Resonances in Few-Particle Systems

Winfried Leidemann

Department of Physics
University of Trento

UNIVERSITÀ DEGLI STUDI DI TRENTO

INFN

Outline

- Resonances in 2-body scattering and in electromagnetic responses of 3- and 4-particle systems
- Integral Transform Approach
- 0^+-resonance in excitation spectrum of ^{4}He in (e,e')
- $(1/2)^+$ and $(5/2)^+$ resonances in excitation spectrum of ^{9}Be (photodisintegration to $\alpha\alpha$n channel)
- Halo-EFT ($\alpha\alpha$ 0^+ and αn $^2P_{3/2}$ resonances)

Integral Transform Approach

Aim: Calculation of Reactions

involving the many-body continuum

- Integral transform methods:

  calculation of continuum wave function can be avoided

- Problem: The necessary inversion of the integral transform is a

  so-called ill-posed problem

KERNEL

$$\Phi(\sigma) = \int d\omega \; K(\omega,\sigma)\; R(\omega)$$

Transform

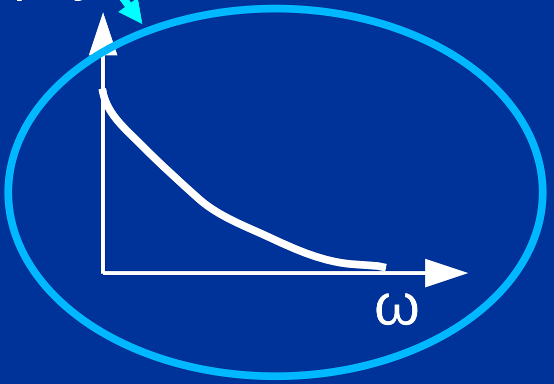
Response Function

One is able to calculate $\Phi(\sigma)$ but wants $R(\omega)$, which is the quantity of direct physical meaning.

Two examples in the literature:

Exponential Kernel: $K(\omega,\sigma) = e^{-\omega\sigma}$ σ real

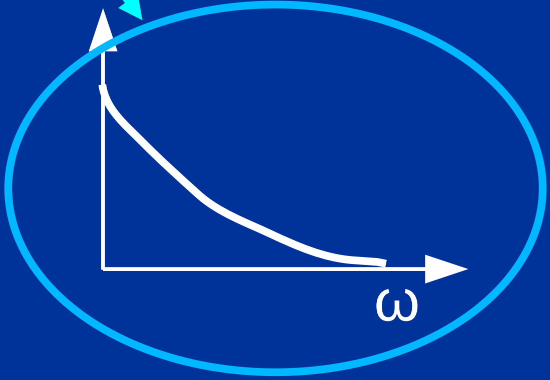
- used in condensed matter physics, nuclear physics, lattice QCD,...



- $\Phi(\sigma)$ calculated by **GFMC**

Exponential Kernel: $K(\omega,\sigma) = e^{-\omega\sigma}$ σ real

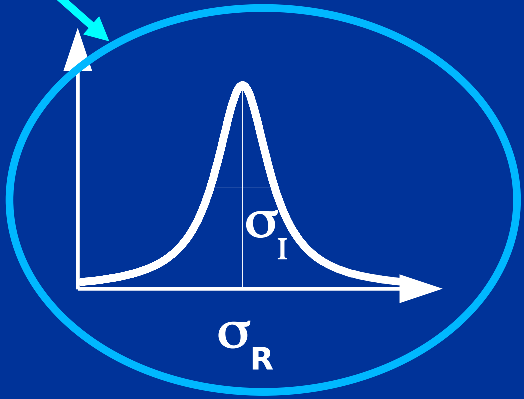
- used in condensed matter physics, nuclear physics, lattice QCD,...



- $\Phi(\sigma)$ calculated by **GFMC**

Lorentzian Kernel: $K(\omega,\sigma) = [(\omega - \sigma)\,(\omega - \sigma)^*]^{-1}$

complex $\sigma = \sigma_R + i\sigma_I$



- used in nuclear physics

- $\Phi(\sigma)$ calculated via **matrix diagonalization on localized** basis functions

Lorentz integral transform $L(\sigma_R,\sigma_I)$ for response function

$$R(\omega) = \int df \ \ |\langle f|O|0\rangle|^2 \ \delta(\omega - E_f + E_0)$$

$$L(\sigma_R,\sigma_I) = \langle \Psi|\Psi\rangle$$

$$(H - E_0 - \sigma_R + i\sigma_I)\ |\Psi\rangle \ = O\ |0\rangle$$

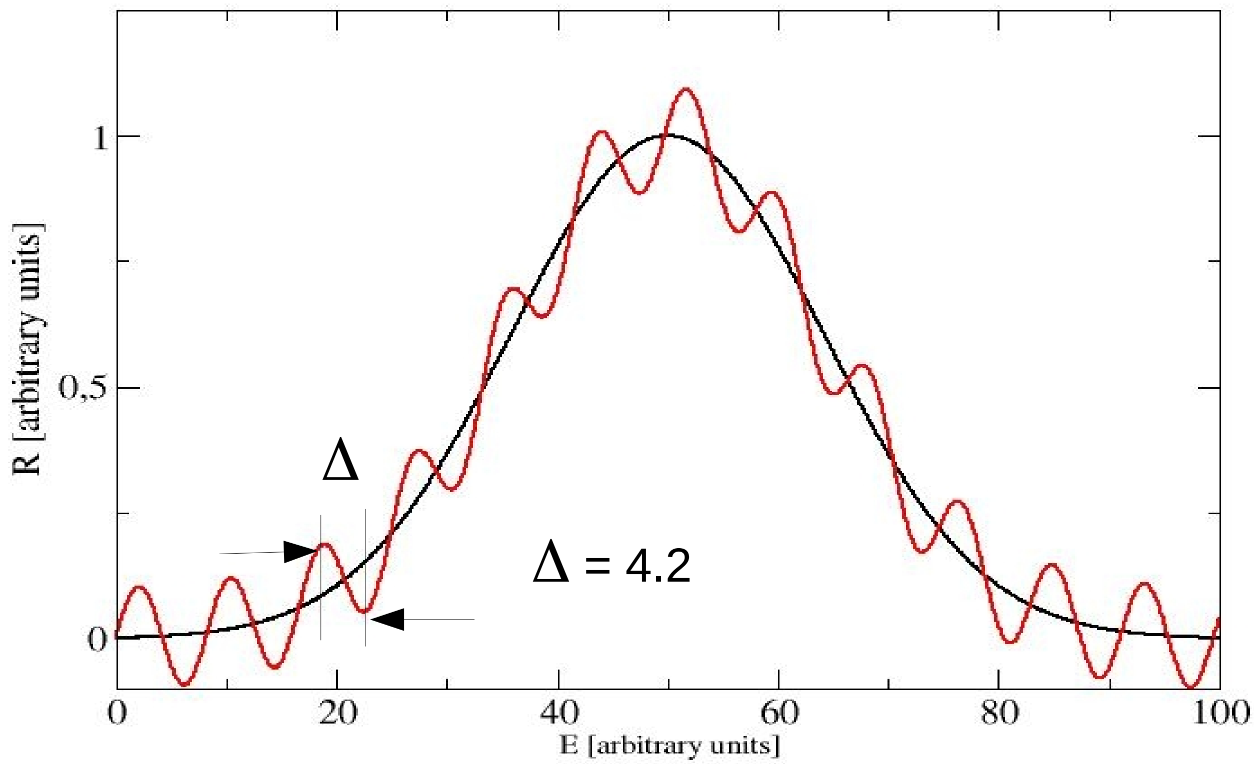
Solution is unique and has bound-state like asymptotic behaviour, one can apply bound-state methods for solution

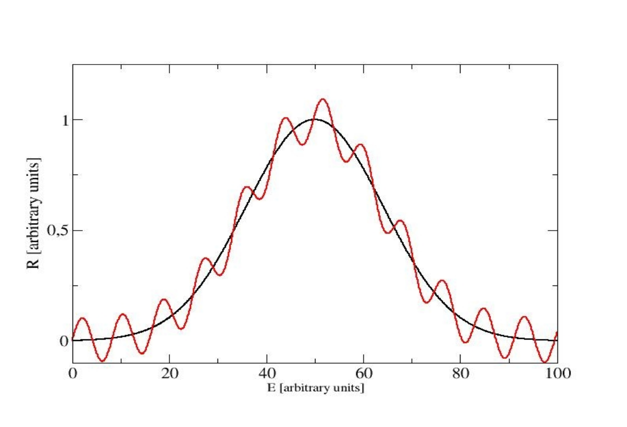
ILL-POSED

What does it mean?

