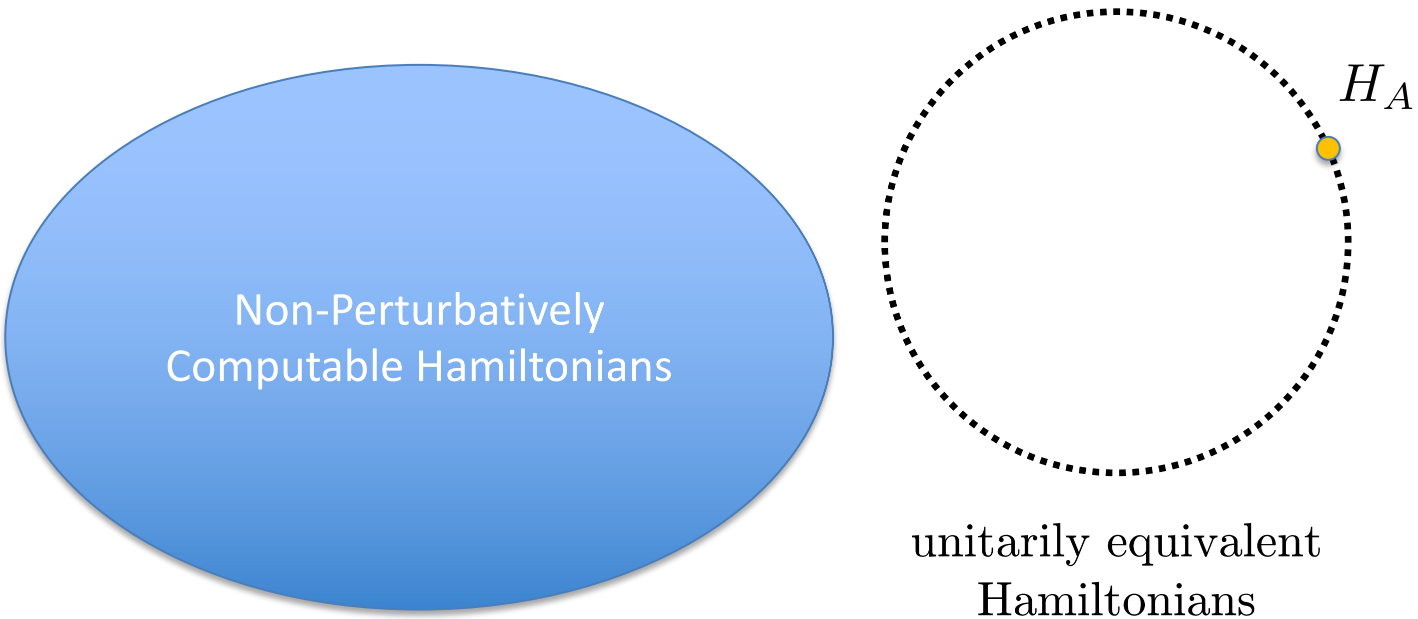


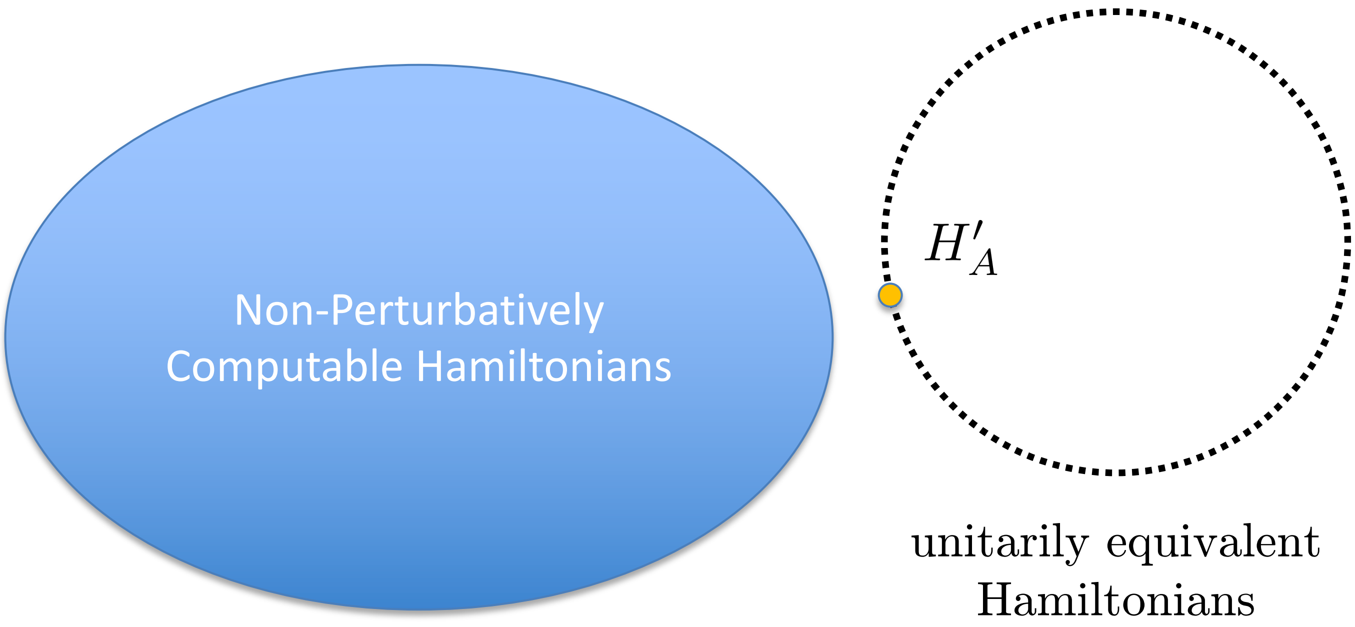
Work in progress: Elhatisari, Bovermann, et al.