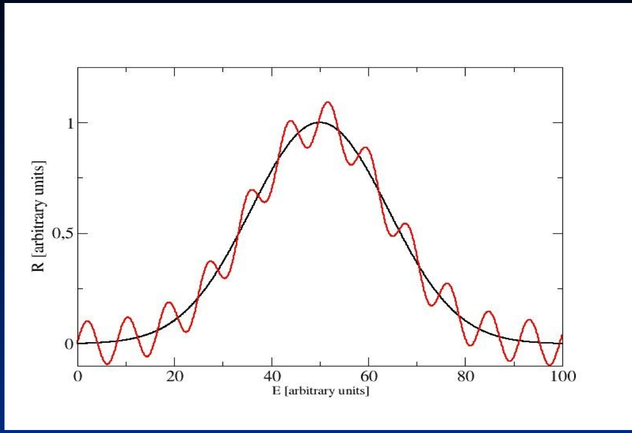
Let us check an example

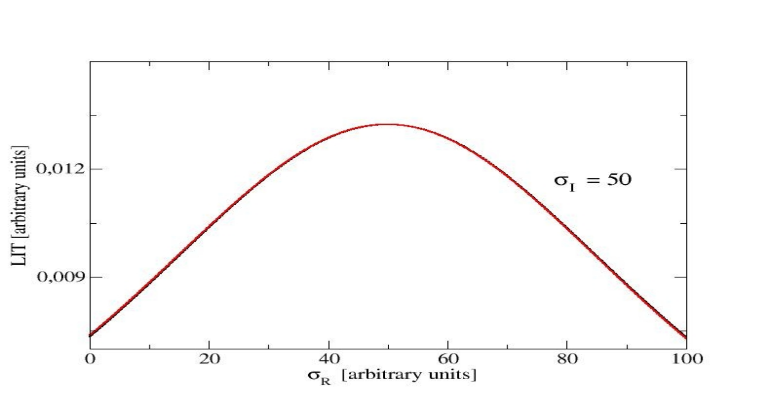
Example: black and red responses

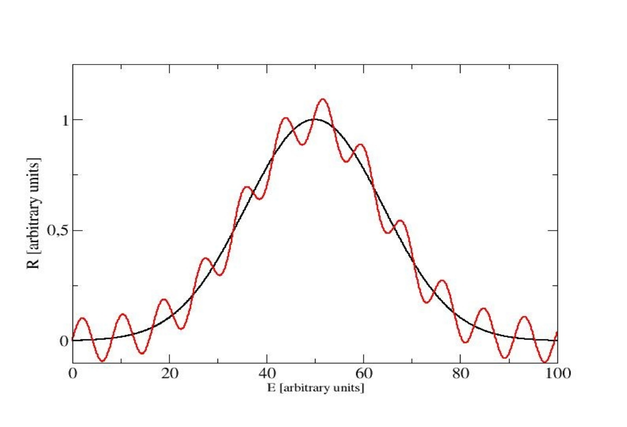


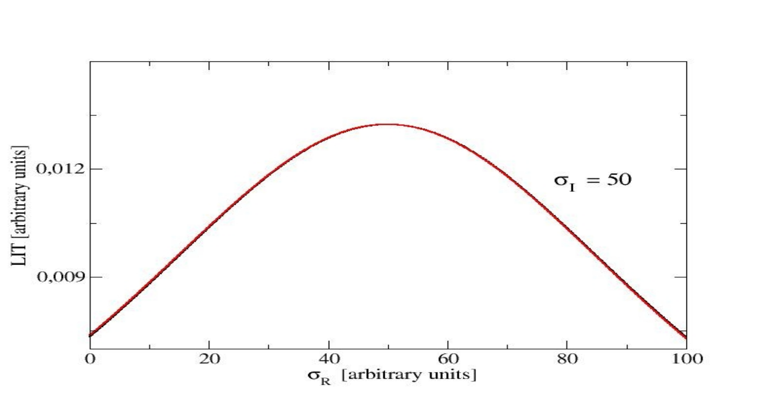


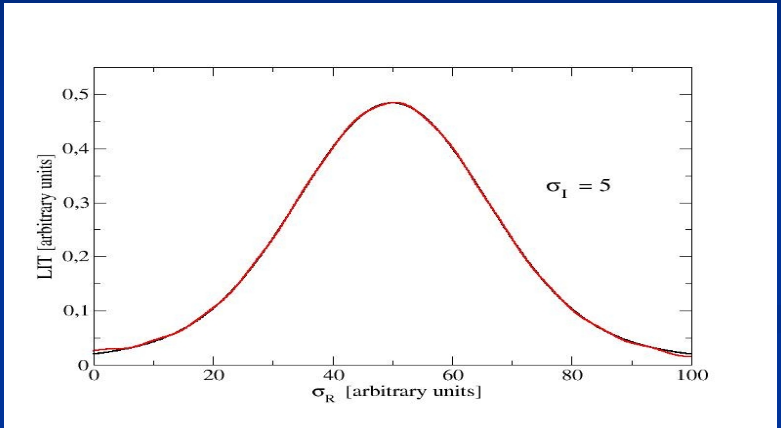
Let us check corresponding LITs with various width parameters $\Gamma = 2\sigma_l$

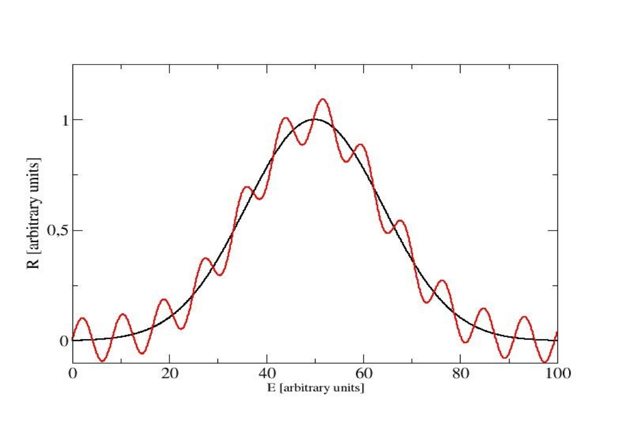

R [arbitrary units]
1
0,5
0
0
20
40
60
80
100
E [arbitrary units]

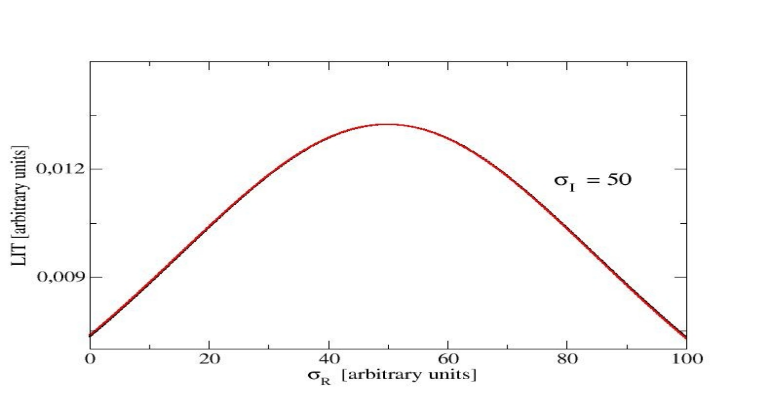

LIT [arbitrary units]
0,012
0,009
$\sigma_I = 50$
0
20
40
60
80
100
σ_R [arbitrary units]

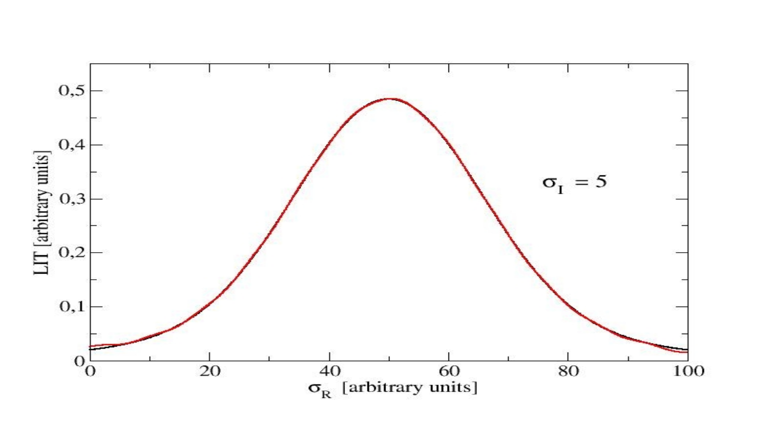

R [arbitrary units]
E [arbitrary units]

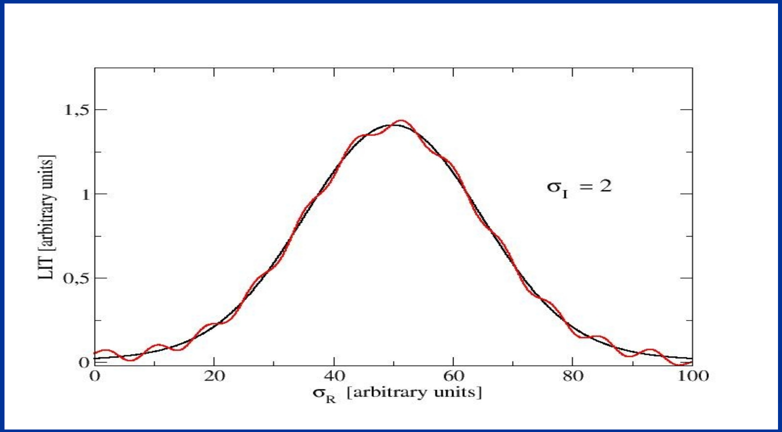

LIT [arbitrary units]
$\sigma_I = 50$
σ_R [arbitrary units]

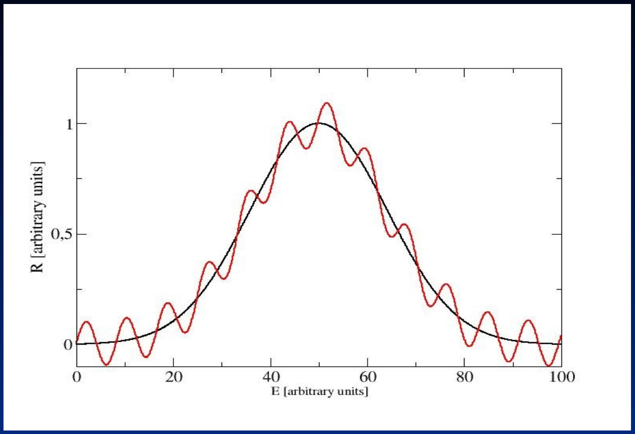

LIT [arbitrary units]
$\sigma_I = 5$
σ_R [arbitrary units]

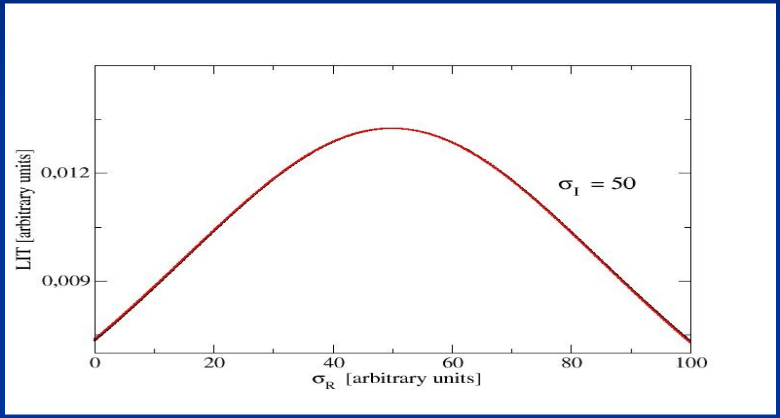

R [arbitrary units]
E [arbitrary units]

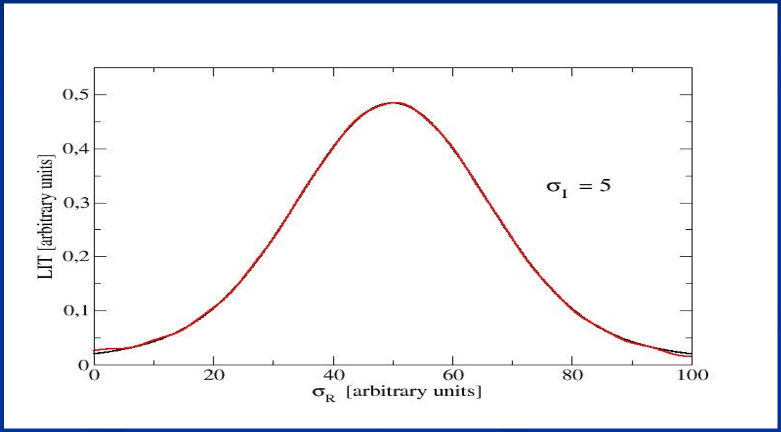

LIT [arbitrary units]
$\sigma_I = 50$
σ_R [arbitrary units]

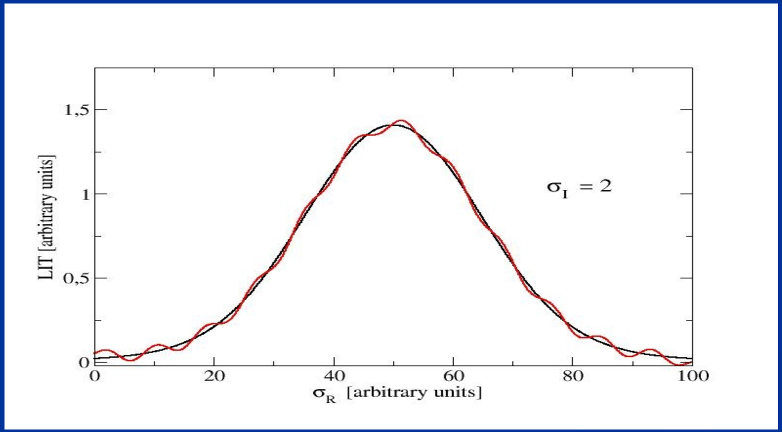

LIT [arbitrary units]
$\sigma_I = 5$
σ_R [arbitrary units]

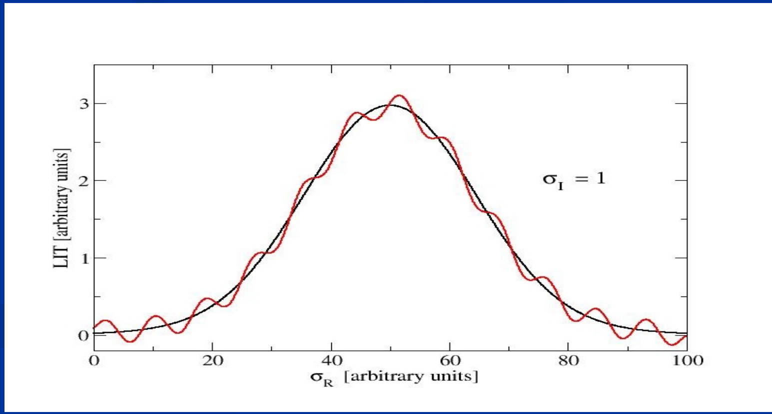

LIT [arbitrary units]
$\sigma_I = 2$
σ_R [arbitrary units]


R [arbitrary units]
E [arbitrary units]
0
0,5
1
0
20
40
60
80
100


LIT [arbitrary units]
σ_R [arbitrary units]
σ_I = 50
0,009
0,012
0
20
40
60
80
100


LIT [arbitrary units]
σ_R [arbitrary units]
σ_I = 5
0
0,1
0,2
0,3
0,4
0,5
0
20
40
60
80
100


LIT [arbitrary units]
σ_R [arbitrary units]
σ_I = 2
0
0,5
1
1,5
0
20
40
60
80
100


LIT [arbitrary units]
σ_R [arbitrary units]
σ_I = 1
0
1
2
3
0
20
40
60
80
100

Conclusion:

LIT method is a method with a controlled resolution

Consequence: discard inversions with structures having a width smaller than σ_I

Inversion of the LIT

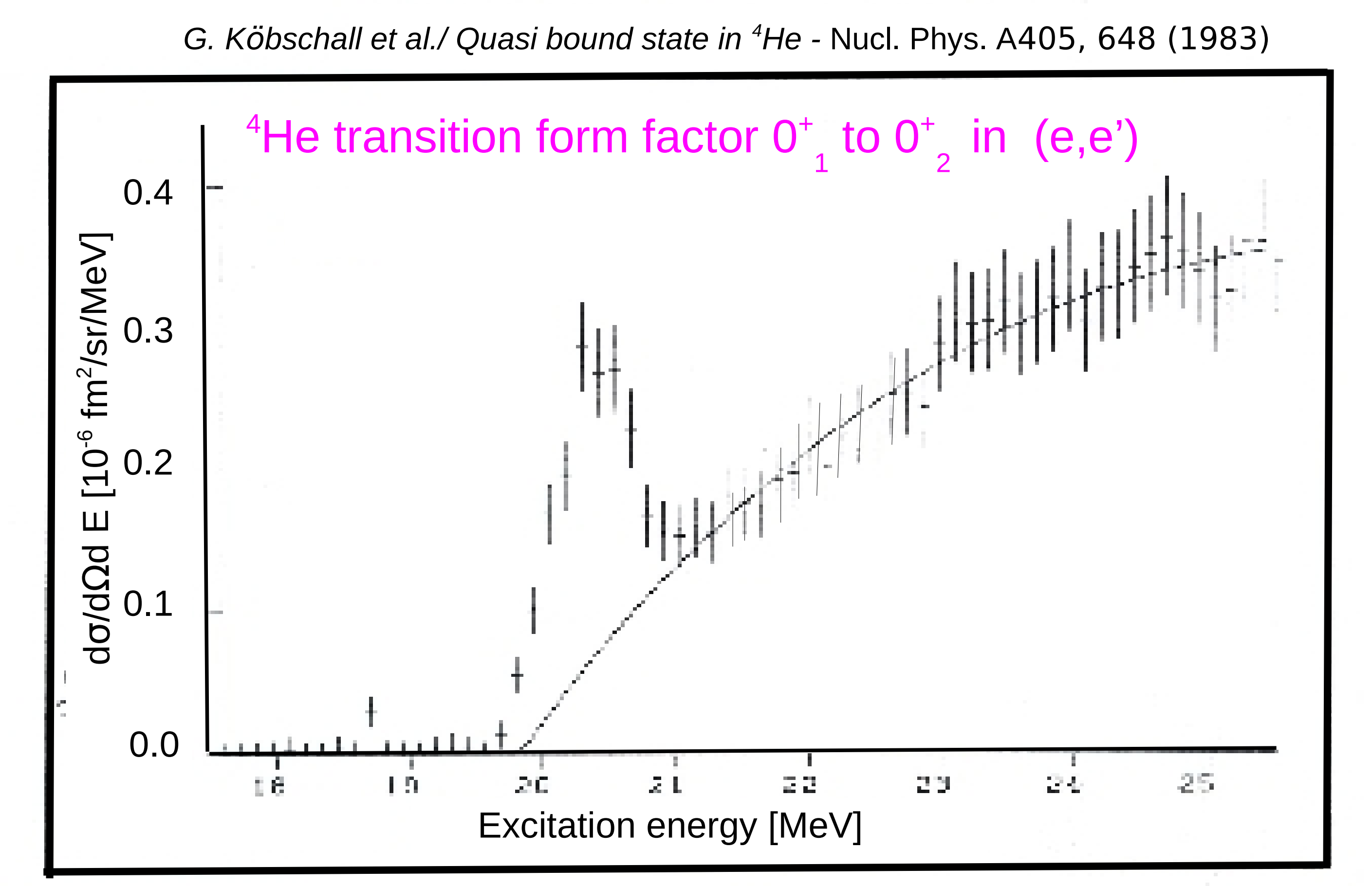
- LIT is calculated for a fixed σ_I in many σ_R points

Express the searched response function formally on a basis set with M basis basis functions $f_m(E)$ and open coefficients c_m with correct threshold behaviour for the $f_m(E)$ (e.g., $f_m = f_{thr}(E) \exp(-\alpha E/m)$). If specific structures, like narrow resonances, are present allow for basis functions $f_m(E)$ with such a structure, e.g. Lorentzians with variable position and width

- Make a LIT of the basis functions and determine coefficents c_m by a fit to the calculated LIT

- Increase M up to the point that a sufficient convergence is obtained (structures with too small widths or uncontrolled oscillations should not be present)

0^+ Resonance in the ^{4}He compound system



Resonance at E_R = -8.2 MeV, i.e. above the ^{3}H-p threshold. Strong evidence in electron scattering off ^{4}He, Γ = 270±50 keV

p-triton scattering length

Experimental data rather old (fit J. Carbonell)

$(a_0 + a_1)/2 \sim -17$ fm with $a_1 \sim 5.5$ fm (Lazauskas, EPJ Web of Conf. 3, 04006 (2010))

$\Rightarrow$ $a_0 \sim -40$ fm (towards unitarity ?) (also interesting: Viviani et al., arXiv:2003.14059)

p-triton scattering length

Experimental data rather old (fit J. Carbonell)

$(a_0 + a_1)/2 \sim -17$ fm with $a_1 \sim 5.5$ fm (Lazauskas, EPJ Web of Conf. 3, 04006 (2010))

$\Rightarrow a_0 \sim -40$ fm (towards unitarity ?)