Lattice Monte Carlo simulations can compute highly nontrivial correlations in nuclear many-body systems. Unfortunately, sign oscillations prevent direct simulations using a high-fidelity Hamiltonian based on chiral effective field theory due to short-range repulsion.

Wave function matching solves this problem by means of unitary transformations and perturbation theory. By using unitary transformations, we construct a high-fidelity Hamiltonian that can be reached by perturbation theory, starting from a Hamiltonian without a sign problem.

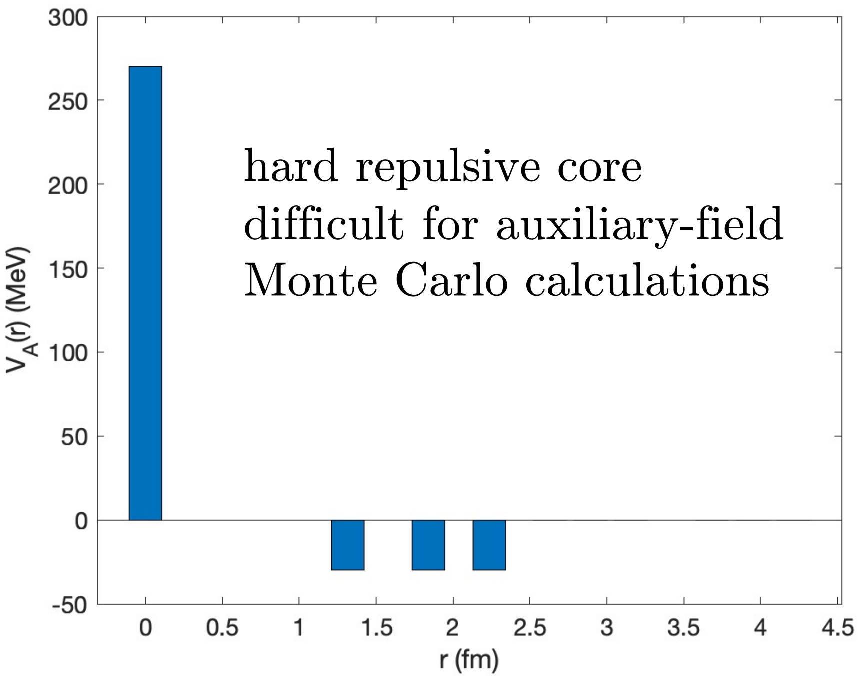

Non-Perturbatively
Computable Hamiltonians
H_A

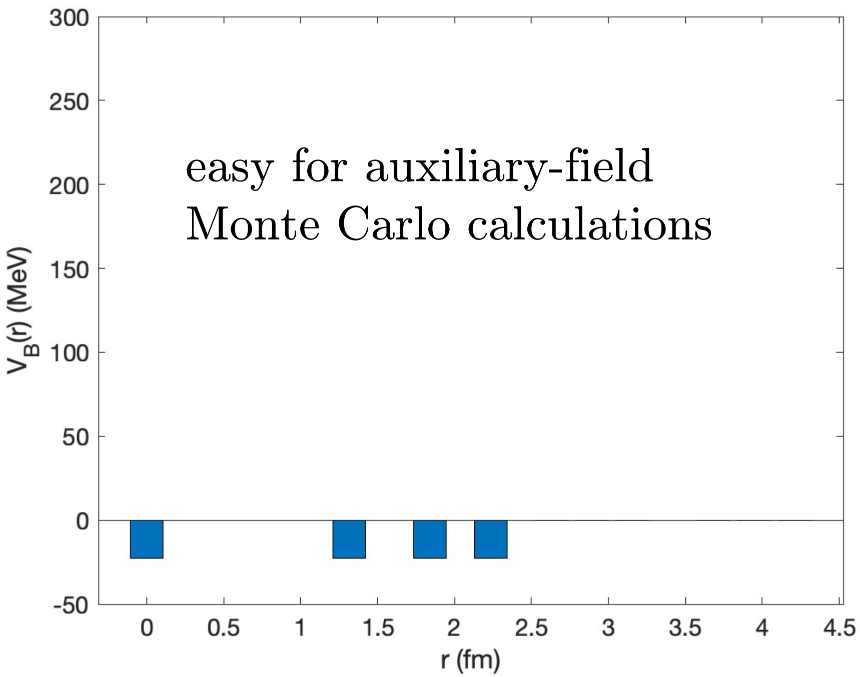

Non-Perturbatively
Computable Hamiltonians
H_A
unitarily equivalent
Hamiltonians


Non-Perturbatively