Theoretical calculations of transition form factor:

p-triton scattering length

Experimental data rather old (fit J. Carbonell)

$(a_0 + a_1)/2 \sim -17$ fm with $a_1 \sim 5.5$ fm (Lazauskas, EPJ Web of Conf. 3, 04006 (2010))

$\Rightarrow a_0 \sim -40$ fm (towards unitarity ?)

Theoretical calculations of transition form factor:

- Hiyama et al., 2004 (resonance treated as bound state): nice agreement with exp. data with potential model AV8' (NN) + hypercentral 3NF

p-triton scattering length

Experimental data rather old (fit J. Carbonell)

$(a_0 + a_1)/2 \sim -17$ fm with $a_1 \sim 5.5$ fm (Lazauskas, EPJ Web of Conf. 3, 04006 (2010))

$\Rightarrow a_0 \sim -40$ fm (towards unitarity ?)

Theoretical calculations of transition form factor:

- Hiyama et al., 2004 (resonance treated as bound state): nice agreement with exp. data with potential model AV8’ (NN) + hypercentral 3NF
- Bacca et al. 2013 (LIT calculation): rather large disagreement with exp. data with two realistic nuclear forces (AV18+UIX, chiral force: N3LO (NN)+N2LO (3NF)

p-triton scattering length

Experimental data rather old (fit J. Carbonell)

$(a_0 + a_1)/2 \sim -17$ fm with $a_1 \sim 5.5$ fm (Lazauskas, EPJ Web of Conf. 3, 04006 (2010))

$\Rightarrow a_0 \sim -40$ fm (towards unitarity ?)

Theoretical calculations of transition form factor:

- Hiyama et al., 2004 (resonance treated as bound state): nice agreement with exp. data with potential model AV8' (NN) + hypercentral 3NF
- Bacca et al. 2013 (LIT calculation): rather large disagreement with exp. data with two realistic nuclear forces (AV18+UIX, chiral force: N3LO (NN)+N2LO (3NF)
- New Mainz measurement of transition form factor (S. Kegel et al., arXiv:2112.10582)

p-triton scattering length

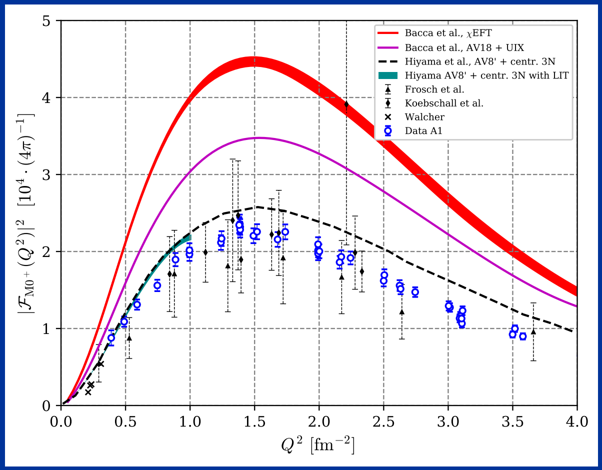
Experimental data rather old (fit J. Carbonell)

$(a_0 + a_1)/2 \sim -17$ fm with $a_1 \sim 5.5$ fm (Lazauskas, EPJ Web of Conf. 3, 04006 (2010))

$\Rightarrow$ $a_0 \sim$ -40 fm (towards unitarity ?)

Theoretical calculations of transition form factor:

- Hiyama et al., 2004 (resonance treated as bound state): nice agreement with exp. data with potential model AV8’ (NN) + hypercentral 3NF
- Bacca et al. 2013 (LIT calculation): rather large disagreement with exp. data with two realistic nuclear forces (AV18+UIX, chiral force: N3LO (NN)+N2LO (3NF)
- New Mainz measurement of transition form factor (S. Kegel et al., arXiv:2112.10582)



p-triton scattering length

Experimental data rather old (fit J. Carbonell)

$(a_0 + a_1)/2 \sim -17$ fm with $a_1 \sim 5.5$ fm (Lazauskas, EPJ Web of Conf. 3, 04006 (2010))

$\Rightarrow a_0 \sim -40$ fm (towards unitarity ?)

Theoretical calculations of transition form factor:

- Hiyama et al., 2004 (resonance treated as bound state): nice agreement with exp. data with potential model AV8' (NN) + hypercentral 3NF
- Bacca et al. 2013 (LIT calculation): rather large disagreement with exp. data with two realistic nuclear forces (AV18+UIX, chiral force: N3LO (NN)+N2LO (3NF)
- New Mainz measurement of transition form factor (S. Kegel et al., arXiv:2112.10582)
- Interesting new feature: low-q expansion of transition form factor $F^{0+}(q^2)$

p-triton scattering length

Experimental data rather old (fit J. Carbonell)

$(a_0 + a_1)/2 \sim -17$ fm with $a_1 \sim 5.5$ fm (Lazauskas, EPJ Web of Conf. 3, 04006 (2010))

$\Rightarrow a_0 \sim -40$ fm (towards unitarity ?)

Theoretical calculations of transition form factor:

- Hiyama et al., 2004 (resonance treated as bound state): nice agreement with exp. data with potential model AV8' (NN) + hypercentral 3NF
- Bacca et al. 2013 (LIT calculation): rather large disagreement with exp. data with two realistic nuclear forces (AV18+UIX, chiral force: N3LO (NN)+N2LO (3NF)
- New Mainz measurement of transition form factor (S. Kegel et al., arXiv:2112.10582)
- $Z|F^{0+}(q^2)|/q^2 = \langle r^2 \rangle_{tr}/6\,\left(1 - q^2 R^2_{tr}/20 + O(q^4)\right)$

p-triton scattering length

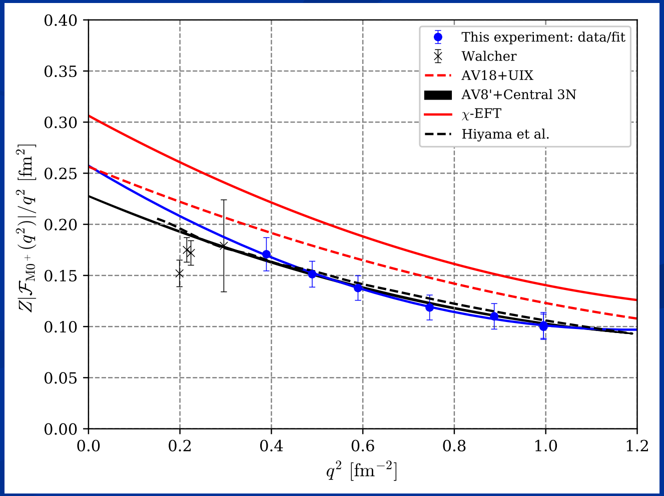
Experimental data rather old (fit J. Carbonell)

$(a_0 + a_1)/2 \sim -17$ fm with $a_1 \sim 5.5$ fm (Lazauskas, EPJ Web of Conf. 3, 04006 (2010))

$\Rightarrow a_0 \sim -40$ fm (towards unitarity ?)

Theoretical calculations of transition form factor:

- Hiyama et al., 2004 (resonance treated as bound state): nice agreement with exp. data with potential model AV8' (NN) + hypercentral 3NF
- Bacca et al. 2013 (LIT calculation): rather large disagreement with exp. data with two realistic nuclear forces (AV18+UIX, chiral force: N3LO (NN)+N2LO (3NF)
- New Mainz measurement of transition form factor (S. Kegel et al., arXiv:2112.10582)
- $Z|F^{0+}(q^2)|/q^2 = \langle r^2 \rangle_{tr} /6 \left(1 - q^2 R^2_{tr}/20 + O(q^4)\right)$



p-triton scattering length

Experimental data rather old (fit J. Carbonell)

$(a_0 + a_1)/2 \sim -17$ fm with $a_1 \sim 5.5$ fm (Lazauskas, EPJ Web of Conf. 3, 04006 (2010))

$\Rightarrow a_0 \sim -40$ fm (towards unitarity ?)

Theoretical calculations of transition form factor:

- Hiyama et al., 2004 (resonance treated as bound state): nice agreement with exp. data with potential model AV8' (NN) + hypercentral 3NF
- Bacca et al. 2013 (LIT calculation): rather large disagreement with exp. data with two realistic nuclear forces (AV18+UIX, chiral force: N3LO (NN)+N2LO (3NF)
- New Mainz measurement of transition form factor (S. Kegel et al., arXiv:2112.10582)
- $Z|F^{0+}(q^2)|/q^2 = \langle r^2\rangle_{tr}/6\,(1 - q^2 R^2_{tr}/20 + O(q^4))$

	$\langle r^2\rangle_{tr}$	R^2_{tr}
Experiment	1.53 ± 0.05	4.56 ± 0.15
Theory (AV8'+ hypcntrl 3NF)	1.36 ± 0.01	4.01 ± 0.05
Theory (AV18+UIX)	1.54 ± 0.01	3.77 ± 0.08
Theory (χEFT)	1.83 ± 0.01	3.97 ± 0.05

Additional observation concerning resonance width Γ :

Additional observation concerning resonance width Γ :

Previous calculations were unable to determine a width
Hiyama et al.: bound-state calculation
Bacca et al.: Hypersperical harmonic expansion of LIT states states with a scarse density of LIT states in resonance region (resonance described by a single LIT state)

Additional observation concerning resonance width Γ :

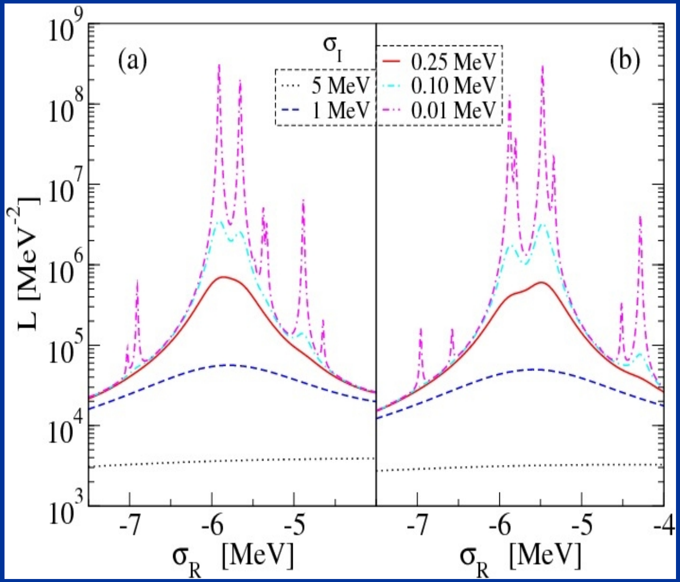
Previous calculations were unable to determine a width
Hiyama et al.: bound-state calculation
Bacca et al.: Hyperspherical harmonic expansion of LIT states states with a scarse density of LIT states in resonance region (resonance described by a single LIT state)

LIT state density can be increased choosing a different basis: 3-body HH + additional expansion of $|\Psi>$ for last Jacobi coordinate as shown in calculation with MT NN potential (central) (WL, PRC 91, 054001 (2015))

Additional observation concerning resonance width Γ :

Previous calculations were unable to determine a width
Hiyama et al.: bound-state calculation
Bacca et al.: Hypersperical harmonic expansion of LIT states states with a scarse density of LIT states in resonance region (resonance described by a single LIT state)

LIT state density can be increased choosing a different basis: 3-body HH + additional expansion of $|\Psi>$ for last Jacobi coordinate as shown in calculation with MT NN potential (central)   (WL, PRC 91, 054001 (2015))



Inversion: Γ = 180(70) keV

New Mainz exp.: 288(39) keV

Halo or Cluster EFT

^{9}Be as $\alpha\alpha$n system

Interactions needed:

LO: s-wave for $\alpha\alpha$ (^{8}Be resonance)

a_0 = -1920 fm

p-wave resonance $2P_{3/2}$ for αn (^{5}He resonance)

a_1 = -62.951 fm^3

3BF $\alpha\alpha$n (hypercentral)

NLO: s-wave for αn

Our Aim: $^{9}\text{Be} + \gamma \longrightarrow \alpha + \alpha + n$

Potentials in momentum space

$$V(\mathbf{p},\mathbf{p}') = \sum_\ell V_\ell(p,p')\,(2\ell+1)\,P_\ell\cos(\Theta_{\mathbf{pp}'})$$

$$V_\ell(p,p') = g(p)\,g(p')\,p^\ell\,p'^\ell\,[\,\lambda_0 + \lambda_1\,(p^2 + p'^2)]$$

where **p** and **p'** are the relative momenta of the 2-body system

and g(p) is a cutoff: $g(p) = \exp(-p^4/\Lambda^4)$

Make similar expansion for t-matrix

$$t_\ell(p,p') = g(p)\,g(p')\,p^\ell\,p'^\ell\,[\,\tau_0 + \tau_1\,(p^2 + p'^2)]$$

Solve resulting Lippmann-Schwinger equation analytically

(because of Coulomb $\alpha\alpha$ more complicated: $T = T_C + T_{SC}$
where T_C is the T-matrix connected to the pure Coulomb interaction,
while T_{SC} is the one associated to the Coulomb-distorted short-range interaction)

Solve resulting Lippmann-Schwinger equation analytically

(because of Coulomb $\alpha\alpha$ more complicated: $T = T_C + T_{SC}$
where T_C is the T-matrix connected to the pure Coulomb interaction,
while T_{SC} is the one associated to the Coulomb-distorted short-range interaction)

Compare on-shell T-matrix to effective range expansion (here given without Coulomb)

$$k^{2\ell}/T_{on}(E) = -\mu/2\pi \;\; (-1/a_\ell + r^2_{\ell,e}\, k^2 - ik^{2\ell+1} + \ldots), \qquad E=k^2/2\mu,$$

$\Rightarrow$ Quadratic eqs. with two solutions for LECs λ_0 and λ_1 for any value of Λ

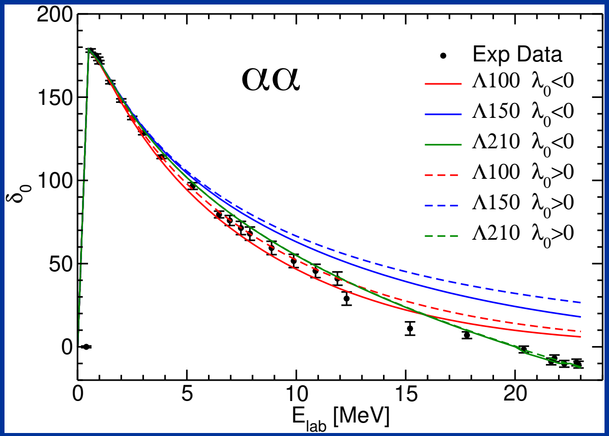
Solve resulting Lippmann-Schwinger equation analytically

(because of Coulomb $\alpha\alpha$ more complicated: $T = T_C + T_{SC}$
where T_C is the T-matrix connected to the pure Coulomb interaction,
while T_{SC} is the one associated to the Coulomb-distorted short-range interaction)

Compare on-shell T-matrix to effective range expansion (here given without Coulomb)

$$k^{2\ell}/T_{on}(E) = -\mu/2\pi \ (-1/a_\ell + r^2_{\ell,e}\, k^2 - ik^{2\ell+1} + \dots), \quad E=k^2/2\mu,$$

$\Rightarrow$ Quadratic eqs. with two solutions for LECs λ_0 and λ_1 for any value of Λ



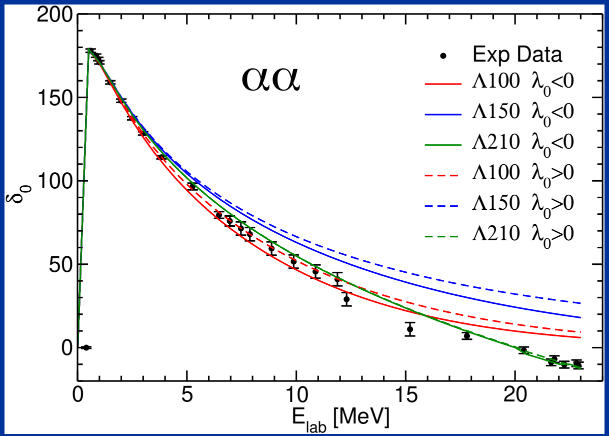
Solve resulting Lippmann-Schwinger equation analytically

(because of Coulomb $\alpha\alpha$ more complicated: $T = T_C + T_{SC}$
where T_C is the T-matrix connected to the pure Coulomb interaction,
while T_{SC} is the one associated to the Coulomb-distorted short-range interaction)

Compare on-shell T-matrix to effective range expansion (here given without Coulomb)

$$k^{2\ell}/T_{on}(E) = -\mu/2\pi \;\; (-1/a_\ell + r^2_{\ell,e}\, k^2 - ik^{2\ell+1} + \ldots), \qquad E=k^2/2\mu,$$

$\Rightarrow$ Quadratic eqs. with two solutions for LECs λ_0 and λ_1 for any value of Λ



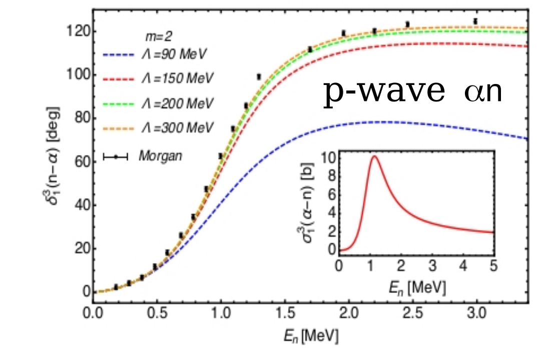


FIGURE 4.2: Phase shifts $\delta_1^3(E_n)$ ($l = 1$, $J = 3/2$) with experimental data from Morgan and Walter [29] and in the inset the cross section $\sigma_1^3(E_n)$ obtained with $\Lambda = 300$ MeV.