Computable Hamiltonians
H'_A
unitarily equivalent
Hamiltonians

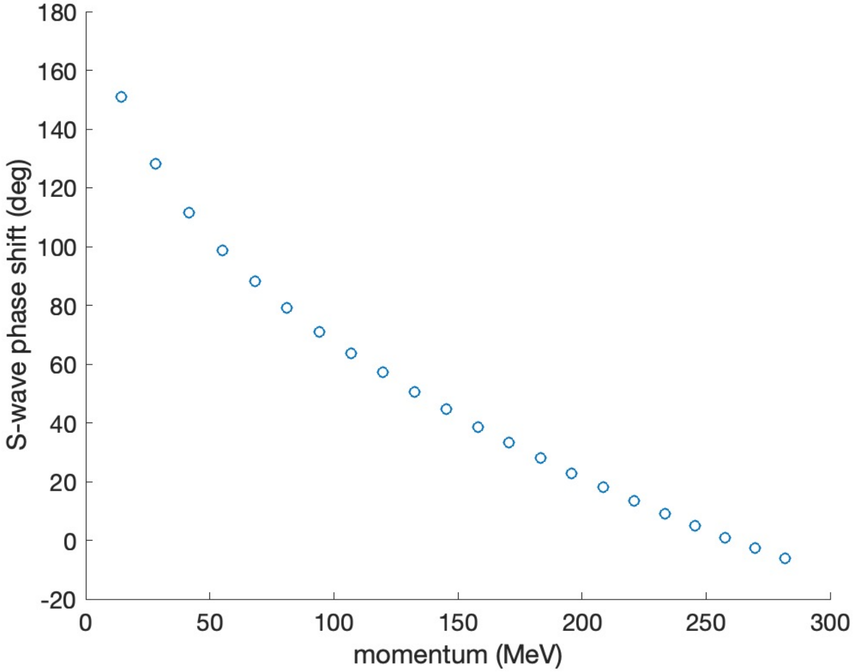
Wave function matching

$V_A(r)$ $\qquad\qquad$ $V_B(r)$



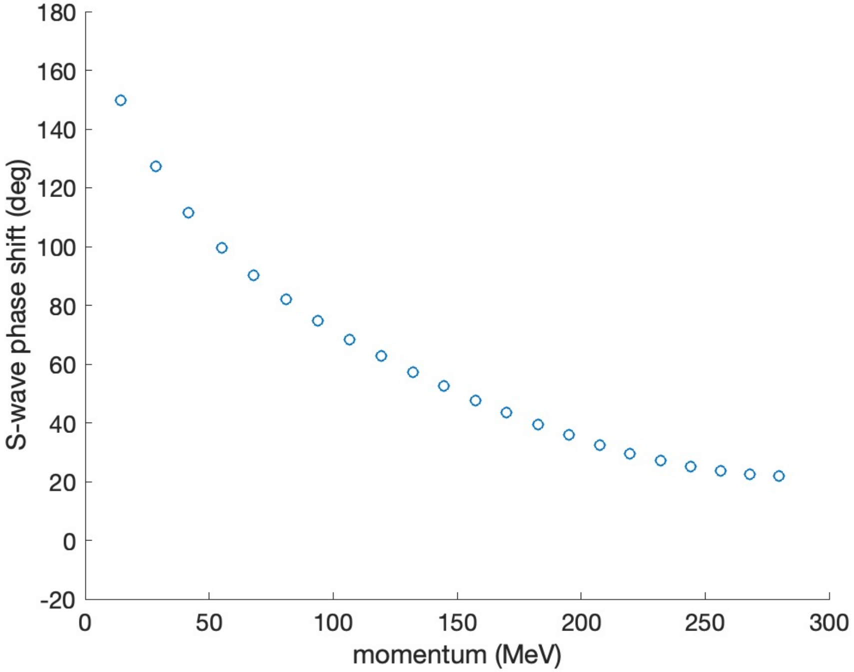


$V_A(r)$



S-wave phase shift (deg)
momentum (MeV)

$V_B(r)$



S-wave phase shift (deg)
momentum (MeV)

Let us write the eigenenergies and eigenfunctions for the two interactions as

$$H_A \left|\psi_{A,n}\right\rangle = (K + V_A) \left|\psi_{A,n}\right\rangle = E_{A,n} \left|\psi_{A,n}\right\rangle$$
$$H_B \left|\psi_{B,n}\right\rangle = (K + V_B) \left|\psi_{B,n}\right\rangle = E_{B,n} \left|\psi_{B,n}\right\rangle$$

We would like to compute the eigenenergies of H_A starting from the eigenfunctions of H_B and using first-order perturbation theory.