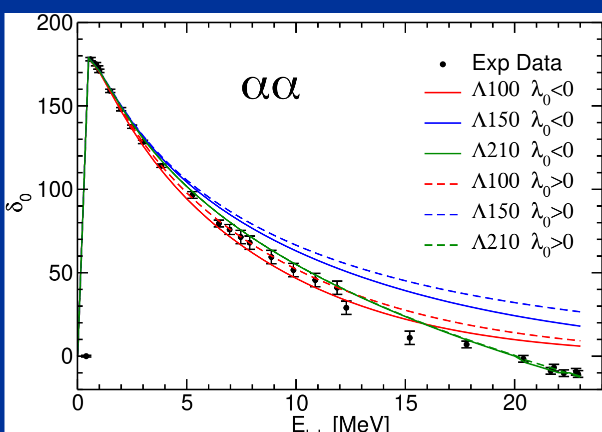
Solve resulting Lippmann-Schwinger equation analytically

(because of Coulomb $\alpha\alpha$ more complicated: $T = T_C + T_{SC}$
where T_C is the T-matrix connected to the pure Coulomb interaction,
while T_{SC} is the one associated to the Coulomb-distorted short-range interaction)

Compare on-shell T-matrix to effective range expansion (here given without Coulomb)

$$k^{2\ell}/T_{on}(E) = -\mu/2\pi \;\; (-1/a_\ell + r^2_{\ell,e}\, k^2 - ik^{2\ell+1} + \dots), \qquad E=k^2/2\mu,$$

$\Rightarrow$ Quadratic eqs. with two solutions for LECs λ_0 and λ_1 for any value of Λ



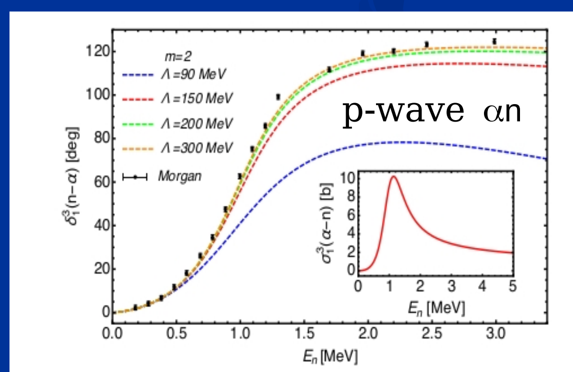
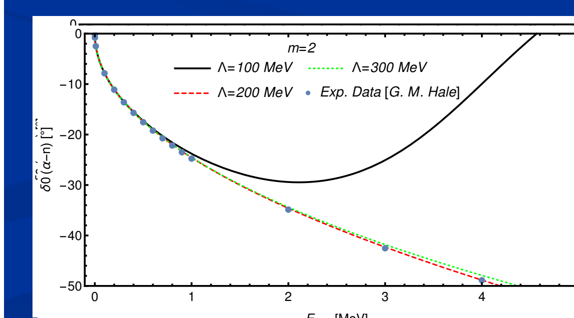
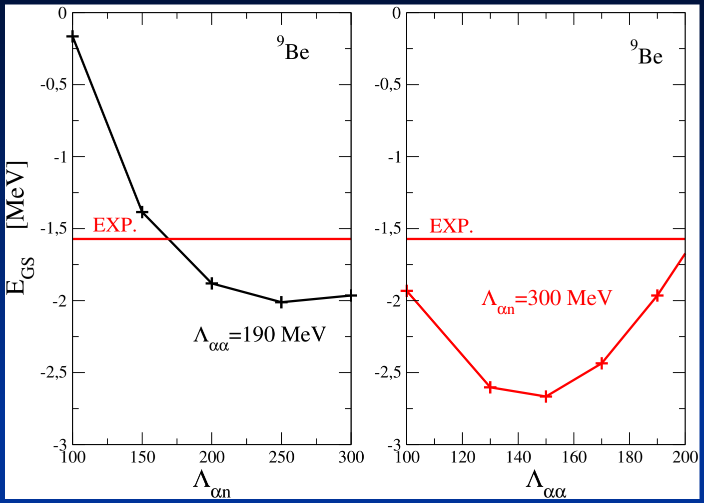


FIGURE 4.2: Phase shifts $\delta_1^3(E_n)$ ($l = 1$, $J = 3/2$) with experimental data from Morgan and Walter [29] and in the inset the cross section $\sigma_1^3(E_n)$ obtained with $\Lambda = 300$ MeV.

S-wave αn

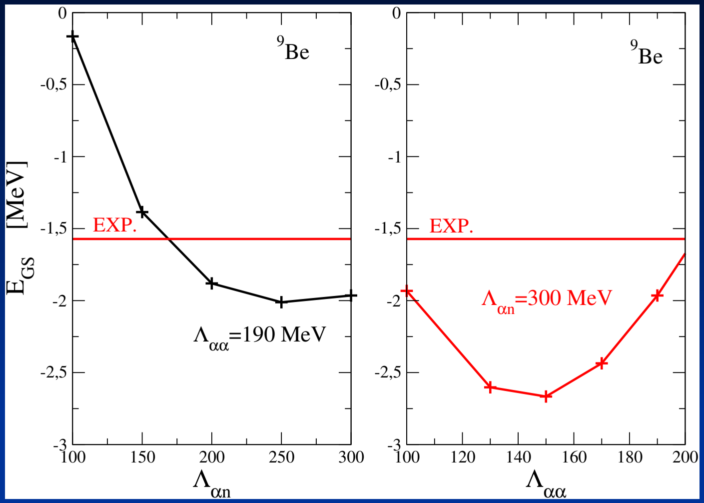


Cutoff dependence of ^{9}Be ground-state energy



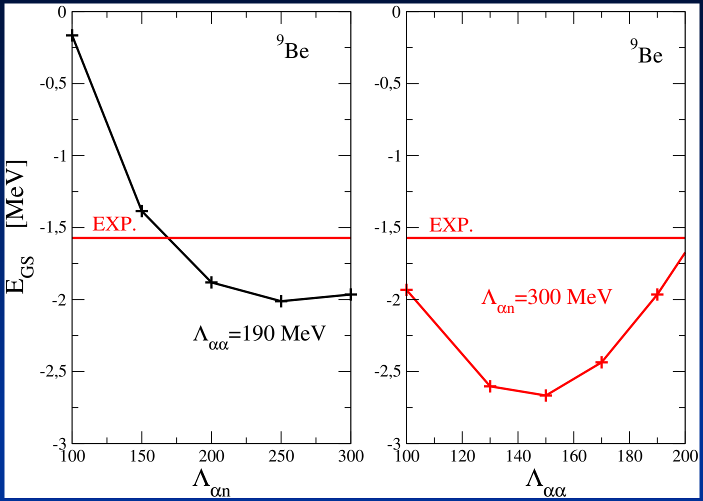
Wave function is calculated via expansion in hyperspherical harmonics (HH) in momentum space

Cutoff dependence of ^{9}Be ground-state energy



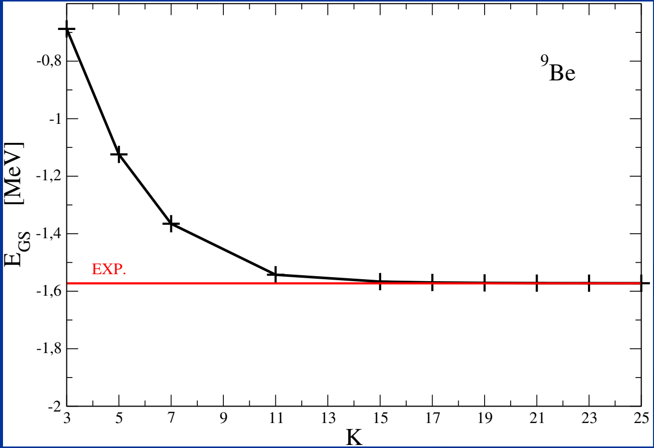
Include 3-body force: $V_3 = \lambda_3 \exp\left[-(p_{12}+p_{23}+p_{31})/\Lambda^2_3\right] \exp\left[-(p`^2_{12}+p`^2_{23}+p`^2_{31})/\Lambda^2_3\right]$

Cutoff dependence of ^{9}Be ground-state energy



Include 3-body force: $V_3 = \lambda_3 \exp[-(p_{12}+p_{23}+p_{31})/\Lambda^2_3] \exp[-(p^{`2}_{12}+p^{`2}_{23}+p^{`2}_{31})/\Lambda^2_3]$

HH convergence in function of grand-angular quantum number K



^{9}Be photodisintegration

Only E1 transitions are considered, since ^{9}Be has $J^{\pi}= (3/2)^{-}$

one has $(1/2)^{+}$, $(3/2)^{+}$ and $(5/2)^{+}$ final states

^{9}Be photodisintegration

Only E1 transitions are considered, since ^{9}Be has $J^{\pi} = (3/2)^{-}$
one has $(1/2)^{+}$, $(3/2)^{+}$ and $(5/2)^{+}$ final states

Current operator (LO) in limit of vanishing photon momentum proportional to

$$e\,(\mathbf{p}_{\alpha_1,\perp} + \mathbf{p}_{\alpha_2,\perp}) \,/\, m_{\alpha}$$

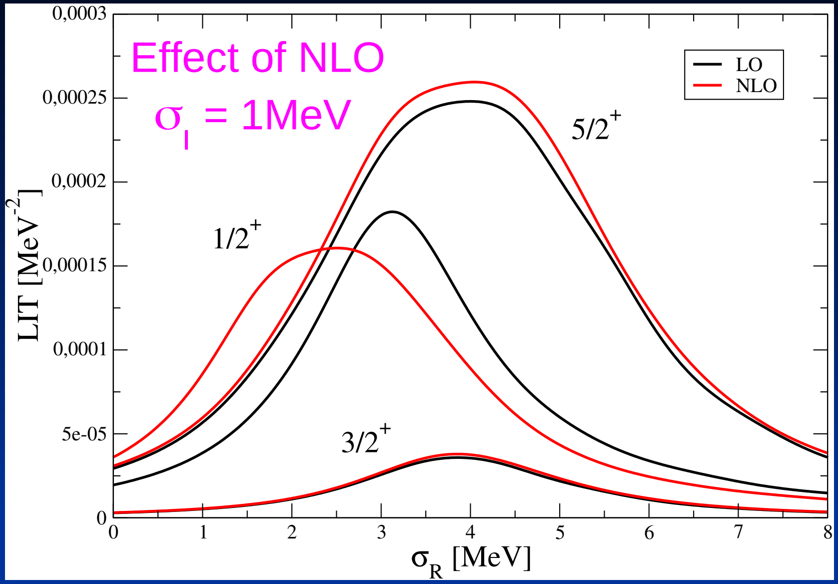
^{9}Be photodisintegration

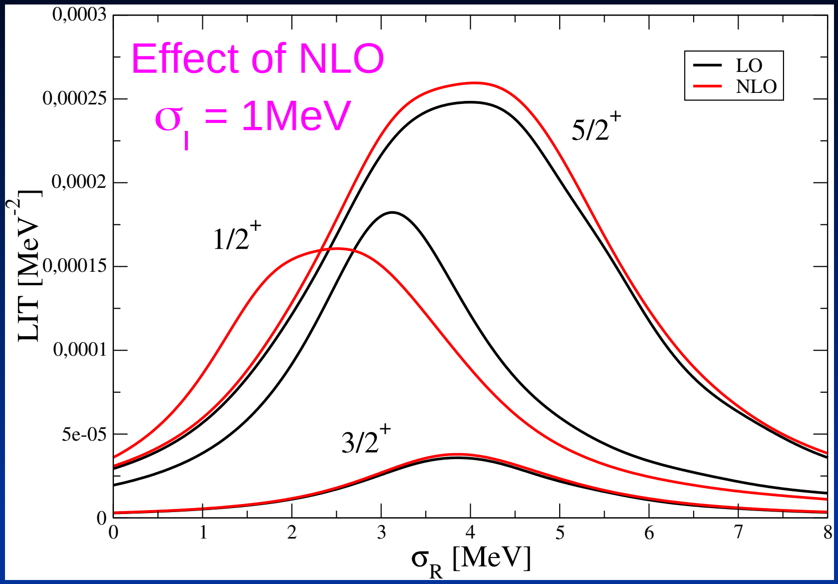
Only E1 transitions are considered, since ^{9}Be has $J^\pi = (3/2)^-$
one has $(1/2)^+$, $(3/2)^+$ and $(5/2)^+$ final states

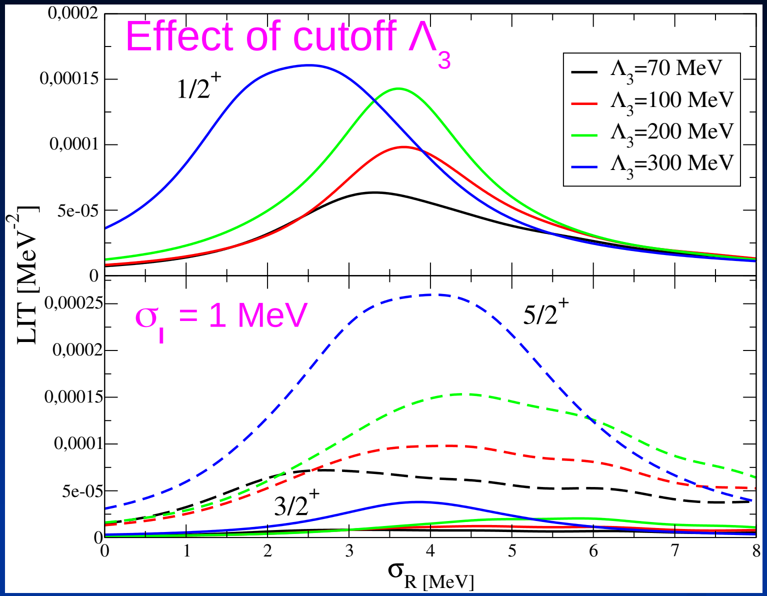
Current operator (LO) in limit of vanishing photon momentum proportional to

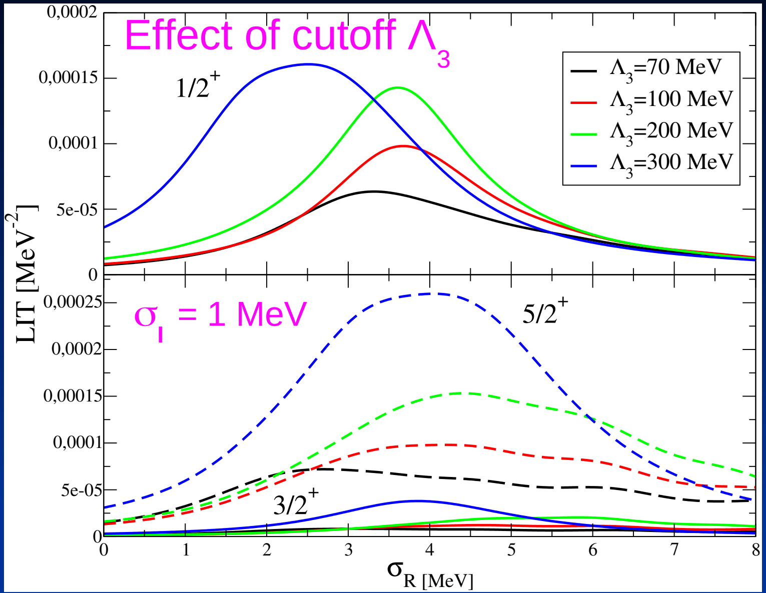
$$e\,(\mathbf{p}_{\alpha_1,\perp} + \mathbf{p}_{\alpha_2,\perp}) \,/\, m_\alpha$$

Leads to following LIT results

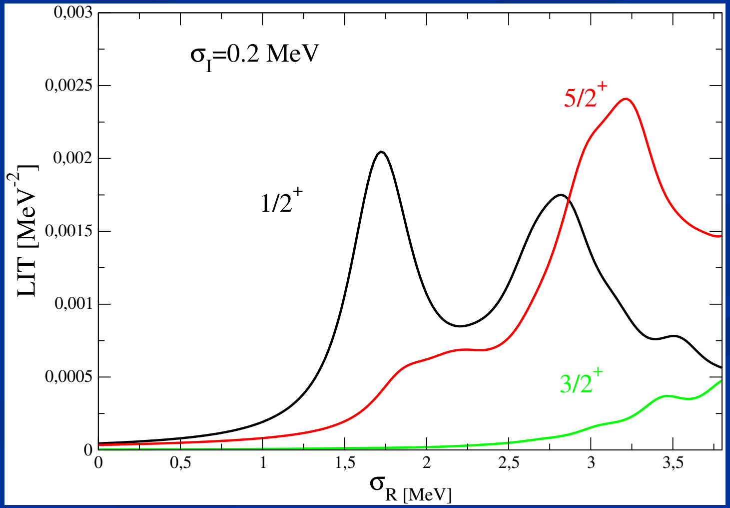

Effect of NLO
σI = 1MeV
LO
NLO
5/2+
1/2+
3/2+
LIT [MeV-2]
σR [MeV]
0,0003
0,00025
0,0002
0,00015
0,0001
5e-05
0
0
1
2
3
4
5
6
7
8

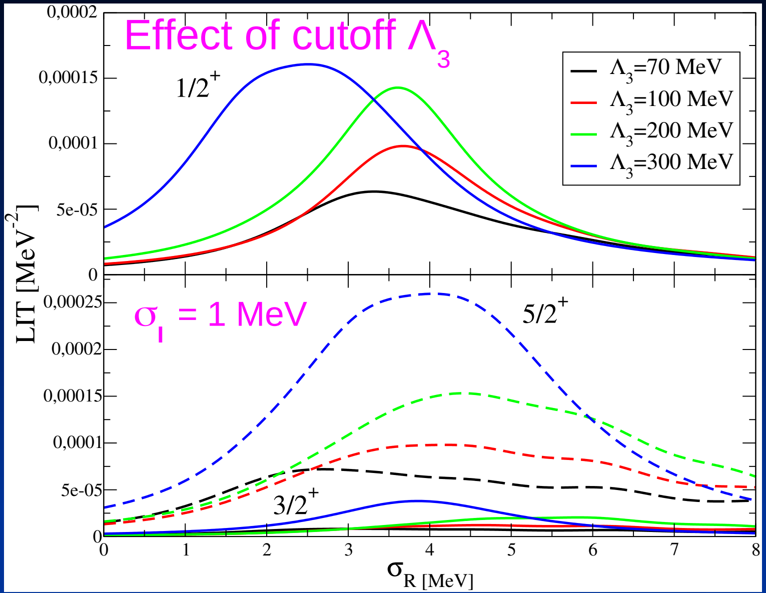

Effect of NLO
σI = 1MeV
LO
NLO
5/2+
1/2+
3/2+
LIT [MeV-2]
σR [MeV]
0,0003
0,00025
0,0002
0,00015
0,0001
5e-05
0
0
1
2
3
4
5
6
7
8


Effect of cutoff Λ3
Λ3=70 MeV
Λ3=100 MeV
Λ3=200 MeV
Λ3=300 MeV
1/2+
σI = 1 MeV
5/2+
3/2+
LIT [MeV-2]
σR [MeV]
0,0002
0,00015
0,0001
5e-05
0
0,00025
0,0002
0,00015
0,0001
5e-05
0
0
1
2
3
4
5
6
7
8