Not surprisingly, this does not work very well. The interactions V_A and V_B are quite different.

$E_{A,n}$ (MeV)	$\langle\psi_{B,n}\|H_A\|\psi_{B,n}\rangle$ (MeV)
-1.2186	3.0088
0.2196	0.3289
0.8523	1.1275
1.8610	2.2528
3.2279	3.6991
4.9454	5.4786
7.0104	7.5996
9.4208	10.0674
12.1721	12.8799
15.2669	16.0458

Let P be a projection operator that is nonzero only for separation distances r less than R. We define a short-distance unitary operator U such that

$$U : P\left|\psi_A^0\right\rangle / \|P\left|\psi_A^0\right\rangle\| \rightarrow P\left|\psi_B^0\right\rangle / \|P\left|\psi_B^0\right\rangle\|$$

There are many possible choices for U. The corresponding action of U on the Hamiltonian is

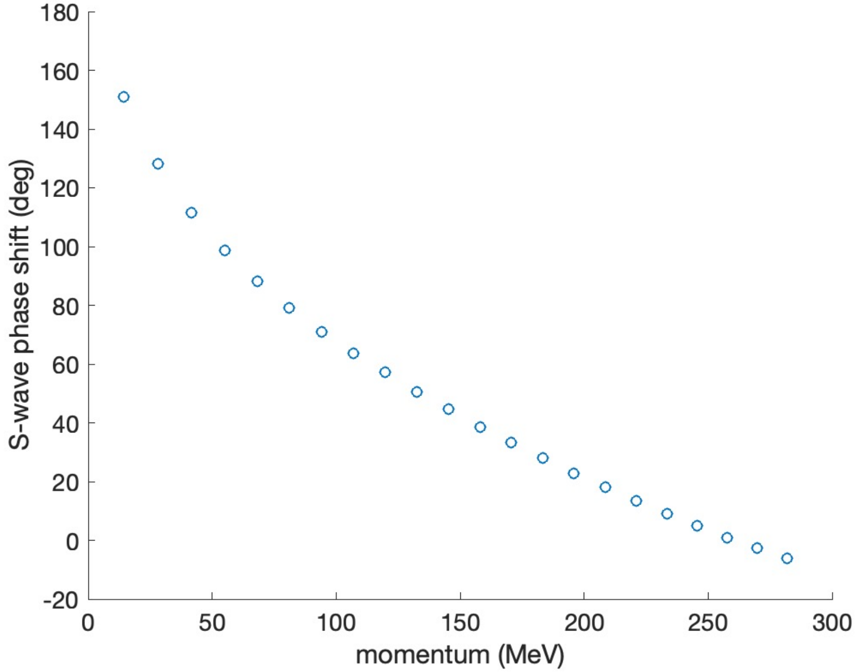
$$U : H_A \rightarrow H_A' = U^\dagger H_A U$$

and the resulting nonlocal interaction is

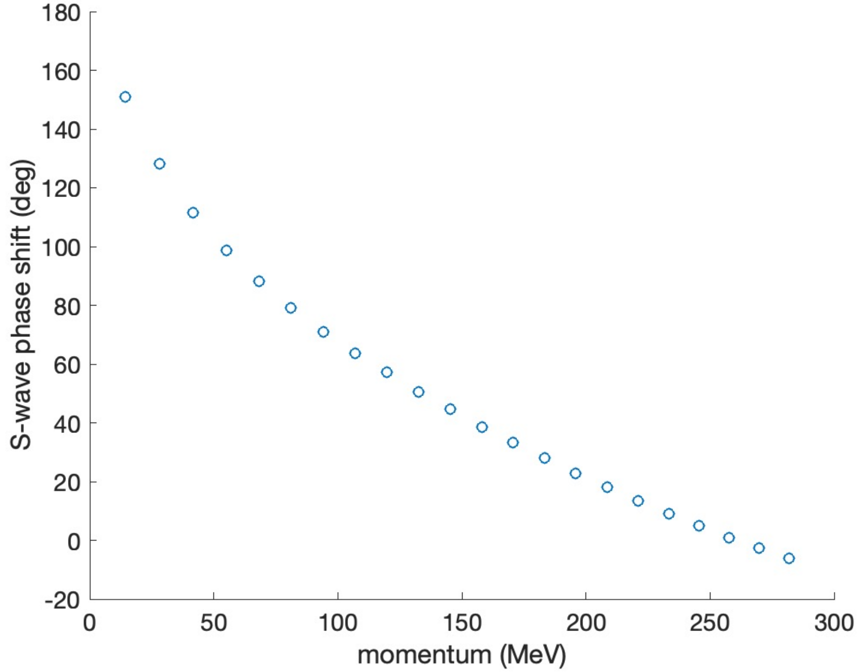
$$V_A' = H_A' - K = U^\dagger H_A U - K$$

Since they are unitarily equivalent, the phase shifts are exactly the same.

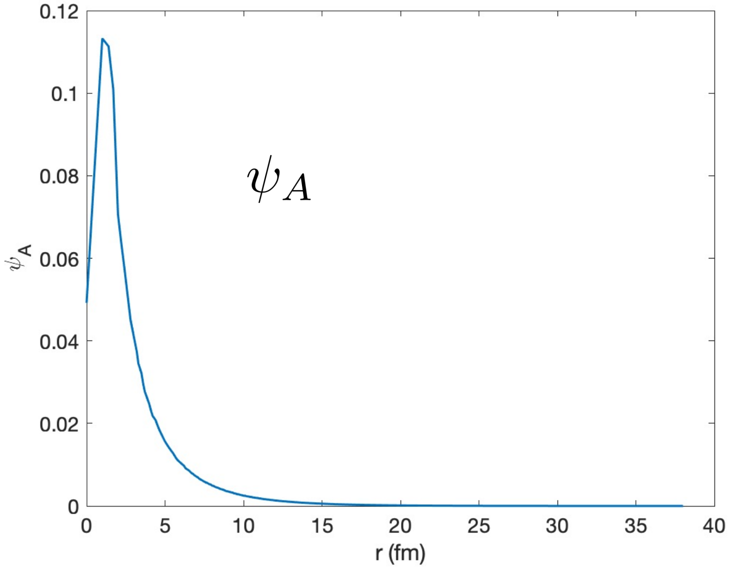
$V_A(r)$

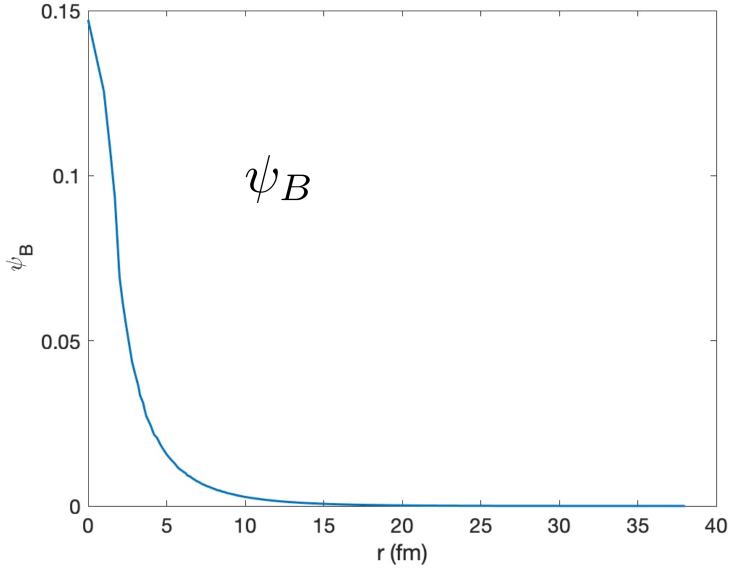


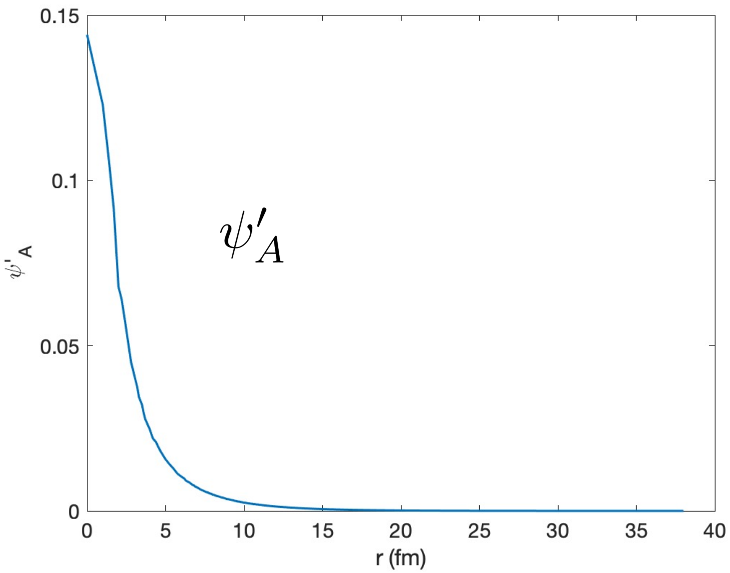
$V_A'(r, r')$



Ground state wave functions