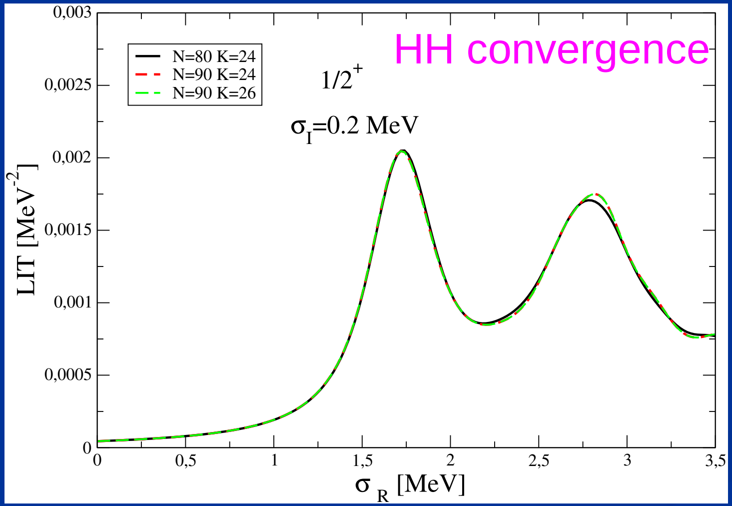


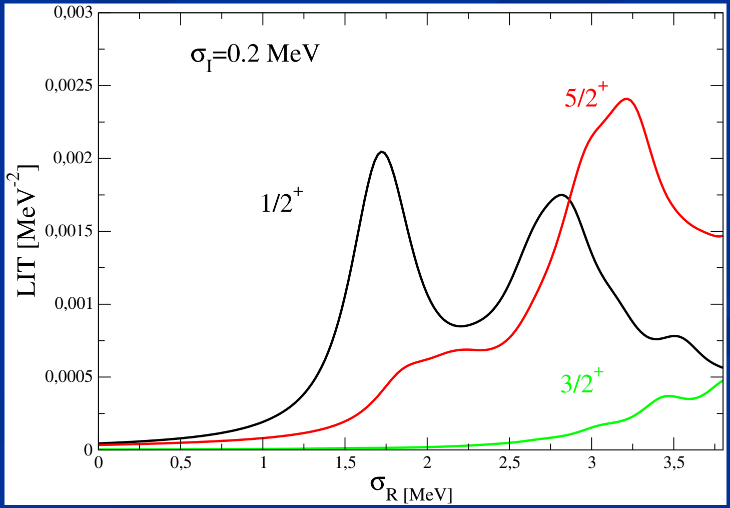
Increase resolution to σ_I = 0.2 MeV for Λ_3 = 300 MeV

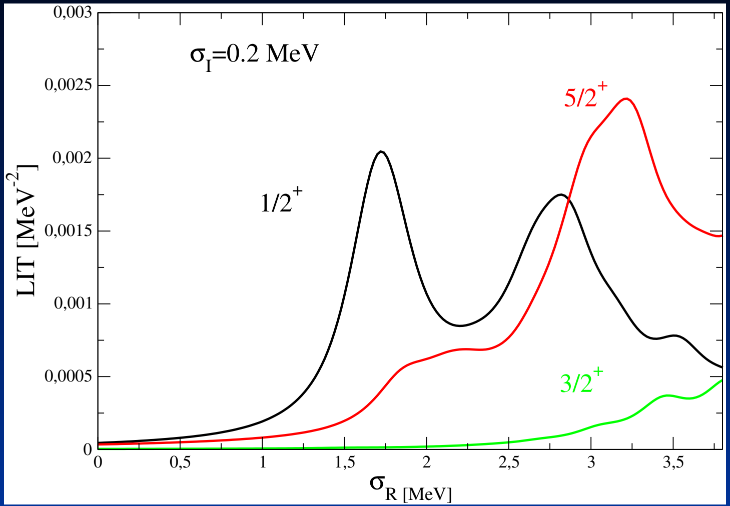




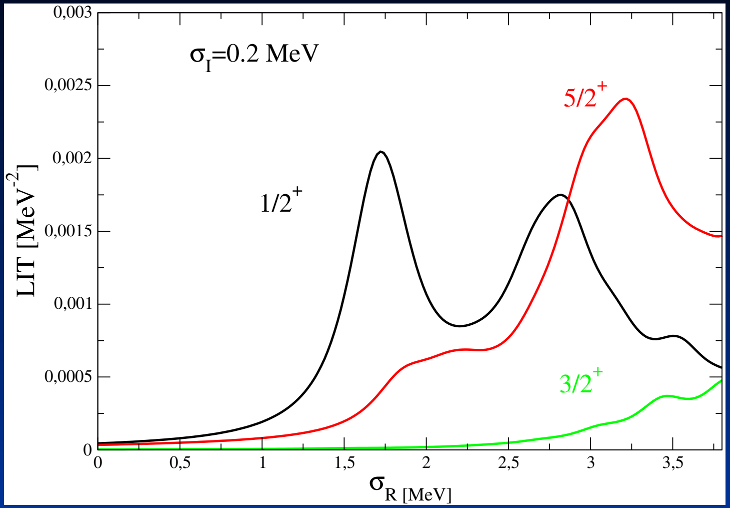
Increase resolution to σ_I = 0.2 MeV for Λ_3 = 300 MeV



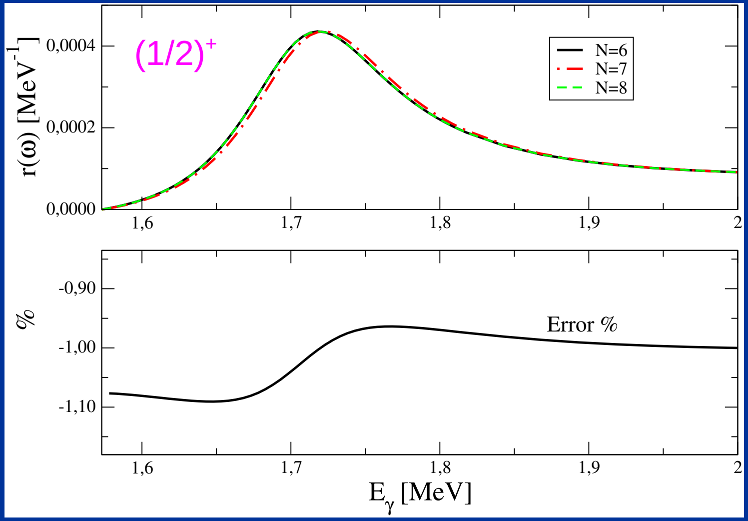


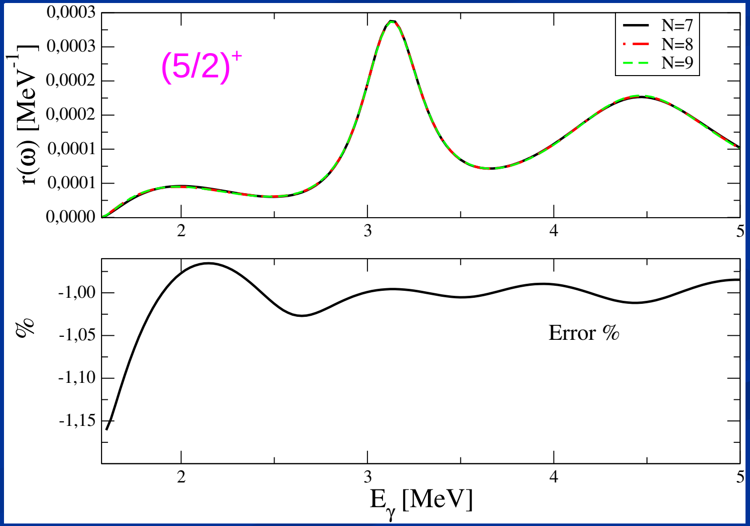


Inversions for dominant multipoles $(1/2)^+$ and $(5/2)^+$

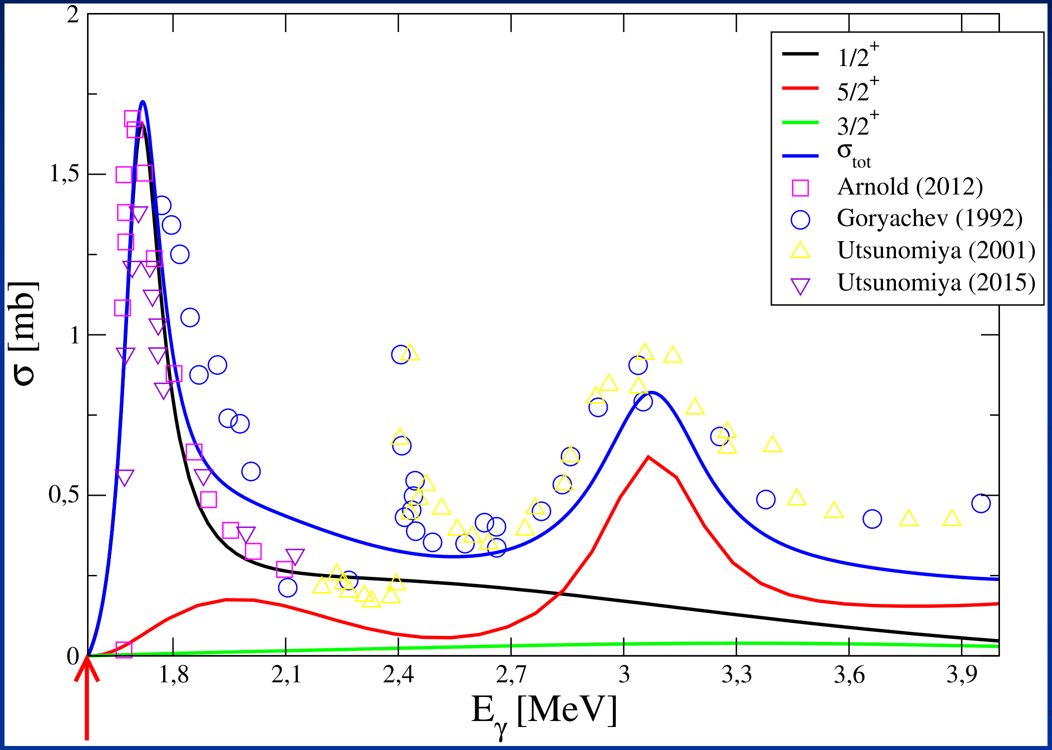


Inversions for dominant multipoles $(1/2)^+$ and $(5/2)^+$





Resulting ^{9}Be photoabsorption cross section



Conclusion concerning ^{9}Be photoabsorption

Fine-tuning of 3-body cutoff such that $(1/2)^+$ resonance position agrees with experiment data leads to

- correct $(1/2)^+$ resonance strength
- correct position of $(5/2)^+$ resonance
- correct $(5/2)^+$ resonance strength

Thanks to collaborators

^{4}He-transition form factor: Sonia Bacca, Nir Barnea, G. Orlandini

Cluster EFT: Elena Filandri, Chen Ji, G. Orlandini