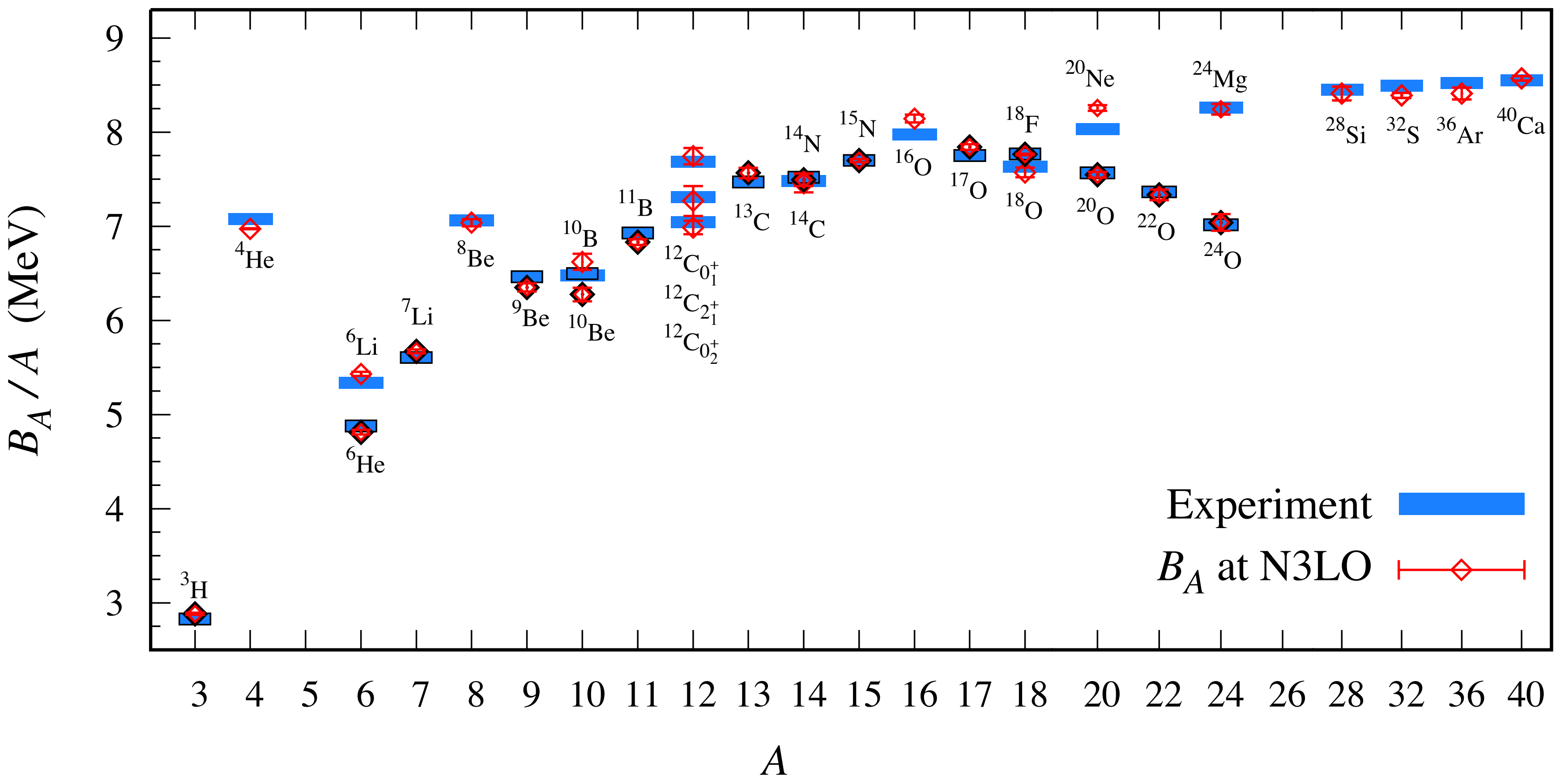


With wave function matching, we can now compute the eigenenergies starting from the eigenfunctions of H_B and using first-order perturbation theory.

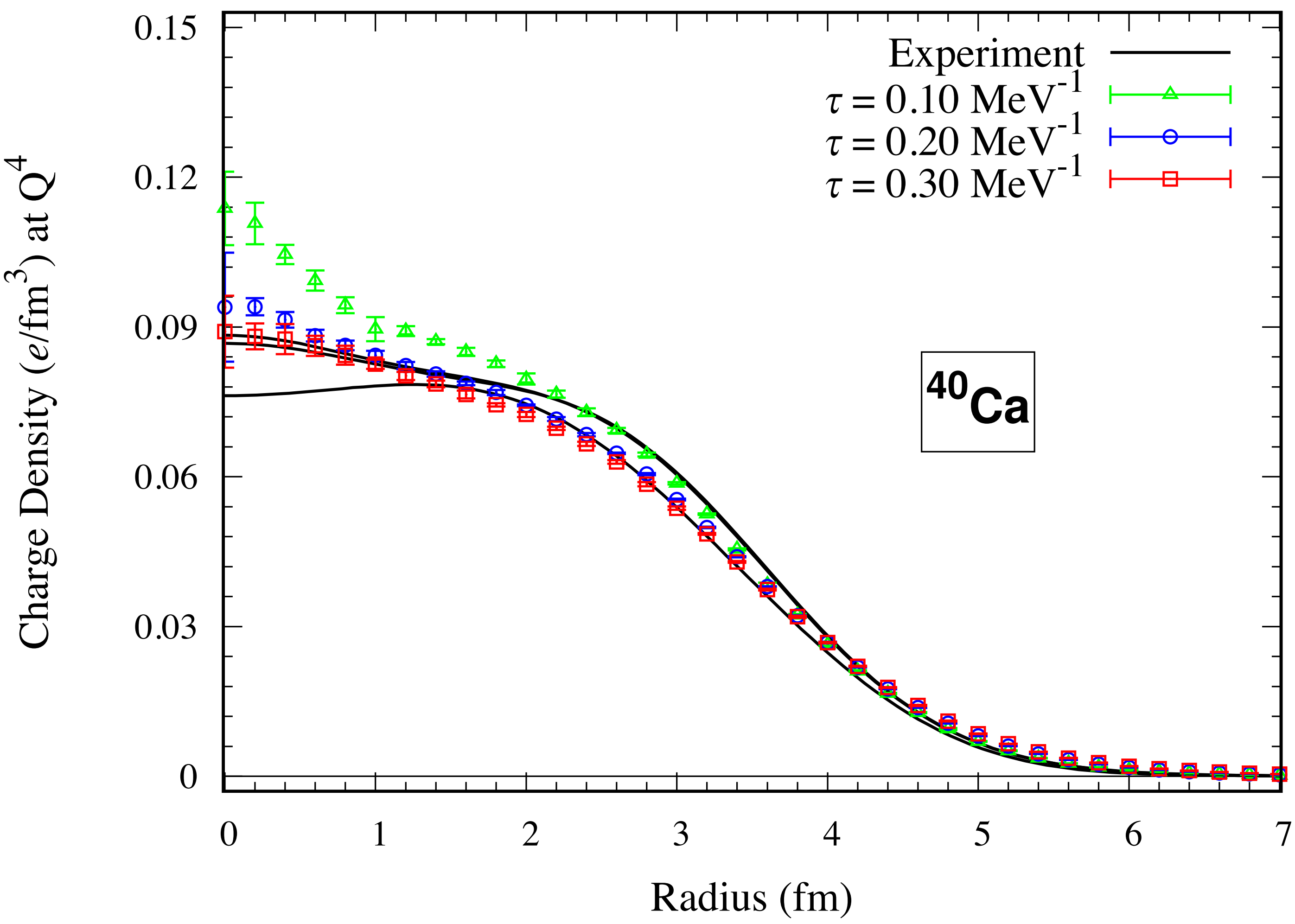
$R = 2.6$ fm

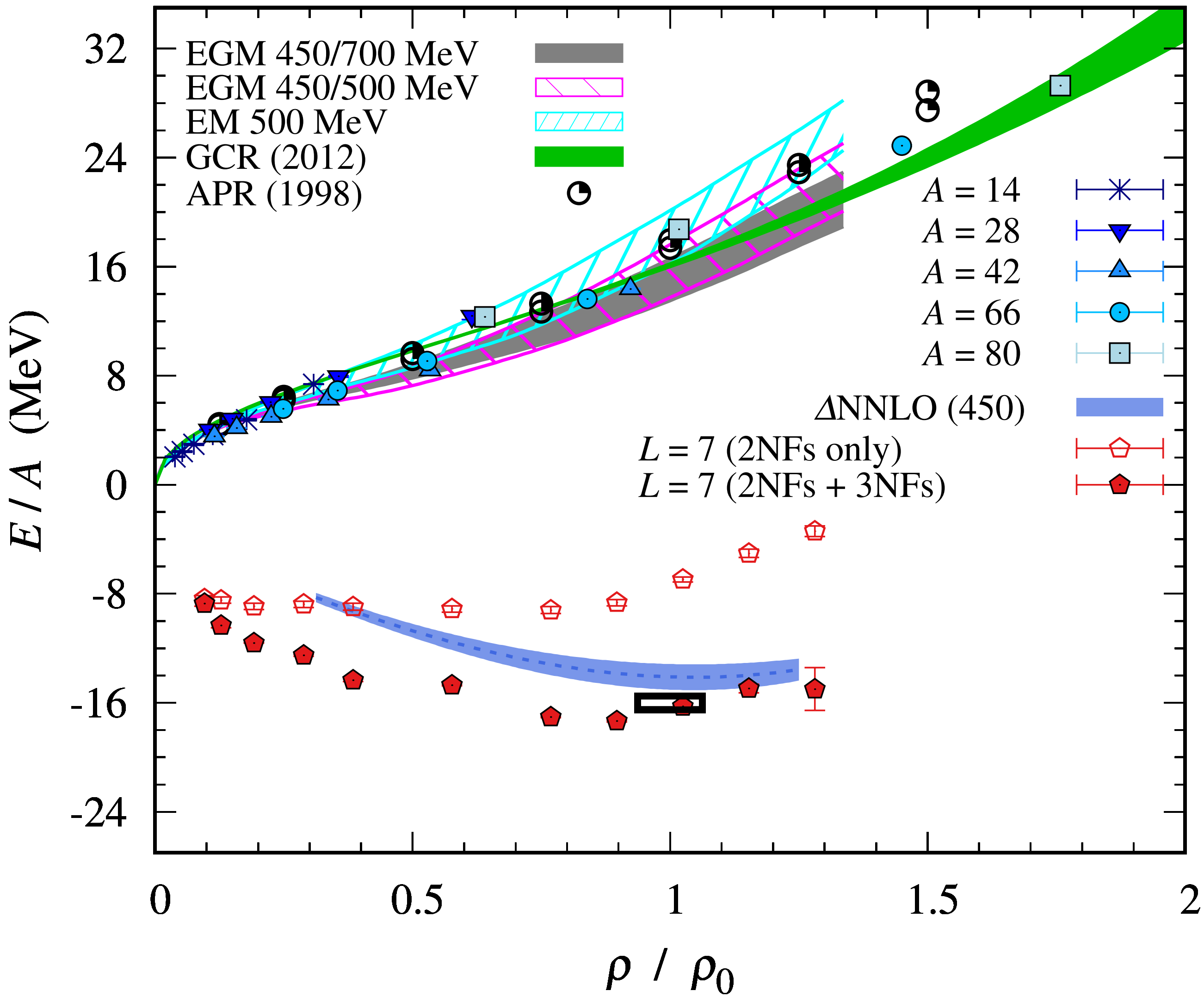
$E_{A,n} = E'_{A,n}$ (MeV)	$\langle \psi_{B,n} \vert H_A \vert \psi_{B,n} \rangle$ (MeV)	$\langle \psi_{B,n} \vert H'_A \vert \psi_{B,n} \rangle$ (MeV)
-1.2186	3.0088	-1.1597
0.2196	0.3289	0.2212
0.8523	1.1275	0.8577
1.8610	2.2528	1.8719
3.2279	3.6991	3.2477
4.9454	5.4786	4.9798
7.0104	7.5996	7.0680
9.4208	10.0674	9.5137
12.1721	12.8799	12.3163
15.2669	16.0458	15.4840

Chiral lattice results at N3LO using wave function matching



Work in progress: Elhatisari, Bovermann, et al.


Experiment
$\tau = 0.10\ \mathrm{MeV}^{-1}$
$\tau = 0.20\ \mathrm{MeV}^{-1}$
$\tau = 0.30\ \mathrm{MeV}^{-1}$
^{40}Ca
Charge Density (e/fm^3) at Q^4
Radius (fm)
0.15
0.12
0.09
0.06
0.03
0
1
2
3
4
5
6
7



EGM 450/700 MeV
EGM 450/500 MeV
EM 500 MeV
GCR (2012)
APR (1998)
A = 14
A = 28
A = 42
A = 66
A = 80
ΔNNLO (450)
L = 7 (2NFs only)
L = 7 (2NFs + 3NFs)
E / A (MeV)
ρ / ρ0
32
24
16
8
0
-8
-16
-24
0
0.5
1
1.5
2

Summary

We started with an introduction to lattice effective field theory. We then discussed the unitarity limit and properties of two-component Fermi gases at unitarity.

We then moved on to fermionic systems with more than two components and asked the question whether nuclear physics is near the unitarity limit. We found that symmetric nuclear matter is near a quantum phase transition, suggesting that the unitarity limit for nucleons likely produces a Bose gas of alpha particles rather than a nuclear liquid.

However, we found numerical evidence and large-N_c arguments that nuclear physics is close to Wigner's SU(4) symmetric limit with alpha-alpha scattering near the unitarity limit. Alpha cluster substructures can be seen in certain nuclei.

We concluded with a discussion of a new method called wave function matching. Using unitary transformations, we construct a high-fidelity Hamiltonian that can be reached by perturbation theory starting from a Hamiltonian that can be computed nonperturbatively.